



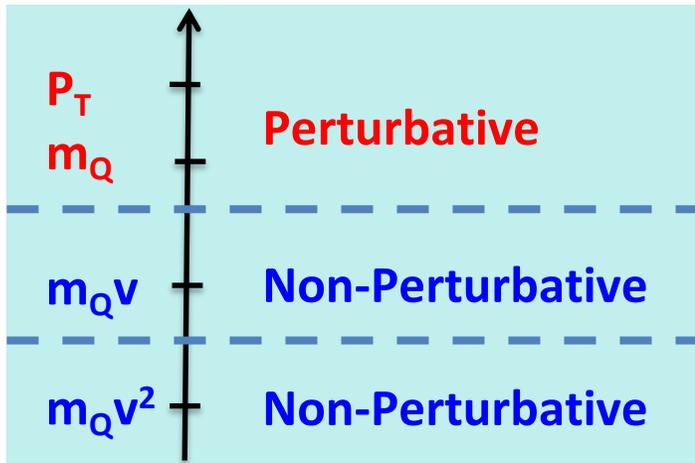
Understanding heavy quarkonium production in terms of QCD factorization

- Challenge to our understanding on how quarkonium is produced?
- A new QCD factorization formalism for pT-distribution of heavy quarkonium production
- Both leading power and next-to-leading power contributions are needed
- Factorized QCD calculations describe well existing data from the LHC and Tevatron
- Summary and outlook

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Heavy quarkonium and its production at high p_T

Well-separated momentum scales – effective theory:



Hard — Production of $Q\bar{Q}$

[pQCD]

Soft — Relative Momentum

[NRQCD]

← Λ_{QCD}

Λ_{QCD} →

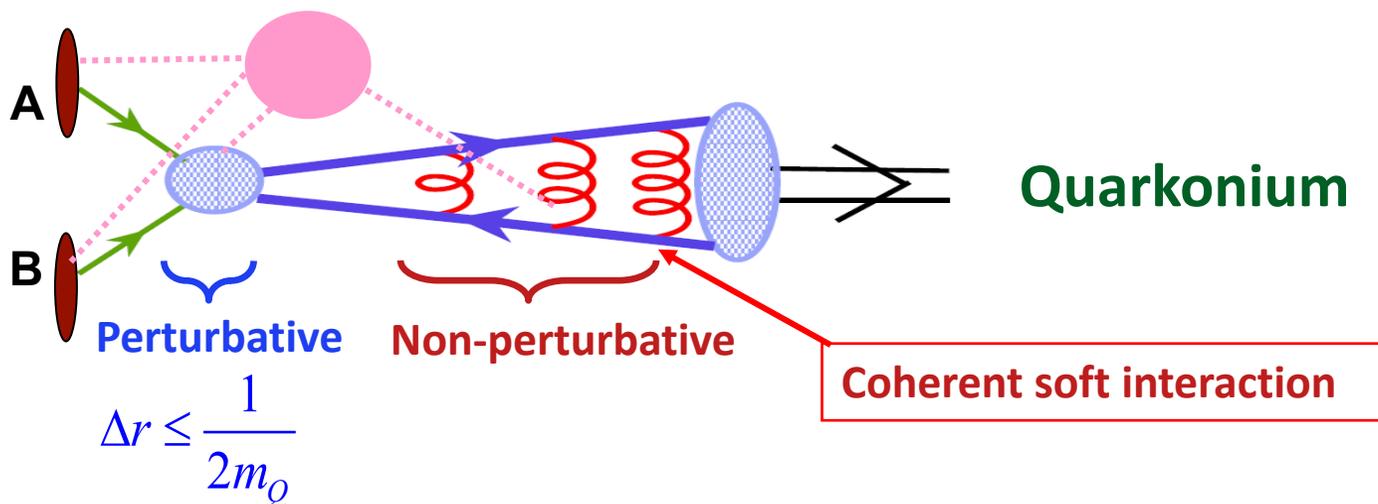
Ultrasoft — Binding Energy

[pNRQCD]

Known quarks

Flavor	Mass
u	1.5 – 4.5 MeV
d	5.0 – 8.5 MeV
s	80 – 155 MeV
c	1.0 – 1.4 GeV
b	4.0 – 4.5 GeV
t	174.3 ± 5.1 GeV

Basic production mechanism:



- QCD Factorization is “expected” to work for the production of heavy quarks
- Difficulty: how the heavy quark pair becomes a quarkonium?

NRQCD factorization and the “lack” of universality of LDMEs

NRQCD factorization:

$$d\sigma_{A+B \rightarrow H+X} = \sum_n d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle$$

Expansion in powers of both α_s and v !

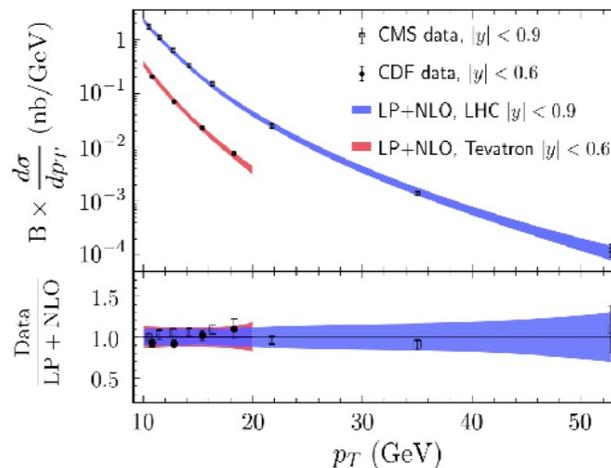
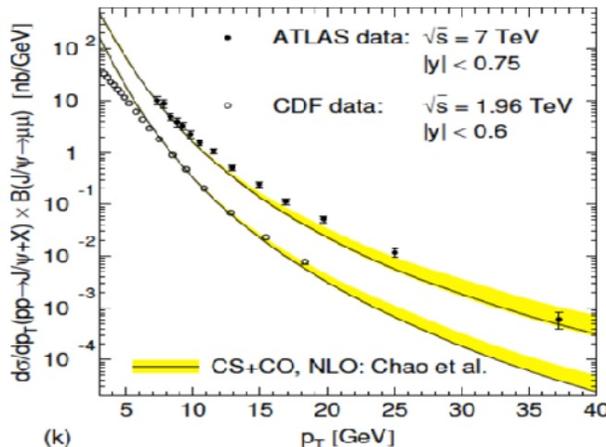
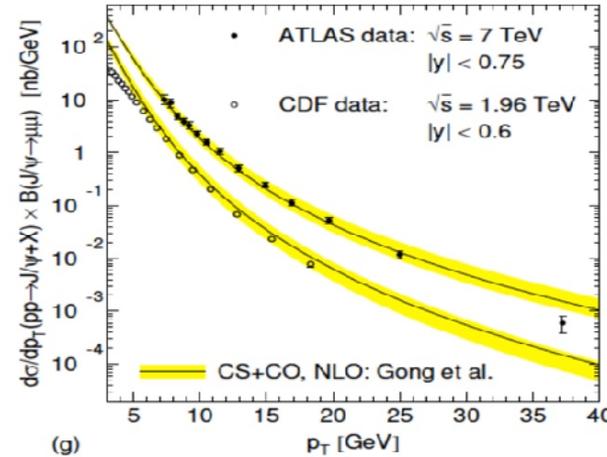
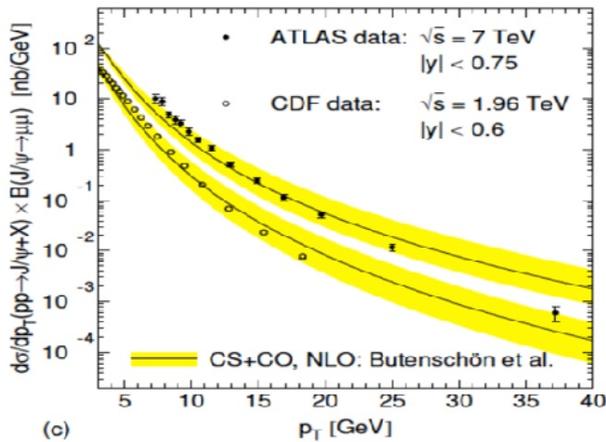
Hadronization

Bodwin, Braaten, Lepage, PRD, 1995

4 leading channels in v :

$${}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}$$

Phenomenology – full NLO in α_s :



	$\langle \mathcal{O}({}^3S_1^{[1]}) \rangle$ GeV ³	$\langle \mathcal{O}({}^1S_0^{[8]}) \rangle$ 10 ⁻² GeV ³	$\langle \mathcal{O}({}^3S_1^{[8]}) \rangle$ 10 ⁻² GeV ³	$\langle \mathcal{O}({}^3P_0^{[8]}) \rangle$ 10 ⁻² GeV ⁵
Set I (Butenschön <i>et al.</i>)	1.32	3.04	0.16	-0.91
Set II (Chao <i>et al.</i>)	1.16	8.9	0.30	1.26
Set III (Gong <i>et al.</i>)	1.16	9.7	-0.46	-2.14
Set IV (Bodwin <i>et al.</i>)	-	9.9	1.1	1.1

LDMEs should be universality, however:

- Numbers are not the same.
- Not even the sign.

