

Coulomb Sum Rule (^4He target)

kai Jin

University of Virginia

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Coulomb Sum Rule

e-P elastic scattering

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{Mott} \left[\frac{G_E^2(Q^2)}{1 + \tau} + \frac{\tau G_M^2(Q^2)}{\epsilon(1 + \tau)} \right]$$

e-Nuclear quasi-elastic scattering

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{Mott} \left[\frac{Q^4}{\vec{q}^4} R_L(|\vec{q}|, \omega) + \frac{Q^2}{2\vec{q}^2 \epsilon} R_T(|\vec{q}|, \omega) \right]$$

- Virtual photon polarization: $\epsilon(|\vec{q}|, \omega) = [1 + \frac{2\vec{q}^2}{Q^2} \tan^2 \frac{\theta}{2}]^{-1}$
- Mott cross-section: $\sigma_M = \frac{\alpha^2 \cos^2(\frac{\theta}{2})}{4E^2 \sin^4 \frac{\theta}{2}}$
- Four momentum square: $Q^2 = \vec{q}^2 - \omega^2 = 4EE' \sin^2(\frac{\theta}{2})$
- $\tau = \frac{Q^2}{4M^2}$

- charge information:
- magnetization information:

Rosenbluth separation

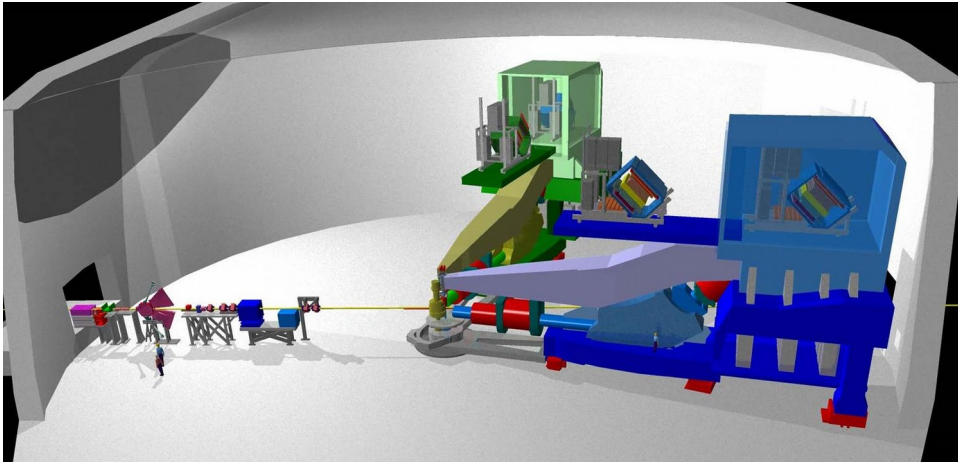


Coulomb Sum

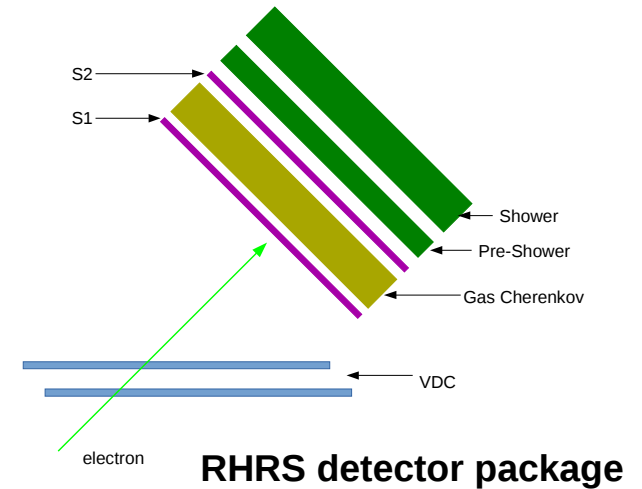
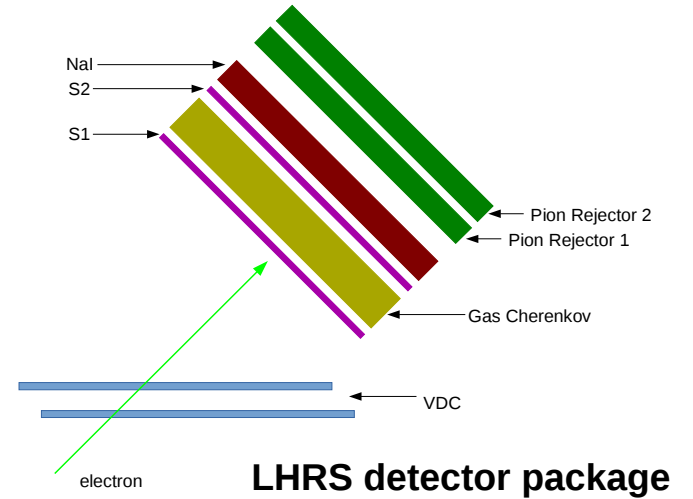
$$S_L(|\vec{q}|) = \int_{\omega^+}^{|\vec{q}|} d\omega \frac{R_L(\omega, |\vec{q}|)}{ZG_{Ep}^2(Q^2) + NG_{En}^2(Q^2)}$$

At large $|q| \gg 2k_f$, S_L should go to 1. Any significant deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

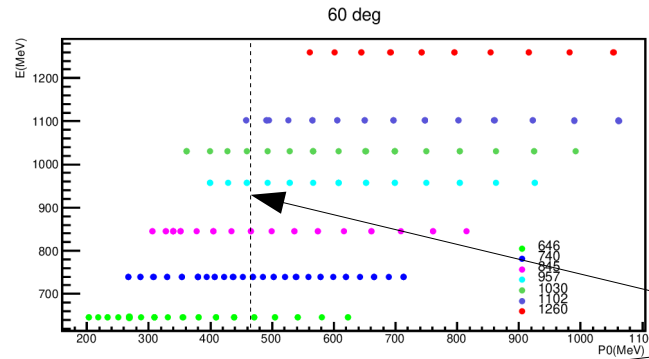
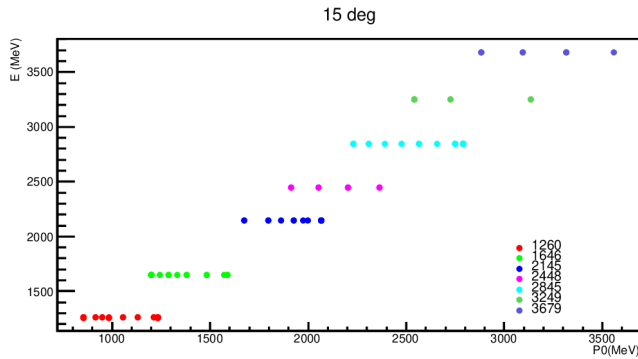
Experiment Setup



- $550 \text{ MeV}/c \leq |\mathbf{q}| \leq 1000 \text{ MeV}/c$
- Beam energy: 0.4 - 4 GeV
- Scattered electron energy: 0.1 - 4 GeV
- Scattering angle: 15° , 60° , 90° and 120°
(The number of kinematics is larger)
- LHRS and RHRS independent(redundant) measurements for most settings
- Targets: ^4He , ^{12}C , ^{56}Fe , ^{208}Pb

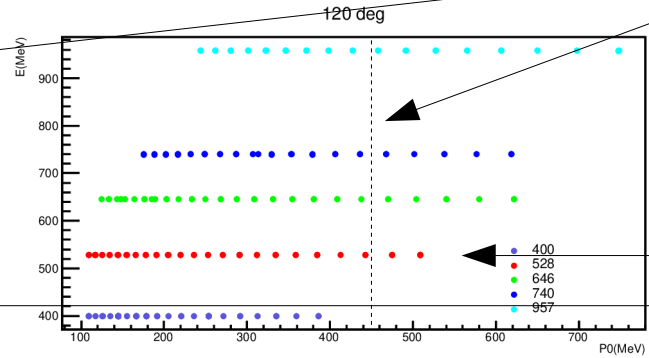
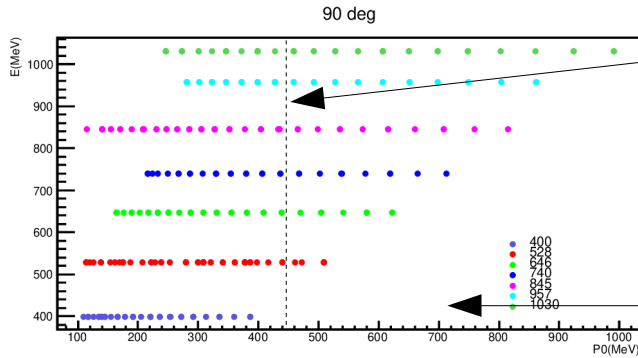


Kinematics

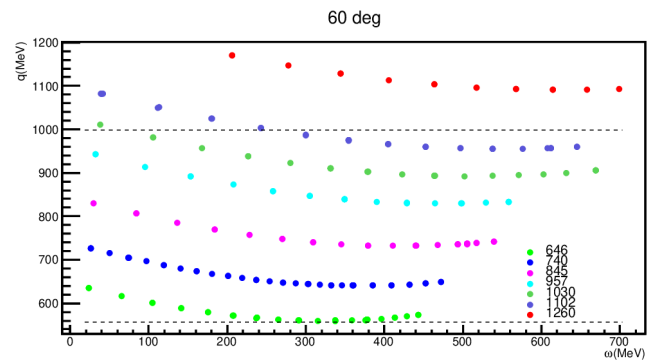
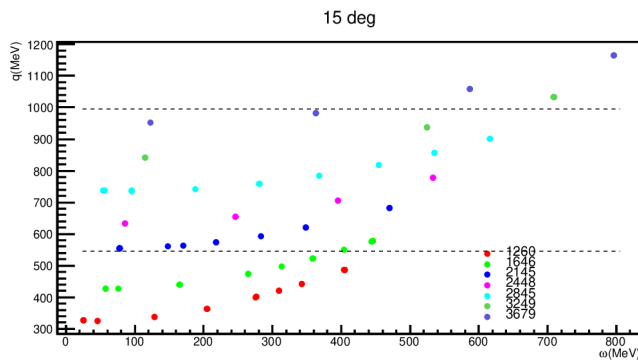


4He kinematics in
E (beam energy) and
P0 (HRS central
momentum)

A lot of kinematics
at low momentum.

