Nucleon Form Factors and Related Matters

Gerald A. Miller, UW

- Definitions, model calculations
- Meaning of form factors
- Shape of the proton

Talk based on personal interests. For other approaches see
Cloet et al, Few Body Syst. 46 (2009) 1-36
Arrington et al, J.Phys. G34 (2007) S23-S52
Punjabi, Perdrisat et al, Eur.Phys.J. A51 (2015) 79
De Teramond et al PRL 120,182001

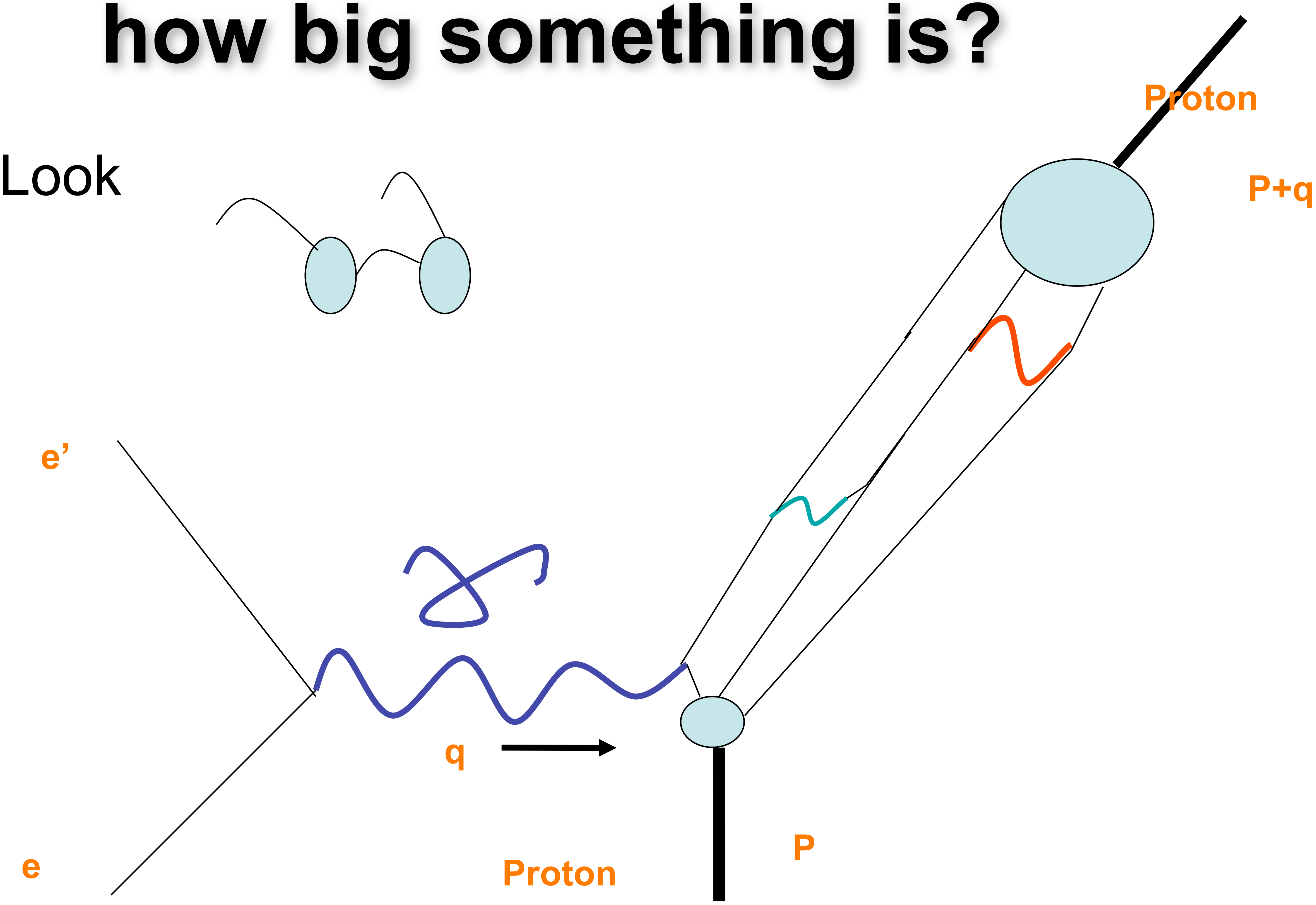
Office of Science
U.S. Department of Energy

Why study the nucleon?

- More than 99 % of matter we see made of neutron and proton
- Neutrons protons made of quarks, gluons
- Quantum Chromodynamics QCD
- CONFINEMENT, test QCD lattice
- Size influences atomic physics tests of QED
- How does the nucleon stick together when struck by photon?
- Where is charge and magnetization density located? Origin of angular momentum?
- What is the shape of the proton?
- Discover new phenomenon - proton G_E/G_M, neutron central charge density

What is a form factor? How to tell how big something is?

- Look



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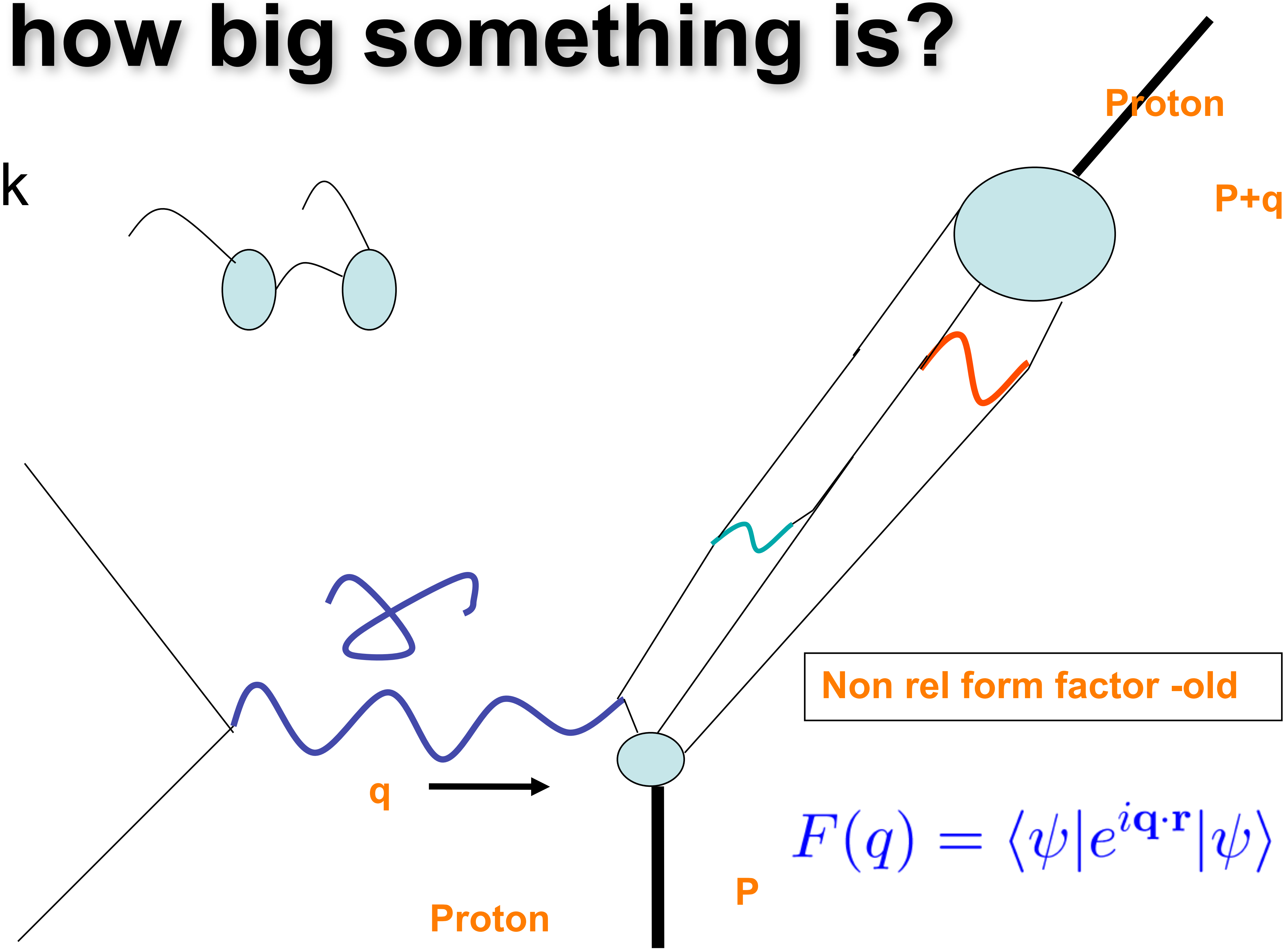
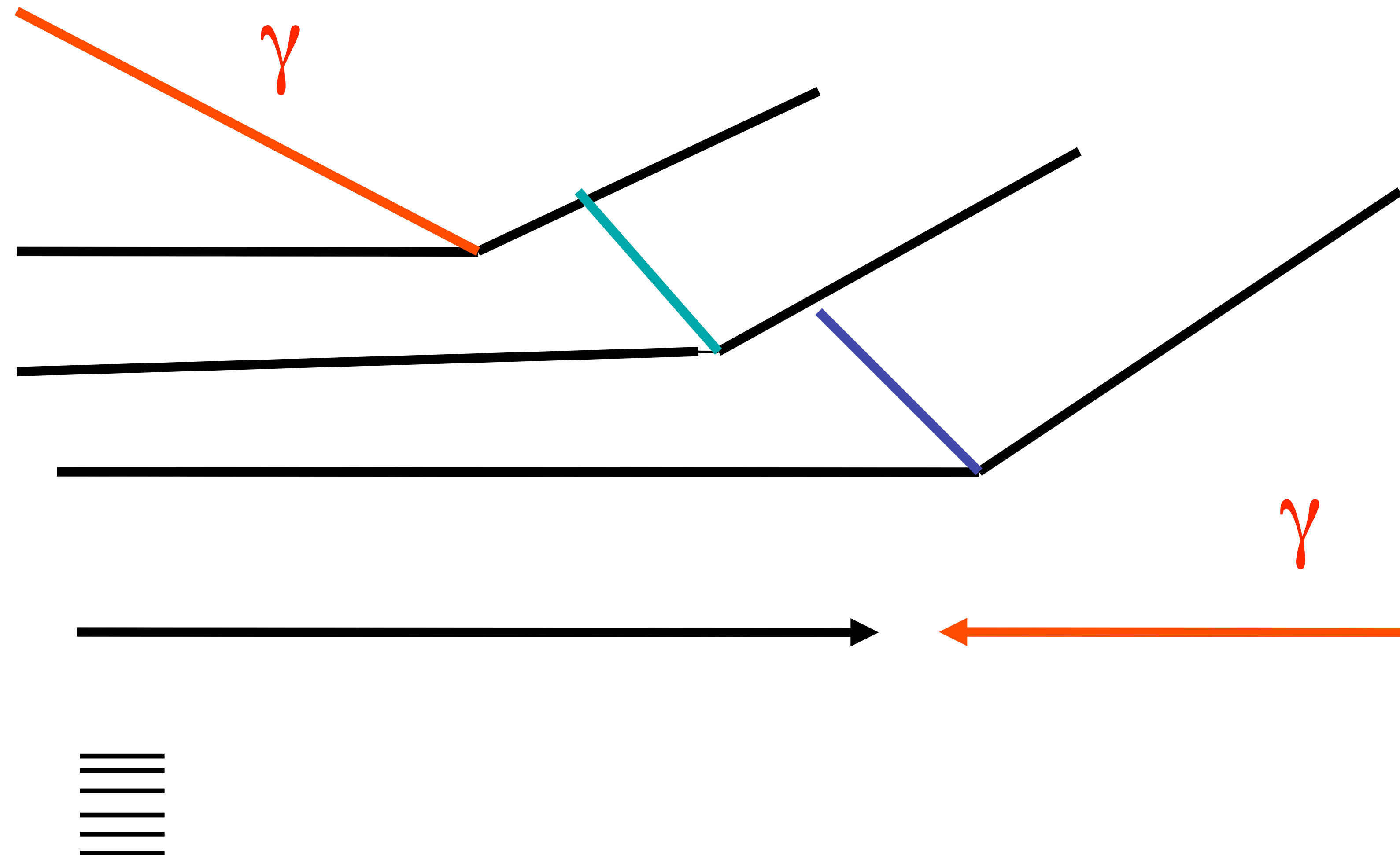


$$F(q) = \langle \psi | e^{i\mathbf{q}\cdot\mathbf{r}} | \psi \rangle$$

Structure factor

How proton holds together-high Q^2

- **pQCD**



Feynman

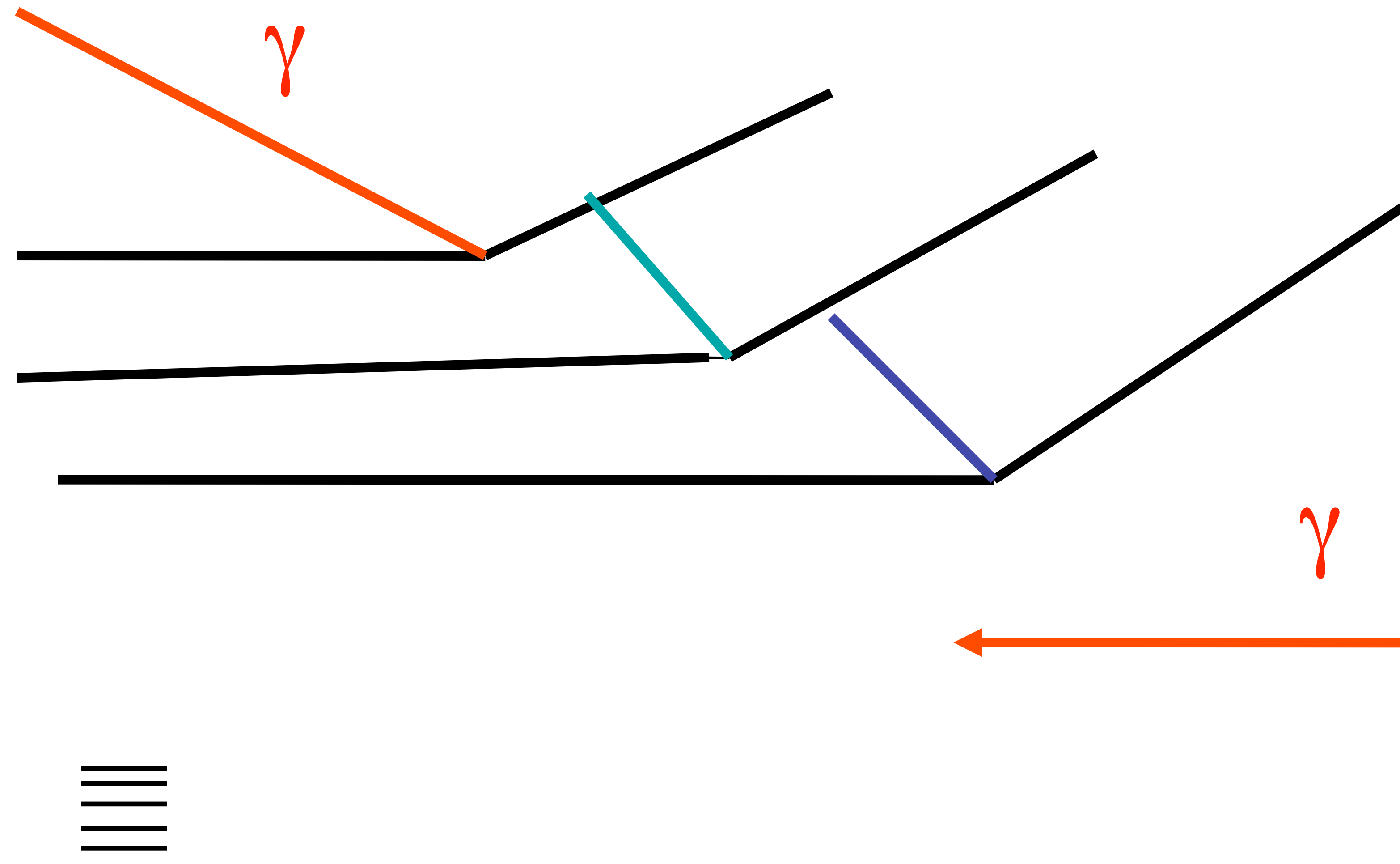
Non perturbative
∞ gluon exch

All reaction mechanisms included in light front wave function of proton

Need GPDs and color transparency to decide

How proton holds together-high Q^2

- pQCD



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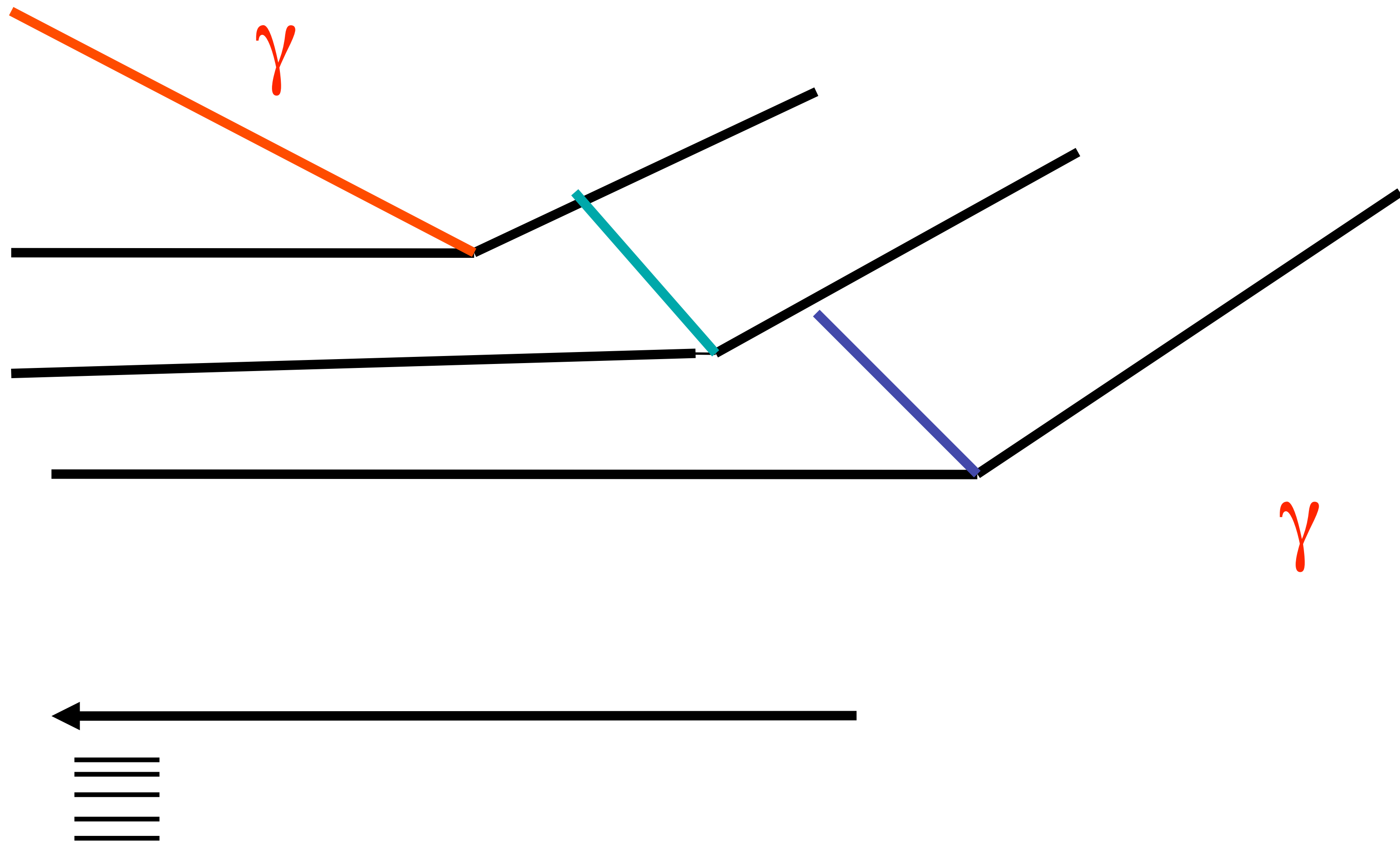

