Single-Pion Photoproduction Contribution to Gerasimov – Drell – Hearn Sum Rule & Related Integrals

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IS, S. Sirca, W.J. Briscoe, A, Deur, R.L. Workman, Phys Rev C 105, 045202 (2022)

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- Sum Rules:
  - GDH.
  - Baldin.
  - *GGT*.
- Photoabsorption cross sections.
- Double-pol observable *E*.
- Differences of total cross sections.
- Running integrals.
  - *GDH*.
  - Baldin.
  - *GGT*.
- Future experiment(s).
- Summary.











### Sum Rules

- Study of *polarized lepton* scattering on *polarized nucleons* provides information on *spin composition of nucleon*.
- In *real-photon limit* of *lepton-nucleon* interaction, process can be characterized either
  - in terms of *integrals* of *cross-sections* with various weights,
  - in terms of *static nucleon properties* or resulting in "*Sum Rules*."
- Relevant quantities of interest are σ<sub>3/2</sub> & σ<sub>1/2</sub>, photon-nucleon total absorption cross-sections for *circularly polarized photons* on *longitudinally polarized nucleons*, with total helicity 3/2 & 1/2 from which either
  - <u>difference</u>,  $\Delta \sigma = \sigma_{3/2} \sigma_{1/2}$ , or
  - <u>sum</u>,  $\sigma_{tot} = \sigma_{3/2} + \sigma_{1/2}$ , can be constructed.
- Our Phys Ref C 105, 045202 (2022) gives *comprehensive review* of status of
  - Gerasimov-Drell-Hearn (GDH),
  - Baldin, &
  - Gell-Mann-Goldberger-Thirring (GGT) sum rules.







### GDH Sum Rule

- Gerasimov Drell Hearn (GDH) sum rule, formulated in 1960s, rests upon *fundamental physics principles* or *first principles* (Lorentz & gauge invariances, crossing symmetry, rotational invariance, causality, & unitarity) & unsubtracted *dispersion relations* applied to forward *Compton* amplitude.
- Because of its *fundamental* character, *GDH* sum rule requires *experimental verification* which has been awaiting technical developments that have only recently been attained.
- GDH sum rule relates proton (neutron) anomalous magnetic moment to integral of total photon-nucleon total absorption cross-sections for circularly polarized photons on longitudinally polarized nucleons, with total helicity 3/2 & 1/2,  $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$ , weighted by photon energy in lab frame,  $E_{\gamma}$ :

$$I_{\rm GDH} = \int_{E_{\gamma}^{\rm thr}}^{\infty} \frac{\Delta\sigma}{E_{\gamma}} \, dE_{\gamma} = \frac{2\pi^2\alpha}{M^2}\kappa^2$$

• For nucleon, all static quantities appearing in Eq. (right-hand) are known very precisely.

204.784482(35) µb for *proton* 232.25159(13) µb for *neutron* 



where  $\kappa_p = \mu_p - 1 \approx 1.793 \,\mu_N \,(\kappa_n = \mu_n \approx -1.913 \,\mu_N)$  is anomalous magnetic moment;  $\mu_N$  is nuclear magneton;

 $E_{\gamma}^{\text{thr}}$  is photon energy corresponding to pion photoproduction threshold;  $\alpha = e_0^2/4\pi$  is fine structure constant;

M is nucleon mass.

- S.B. Gerasimov, Sov J Nucl Phys 2, 430 (1966)
- S.D. Drell & A.C. Hearn, Phys Rev Lett 16, 908 (1966)
- L.I. Lapidus & Chou Kuang-chao [Zhou Guangzhao], Sov Phys JETP 14, 352 (1962)









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# Possible Causes of Violation of GDH

• *Gerasimov* – *Drell* – *Hearn* sum rule rests upon *first principles*:

*Lorentz* & gauge invariances, crossing symmetry, rotational invariance, causality, unitarity & unsubtracted *dispersion relations* applied to forward *Compton* amplitude.

- Possible causes for *GDH sum rule violation* or its apparent violation when integral is "measured" over finite  $E_{\gamma}$  range.
- Most often considered are (all proposed mechanisms would manifest themselves at high  $E_{\gamma}$ ):
  - Existence of unknown high-energy phenomena, such as quark substructure (non-zero quark anomalous moments)
  - Existence of **J** = 1 pole of nucleon *Compton* amplitude, &
  - Chiral anomaly.

 Weinberg stated that usefulness of GDH sum rule was really underappreciated & community should realize that.



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• Wealth of new data has been both informative & challenging for theory, & we look forward to further experimental & theoretical advances in our *quest* to understand *nucleon's spin structure* in realm of nonperturbative *QCD*.



