Mechanical properties of the proton (from lattice QCD)

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Outline

Gravitational form factors

- What are they?
- Relation to GPDs & experimental results

GFFs on the lattice

- Previously: computed glue GFFs at 450 MeV [Pefkou DH Shanahan 2022]
- Ongoing: full set of GFFs at 170 MeV
- Preliminary results
- Spatial densities

Gravitational form factors (GFFs)

$a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$
$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$
$u, \bar{u}$ = Dirac spinors
$P = (p' + p)/2$
$\Delta = p' - p$
$t = \Delta^2$

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\,q$

Momentum fraction

$A_{q,g}(0) = \langle x \rangle_{q,g}$

$A_g(0) + \sum_q A_q(0) = 1$

Spin fraction

$J = (A + B)/2$

$J_g(0) + \sum_q J_q(0) = \frac{1}{2}$

$$\left\langle N(p')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p)\right\rangle = \bar{u}(p')\left[A_{g,q}(t)\gamma^{\{\mu}P^{\nu\}} + B_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)M g^{\mu\nu}\right]u(p)$$

Internal forces

$D(0) = D_g(0) + \sum_q D_q(0)$

"the last global unknown"

$\sim$ trace anomaly

Power-divergent mixing

Not conserved $\sum_q \dot{c}_q + \dot{c}_g = 0$

$D(0)$: ”the last global unknown”

[Polyakov Schweitzer 2018]

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\textbf{em}} \vert N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$
				$\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\textbf{weak}} \vert N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$
				$g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\textbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\textbf{grav}} \vert N \rangle$	$\longrightarrow$	$m = 938.272013(23)\,\text{MeV}/c^2$
				$J = \frac{1}{2}$
				$\boldsymbol{D} = \textbf{?}$

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

Experimental access?

GFFs related to Mellin moments of GPDs

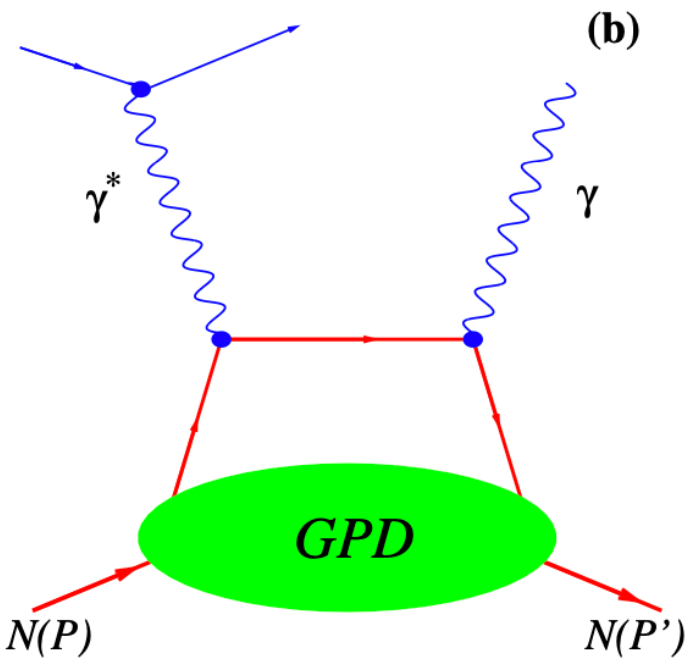
$$\int dx\, x^{n-1}\, \mathrm{GPD}(\mathrm{x}, \xi, t)$$

Unpolarized twist 2 nucleon GPDs, $n = 2$:

$$\int dx\, x\, H_{q,g}(x, \xi, t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$

