Nearly model-independent constraints on dense matter equation of state in a Bayesian approach

N. K. Patra, Sk Md Adil Imam, B. K. Agrawal, Arunava Mukharjee and Tuhin Malik

Based on:- Phys.Rev.D **106**, 043024 (2022) [arXiv:2203.08521]

Department of Physics, BITS Pilani Goa Campus

September 5, 2022

Out Lines

Some bounds on neutron star structure

- For the first time National Radio Astronomy Observatory (NRAO) detected a massive neutron star of mass $2.01 \pm 0.04\ M_\odot$ and the radius of 13 ± 2 km from PSR J0348+0432 in 2007.

  J. Antoniadis et al., Science 340, 6131 (2013)

- Recently in 2019, Neutron Star Interior Composition Explorer (NICER) detected a most massive neutron stars of mass $2.072^{+0.067}_{-0.066} M_\odot$ and radius of $12.39^{+1.30}_{-0.98} km$ from PSR J0740+6620.

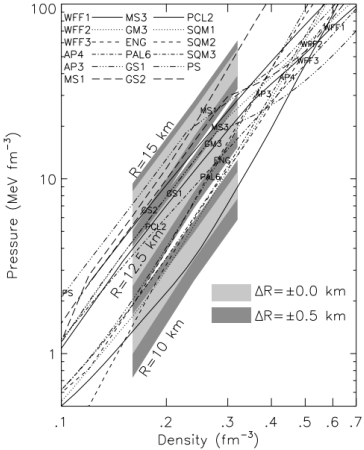
  T. E. Riley et al., Astrophys. J. Lett. 918, L27 (2021)

Some important aspects of neutron star structure

- The neutron star radius is primarily determined by the behavior of the pressure of matter in the vicinity of nuclear matter equilibrium density.

$$\boxed{R_M = C(n, M)[P(n)]^{0.23-0.26}} \qquad (1)$$

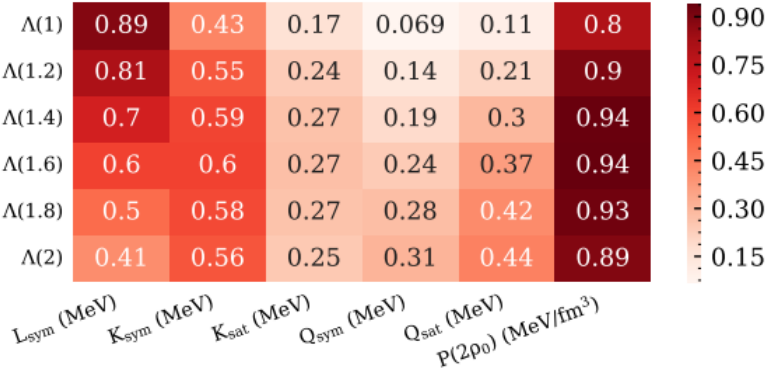
  - P(n) is the total pressure
  - C(n,M) is a number that depends on the density n at which the pressure was evaluated and the stellar mass M.
  - The correlations seen tighter in case of baryon density n = 1.5 n_s and 2 n_s.



J. M. Lattimer & **M. Prakash**, Astrophys.J. 550,426 (2001)

Some important aspects of neutron star structure

- The nuclear equation of state (EOS) is very sensitivity to the neutron star tidal deformability (Λ) on 1 to 2 solar masses neutron stars.

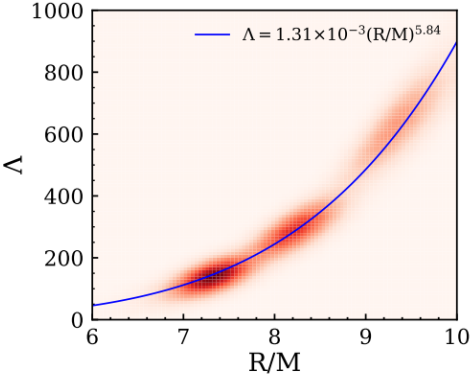


C.Y. Tsang et al., Phys.Rev.C 102, 045808 (2020)

Some important aspects of neutron star structure

- A global power law dependence between tidal deformability and compactness parameter (M/R) is verified over the mass region 1 to 2 solar masses neutron stars.

$$\Lambda = 1.31 \times 10^{-3} (\frac{R}{M})^{5.84} \qquad (2)$$



C.Y. Tsang et al., Phys.Rev.C 102, 045808 (2020)

Motivations

- Find out the correlations of various NS properties with the key parameter of EOS in a model-independent manner.

