



Hadron structure and spectroscopy with functional methods

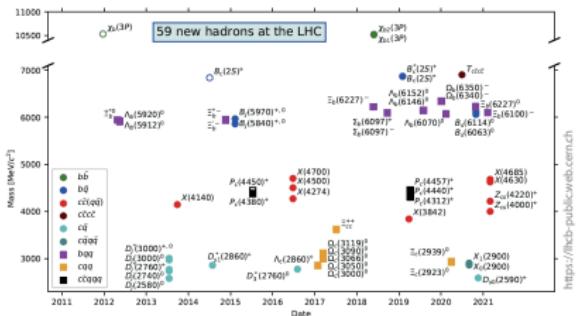
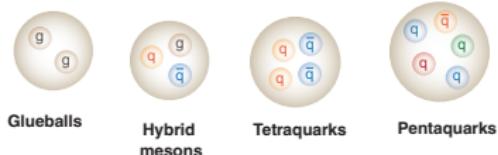
Gernot Eichmann

QNP 2022 - 9th International Conference on Quarks and Nuclear Physics
Sep 6, 2022

Many open questions

- **Understanding exotic hadrons:**

Hadron spectroscopy at [LHC](#), [Belle II](#), [BES III](#), [PANDA](#), [JLab](#), [ELSA](#), ...



- **Mass generation and confinement?**

Higgs

QCD

- **Quark-gluon structure of hadrons and nuclei:**
Hadron tomography at [EIC](#), [JLab](#), [COMPASS/AMBER](#), ...

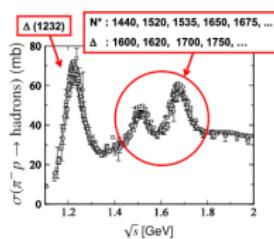


Theory tools

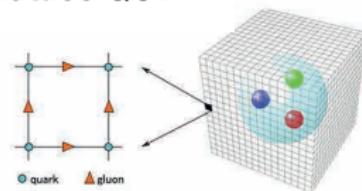
Functional methods (DSEs & BSEs, FRG, ...)



Amplitude analyses



Lattice QCD



Phenomenological models



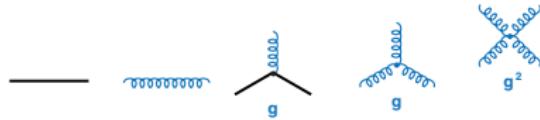
Effective theories (ChPT, ...)



Functional methods

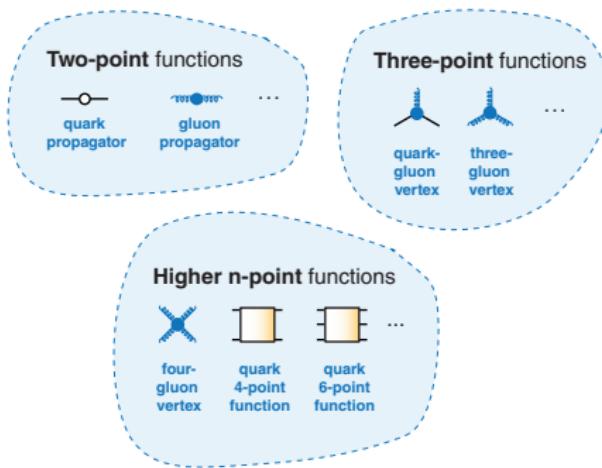
Classical Lagrangian of QCD:

$$\mathcal{L} = \bar{\psi} (i\cancel{\partial} + g\cancel{A} - \cancel{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$



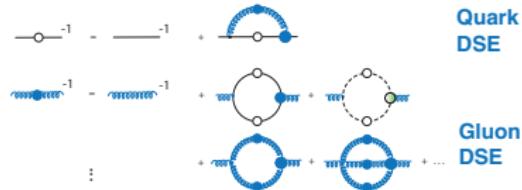
Quantum field theory

Correlation functions in QCD:



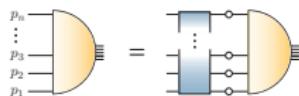
Can be calculated ...

- in **perturbation theory**
- in **lattice QCD**
- with **functional methods:**
DSEs (Dyson-Schwinger equations),
FRG (functional renormalization group)



Functional methods

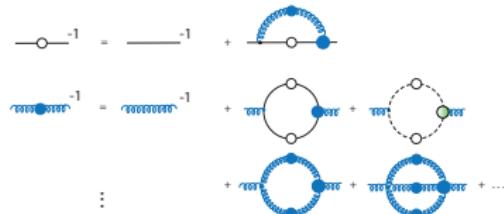
- Hadronic bound-state equations
(Bethe-Salpeter & Faddeev eqs)



"QFT analogue of Schrödinger eq."

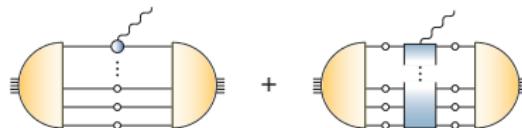
- hadron masses & "wave functions"
- **spectroscopy calculations**

- Ingredients: **QCD's n-point functions**,
Satisfy quantum eqs. of motion (DSEs)

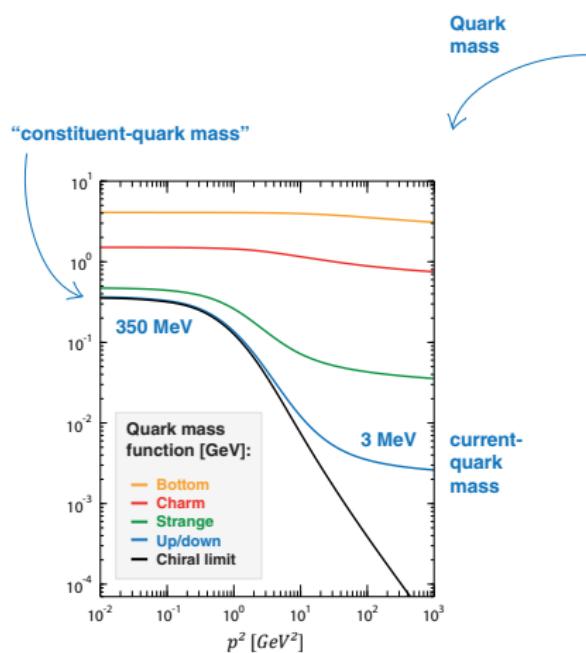


- Dynamical mass generation,
gluon mass gap, confinement, ...

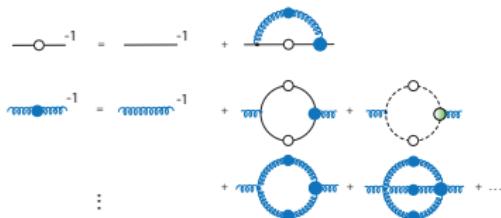
- Structure calculations:** form factors, PDFs, GPDs, TMDs, two-photon processes, ...



Functional methods

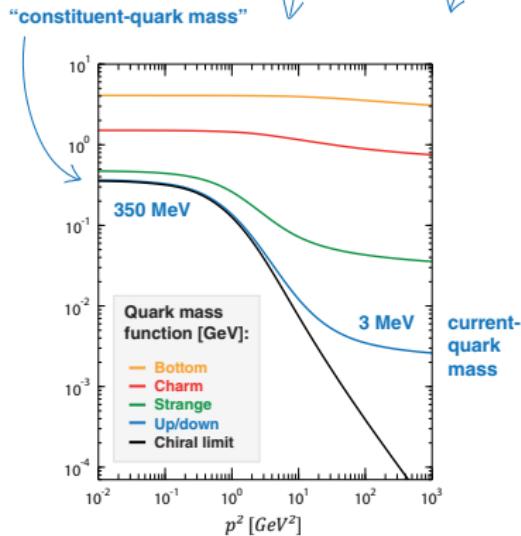
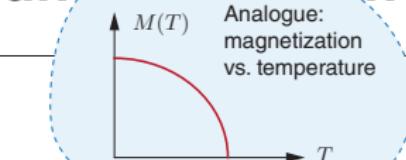


- Ingredients: **QCD's n-point functions**,
Satisfy quantum eqs. of motion (DSEs)



→ Dynamical mass generation,
gluon mass gap, confinement, ...

Functional methods



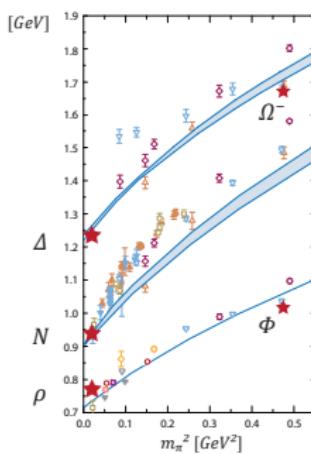
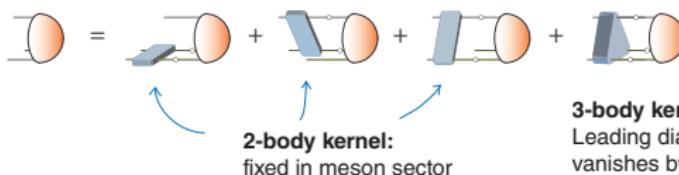
- Ingredients: **QCD's n-point functions**, Satisfy quantum eqs. of motion (DSEs)

→ Dynamical mass generation, gluon mass gap, confinement, ...