More work is needed!

Fits in NRQCD

Butenschön, Kniehl, PRD84, 051501 (2011).

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).

Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).

Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).

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Fits in pNRQCD

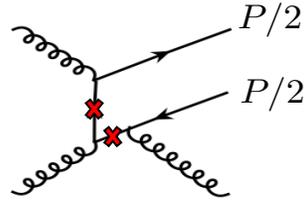
Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

Heavy quarkonium production of high p_T

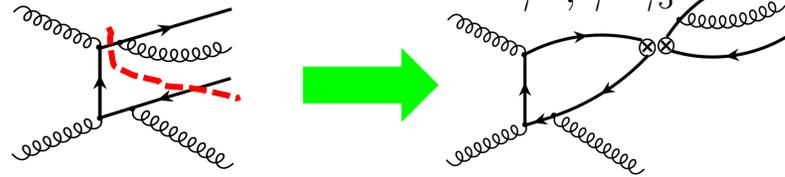
Z.B. Kang, et al., PRL 2011

□ $O(\alpha_s)$ expansion vs. $1/p_T$ expansion:

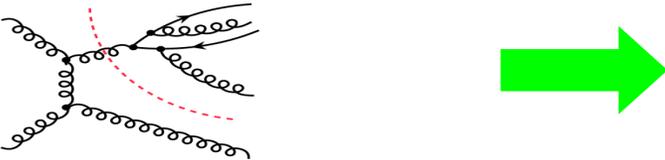
LO in α_s :



NLO in α_s :



NNLO in α_s :



CS channel as a case study

$$\hat{\sigma}^{\text{LO}} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

p_T Power!

$$\hat{\sigma}^{\text{NLO}} \rightarrow \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu) \log(\mu^2/\mu_0^2)$$

$\mu_0 \gtrsim 2m_Q$

$$\hat{\sigma}^{\text{NNLP}} \rightarrow \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu) \log^m(\mu^2/\mu_0^2)$$

- When $p_T \gg m_Q$, the expansion in powers of α_s is not reliable!
- Leading order in α_s -expansion \neq leading power in $1/p_T$ -expansion!

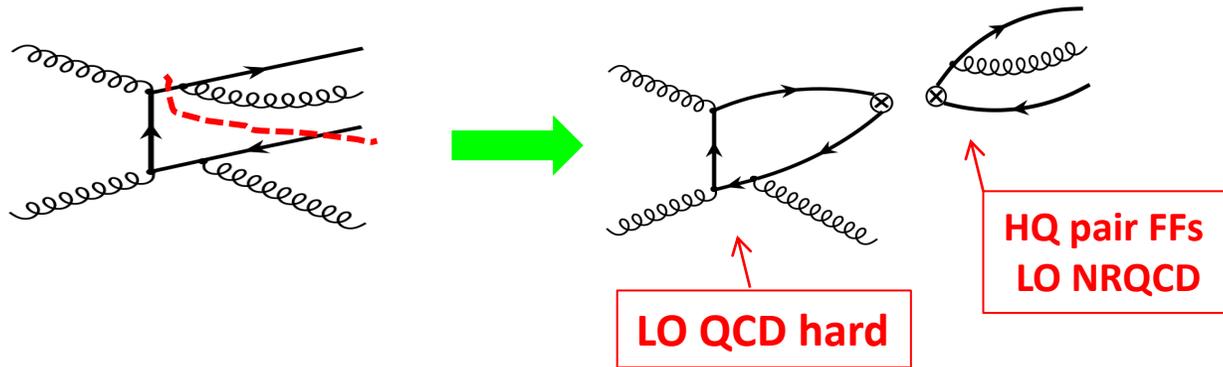
□ PQCD factorization:

- $1/p_T$ expansion first: leading power (LP) & next-to-leading power (NLP) are factorizable!
- $O(\alpha_s)$ -expansion: leading order (LO) & next-to-leading order (NLO) are calculated

QCD factorization + NRQCD factorization

Kang, Qiu and Sterman, 2011

□ Color singlet as an example:

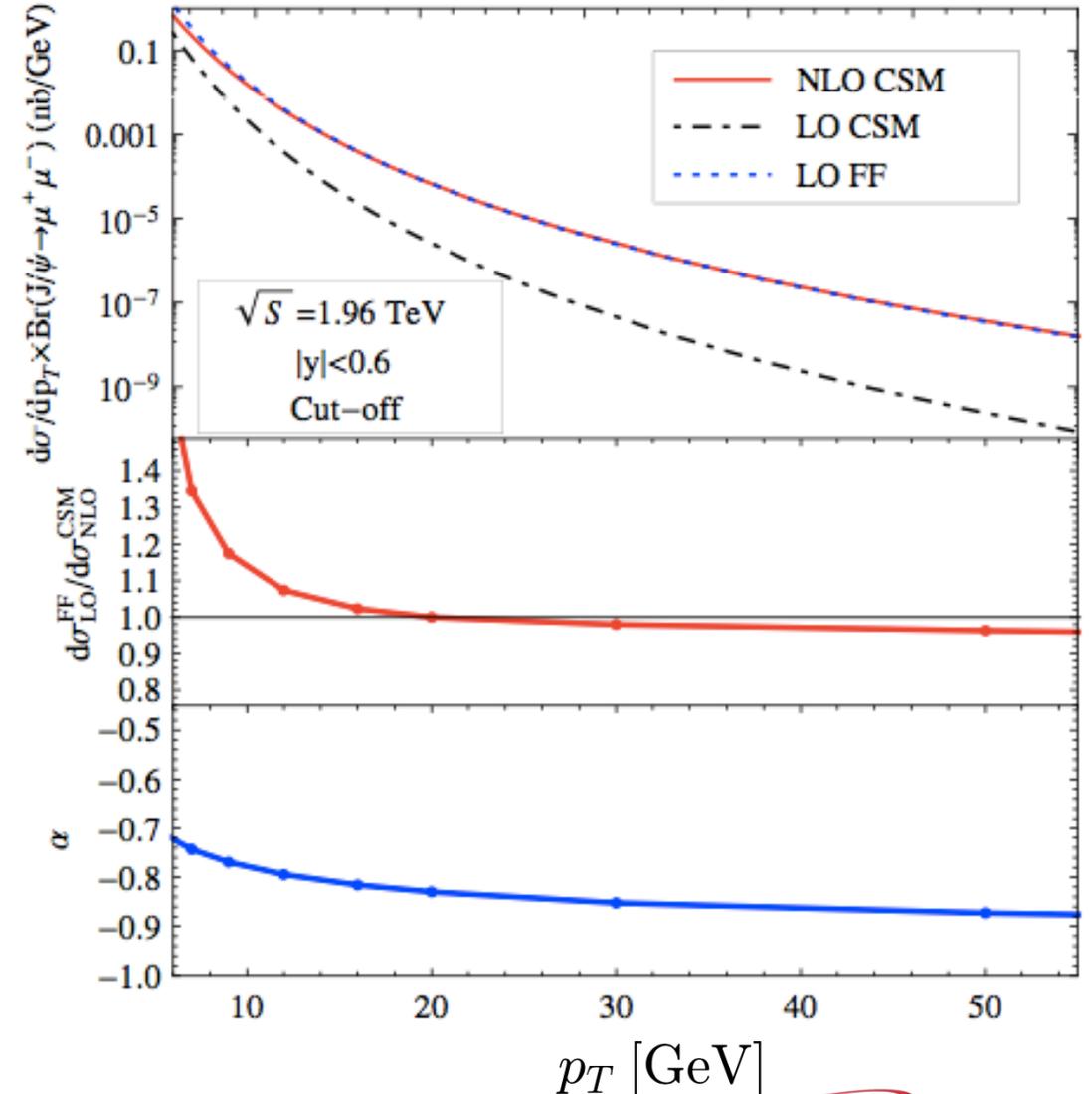


$$\sigma_{\text{NRQCD}}^{(\text{NLO})} \propto \left[d\hat{\sigma}_{ab \rightarrow [Q\bar{Q}(v8)]}^{V(\text{LO})} \otimes \mathcal{D}_{[Q\bar{Q}(v8)] \rightarrow J/\psi}^{(\text{LO})} + d\hat{\sigma}_{ab \rightarrow [Q\bar{Q}(a8)]}^{A(\text{LO})} \otimes \mathcal{D}_{[Q\bar{Q}(a8)] \rightarrow J/\psi}^{(\text{LO})} \right]$$

Reproduce NLO CSM for $p_T > 10$ GeV!

Cross section + polarization

**Different kinematics, different approximation,
Dominance of different production channels!**



Heavy quarkonium production of high p_T

Lee, Qiu, Sterman, Watanabe, 2022

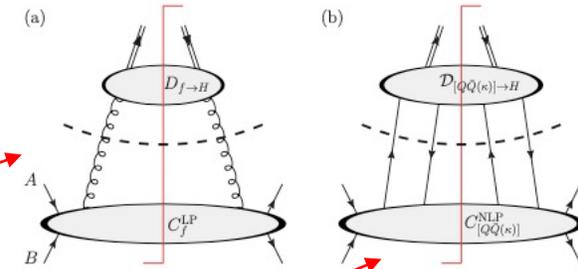
□ PQCD + NRQCD factorization:

$$E \frac{d\sigma_{hh' \rightarrow J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} \langle O_{c\bar{c}[n]}^{J/\psi}(0) \rangle \sum_{a,b} \int dx_a f_{a/h}(x_a, \mu_f^2) \int dx_b f_{b/h'}(x_b, \mu_f^2) \\ \times \left[E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Resum}}}{d^3P} + E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{NRQCD}}}{d^3P} - E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Asym}}}{d^3P} \right]$$

$$c\bar{c}[n] = c\bar{c}[^{2S+1}L_J^{[1,8]}]$$

- PQCD factorization + NRQCD FFs: $\kappa = (v, a, t)^{[1,8]}$

$$E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Resum}}}{d^3P} \approx \sum_f \int \frac{dz}{z^2} D_{f \rightarrow c\bar{c}[n]}(z, \mu_f^2) E_f \frac{d\hat{\sigma}_{ab \rightarrow f(p_f)X}}{d^3p_f}(z, p_f = P/z, \mu_f^2) \\ + \sum_{[c\bar{c}(\kappa)]} \int \frac{dz}{z^2} D_{[c\bar{c}(\kappa)] \rightarrow c\bar{c}[n]}(z, \mu_f^2) E_c \frac{d\hat{\sigma}_{ab \rightarrow [c\bar{c}(\kappa)](p_c)X}}{d^3p_c}(z, p_c = P/z, \mu_f^2)$$



Kang, Ma, Qiu, Sterman, 2014

- NRQCD fixed-order:

$$E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{NRQCD}}}{d^3P}$$

When $P_T \gg m_c$, $E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Asym}}}{d^3P}$ **cancels** $E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{NRQCD}}}{d^3P}$

- Asymptotic contribution:

$$E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Asym}}}{d^3P} = E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Resum}}}{d^3P} \Big|_{\text{fixed order}}$$

When $P_T \gtrsim m_c$, $E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Asym}}}{d^3P}$ **cancels** $E \frac{d\tilde{\sigma}_{ab \rightarrow c\bar{c}[n](P)X}^{\text{Resum}}}{d^3P}$

Renormalization group improvement

- Twist-2 evolution equation: DGLAP + **nonlinear quark pair corrections**

Kang, Ma, Qiu, Sterman, PRD 90, 3, 034006 (2014)

$$\frac{\partial D_{[f] \rightarrow H}}{\partial \ln \mu^2} = \gamma_{[f] \rightarrow [f']} \otimes D_{[f'] \rightarrow H} + \frac{1}{\mu^2} \gamma_{[f] \rightarrow [Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}$$

v: vector
a: axial vector
t: tensor

The inhomogeneous term is added to the **slope**, not to the FF itself.

- Twist-4 “DGLAP like” evolution equation:

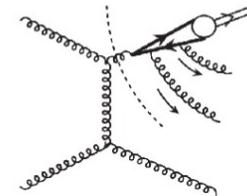
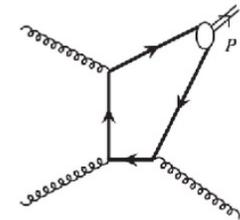
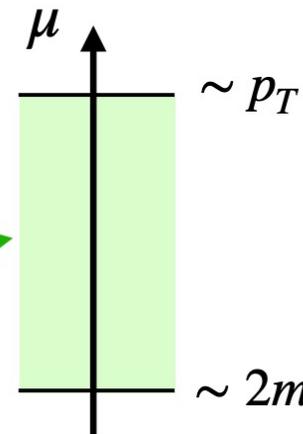
$$\frac{\partial \mathcal{D}_{[Q\bar{Q}(n)] \rightarrow H}}{\partial \ln \mu^2} = \Gamma_{[Q\bar{Q}(n)] \rightarrow [Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}$$

$$f, f' = q, Q, g$$

$$\kappa, n = v^{[8]}, v^{[1]}, a^{[8]}, a^{[1]}, t^{[8]}, t^{[1]}$$

The RG improved factorized cross section covers all events in which the heavy quark pair can be produced:

- at the short-distance: early stage (**NLP**)
- at the input scale: later stage (**LP**)
- in-between (**Nonlinear quark pair correction**)



Evolution of $c\bar{c}$ -fragmentation function in μ, ν space

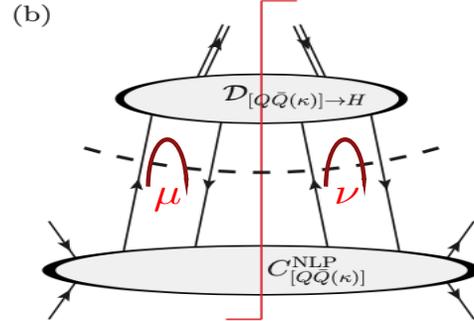
Consider the derivative of a test function:

$$D'_{\kappa \rightarrow n}(z, u, v) \equiv \frac{2\pi}{\alpha_s} \frac{dD_{\kappa \rightarrow n}(z, u, v)}{d \ln \mu^2},$$