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Definitions

$$\langle N,\lambda' p'\,|J^\mu|\,N,\lambda p\rangle = \bar{u}_{\lambda'}(p')[F_1(Q^2)\gamma^\mu + F_2(Q^2)\sigma^{\mu\nu}i\frac{(p'-p)_\nu}{M_p}]u_\lambda(p)$$

$$G_E = F_1 - \frac{Q^2}{4M_N^2}F_2\,, \qquad G_M = F_1 + F_2$$

$$F_1(Q^2) = \langle N,\uparrow|J^+|N,\uparrow\rangle, \qquad QF_2(Q^2) = -2M_p\langle N,\uparrow|J^+|N,\downarrow\rangle$$

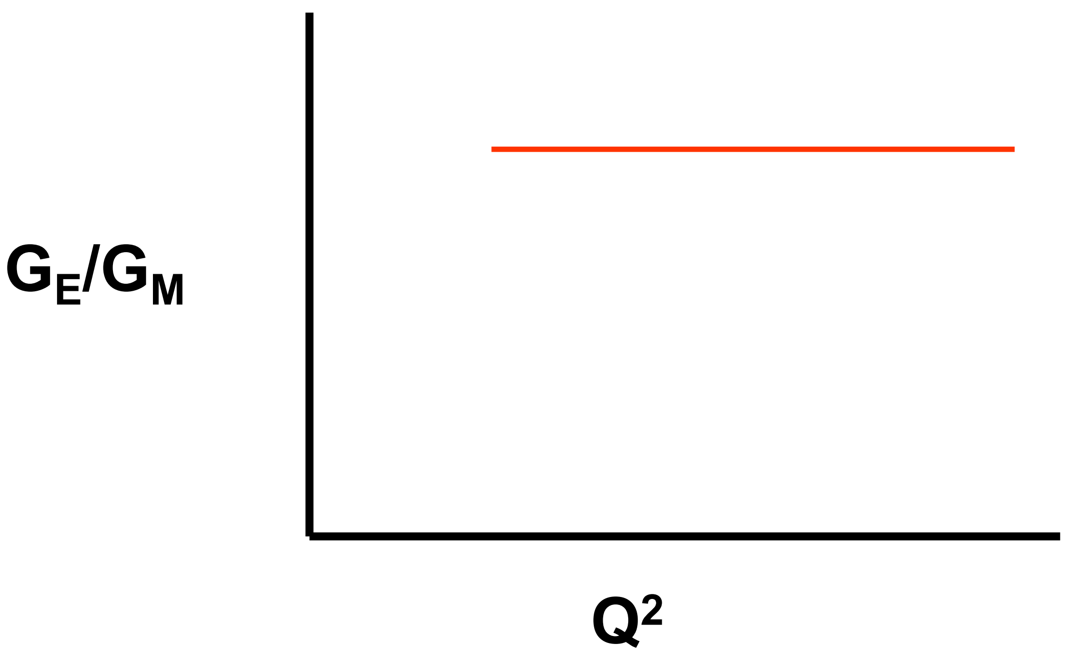
$$\frac{QF_2}{2M_pF_1} = \frac{\text{spin flip}}{\text{spin non flip}}$$

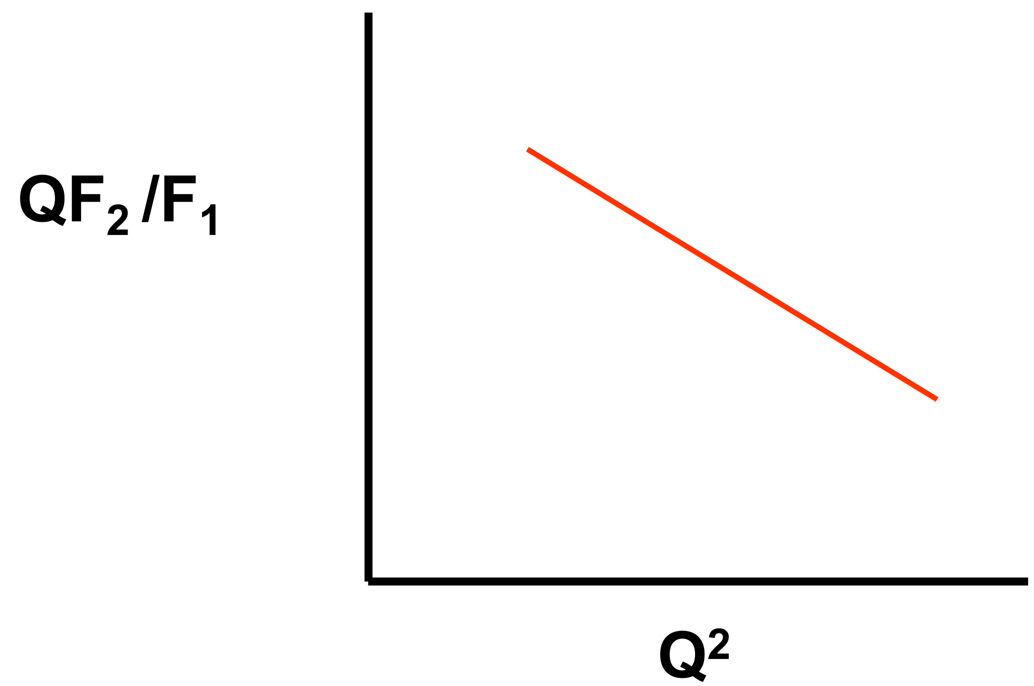
old pQCD:

$$\frac{QF_2(Q^2)}{2M_NF_1} \sim \frac{m_{\text{quark}}}{Q} \rightarrow \frac{G_E}{G_M} = \text{const}$$

Same as non-relativistic

Before JLab





New Phenomenon!

Proton recoil polarization technique

www.scholarpedia.org/article/Nucleon_Form_factors

Punjabii, Perdrisat

Conclusion -non-relativistic quark model is not correct

Gell-Mann used non-rel to predict Ω^- -Nobel

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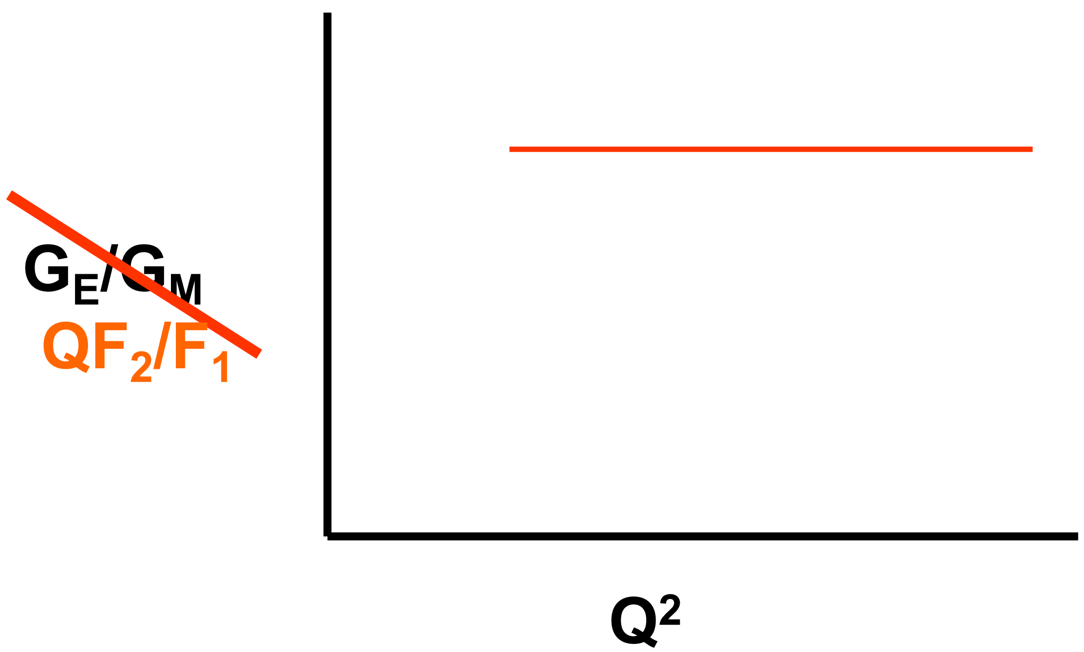

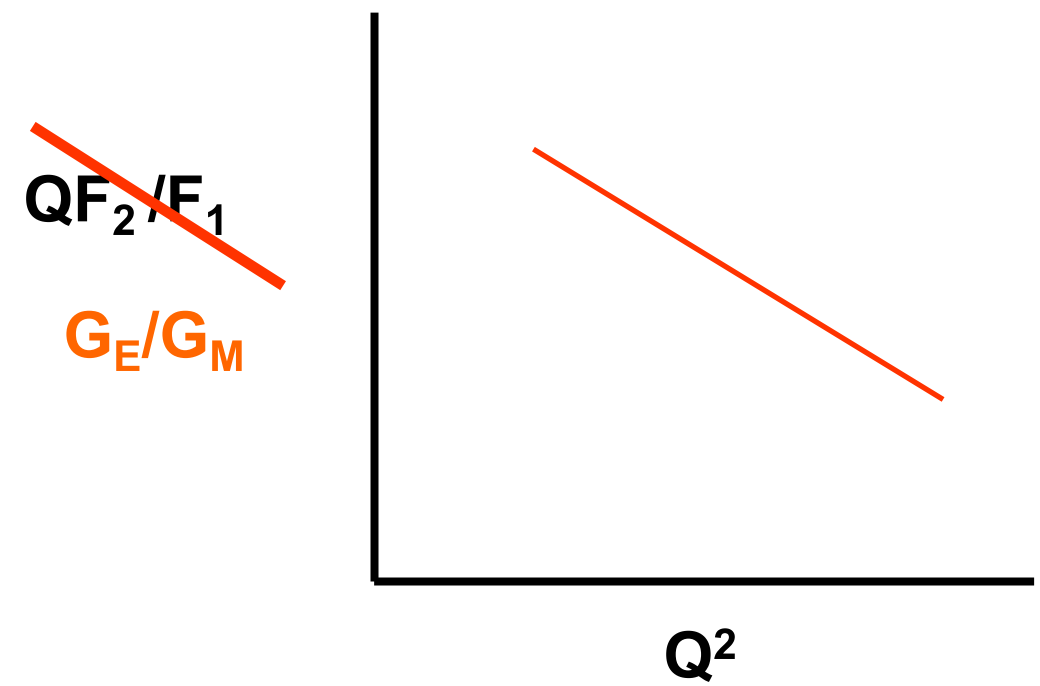

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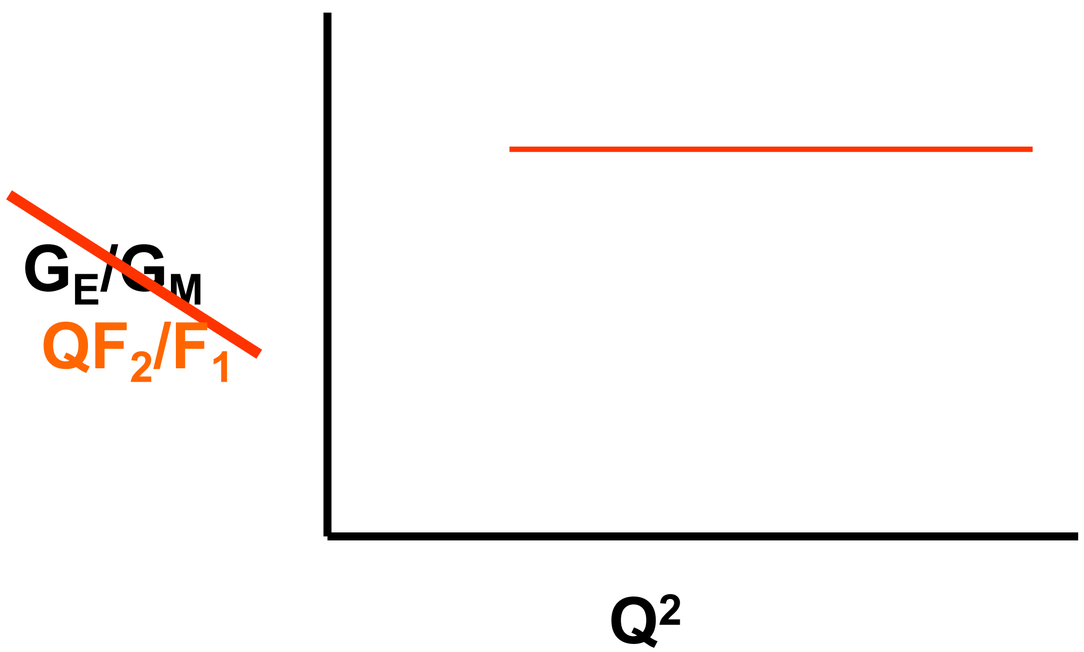

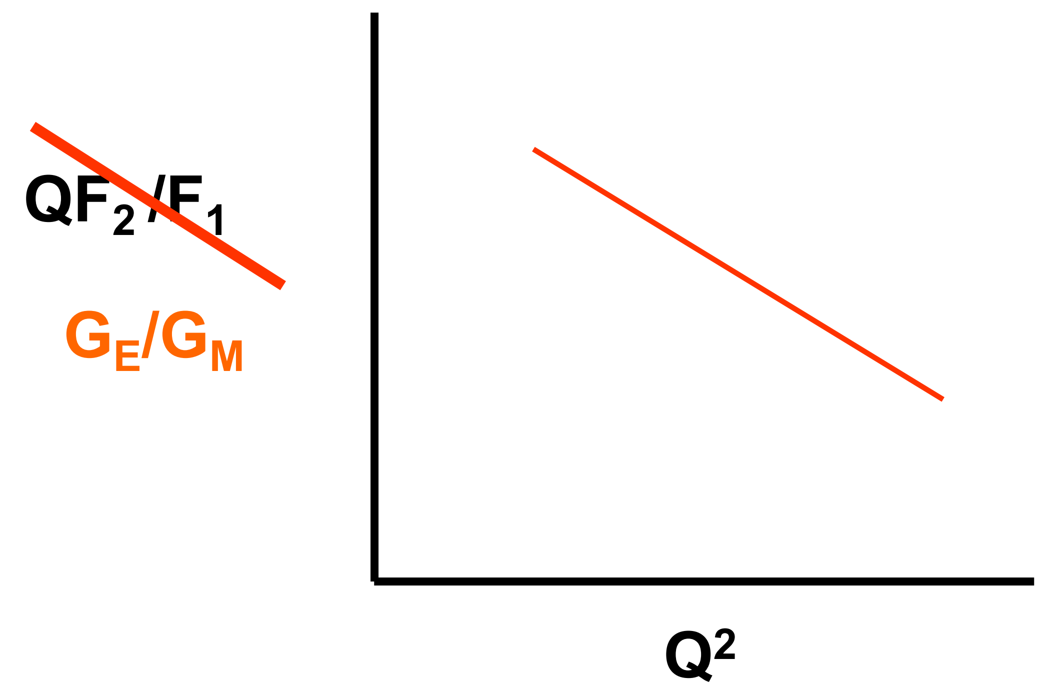

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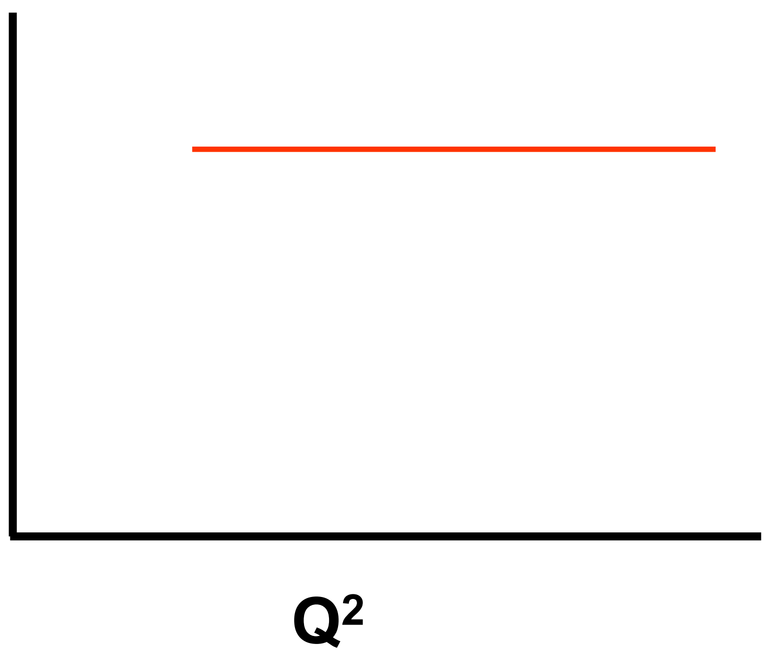
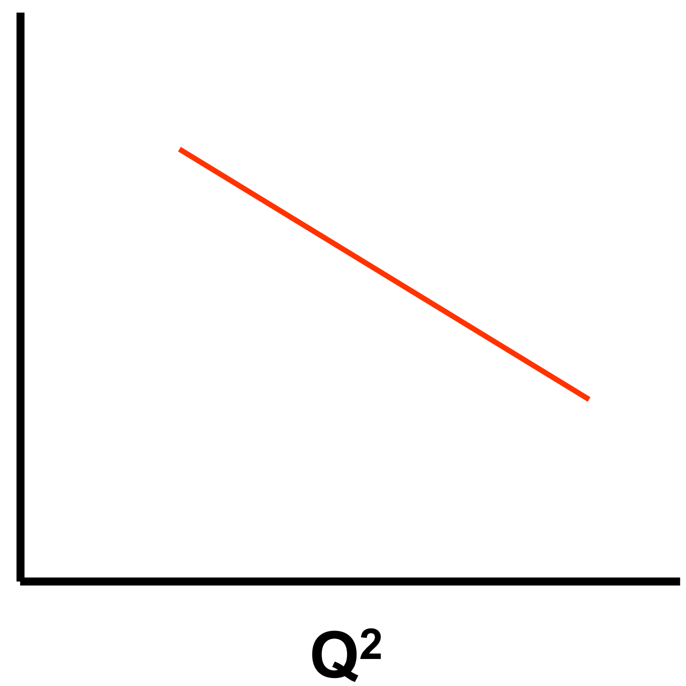




Relativistic quark model is needed

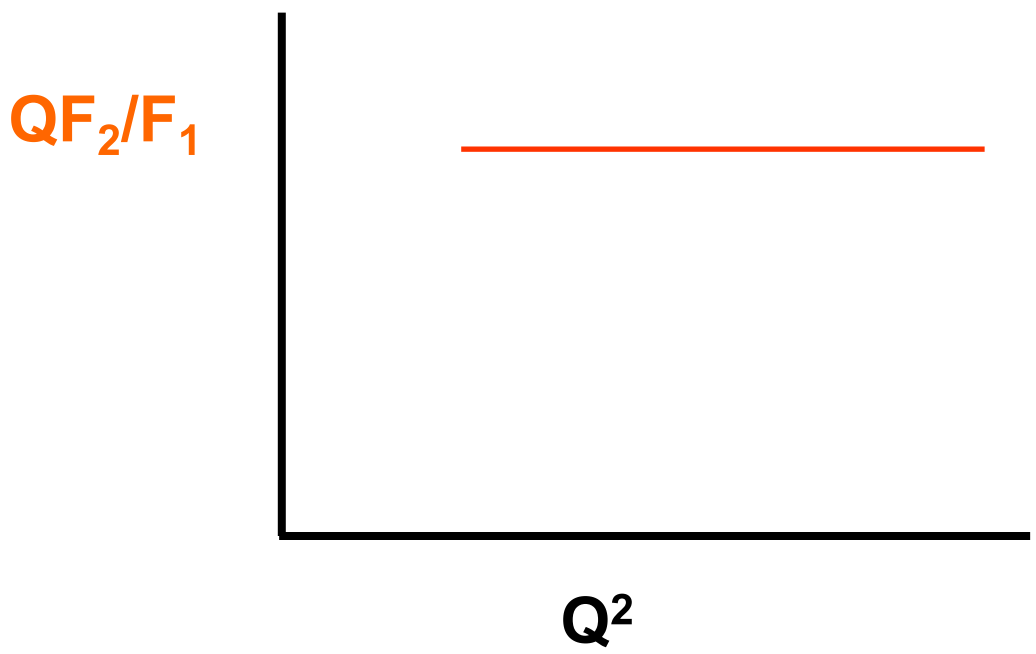
Our prediction:

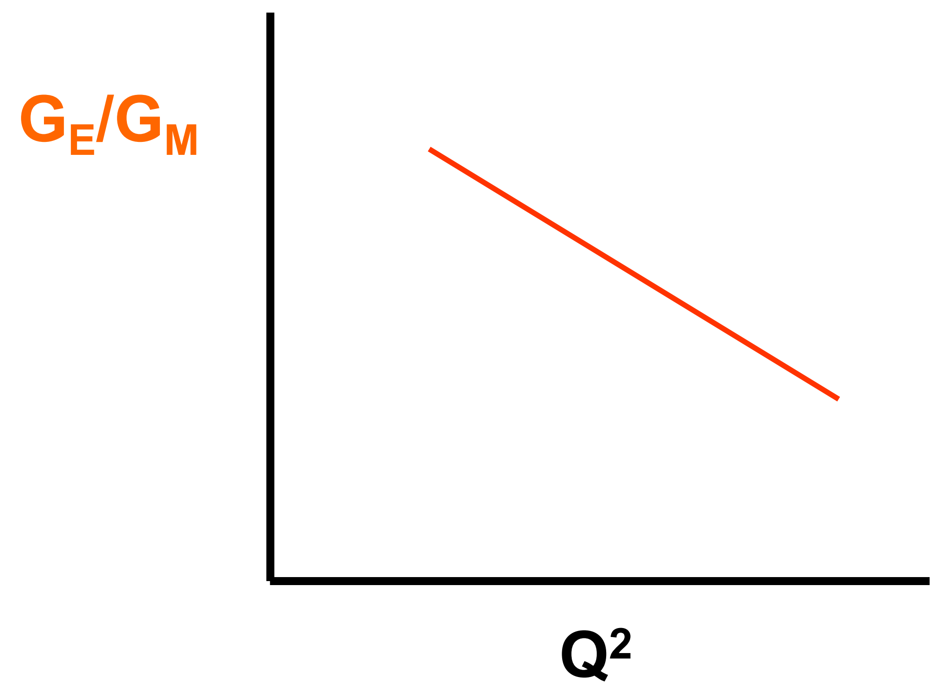
M. R. Frank, B. K. Jennings, and G. A. Miller
Phys. Rev. C 54, 920 – 1 August 1996

Our explanation - relativistic quark model needed

Gerald A. Miller and Michael R. Frank
Phys. Rev. C 65, 065205 –

Jlab





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Implement Relativity: Light front, Infinite momentum

$$\text{“}Time\text{”},\ x^+ = x^0 + x^3, \quad \text{“}Evolve\text{”},\ p^- = p^0 - p^3$$

$$\text{“}Space\text{”},\ x^- = x^0 - x^3, \quad \text{“}Momentum\text{”},\ p^+ (\text{Bjorken})$$

$$Transverse\ position,\ momentum \quad \mathbf{b}, \mathbf{p} \qquad x_{\text{Bj}} = \frac{p^+(\text{quark})}{p^+(\text{traget})}$$

These variables are used in GPDs, TMDs, standard variables

transverse boosts in kinematic subgroup

$$\mathbf{k} \rightarrow \mathbf{k} - k^+ \mathbf{v} \qquad |\mathbf{R} = \mathbf{0}, \lambda\rangle = \int d^2p |\mathbf{p}, \lambda\rangle$$

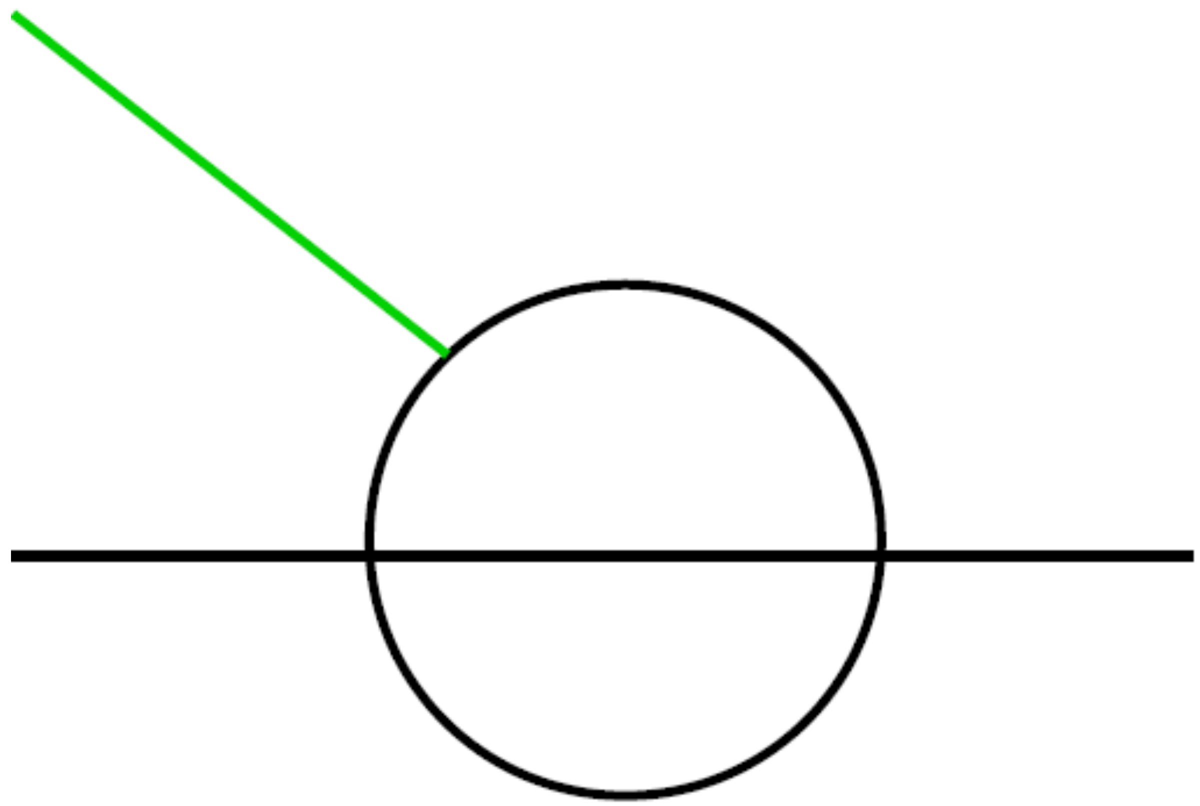
Momentum transfer in transverse direction

then density is 2 Dimensional Fourier Transform

Frank Jennings Miller PRC54, 920, 1996
Impulse Approximation

Three particles independent spatial variables

Model proton wave function $\Psi(\mathbf{k}_\perp, \mathbf{K}_\perp, \xi, \eta)$



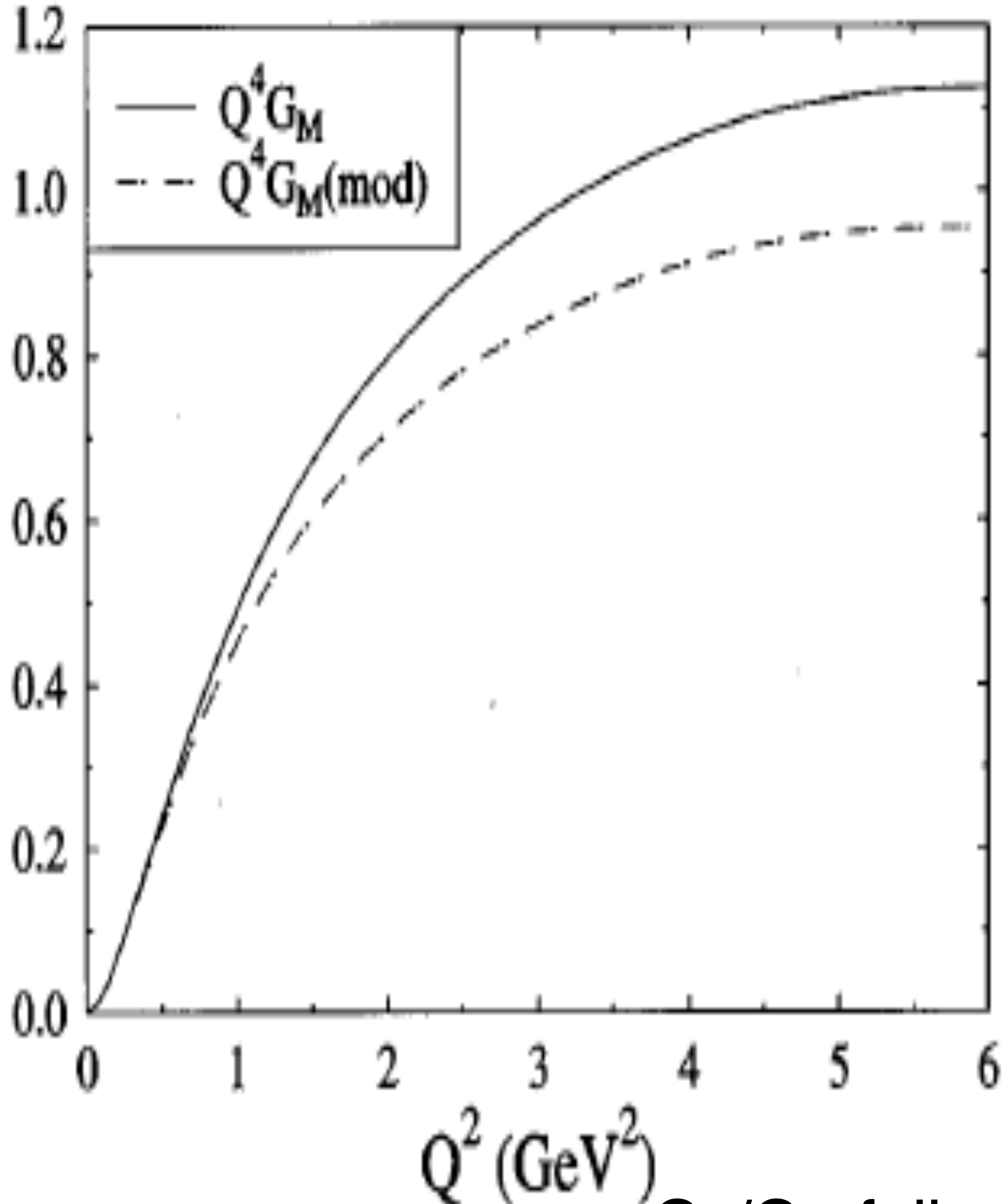
Poincare invariant

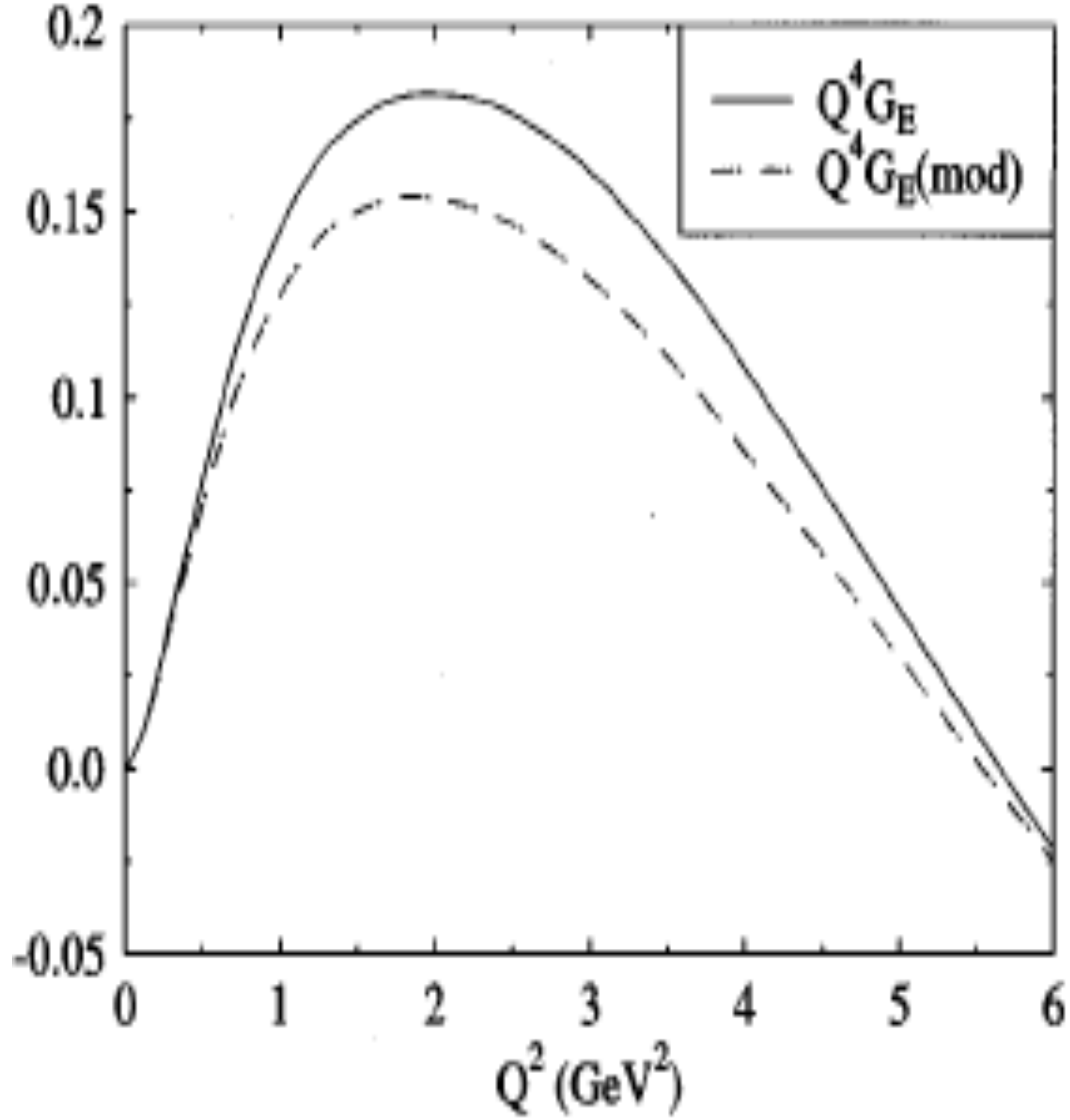
Light front variables for boost: $\mathbf{K} \rightarrow \mathbf{K} + \eta \mathbf{q}_\perp$

Dirac spinors

Purpose of model wave function - learn phenomena
Please know the difference between using wave functions and fits
Models give predictions and fits do not

1996 Frank, Jennings, Miller



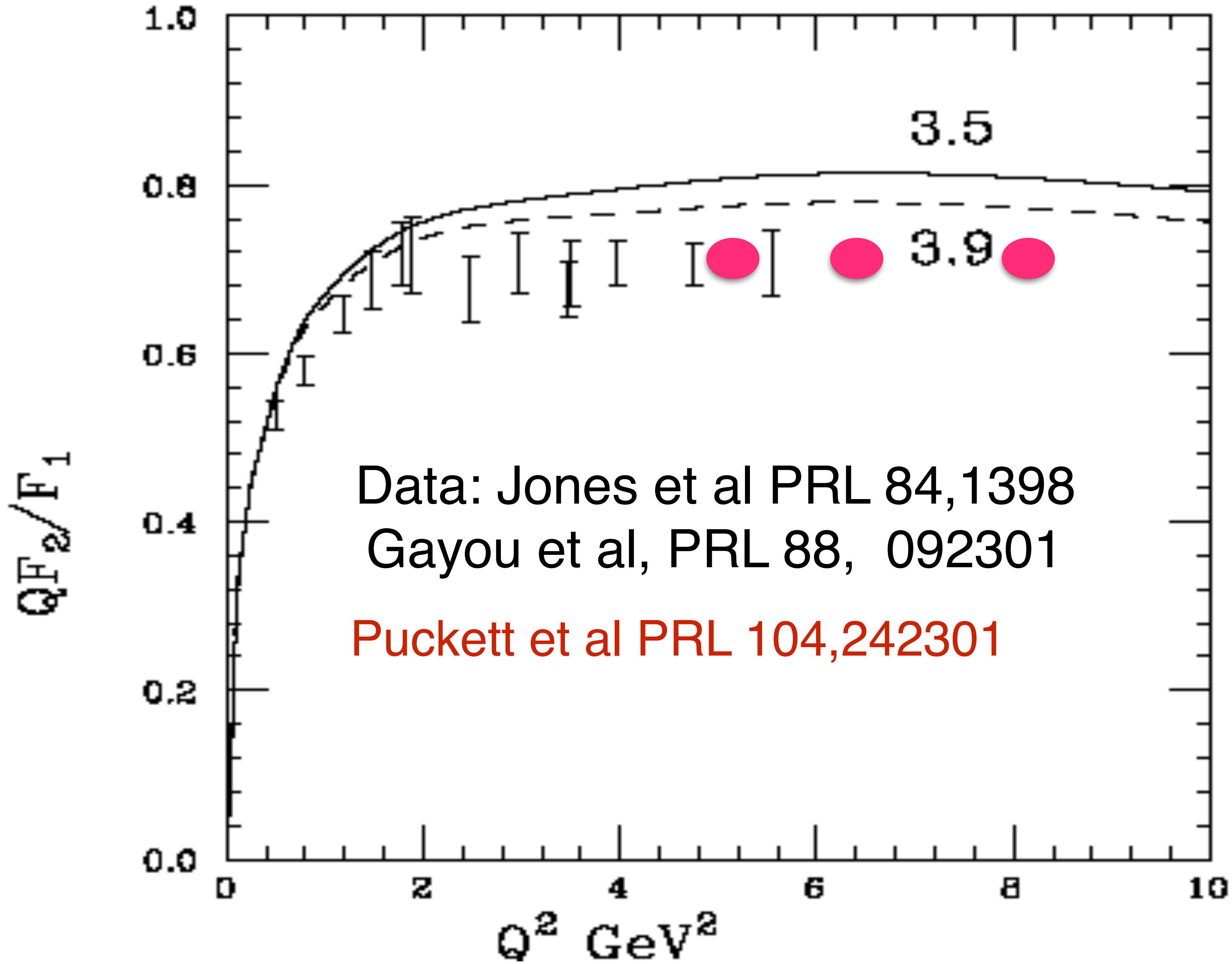


G_E/G_M falls with increasing Q^2

Ratio of Pauli to Dirac Form Factors Theory 1996

Miller & Frank PRC 65, 920

Will flat trend continue?