Baryons

Three-quark BSE (Faddeev equation) for baryons:

GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)



GE, Sanchis-Alepuz,
Williams, Alkofer, Fischer,
Prog. Part. Nucl. Phys. 91 (2016)

3-body kernel:
 Leading diagram (3-gluon vertex)
 vanishes by color trace,
 higher-order diagrams small (?)
 Sanchis-Alepuz, Williams, *PLB* 749 (2015)

2-quark correlations dominant?

$$\Psi_{\alpha\beta\gamma\delta}(p, q, P) = \sum_i f_i(p^2, q^2, p \cdot q, p \cdot P, q \cdot P) \tau_i(p, q, P)_{\alpha\beta\gamma\delta} \otimes \text{Flavor} \otimes \text{Color}$$

i Lorentz-invariant Dirac-Lorentz tensors:
 dressing functions 64 for $J=1/2$

Relativistically, nucleon also has **p waves**

s waves

p wave:

see also:
Oin, Roberts, Schmidt, PRD 97 (2018)

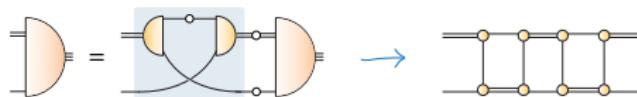
Dirac-Lorentz tensors:

64 for $J = 1/2$,
128 for $J = 3/2$

Diquark correlations

- Quark-diquark (two-body) equation

Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)

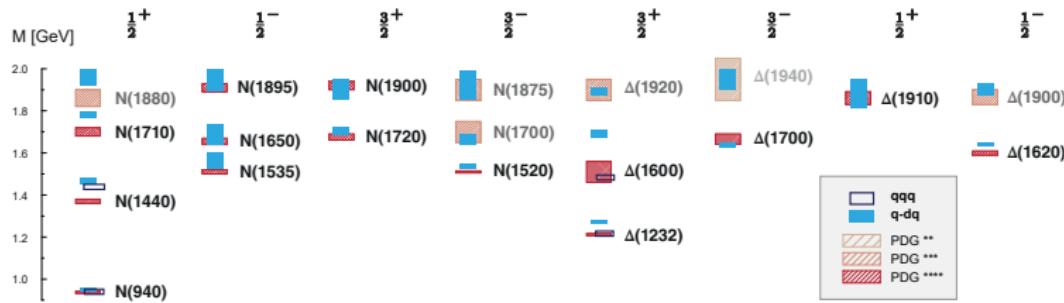


- Three-quark and quark-diquark results very similar

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

Diquark clustering in baryons?

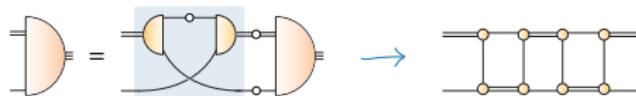
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)



Diquark correlations

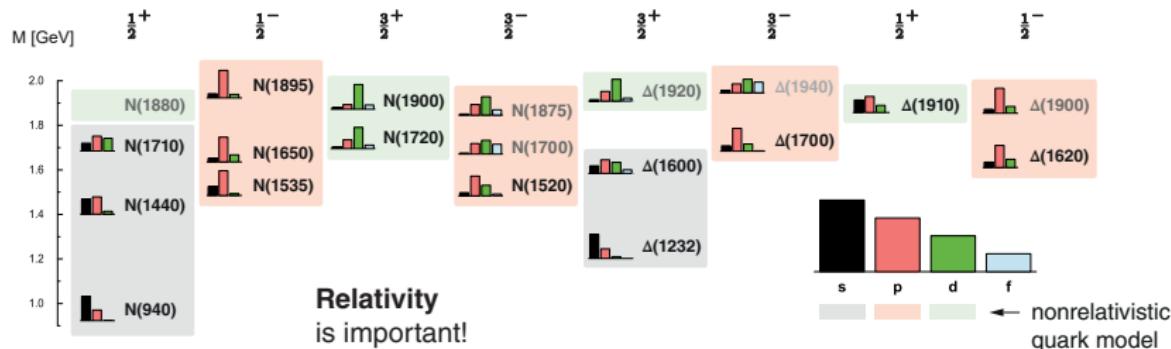
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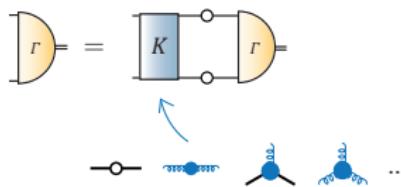


Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)



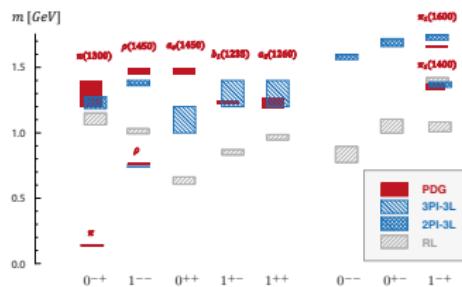
Towards ab-initio

- Going **beyond rainbow-ladder** means calculating higher n-point functions



- Beyond rainbow-ladder calculations improve **light-meson spectrum**

Williams, Fischer, Heupel, PRD 93 (2016)



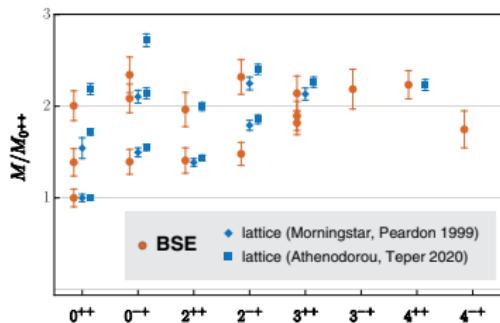
- Lots of activity with **DSEs, FRG, lattice QCD**

..., Williams, Fischer, Heupel, PRD 93 (2016), Cyrol et al., PRD 97 (2018), Oliveira, Silva, Skullerud, Stemberk, PRD 99 (2019), Huber, PRD 101 (2020), Aguilar et al., EPJ C 80 (2020), Qin, Roberts, Chin. Phys. Lett. 38 (2021), ...

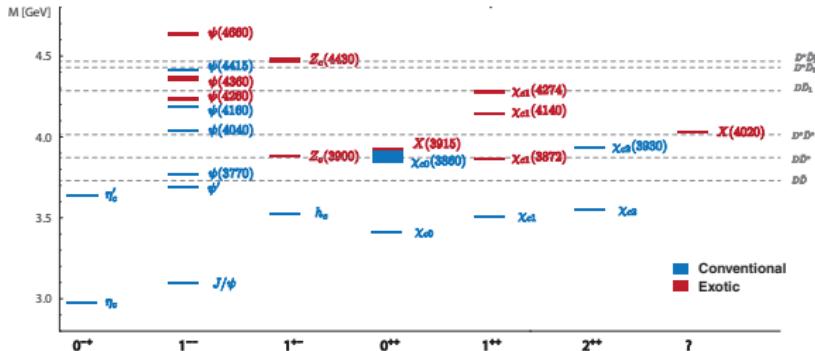


- **Glueball spectrum** agrees with lattice QCD

Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)



Exotic mesons



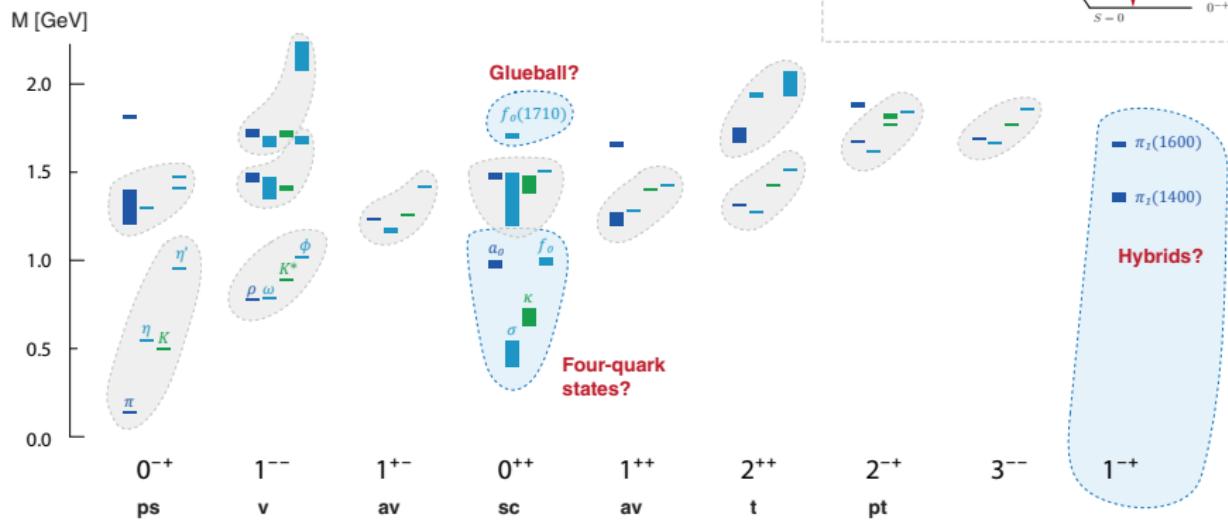
- Several tetraquark candidates in **charmonium spectrum**: $X(3872)$, $X(3915)$, $Z_c(3900)$, ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Recent additions: all-charm $X(6900)$, open-charm T_{cc}^+ , ...
- Oldest tetraquark candidates: **light scalar mesons**

Reviews:

- Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092
- Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528
- Esposito, Pilloni, Polosa,
Phys. Rept. 668 (2017), 1611.07920
- Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141
- Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610
- Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2018), 1708.04012
- Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976
- Brambilla, Eidelman, Hanhart et al.,
Phys. Rept. 873 (2020)
- ...

