

# Equations of state of dense matter in the light of present and future nuclear physics and astrophysics constraints

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# Overview

- Neutron Stars (NS): formation, properties, structure, composition
- Equation of State (EOS): definition, constraints, models, uncertainties
- NS with heavy baryons: particle abundances, maximum mass, radii, tidal deformabilities
- Core-Collapse SuperNovae (CCSN), Binary NS Mergers (BNS): domains of temperature, density, proton fraction
- Hot EOS: thermal energy density and pressure, entropy, specific heats, adiabatic and thermal index, speed of sound; CompOSE (<https://compose.obspm.fr/>)
- Model dependence; effective masses
- Nucleons, Heavy Baryons, Quarks
- Challenges and Future

# What is a Neutron Star?

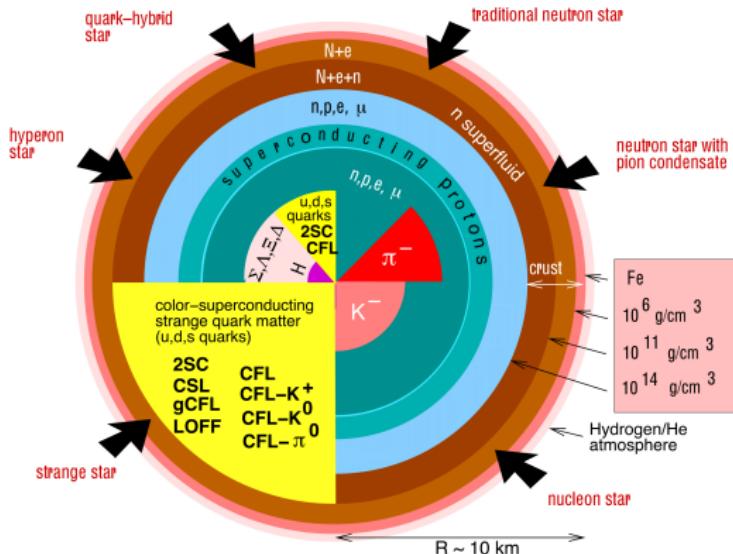


Image credit: F. Weber

- residue of **core collapse supernova**, usually observed as **pulsars**
- $1 \lesssim M/M_\odot \lesssim 2$ ,  $10 \lesssim R [\text{km}] \lesssim 12$
- average density  $\approx n_{\text{sat}}$ , usually referred to as **compact objects**
- highly non-uniform  
 $0 \lesssim n \lesssim 5 - 10 n_{\text{sat}}$
- fast spinning:  $v = 716 \text{ Hz}$  (PSR J1748-2446)
- magnetic field  $10^6 \lesssim B [\text{G}] \lesssim 10^{16}$
- temperature  $10^6 \lesssim T [\text{K}] \lesssim 10^{11}$

**Chemical composition according to  $\beta$ -equilibrium:**  $\mu_n = \mu_p + \mu_e$

**Astrophys. observations:** masses, radii, rotation frequencies, gravitational waves, surface temperatures, etc.

# Nuclear Equation of State (EOS)

EOS=thermodyn. concept. The nuclear EOS at  $T=0$ :  $E/A(n_n, n_p)$ ;

Taylor expansion in terms of deviation from isospin asymmetry,  $\delta = (n_n - n_p)/n$ , and saturation density,  $\chi = (n - n_{sat})/3n_{sat}$ , with  $n = n_n + n_p$ .

$$\begin{aligned} E/A(n, \delta) &= E/A(n, 0) + S(n) \delta^2 + \dots \\ &= \sum_{i \geq 0} \frac{1}{i!} X_{sat}^{(i)} \chi^i + \sum_{j \geq 0} \frac{1}{j!} X_{sym}^{(j)} \chi^j \delta^2 + \dots \\ &\quad \text{energy SNM} \quad \text{symmetry energy} \end{aligned}$$