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### Baldin Sum Rule

*Baldin* sum rule, formulated in 1960s, relates sum of *electric* α & *magnetic* β *polarizabilities* of *nucleon* to total *photo-absorption* cross section σ<sub>tot</sub> = σ<sub>3/2</sub> + σ<sub>1/2</sub>,

$$I_{\text{Baldin}} = \frac{1}{4\pi^2} \int_{E_{\gamma}^{\text{thr}}}^{\infty} \frac{\sigma_{\text{tot}}}{E_{\gamma}^2} \ dE_{\gamma} = \alpha + \beta$$

As pion photoproduction *PWA* does not allow for model independent separation of *electric & magnetic polarizabilities* of *nucleon*, we shall compare our results for *sum* of α & β, determined from *Compton* scattering:

$$\frac{\alpha + \beta \text{ is } (14.2 \pm 0.5) \times 10^{-4} \text{ fm}^3}{(15.5 \pm 1.6) \times 10^{-4} \text{ fm}^3} \text{ for } proton \text{ for } neutron \text{ for }$$

A.M. Baldin, Nucl Phys **18**, 310 (1960) L.I. Lapidus, Sov Phys JETP **16**, 964 (1963)

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# GGT Sum Rule

Third sum rule, introduced by *Gell-Mann–Goldberger–Thirring* (*GGT*) in 1950s, involves Δσ = σ<sub>3/2</sub> - σ<sub>1/2</sub> in integrand, & results in *forward spin polarizability*, γ<sub>0</sub>, of *nucleon*:

$$I_{\rm GGT} = -\frac{1}{4\pi^2} \int_{E_{\gamma}^{\rm thr}}^{\infty} \frac{\Delta\sigma}{E_{\gamma}^3} dE_{\gamma} = \gamma_0$$

M. Gell-Mann, M.L. Goldberger, & W.E. Thirring, Phys. Rev. **95**, 1612 (1954) M. Gell-Mann & M.L. Goldberger, Phys Rev **96**, 1433 (1954)





### Photoabsorption Cross-Section



• Sum rules considered in our Phys Ref C 105, 045202 (2022), call for both  $\Delta \sigma \& \sigma_{tot}$ , & it is important to note that @ energies far above *nucleon resonance region* 







### Photoabsorption Cross-Section





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### Photoabsorption Cross-Section

- Sum rules considered in our Phys Ref C 105, 045202 (2022), call for both  $\Delta\sigma \& \sigma_{tot}$ , & it is important to note that @ energies far above *nucleon resonance region* (not far beyond invariant masses of several GeV), unpolarized total photoabsorption cross-section,  $\sigma_{tot}$ , appears to rise indefinitely.
- In *Regge theory*, this rise can be explained in terms of processes involving *pomeron exchange*, but it results in non-convergent integral of  $\sigma_{tot}$ , which puts into question asymptotic behavior of  $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$  as well.







Circular Polarized PhotonBeamLongitudinally Polarized NucleonTarget



• Difference of cross-sections for helicity states 3/2 & 1/2, that is,  $\Delta(d\sigma/d\Omega) = (d\sigma_{3/2}/d\Omega - d\sigma_{1/2}/d\Omega)$  for  $\vec{\gamma}\vec{N} \to \pi N$ , is given in terms of helicity amplitudes:

$$\frac{d\sigma_{3/2}}{d\Omega} = \frac{q}{k} \left( |H_1|^2 + |H_3|^2 \right)$$
$$\frac{d\sigma_{1/2}}{d\Omega} = \frac{q}{k} \left( |H_2|^2 + |H_4|^2 \right)$$

$$E = \frac{|H_2|^2 + |H_4|^2 - |H_1|^2 - |H_3|^2}{|H_2|^2 + |H_4|^2 + |H_1|^2 + |H_3|^2}$$









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$$E = \frac{|H_2|^2 + |H_4|^2 - |H_1|^2 - |H_3|^2}{|H_2|^2 + |H_4|^2 + |H_1|^2 + |H_3|^2}$$





#### <mark>715 – 1450</mark> MeV



Circular Polarized Photon Beam Longitudinally Polarized Nucleon Target



• Difference of cross-sections for helicity states 3/2 & 1/2, that is,  $\Delta(d\sigma/d\Omega) = (d\sigma_{3/2}/d\Omega - d\sigma_{1/2}/d\Omega)$  for  $\vec{\gamma}\vec{N} \to \pi N$ , is given in terms of helicity amplitudes:

$$\frac{d\sigma_{3/2}}{d\Omega} = \frac{q}{k} \left( |H_1|^2 + |H_3|^2 \right)$$
$$\frac{d\sigma_{1/2}}{d\Omega} = \frac{q}{k} \left( |H_2|^2 + |H_4|^2 \right)$$



where *q* & *k* are *pion* & *photon* c.m. momenta. Their sum & difference can then be used to construct *beam-target* polarization quantity *E*:

 $E = \frac{|H_2|^2 + |H_4|^2 - |H_1|^2 - |H_3|^2}{|H_2|^2 + |H_4|^2 + |H_1|^2 + |H_3|^2}$ 

This solution uses E DB in fit.