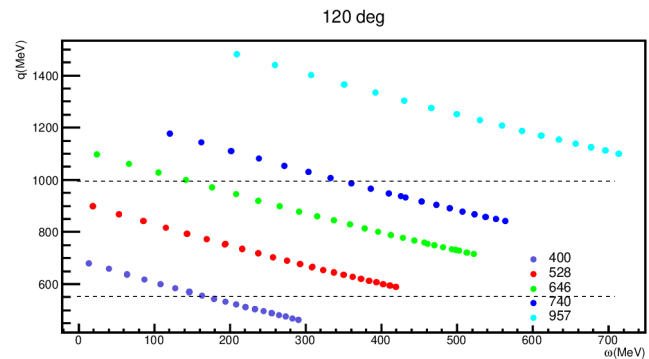
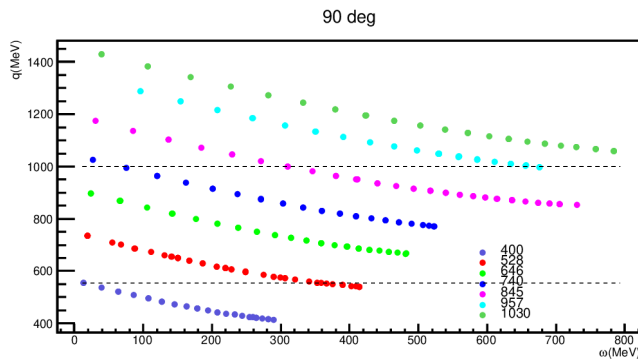


Runs at large angle
(90deg, 120deg) reach
very low central
momentum P0.



4He kinematics in
 ω and q

The spectrums are not
along constant q :
interpolation between
spectrums is necessary!



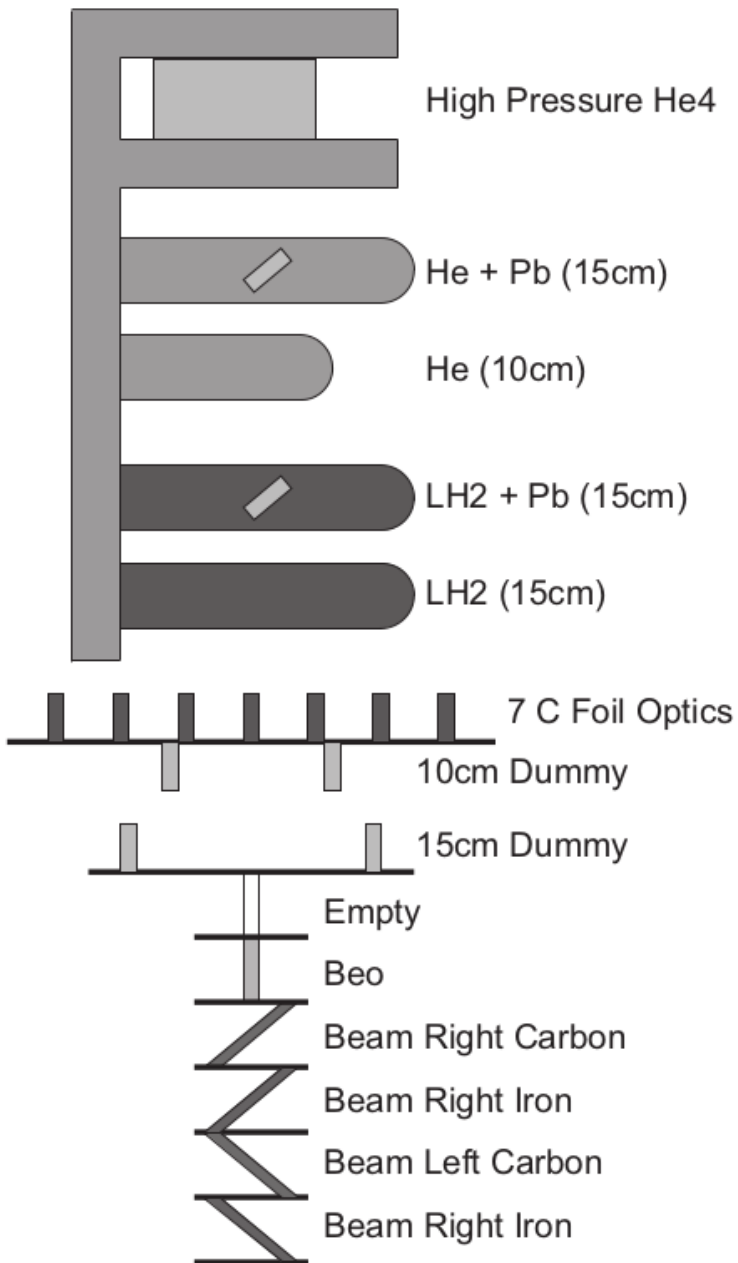
Extended Targets

Loop1(high pressure ^4He gas): racetrack, 10 cm, 0.12 g/cm^3 6.3K, 1.4 MPa



Loop3, top (Pb foil kept in liquid H_2): 15 cm

Loop3, bottom (liquid H_2): 15 cm, 0.0723 g/cm^3
7.0 K, 170 psi.



Acceptance at Low Momentum

The low momentum acceptance is important for CSR experiment:

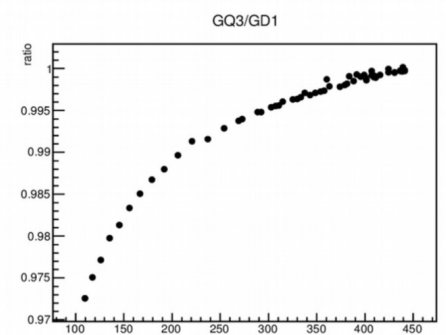
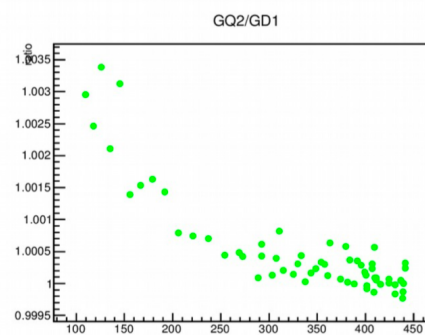
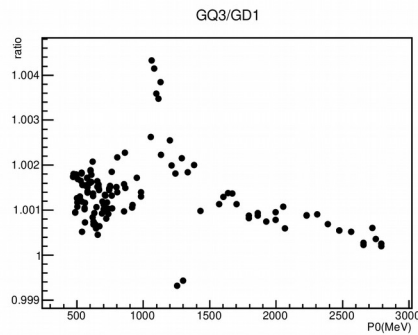
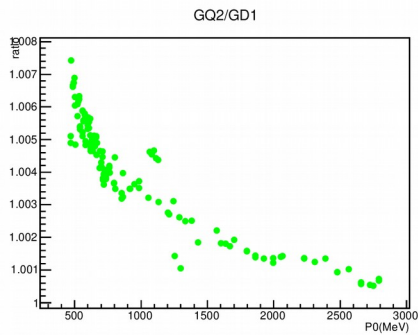
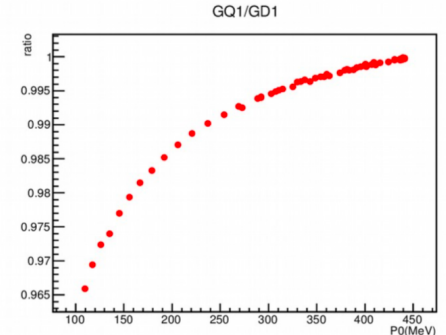
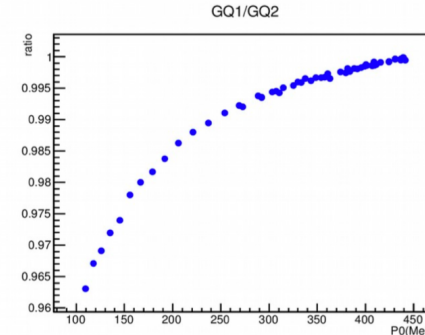
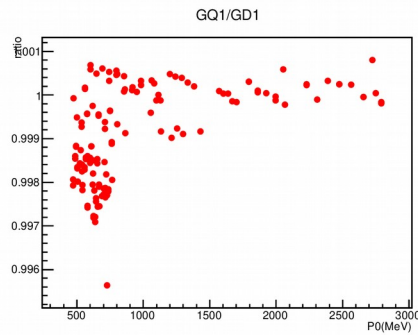
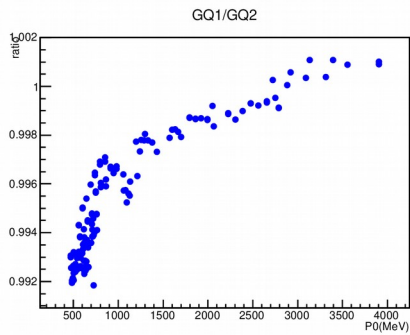
- The HRS behavior is different when $P_0 < 450$ MeV:

$P_0 > 450$ MeV, field ratio between magnets kept nearly constant so optics property does not change with P_0 .
 $P_0 < 450$ MeV, field ratio between magnets change with P_0 by $\sim 3\%$.