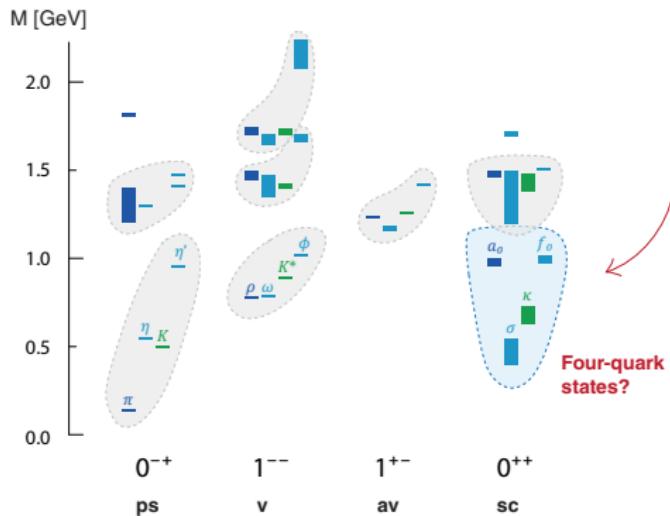
Light exotic mesons

Light meson spectrum
(PDG 2020)

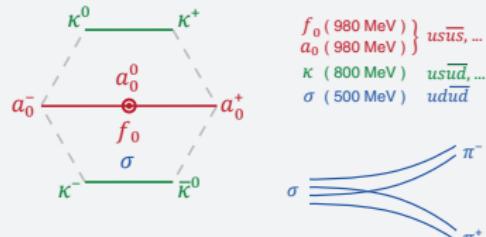


Light exotic mesons

Light meson spectrum
(PDG 2020)



- **Diquark-antidiquark?**
Explains mass ordering & decay widths
Jaffe 1977, Close, Tornqvist 2002,
Maiani, Polosa, Riquer 2004



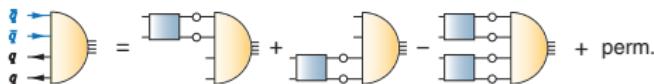
- **Meson molecules?**
Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non- $q\bar{q}$ nature supported by various approaches**
Pelaez, Phys. Rept. 658 (2016)

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states**:

GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p, q, k, P) = \sum_i f_i(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}) \tau_i(p, q, k, P) \otimes \text{Color} \otimes \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$$

$$\omega_1 = p \cdot k \quad \eta_1 = p \cdot P$$

$$\omega_2 = p \cdot k \quad \eta_2 = q \cdot P$$

$$\omega_3 = p \cdot q \quad \eta_3 = k \cdot P$$

256 Dirac-Lorentz tensors

2 Color tensors:
3 \otimes 3, 6 \otimes 6 or
1 \otimes 1, 8 \otimes 8
(Fierz-equivalent)

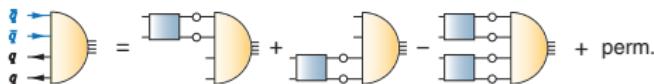
$$K \psi_i = \lambda_i \psi_i$$

	$\dim K$	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

Four-quark states

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9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$$

$$\omega_1 = \mathbf{k} \cdot \mathbf{k} \quad \eta_1 = \mathbf{p} \cdot \mathbf{P}$$

$$\omega_2 = \mathbf{p} \cdot \mathbf{k} \quad \eta_2 = \mathbf{q} \cdot \mathbf{P}$$

$$\omega_3 = \mathbf{p} \cdot \mathbf{q} \quad \eta_3 = \mathbf{k} \cdot \mathbf{P}$$

256 Dirac-
Lorentz tensors

2 Color
tensors:

$$3 \otimes \overline{3}, \quad 6 \otimes \overline{6} \quad \text{or} \\ 1 \otimes 1, \quad 8 \otimes 8 \\ (\text{Fierz-equivalent})$$

- Group momentum variables into multiplets of **permutation group S4**: can switch off groups of variables without destroying symmetries

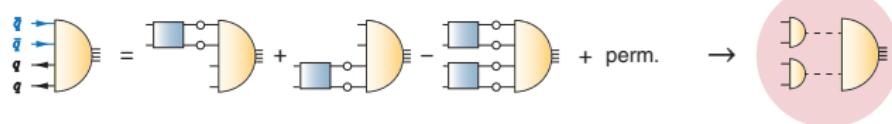
GE, Fischer, Heupel, PRD 92 (2015)

$$f_i(S_0, \nabla, \triangle, \circ)$$

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states**:

GE, Fischer, Heupel, PLB 753 (2016)



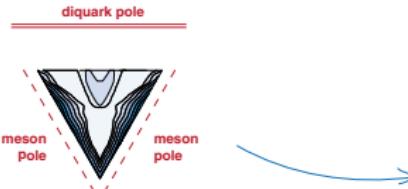
- BSE dynamically generates **meson poles** in BS amplitude:

$$f_i(S_0, \nabla, \Delta, \circ) \rightarrow 1500 \text{ MeV}$$

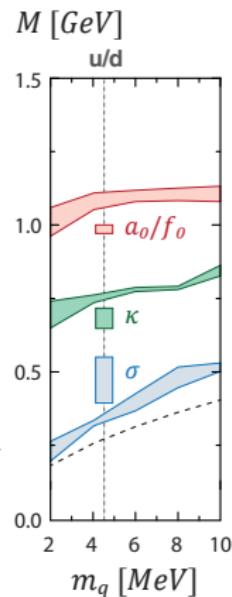
$$f_i(S_0, \nabla, \Delta, \circ) \rightarrow 1500 \text{ MeV}$$

$$f_i(S_0, \nabla, \Delta, \circ) \rightarrow 1200 \text{ MeV}$$

$$f_i(S_0, \nabla, \Delta, \circ) \rightarrow 350 \text{ MeV} !$$



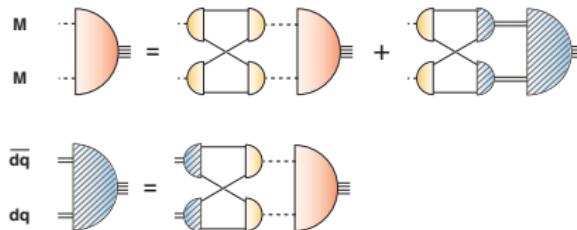
- "Light scalar mesons" look like **meson molecules**, diquark-antidiquark components almost negligible.
Lightness is inherited from pseudoscalar Goldstone bosons!



Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark**,
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)

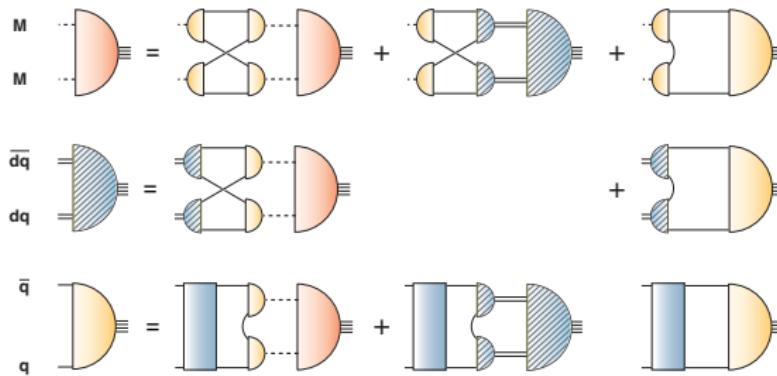


- Interaction by **quark exchange**
- System 'wants' to be **meson-meson-like** (no diagonal $d\bar{q}-d\bar{q}$ term)
- Similar results as in 4-quark approach:
 $m_\sigma \sim 400$ MeV, etc.

Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark**,
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)



Include mixing with $q\bar{q}$:
 $\pi\pi$ still dominant

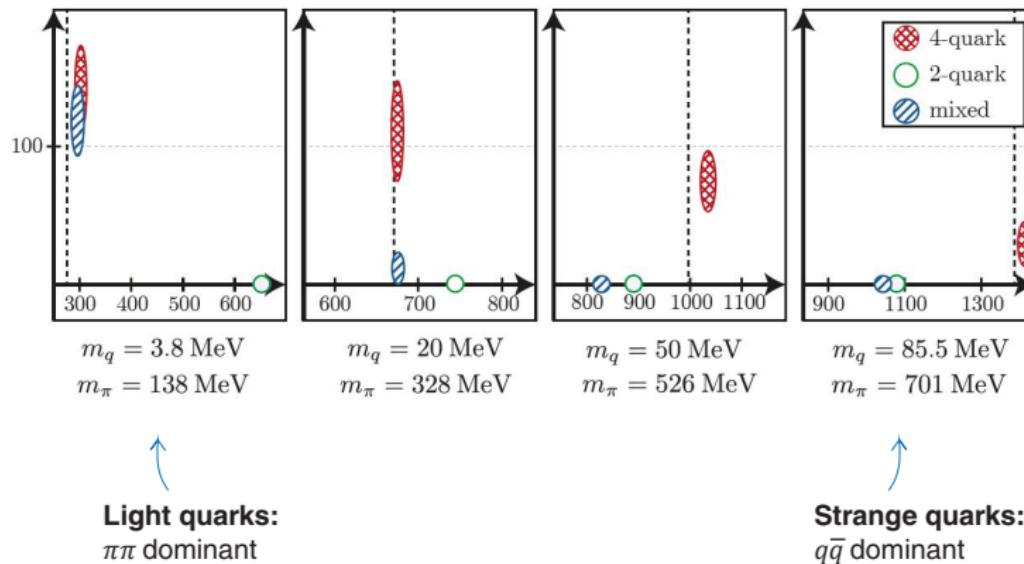
Santowsky, GE, Fischer, Wallbott,
Williams, PRD 102 (2020)

[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+ 0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+ 0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Four-quark states

Four-quark vs. $q\bar{q}$ dominance

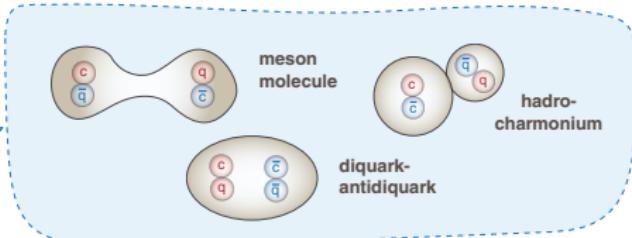
Santowsky, Fischer, PRD 105 (2022)



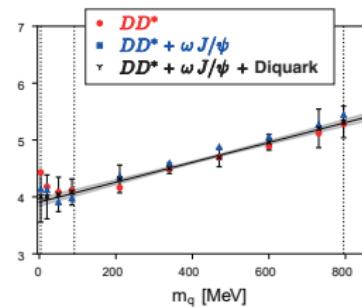
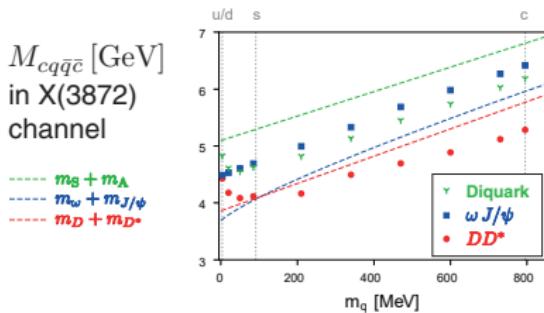
Four-quark states

- Heavy-light four-quark states:
what is their internal decomposition?

