The role of meson interactions in the $D_s^+ \to \pi^+ \pi^- \eta$ decay

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Introduction

- **D** Theoretical formalism
- **D** Results and discussion
- **D** Summary



□ Introduction

- **Theoretical formalism**
- **C** Results and discussion
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Introduction

Experiment from *BESIII collaboration Phys.Rev.D* 104 071101(2021)

Study of the Decay $D_s^+ \to \pi^+ \pi^- \eta$ and Observation of the W-annihilation Decay $D_s^+ \to a_0(980)^+ \rho^0$





Motivation and goal: reproduce the six invariant mass distributions



TABLE I. Phases and FFs for various intermediate processes. The first and the second uncertainties are statistical and systematic, respectively.

Amplitude	Phase	$\mathrm{FF}(\%)$
$a_1(1260)^+(\rho(770)^0\pi^+)\eta$	0.0(fixed)	$55.4\pm3.9\pm2.0$
$a_1(1260)^+(f_0(500)\pi^+)\eta$	$5.0\pm0.1\pm0.1$	$8.1\pm1.9\pm2.1$
$a_0(980)^+ ho(770)^0$	$2.5\pm0.1\pm0.1$	$6.7\pm2.5\pm1.5$
$\overline{\eta(1405)(a_0(980)}^-\pi^+)\pi^+$	$0.2\pm0.2\pm0.1$	$0.7\pm0.2\pm0.1$
$\eta(1405)(a_0(980)^+\pi^-)\pi^+$	$0.2\pm0.2\pm0.1$	$0.7\pm0.2\pm0.1$
$f_1(1420)(a_0(980)^-\pi^+)\pi^+$	$4.3\pm0.2\pm0.4$	$1.9\pm0.5\pm0.3$
$f_1(1420)(a_0(980)^+\pi^-)\pi^+$	$4.3\pm0.2\pm0.4$	$1.7\pm0.5\pm0.3$
$[a_0(980)^-\pi^+]_S\pi^+$	$0.1\pm0.2\pm0.2$	$5.1\pm1.2\pm0.9$
$[a_0(980)^+\pi^-]_S\pi^+$	$0.1\pm0.2\pm0.2$	$3.4\pm0.8\pm0.6$
$[f_0(980)\eta]_S\pi^+$	$1.4\pm0.2\pm0.3$	$6.2\pm1.7\pm0.9$
$[f_0(500)\eta]_S \pi^+$	$2.5\pm0.2\pm0.3$	$12.7\pm2.6\pm2.0$

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Outline

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D Theoretical formalism

- 1. Hadronization of one $q\bar{q}$ pair
- 2. Rescattering from the tree level $\eta \rho^0 \pi^+$ component
- 3. Two hadronizations: external and internal emission
- 4. Rescattering from the tree level $\eta f_0 \pi^+$ component
- 5. Evaluation of the differential cross section: integral

1. Hadronization of one $q\bar{q}$ pair

✓ Final states: G-parity: "–"; $\rho^0 \pi \eta$ (**PPV**).

$$P = \begin{pmatrix} \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^{+} & K^{+} \\ \pi^{-} & \frac{-\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{3}} \end{pmatrix}, \quad V = \begin{pmatrix} \frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+} \\ \rho^{-} & \frac{-\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

External emission mechanism



Internal emission mechanism

c

 \overline{s}

 D_s^+

 W^+



 \overline{s}



 \overline{s}

 \overline{s}

1. Hadronization of one $q\bar{q}$ pair

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Internal emission mechanism



- 1. Hadronize $s\bar{d}$ with PP and $u\bar{s}$ is a vector;
- 2. Hadronize $u\bar{s}$ to PP and $s\bar{d}$ is a vector;
- 3. Hadronize $s\bar{d}$ to VP, PV and $u\bar{s}$ is a pseudoscalar;
- 4. Hadronize $u\bar{s}$ to VP, PV and $s\bar{d}$ is a pseudoscalar.