- The multiple scattering is inverse-proportional to particle's momentum:

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta_{cp}} z \sqrt{x/X_0} \left[1 + 0.038 \ln(x/X_0) \right]$$

at very low momentum (< 200 MeV), the effect of multiple scattering is big.



field ratio above 450 MeV

field ratio below 450 MeV

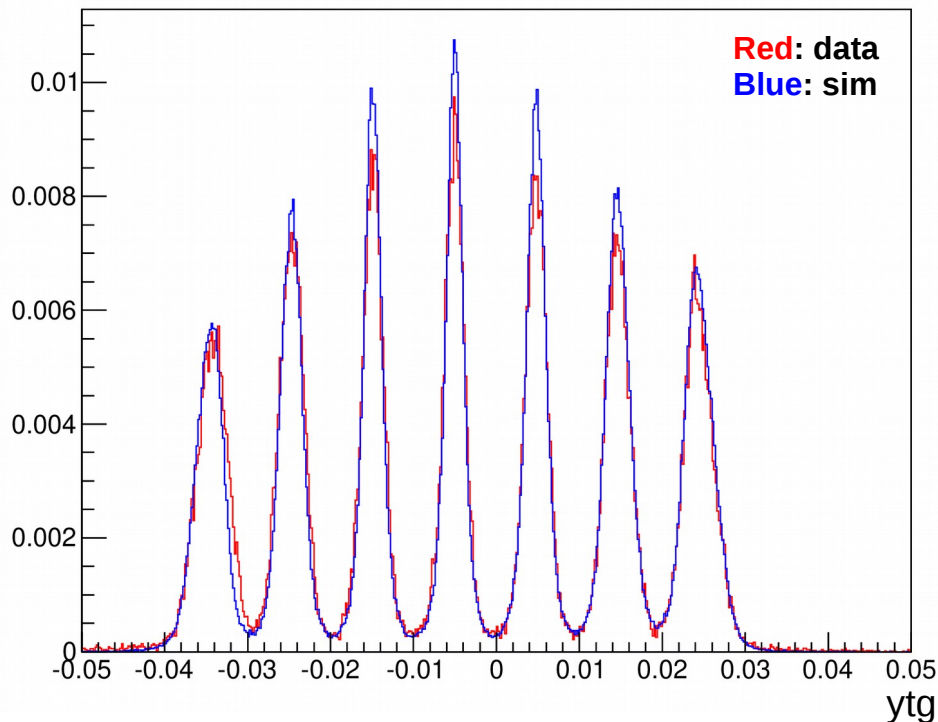
Extended Target Acceptance

Acceptance is a function $A(\delta, \theta_{tg}, \phi_{tg}, y_{tg})$ which tells the possibility of a particle passes through spectrometer and reaches detectors. The main difference between acceptance used for foil target (^{12}C and ^{56}Fe) and extended target (^4He and ^1H) is the extra dimension y_{tg} . The extended target acceptance is extracted from Monte-Carlo simulation program SAMC.

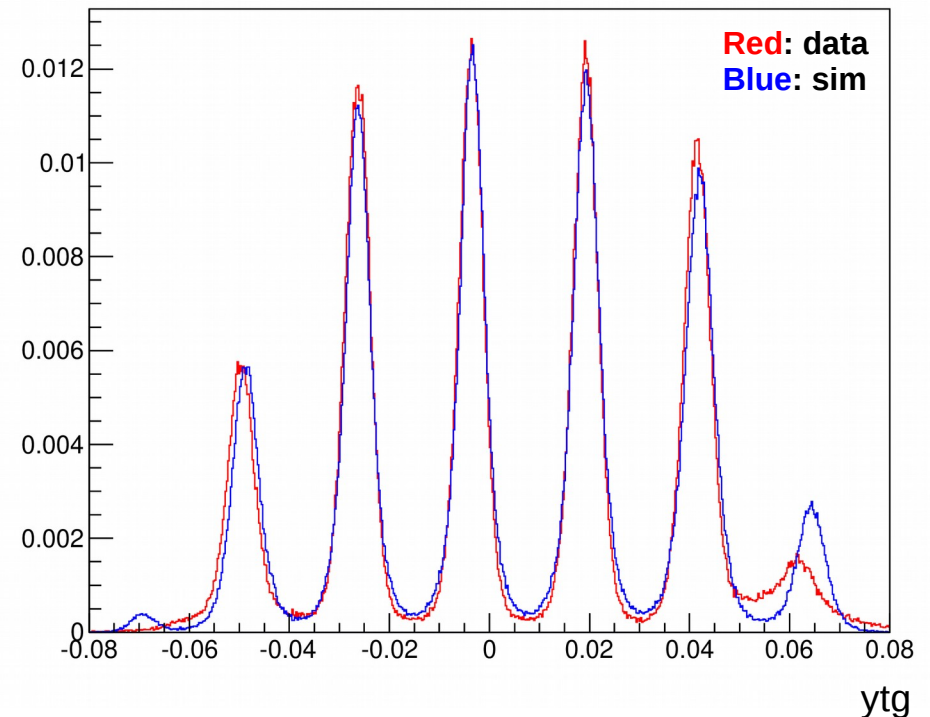
A comparison between data and simulation on optics target (sieve slit not inserted) is done to check the performance of monte-carlo:

- Events generated outside target, and include multiple scattering, energy loss before and after main scattering;
- Radiative effects includes internal and external bremsstrahlung are included;
- Events are weighted by cross sections calculated Sum-Of-Gauss fit to world data.