$$\int dx\, x\, E_{q,g}(x, \xi, t) = B_{q,g}(t) - \xi^2 D_{q,g}(t)$$

→ relate to experiment via factorization

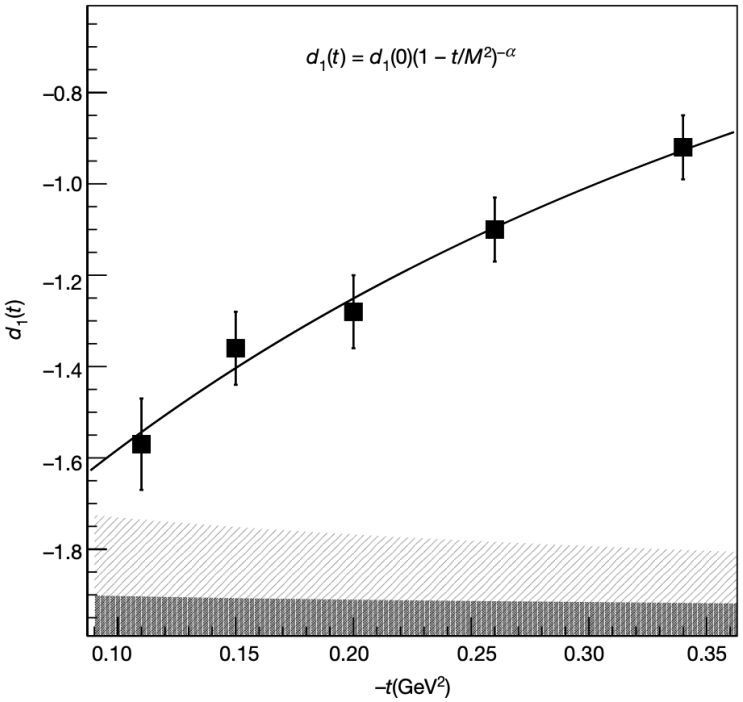


[Polyakov Schweitzer 2018]

Experimental results

Experimental quark extraction from DVCS

[Burkert Elouadrhiri Girod 2018]

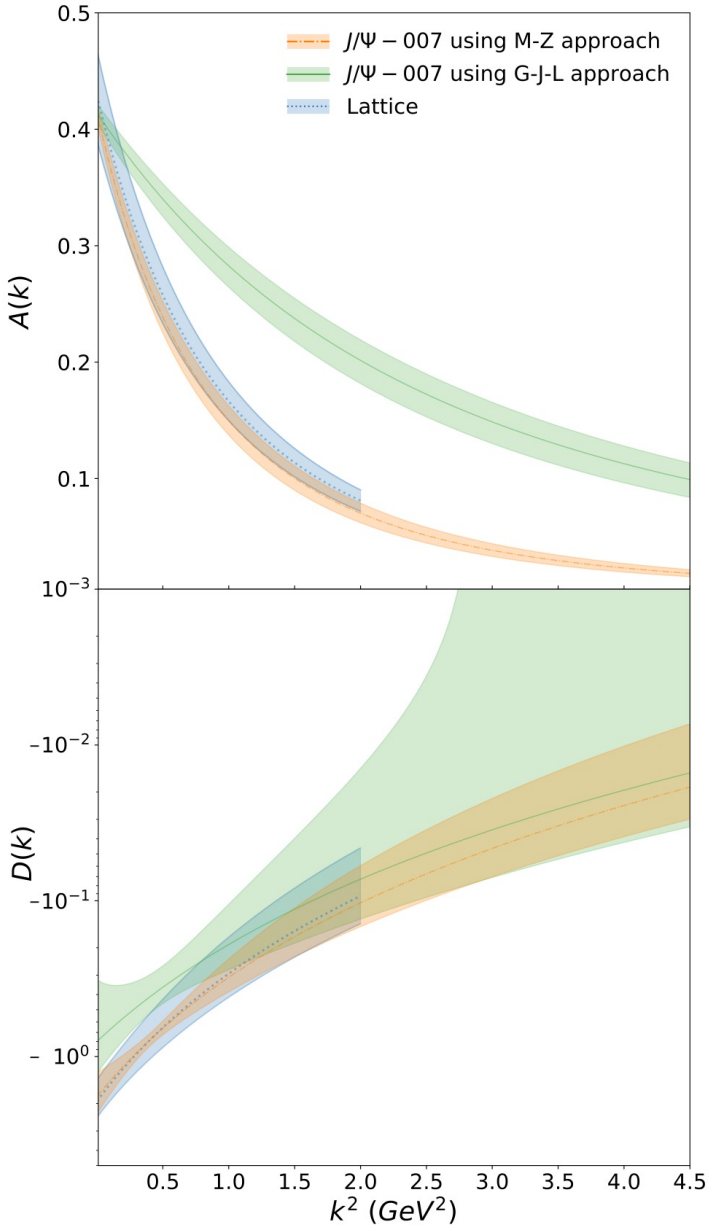


Experimental glue extraction from J/Ψ photoproduction

[JLab HallC 2207.05212]