Objectives

- To construct large number of minimally constrained equations of state (EOSs) and study their correlations with a few selected properties of a neutron star.
- To see the nearly model-independent manner above the saturation density on the correlations of NS properties and the pressure of β-equilibrated matter.
- Formulate a parametrize form of the pressure for β-equilibrated matter, around $2\rho_0$, as a function of neutron star mass and the corresponding tidal deformability.

Equation of States

- We consider n,p,e,μ in the core part of neutron star.
- The three conditions such as,
  - Conservation of baryon no.:- $\rho = \rho_n + \rho_p$
  - Charge neutrality :- $\rho_p = \rho_e + \rho_\mu$
  - Beta-equilibrium :- $\mu_n = \mu_p + \mu_e$
- Using these conditions, we calculate proton fraction and isospin asymmetry parameter ($\delta = \frac{\rho_n - \rho_p}{\rho}$).
- The energy per nucleon for neutron star matter $E(\rho, \delta)$ a given total nucleon density ρ decomposed as,

$$E(\rho, \delta) = E(\rho, 0) + E_{\rm sym}(\rho)\delta^2 + ..., \quad (3)$$

- where, $E(\rho, 0)$ is for symmetric nuclear matter and $E_{\rm sym}(\rho)$ for density-dependent symmetry energy.

Models

- We have taken two different models such as Taylor and $\frac{n}{3}$ which are based on expansions of $E(\rho, 0)$ and $E_{\mathrm{sym}}(\rho)$.

Models

- We have taken two different models such as Taylor and $\frac{n}{3}$ which are based on expansions of $E(\rho, 0)$ and $E_{\mathrm{sym}}(\rho)$.

Taylor's expansion

$$E(\rho,0) = \sum_{n=0}^{4} \frac{a_n}{n!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (4)$$

$$E_{\mathrm{sym}}(\rho) = \sum_{n=0}^{4} \frac{b_n}{n!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (5)$$

$$E(\rho,\delta) = \sum_{n=0}^{4} \frac{1}{n!}(a_n + b_n\delta^2)\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (6)$$

Models

- We have taken two different models such as Taylor and $\frac{n}{3}$ which are based on expansions of $E(\rho,0)$ and $E_{\rm sym}(\rho)$.

Taylor's expansion

$$E(\rho,0) = \sum_{n=0}^{4} \frac{a_n}{n!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (4)$$

$$E_{\rm sym}(\rho) = \sum_{n=0}^{4} \frac{b_n}{n!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (5)$$

$$E(\rho,\delta) = \sum_{n=0}^{4} \frac{1}{n!}(a_n + b_n\delta^2)\left(\frac{\rho-\rho_0}{3\rho_0}\right)^n, \quad (6)$$

$\frac{n}{3}$ expansion

$$E(\rho,0) = \sum_{n=2}^{6}(a'_{n-2})\left(\frac{\rho}{\rho_0}\right)^{\frac{n}{3}}, \quad (7)$$

$$E_{\rm sym}(\rho) = \sum_{n=2}^{6}(b'_{n-2})\left(\frac{\rho}{\rho_0}\right)^{\frac{n}{3}}, \quad (8)$$

$$E(\rho,\delta) = \sum_{n=2}^{6}(a'_{n-2} + b'_{n-2}\delta^2)\left(\frac{\rho}{\rho_0}\right)^{\frac{n}{3}}. \quad (9)$$

- a_n, $a'_n = \epsilon_0, 0, K_0, Q_0, Z_0$
- b_n, $b'_n = J_0, L_0, K_{\rm sym,0}, Q_{\rm sym,0}, Z_{\rm sym,0}$