optics target, E=1102 MeV, $\theta=15$ deg

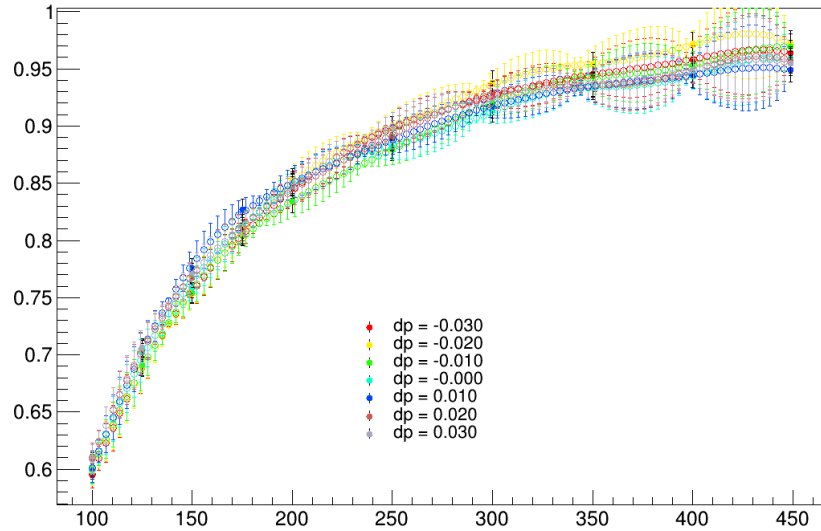


optics target, E=399 MeV, $\theta=35$ deg

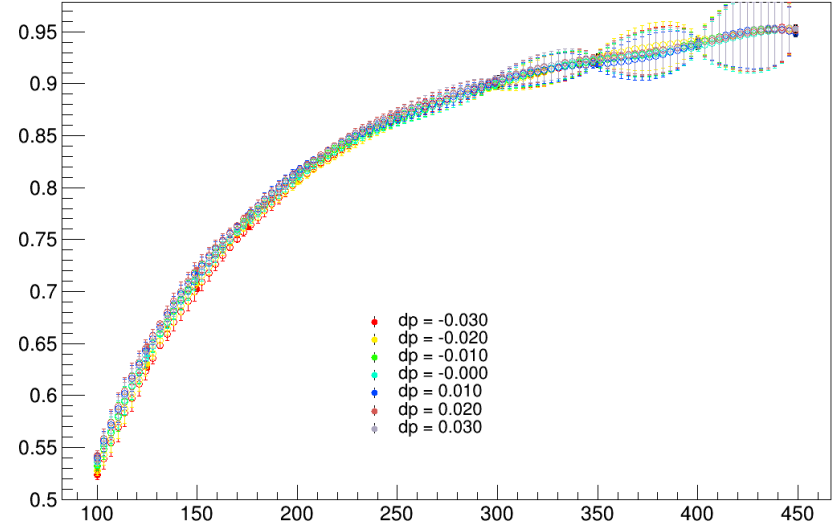


Extended Target Acceptance

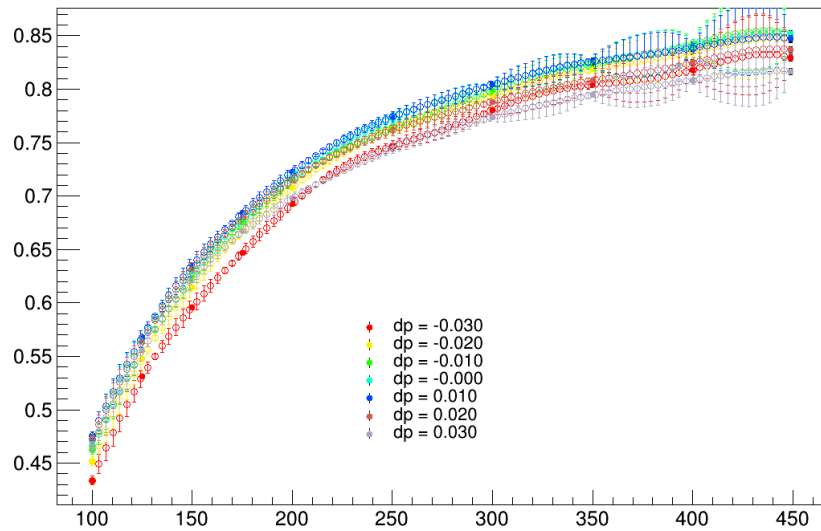
-1cm < ytg < 1cm



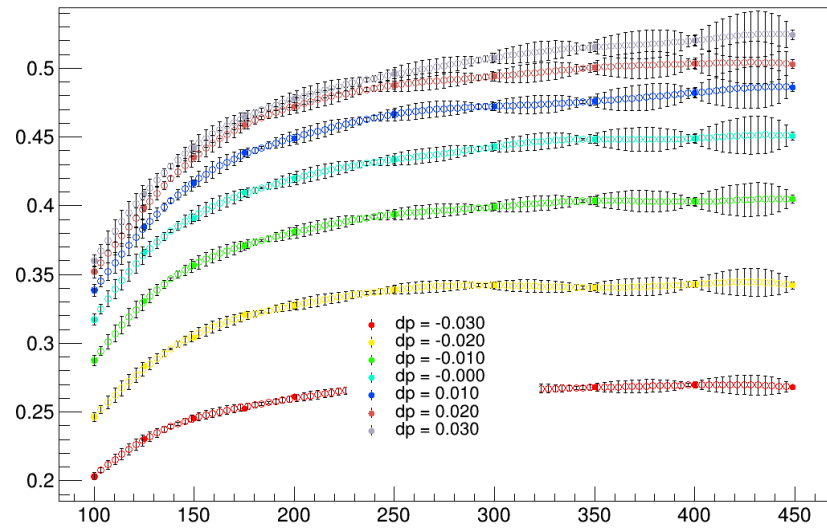
1 cm < ytg < 3 cm



3 cm < ytg < 5 cm



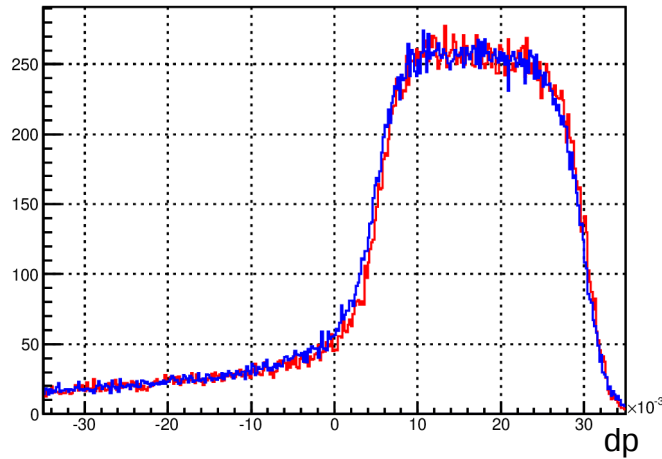
5 cm < ytg < 7 cm



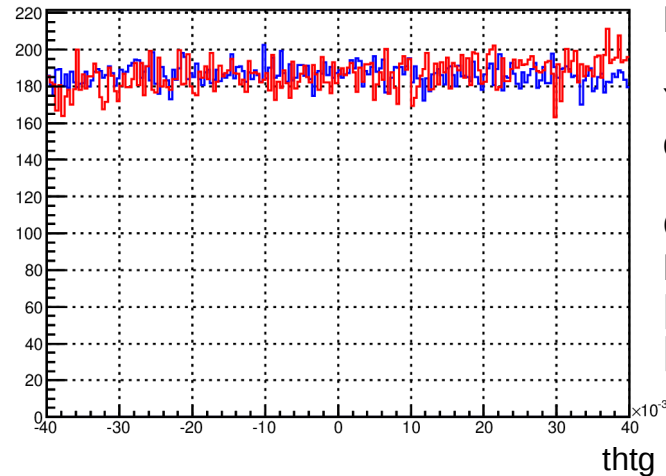
- Monte-Carlo simulations with P0 from 450 MeV to 100 MeV;
- Acceptance sum over thtg and phtg, bin with dp and ytg;
- An interpolation is done between acceptance at different P0;

LH2 Elastic Cross Section

Run=2026, E=1030.3, P=652.1, th=60.0



th comapre



LH2 elastic run at 60 deg, E=1030.3 MeV, P0=652.1 MeV

$\gamma_{Data}/\gamma_{MC} = 0.9711 \pm 0.0221$
cross section = $8.240 \times 10^{-33} \text{ cm}^2$

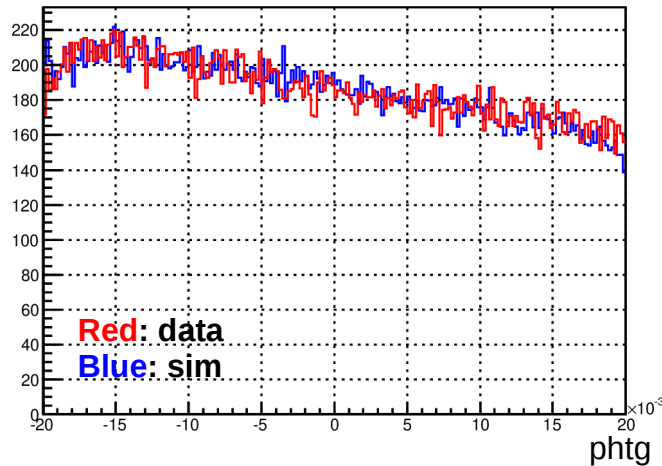
Cuts:

PID cut,

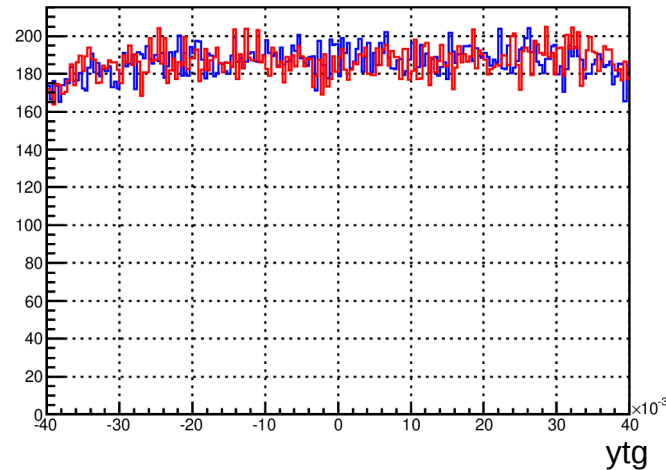
$|\delta| < 3.5\%$, $|thtg| < 40 \text{ mrad}$, $|phtg| < 20 \text{ mrad}$

$|ytg| < 0.04$

ph comapre



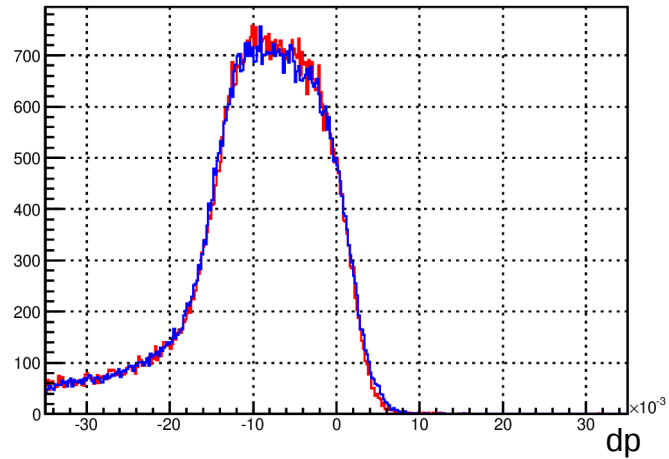
ytg compare



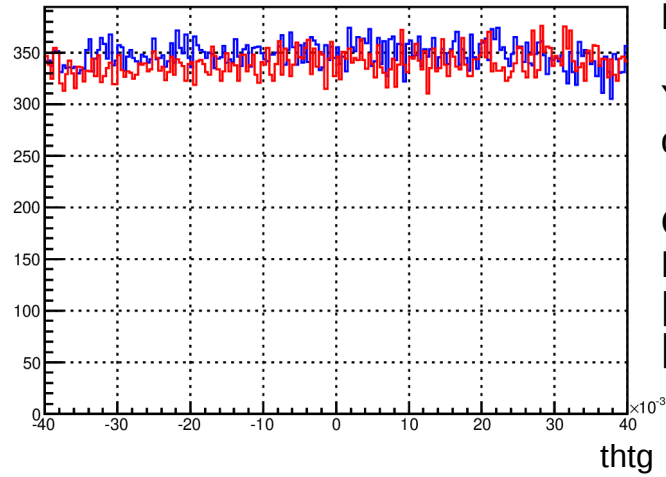
There are many LH2 elastic runs at different angles, with momentum down to ~ 230 MeV. They can be used to check the quality of acceptance and other correction of data at very low mom. The Monte-Carlo is weighted by cross sections calculated from proton elastic form factor fit of world data. The comparison between data and simulation is absolute.

LH2 Elastic Cross Section

Run=3227, E=646.5, P=383.2, th=90.0



th comapre

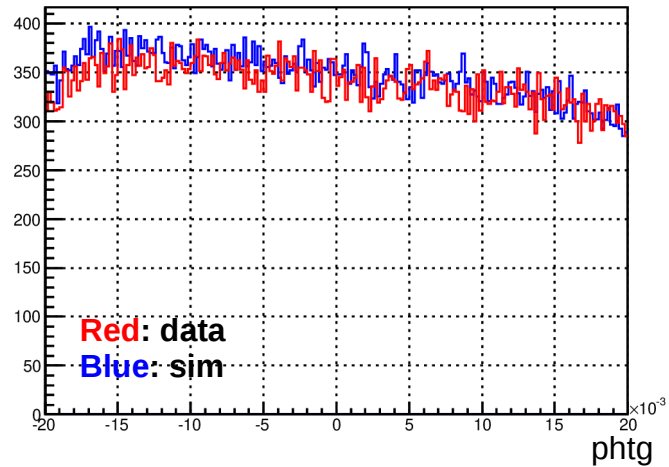


LH2 elastic run at 90 deg, E=646.5 MeV, P=383.2 MeV

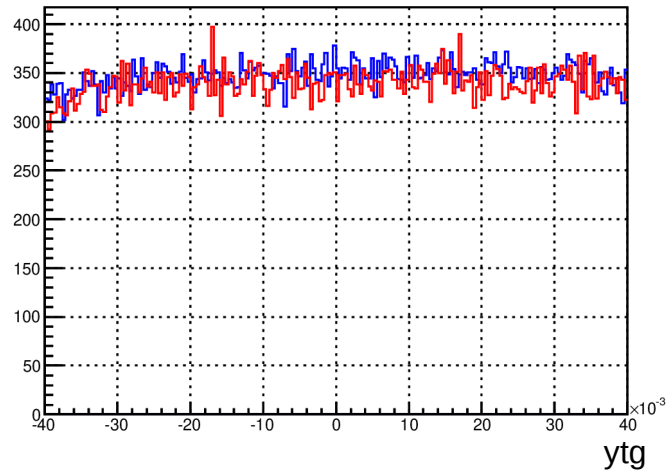
$\gamma_{Data}/\gamma_{MC} = 0.9753 \pm 0.0259$
cross section = $7.316 \times 10^{-33} \text{ cm}^2$