Note: unpublished at present

Note': strong method dependence



Lattice calculation

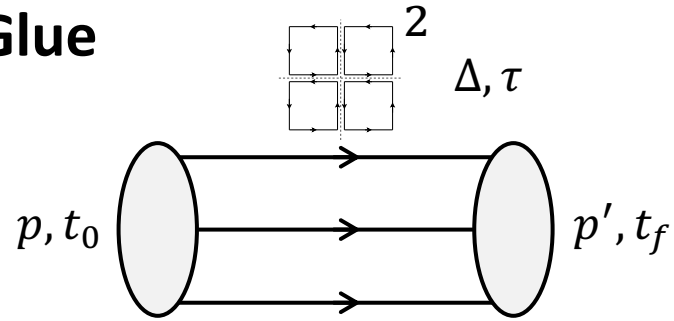
Ensemble [“a091m170” (JLab/W&M/MIT/LANL)]

- Tree-level tadpole-improved Symanzik gauge
- 2+1 stout-smeared Wilson clover
- $M_\pi = 170\ \mathrm{MeV}$
- $a = 0.091\ \mathrm{fm}$
- $48^3 \times 96$

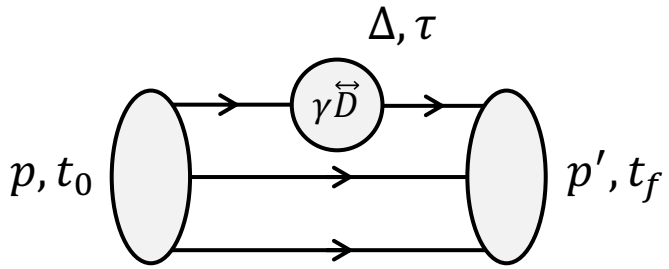
Sketch of calculation:

1. Compute three-point functions
2. Fit to extract bare matrix elements
3. Analyze to extract bare GFFs
4. Renormalize (non-perturbative)

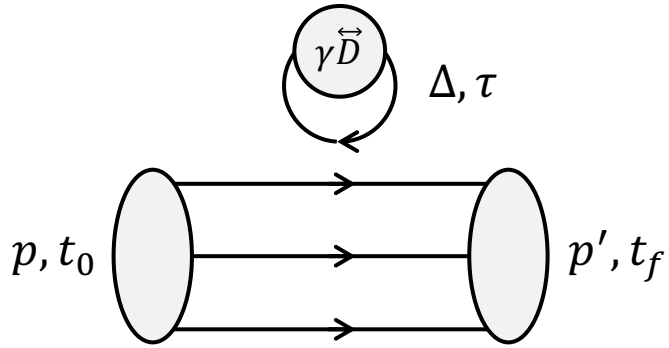
Glue



Connected quark (u, d)