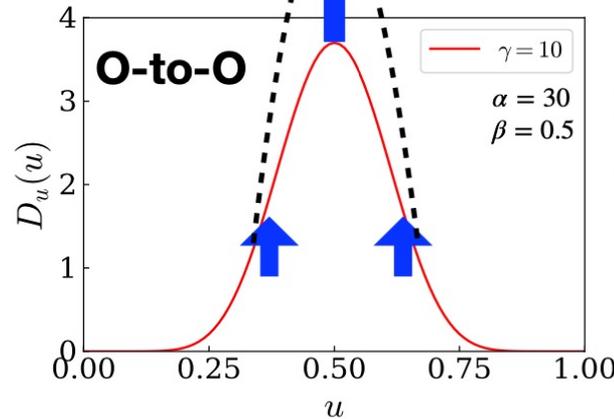
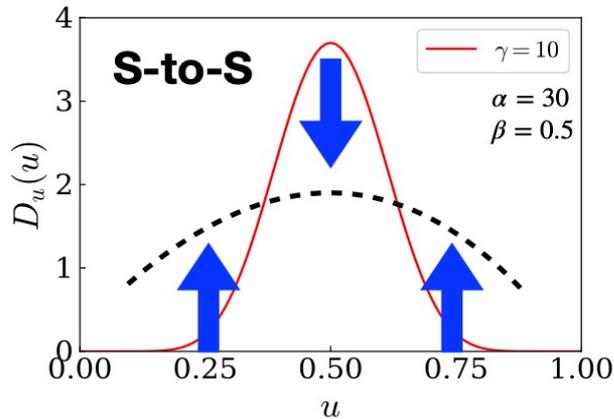
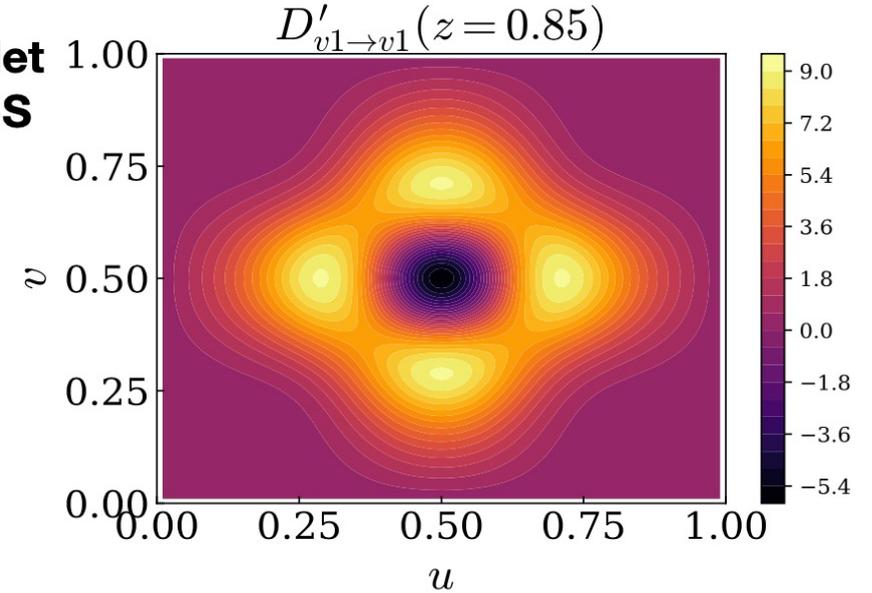
$$D(z, u, v) \rightarrow D_z(z)D_u(u)D_v(v),$$

$$D_z(z, \alpha) = \frac{z^\alpha(1-z)^\beta}{B[1+\alpha, 1+\beta]},$$

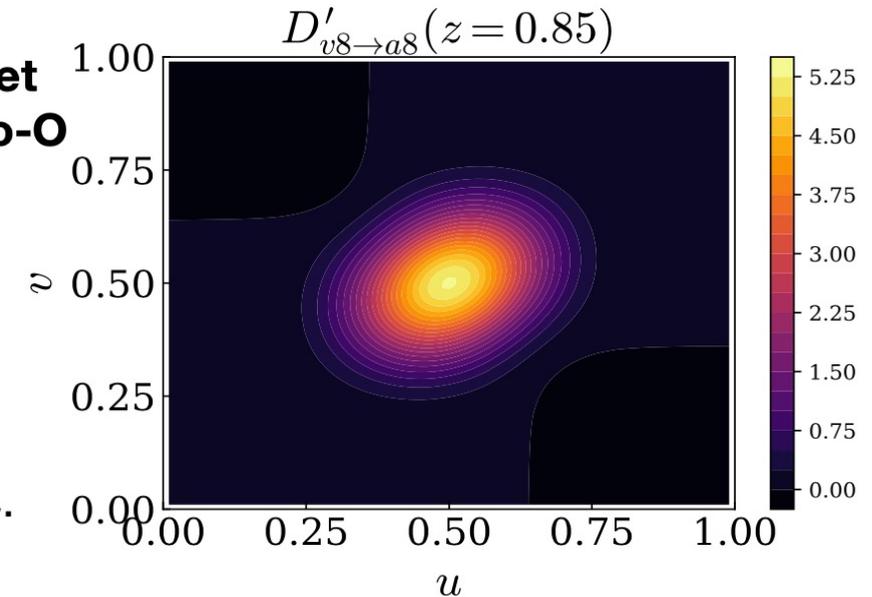
$$D_{u,v}(x, \gamma) = \frac{x^\gamma(1-x)^\gamma}{B[1+\gamma, 1+\gamma]},$$



Diagonal singlet channel: S-to-S

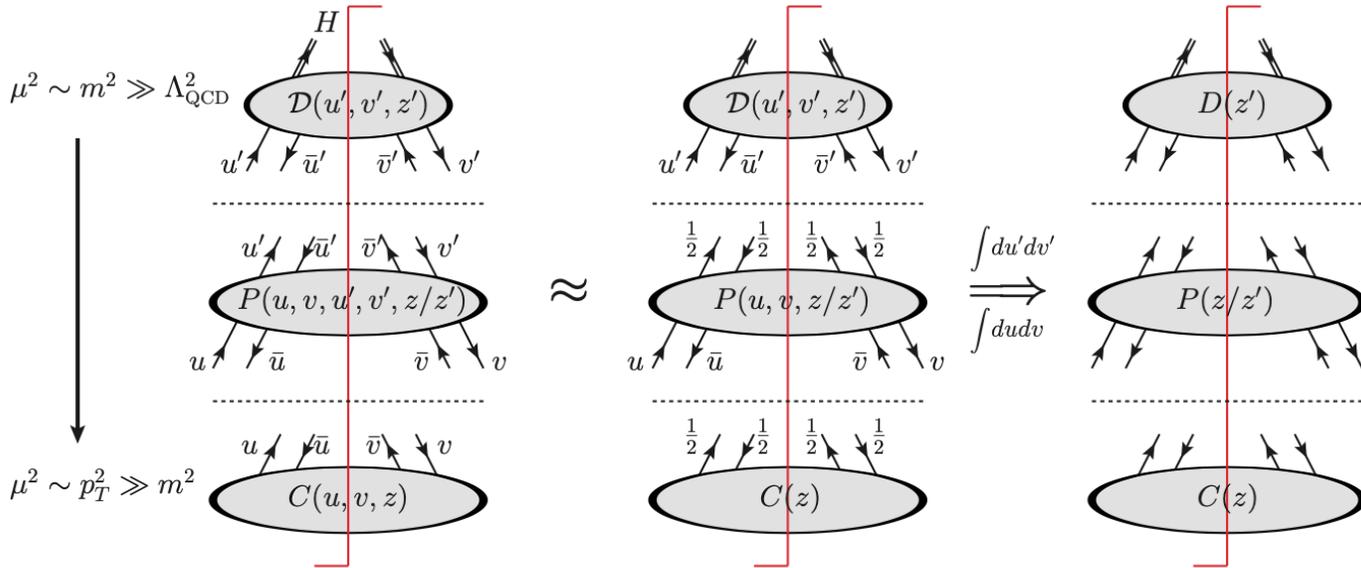


Diagonal octet channel: O-to-O



- S-to-S DP FFs get **broader** in u, ν -space after evolution.
- O-to-O DP FFs become **narrower** with a large peak around $u = \nu = 1/2$.
- Off-diagonal channels: similar to O-to-O.