Cuts:
PID cut,
 $|\delta| < 3.5\%$, $|thtg| < 40 \text{ mrad}$, $|phtg| < 20 \text{ mrad}$,
 $|ytg| < 0.04$

ph comapre

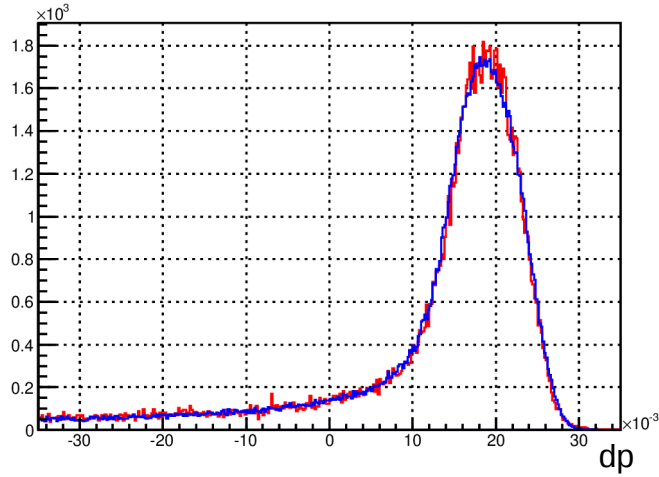


ytg compare

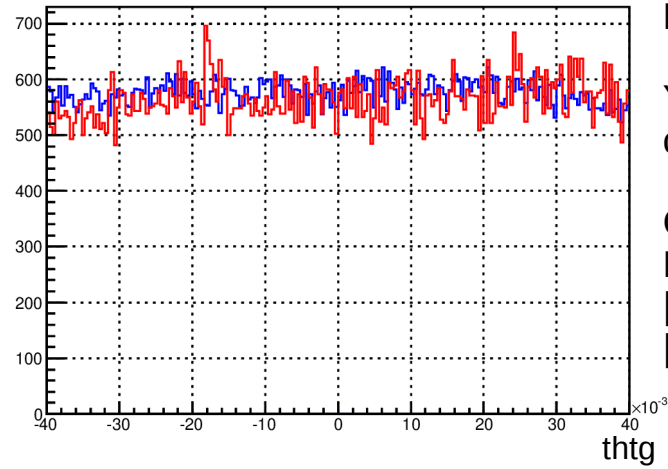


LH2 Elastic Cross Section

Run=5035, E=399.5, P=237.7, th=120.0



th comapre



LH2 elastic run at 120 deg, E=399.5 MeV, P=237.7 MeV

$\gamma_{Data}/\gamma_{MC} = 0.9716 \pm 0.0205$
cross section = $1.129 \times 10^{-32} \text{ cm}^2$

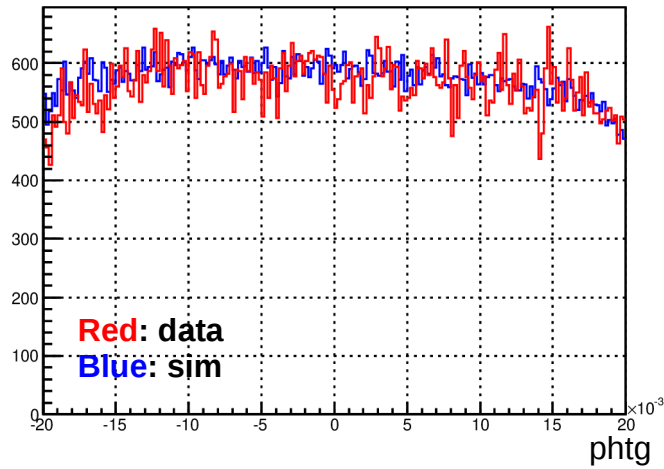
Cuts:

PID cut,

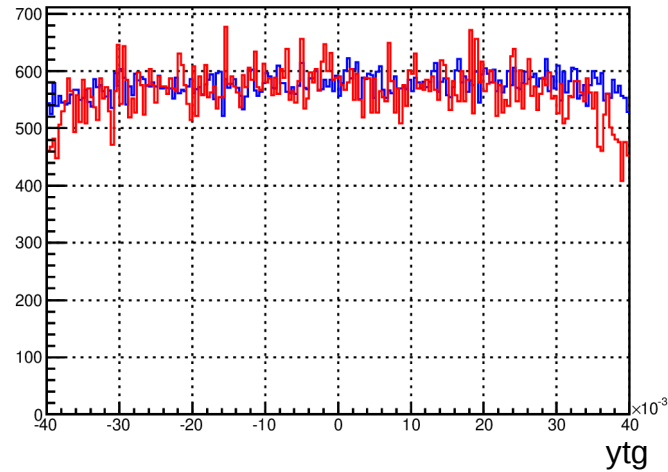
$|\delta| < 3.5\%$, $|thtg| < 40 \text{ mrad}$, $|phtg| < 20 \text{ mrad}$

$|ytg| < 0.04$

ph comapre



ytg compare



Window Subtraction

dummy target

each foil thickness: 0.259 g/cm²

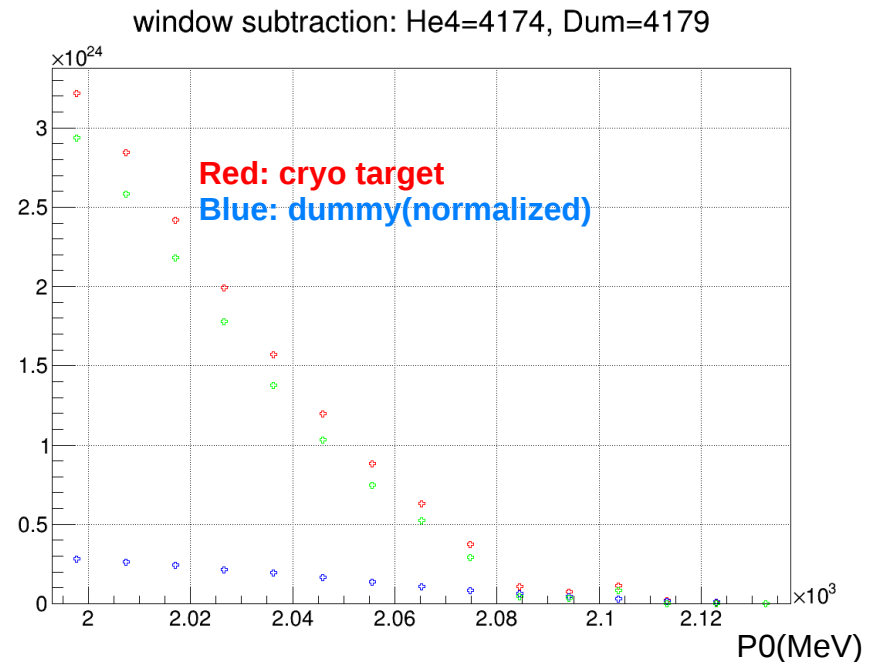
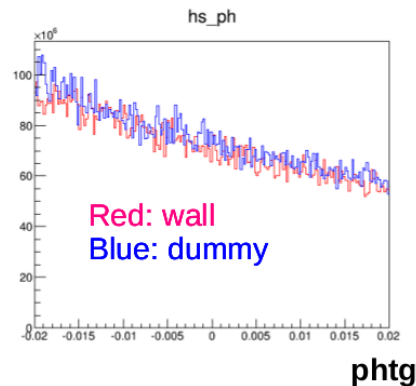
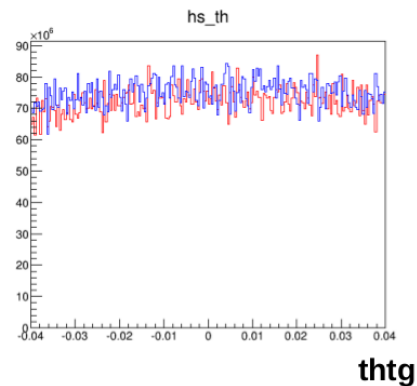
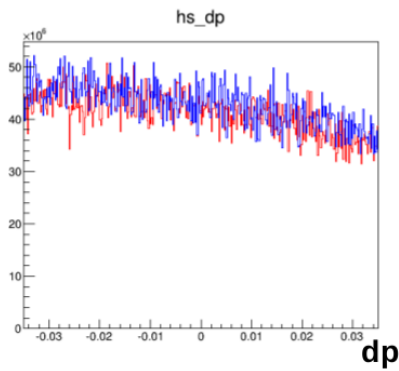
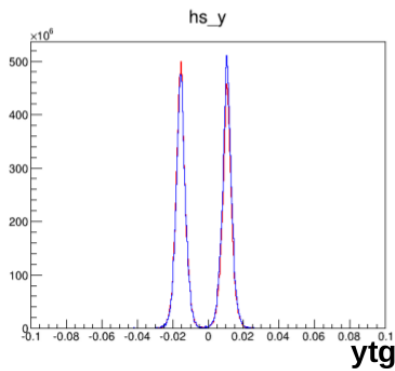
Use SAMC to simulate the external radiation effect of different aluminum thickness and He4 gas before or after scattering on windows.
The simulation runs are weighted by F1F209 fitting.

The radiation factor was put into the following formula:

$$Y_{corrected} = Y_{cryotarget} - Y_{dummy} \frac{T_{wall}}{T_{dummy}} \frac{R_{wall}}{R_{dummy}}$$