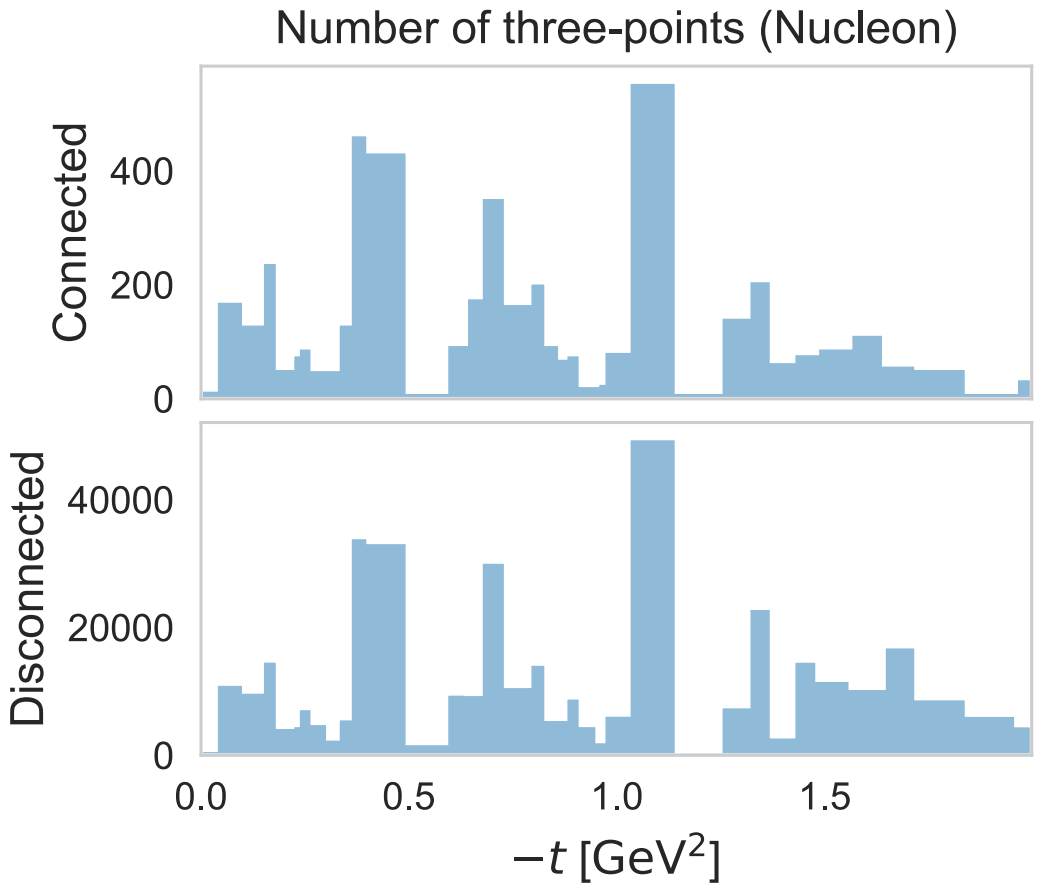
Disconnected quark $(u + d, s)$

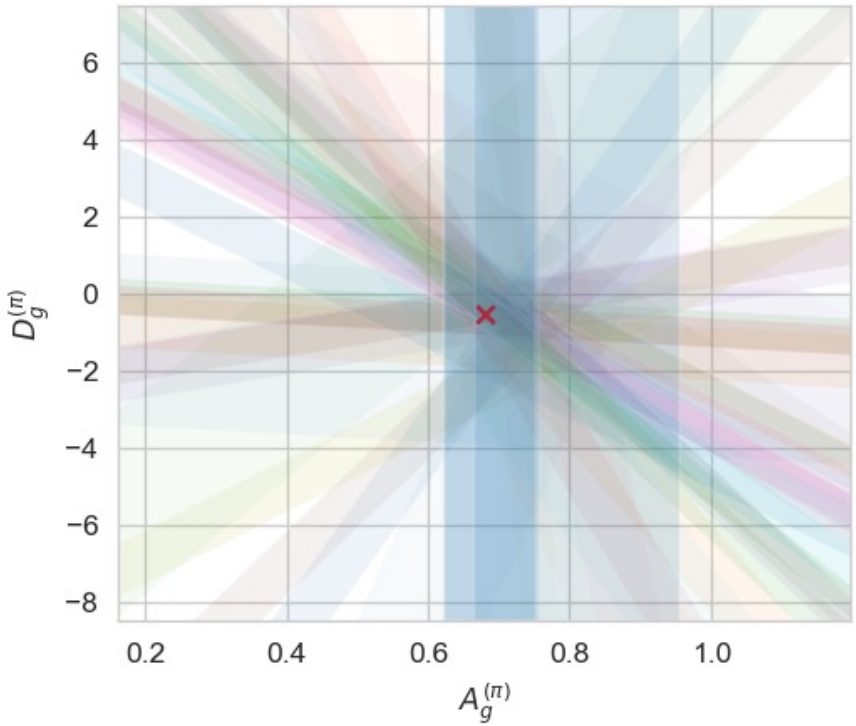


Extracting the GFFs

$$\langle p'|T(\Delta)|p\rangle = (\text{kinematic coeffs}) \cdot (\text{GFFs})(t) + (\text{excited states})$$