Evolution equations in a simplified situation



- Since the produced heavy quark pair is dominated by its on-shell state, we may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around $u = v = 1/2$.
- This can be a reasonable approximation suggested by the evolution of DP FFs in u, v -space.

$$\frac{d\sigma_{\text{NLP}}^H}{dyd^2p_T} = \int dzdudv C_{[Q\bar{Q}]}(p_Q, p_{\bar{Q}}, \mu) \mathcal{D}_{[Q\bar{Q}] \rightarrow H}(u, v, z, \mu) \approx \int dz C_{[Q\bar{Q}]}(\hat{p}_Q^+ = \frac{1}{2}p_c^+, \hat{p}_{\bar{Q}}^+ = \frac{1}{2}p_c^+, \mu) \underbrace{\int dudv \mathcal{D}_{[Q\bar{Q}] \rightarrow H}(u, v, z, \mu)}_{\equiv D_{[Q\bar{Q}] \rightarrow H}(z, \mu)}$$

$$\frac{\partial D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z, \mu)}{\partial \ln \mu^2} \approx \sum_n \int_z^1 \frac{dz'}{z'} \int_0^1 du \int_0^1 dv \Gamma_{[Q\bar{Q}(n)] \rightarrow [Q\bar{Q}(\kappa)]} \left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'} \right) D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z', \mu),$$

$$\frac{\partial D_{f \rightarrow H}(z, \mu)}{\partial \ln \mu^2} \approx \frac{\alpha_s}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f \rightarrow f'}(z/z') D_{f' \rightarrow H}(z') + \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \int_z^1 \frac{dz'}{z'} P_{f \rightarrow [Q\bar{Q}(\kappa)]} \left(u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'} \right) D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z', \mu)$$

Input fragmentation functions at $\mu_0 \sim \# m_c$

Ma, Qiu, Zhang, PRD89 (2014) 094029; ibid. 94030
Lee, Qiu, Steerman, Watanabe, SciPost Phys. Proc. 8, 143 (2022)

$$D_{f \rightarrow H}(z; m, \mu_0) = \sum_{[Q\bar{Q}(n)]} \pi \alpha_s \left\{ \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}^{(1)}(z; m, \mu_0, \mu_\Lambda) + \frac{\alpha_s}{\pi} \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}^{(2)}(z; m, \mu_0, \mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}_{[Q\bar{Q}(n)]}^H(\mu_\Lambda) \rangle}{m^{2L+3}}$$

$$D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z; m, \mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}_{[Q\bar{Q}(\kappa)] \rightarrow [Q\bar{Q}(n)]}^{(0)}(z; m, \mu_0, \mu_\Lambda) + \frac{\alpha_s}{\pi} \hat{d}_{[Q\bar{Q}(\kappa)] \rightarrow [Q\bar{Q}(n)]}^{(1)}(z; m, \mu_0, \mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}_{[Q\bar{Q}(n)]}^H(\mu_\Lambda) \rangle}{m^{2L+1}}$$

$$\mu_0 = \mathcal{O}(2m): \text{input scale}, \mu_\Lambda = \mathcal{O}(m): \text{NRQCD factorization scale} \quad \kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$$

Perturbative SDCs of input FFs in α_s and v expansion in the NRQCD are reliable only when SDCs $\ll \mathcal{O}(1)$. Indeed, the NRQCD factorization is not reliable as $z \rightarrow 1$ where SDCs $\hat{d}(z)$ include the following terms:

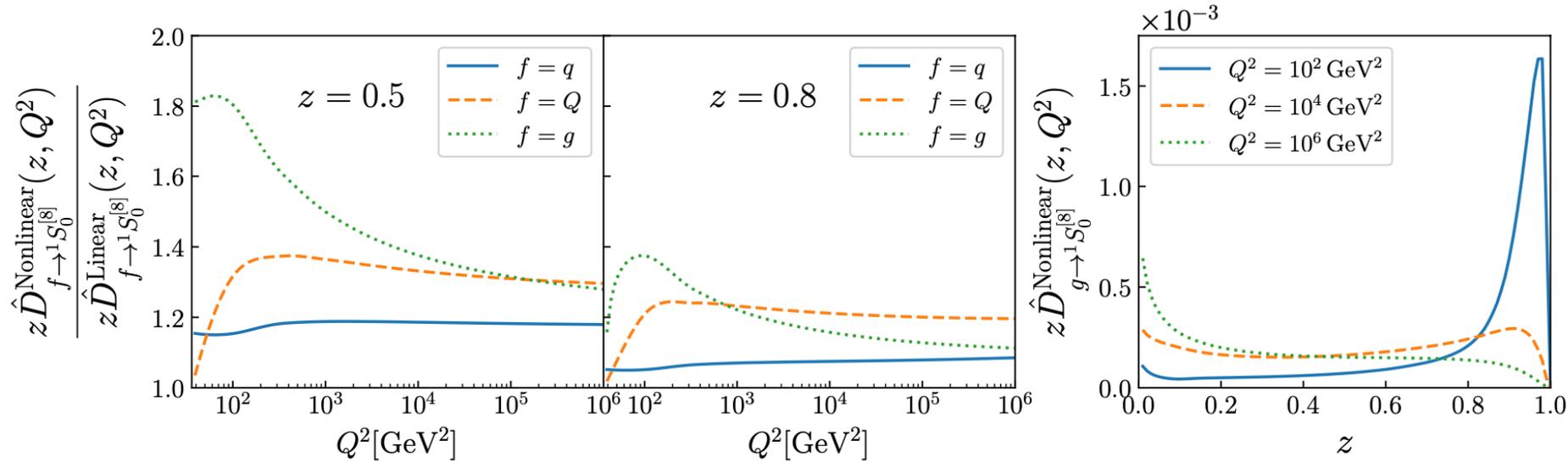
1. $\delta(1 - z)$ at LO in α_s expansion
2. $f(z)\ln(1 - z)$ with $f(z)$ being a regular function
3. $\frac{f(z)}{[1 - z]_+}$, $f(z) \left[\frac{\ln(1 - z)}{1 - z} \right]_+$ due to the perturbative cancelation of IR divergences

In our current analysis, we use analytic results if those vanish as $z \rightarrow 1$, otherwise, singular or negative input FFs are cast into

$$D_{[Q\bar{Q}(n)]}(z) = C_{[Q\bar{Q}(n)]}(\alpha_s) \frac{z^\alpha (1 - z)^\beta}{B[1 + \alpha, 1 + \beta]} \quad (\alpha \gg 1, 1 > \beta > 0) \quad C_{[Q\bar{Q}(n)]}: \text{abs. value of the first moment}$$

NLP contribution to single parton fragmentation functions

Lee, Qiu, Steerman, Watanabe, *SciPost Phys. Proc. 8, 143 (2022)*



$\alpha = 30, \beta = 0.5$
 $\mu_0 = 4m_c = 6 \text{ GeV}$
 $\mu_\Lambda = m_c = 1.5 \text{ GeV}$

The nonlinear quark pair corrections remain significant even at high $Q^2 = \mu^2 \sim p_T^2$.

$$\frac{\partial D_{[f] \rightarrow H}}{\partial \ln \mu^2} = \gamma_{[f] \rightarrow [f]} \otimes D_{[f] \rightarrow H} + \frac{1}{\mu^2} \gamma_{[f] \rightarrow [Q\bar{Q}(\kappa)]} \otimes \cancel{D_{[Q\bar{Q}(\kappa)] \rightarrow H}} \longrightarrow \gamma_{[f] \rightarrow [f]} \otimes D_{[f] \rightarrow H}$$