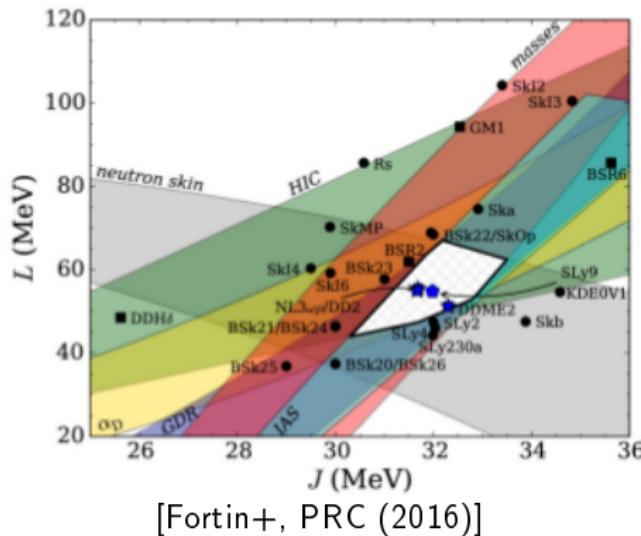
$$X_{sat}^{(i)} = 3^i n_{sat}^i \left( \frac{\partial^i (E/A)}{\partial n^i} \right)_{n=n_{sat}, \delta=0}; \quad X_{sym}^{(j)} = 3^j n_{sat}^j \left( \frac{\partial^j S(n)}{\partial n^j} \right)_{n=n_{sat}, \delta=0}$$

$i=0, 1, 2, 3, 4, \dots$  binding energy per nucleon  $E_{sat}$ , incompressibility  $K_{sat}$ , skewness  $Q_{sat}$ , kurtosis  $Z_{sat}$ , etc. at  $n_{sat}$

$j=0, 1, 2, 3, 4, \dots$  symmetry energy  $J_{sym}$  and its slope  $L_{sym}$ , curvature  $K_{sym}$ , etc. at  $n_{sat}$

# Constraints from nuclear physics experiments

- nuclear masses:  $E_{sat} = -15.8 \pm 0.3$  MeV [Margueron+, PRC (2018)]
- isoscalar giant monopole resonances:  $K_{sat} = 230 \pm 40$  MeV [Khan+, PRL (2012)]
- nuclear masses, isobaric analog state, neutron skin thickness, heavy ion collisions:  
 $J_{sym} = 31.7 \pm 3.2$  MeV [Oertel+, RMP (2017)]
- neutron skin thickness, dipole polarizability, dipole resonance:  $L_{sym} = 58.7 \pm 28.1$  MeV [Oertel+, RMP (2017)]
- higher order coefficients  $Q_{sat}$ ,  $Z_{sat}$ ,  $K_{sym}$ ,  $Q_{sym}$ ,  $Z_{sym}$  are highly uncertain



⚠ parameter values and their correlations are model dependent  
[Margueron+, PRC97 (2018)]

- extra constraints from neutron stars, in particular the isovector channel ( $L_{sym}$ )

# Constraints from NS measurements

- **masses of massive NS**, e.g.  $M/M_\odot \geq 2.01 \pm 0.04$  [Antoniadis+ (2013)]; lower bound on  $M_{max}$ ; **extra particle d.o.f.**
- **mass and radius accurately** [NICER, Athena, LOFT,...]
  - ▶ **intermediate mass NS**: **isovector** channel over  $1 \lesssim n/n_{sat} \lesssim 2 - 3$   
e.g. PSR J0030 + 0451 [NICER]  
 $R(1.44^{+0.15}_{-0.14} M_\odot) = 13.02^{+1.24}_{-1.06} \text{ km}$  [Miller+, 2019];  
 $R(1.34^{+0.15}_{-0.16} M_\odot) = 12.71^{+1.14}_{-1.19} \text{ km}$  [Riley+, 2019];
  - ▶ **massive NS**: EOS at high densities;  
e.g. PSR J0740 + 6620 [NICER]  
 $R(2.08 \pm 0.07 M_\odot) = 13.7^{+2.6}_{-1.5} \text{ km}$  [Miller+, 2021];  
 $R(2.072^{+0.067}_{-0.066} M_\odot) = 12.39^{+1.30}_{-0.98} \text{ km}$  [Riley+, 2021]
- **tidal deformability** from GW  
e.g. late inspiral of  $1.36 \leq M_1/M_\odot \leq 1.6$ ,  $1.16 \leq M_2/M_\odot \leq 1.36$ :  $\Lambda = 190^{+390}_{-120}$  [LIGO and VIRGO, PRL 119, 161101]; **isovector** channel over  $1 \lesssim n/n_{sat} \lesssim 2 - 3$
- **thermal evolution**: dURCA: composition: **isovector** or extra particles;
- **quasi-periodic oscillations, moment of inertia**, etc.