Bayesian estimation

- This approach is mainly based on the Bayes theorem which states that,

$$P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}, \tag{10}$$

  - $\boldsymbol{\theta}$ is model parameters.
  - D is the data.
  - $\mathcal{Z}$ is the evidence.
  - $P(\boldsymbol{\theta})$ is the prior for the model parameters.
  - The likelihood function,

$$\mathcal{L}(D|\boldsymbol{\theta}) = \prod_j \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{1}{2}\left(\frac{d_j - m_j(\boldsymbol{\theta})}{\sigma_j}\right)^2}. \tag{11}$$

  - The marginalized posterior distribution for a parameter θ_i can be obtained as,

$$P(\theta_i|D) = \int P(\boldsymbol{\theta}|D) \prod_{k \neq i} d\theta_k. \tag{12}$$

Construction of EOSs

- For construction large sets of EOSs, we use Bayesian approach. It needs,
  - Data(D)
  - Model (M($\boldsymbol{\theta}$))
  - Prior (P($\boldsymbol{\theta}$))
- We have considered the data as energy per neutron of PNM from Ref. **K. Hebeler et al.** Astrophys. J. 773, 11 (2013).

Construction of EOSs

- For construction large sets of EOSs, we use Bayesian approach. It needs,
  - Data(D)
  - Model (M($\boldsymbol{\theta}$))
  - Prior (P($\boldsymbol{\theta}$))
- We have considered the data as energy per neutron of PNM from Ref. **K. Hebeler et al.** Astrophys. J. 773, 11 (2013).

- The prior distributions of nuclear matter parameters given in Table as,

NMPs (in MeV)	Pr-Dist	μ	σ
ε_0	G	-16	0.3
K_0	G	240	50
Q_0	G	-400	400
Z_0	G	1500	1500
J_0	G	32	5
L_0	G	50	50
$K_{\mathrm{sym},0}$	G	-100	200
$Q_{\mathrm{sym},0}$	G	550	400
$Z_{\mathrm{sym},0}$	G	-2000	2000

Construction of EOSs

- The nuclear matter parameters are filtered by demanding,
  - pressure for the β-equilibrated matter should increase monotonically with density and symmetry energy should not be negative (thermodynamic stability).
  - speed of sound must not exceed the speed of light (causality).
  - maximum mass of neutron star must exceeds $2M_{\odot}$ (observational constraint).
- The causality breaks down at higher density mostly for the Taylor EOS. In such cases, we use the stiffest EOS, $P(\epsilon) = P_m + (\epsilon - \epsilon_m)$.
- Once the EOS for the core and crust are known the values of NS mass, radius and tidal deformability corresponding to given central pressure can be obtained by solving Tolman-Oppenheimer-Volkoff equations.

