

Theory overview of GPDs - fits and modelization

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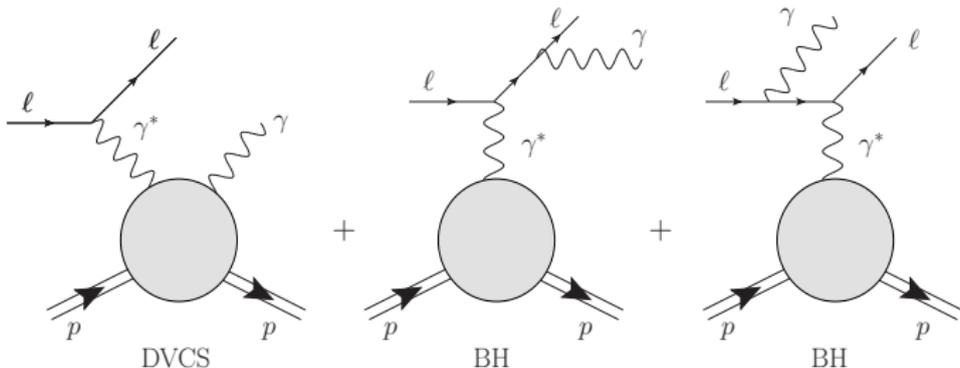


Outline

- ➊ Introduction
- ➋ Theory points
- ➌ Modelling
- ➍ DVCS
- ➎ DVCS+DVMP
- ➏ Gepard

Deeply virtual Compton scattering

- **Deeply virtual Compton scattering (DVCS)** — “gold plated” process of exclusive physics
- DVCS is measured via lepton production of a photon



- **Interference** with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

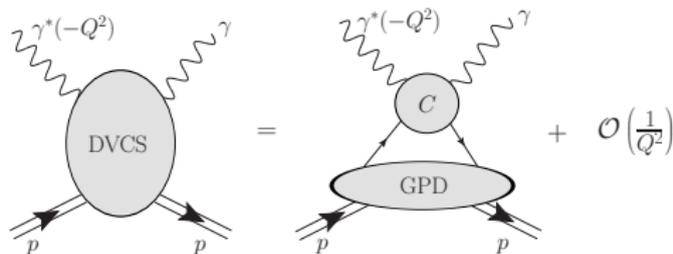
DVCS \longrightarrow CFFs \longrightarrow GPDs

- At leading order DVCS cross-section depends on four complex

Compton form factors (CFFs)

$$\mathcal{H}(\xi, t, Q^2), \quad \mathcal{E}(\xi, t, Q^2), \quad \tilde{\mathcal{H}}(\xi, t, Q^2), \quad \tilde{\mathcal{E}}(\xi, t, Q^2)$$

- [Collins et al. '98]



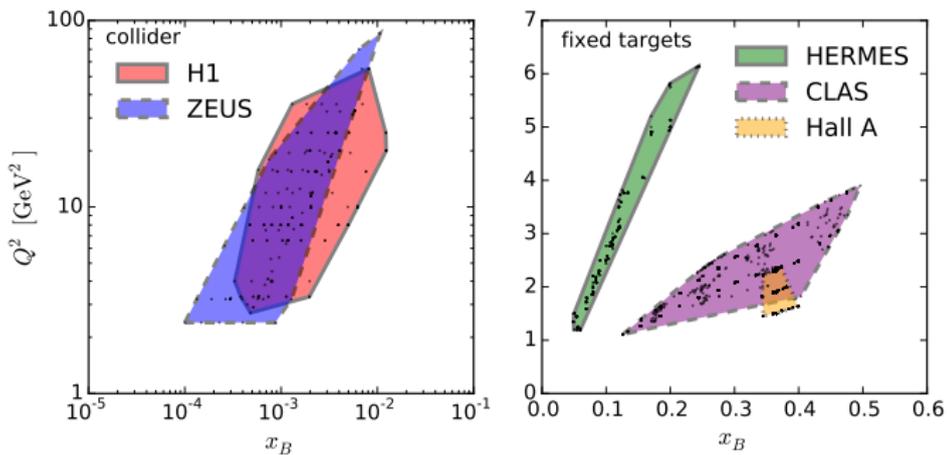
- CFFs are convolution:

$${}^a\mathcal{H}(\xi, t, Q^2) = \int dx C^a(x, \xi, \frac{Q^2}{Q_0^2}) H^a(x, \eta = \xi, t, Q_0^2) \quad a=q, G$$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

[Müller '92, et al. '94, Ji, Radyushkin '96]