1. Hadronization of one $q\overline{q}$ pair

> Ingredients



> Suitable states and the weights (α , β , γ)

$$|H1\rangle \equiv C \left[-\sqrt{\frac{2}{3}} \eta \rho^0 \pi^+ + \frac{\eta}{\sqrt{3}} (1 + \alpha + \beta) (\bar{K}^{*0} K^+ - K^{*+} \bar{K}^0) + \sqrt{2} \beta \rho^0 K^+ \bar{K}^0 + \alpha \pi^+ (K^{*-} K^+ - K^{*0} \bar{K}^0) - \gamma \pi^+ (\bar{K}^{*0} K^0 - K^{*+} K^-) \right]$$

1. Hadronization of one $q\bar{q}$ pair

> Couplings

L. Roca et al., Phys. Rev. D 72, 014002 (2005)

a_1		b_1	
$g_{ar{K}^*K}$	$g_{ ho\pi}$	$g_{K^*ar K}$	$g_{ ho\eta}$
1872 - i1486	-3.795 + i2330	-3041 + i498	6172 - i75

> Amplitude

$$t_{H1}(\pi^{+}\rho^{0}\eta) = C \left[-\sqrt{\frac{2}{3}} + \frac{\eta}{\sqrt{3}} (1 + \alpha + \beta) G_{K^{*}\bar{K}}(M_{\mathrm{inv}}(\rho^{0}\pi^{+})) \frac{g_{a_{1},K^{*}\bar{K}}g_{a_{1},\rho\pi}}{M_{\mathrm{inv}}^{2}(\rho^{0}\pi^{+}) - M_{a_{1}}^{2} + iM_{a_{1}}\Gamma_{a_{1}}} - \sqrt{2\beta}G_{K\bar{K}}(M_{\mathrm{inv}}(\pi^{+}\eta)) \frac{g_{a_{0},K\bar{K}}g_{a_{0},\pi\eta}}{M_{\mathrm{inv}}^{2}(\pi^{+}\eta) - M_{a_{0}}^{2} + iM_{a_{0}}\Gamma_{a_{0}}} - (\alpha + \gamma)G_{K^{*}\bar{K}}(M_{\mathrm{inv}}(\rho^{0}\eta)) \frac{g_{b_{1},K^{*}\bar{K}}g_{b_{1},\rho\eta}}{M_{\mathrm{inv}}^{2}(\rho^{0}\eta) - M_{b_{1}}^{2} + iM_{b_{1}}\Gamma_{b_{1}}} \right]$$

> Ingredients



> Couplings

L. Roca et al., Phys. Rev. D 72, 014002 (2005)

a_1		b_1	
$g_{ar{K}^*K}$	$g_{ ho\pi}$	$g_{K^*ar K}$	$g_{ ho\eta}$
1872 - i1486	-3.795 + i2330	-3041 + i498	6172 - i75

> Amplitude

$$t_{\text{RES}}(\rho^{0}\pi^{+}\eta) = -C\sqrt{\frac{2}{3}}G_{\rho\pi}(M_{\text{inv}}(\rho^{0}\pi^{+}))\frac{\frac{1}{\sqrt{2}}g_{a_{1},\rho\pi}\frac{1}{\sqrt{2}}g_{a_{1},\rho\pi}}{M_{\text{inv}}^{2}(\rho^{0}\pi^{+}) - M_{a_{1}}^{2} + iM_{a_{1}}\Gamma_{a_{1}}} - C\sqrt{\frac{2}{3}}G_{\rho\eta}(M_{\text{inv}}(\rho^{0}\eta))\frac{g_{b_{1},\rho\eta}g_{b_{1},\rho\eta}}{M_{\text{inv}}^{2}(\rho^{0}\eta) - M_{b_{1}}^{2} + iM_{b_{1}}\Gamma_{b_{1}}} - C\sqrt{\frac{2}{3}}G_{\pi^{+}\eta}(M_{\text{inv}}(\pi^{+}\eta))\frac{g_{a_{0},\pi\eta}g_{a_{0},\pi\eta}}{M_{\text{inv}}^{2}(\pi\eta) - M_{a_{0}}^{2} + iM_{a_{0}}\Gamma_{a_{0}}}$$

> Symmetrized amplitude with ρ decaying to two pions.