Relativistic Explanation

J^+ acts on third quark, other two have 0 spin

$$u(K,s) = \begin{pmatrix} (E(K)+m)|s\rangle \\ \boldsymbol{\sigma}\cdot\mathbf{K}|s\rangle \end{pmatrix}$$

Lower component
signature of relativistic effects

$\boldsymbol{\sigma}_y|s\rangle$: quark spin $\neq$ proton ang mom

lower components $\equiv L_z \neq 0$

$\bar{u}(K',s')\gamma^+ u(K,s) \sim \langle s'|K^+ + i\sigma_y Q|s\rangle$ Large Q

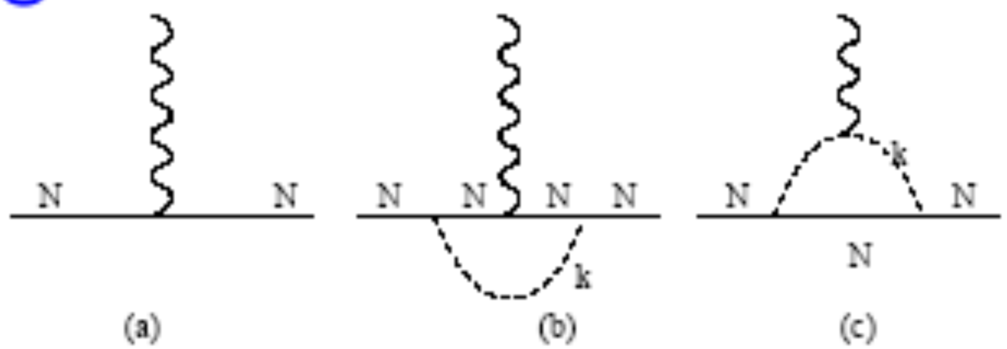
spin non-flip $F_1(Q^2) = \int \cdots Q\Phi\Phi$, flip $QF_2 = \int \cdots Q\Phi\Phi$

$\frac{QF_2}{F_1} \sim Constant$ Over range of existing data, future?

Neutron: Need π cloud effect at low Q

$G_E \neq 0$

Cloudy Bag Model 1980 PRD 24, 216



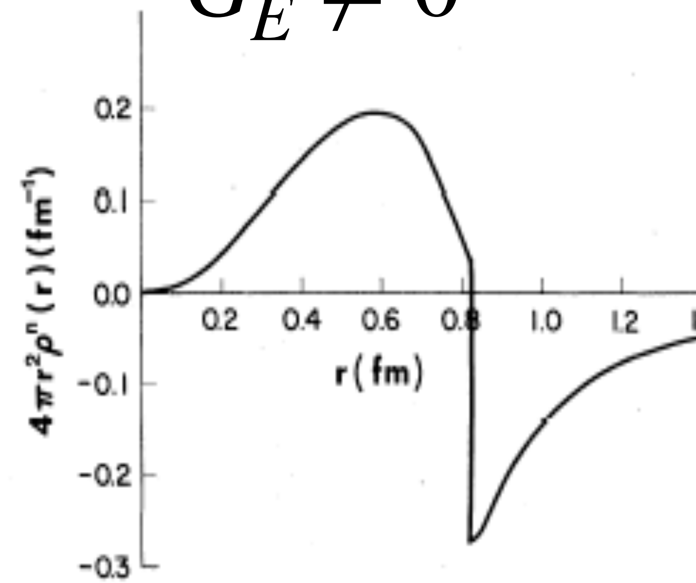


FIG. 11. Neutron charge density.

From 3D Fourier Transform of G_E

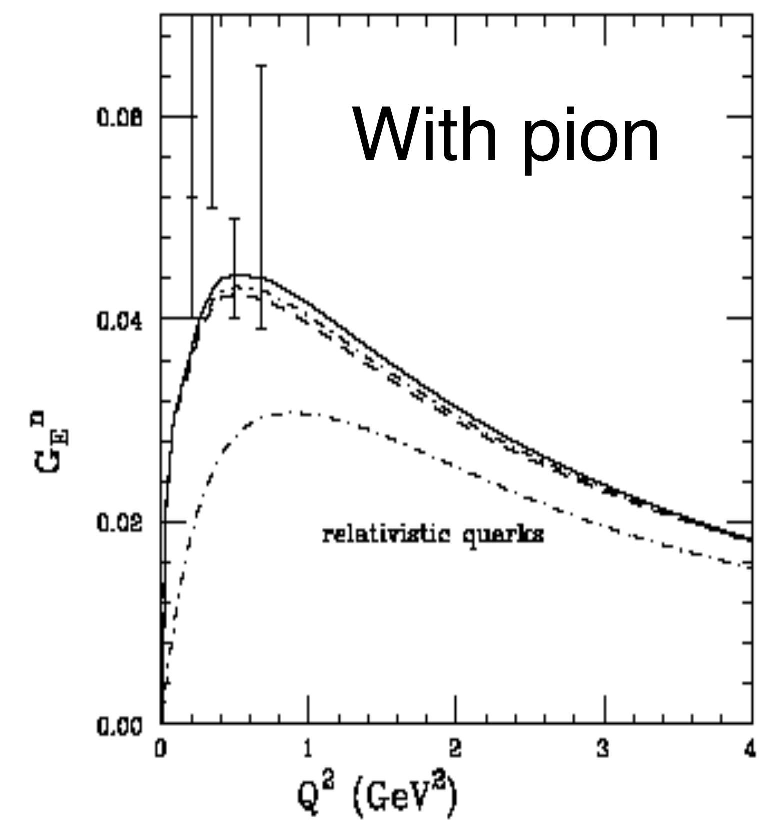
Relativistic treatment needed Feynman graphs, $\int dk^-$

Updated : Miller, Phys.Rev. C66 (2002) 032201

Light front cloudy bag model LFCBM 2002

- γN form factors from model
- rel. πN form factor $\Lambda_{\pi N}$
- Model parameters: $m, \beta, \gamma, \Lambda_{\pi N}$

Good fits to other form factors mag. moments



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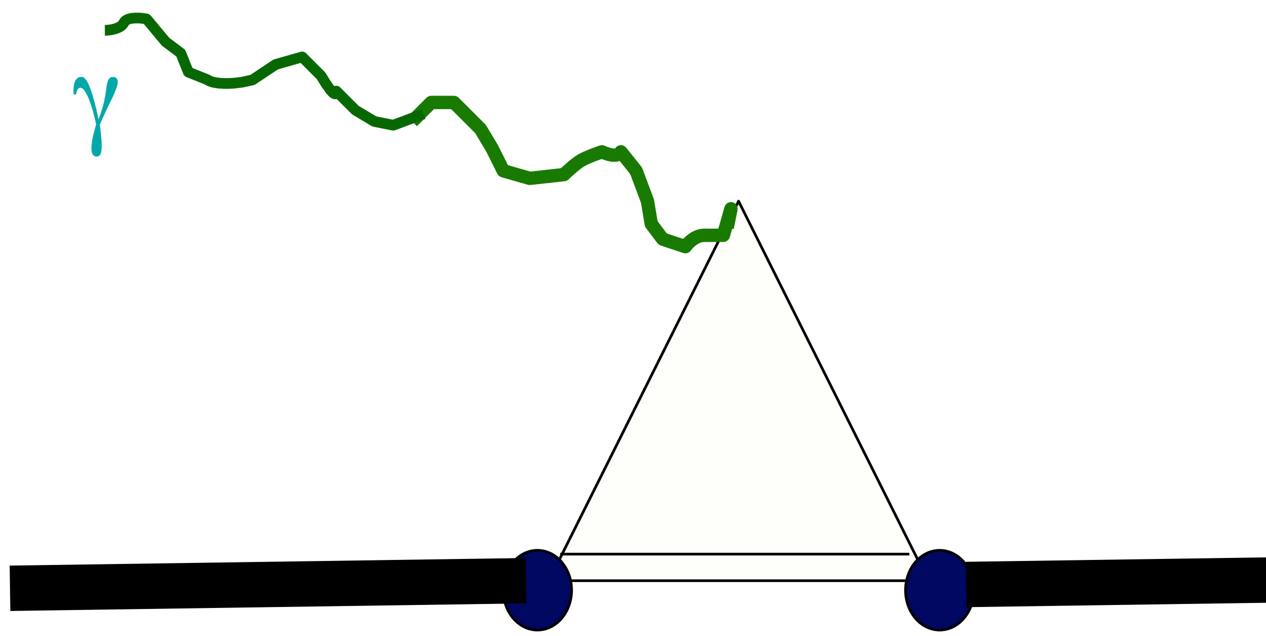
PRC86,015208

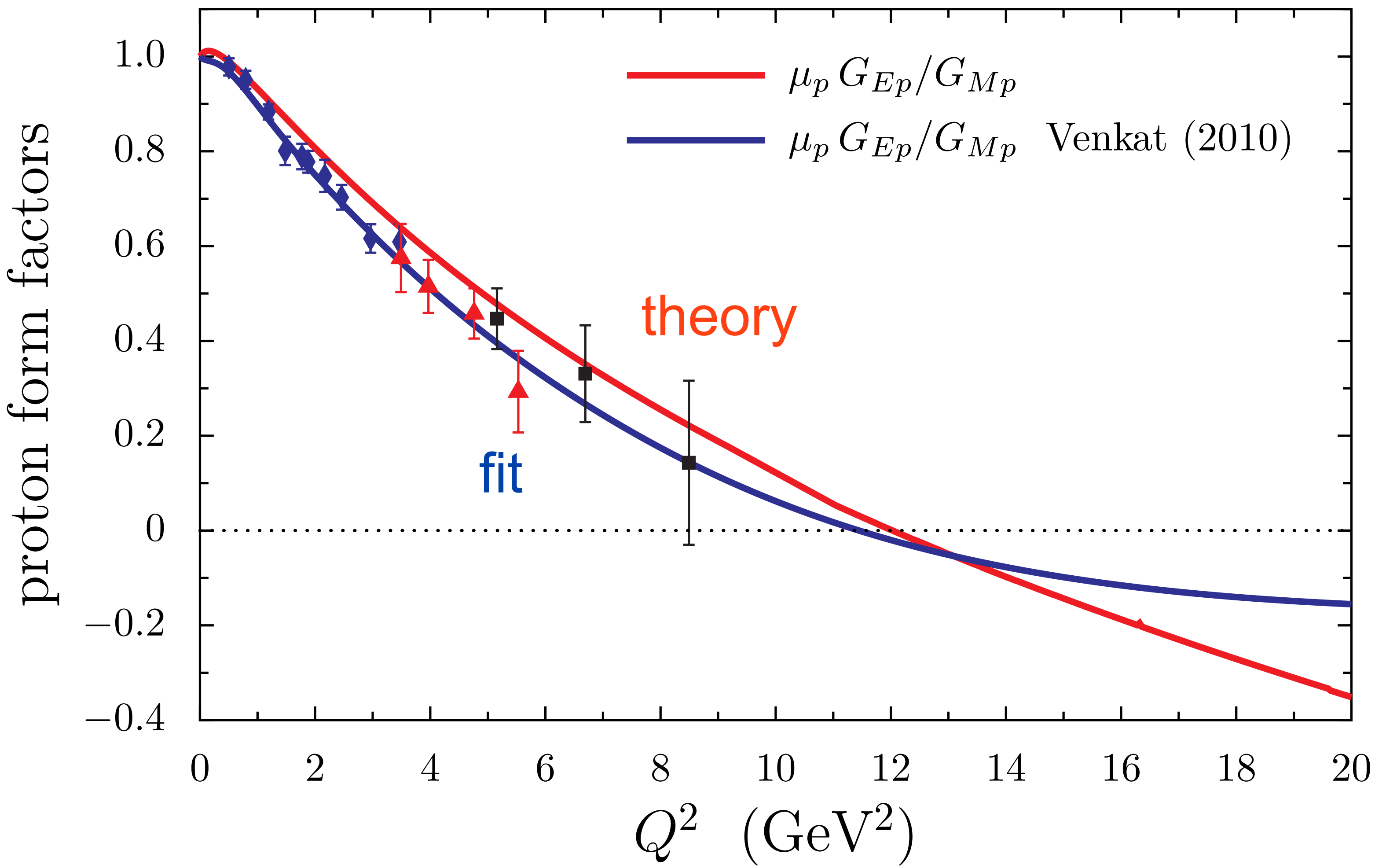
Model proton wave function: quark-diquark

Lorentz and rotationally invariant-more different forms!

Light front variables

Dirac spinors-orbital angular momentum





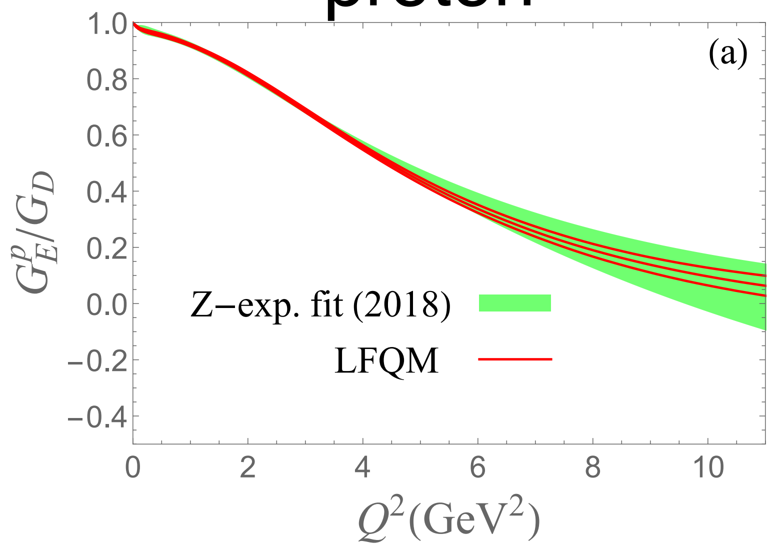
Quark spin is 35 % of proton total angular momentum

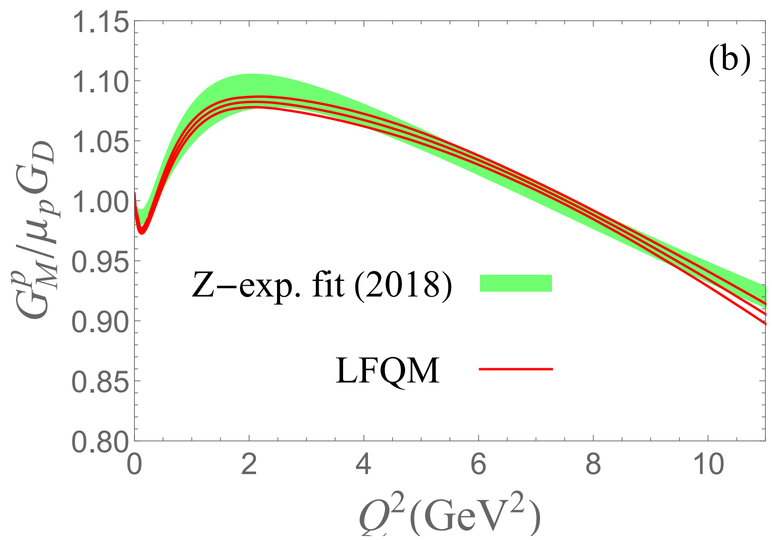
Unified model of nucleon elastic form factors and implications for neutrino oscillation experiments
Zhang, Hobbs, Miller arXiv: 1912:08479

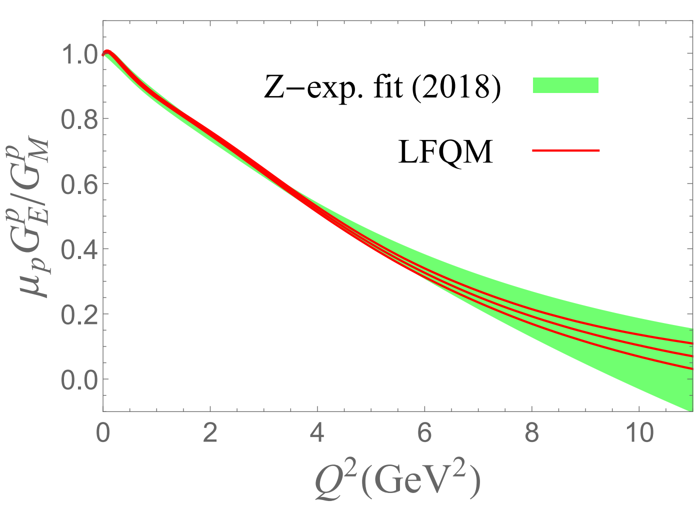
- Axial current form factors needed for neutrino-nucleus interactions
- Light front wave function model- includes $\Delta - \pi$
- Fit parameters to electromagnetic (vector) form factors
- Compute axial form factors and consequences for neutrino-nucleus interactions physics

Vector FF's: Light Front Quark Model (LFQM) arXiv: 1912:08479
Z-exp. Fit: Ye et al PLB 777, 8 ('18)