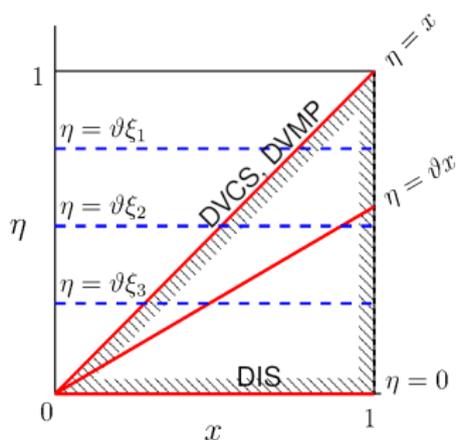
Experimental coverage



GPD skewness

- GPD "skewness": ratio of GPD to the corresponding PDF

$$r = \frac{H(x, \eta = x)}{q(x)}$$



- Conformal (Shuvaev) values, with GPDs completely specified by PDFs:

$$r^{\text{Quark}} \approx 1.65, \quad r^{\text{Gluon}} \approx 1.0.$$

Three “classical” objectives of GPD studies

- Both meanings are valid:
 - “classical” = well known, venerable
 - “classical” = understandable from non-quantum viewpoint

1 Ji's “sum rule”

$$J_z^a = \frac{1}{2} \int_{-1}^1 dx x \left[H^a(x, \xi, t) + E^a(x, \xi, t) \right]_{t \rightarrow 0} \quad [\text{Ji '96}]$$

2 3D tomography

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{b}_\perp \cdot \vec{\Delta}_\perp} H(x, 0, -\vec{\Delta}_\perp^2) \quad [\text{Burkardt '00}]$$

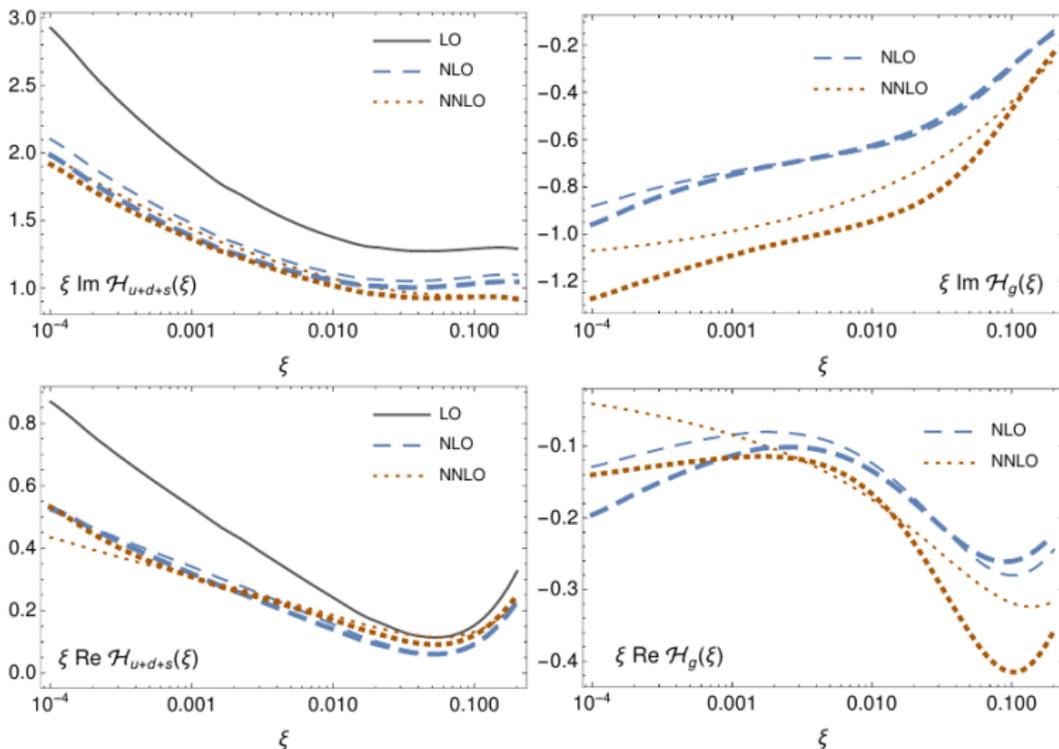
- ## 3 Pressure distribution in the nucleon — directly related to subtraction constant $\Delta(t)$ of CFF dispersion relation — directly related to GPD “D-term” [Polyakov '03, Teryaev '05]

$$\Delta(t) = \Re \mathcal{H}(\xi, t) - \frac{1}{\pi} \text{P.V.} \int_0^1 dx \frac{2x}{\xi^2 - x^2} \Im \mathcal{H}(x, t)$$

(See [Ji and Liu '21] for critique.)