Posterior Distribution of NM Parameters

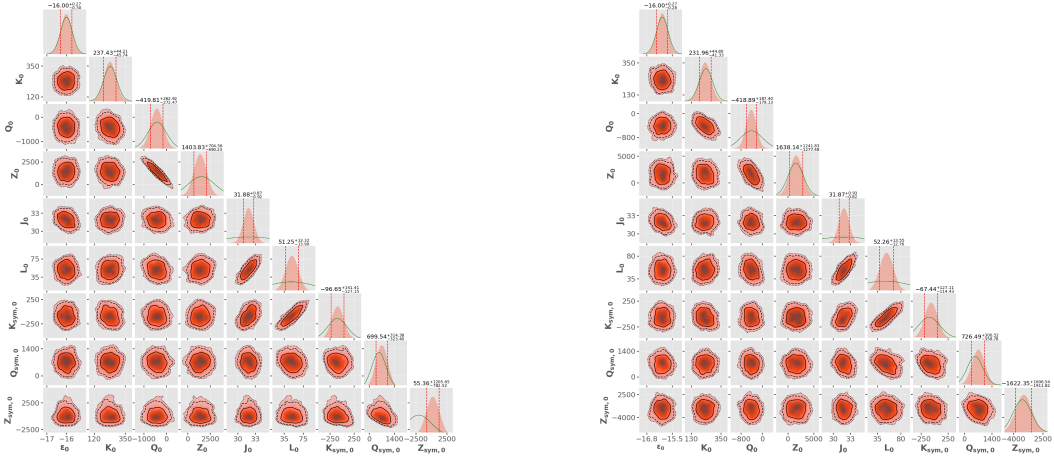


Fig.1: Corner plots for the nuclear matter parameters (in MeV) obtained for Taylor (left) and $\frac{n}{3}$ expansions (right). The one dimensional marginalized posterior distributions (salmon) and the prior distributions (green lines) are displayed along the diagonal plots . The vertical lines indicate 68% confidence interval of nuclear matter parameters.

Posterior Distribution of NM Parameters

NMPs (in MeV)	without PNM		with PNM	
	Taylor	$\frac{n}{3}$	Taylor	$\frac{n}{3}$
ε_0	$-16.02^{+0.23(0.41)}_{-0.28(0.56)}$	$-15.99^{+0.27(0.43)}_{-0.27(0.51)}$	$-16.00^{+0.27(0.42)}_{-0.30(0.54)}$	$-16.00^{+0.27(0.44)}_{-0.28(0.56)}$
K_0	$236.42^{+42.78(74.34)}_{-42.58(79.62)}$	$233.38^{+48.94(76.14)}_{-42.73(83.95)}$	$237.43^{+44.24(72.25)}_{-45.75(83.22)}$	$231.96^{+44.80(72.94)}_{-41.33(76.63)}$
Q_0	$-436.23^{+273.36(419.17)}_{-306.50(603.76)}$	$-411.84^{+207.53(301.56)}_{-210.88(409.00)}$	$-419.81^{+262.95(437.69)}_{-272.47(531.58)}$	$-418.89^{+187.43(300.76)}_{-179.25(377.42)}$
Z_0	$1441.51^{+792.45(1298.64)}_{-696.39(1381.30)}$	$1600.07^{+1067.33(1883.00)}_{-1362.28(2615.10)}$	$1403.84^{+704.56(1133.85)}_{-690.82(1386.25)}$	$1638.14^{+1241.83(1906.75)}_{-1277.48(2244.23)}$
J_0	$32.37^{+4.08(6.79)}_{-4.26(8.83)}$	$32.37^{+4.69(7.22)}_{-4.71(10.23)}$	$31.88^{+0.87(1.43)}_{-0.92(-1.85)}$	$31.87^{+0.93(1.49)}_{-0.82(1.68)}$
L_0	$59.88^{+41.14(65.90)}_{-39.84(78.17)}$	$55.60^{+37.59(63.89)}_{-43.88(84.62)}$	$51.25^{+13.32(21.60)}_{-13.91(25.54)}$	$52.25^{+13.55(22.73)}_{-12.76(23.04)}$
$K_{\mathrm{sym},0}$	$-85.86^{+192.67(327.83)}_{-151.57(266.76)}$	$-40.03^{+161.60(271.89)}_{-135.08(234.67)}$	$-96.65^{+141.41(225.69)}_{-127.49(216.74)}$	$-67.44^{+127.18(206.09)}_{-114.80(200.38)}$
$Q_{\mathrm{sym},0}$	$731.13^{+308.54(543.01)}_{-347.82(669.47)}$	$705.36^{+311.23(511.39)}_{-352.72(727.86)}$	$699.56^{+324.38(521.95)}_{-323.52(639.30)}$	$726.49^{+300.40(510.33)}_{-358.51(631.86)}$
$Z_{\mathrm{sym},0}$	$-2.07^{+1190.67(2153.84)}_{-820.92(1473.09)}$	$-1390.39^{+1518.69(2526.53)}_{-1856.18(3623.74)}$	$55.34^{+1205.62(2255.28)}_{-782.52(1415.84)}$	$-1622.35^{+1606.61(2788.70)}_{-1911.81(3468.40)}$