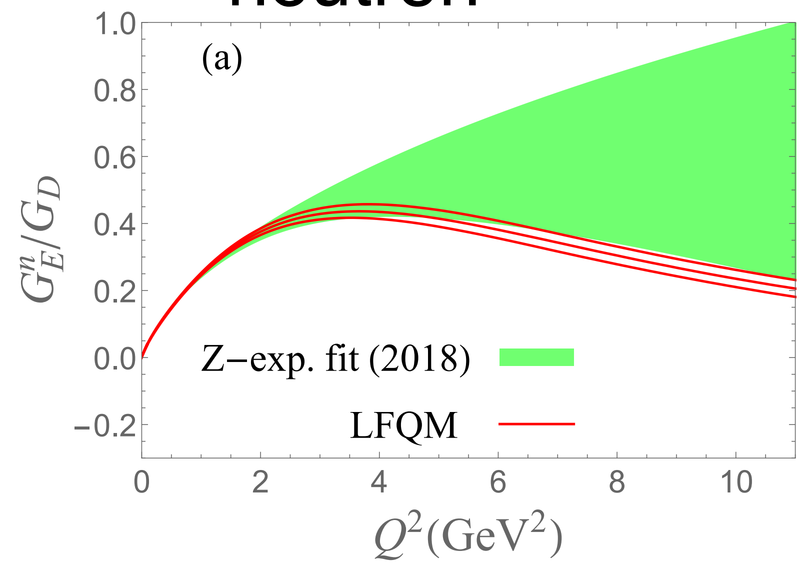
proton

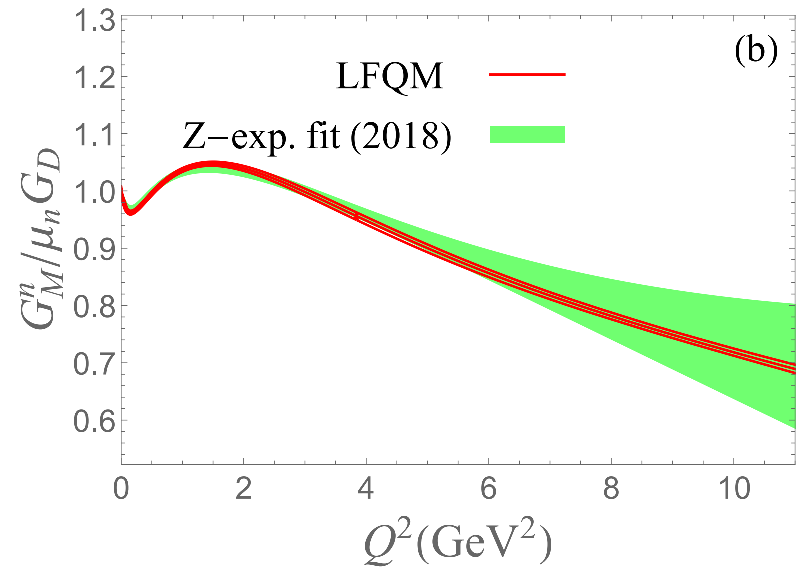




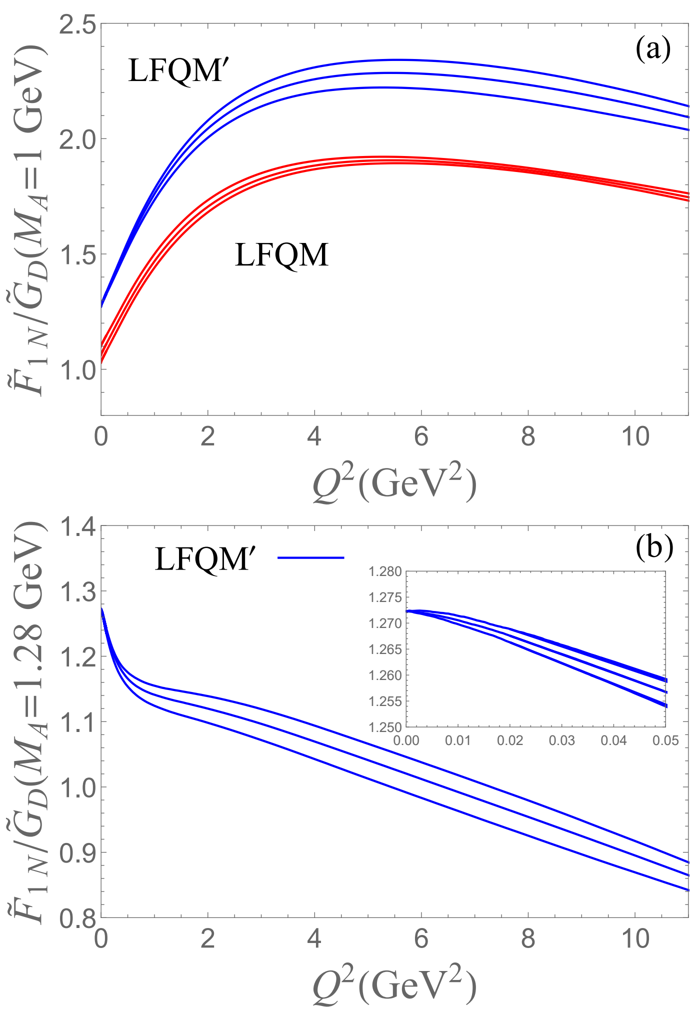


neutron

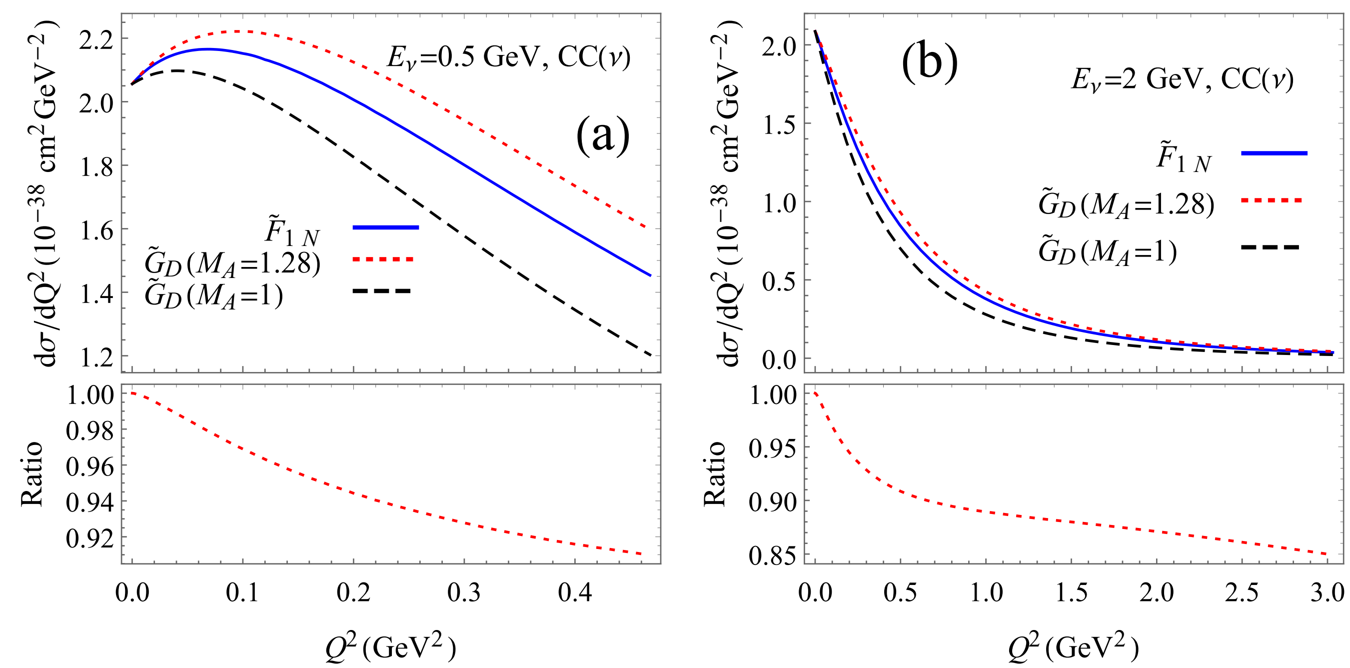




Axial form factor ratio to dipole



Neutrino-nucleon scattering



Using non-dipole form factors
Widely used dipole ansatz inadequate
Gives 5-10% overestimate of total cross section

Future JLab measurements will impact knowledge of axial form factor!

Meaning of form factors

- General idea- give charge and magnetization densities
- What they do NOT mean and what they DO mean

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

WRONG

$$G_E(\vec{q}^{\,2}) = \int d^3r \rho(r) e^{i\vec{q}\cdot\vec{r}} \rightarrow \int d^3r \rho(r)(1 - \vec{q}^{\,2} r^2/6 + \cdots)$$

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

interpretation of Sachs FF is wrong

Final wave function is boosted from initial

Need relativistic treatment

$$\rho_{\rm NR}(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\mathbf{Q}\cdot\mathbf{r}} G_E(Q^2).$$ not a density-physics

Charge radius of neutron magnetic moment $=\mu_n$:

$$G_{En}(Q^2) = F_{1n}(Q^2) - \frac{Q^2}{4M^2}\mu_n F_2(Q^2) \to -Q^2 R_1^2/6 = \frac{Q^2}{4M^2}\mu_n$$

Charge radius of neutron determined by its magnetic moment. Gasp!

$r_n^2 = -\frac{6}{4M^2}\mu_n = -0.126\,\text{fm}^2$. Experimental value $= -0.116\,\text{fm}^2$

Charge radius of pion

$$F_\pi(Q^2) \approx \frac{1}{1+\frac{Q^2}{m_\rho^2}}$$

Fourier transform $\rho_{\rm NR}(r) \propto \frac{e^{-m_\rho r}}{r}$

Singular at $r = 0$. Gasp!

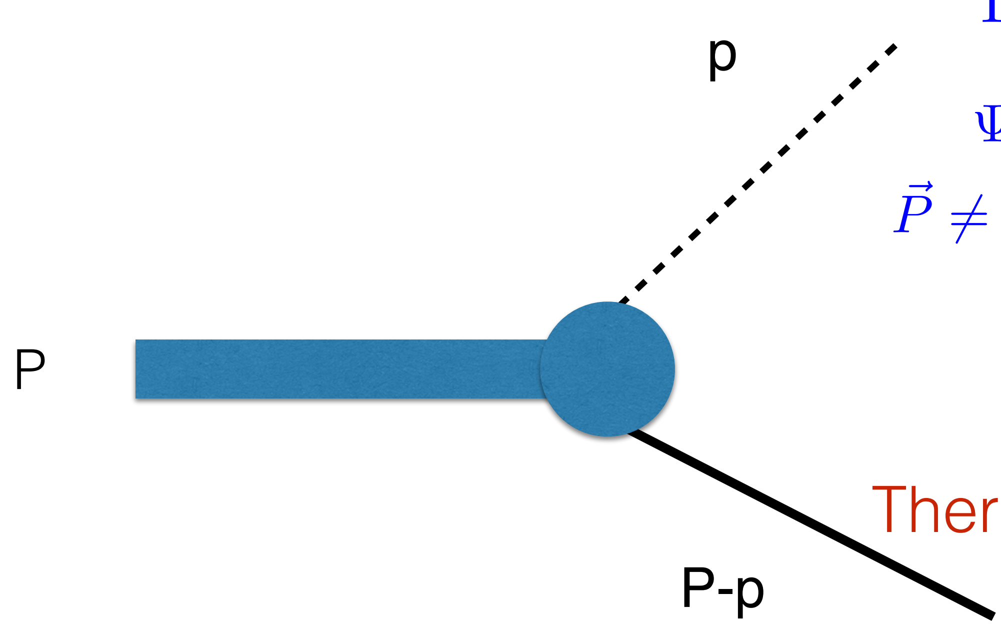
$$\rho_{\rm NR}(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\mathbf{Q}\cdot\mathbf{r}} G_E(Q^2).$$ not a density formal reasons

- Correct non-relativistic because center-of-mass and internal coordinates are independent - factorize
- Relativistic: Internal wave functions depend on total momentum. No factorization.
- Momentum q is absorbed by proton of momentum p. Initial and final wave functions have different momenta, so no square of wave function appears
- NO $\Psi^*\Psi$, NO density

Initial state 4-momentum =P, final state =P+q

$$\Psi_{\rm B.S.}(P,p) = \frac{1}{p^2-m_q^2+i\epsilon}\frac{1}{(P-p)^2-M_S^2+I\epsilon}$$

$\vec{P} \neq 0$ is called a 'Boost'. Different boosts different wave functions



Form factor is overlap of different initial and final wave functions. There is no density in the square of wave function sense

The only derivation of the three-dimensional density is due to Sachs PR 126, 2256 (1962)

Where Sachs (1962) went wrong

Appendix claims three-dimensional density exists

Tries to get around boost by using wave packet

$|\Psi\rangle = \int d^3P g(\vec{P})|P,s\rangle$

$|g(\vec{P})|^2 = \delta(\vec{P})$

Gets rid of boost- both initial and final wave function are at 0 momentum

But defining momentum precisely means position is spread over all of space!

Technically- Sachs ignored derivative of $\delta(\vec{P})$, dropping an infinite term

For details see G A Miller Phys.Rev. C99 (2019) no.3, 035202

Next - doing it right, a true density

Implement Relativity: Light front, Infinite momentum

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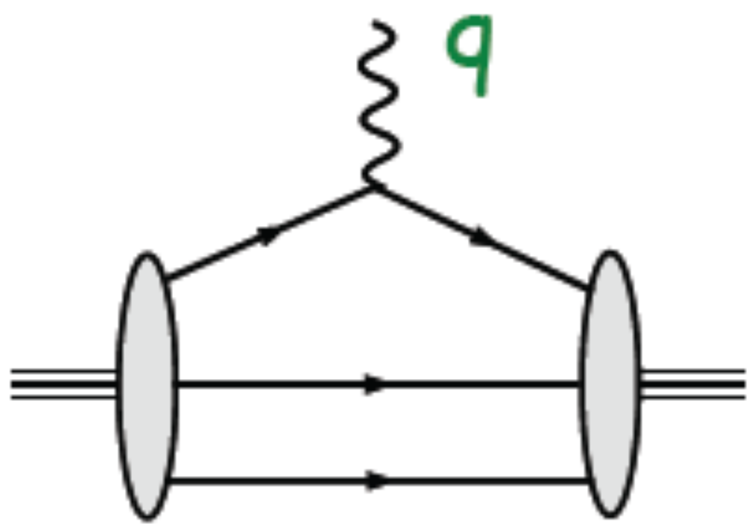
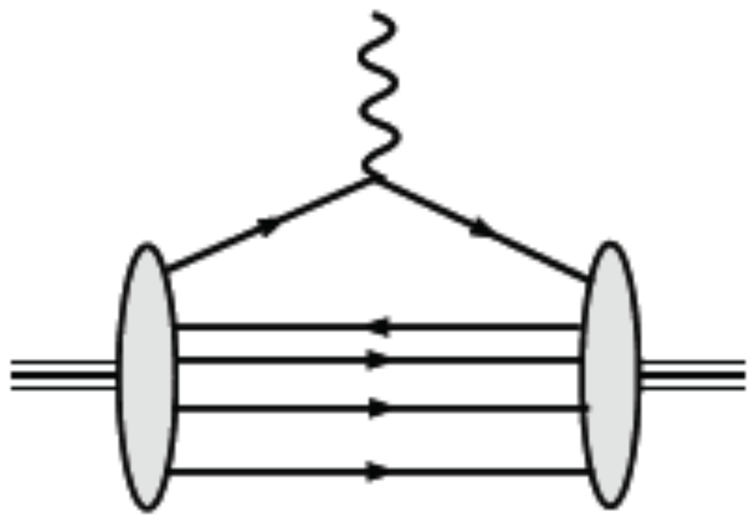
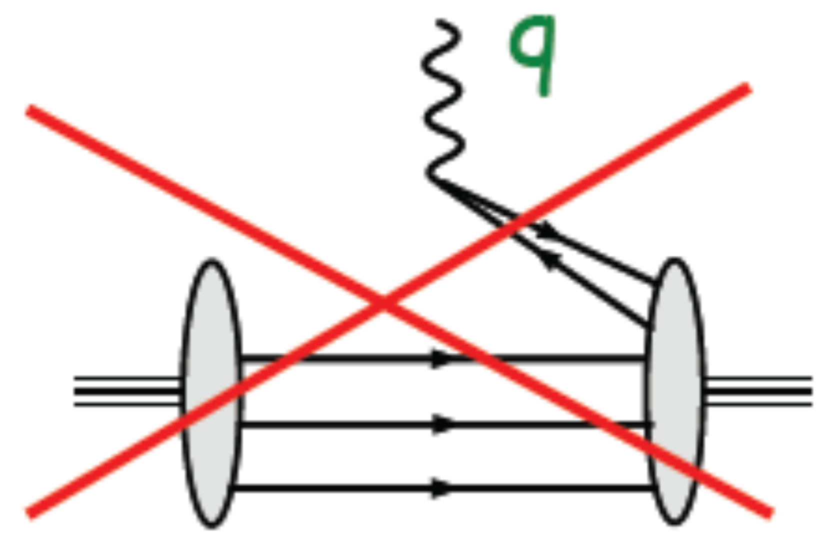
interpretation of FF as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

Absent in a Drell-Yan Frame

$$q^+ = q^0 + q^3 = 0$$

From Marc Vanderhaeghen

GAM Phys.Rev.Lett.99:112001,2007

Transverse Charge Density

$$\widehat{\rho}_\infty(x^-,\mathbf{b}) = \sum_q e_q \bar{q}(x^-,\mathbf{b})\gamma^+ q(x^-,\mathbf{b}) \quad = J^+(x^-,\mathbf{b})$$

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\,\hat{\rho}_\infty(x^-,\mathbf{b})\,|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

Diagonal matrix element of density operator

Integrate over x^-, use momentum expansion, definition of F_1:

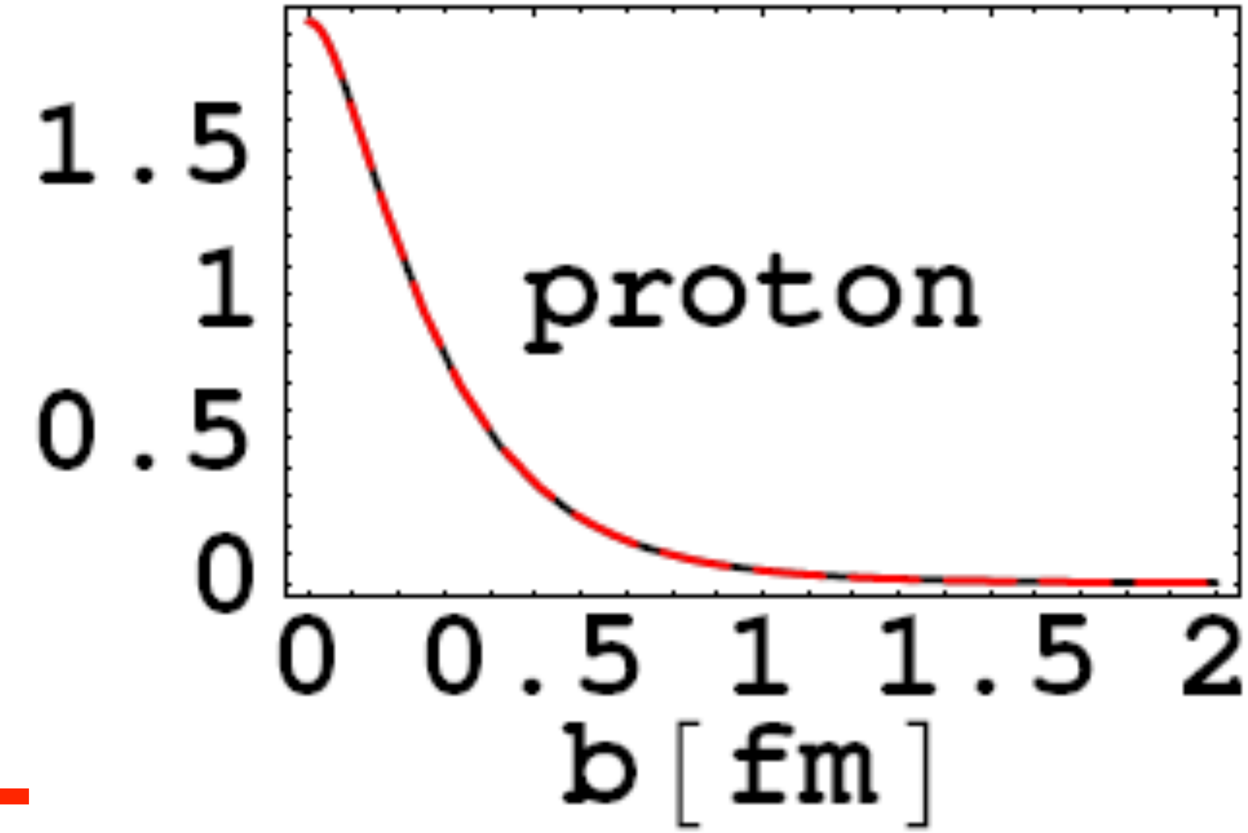
$$\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} F_1(Q^2=\mathbf{q}^2) e^{-i\mathbf{q}\cdot\mathbf{b}}$$

Transverse charge density

b is distance between struck quark and $R=0$

Results

$\varrho(\mathbf{b})\ [\mathrm{fm}^{-2}]$



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Kelly

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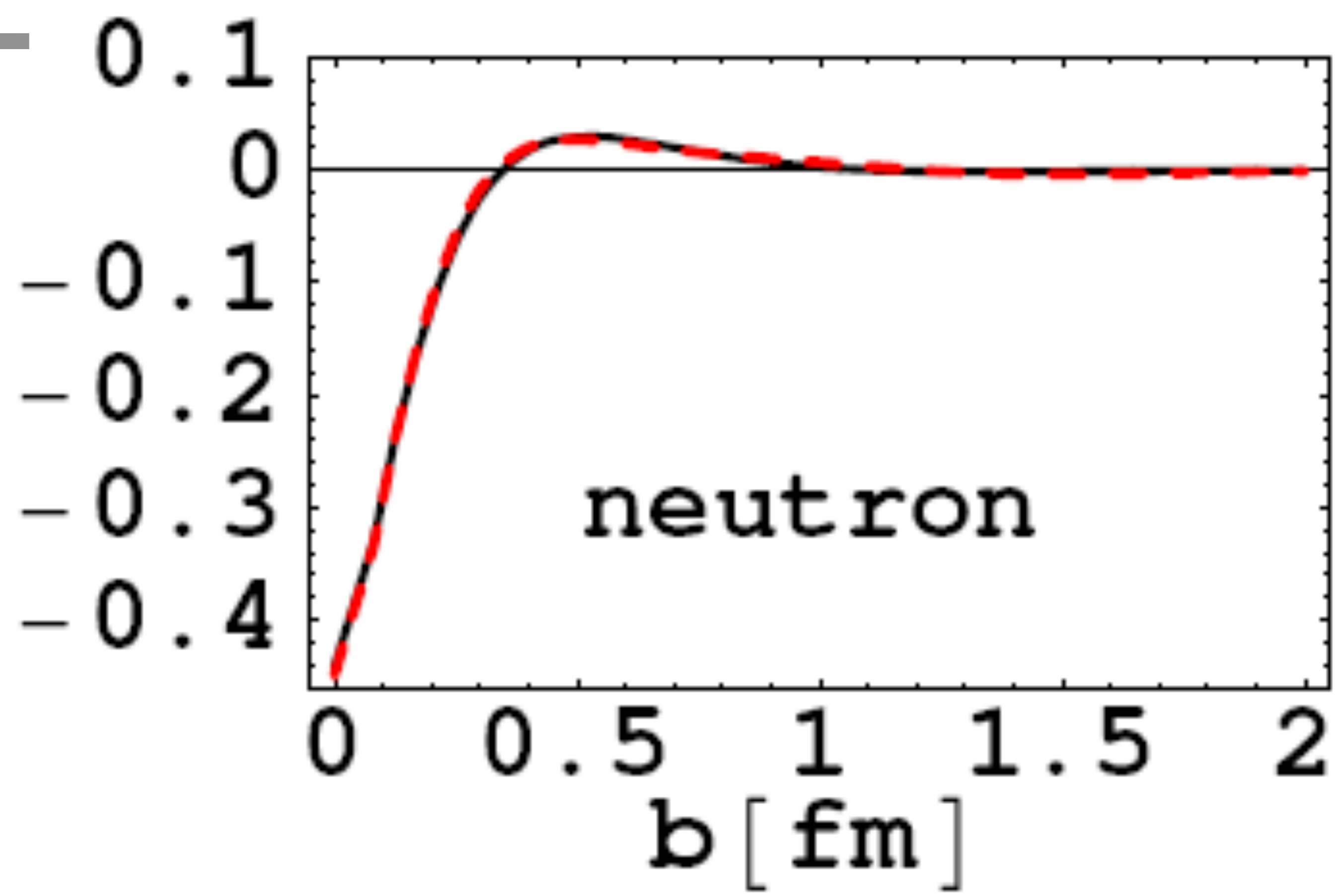