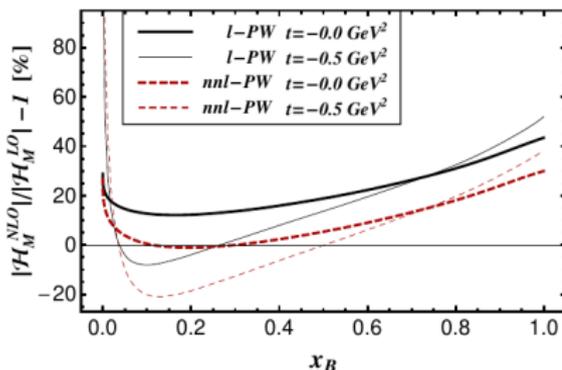
(N)NLO corrections to DVCS

- [Braun, Ji, Schoenleber '22] (NNLO in $\overline{\text{MS}}$ scheme)

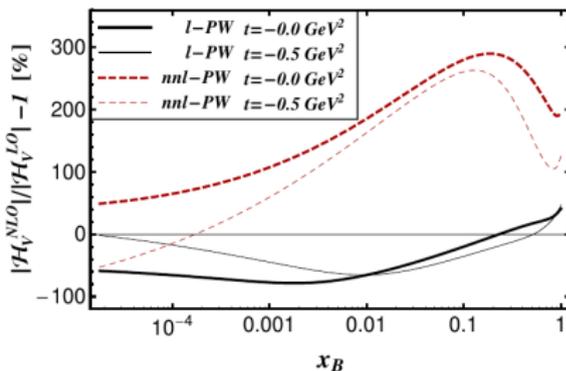


NLO corrections - DVMP

- [Belitsky and Müller '01], [Ivanov, Szymanowski, Krasnikov '04], [Müller, Lautenschlager, Passek-K., Schäfer '13], [Duplančić, Müller, Passek-K. '16]
- nonsinglet:



singlet:



Recent developments and status

- New GPD-relevant exclusive processes are added to the mix, many of them calculated at NLO
 - timelike DVCS [Berger, Diehl, Pire, '01]
 - double DVCS [Guidal, Vanderhaeghen '02]
 - diphoton production [Grocholski et al. '22]
 - photon-meson production [Duplančić et al. '18]
- Global GPD extraction efforts should be improved
 - latest global fits don't even use proper full LO evolution!

Modelling

Two representations

1 Momentum fraction space (x-space)

$$H(x, \eta, t) = \int_{\Omega} d\beta d\alpha F_{DD}(\beta, \alpha, t) \delta(x - \beta - \alpha\eta) + D\left(\frac{x}{\eta}, t\right).$$

$$F_{DD}(\beta, \alpha, t) = \pi_N(\beta, \alpha) \underbrace{q(\beta, t)}_{\text{PDF} + t \text{ dep.}} \quad \text{– double distribution}$$

2 Conformal moment space (j-space)

$$H(x, \eta, t) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj \frac{1}{\sin \pi j} p_j(x, \eta) H_j(\eta, t),$$

p_j – proportional to Gegenbauer polynomial

(relation between the two studied in [Müller, Polyakov, Semenov-Tian-Shansky '15])

Cheaper workaround: modelling CFFs directly

- LO relation:

$$\Im \mathcal{H}(\xi, t) = \pi \left[\frac{4}{9} H^{u_{\text{val}}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\text{val}}}(\xi, \xi, t) \right]$$

- enables direct modelling of CFFs

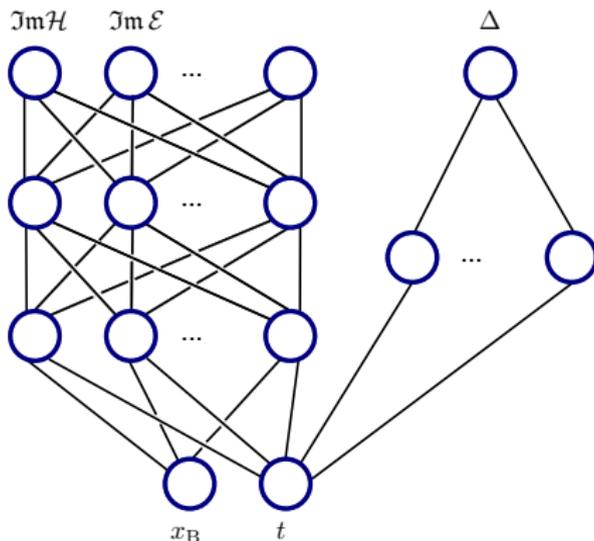
$$H(x, x, t) = n r 2^{\alpha} \left(\frac{2x}{1+x} \right)^{-\alpha(t)} \left(\frac{1-x}{1+x} \right)^b \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^{\rho}}.$$