# Models of nuclear EOS

**Phenomenological:** density-dependent effective interactions adjusted to nuclear observables and neutron star observations;  $0 \lesssim n/n_{sat} \lesssim 5 - 10$ ;

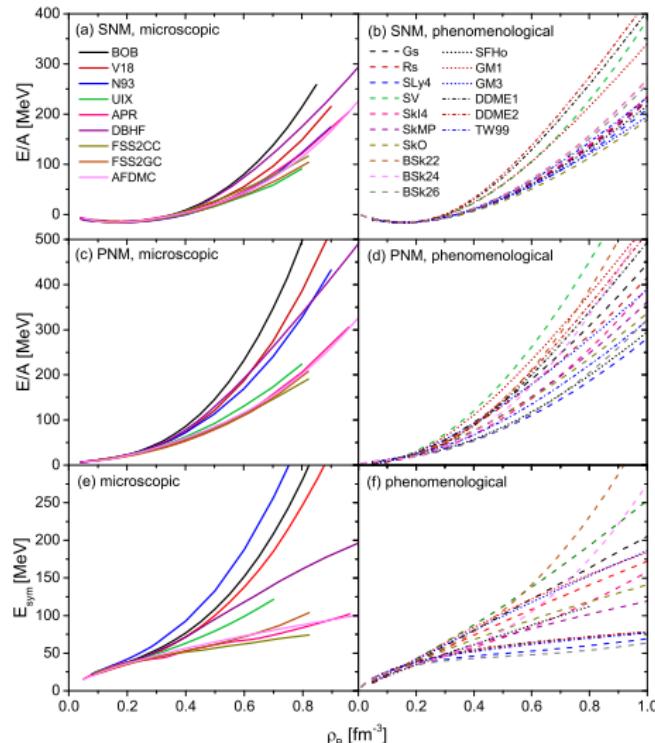
e.g.: non-relativistic EDF (Skyrme, Gogny), relativistic mean-field, relativistic Hartree-Fock [Dutra+, PRC 85, 035201; Dutra+, PRC 90, 055203]

**Microscopic Ab-initio:** the many-body problem is solved starting from two- and three-body interactions;  $0 \lesssim n/n_{sat} \lesssim 1 - 2$ ;

e.g.: variational (APR, TNTYST), quantum Monte Carlo (VMC, AFDMC, GFDMC), coupled cluster expansion, diagrammatic: BBG (BHF), lattice, chiral effective, etc.

**Agnostic:** piecewise polytrops, parametrization of the speed of sound

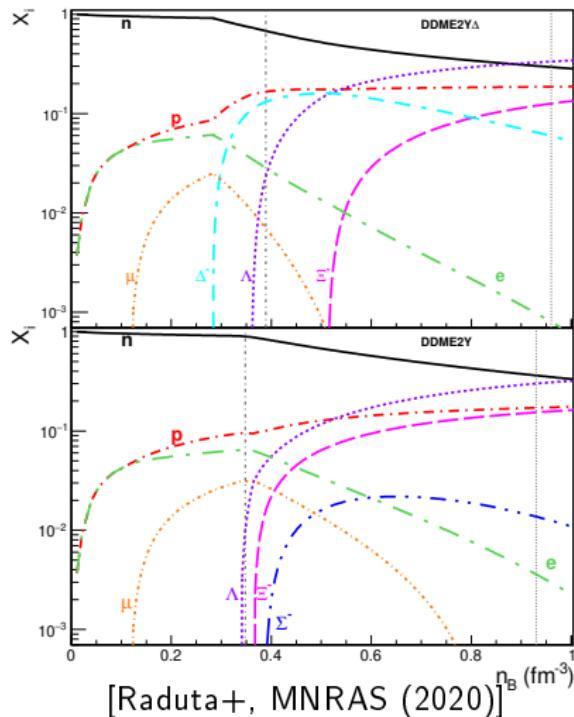
**Large uncertainties** for  $n \gtrsim 2n_{sat}$  and  $\delta \neq 0$