$cq\bar{q}\bar{c}$



- Four-quark BSE: all mix together

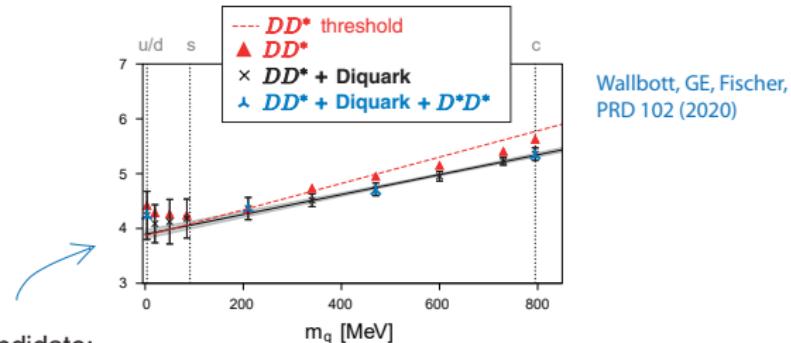
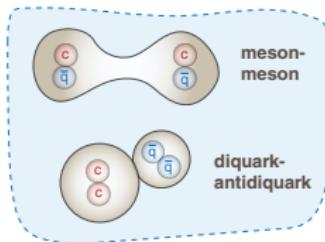


Wallbott, GE, Fischer,
PRD 100 (2019),
PRD 102 (2020)

$cq\bar{q}\bar{c} \rightarrow$ strong meson-meson component: DD^* for $X(3872)$, $Z_c(3900)$

Four-quark states

- Open-charm states: $cc\bar{q}\bar{q}$

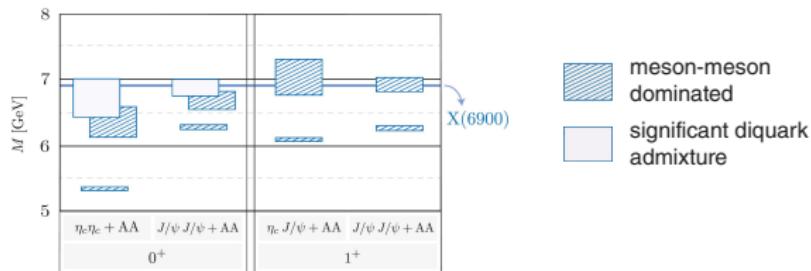


Experimental candidate:
 $T_{cc}^+, 0(1^+)$, 3875 MeV

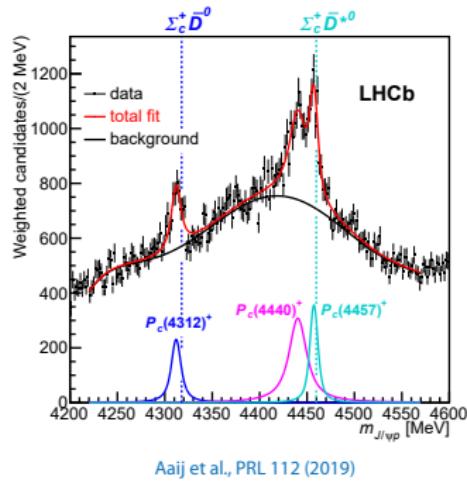
- All-charm state: $cccc$
 $X(6900)$

Results so far available
in two-body approach,
1st radial excitation?

Santowsky, Fischer, EPJC 82 (2022)

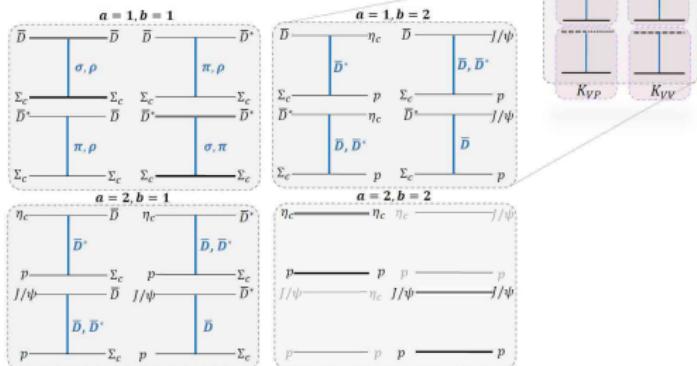
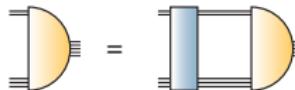


Pentaquarks?



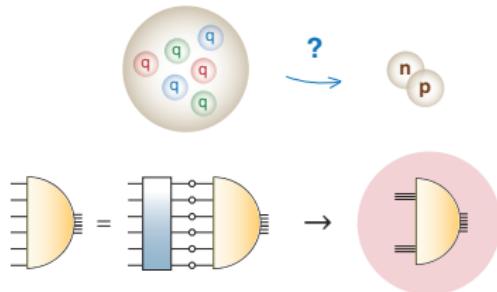
Aaij et al., PRL 112 (2019)

- Meson-baryon equation with hadronic exchanges
GE, Lourenco, Stadler, Torres, in preparation



Nucleons in nuclei?

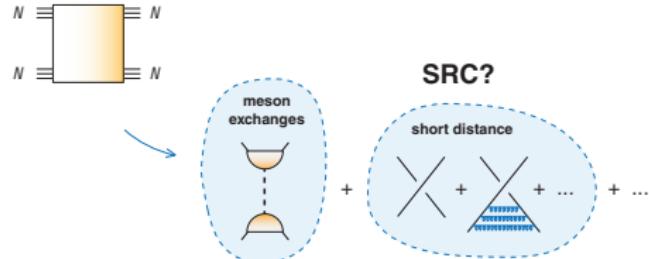
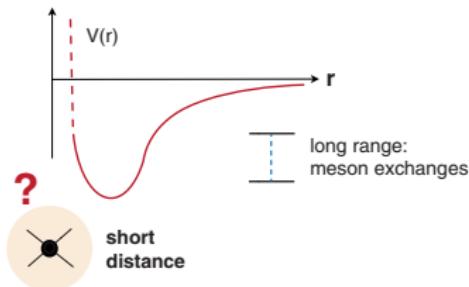
Transition from quarks & gluons to **light nuclei**:



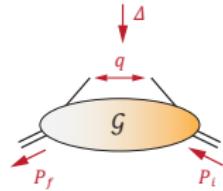
- Relativistic structure of **deuteron**
- Exotic dibaryons and hypernuclei
- **Short-range correlations**
- **EMC effect:** overlapping nucleons in nuclei?

Hen, Miller, Piasetzky, Weinstein, Rev. Mod. Phys. 89 (2017),
Cloet et al., J. Phys. G 46 (2019)

Microscopic origins of **short-range nuclear force**?

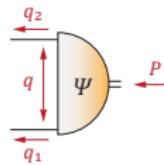


Hadron structure



Hadron-to-hadron correlator

$$\mathcal{G}(z, P, \Delta) = \langle P_f | \mathcal{T} \Phi(z) \mathcal{O} \Phi(0) | P_i \rangle$$



Bethe-Salpeter WF:
vacuum-to-hadron correlator

$$\Psi(z, P) = \langle 0 | \mathcal{T} \Phi(z) \Phi(0) | P \rangle$$

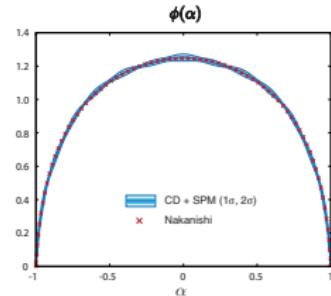
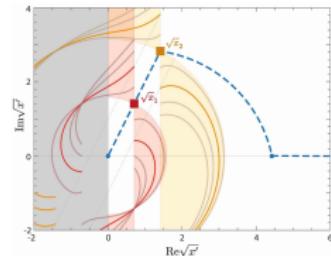
	$\mathcal{G}(q, P, \Delta = 0)$	$\mathcal{G}(q, P, \Delta)$	$\Psi(q, P)$
$\int dq^-$	TMD	GTMD	LFWF
$\int d^2\mathbf{q}_\perp \int dq^-$	PDF	GPD	PDA

Diehl, Phys. Rept. 388 (2003)

Belitsky, Radyushkin,
Phys. Rept. 418 (2005)
Lorcé, Pasquini, Vanderhaeghen,
JHEP 05 (2011)

...