Negative

Results

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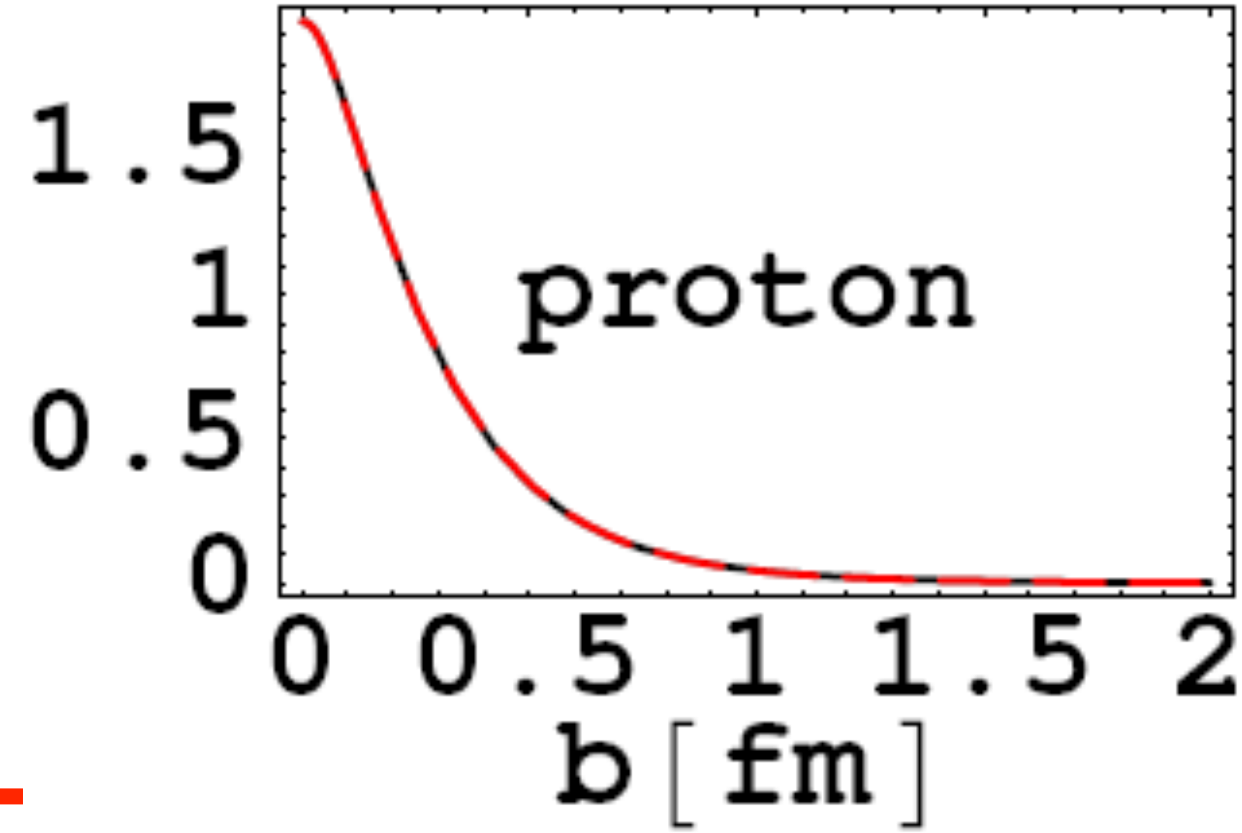

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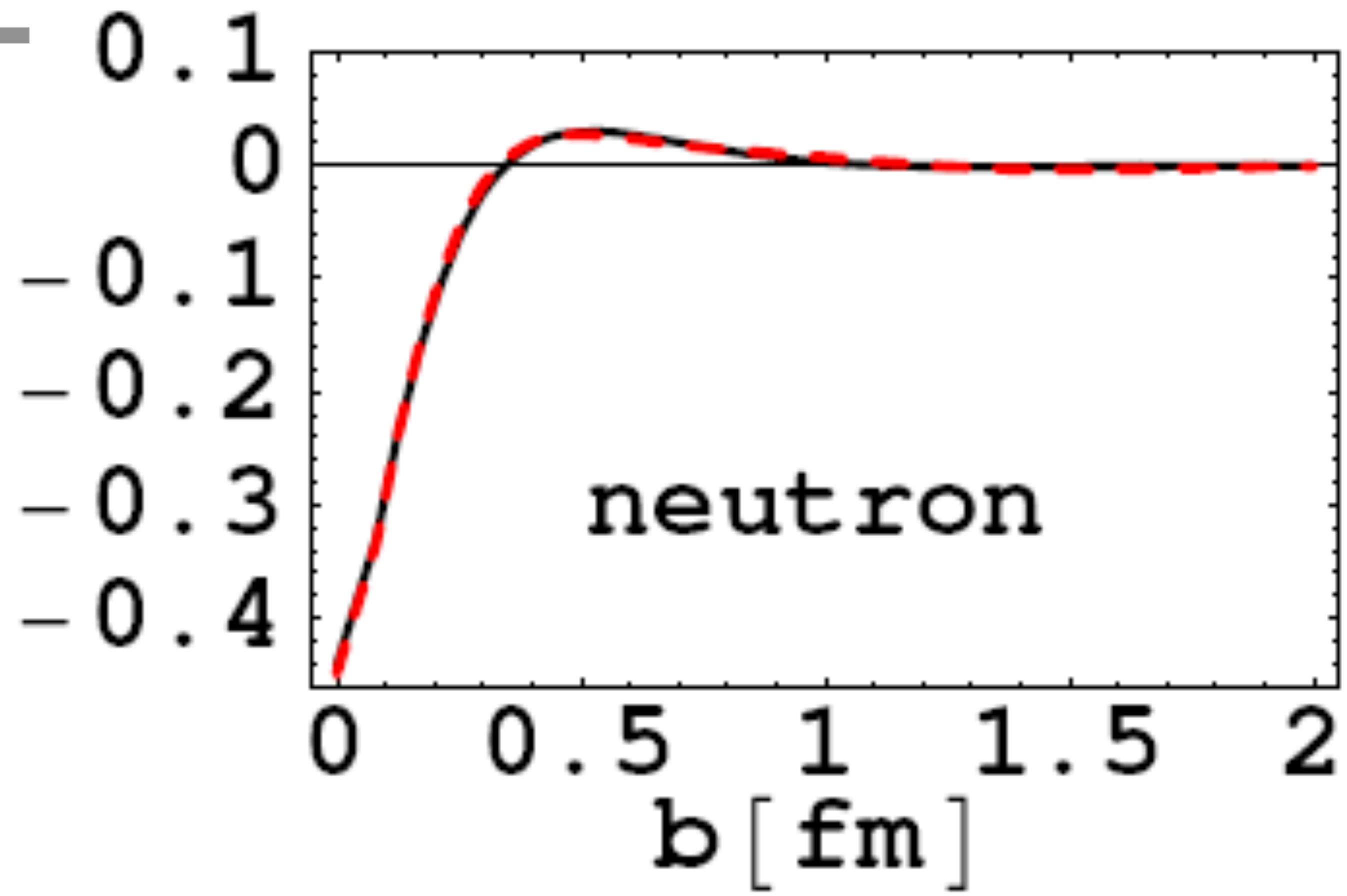
$\varrho(\mathbf{b})\ [\mathrm{fm}^{-2}]$

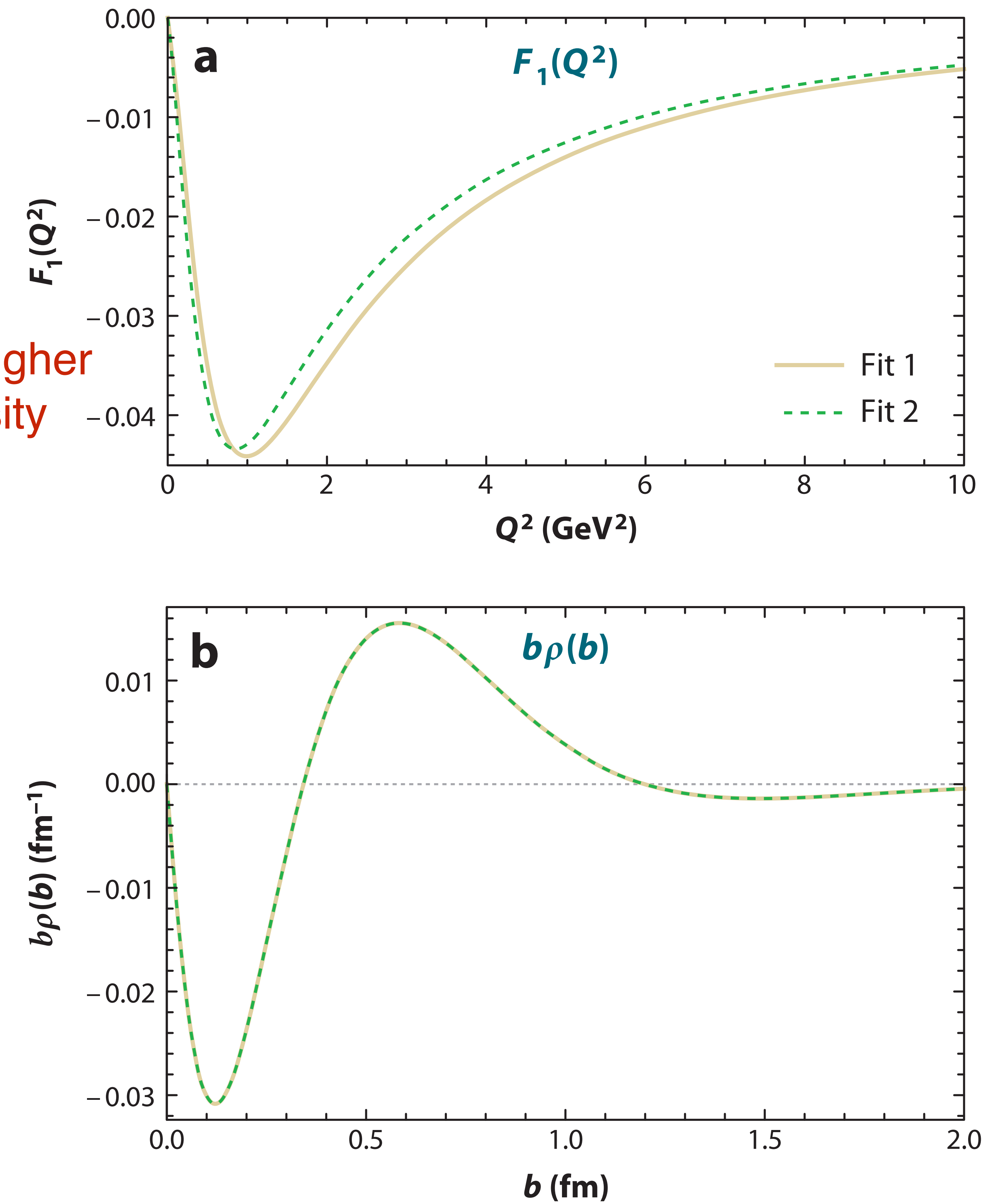
Negative



Neutron

F_1 is negative, so is central density

If F_1 goes positive at higher Q^2 then central density might be positive



Details in GAM ARNPS 60,1 ('15)

Meaning of b:

- Distance between struck quark and transverse center of momentum
- Distance between struck quark and spectator system is r, with $r(1-x)=b$.
- $b=0$ corresponds to either $r=0$ or $x=1$
- My opinion is that having a single quark carrying all momentum ($x= 1$) is very rare, so
- $b=0$ corresponds to $r=0$
- Need measurements of form factors and GPD $H(x,t)$ to know (so far many extractions of H use form factors)

Shapes of the proton

- How to learn influence of quark orbital angular momentum?
- OAM present in all relativistic wave functions
- Proton has spin 1/2, no quadrupole moment
- Spin-dependent density (SDD) G A Miller PRC68,022201(2003)
- SDD is probability that struck quark has a given momentum and also a spin in a given direction
- Direction defined by proton angular momentum $\hat{\mathbf{s}}$

Proton

$$\hat{\rho}(\mathbf{K},\mathbf{n}) = \int \frac{d^3r}{(2\pi)^3} e^{i\mathbf{K}\cdot\mathbf{r}} \bar{\psi}(\mathbf{r}) \frac{\widehat{Q}}{e} (\gamma^0 + \boldsymbol{\gamma}\cdot\mathbf{n}\gamma_5)\psi(\mathbf{0})$$

Spin projection operator

Take expectation value

$\gamma(K)$ from lower component Dirac spinor

Same mechanism that gave QF_2/F_1 ~constant

$$\rho(K,\mathbf{n}=\hat{\mathbf{s}}) = \rho(K)\left(1+\gamma(K)\cos^2\theta\right)$$

$$\rho(K,\mathbf{n}=-\hat{\mathbf{s}}) = \rho(K)\gamma(K)\sin^2\theta$$

$\rho(K)$ from wave function

$$\gamma(K) \sim \frac{K^2}{4m^2}(K\to 0),\ \sim 1(K\to\infty)$$

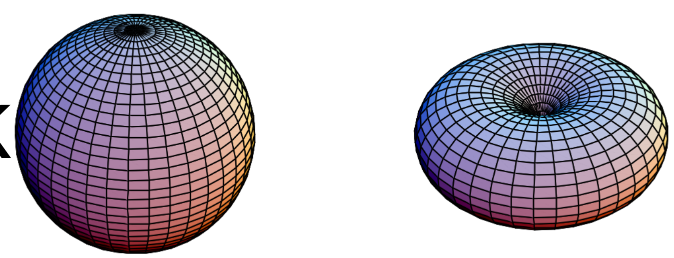
Shapes independent of Q^2, property of wavefunction

- G A Miller PRC68,022201(2003)

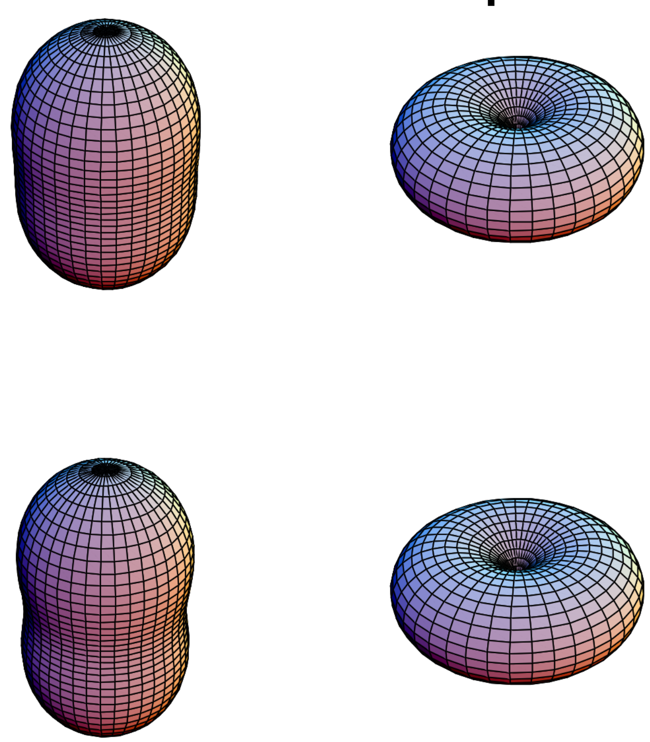
Shapes of the proton

Momentum space

Increase K

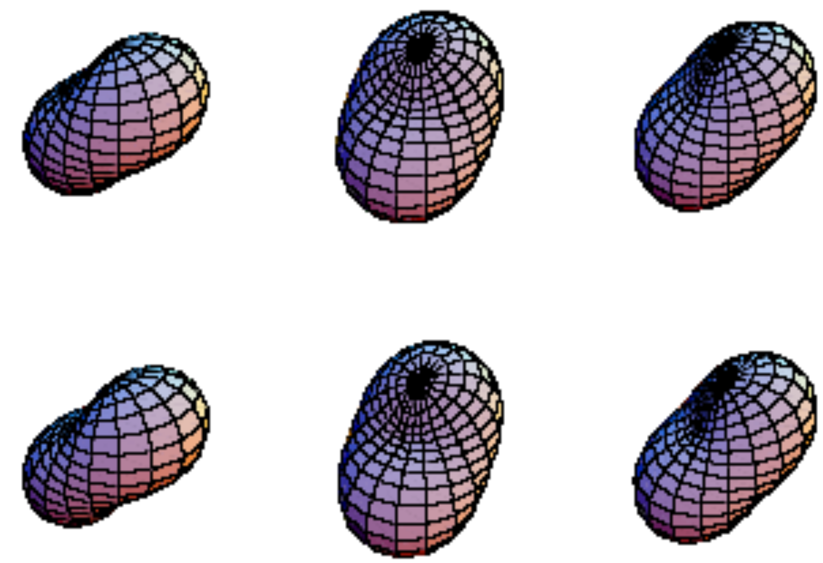


Quark spin opposes proton spin

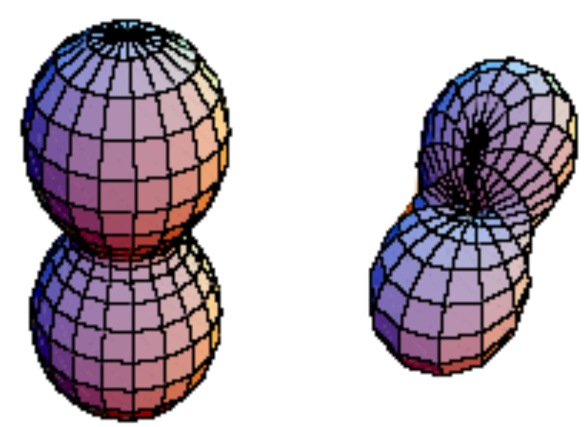


Quark spin II proton spin

Coordinate space



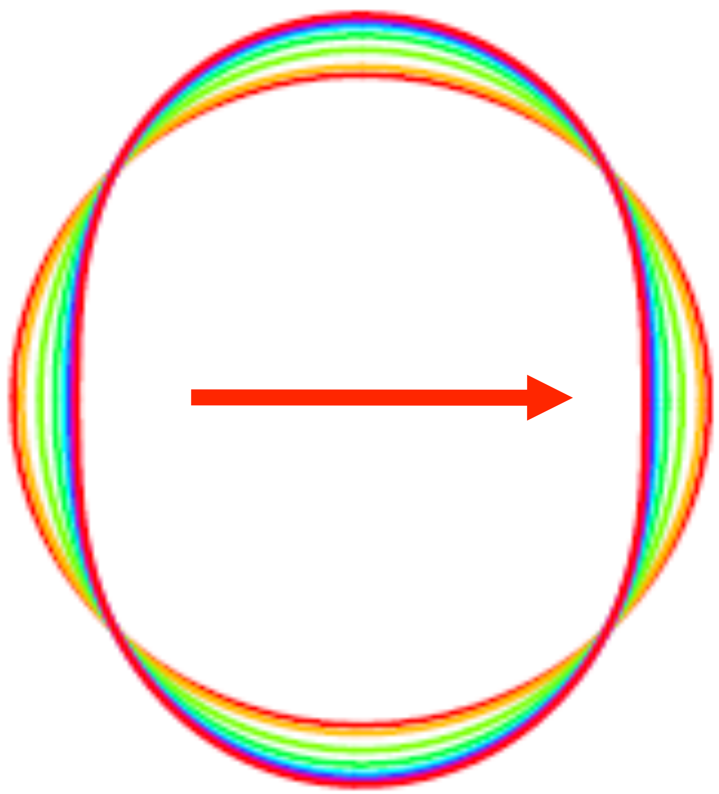
Origin of term “pretzelocity”