[Burgio+, PPNP (2021)]

# Exotic d.o.f.: hyperons and $\Delta$ s

- Onset of the  $X$ -species at  $T = 0$ :  
 $\mu_X = B_X \mu_B + Q_X \mu_Q \geq m_X$ ;  
 $B_X$ ,  $Q_X$ ,  $\mu_X$ ,  $m_X$  are baryon and charge quantum nr. chemical potential, rest mass
- exp. data on YN and YY eff. int.  
[Gal+, RMP (2016)]:  $U_{\Lambda}^{(N)} \approx -28$  MeV,  
 $U_{\Xi}^{(N)} \approx -18$  MeV,  $U_{\Sigma}^{(N)} \approx 30$  MeV
- exp. data on  $\Delta N$  eff. int. [Drago+, PRC (2014); Kolomeitsev+, NPA (2017)]:  
 $-30 \text{ MeV} + U_N^{(N)} \lesssim U_{\Delta}^{(N)} \lesssim U_N^{(N)}$
- composition determined from **baryon nr. cons.**, **net charge neutrality**,  $\beta$ -equil.

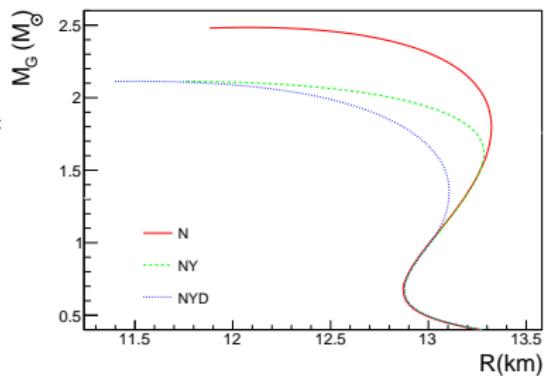
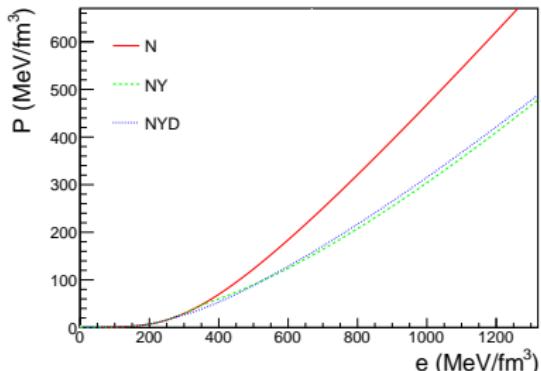


in cold NS only  $\Lambda$ ,  $\Xi^-$  and  $\Sigma^-$  ( $\Delta^-$ )

# NS with heavy baryons: EOS softening and $M_{max}$ decrease

- extra new species softens EOS

Compo.	$P_1$	$n_{P_1}$ [fm $^{-3}$ ]	$M_{max}$ [ $M_\odot$ ]	$R_{1.4}$ [km]	$\Lambda_{1.4}$
N	-	-	2.485	13.24	723
NY	$\Lambda$	0.34	2.113	13.25	712
NY $\Delta$	$\Delta^-$	0.28	2.111	13.08	653



- $\gamma$  diminishes  $M_{max}$
- $\Delta$  diminishes  $R_{1.4}, \Lambda_{1.4}$