- $\Re \mathcal{H}$ determined by dispersion relations

$$\Re \mathcal{H}(\xi, t) = \frac{1}{\pi} \text{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im \mathcal{H}(\xi', t) - \frac{C}{\left(1 - \frac{t}{M_c^2}\right)^2}$$

- Typically 10-15 free parameters
- [K.K., Müller '09] (Hybrid “KM model”)

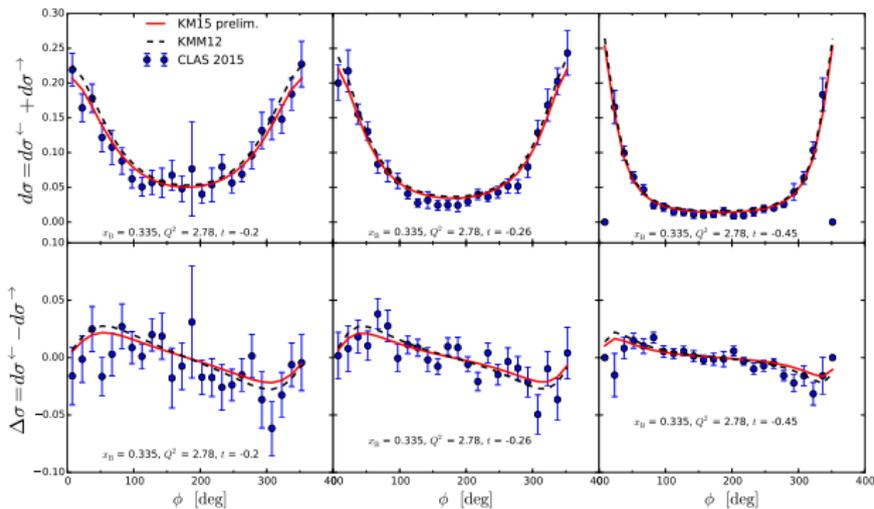
Unbiased model: Neural Networks



- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets
- Real parts are then fixed by dispersion relations