Posterior Distribution of NS Properties

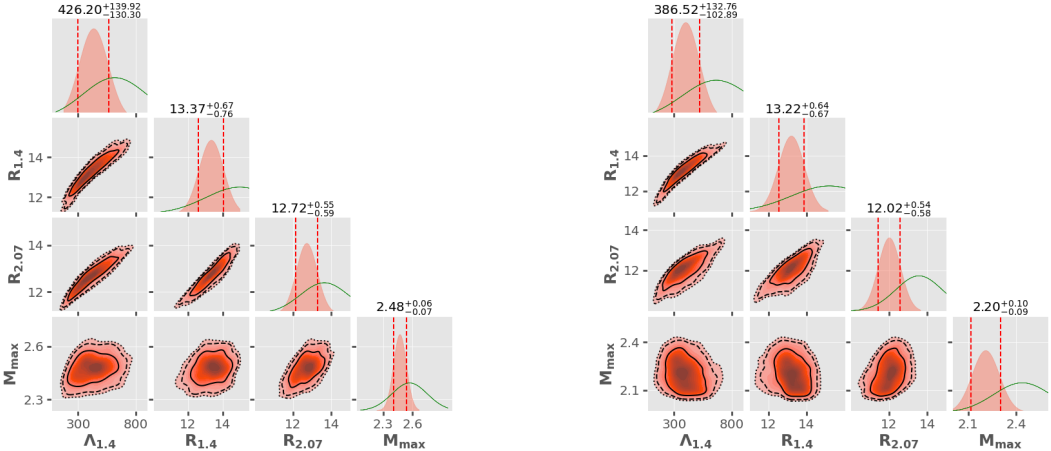


Fig.2: Corner plots for the marginalized posterior distributions (salmon) of the tidal deformability $\Lambda_{1.4}$, radii $R_{1.4}$ (km) and $R_{2.07}$ (km) and the maximum mass $M_{\rm max}$ ($M_\odot$) for Taylor (left) and $\frac{n}{3}$ (right) expansions. The green lines represent effective priors obtained using the priors for nuclear matter parameters.

Joint Probability Distributions of M,R

NS properties	without PNM		with PNM	
	Taylor	$\frac{n}{3}$	Taylor	$\frac{n}{3}$
$\Lambda_{1.4}$	$527.72^{+250.72(477.68)}_{-186.11(292.57)}$	$455.85^{+223.65(465.72)}_{-163.05(243.23)}$	$426.20^{+139.93(224.58)}_{-130.32(205.18)}$	$386.52^{+132.76(213.24)}_{-102.84(199.09)}$
$R_{1.4}$(km)	$14.69^{+1.78(3.43)}_{-1.63(2.74)}$	$14.15^{+1.87(3.34)}_{-1.69(2.58)}$	$13.37^{+0.67(1.03)}_{-0.75(1.60)}$	$13.22^{+0.64(0.99)}_{-0.67(1.59)}$
$R_{2.07}$(km)	$13.24^{+0.82(1.49)}_{-0.82(1.42)}$	$12.27^{+0.88(1.52)}_{-0.80(1.52)}$	$12.72^{+0.55(0.85)}_{-0.59(1.07)}$	$12.02^{+0.54(0.88)}_{-0.58(1.23)}$
$M_{\max}(M_\odot)$	$2.45^{+0.07(0.11)}_{-0.06(0.13)}$	$2.19^{+0.10(0.19)}_{-0.09(0.09)}$	$2.48^{+0.06(0.10)}_{-0.07(0.14)}$	$2.20^{+0.10(0.16)}_{-0.09(0.11)}$

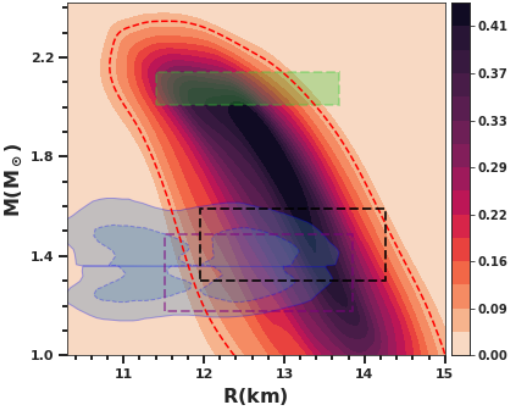


Fig.3: Plot for joint probability distribution $P(M, R)$ as a function of mass and radius of neutron star obtained for $\frac{n}{3}$ expansion. The red dashed line represents the 90% confidence interval