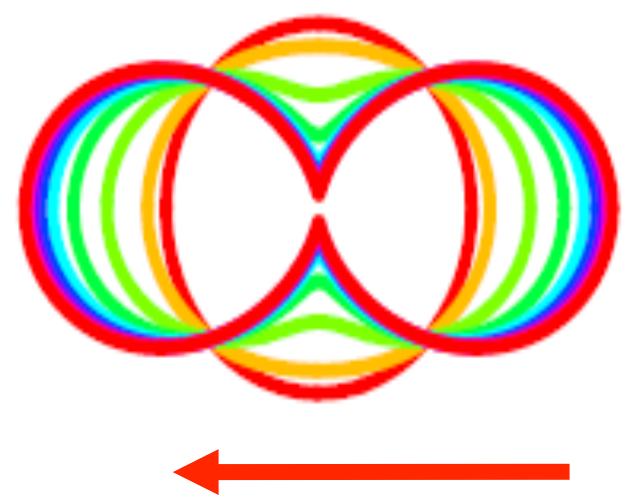


Published in PRC and NY Times

Problem with previous

- Three dimensional- not two
- Used three dimensional vectors, need to use transverse momentum **k** and fraction x=k⁺/P^⁺
- Use language of TMDs and GPDs! GAM Phys.Rev. C76 (2007) 065209





This corresponds to the TMD

$$h_{1T}^{\perp}$$

Transverse Momentum Distributions TMDs

$$\Phi^{[\Gamma]}(x, \mathrm{K}_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{iK\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\Gamma\,\mathcal{L}(0,\xi;n_-)\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

- $x=K^+/P^+$

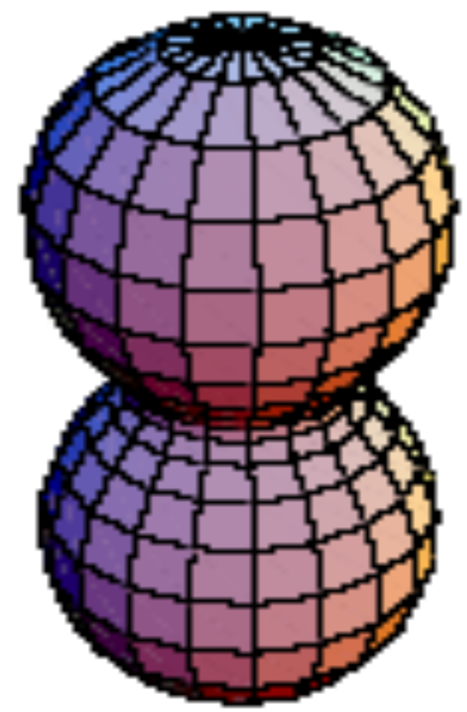
$\xi^+=t+z=0$

- Mulders & Tangerman ’96
- 11 possible choices of Γ
- one is like the SDD
- the one TMD is called $h_{1T}^{\perp}$

Summary of SDD

- **SDD are closely related to TMD's**
- **If $h_{1T}^{\perp}$ is not 0, proton is not round. Lattice calculations show not zero.**

Experiment should show the same



See GAM Nucl.Phys.News 18 (2008) 12-16
For further explanations

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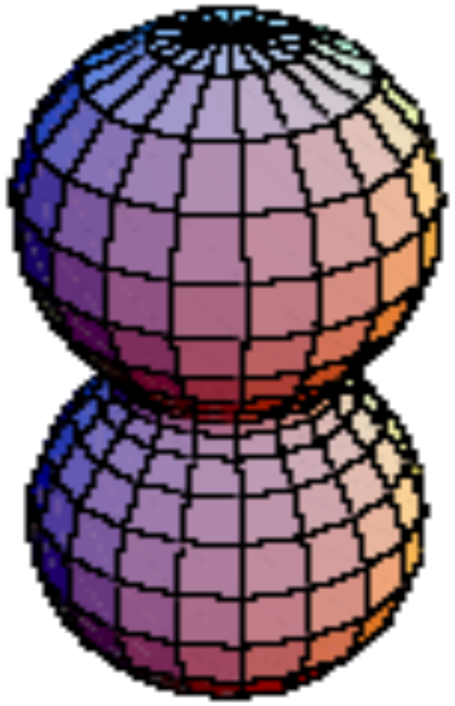
The Proton



See GAM Nucl.Phys.News 18 (2008) 12-16
For further explanations

Summary

- Model wave functions allow an understanding of phenomena and relations between different quantities, example: vector form factors-> axial vector form factors
- The charge density (and also magnetization density) are 2 dimensional Fourier- transforms of form factors
- The three-dimensional Fourier transform of G_E is NOT a density
- Proton is not round
- The upcoming form factor program at JLab is very exciting: a) measures fundamental properties of Nature b) Mechanism of form factors teach us about confinement c) pin down charge density of neutron

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p | s \rangle$$

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1+\boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$

$\mathbf{n} \parallel \hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2 \theta$

$\mathbf{n} \parallel -\hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2 \theta$

non-spherical shape depends on spin direction

Spares follow

Generalized densities

$$\mathcal{O}_q^\Gamma(px,\mathbf{b}) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger(0,\mathbf{b})\Gamma q_+(x^-,\mathbf{b})$$

$$\rho^\Gamma(b) = \int dx \sum_q e_q \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\mathcal{O}_q^\Gamma(p^+x,\mathbf{b})|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

$\int dx$ sets $x^- = 0$, get $q_+^\dagger(0,\mathbf{b})\Gamma q_+(0,\mathbf{b})$ **Density!**

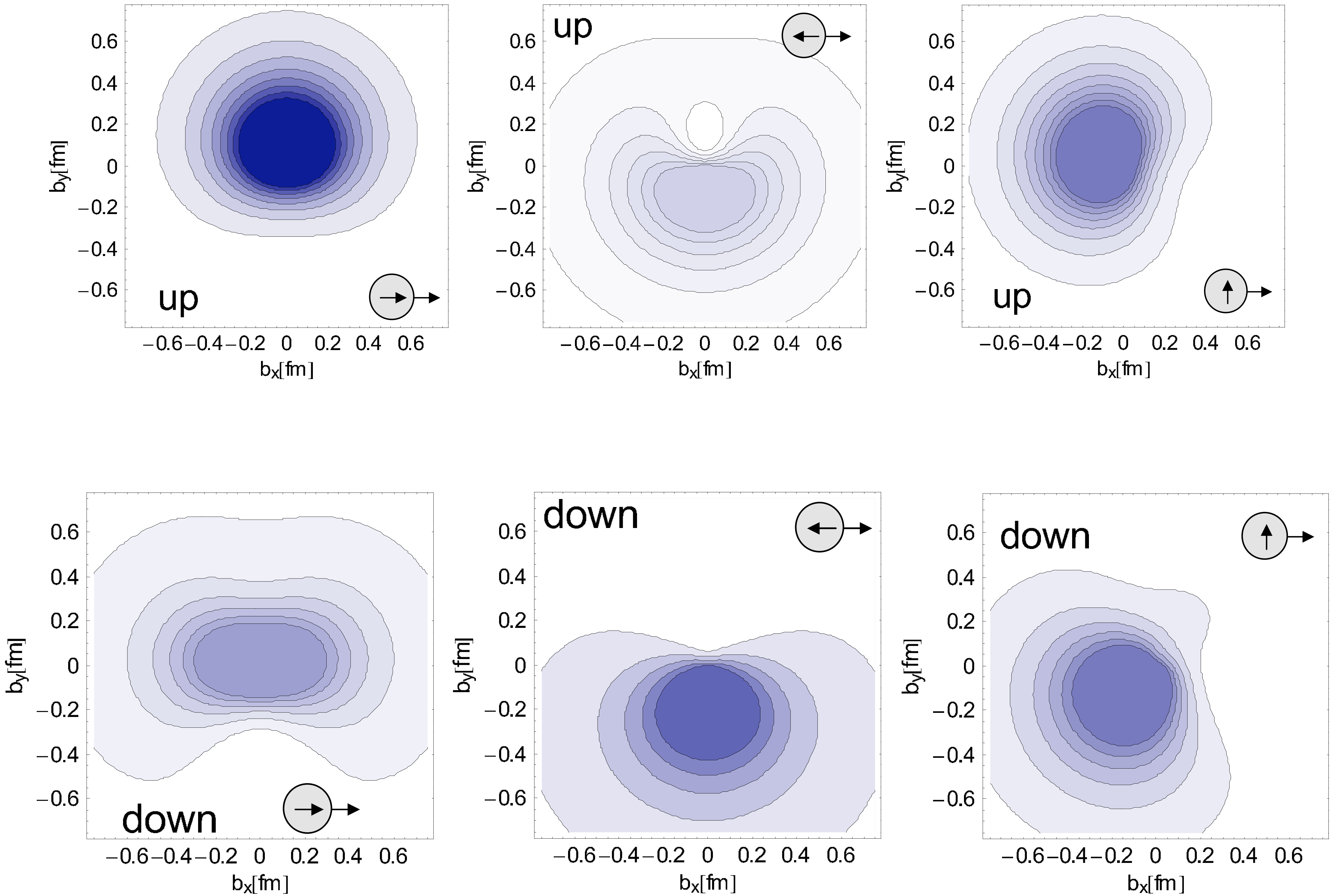
$\Gamma = 1/2(1+\mathbf{n}\cdot\gamma)$ gives spin-dep density

Local operators calculable on lattice M. Göckeler et al PRL98,222001 $\widetilde{A}''_{T10} \sim \mathrm{sdd}$ spin-dependent density

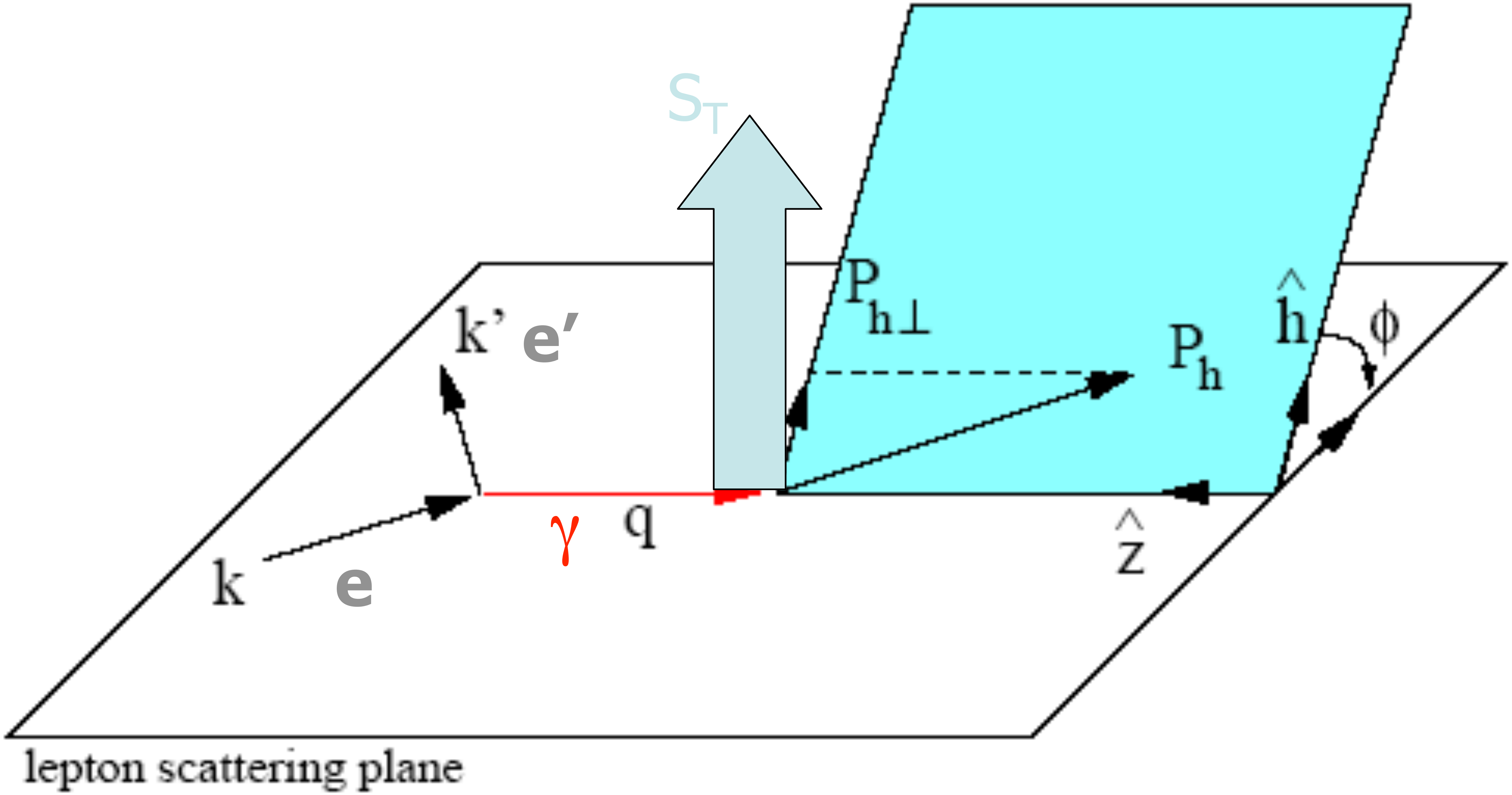
Schierholtz, 2009 -this quantity is not zero, proton is not round

Transverse spin dependent densities

darker is larger density QCDSF, UKQCD



Measure $h_{1T}^{\perp}$: $e, \uparrow p \to e' \pi X$

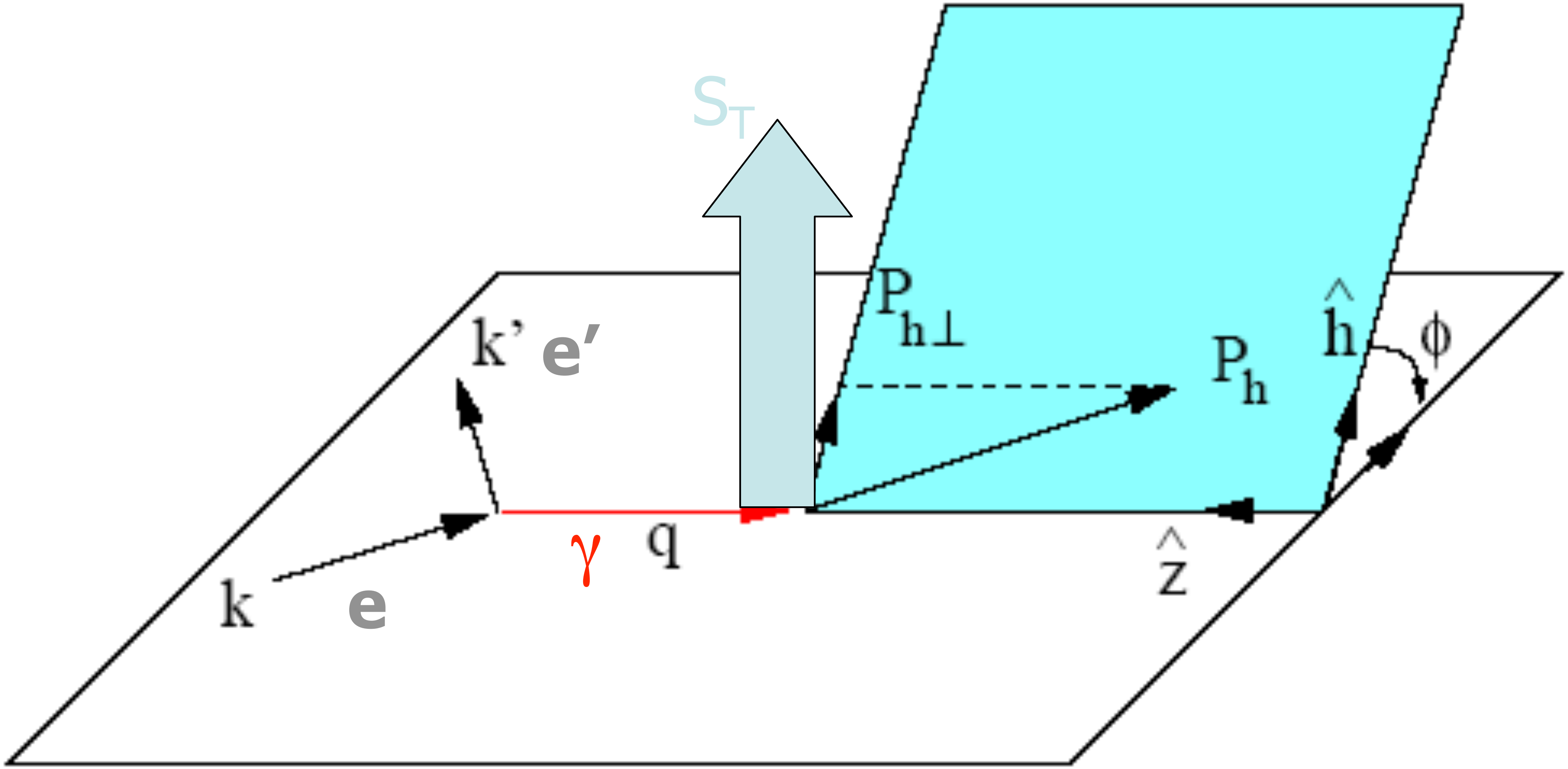


Cross section has term proportional to cos 3φ

Bo**er Mulders '98**

Measure $h_{1T}^{\perp}$: $e, \uparrow p \rightarrow e'\pi X$

H. Avakian, *et al.* "Transverse Polarization Effects in Hard Scattering at CLAS12 Jefferson Laboratory", LOI12-06-108, and H. Avakian private communication.