# Core-Collapse SuperNovae, Proto-NS evolution, Binary NS Mergers, stellar BH formation

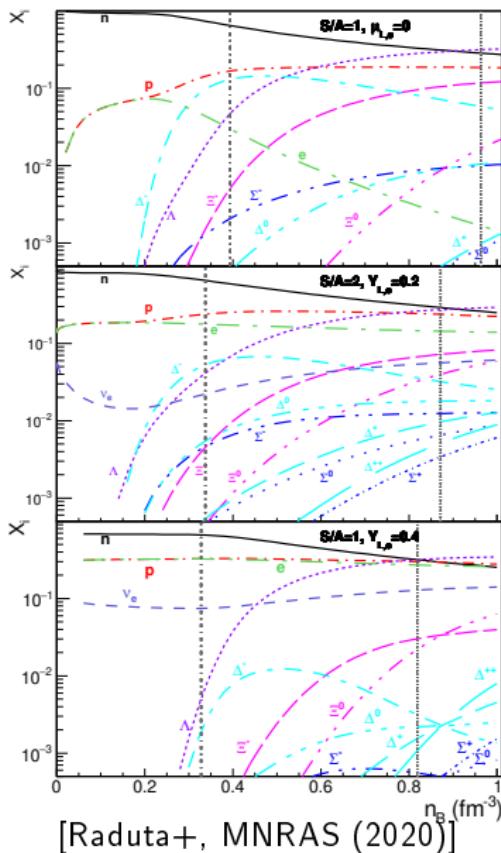
- wide ranges of
  - ▶ baryonic densities [ $10^{-10} \leq n_B \leq 1 - 10 \text{ fm}^{-3}$ ],
  - ▶ temperature [ $0 \leq T \leq 100 \text{ MeV}$ ],
  - ▶ charge fraction [ $0 \leq Y_q \leq 0.6$ ]

are populated

[Pons+, ApJ 667, 282; Janka+, Phys Rep 442, 38; Fischer+, AA 499, 1; Shibata+, Living Rev. Rel.14, 6; O'Connor+, ApJ 730, 70; Hempel+, ApJ 48, 70; Mezzacappa+, 1507.05680; Rosswog, Int J Mod Phys D24, 1530012; Baiotti+, Rep Prog Phys 80, 096901; O'Connor+, ApJ 865, 81; Burrows+, MNRAS 491, 2715; Ruiz+, PRD101, 064042; Janka, Ann Rev Nucl Part Phys 62, 407; Bauswein+, PRD86, 063001; Koppel+, ApJ872, L16; Bauswein+, PRL125]

- EOS:  $X(n_B, Y_p, T)$ ,  $X = e, P, \mu_B, \mu_Q, \dots$   
also microscopic and composition info. is provided;  
3D tables, see CompOSE (<https://compose.obspm.fr/>)
- simulation results depend on effective interaction, particle d.o.f., modeling
- extra uncertainties due to **thermal** behavior; **model dependence**

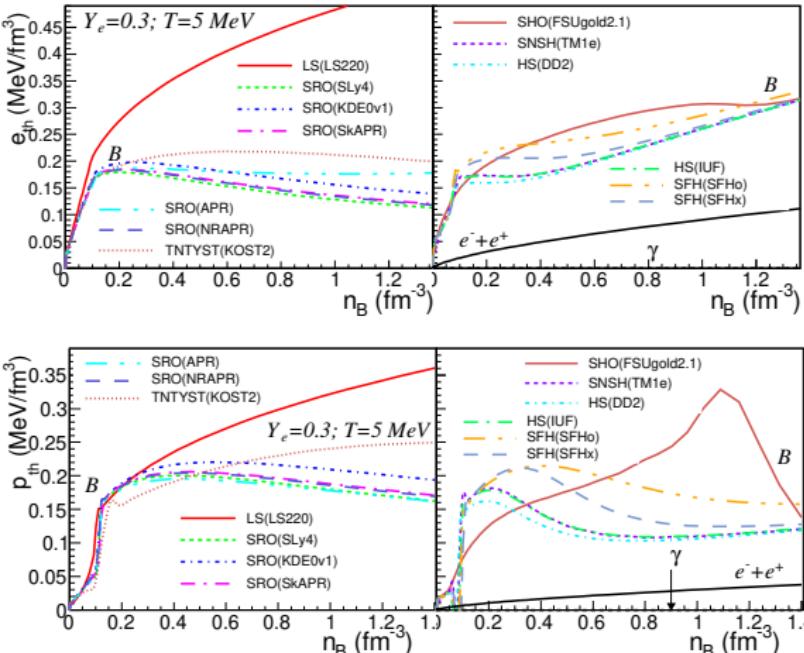
# Heavy baryons in hot and dense matter



- thermal excitation of new d.o.f.
- $\nu_e$  trapping modifies the composition
- high  $T$ : hyperons and  $\Delta$ s appear at  $n_B < n_{sat}$
- high  $T$  favor exotic species
- $\Lambda$  and  $\Delta^-$  dominate
- thermodyn. potentials, microscopic quantities will depend on  $T$ ,  $Y_{p/L}$ , particle d.o.f. and nucleonic EOS
- effects on properties and stability of hot stars