Novel method to compute
light-front wave functions
via contour deformations
GE, Ferreira, Stadler, PRD 105 (2022)



Summary & outlook

- **Baryons:**

GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), arXiv:1606.09602

- **Four-quark states:**

GE, Fischer, Heupel, Santowsky, Wallbott, FBS 61 (2020), arXiv:2008.10240

- Towards **ab-initio calculations:**

higher n-point functions, gluon mass generation, resonances

- **Exotic hadrons:** glueballs, hybrids, tetraquarks, pentaquarks

- **Hadron structure:** PDFs, GPDs, TMDs

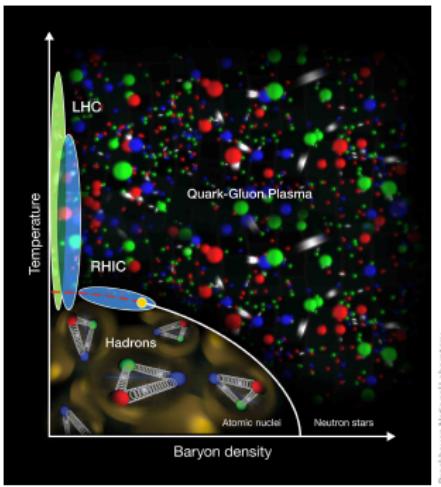
- **Flavor matrix elements:** constrain QCD contributions

- **Nuclei** from quarks and gluons

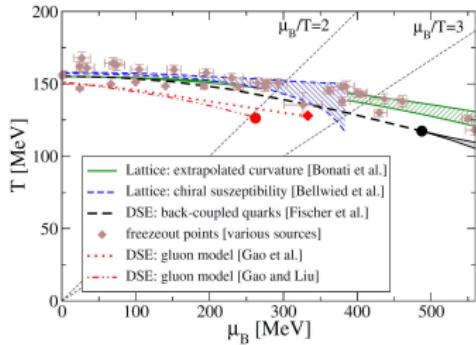
Thank you!

Backup slides

QCD phase diagram



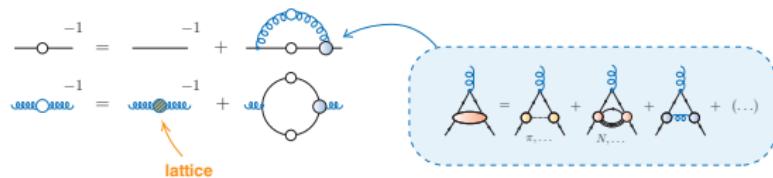
Search for **critical endpoint (CEP)** from DSEs & lattice:



Fischer, Prog. Part.
Nucl. Phys. 105 (2019)

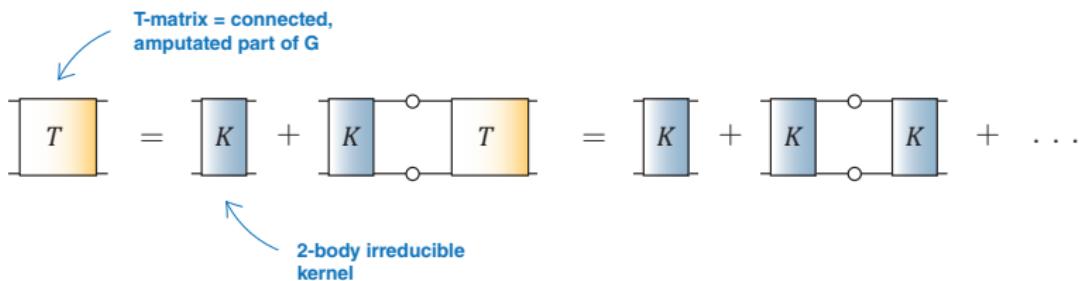
Location of CEP sensitive to baryons?

GE, Fischer, Welzbacher, PRD 93 (2016)



Bethe-Salpeter equations

Write down inhomogeneous BSE:



Analogy: **geometric series**

$$\begin{aligned} f(x) &= 1 + x f(x) \\ &= 1 + x + x^2 f(x) \quad \Rightarrow \quad f(x) = \frac{1}{1-x} \quad \left. \right\} \text{“non-perturbative”} \\ &= 1 + x + x^2 + x^3 f(x) \\ f(x) &\approx 1 + x + x^2 + x^3 + \dots \quad \text{only for } |x| < 1 \quad \left. \right\} \text{“perturbative”} \end{aligned}$$

Bethe-Salpeter equations

Write down inhomogeneous BSE:

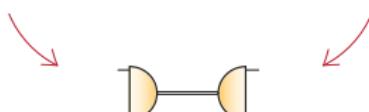
$$T = K + K \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} T$$

T-matrix = connected, amputated part of G

Homogeneous BSE at pole:

$$\Gamma = K \quad \text{compare pole residues}$$

$p^2 \rightarrow -m^2$



- $q\bar{q}$ irreducible kernel
- chiral symmetry constraints ($V + AV$ WTI)
- can be systematically derived from effective action, depends on QCD's n-point functions



- Analogue of Schrödinger equation in QFT!
- $\Gamma = \text{Bethe-Salpeter amplitude}$

Bethe-Salpeter equations

Write down inhomogeneous BSE:

$$T = K + K \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} T$$

T-matrix = connected, amputated part of G

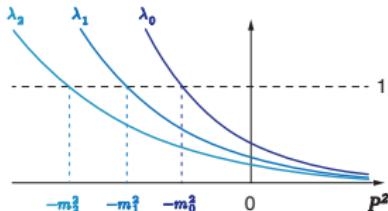
Homogeneous BSE at pole:

$$\Gamma = K \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \Gamma$$

compare pole residues

BSE = eigenvalue equation,
pole in $T \Leftrightarrow$ eigenvalue = 1

$$KG_0\Gamma_i = \lambda_i\Gamma_i$$



Explicitly:

$$\Gamma(p, P) = \int \frac{d^4 q}{(2\pi)^4} \mathbf{K}_{\alpha\gamma,\delta\beta}(p, q, P) [S(q_+) \Gamma(q, P) S(q_-)]_{\gamma\delta}$$

Basis decomposition:

$$\Gamma(p, P) = \sum_{i=1}^n f_i(p^2, \hat{p} \cdot \hat{P}, P^2) \tau_i(p, P)$$

⇒ Coupled Lorentz-invariant equations
for the dressing functions f_i

Ladder

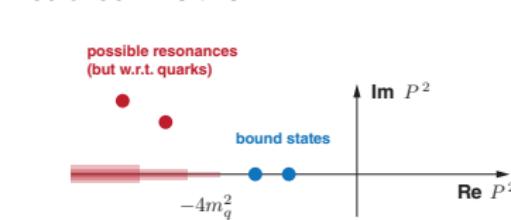
Simplest attempt:

$$\boxed{G} = \boxed{\Gamma} \quad \Leftrightarrow \quad \boxed{T} = \boxed{T} + \boxed{\Gamma} + \dots$$

free propagators

$$\frac{-i\cancel{p} + m}{p^2 + m^2}$$

Analytic structure of G , T , etc.
would look like this:



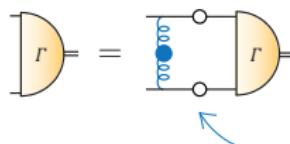
- breaks chiral symmetry
(free propagators \Leftrightarrow contact interaction)
- generates bound-state poles in G and T ,
possibly also resonances
- but also quark thresholds & cuts:
“hadrons” decay into quarks,
no confinement

would be ok if elementary d.o.f. were
not quarks but **hadrons** (\rightarrow EFTs)

Rainbow-ladder

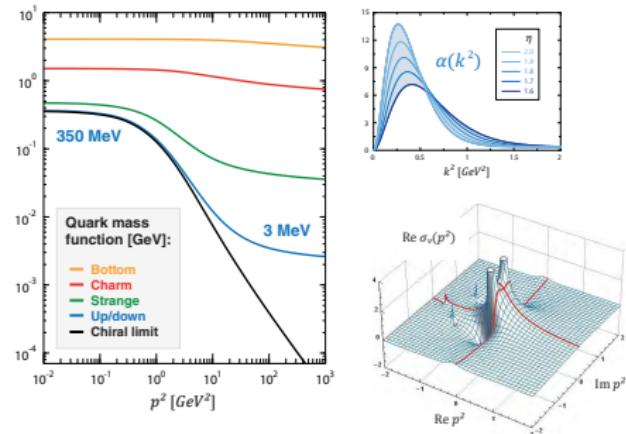
Better: rainbow-ladder truncation

Maris, Roberts, PRC 56 (1997), Maris, Tandy, PRC 60 (1999)



gluon = effective interaction,
dressed propagators
from quark DSE

$$\frac{1}{A(p^2)} \frac{-i\cancel{p} + \mathbf{M}(p^2)}{p^2 + \mathbf{M}(p^2)^2}$$



Analytic structure of G , T , etc.
would look like this:

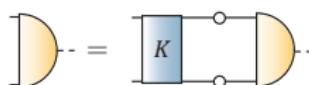


- chiral symmetry ✓
- dynamical propagators do not have real poles \Rightarrow no quark thresholds ✓
- but no resonances: **bound states**
(need to go beyond rainbow-ladder)

Diquark correlations

Mesons and diquarks closely related through BSE

Maris, FBS 32 (2002)

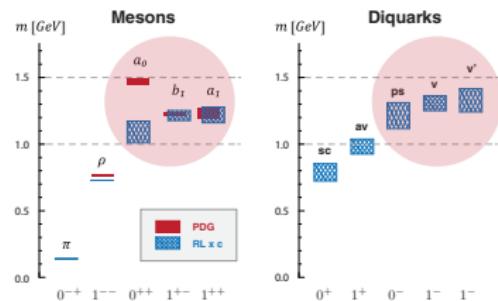


Lowest-lying diquarks are dominant for ground-state octet & decuplet baryons

pseudoscalar mesons \Leftrightarrow **scalar diquarks** (~0.8 GeV)
vector mesons \Leftrightarrow **axialvector diquarks** (~1 GeV)