Amplitude

$$t_{\rho} = -C \left[P_{D_s} \cdot (p_4 - p_2) \frac{1}{M_{\text{inv}}^2(\rho, a) - M_{\rho}^2 + iM_{\rho}\Gamma_{\rho}} t^{(a)} + P_{D_s} \cdot (p_4 - p_3) \frac{1}{M_{\text{inv}}^2(\rho, b) - M_{\rho}^2 + iM_{\rho}\Gamma_{\rho}} t^{(b)} \right]$$

• ρ propagator: the sum over the ρ polarization $\sum_{pol} P_D^{\mu} \epsilon_{\mu} \epsilon_{\nu} (p_2 - p_4)^{\nu} = P_D^{\mu} (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_{\rho^2}}) (p_2 - p_4)^{\nu}$ $q = p_2 + p_4; q(p_2 - p_4) = m_{\pi}^2 - m_{\pi}^2 = 0$

3. Two hadronizations: external emission

Two hadronizations with external emission: $f_0(980)$ contribution



Final states: directly four pseudoscalars(**PPPP**) G-parity: "-" 2.

Diagrams stemming from two hadronizations producing the $f_0(980)$ resonance.



Amplitude and the weight (μ)

 $t_1(f_0) = C\mu[\widetilde{D}_{f_0}(M_{\text{inv}}(\pi^+(3)\pi^-)) + \widetilde{D}_{f_0}(M_{\text{inv}}(\pi^+(2)\pi^-))]$

• Ingredients:



➤ Amplitude: $t_2(f_0) = t_{2a} + t_{2d} + t_{2b} + t_{2e} + t_{2c} + t_{2f}$

> Ingredient 1:



> Amplitude and the weight (μ)

> Ingredient 2:



> Amplitude and the weight (μ)

$$t_{2b}(f_0) = C\mu \tilde{D}_{f_0} \left(M_{inv}(\pi^- \pi_{int}^+) \right) G_{\pi\eta} \left(M_{inv}(\pi^+(3)\eta) \right) t_{\pi^+\eta,\pi^+\eta} \left(M_{inv}(\pi^+(3)\eta) \right) t_{2c}(f_0) = C\mu \tilde{D}_{f_0} \left(M_{inv}(\pi^+(3)\pi_{int}^-) \right) G_{\pi\eta} \left(M_{inv}(\pi^-\eta) \right) t_{\pi^-\eta,\pi^-\eta} \left(M_{inv}(\pi^-\eta) \right$$

> Ingredient 3:



> Amplitude and the weight (μ)

$$t_{2e}(f_0) = C\mu \tilde{D}_{f_0} \left(M_{inv}(\pi^- \pi_{int}^+) \right) G_{\pi\eta} \left(M_{inv}(\pi^+ (2)\eta) \right) t_{\pi^+\eta,\pi^+\eta} \left(M_{inv}(\pi^+ (2)\eta) \right) t_{2f}(f_0) = C\mu \tilde{D}_{f_0} \left(M_{inv}(\pi^+ (2)\pi_{int}^-) \right) G_{\pi\eta} \left(M_{inv}(\pi^-\eta) \right) t_{\pi^-\eta,\pi^-\eta} \left(M_{inv}(\pi^-\eta$$

3. Two hadronizations: internal emission



> Amplitude and the weight (ν)

$$t_{\text{DIE}} = C \nu G_{K\overline{K}}(M_{\text{inv}}(\pi^-\eta)) t_{\pi^-\eta, K^0K^-}$$