# Thermal energy and pressure: nucleonic models

Thermal contrib.:  $X_{th} = X(n_B, Y_e, T) - X(n_B, Y_e, 0)$



- $S/A, C_V$  have strong correl. with  $m_L^*$

[Raduta+, EPJA (2021)]

- low  $n_B$ , high  $T$ : ideal gas
- high  $n_B$ : strong  $n_B$ -,  $T$ -dep.  
strong EoS-dep.
- qualitative differences among non-rel. and RMF models;  
due to single particle en.

$$\text{RMF: } \epsilon(k) = \sqrt{k^2 + m_D^{*2}} + \Sigma_V$$

Skyrme:

$$\epsilon(k) = k^2 / 2m_L^* + V(n, \tau)$$

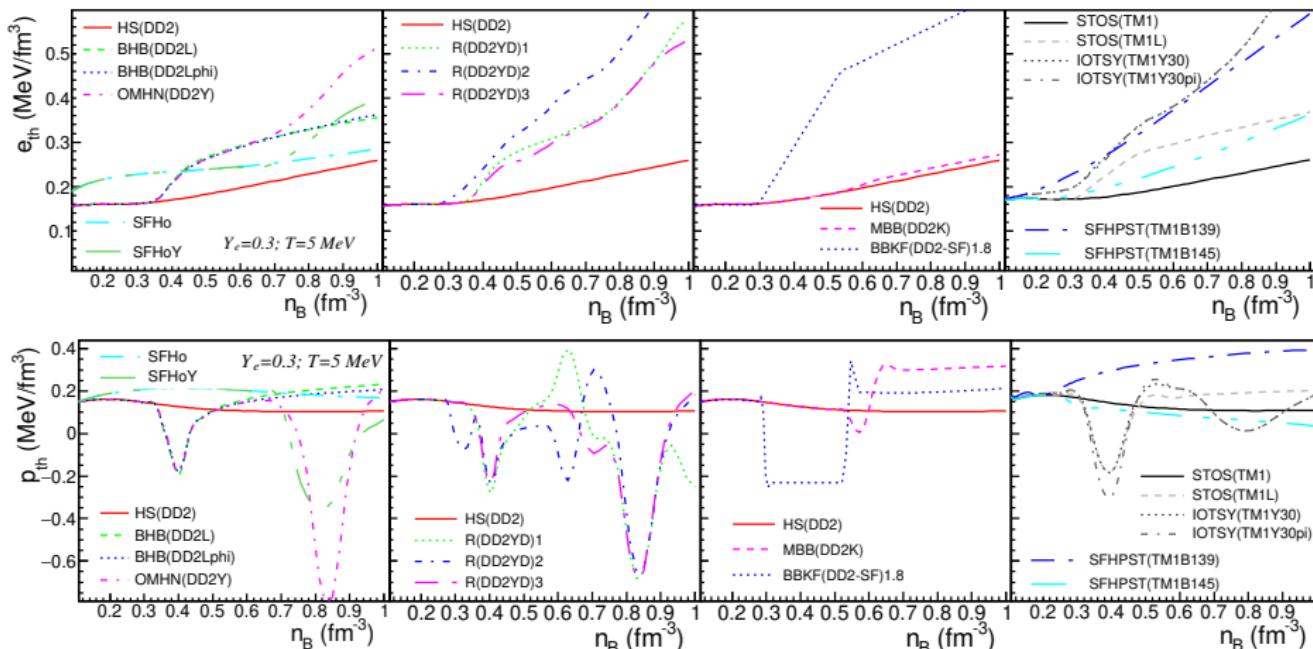
- $e_{th}, p_{th}$  depend on  $m_L^*$ ;
- $p_{th}$  depends on  $dm_L^*/dn$