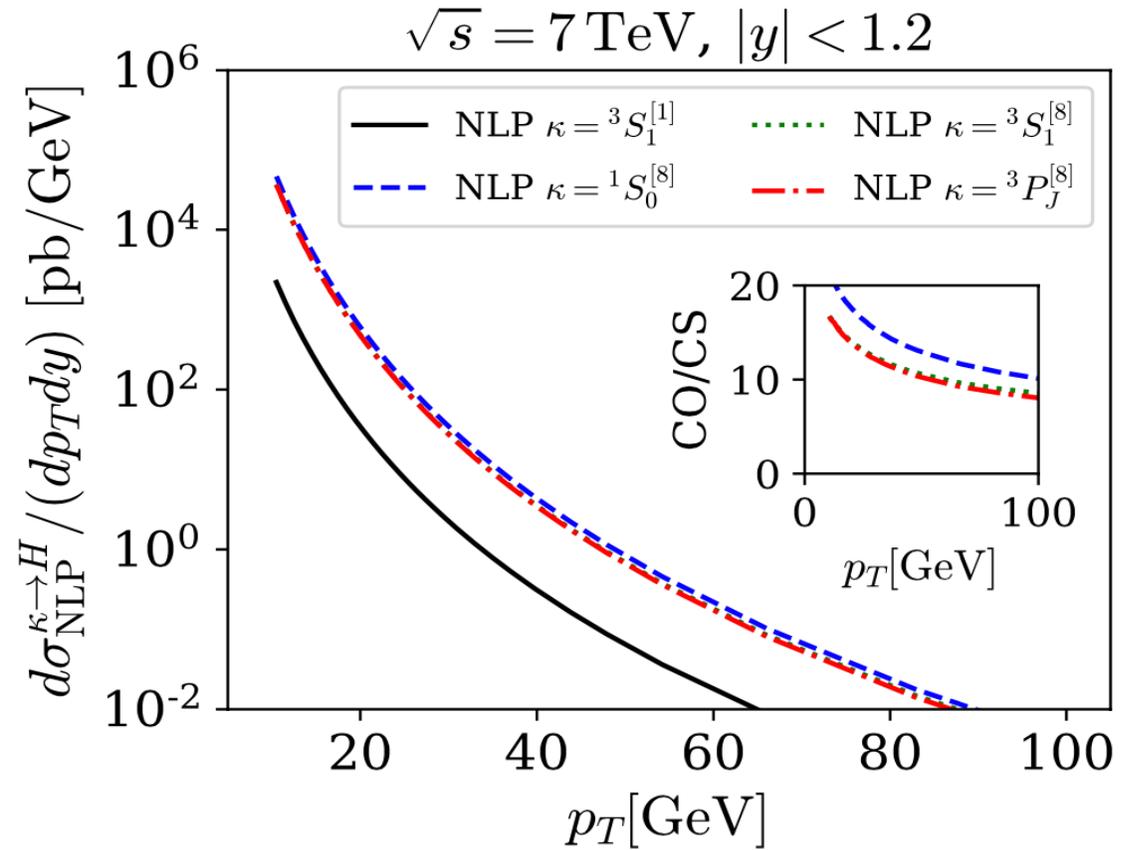
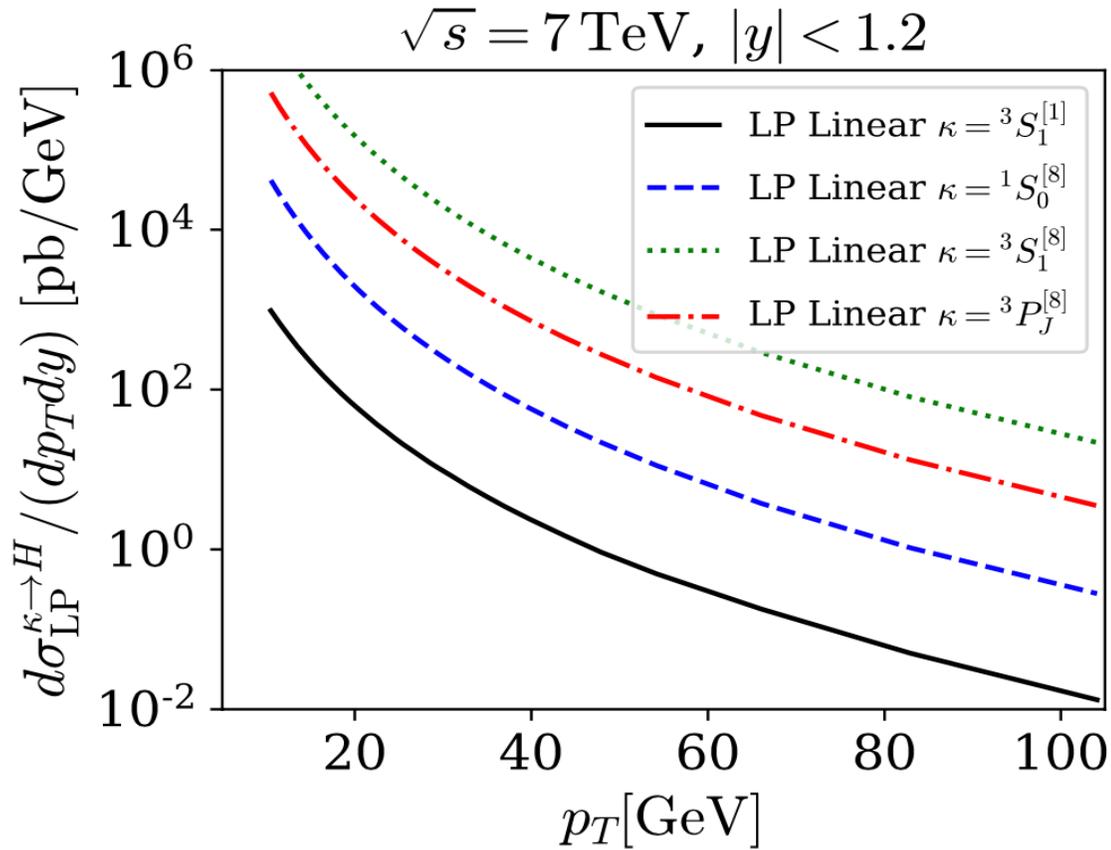
$$\frac{\partial D_{[f] \rightarrow H}^{\text{Nonlinear}}}{\partial \ln \mu^2} \sim \frac{\partial D_{[f] \rightarrow H}^{\text{Linear}}}{\partial \ln \mu^2} \quad \text{at high } \mu^2$$

The power corrections effect at low μ^2 does not go away fast:
analogous to nonlinear gluon recombination effects to gluon PDF at small- x and large μ^2 .

Mueller and Qiu, NPB268, 427 (1986)
 Qiu, NPB291, 746 (1987)
 Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