Higher-lying diquarks are subleading, but contribute to excited states & remaining channels

scalar mesons \Leftrightarrow **pseudoscalar diquarks** (~1.2 GeV)
axialvector mesons \Leftrightarrow **vector diquarks** (~1.3 GeV)



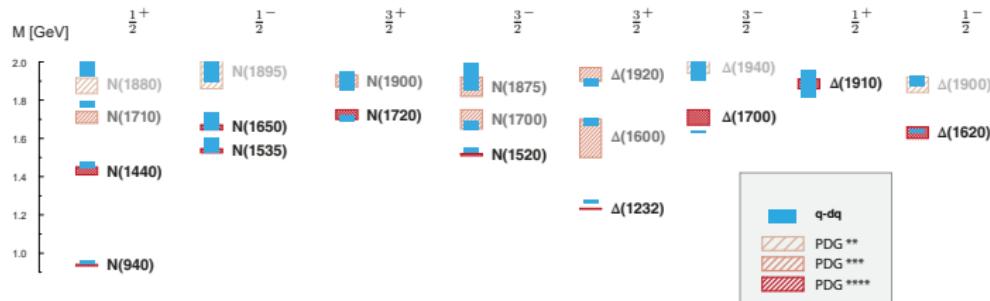
In RL, these are too strongly bound;
simulate beyond-RL effects
by (one) strength parameter c

Roberts, Chang, Cloet, Roberts, FBS 51 (2011)
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

Diquark correlations

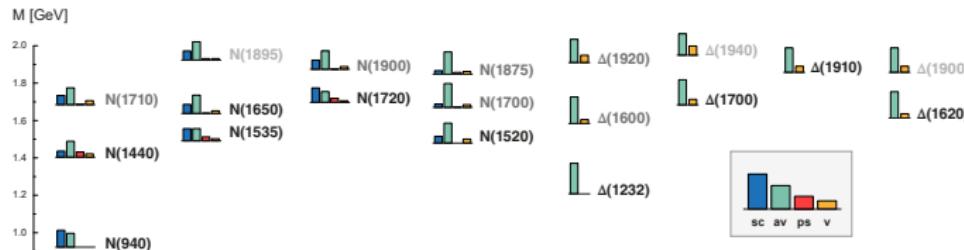
Light baryon spectrum

GE Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquark content:

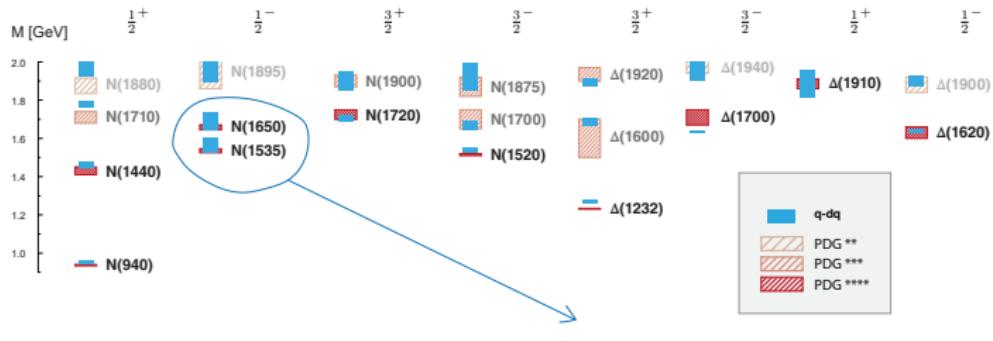
Barabanov et al., PPNP 116 (2021)



Diquark correlations

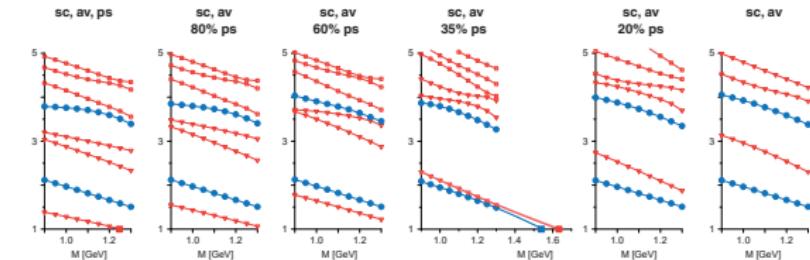
Light baryon spectrum

GE Fischer, Sanchis-Alepuz, PRD 94 (2016)



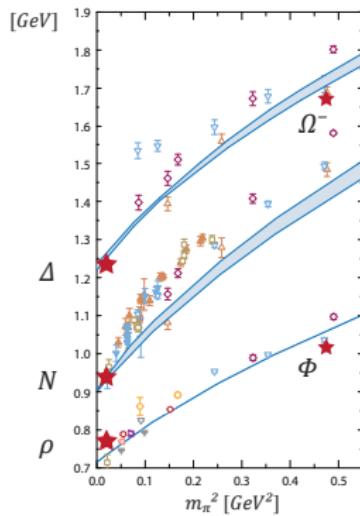
- **Level ordering** determined by diquark dynamics
- Diquarks are not pointlike, also here **rich spectrum!**

Barabanov et al., PPNP 116 (2021)



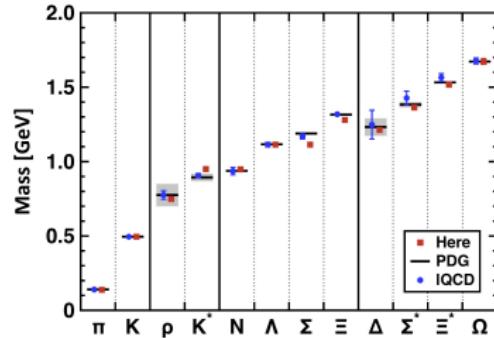
Baryons

Baryon spectrum from three-body equation (in rainbow-ladder)



GE, Alkofer, Nicmorus,
Krassnigg, PRL 104 (2010)

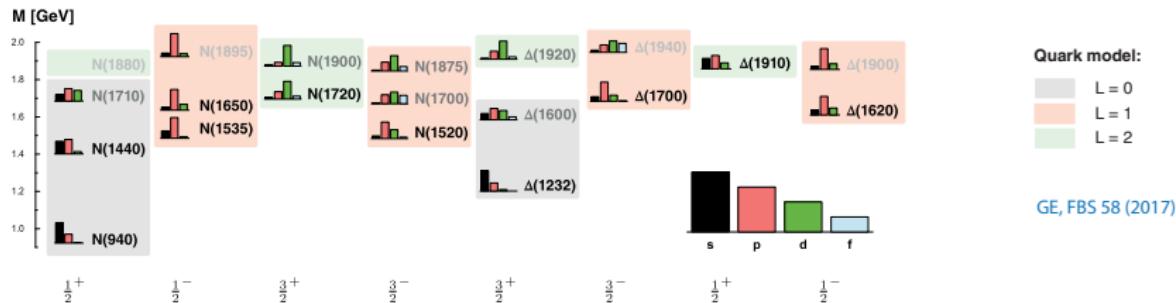
GE, Sanchis-Alepuz,
Williams, Alkofer, Fischer,
Prog. Part. Nucl. Phys. 91 (2016)



Qin, Roberts, Schmidt,
PRD 97 (2018), FBS 60 (2019)

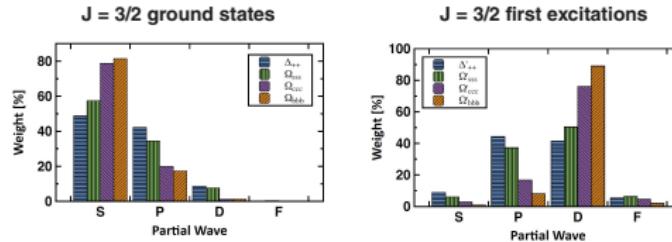
Relativistic effects

Orbital angular momentum: clear traces of nonrelativistic quark model, but strong relativistic effects (in some cases even dominant)



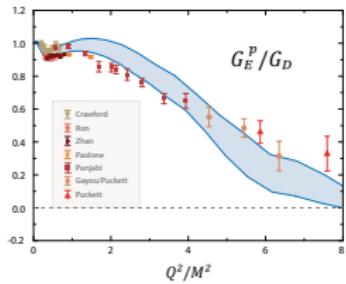
Relativistic contributions
even up to bottom baryons!

Qin, Roberts, Schmidt, PRD 97 (2018)

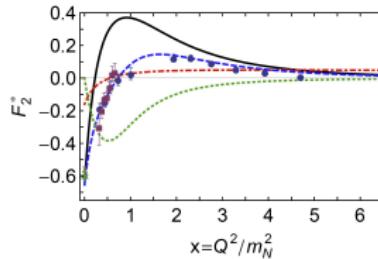


Baryon structure

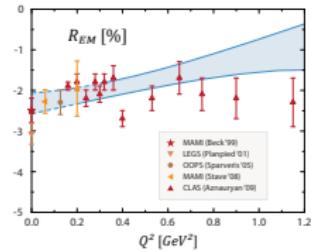
Nucleon electromagnetic FFs
GE, PRD 84 (2011)



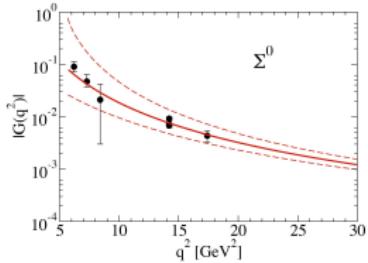
Roper em. transition FFs
Segovia et al., PRL 115 (2015)



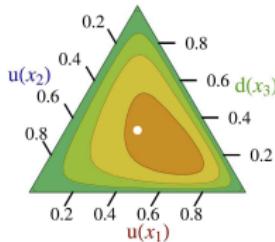
Δ em. transition FFs
GE, Nicmorus, PRD 85 (2012)



Timelike em. strangeness FFs
Ramalho, Peña, PRD 101 (2020)



Distribution amplitudes
Mezrag, Segovia, Chang, Roberts, PLB 783 (2018)



Towards ab-initio

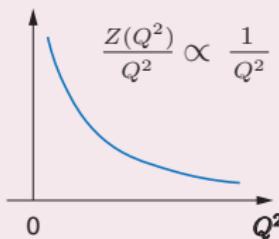
Gluon propagator:



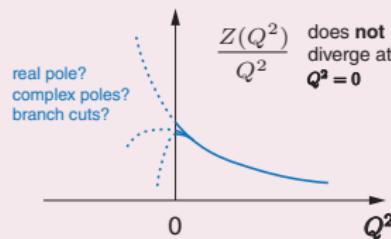
$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \frac{L(Q^2)}{Q^2} \frac{Q^\mu Q^\nu}{Q^2}$$

transverse dressing longitudinal dressing = 1

- Perturbation theory:
Massless gluon pole



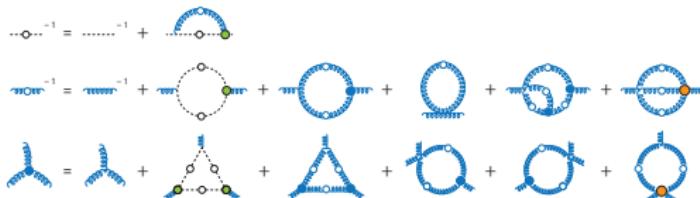
- Nonperturbative calculations:
Massless pole disappears!