- different  $\epsilon(k)$  and effective masses

see also [Constantinou+, PRC (2014); PRC (2015)]

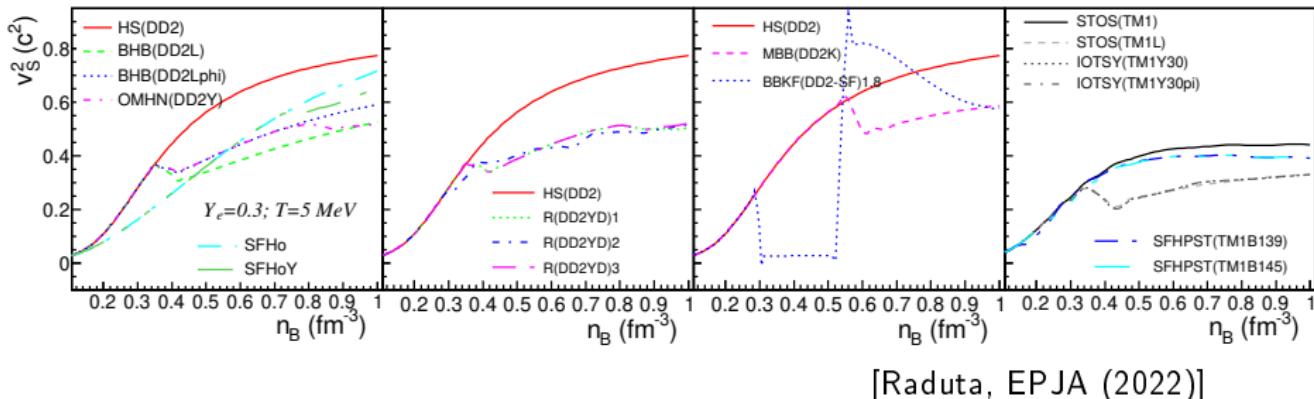
# Thermal energy and pressure: nucleons vs. exotica

$$X_{th} = X(n_B, Y_e, T) - X(n_B, Y_e, 0)$$



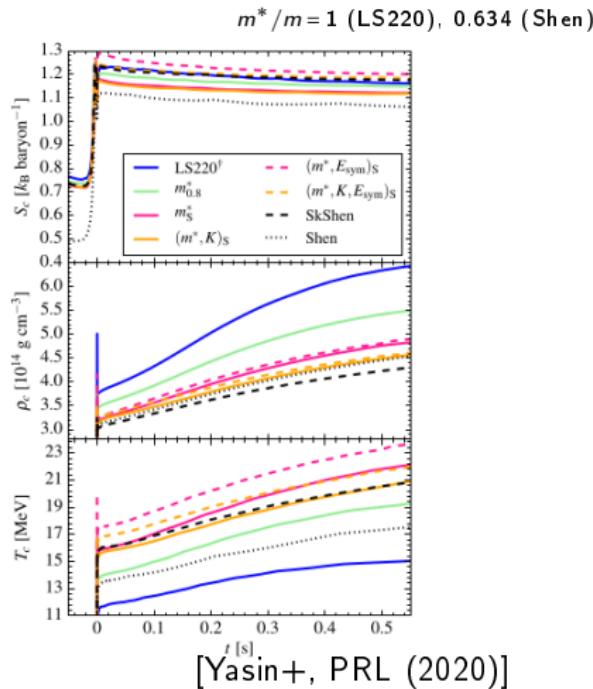
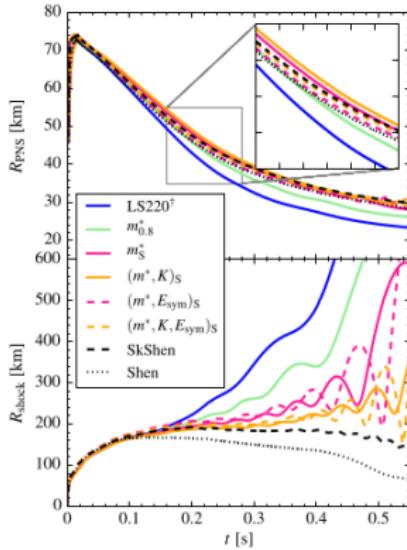
- the larger the number of particle d.o.f. the larger  $e_{th}$ ; [Raduta, EPJA (2022)]
- nucleation of exotic d.o.f. diminishes  $p_{th}$ ; under specific conditions  $p_{th} < 0$
- Gibbs versus Glendenning construction for hadron to quark phase transition