DVCS results

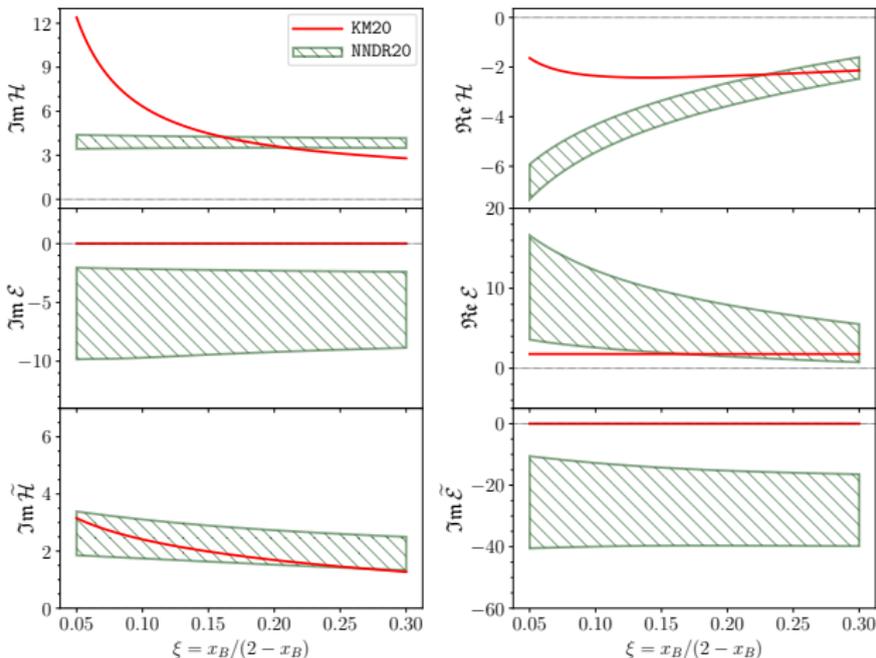
Example: CLAS cross-sections



-
- $\chi^2/\text{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

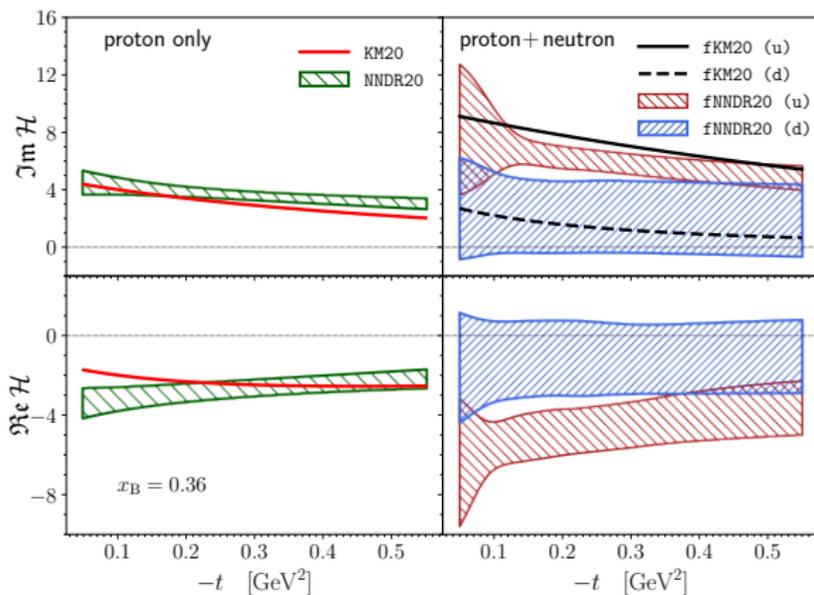
Extraction of 6 (out of 8) CFFs

[M. Čuić, K.K., A. Schäfer, '20], from JLab data



Separating flavored CFFs

- Using neutron DVCS data [Benali et al. '20], contributions of u and d quarks to CFF \mathcal{H} can be cleanly separated [M. Čuić, K.K., and A. Schäfer, PRL '20],



Going multichannel, adding DVMP

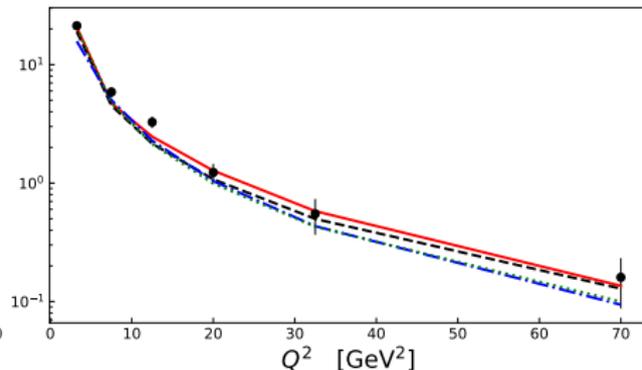
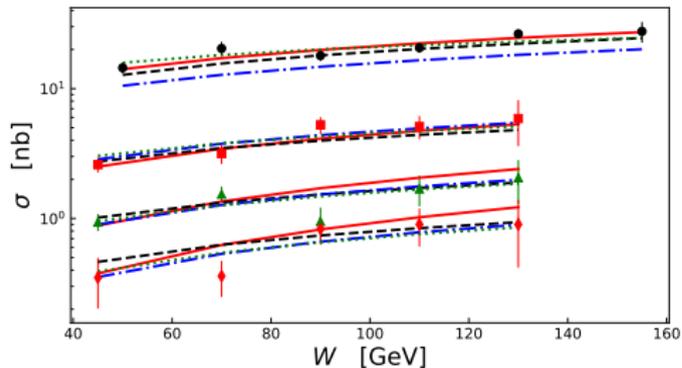
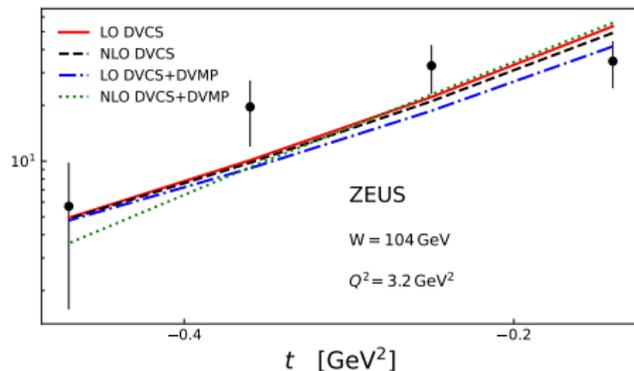
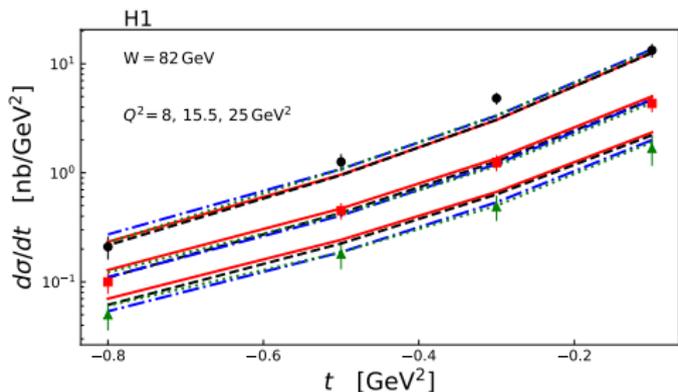
- This should provide:
 - true flavor separation
 - truly universal GPDs
- This likely requires going beyond LO