Correlations of NS properties with EOS

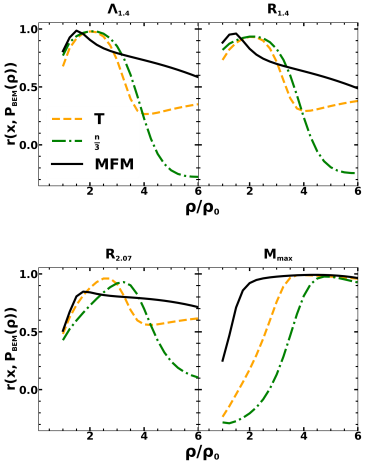


Fig.4: The correlation coefficients r(x,$P_{\mathrm{BEM}}(\rho)$) of both Taylor and $\frac{n}{3}$ expansions along with the mean-field theory calculations are shown.

Correlations of NS properties with EOS

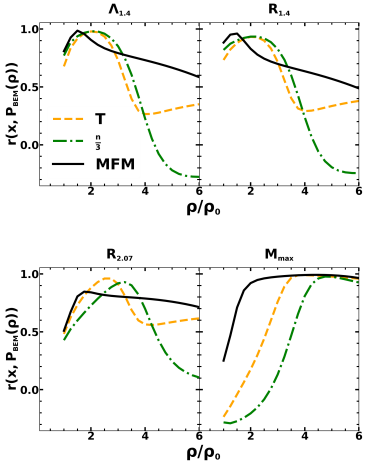


Fig.4: The correlation coefficients r(x,$P_{\rm BEM}(\rho)$) of both Taylor and $\frac{n}{3}$ expansions along with the mean-field theory calculations are shown.

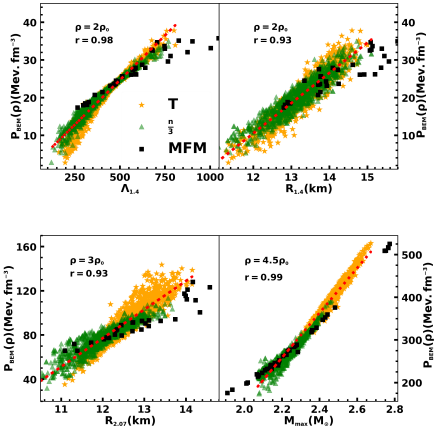


Fig.5: The variations of pressure for β-equilibrated matter ($P_{\rm BEM}(\rho)$) at selected densities versus NS properties.

Correlations of Tidal deformability with Pressure

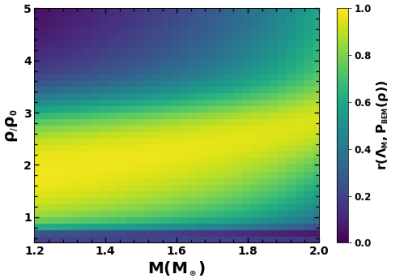


Fig.6: Here is the plot of correlation coefficients between tidal deformability (Λ_M) and the pressure of β-equilibrated matter ($P_{\rm BEM}(\rho)$) on neutron star mass (M) and density (ρ)

Correlations of Tidal deformability with Pressure

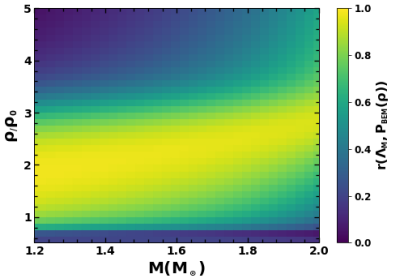


Fig.6: Here is the plot of correlation coefficients between tidal deformability (Λ_M) and the pressure of β-equilibrated matter ($P_{\rm BEM}(\rho)$) on neutron star mass (M) and density (ρ)

- The $P_{\rm BEM}(\rho)$ at $\rho \sim 1.5 - 2.5\rho_0$ are strongly correlated ($r \sim 0.8 - 1$) with tidal deformability for NS masses in the range $1.2 - 2.0 M_\odot$.