# Speed of sound: $c_S^2 = dP/d\epsilon_{S,A,Y_Q}$ : nucleons vs. exotica



- strong  $n_B$ - and EOS- dependence;
- for Gibbs treatment of phase coex.,  $c_S^2 = 0$
- heavy baryons, mesons:  $c_S^2$  decreases over a narrow  $n_B$  domain
- transition to quarks:  $c_S^2$  decreases over large  $n_B$  domain

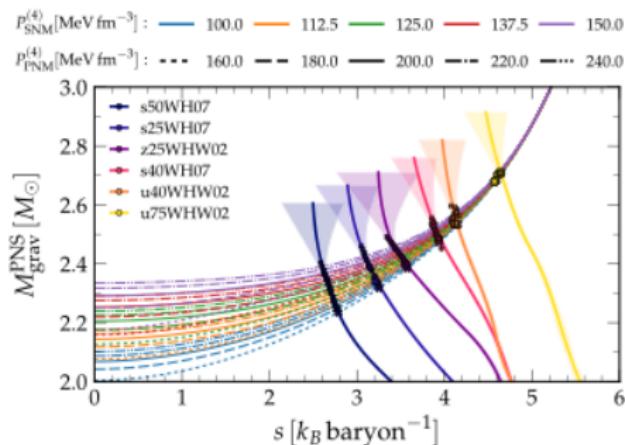
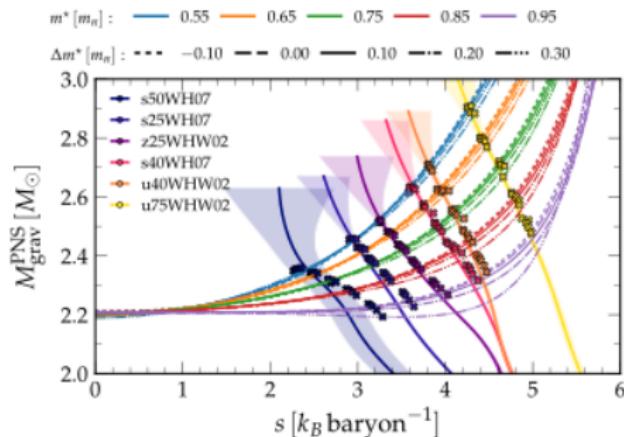
# $m^*$ -effect in early post-bounce evolution



large  $m^* \rightarrow$  fast explosion; fast contraction of PNS; high (low)  $\rho_c$  ( $T$ )

large  $m^* \rightarrow$  high (low)  $T$  ( $n_B$  and  $R$ ) in the  $\nu$ -sphere;

# $m^*$ -effect in the onset of collapse



[Schneider et al., ApJ (2020)]  
failed CCSN; stellar BH formation;

simulation results: collapse begins when hot core's gravitational mass exceeds the maximum gravitational mass predicted by the EoS under the specific thermo conditions;  
most important ingredients: progenitor and  $m^*$

# Conclusions and Challenges

- EOS is essential for describing static and dynamic properties of NS and evolution of proto-NS, core-collapse supernovae, binary NS mergers, formation of stellar BH, etc.
  - ▶ isoscalar behavior at supra-saturation - role and treatment of three-body forces
  - ▶ isovector behavior in isospin asymmetric matter - the symmetry energy
  - ▶ exotic particles; density dependent effective interactions
  - ▶ finite- $T$  behavior
- further constraints
  - ▶ ab-initio calculations
  - ▶ experimental data from nuclear and hypernuclear physics
  - ▶ multi-messenger astrophysics, including gravitational wave astronomy