Full amplitude:

$$t = t_{H1} + t_{RES} + t_1(f_0) + t_2(f_0) + t_{DIE}$$

5. Evaluation of the differential cross section: integral

• The width for the D_s decay into $\pi^+\pi^+\pi^-\eta$

- Define the η in the *z* direction:
 - $\delta() \text{ condition allows us to obtain the relation of } \cos\theta \text{ and variables:} \\ \cos\theta \equiv A = \frac{1}{2P_{\pi}p_{\eta}} \left[(M_{D_s} E_{\eta} E_{\pi^+} E'_{\pi^+})^2 m_{\pi^-}^2 \mathbf{P}_{\pi}^2 \mathbf{p}_{\eta}^2 \right]$
- Define the P_{π} in the *z* direction:
- a. Rotation of angle θ in the *xz* plane, i.e. the q_{π} along to *y*-axis,

$$\mathbf{x} \qquad \mathbf{R}_{\theta} \begin{pmatrix} 1\\ 0\\ 0 \end{pmatrix} = \begin{pmatrix} \sin\theta\\ 0\\ \cos\theta \end{pmatrix} \qquad \mathbf{R}_{\theta} \begin{pmatrix} 0\\ 1\\ 0 \end{pmatrix} = \begin{pmatrix} 0\\ 1\\ 0 \end{pmatrix} \qquad \mathbf{R}_{\theta} \begin{pmatrix} 1\\ 0\\ 0 \end{pmatrix} = \begin{pmatrix} \sin\theta\\ 0\\ \cos\theta \end{pmatrix}$$

b. Rotation of angle φ in the xy plane, i.e. the q_{π} along to z-axis,

$$\mathbf{x} \quad \mathbf{\varphi} \quad \mathbf{y} \quad R_{\phi} \begin{pmatrix} 1\\0\\0 \end{pmatrix} = \begin{pmatrix} \cos\phi\\\sin\phi\\0 \end{pmatrix} \quad R_{\phi} \begin{pmatrix} 0\\1\\0 \end{pmatrix} = \begin{pmatrix} -\sin\phi\\\cos\phi\\0 \end{pmatrix} \quad R_{\phi} \begin{pmatrix} 0\\0\\1 \end{pmatrix} = \begin{pmatrix} 0\\0\\1 \end{pmatrix}$$

5. Evaluation of the differential cross section: integral

• The complete rotation: $q = R\widetilde{q}$

$$R = R_{\phi}R_{\theta} = \begin{pmatrix} \cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta\\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta\\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$
where, the other variables are:

$$\mathbf{p}_{\eta} = p_{\eta} \begin{pmatrix} 0\\0\\1 \end{pmatrix}; \ \mathbf{P}_{\pi} = P_{\pi} \begin{pmatrix} \sin\theta\cos\phi\\\sin\theta\sin\phi\\\cos\theta \end{pmatrix}, \ (\cos\theta = A, \ \sin\theta = \sqrt{1 - A^2})$$

$$\tilde{\mathbf{q}} = q \begin{pmatrix} \sin \tilde{\theta}_{q} \cos \tilde{\phi}_{q} \\ \sin \tilde{\theta}_{q} \sin \tilde{\phi}_{q} \\ \cos \tilde{\theta}_{q} \end{pmatrix} \qquad E_{\pi^{+}} + E_{\pi^{+}}' = \sqrt{m_{\pi}^{2} + \frac{1}{4}(\mathbf{P}_{\pi} + \mathbf{q})^{2}} + \sqrt{m_{\pi}^{2} + \frac{1}{4}(\mathbf{P}_{\pi} - \mathbf{q})^{2}} \\ = \sqrt{m_{\pi}^{2} + \frac{1}{4}P_{\pi}^{2} + \frac{1}{4}Q^{2} + \frac{1}{2}P_{\pi}q\cos\tilde{\theta}_{q}} + \sqrt{m_{\pi}^{2} + \frac{1}{4}P_{\pi}^{2} + \frac{1}{4}Q^{2} - \frac{1}{2}P_{\pi}q\cos\tilde{\theta}_{q}} \end{pmatrix}$$