Coupled Yang-Mills DSEs

GE, Pawłowski, Silva, PRD 104 (2021)

→ Test confinement
in hadron observables!



Family of “decoupling” solutions,
also seen in lattice QCD

Cucchieri, Maas, Mendes, PRD 77 (2008)

Boucaud et al., JHEP 06 (2008)

Bogolubsky et al., PLB 676 (2009)

Fischer, Maas, Pawłowski, Ann. Phys. 324 (2009)

Duarte, Oliveira, Silva, PRD 94 (2016)

Aguilar et al., EPJ C 80 (2020)

Endpoint is “scaling” solution,
confinement manifest

Lerche, Smekal, PRD 65 (2002)

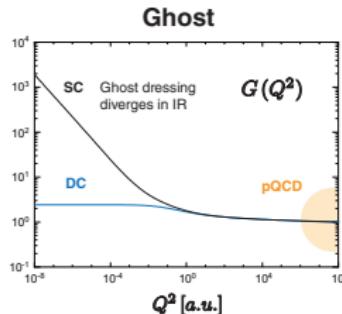
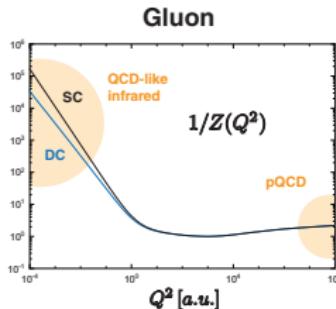
Fischer, Alkofer, PLB 536 (2002)

Alkofer, Fischer, Llanes-Estrada, MPLA 23 (2008)

All solutions show gluon mass gap

$$\lim_{r \rightarrow \infty} \int \frac{d^3 Q}{(2\pi)^3} \frac{Z(Q^2)}{Q^2} e^{i \mathbf{x} \cdot \mathbf{Q}} \propto e^{-m_{\text{gap}} r}$$

Gluon mass gap



- What distinguishes SC + DC?
- What is the “true” solution? Are all solutions physically equivalent?

Scaling (SC) solution:

- n-point functions scale with IR power laws
[Lerche, Smekal, PRD 65 \(2002\)](#), [Fischer, Alkofer, PLB 536 \(2002\)](#)
- Confinement
[Alkofer, Fischer, Llanes-Estrada, Mod. Phys. Lett. A 23 \(2008\)](#)

Decoupling (DC) solution:

- Seen in lattice QCD
[Cucchieri, Maas, Mendes, PRD 77 \(2008\)](#), [Bogolubsky et al., PLB 676 \(2009\)](#),
[Duarte, Oliveira, Silva, PRD 94 \(2016\)](#), [Aguilar et al., EPJ C 80 \(2020\)](#)
- Functional methods: family of DC solutions with SC solution as endpoint
[Boucaud et al., JHEP 06 \(2008\)](#), [Fischer, Maas, Pawłowski, Ann. Phys. 324 \(2009\)](#), [Reinosa et al., PRD 96 \(2017\)](#)

Gluon mass gap

Gluon propagator:

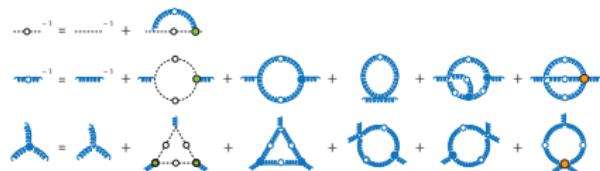


$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \frac{L(Q^2)}{Q^2} \frac{Q^\mu Q^\nu}{Q^2}$$

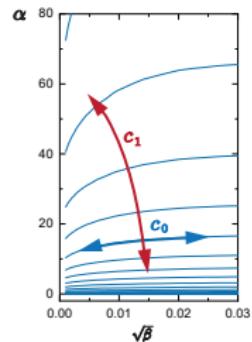
transverse dressing longitudinal dressing = 1

Coupled Yang-Mills DSEs

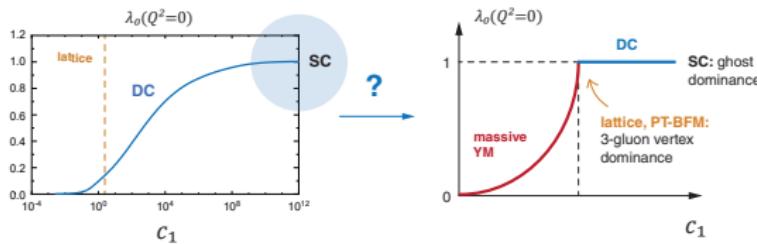
Huber, PRD 101 (2020), GE, Pawłowski, Silva, PRD 104 (2021)



- SC & DC solutions distinguished by c_1 (combination of coupling α and mass parameter β)
- Scaling solution: $c_1 \rightarrow \infty$



Gauge consistency ($L = 1$) satisfied only by SC solution

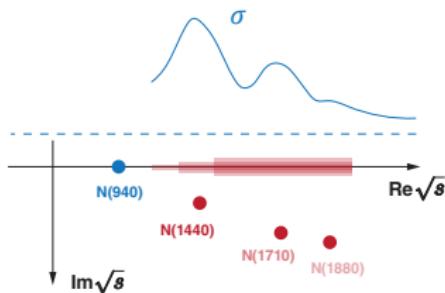


Possible scenario for consistency of DC solutions:

Then all massless (SC, DC) solutions would be physically equivalent; distinguished from massive solutions

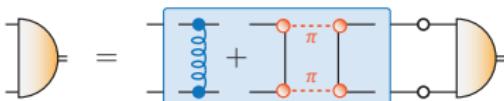
Resonances

- Most hadrons are **resonances** and decay
 \Leftrightarrow poles in complex momentum plane



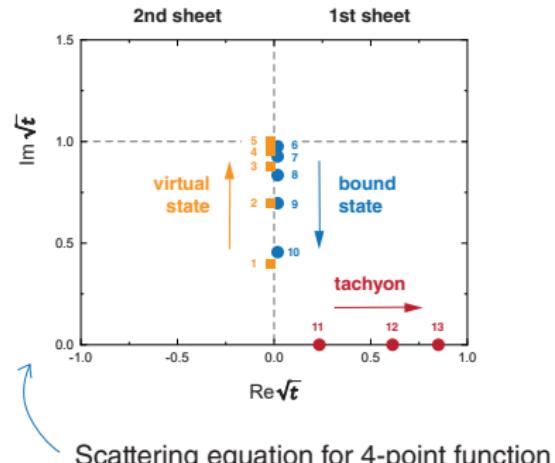
- BSE kernel must include decay channels:
 ρ meson becomes resonance

Williams, PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019),
Santowsky, GE, Fischer, Wallbott, PRD 102 (2020),
Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)



- Contour deformations** as tool to go beyond thresholds

GE, Duarte, Peña, Stadler, PRD 100 (2019)

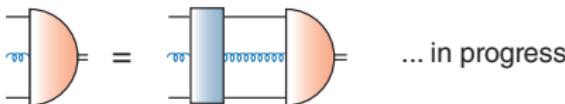


Scattering equation for 4-point function

$$\boxed{\text{I}} = \boxed{\text{I}} + \boxed{\text{X}} + \boxed{\text{II}} + \boxed{\text{XII}}$$

Hybrids

- **Three-body equation** (quark, antiquark, gluon)



- **Two-body equation** [quark–gluon]–antiquark
with model ansätze

Xu, Cui, Chang, Papavassiliou, Roberts, EPJA 55 (2019)

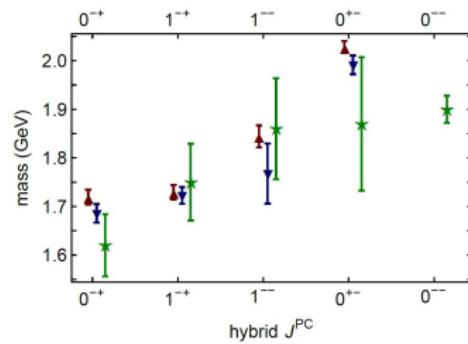
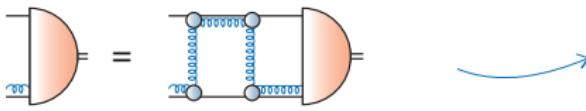
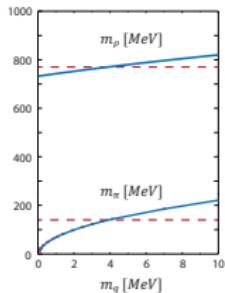


FIG. 2. Comparison between our ACM-improved spectrum (stars, green), Row 2 in Table I, and the rescaled lQCD results in Rows 3 (up-triangles, red) and 4 (down-triangles, blue).