DVCS + DVMP

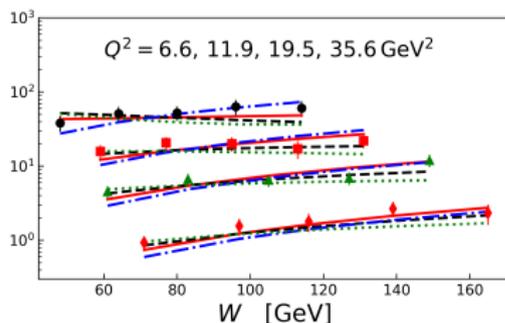
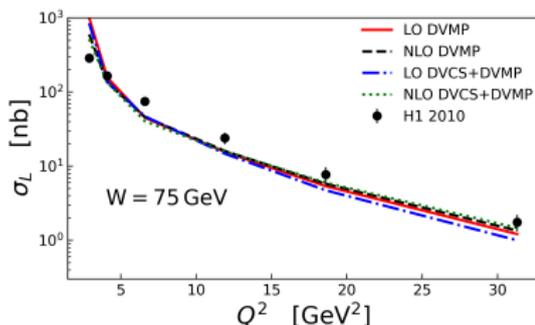
New NLO DIS+DVCS+DVMP small-x global fit

- First global fits to **DIS+DVCS+DVMP** HERA collider data [Lautenschlager, Müller, Schäfer '13, unpublished!]
- hard scattering amplitude corrected in the meantime [Duplančić, Müller, Passek-K. '17]
- new DIS+DVCS+DVMP fit [M. Čuić et al.], preliminary results
- NLO DIS+DVCS+DVMP fit to **HERA collider data** $\chi^2/n_{\text{d.o.f}} = 254.3/231$
- For comparison, in the following, we also show
 - LO fits
 - fits without DVMP data
 - fits without DVCS data