Image: Constraint of the solution of the solu



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### Differences of Total Cross Section for Helicity States 3/2 & 1/2



$$\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$$

- In SAID & MAID PWAs, second (& η-threshold) as well as third resonance regions around 700 & 1000 MeV are very pronounced for charged pions in final state, whereas they are weak in case of neutral pions.
- In ∆-*isobar* region, predictions for both cases are the same.
- Around η-threshold, deviation becomes more apparent.





### GDH Running Integral







# **GDH Running** Integral



• How we know *anomalous magnetic moment* of nucleon:

- *Penning trap* is device for storage of *charged* particles using axial magnetic field & quadrupole electric field.
- Measuring *magnetic moment* of <sup>11</sup>Be & detecting *nuclear magnetic resonance* signals in beryllium crystal lattice.



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### **GDH Running** Integral





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# GDH Running Integral with Regge



### **Baldin Running** Integral



 $\sigma_{\rm tot} = \sigma_{3/2} + \sigma_{1/2}$ 







# **Baldin Running** Integral





VALUE $(10^{-4} \text{ fm}^3)$	DOCUMENT ID	TECN
11.8 ± 1.1 OUR AVERAGE		

#### • *n* MAGNETIC POLARIZABILITY $\beta_n$

<u>VALUE (10<sup>-4</sup> fm<sup>3</sup>)</u> 3.7 ±1.2 OUR AVERAGE DOCUMENT ID \_\_\_\_\_ TECN \_\_\_\_\_ COMMENT

COMMENT







# **Baldin Running** Integral











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# Future Experiment(s)

• New determination of *Regge* theory parameters by extending  $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$  measurements both on *proton* & *neutron* (*deuteron*) targets to  $E_{\nu} \approx 12$  GeV is one of aims of recently approved REGGE *Experiment* in Hall D of Jefferson Lab.

> M.M. Dalton, A. Deur, C.D. Keith, S. Sirca, & J. Stevens, *Measurement of the high-energy contribution to the Gerasimov-Drell-Hearn sum rule*, JLab Proposal **E12-20-011** [arXiv:2008.11059 [nucl-ex]].

### S&T visit: Hall D plans - summary

#### Scheduling Outlook

2027 2021 2022 2023 2024 2025 2026 2028 2029 2030 Activity, experiment scheduled running Run PRIMEX-ŋ Run SRC Installation CPP Run CPP-NPP Run GlueX-II Installation FCAL2 Run GlueX-II+JEF Installation KLF (K, beam) Commissioning, Run KLF Back to photon beam Installation of REGGE 33 PAC days Commissioning, Run REGGE Assumed 25 weeks/year for Hall D running · Assumed timely construction of Assumed timely budgeting for KLF and REGGE JEF, KLF, GDH Jefferson Lab 13 GlueX Meeting, 2022 Feb Hall D Status Update 12/12



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• Evaluation of *GDH*, *Baldin*, & *GGT sum rules* involving difference of helicity dependent total photoabsorption cross-sections,  $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$ , shows that *single-pion* photoproduction off nucleon is dominant contribution to these *sum rules*.

• In all cases, single-pion contribution converges above  $E_{\gamma} \sim 1.7$  GeV.

- In *GDH* sum rule for *proton*, *single-pion* contribution saturates *sum rule* to 90%, while in *neutron* case, missing strength amounts to 44%.
- Situation is most favorable for *Baldin* sum rule where to attainable levels of precision *single-pion* photoproduction off nucleon comes closest to agreeing with *Compton* scattering & *EFT* calculations for *neutron* case.
- In case of the *GGT* sum rule, lack of precise calculations & size of experimental uncertainties preclude clear statement of agreement.
- Direct experimental *verification* of *GDH* sum rule are difficult because of need to extend measurements to sufficiently high *E<sub>γ</sub>* despite 1/*E<sub>γ</sub>* weighting, & to reliably cover not only *single-* & *double*-pion photoproduction but all possible *photon-induced* processes.

THANKS



UMMARY