Cross section has term proportional to cos 3ϕ

Bo**er Mulders '98**

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

$$R^2 = -6 \frac{dG_E(Q^2)}{dQ^2}|_{Q^2=0}$$

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

interpretation of Sachs FF is wrong

-

Final wave function is boosted from initial

Need relativistic treatment

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

$$R^2 = -6\frac{dG_E(Q^2)}{dQ^2}|_{Q^2=0}$$

WRONG

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

interpretation of Sachs FF is wrong

Final wave function is boosted from initial

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Meaning of form factor

- $G_E(Q^2)$ is NOT Fourier transform of charge density
- Relativistic treatment needed- wave function is frame-dependent, initial and final states differ, no density
- Light front coordinates, ∞ momentum frame

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2},\ \text{“evolution”}\ \mathrm{p}^- = (\mathrm{p}^0-\mathrm{p}^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (x^0-x^3)/\sqrt{2},\ \text{“Momentum”}\ p^+ = (p^0+p^3)/\sqrt{2}$$

$$\text{“Transverse position, momentum, } \mathbf{b}, \mathbf{p}$$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs,TMDS

Generalized Coordinate Space Densities

$$\rho^{\Gamma}(\mathbf{b}) = \sum_q e_q \int dx^- q_+(x^-, \mathbf{b}) \gamma^+ \Gamma q_+(x^-, \mathbf{b})$$

$$\Gamma = \frac{1}{2}(1 + \mathbf{n} \cdot \boldsymbol{\gamma}) \text{ gives spin} - \text{dependent density}$$

PRL **98,** 222001 (2007) PHYSICAL REVIEW LETTERS week ending 1 JUNE 2007

Transverse Spin Structure of the Nucleon from Lattice-QCD Simulations

M. Göckeler,[1] Ph. Hägler,[2,*] R. Horsley,[3] Y. Nakamura,[4] D. Pleiter,[4] P. E. L. Rakow,[5] A. Schäfer,[1] G. Schierholz,[6,4] H. Stüben,[7] and J. M. Zanotti[3]

$$\begin{aligned}\rho^n &= \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) \\ &= \frac{1}{2}\Big\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \Big(A_{Tn0}(b_\perp^2) - \frac{\Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2)}{4m^2}\Big) \\ &\quad + \frac{b_\perp^j \epsilon^{ji}}{m}[S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2)] \\ &\quad + s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2)\Big\}, \qquad (1)\end{aligned}$$

spin-dependent density -depends on direction of **b**: proton is not round

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spin-dependent density
-depends on direction of
b: proton is not round

Model independent transverse charge density

$$J^+(x^-,\mathbf{b}) = \sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)$$ Charge Density

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

$$F_1 = \langle p^+,\mathbf{p}',\lambda|J^+(0)|p^+,\mathbf{p},\lambda\rangle$$

$$\rho(b) \equiv \int dx^-\rho_\infty(x^-,\mathbf{b}) = \int \frac{QdQ}{2\pi}F_1(Q^2)J_0(Qb)$$

Density is $u-\bar{u},\ d-\bar{d}$

"Spin Crisis"

- Proton Spin (total angular momentum) is ~1/2 $\hbar$
- Experiments show quarks carry only 30 %

- three ideas-

u,d quarks surrounded by $s \bar{s}$

gluons carry angular momentum

quarks carry orbital angular momentum

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gluons carry angular momentum

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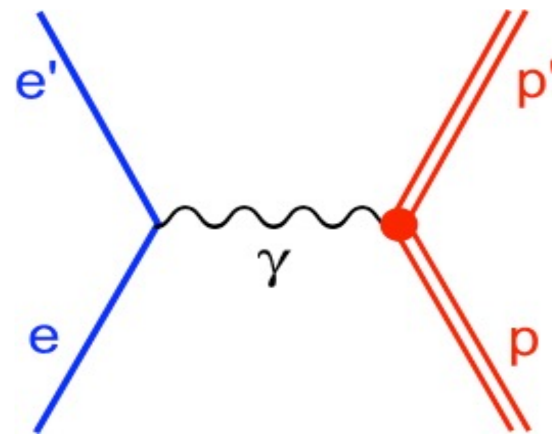
Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x,b)$
- From Drell-Yan-West relation between high x DIS and high Q^2 elastic scattering
- High x related to low b, not uncertainty principle
- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or π^-

$$\text{Density is } u-\bar{u},\ d-\bar{d}$$
$$\pi^- \text{ is } \bar{u}d$$
$$\text{decreases } u \text{ contribution}$$
$$\text{enhances } d \text{ contribution}$$

Electron-nucleon scattering

$j_\mu = \langle e' | \gamma_\mu | e \rangle$



$J_\mu = \langle p' | \Gamma_\mu | p \rangle$

Nucleon vertex:

$$\Gamma_\mu(p,p') = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M} F_2(Q^2)$$

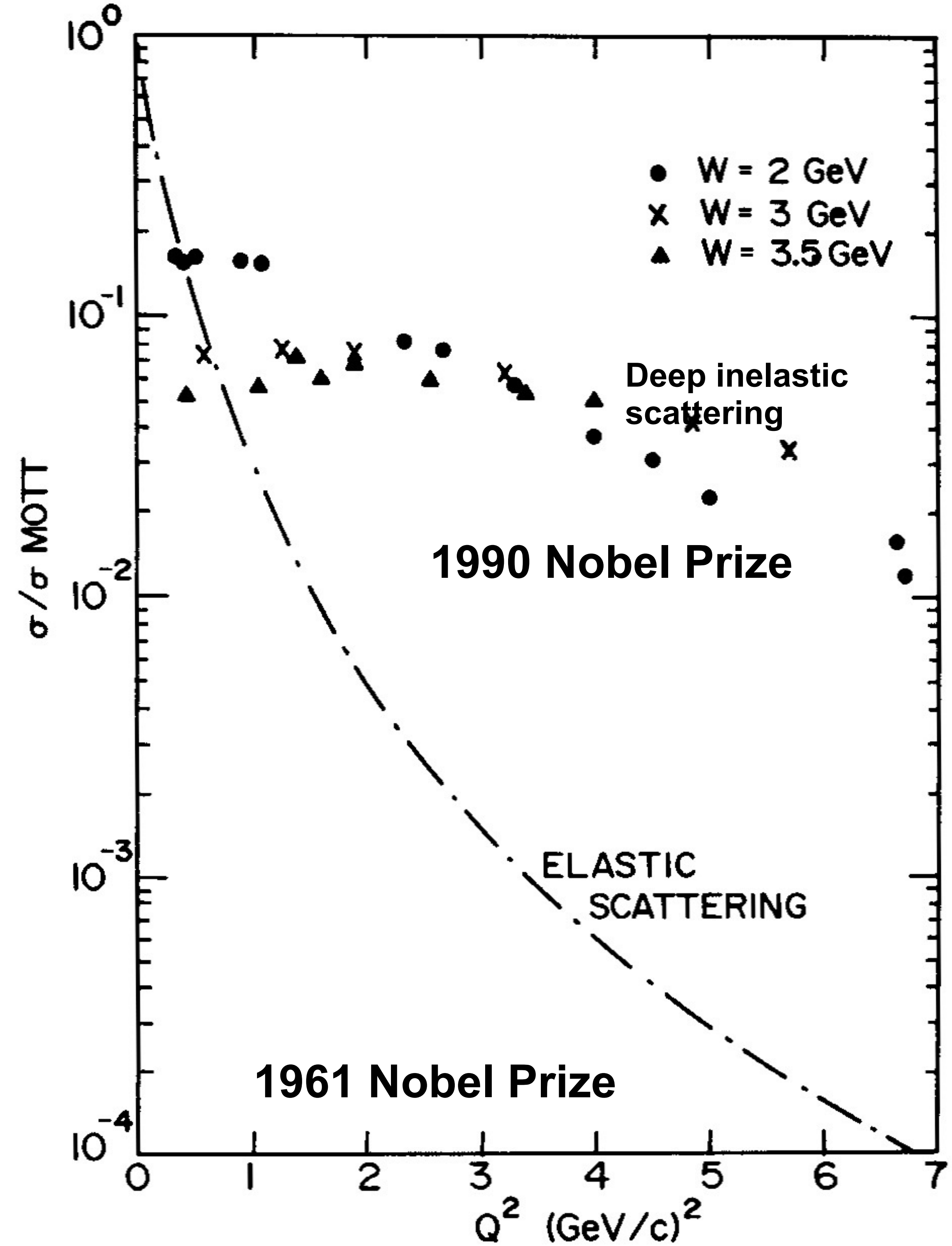
Dirac Pauli

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(G_E^2 + \frac{\tau}{\epsilon} G_M^2\right) / (1+\tau)$$

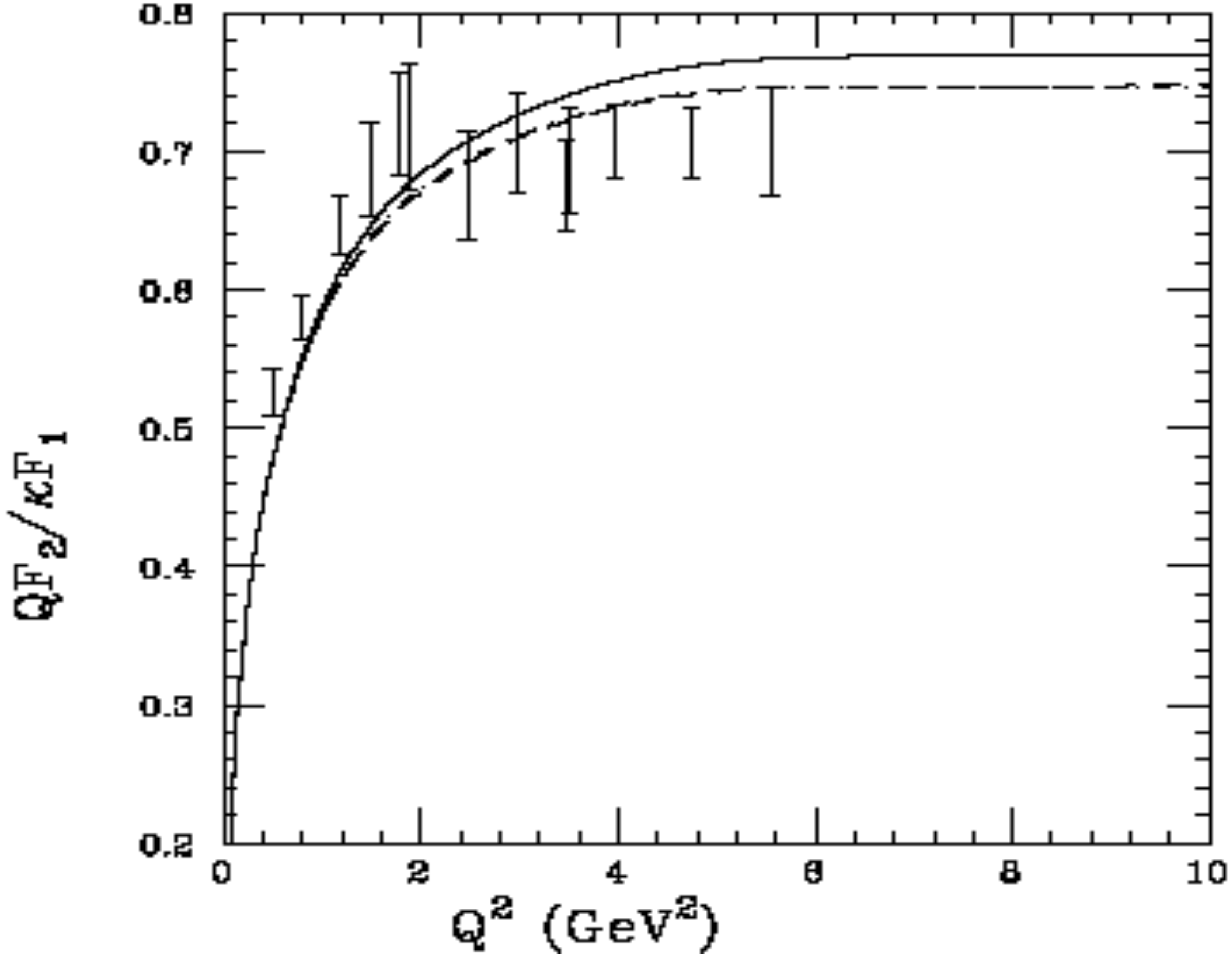
Cross section for scattering from a point-like object

Form factors describing nucleon shape/structure

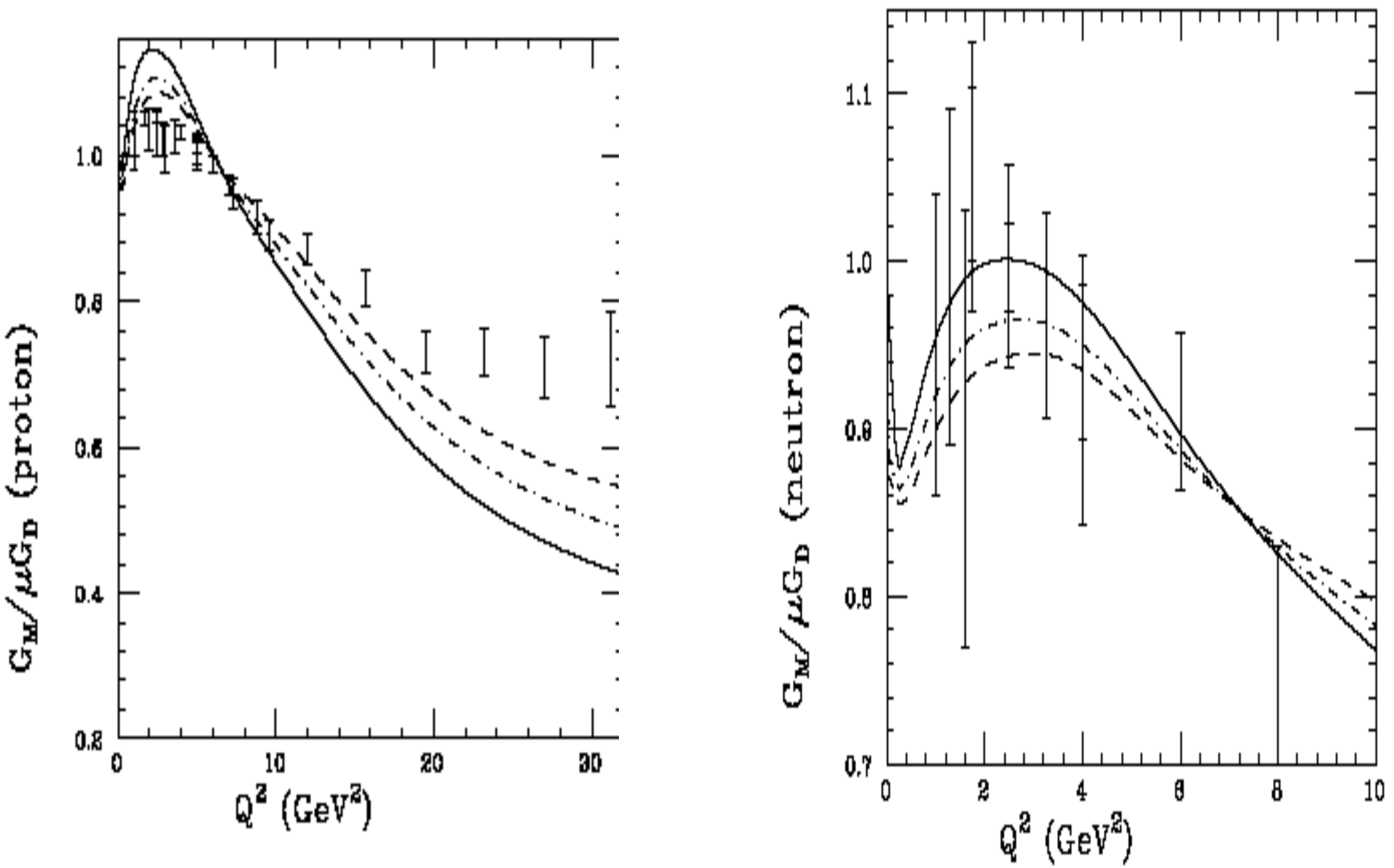
$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \; G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$



Ratio of Pauli to Dirac Form Factors 2003



Two More Form Factors Needed



Impact parameter dependent GPD Burkardt

Probability that quark at b from CTM has long momentum fraction x: $\rho(x,b)$

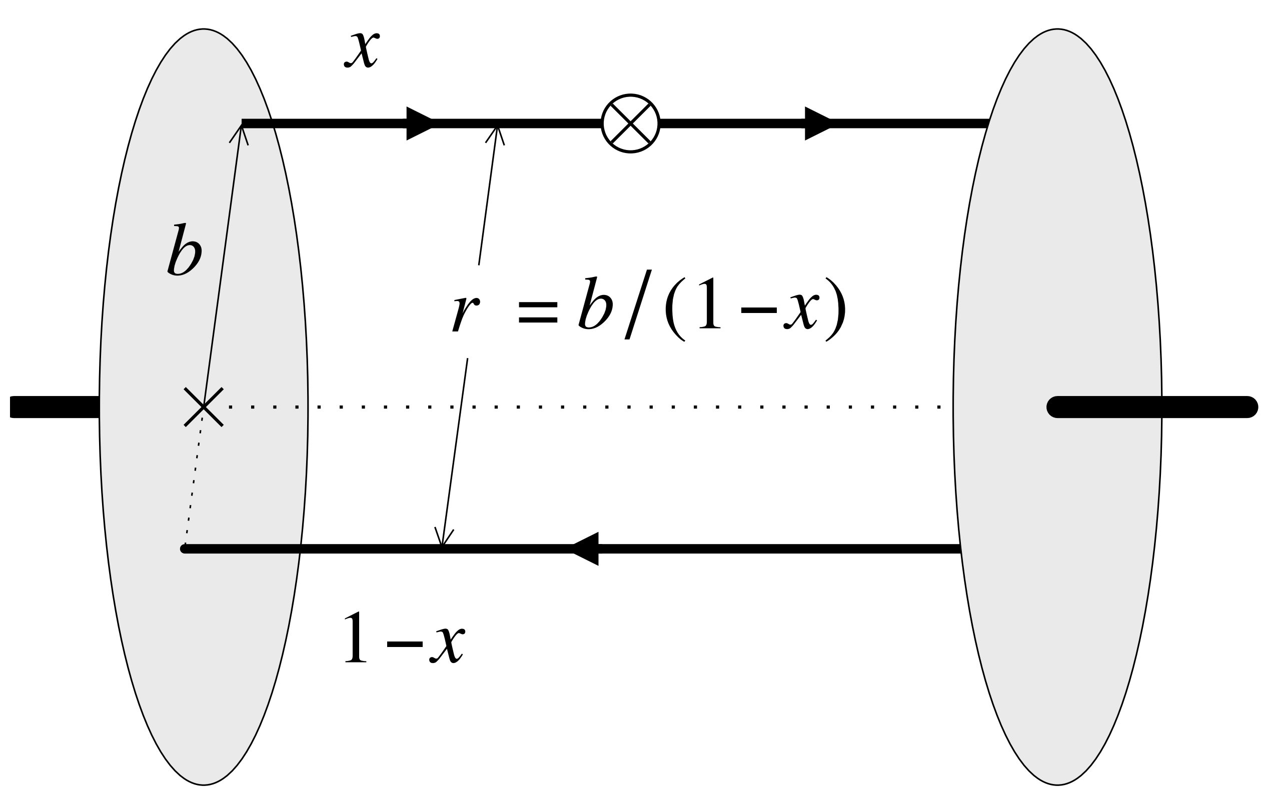
$$\rho(b) = \int dx\, \rho(x,b)$$

Transverse density is integral over longitudinal position or momenta
example of Parseval's theorem

$$\mathbf{R} = 0 = \sum_i^N x_i \mathbf{b}_i$$

Quark of x=1, must have b=0

x=1 is rare



What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

Fermi: n fluctuates to $p\pi^-$

p at center, pion floats to edge

One gluon exchange favors dud

Other ways to observe $h_{1T}^{\perp}$

Another interesting possibility occurs in the Drell-Yan reaction $pp(\uparrow) \to l\bar{l}X$, using one transversely polarized proton [23]. Here the term $h_{1T}^{\perp}$ causes a distinctive oscillatory dependence on the angle $3\phi - \phi_{S_1}$, where ϕ is the angle between the momentum of the outgoing lepton and the reaction plane in the lepton center of mass frame and ϕ_{S_1} denotes the direction of polarization with respect to the reaction plane.

[23] D. Boer, Phys. Rev. D **60**, 014012 (1999).