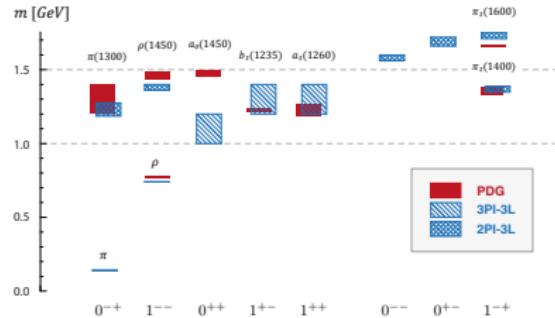
Lattice: Dudek, Edwards, Peardon, Richards, Thomas, PRD 82 (2010)

Mesons

- Pion is **Goldstone boson**: $m_\pi^2 \sim m_q$



- Light meson spectrum beyond rainbow-ladder**

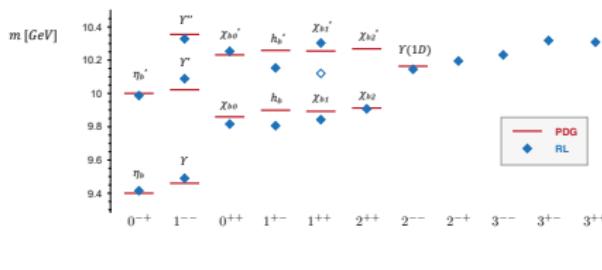


Williams, Fischer, Heupel,
PRD 93 (2016)

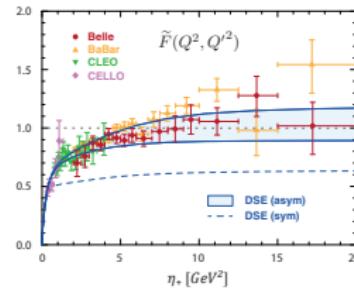
GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016)

- Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)

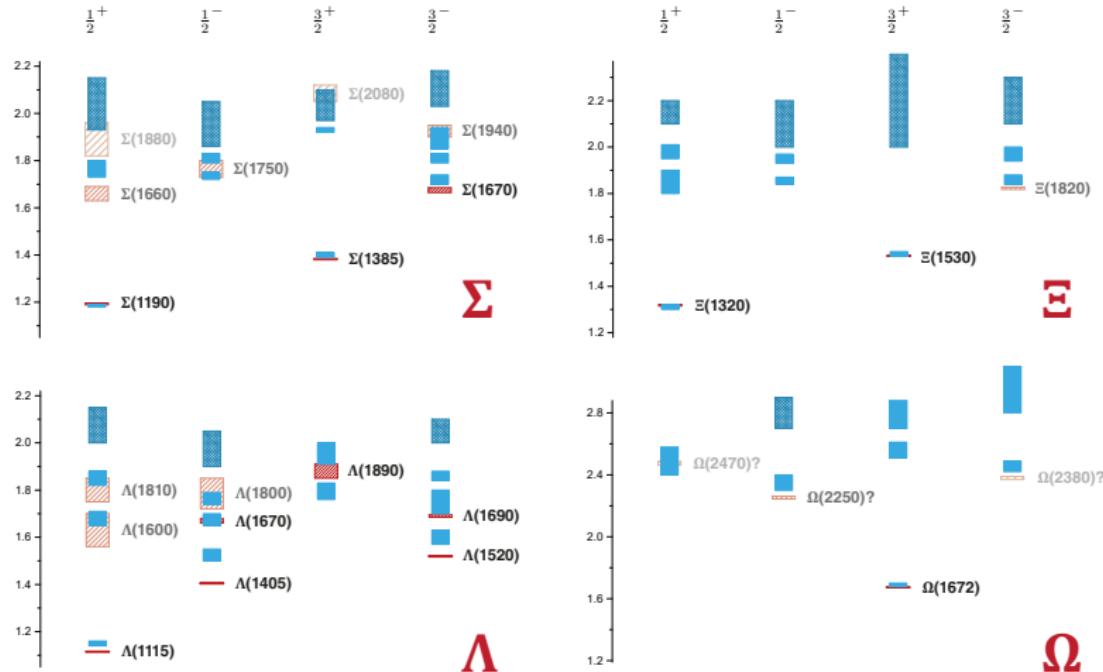


- Pion transition form factor**



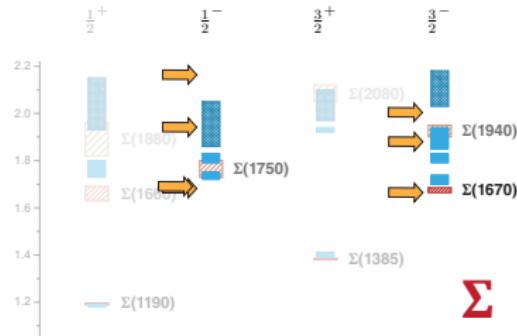
GE, Fischer, Weil, Williams,
PLB 774 (2017)

Strange baryons



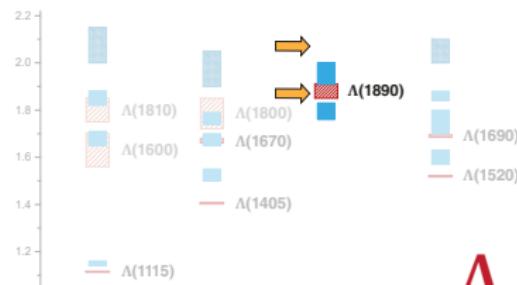
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina
Sarantsev et al., 1907.13387 [nucl-ex]

Σ

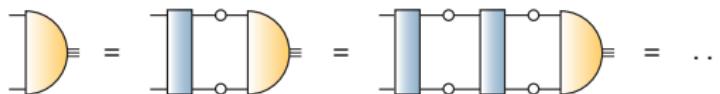


Λ

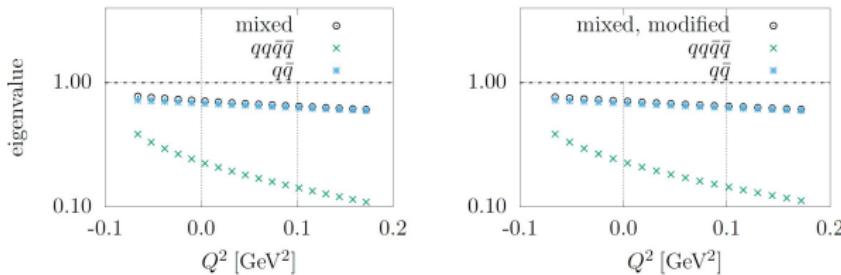
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Reply to comment

- Recent comment: BSE kernel is reducible
[Blankleider & Kvinikhidze, 2102.05818](#)
- Irrelevant for homogeneous BSE: same spectrum

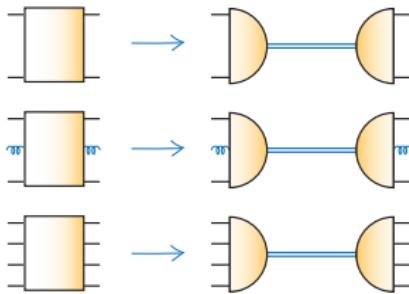


- Results are identical within numerical errors
[Santowsky, GE, Fischer, Wallbott, Williams, 2103.14673](#)

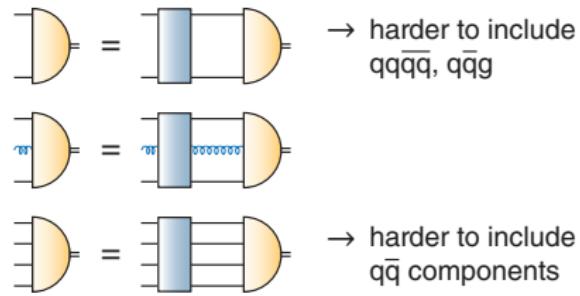


Some thoughts

- Same spectral representation for **all** correlation functions that produce $q\bar{q}$:



- Without** truncations, BSEs for qq , $qq\bar{q}\bar{q}$, $q\bar{q}g$, ... should produce **same spectrum**



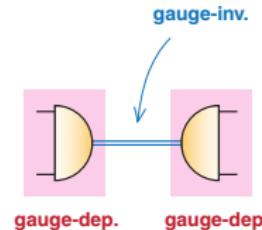
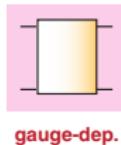
With truncations, $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) BSE should give more reliable spectrum for $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) dominated states

Exotic quantum numbers not excluded in principle (need gluon-rich kernel in $q\bar{q}$ BSE)

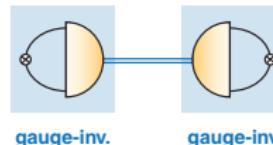
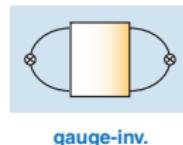
Some thoughts

- Why don't we see exotic quantum numbers in **lattice calculations** with $q\bar{q}$ operators?

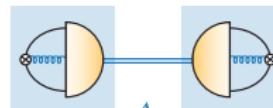
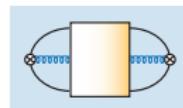
$$\langle q_\alpha \bar{q}_\beta q_\gamma \bar{q}_\delta \rangle =$$



$$\langle (\bar{q} \Gamma q)(\bar{q} \Gamma q) \rangle =$$



= 0 for exotic quantum nrs (?)



$\neq 0$ for exotic quantum nrs

Could test with **gauge-fixed** lattice calculations: same spectrum?