DVCS data description

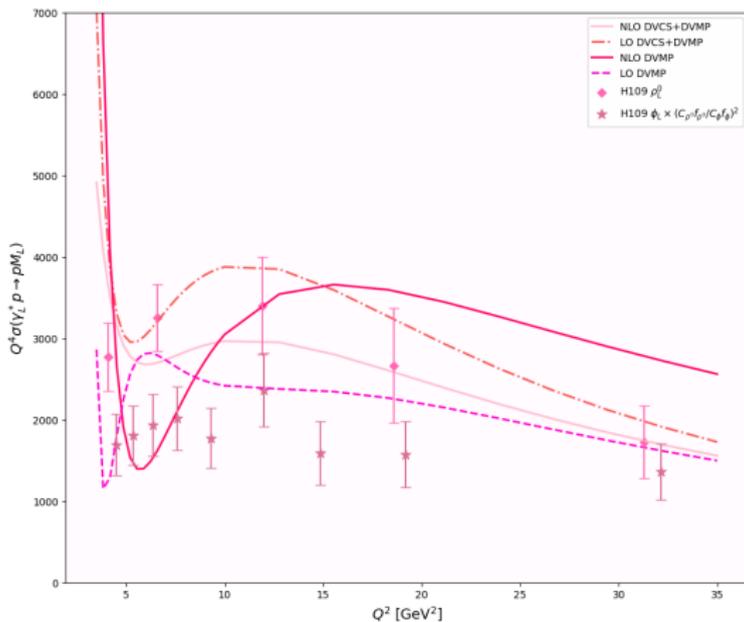


DVMP data description



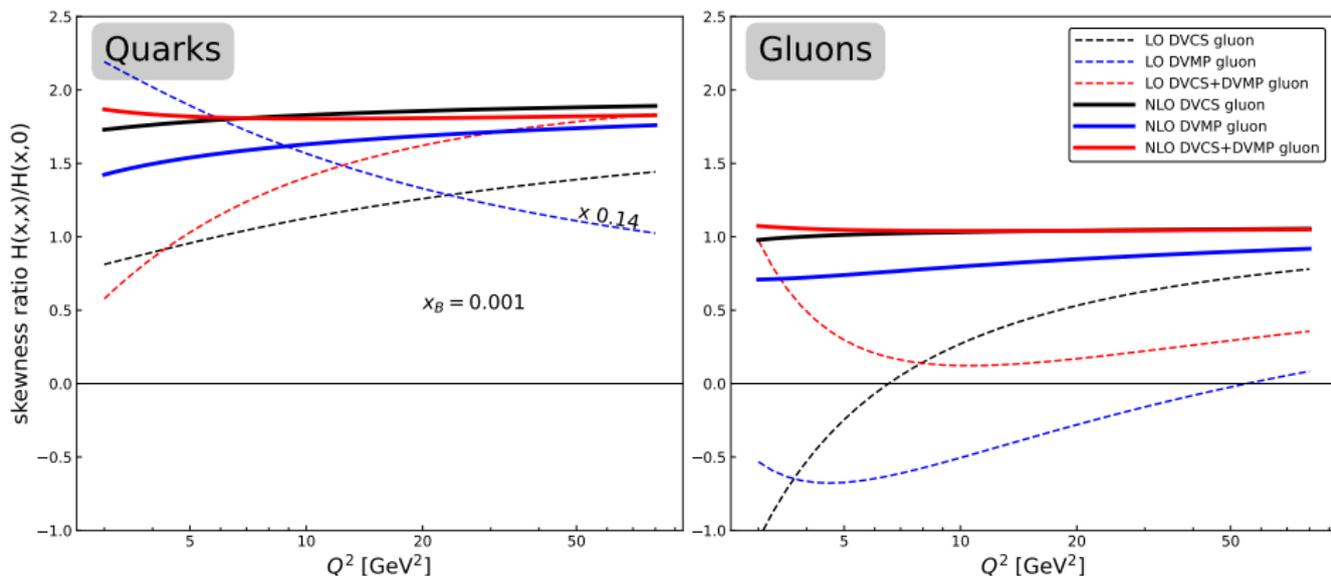
- Things we can learn from fits:
 - Effects of NLO corrections
 - Universality of GPD shape — separately from DVCS and DVMP

DVMP and scaling



- Theory starts behaving and scaling after ~ 10 GeV²
- \rightarrow JLab@20+ opportunity

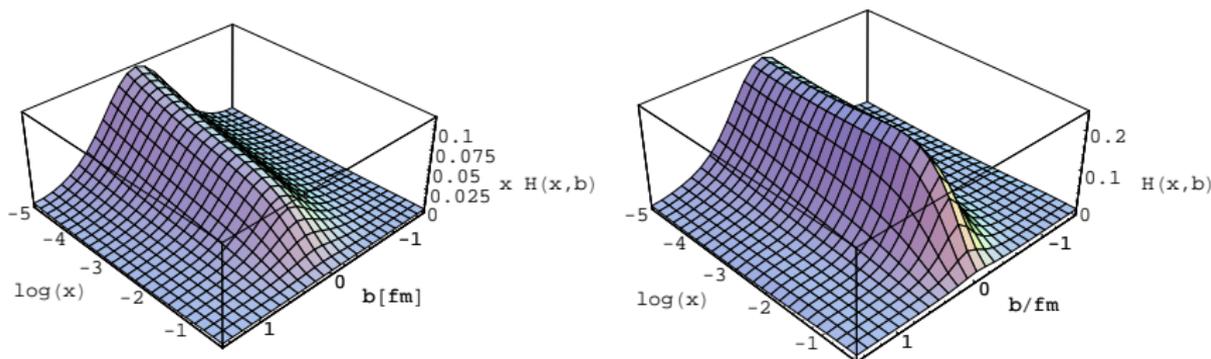
GPD skewness at LO and NLO



- Universal GPD structure emerges **at NLO!**

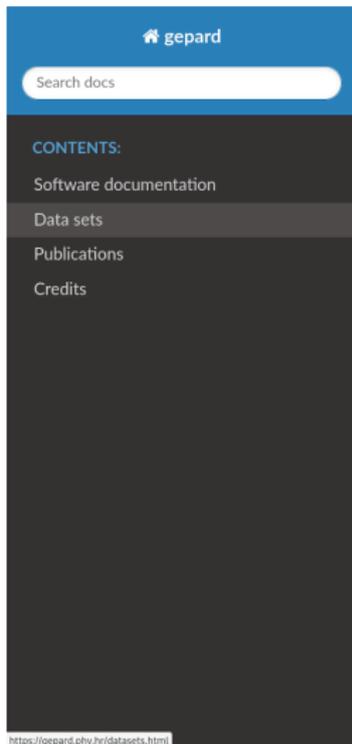
Tomography

- Resulting sea quark and gluon distributions $H(x, \vec{b}_\perp)$:



- (large- x part is still very model dependent)

Gepard - public code for GPD analysis



The screenshot shows the left-hand navigation menu of the Gepard website. At the top is a blue header with the Gepard logo and a search bar labeled 'Search docs'. Below this is a dark grey sidebar with the heading 'CONTENTS:' and four menu items: 'Software documentation', 'Data sets', 'Publications', and 'Credits'. At the bottom of the sidebar, a small URL is visible: 'https://gepard.phy.hr/datasets.html'.