J/ ψ -production in hadronic collisions

Lee, Qiu, Steerman, Watanabe, in preparation

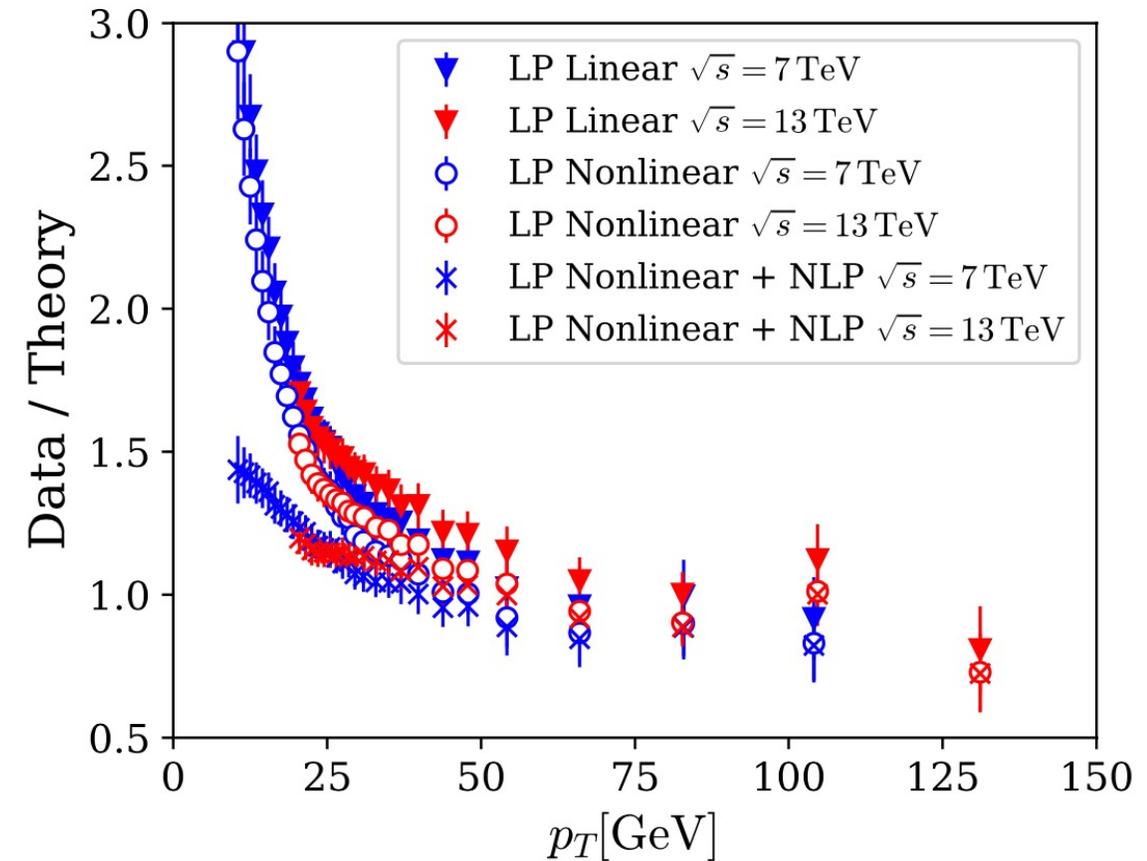


- Unweighted results: $\langle \mathcal{O}({}^3S_1^{[1]}) \rangle / \text{GeV}^3 = \langle \mathcal{O}({}^1S_0^{[8]}) \rangle / \text{GeV}^3 = \langle \mathcal{O}({}^3S_1^{[8]}) \rangle / \text{GeV}^3 = \langle \mathcal{O}({}^3P_0^{[8]}) \rangle / \text{GeV}^5 = 1$.
- $\alpha = 30, \beta = 0.5$ are fixed for both SP and DP FFs.
- NLP contributions steeply fall with p_T .

J/ψ -production in hadronic collisions

Lee, Qiu, Steerman, Watanabe, *SciPost Phys. Proc.* 8, 143 (2022)

- Fitting the LP formalism with the linear evolution eq. to CMS data on high p_T prompt J/ψ at $\sqrt{s} = 7, 13$ TeV in the bin, $|y| < 1.2$.
- # of data points in a fit: 3@7TeV + 4@13TeV = 7 for $p_T \geq 60$ GeV.
- Only the $^1S_0^{[8]}$ channel is considered, yielding unpolarized J/ψ . The other three leading LDMEs = 0.
- $\langle \mathcal{O}(^1S_0^{[8]}) \rangle / \text{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ fitted by high p_T data is similar to the one extracted using fixed order NRQCD at NLO. [Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 \(2012\)](#)

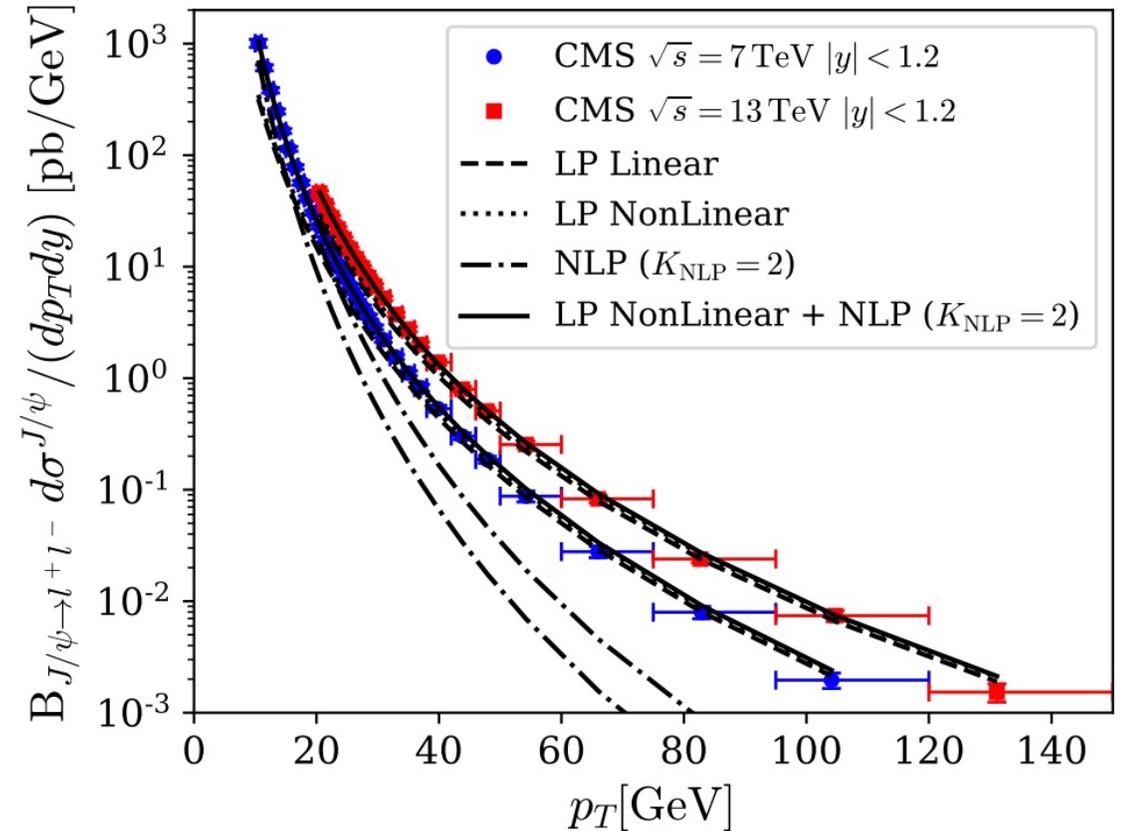


The power corrections do not vanish even at the highest p_T , giving **10-30% corrections**.
At $p_T = 30$ GeV and below, the NLP corrections become significant.

J/ψ -production in hadronic collisions

Lee, Qiu, Steerman, Watanabe, *SciPost Phys. Proc.* 8, 143 (2022)

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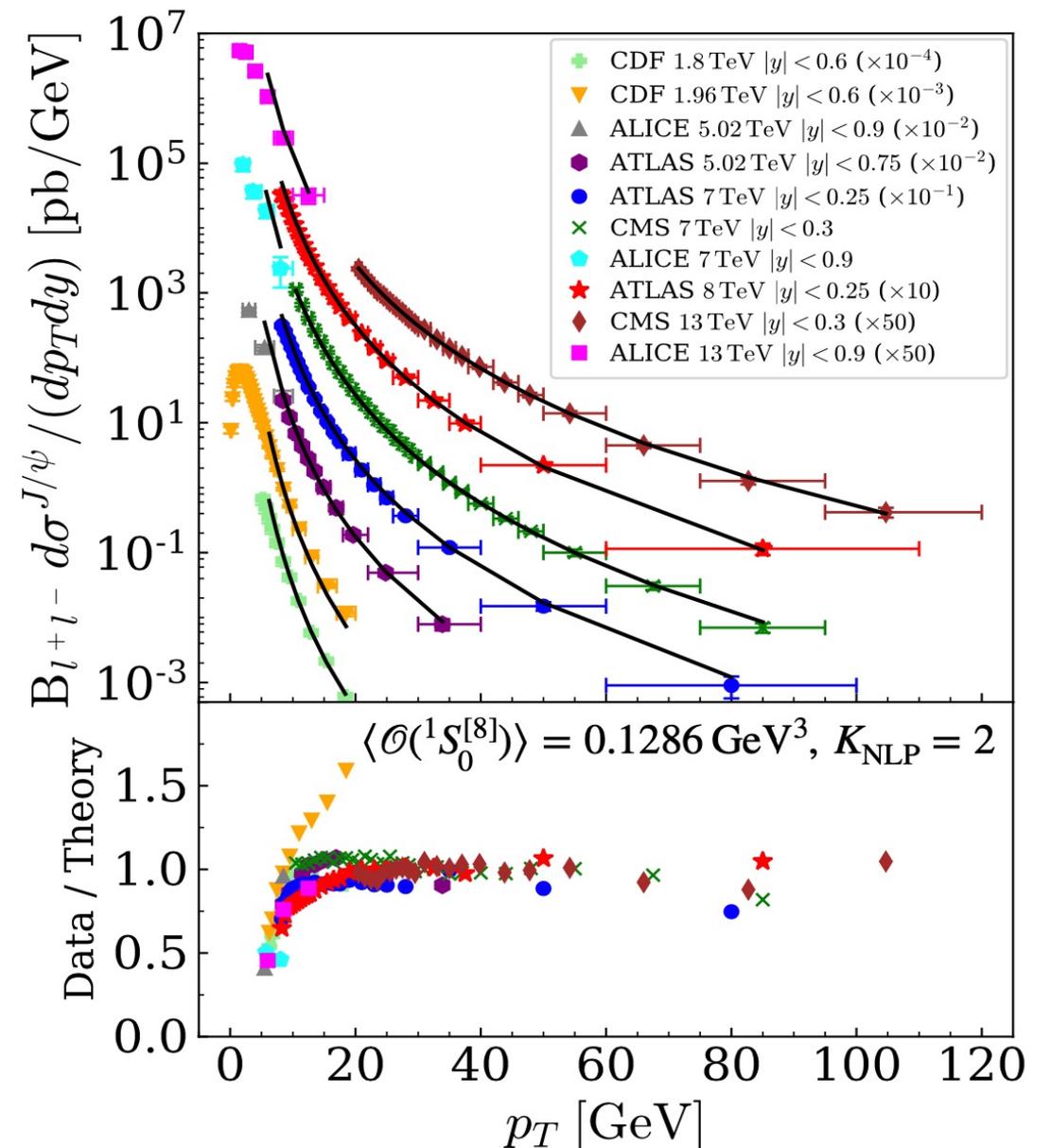
J/ψ -production in hadronic collisions