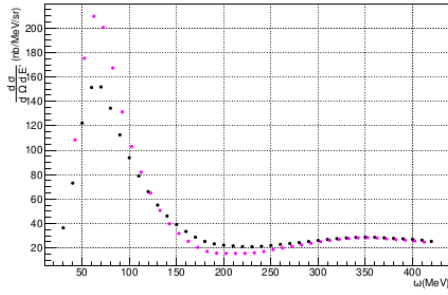
He4 target
He4 density=0.12 g/cm³

entr window: 0.073 g/cm²
exit window: 0.076 g/cm²

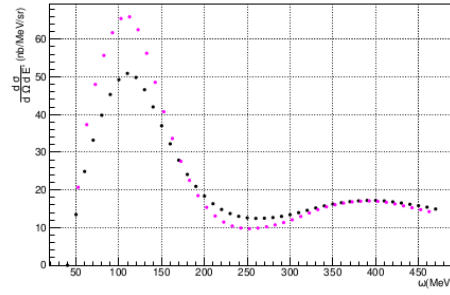


^4He Spectrum

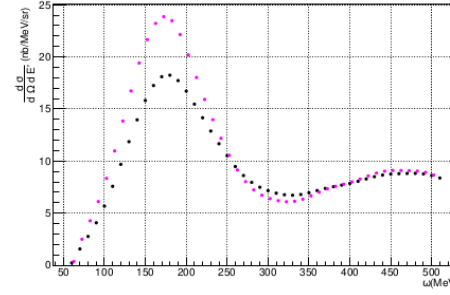
radcor L 15deg E=1260 MeV



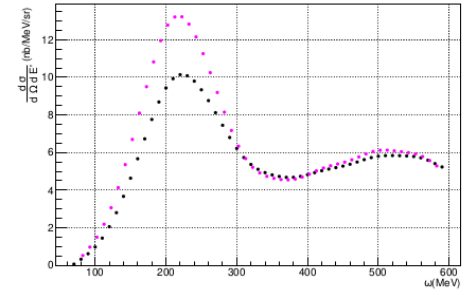
radcor L 15deg E=1646 MeV



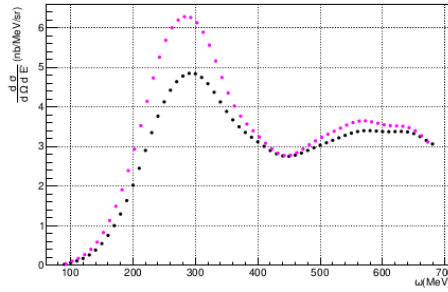
radcor L 15deg E=2145 MeV



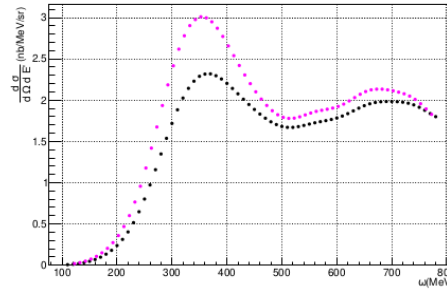
radcor L 15deg E=2448 MeV



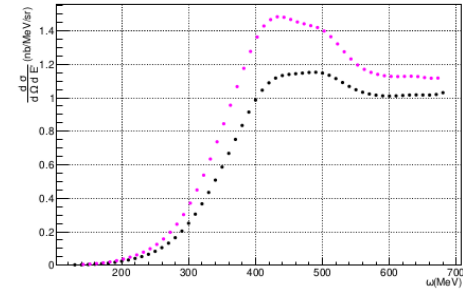
radcor L 15deg E=2845 MeV



radcor L 15deg E=3249 MeV

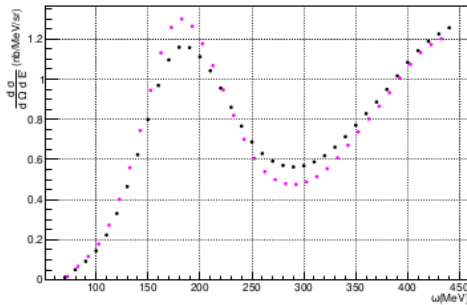


radcor L 15deg E=3679 MeV

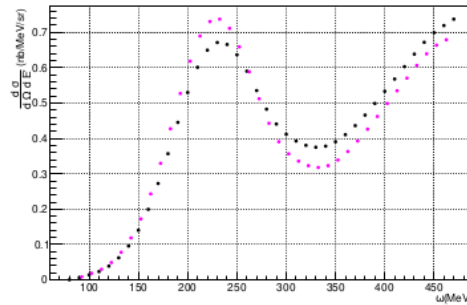


^4He raw and radiative corrected spectrum at 15 deg.

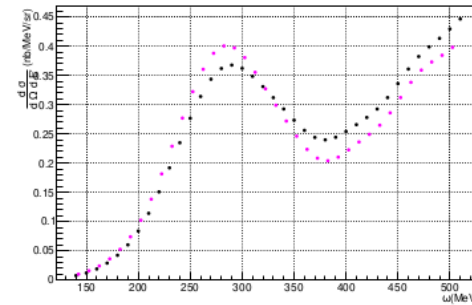
radcor L 60deg E=646 MeV



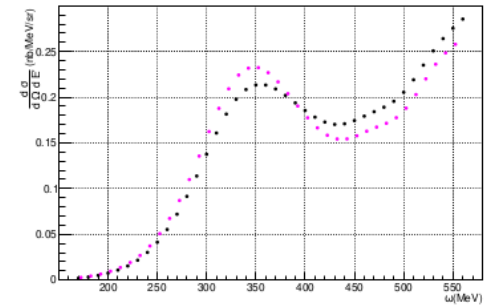
radcor L 60deg E=740 MeV



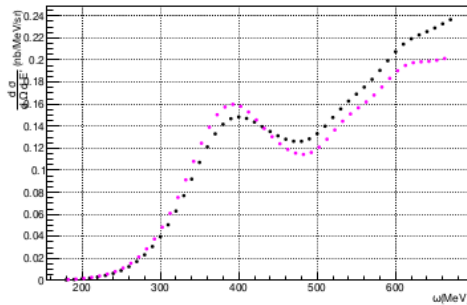
radcor L 60deg E=845 MeV



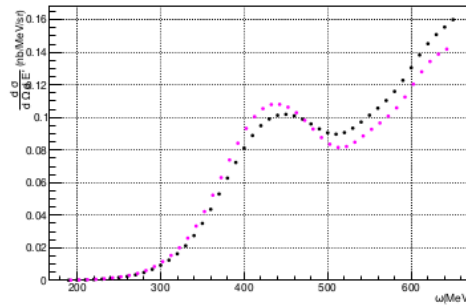
radcor L 60deg E=958 MeV



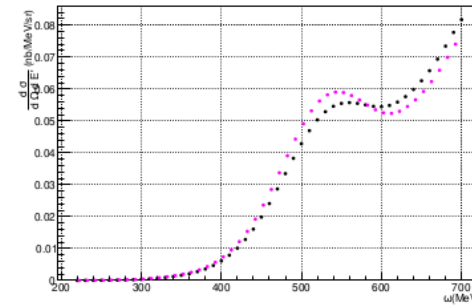
radcor L 60deg E=1030 MeV



radcor L 60deg E=1102 MeV



radcor L 60deg E=1260 MeV



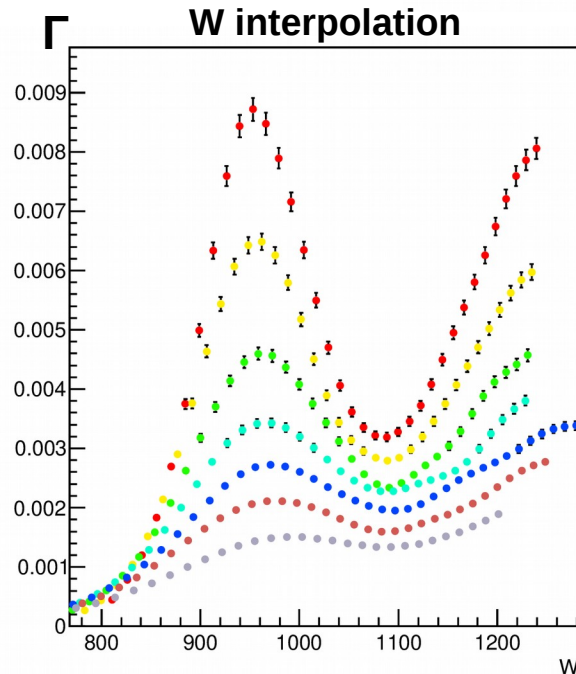
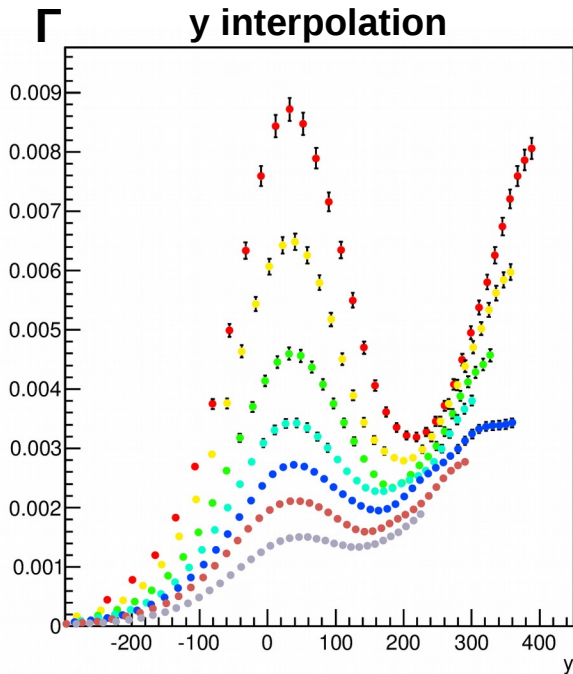
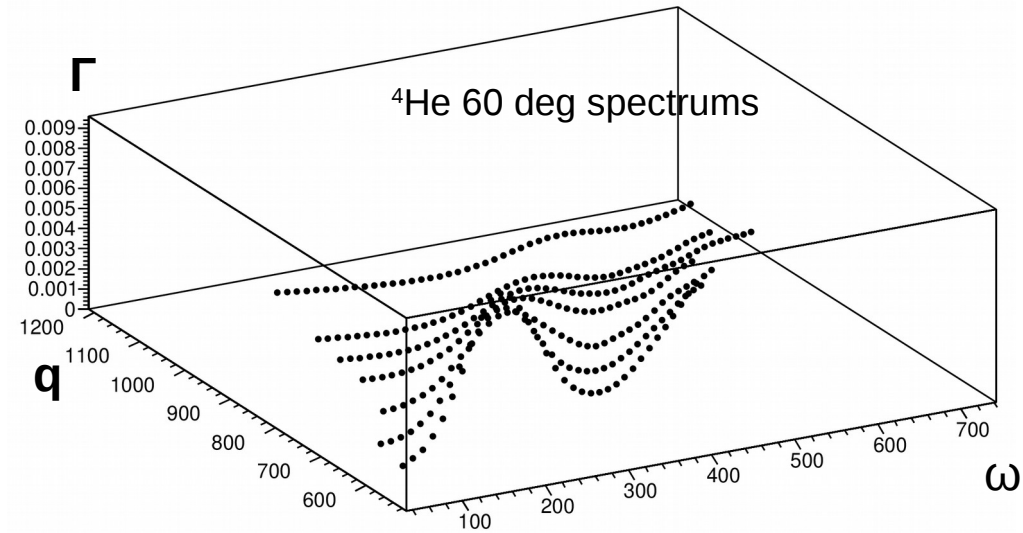
^4He raw and radiative corrected spectrum at 90 deg.