• Simplify, angle rotations, and variable substitution

$$\Gamma = \frac{1}{8M_{D_s}} \frac{1}{4\pi} \int dE_{\eta} \int \frac{P_{\pi} dP_{\pi} d\phi}{(2\pi)^3} \int \frac{q^2 dq d\cos\tilde{\theta}_q d\tilde{\phi}_q}{(2\pi)^3} \frac{1}{2E_{\pi^+}} \frac{1}{2E'_{\pi^+}} |t|^2 \theta (1 - A^2) \theta (M_{D_s} - E_{\eta} - E_{\pi^+} - E'_{\pi^+})$$

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□ Results and discussion

D Summary

Results Phase space and the tree-level of ρ

Phase space: missing all the different structures which are visible in the experimental data.



• the contribution of the $\eta \rho^0 \pi^+$ term alone at the tree level.

- 1. ρ peak in the $M_{\pi^+\pi^-}$ distribution
- 2. A broad bump in $M_{\pi^+\pi^-}$ at 0.5 GeV. Not $f_0(500)! (\rho \text{ term})$

Results the different mass distributions

• Formalism comes from the systematic consideration of all possible mechanisms



• A best fit to all the six mass distributions

1. $\alpha = 4.2387, \beta = 2.8878, \gamma = -3.9334, \mu = -31.161, \nu = -39.092 (\chi^2/d. o. f = 1.77)$ 2. The improvement over the mass distributions of the tree level $\eta \rho^0 \pi^+$ term is remarkable.

Results the different mass distributions

• Formalism comes from the systematic consideration of all possible mechanisms



 \blacklozenge The low and high energy bump of the mass distribution is well reproduced

3. ρ^0 , $f_0(980)$, $a_0^+(980)$, $a_0^-(980)$ peak in the $M_{\pi^+\pi^-}$, $M_{\pi^+\eta}$, $M_{\pi^-\eta}$ distribution;

4. $M_{\pi^+\pi^-\pi^-}$, $M_{\pi^+\pi^-\eta}$ distributions are also in good agreement with the experiment.

!! A peak around 0.85 GeV that we cannot reproduce and do not know its dynamical origin

Results The contributions of a_1 , a_0^+ , a_0^- , b_1 , f_0 , and external emission

• The contribution of the different resonances



• The contributions to the mass distributions: interferences among the different amplitudes

- 1. The dominant contribution comes from a_0^+
- 2. $\alpha = \beta = \gamma = \mu = \nu = 0$, a_1 term: ρ peak appears
- 3. The strength of b_1^+ is practically negligible(mechanisms excitation without experiment!).

Results | Theoretical error bands

• Theoretical error bands vs. experimental data



• The parameters are different for each fit

- 1. The bands are so narrow: small uncertainties in the fit results.
- 2. Take care of the value of the parameters: the strong correlations of the parameters.

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Summary

• The study of $D_s^+ \to \pi^+ \pi^- \eta$ reaction: final state interaction of pairs of mesons

- 1. The first time of $D_s^+ \rightarrow$ in quark level with external and internal emission.
- 2. We have made a theoretical study of the $D_s^+ \to \pi^+ \pi^- \eta$ reaction taking into account the **final state** interaction of pairs of mesons. (the filter of **G-parity**)
- 3. A support for the amplitudes studied by **chiral unitary approach**: we obtain a good reproduction of the 6 invariant mass distributions with **less freedom(5) than** the experiment analysis(19).
- 4. In these mass distributions: we could clearly see peaks for the ρ^0 , a_1 , a_0^+ , a_0^- , f_0 in reasonable agreement with experiment.
- 5. The fit parameters have large uncertainties but there are strong correlations between them such that the uncertainties in the fit are small.

Thanks for your attention!

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