- Given that the overall normalization factor is fixed, QCD factorization approach describes LHC data on prompt J/ψ production in hadronic collisions.

→ **QCD global data analysis is possible.**

- K -factor is included to account for higher order corrections of the NLP partonic cross section.
- At Tevatron energies, we need to modify the normalization factor as K depends on \sqrt{s} .

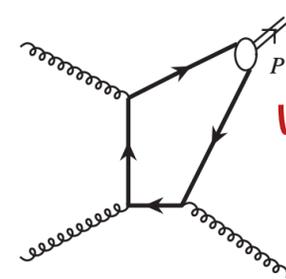
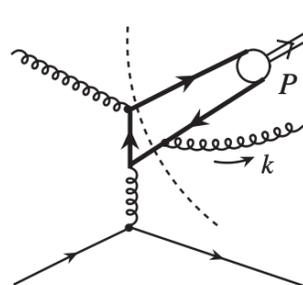
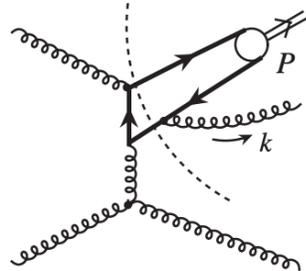
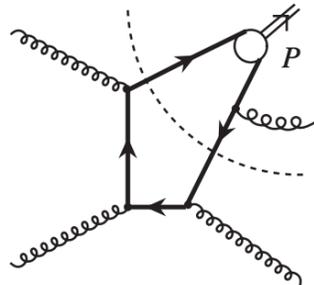
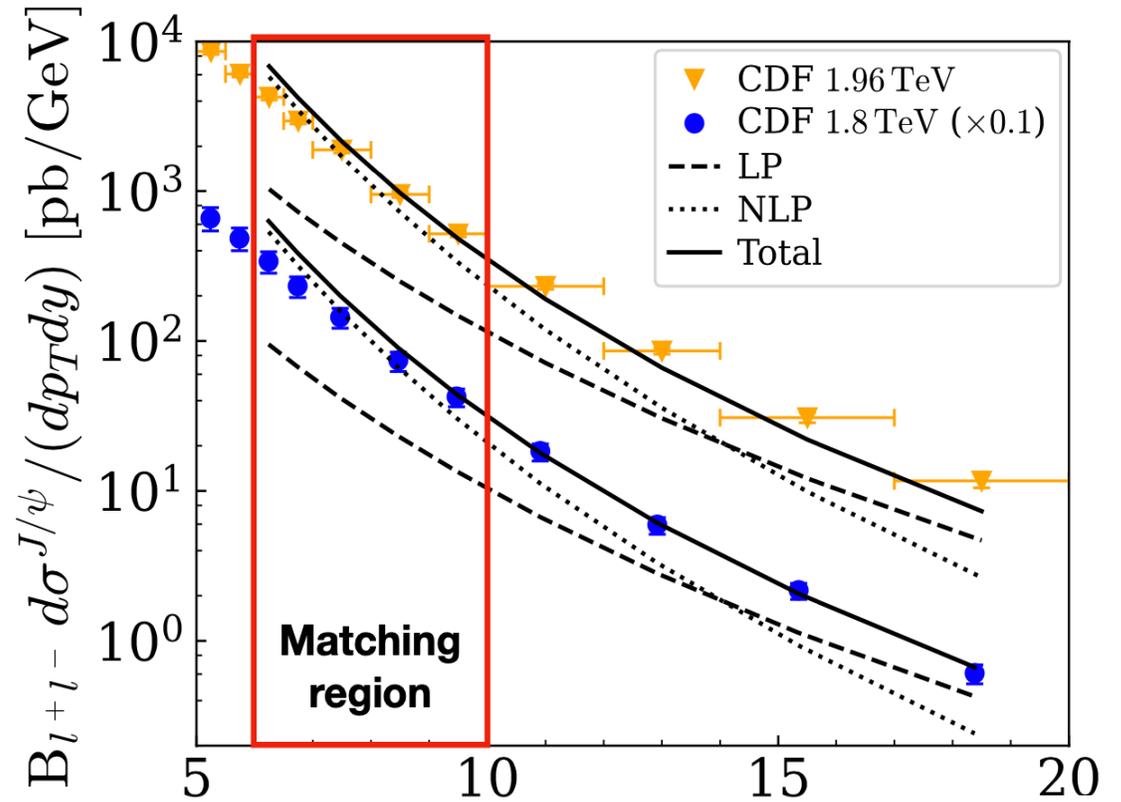
Lee, Qiu, Steerman, Watanabe, in preparation



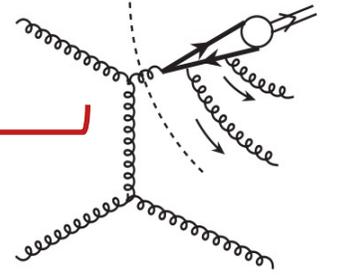
Matching to fixed-order NRQCD calculation

Lee, Qiu, Steerman, Watanabe, in preparation

1. $\ln(p_T^2/m^2)$ -type logarithmically enhanced contributions start to dominate when $p_T \gtrsim 5$ (or 7) $(2m_c) \sim 15 - 20$ GeV, where the LP is significant, power corrections are small.
2. The NLP contribution is important at $p_T = \mathcal{O}(2m_c) \lesssim 10$ GeV, where matching between QCD factorization and NRQCD factorization can be made.
3. Further exploration of the shape of the FFs at large- z would help us understand the quarkonium production mechanism.



p_T [GeV]



Summary and Outlook

- It has been almost 50 years since the discovery of J/Ψ , but, we are still not completely sure about its production mechanism
- We have studied the QCD factorization for hadronic quarkonium production at high p_T
- We demonstrated that the LP contributions are significant for hadronic quarkonium production at high p_T while the NLP contributions are sizable at lower p_T but different in shape, and both are needed, leading to a smooth matching to fixed-order calculations
- Power corrections to the evolution of LP FFs are important even at high p_T , impacting quarkonium polarization
- QCD factorization formalism should make possible a new global data analysis. There is sufficient room to improve the input FFs
- Matching between the QCD factorization and fixed order NRQCD factorization should enable us to describe quarkonium production not only in hadronic collisions but also in other scattering processes in a broader p_T region.

Thanks!