Interpolation

- The R_L should be integrated along a constant $|\mathbf{q}|$
- The measured spectrum is along constant beam energy not constant $|\mathbf{q}|$
- Interpolation between measured spectrum in $(|\mathbf{q}|, \omega)$ is necessary
- The interpolation method should follow paths that pass through corresponding features in each spectrum.
- Two interpolation methods are used:
- (1) y^* interpolation, y^* is quasi-elastic scaling variable:

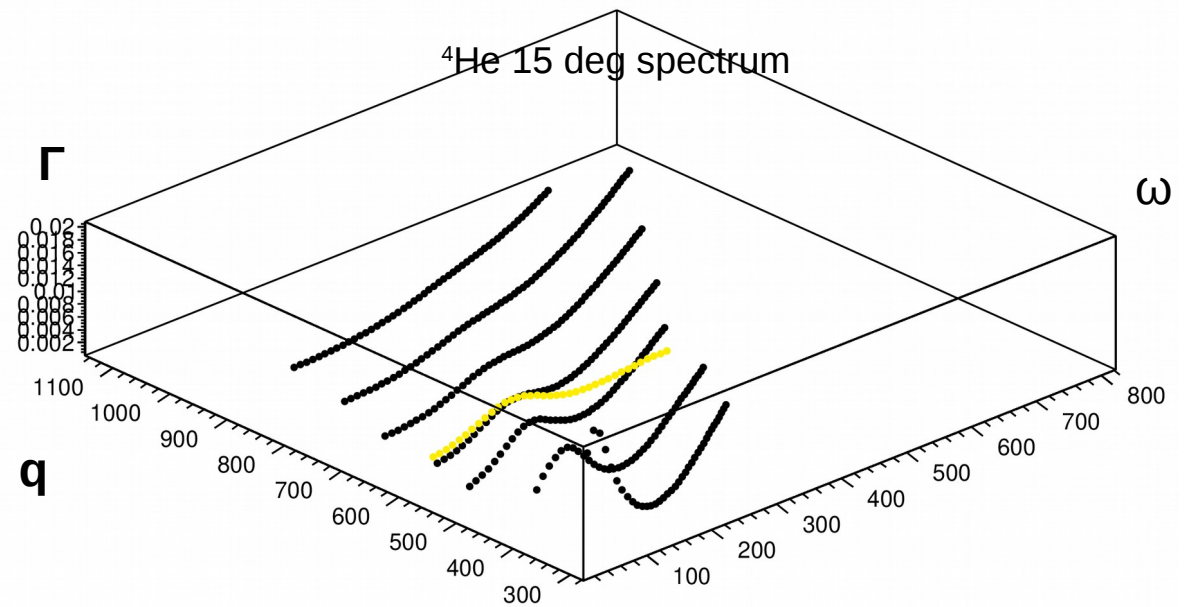
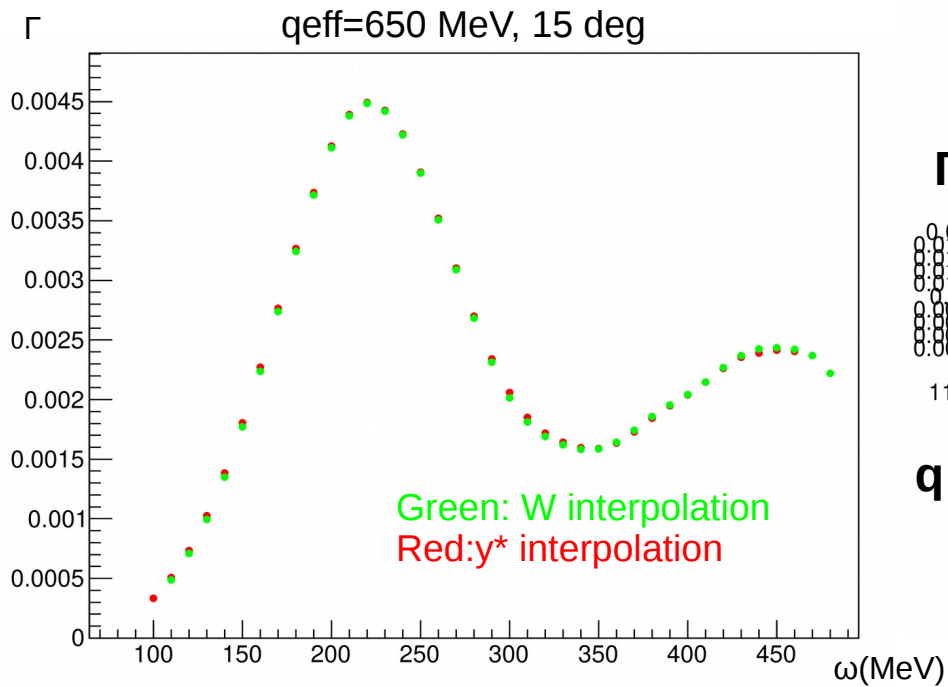
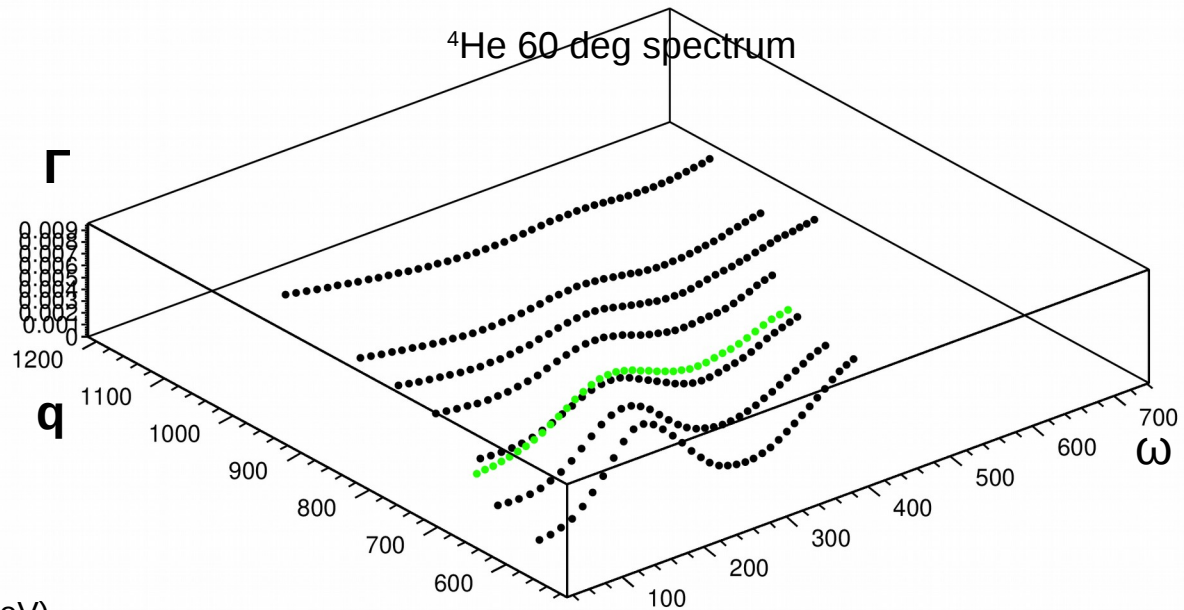
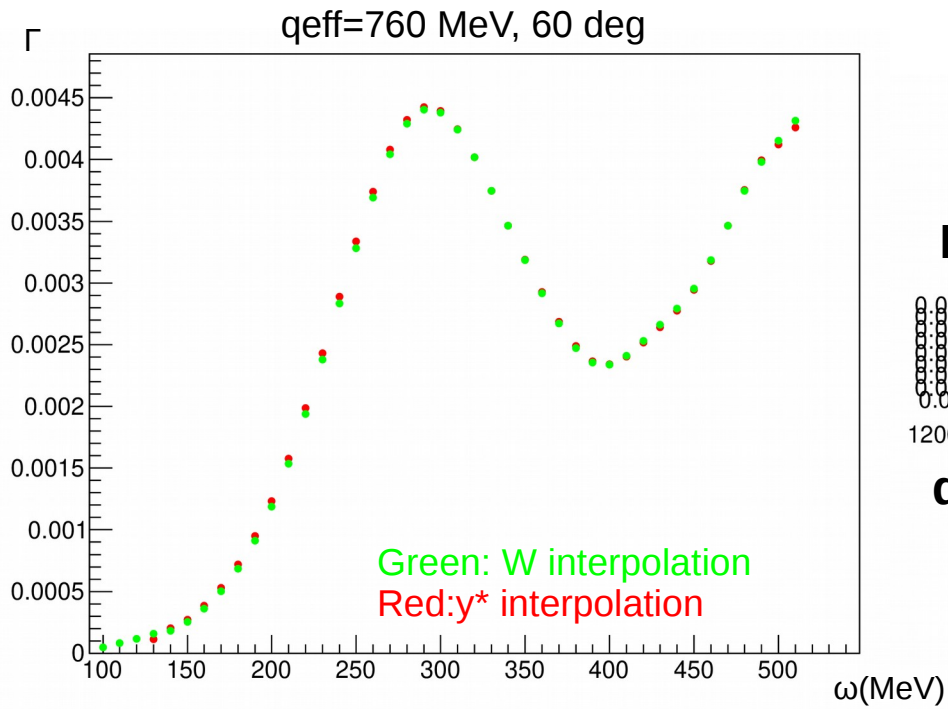
$$\omega + M_A = (y^2 + 2y|\mathbf{q}| + M^2 + \mathbf{q}^2)^{1/2} + (y^2 + M_{A-1}^2)^{1/2}$$

(2) W interpolation, W is invariant mass.



y interpolation aligns the quasi-elastic peak better, while W interpolation aligns the dip region better.