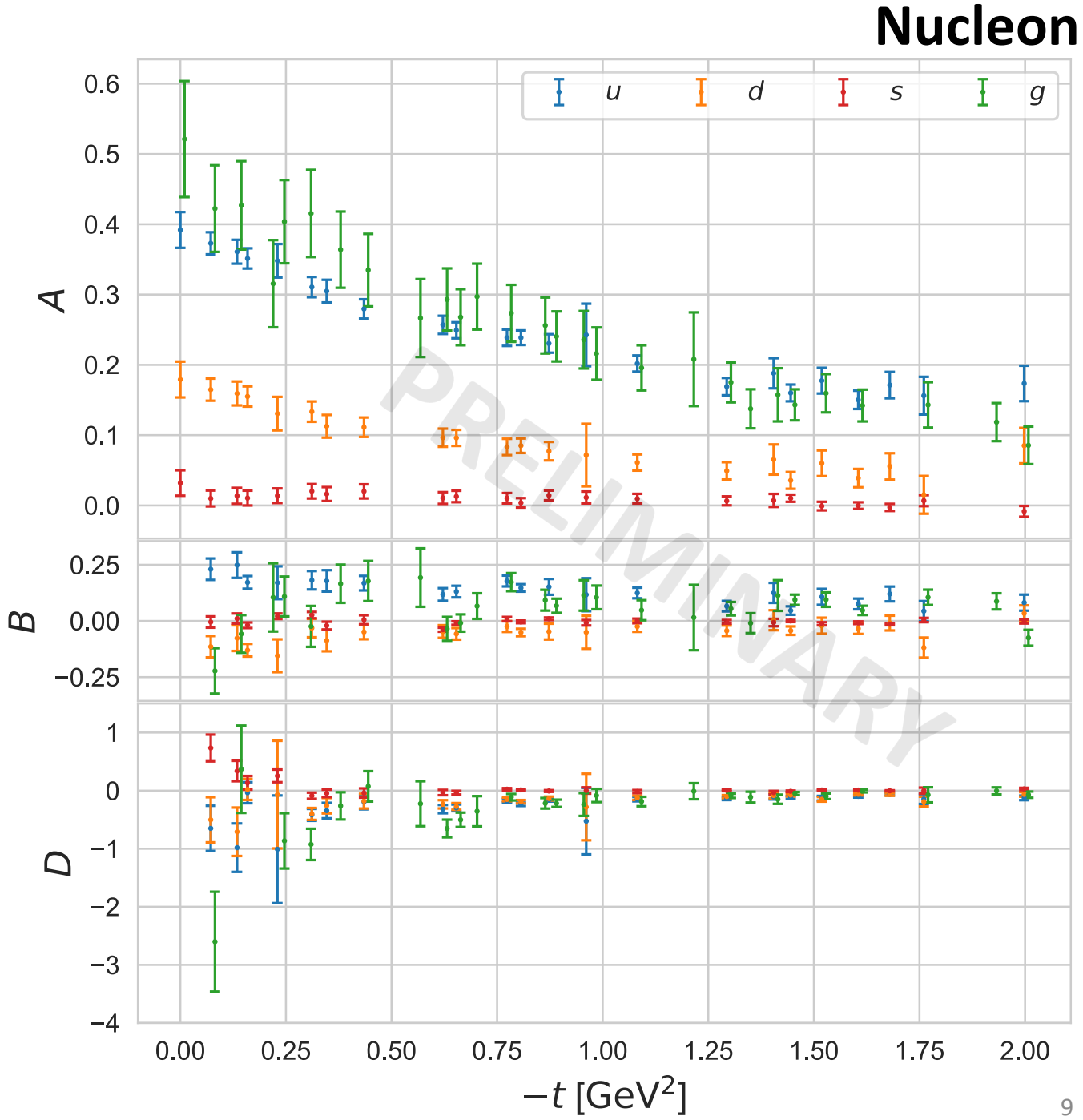
$\Rightarrow$ Bin constraints by $t = \Delta^2$ and fit





Preliminary results

Note: quark/glue mixing not accounted for!