Correlations of Tidal deformability with Pressure

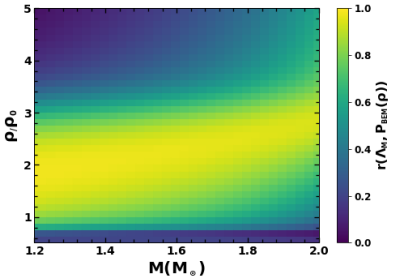


Fig.6: Here is the plot of correlation coefficients between tidal deformability (Λ_M) and the pressure of β-equilibrated matter ($P_{\rm BEM}(\rho)$) on neutron star mass (M) and density (ρ)

- The $P_{\rm BEM}(\rho)$ at $\rho \sim 1.5 - 2.5\rho_0$ are strongly correlated ($r \sim 0.8 - 1$) with tidal deformability for NS masses in the range $1.2 - 2.0 M_\odot$.

Hence, $P_{\rm BEM}(\rho)$ can be parametrized at a given ρ as,

$$\frac{P_{\rm BEM}(\rho)}{\rm MeVfm^{-3}} = a(M) + b(M)\Lambda_M, \quad (13)$$

The mass-dependent coefficients $a(M)$ and $b(M)$ expanded as

$$a(M) = (a_0 + a_1(M - M_0) + a_2(M - M_0)^2), \quad (14)$$
$$b(M) = (b_0 + b_1(M - M_0) + b_2(M - M_0)^2), \quad (15)$$

where M_0 is taken to be $1.4 M_\odot$.

- The values of a_i and b_i are estimated using a Bayesian approach with the help of $P_{\rm BEM}(\rho)$ and tidal deformability obtained for Taylor and $\frac{n}{3}$ expansions.

Correlations of Tidal deformability with Pressure

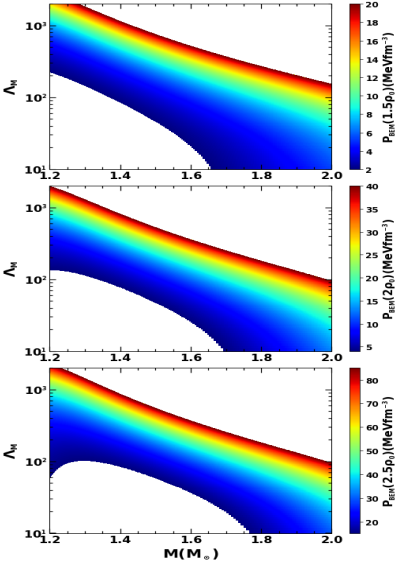
- The priors for a_i and b_i are taken to be uniform in the range of -100 to 100.
- The calculations are performed for ρ= 1.5, 2.0 and 2.5 ρ_0.

Correlations of Tidal deformability with Pressure

- The priors for a_i and b_i are taken to be uniform in the range of -100 to 100.
- The calculations are performed for ρ= 1.5, 2.0 and 2.5 ρ_0.
- The average deviation of $P_{\rm BEM}(2\rho_0)$, obtained using Eq. (13), from the actual values is about 10%.
- We find marginal improvement when the terms corresponding to quadratic in tidal deformability are included in Eq. (13).

Correlations of Tidal deformability with Pressure

- The priors for a_i and b_i are taken to be uniform in the range of -100 to 100.
- The calculations are performed for ρ= 1.5, 2.0 and 2.5 ρ_0.
- The average deviation of $P_{\rm BEM}(2\rho_0)$, obtained using Eq. (13), from the actual values is about 10%.
- We find marginal improvement when the terms corresponding to quadratic in tidal deformability are included in Eq. (13).



- Here, we display the variations of tidal deformability as a function of mass and pressure for β-equilibrated matter at ρ=1.5, 2.0 and 2.5 ρ_0.
- One can easily estimate the values of $P_{\rm BEM}(\rho)$ for $\rho \sim 2\rho_0$ once the values of tidal deformability known in NS mass ranges $1.2 - 2.0 M_\odot$.