Interpolation



Rosenbluth Separation

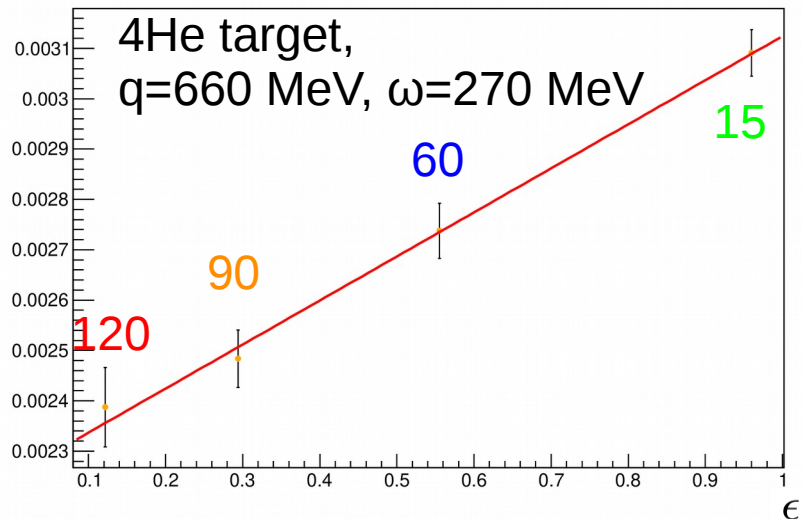
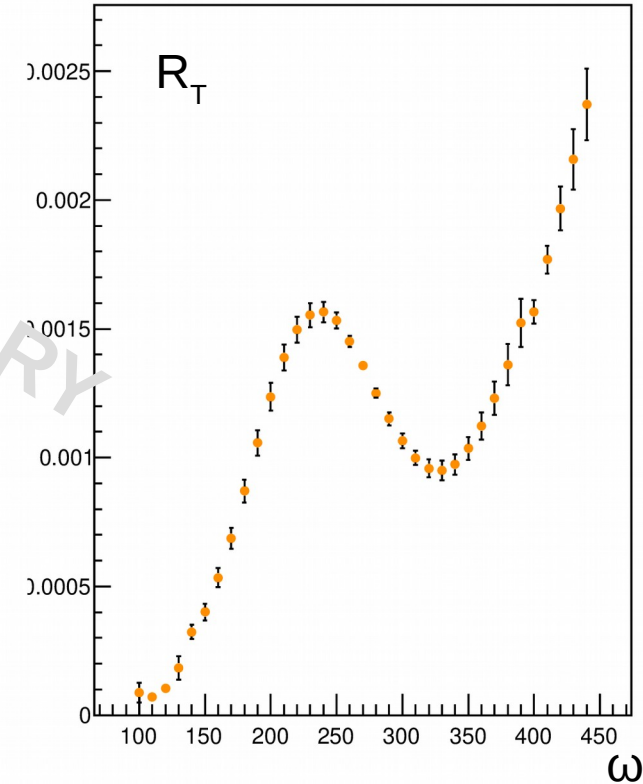
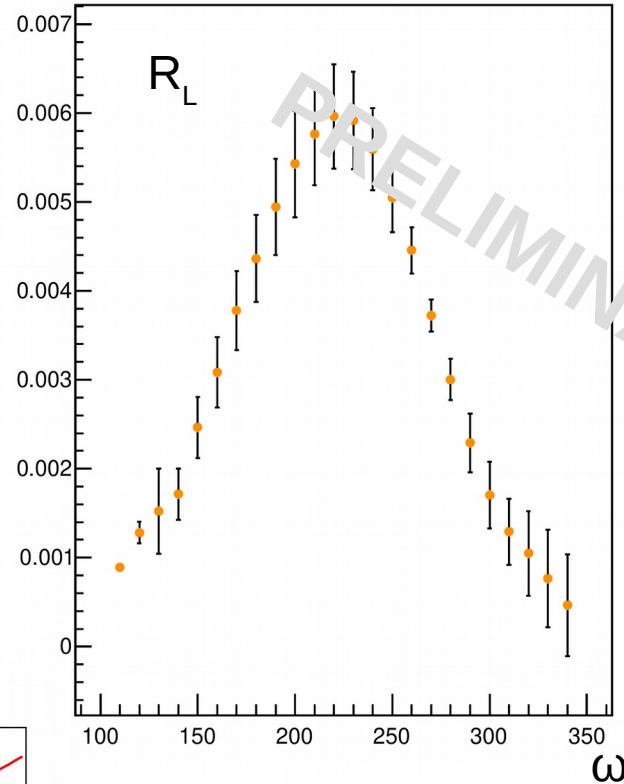
$$\frac{d^2\sigma}{d\Omega d\omega} \frac{\epsilon}{\sigma_{Mott}} = \epsilon \frac{Q^4}{\vec{q}^4} R_L(|\vec{q}|, \omega) + \frac{Q^2}{2\vec{q}^2} R_T(|\vec{q}|, \omega)$$

^4He CSR at $q=660$ MeV

XS at same $|q|$ and ω but different angles from interpolation are plotted together and fitted with linear fit:

$$\text{Slope} = \frac{Q^4}{\vec{q}^4} R_L$$

$$\text{Intercept} = \frac{Q^2}{2\vec{q}^2} R_T$$



Summary

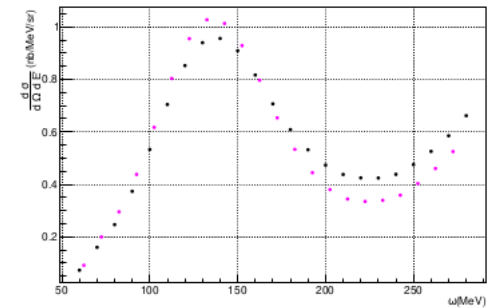
Recent work:

- Get extended target acceptance at low momentum, and check with LH2 elastic;
- Extract ^4He spectrums for 4 angles;

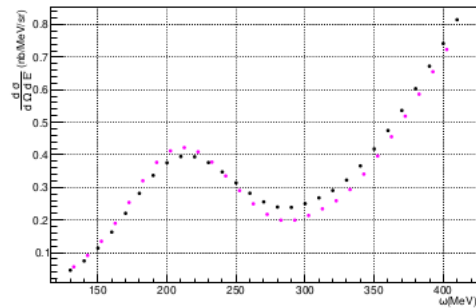
Plans:

- Interpolation and LT separation;
- Compare with world data;

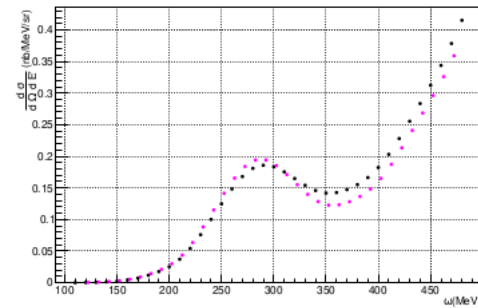
radcor L 90deg E=399 MeV



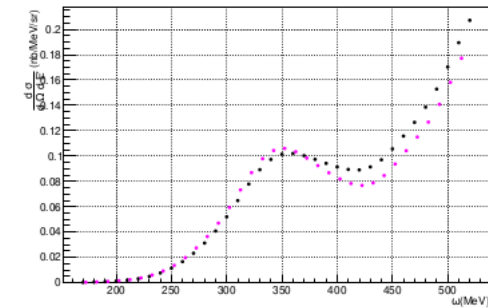
radcor L 90deg E=528 MeV



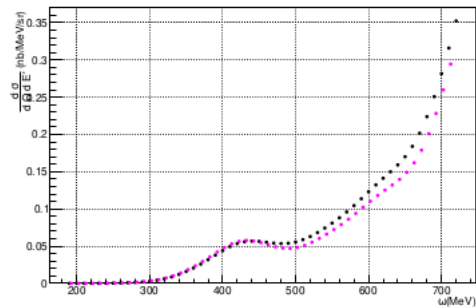
radcor L 90deg E=646 MeV



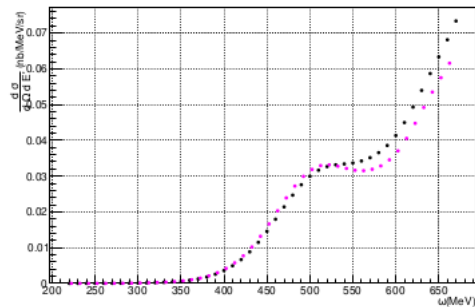
radcor L 90deg E=740 MeV



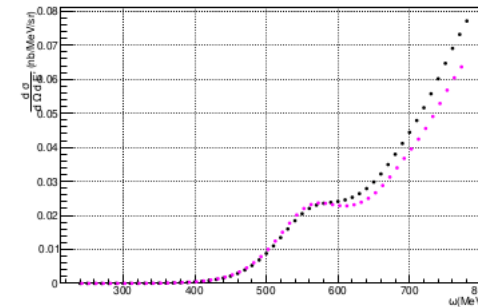
radcor L 90deg E=845 MeV



radcor L 90deg E=958 MeV

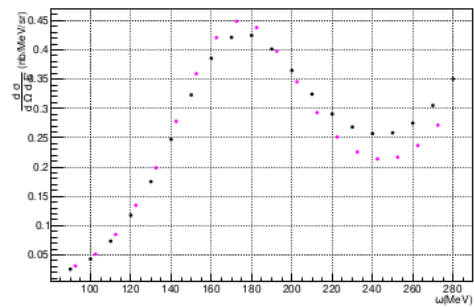


radcor L 90deg E=1030 MeV

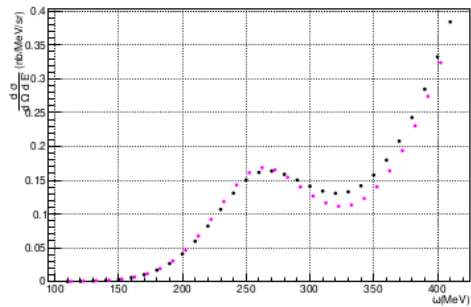


^4He raw and radiative corrected spectrum at 90 deg.

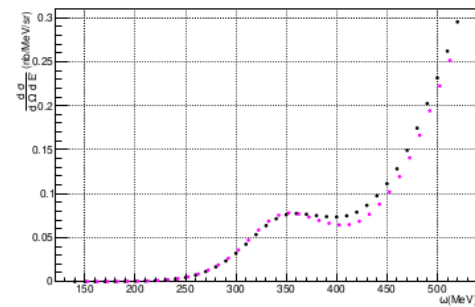
radcor L 120deg E=399 MeV



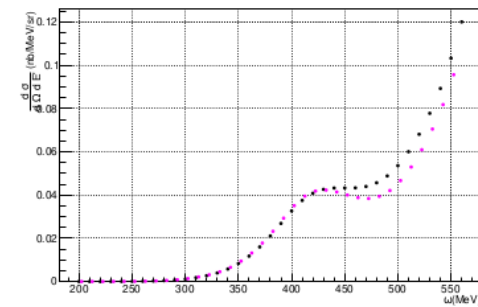
radcor L 120deg E=528 MeV



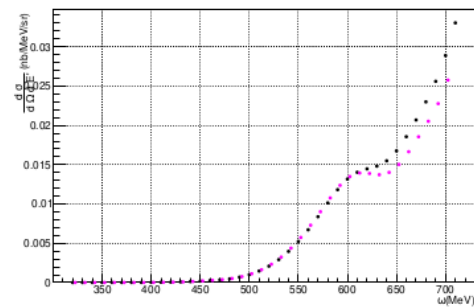
radcor L 120deg E=646 MeV



radcor L 120deg E=740 MeV



radcor L 120deg E=958 MeV



^4He raw and radiative corrected spectrum at 120 deg.