🏠 » Tool for studying the 3D quark and gluon distributions in the nucleon

[View page source](#)

Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

- [Software documentation](#)
 - [Installation](#)
 - [Quickstart](#)
 - [Tutorial](#)
 - [Data points, sets and files](#)

Gepard - publications

- Aiming for **full reproducibility** of results.

The screenshot shows the top navigation bar of the Gepard website. It features a blue header with the 'gepard' logo and a search bar labeled 'Search docs'. Below the header is a dark grey 'CONTENTS:' section with links for 'Software documentation', 'Data sets', 'Publications', 'Accompanying code runs with the latest version of Gepard package', 'Accompanying code runs only with old versions Gepard package', and 'Credits'. The 'Publications' link is highlighted with a white background.

Accompanying code runs with the latest version of Gepard package

These papers have accompanying Jupyter notebooks, published on the github, which are easily runnable after installing the latest version of Gepard:

- K. Kumerički, D. Mueller, K. Passek-Kumerički and A. Schaefer, *Deeply virtual Compton scattering beyond next-to-leading order: the flavor singlet case*, Phys. Lett. B **648** (2007), 186-194, arXiv:[hep-ph/0605237](https://arxiv.org/abs/hep-ph/0605237) [Code at [github](#)]
- K. Kumerički, D. Mueller, and K. Pasek-Kumerički, *Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond*, Nucl. Phys. B **794** (2008) 244-323, arXiv:[hep-ph/0703179](https://arxiv.org/abs/hep-ph/0703179) [Code at [github](#)]
- K. Kumerički and D. Mueller, *Deeply virtual Compton scattering at small x_B and the access to the GPD H* , Nucl. Phys. B **841** (2010) 1-58, arXiv:[0904.0458](https://arxiv.org/abs/0904.0458) [Code at [github](#)]

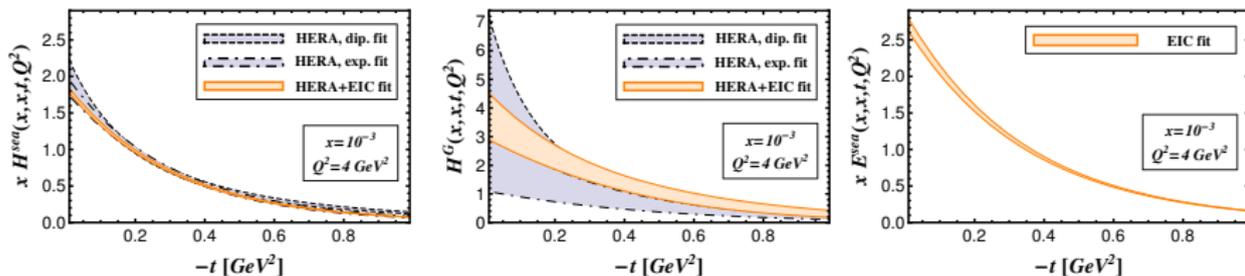
Accompanying code runs only with old versions Gepard package

These papers have accompanying Jupyter notebooks, published on the github, but need old version of Gepard (available as *pyfortran* branch on the Gepard's github page), which can be tricky to compile and run

- M. Čuić, K. Kumerički, and A. Schäfer, *Separation of Quark Flavors using DVCS Data*, Phys. Rev. Lett. **125** (2020) 23, 232005, arXiv:[2007.00029](https://arxiv.org/abs/2007.00029) [Code at [github](#)]
- K. Kumerički, *Measurability of pressure inside the proton*, Nature, 570 (2019) no. 7759, E1-E2,

DVCS at EIC

- This framework was used to estimate impact of EIC [Aschenauer, Fazio, K.K., Müller '13], [EIC white paper]
- Fit to simulated DVCS data ($d\sigma^{\text{DVCS}}/dt$ and A_{UT}) at 20 GeV \times 250 GeV taking $E_{\text{sea}}(x, \xi, t) = \kappa_{\text{sea}} H_{\text{sea}}(x, \xi, t)$



- Improved knowledge of low- t quark and gluon GPDs H (\Rightarrow 3D parton imaging)
- Improved knowledge of sea quark GPD E
- **TODO:** Do the same for DVMP