Conclusions

- A few low-order NMPs such as ϵ_0, K_0, J_0 and L_0 constrained in narrow windows.

Conclusions

- A few low-order NMPs such as ϵ_0, K_0, J_0 and L_0 constrained in narrow windows.
- The correlations of neutron star properties over a wide range of mass with various key quantities characterizing the EOS are investigated.
  - The values of tidal deformability and radius for the NS with $1.4M_\odot$ are strongly correlated with the pressure for the β-equilibrated matter at density $\sim 2\rho_0$.
  - The radius for $2.07M_\odot$ NS is strongly correlated with the pressure for β-equilibrated matter at density $\sim 3\rho_0$.
  - The maximum mass of NS is correlated with the pressure for the β-equilibrated matter at density $\sim 4.5\rho_0$.

Conclusions

- A few low-order NMPs such as ϵ_0, K_0, J_0 and L_0 constrained in narrow windows.
- The correlations of neutron star properties over a wide range of mass with various key quantities characterizing the EOS are investigated.
  - The values of tidal deformability and radius for the NS with $1.4M_\odot$ are strongly correlated with the pressure for the β-equilibrated matter at density $\sim 2\rho_0$.
  - The radius for $2.07M_\odot$ NS is strongly correlated with the pressure for β-equilibrated matter at density $\sim 3\rho_0$.
  - The maximum mass of NS is correlated with the pressure for the β-equilibrated matter at density $\sim 4.5\rho_0$.
- These correlation systematics are similar with those obtained for unified EOSs for a diverse set of non-relativistic and relativistic mean-field models.

Conclusions

- A few low-order NMPs such as ϵ_0, K_0, J_0 and L_0 constrained in narrow windows.
- The correlations of neutron star properties over a wide range of mass with various key quantities characterizing the EOS are investigated.
  - The values of tidal deformability and radius for the NS with $1.4M_\odot$ are strongly correlated with the pressure for the β-equilibrated matter at density $\sim 2\rho_0$.
  - The radius for $2.07M_\odot$ NS is strongly correlated with the pressure for β-equilibrated matter at density $\sim 3\rho_0$.
  - The maximum mass of NS is correlated with the pressure for the β-equilibrated matter at density $\sim 4.5\rho_0$.
- These correlation systematics are similar with those obtained for unified EOSs for a diverse set of non-relativistic and relativistic mean-field models.
- We exploit the model independence of correlations to the pressure for β-equilibrated matter, in the density range $1.5 - 2.5\rho_0$, in terms of the mass and tidal deformability of neutron star.

Conclusions

- A few low-order NMPs such as ϵ_0, K_0, J_0 and L_0 constrained in narrow windows.
- The correlations of neutron star properties over a wide range of mass with various key quantities characterizing the EOS are investigated.
  - The values of tidal deformability and radius for the NS with $1.4M_\odot$ are strongly correlated with the pressure for the β-equilibrated matter at density $\sim 2\rho_0$.
  - The radius for $2.07M_\odot$ NS is strongly correlated with the pressure for β-equilibrated matter at density $\sim 3\rho_0$.
  - The maximum mass of NS is correlated with the pressure for the β-equilibrated matter at density $\sim 4.5\rho_0$.
- These correlation systematics are similar with those obtained for unified EOSs for a diverse set of non-relativistic and relativistic mean-field models.
- We exploit the model independence of correlations to the pressure for β-equilibrated matter, in the density range $1.5 - 2.5\rho_0$, in terms of the mass and tidal deformability of neutron star.
- This parametric form facilitate estimation of the pressure at densities around $2\rho_0$ for a given value of tidal deformability of neutron stars with mass in the range of $1.2 - 2.0M_\odot$.

Acknowledgement

- The Department of Science and Technology, Ministry of Science and Technology, India, for the support of DST/INSPIRE Fellowship/2019/IF190058.

Collaborations list

- T. K. Jha
- B. K. Agrawal
- Hiranmaya Mishra
- Tuhin Malik
- Arunava Mukharjee
- Chiranjib Mondal
- Sk Md Adil Imam
- Debashree Sen

Thank You!