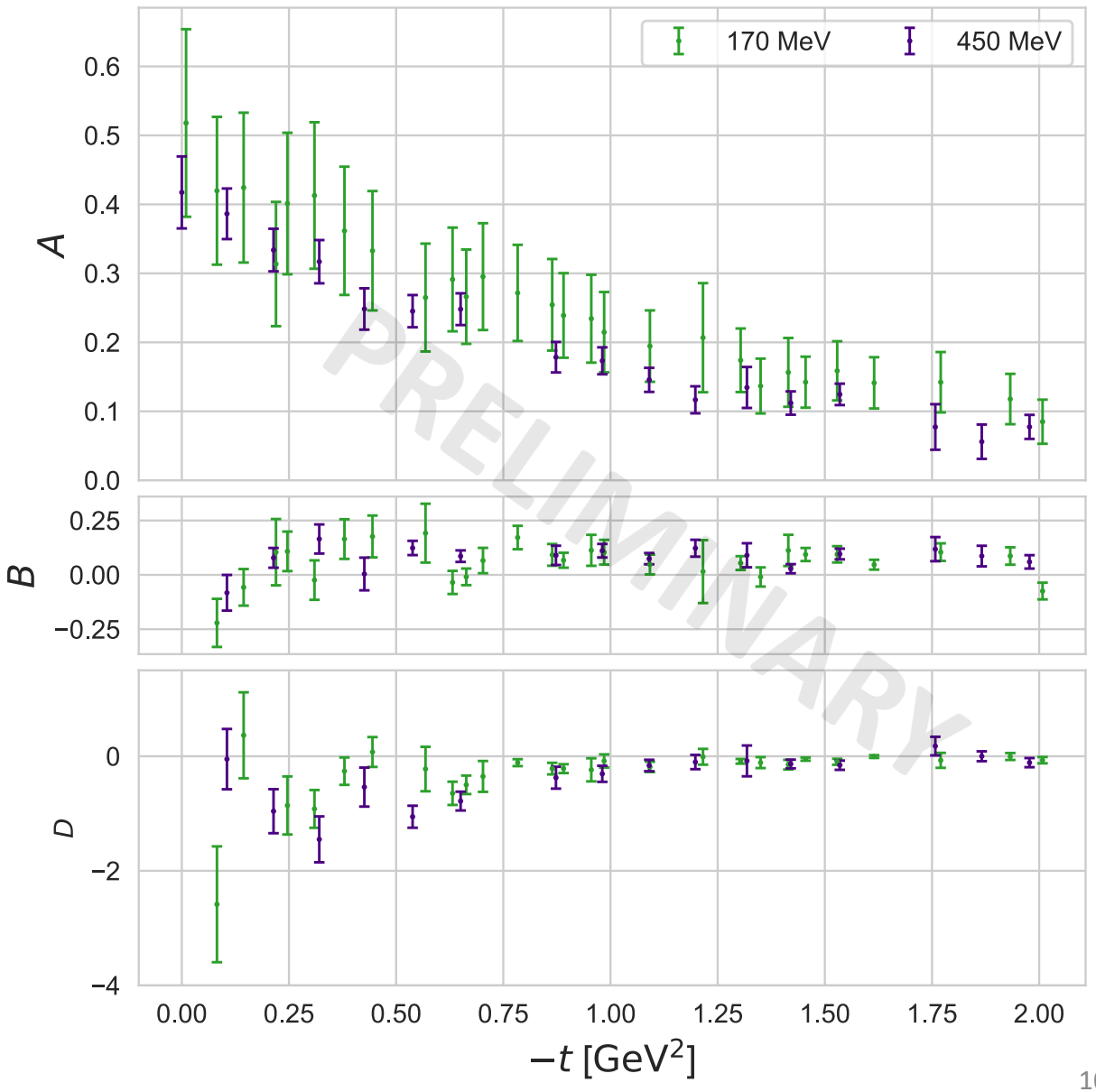
Mass dependence in gluon GFFs(?)

Compare: 170 MeV vs 450 MeV (similar a)

[Pefkou DH Shanahan 2107.10368]

Note: quark/glue mixing not accounted for in either result

Nucleon

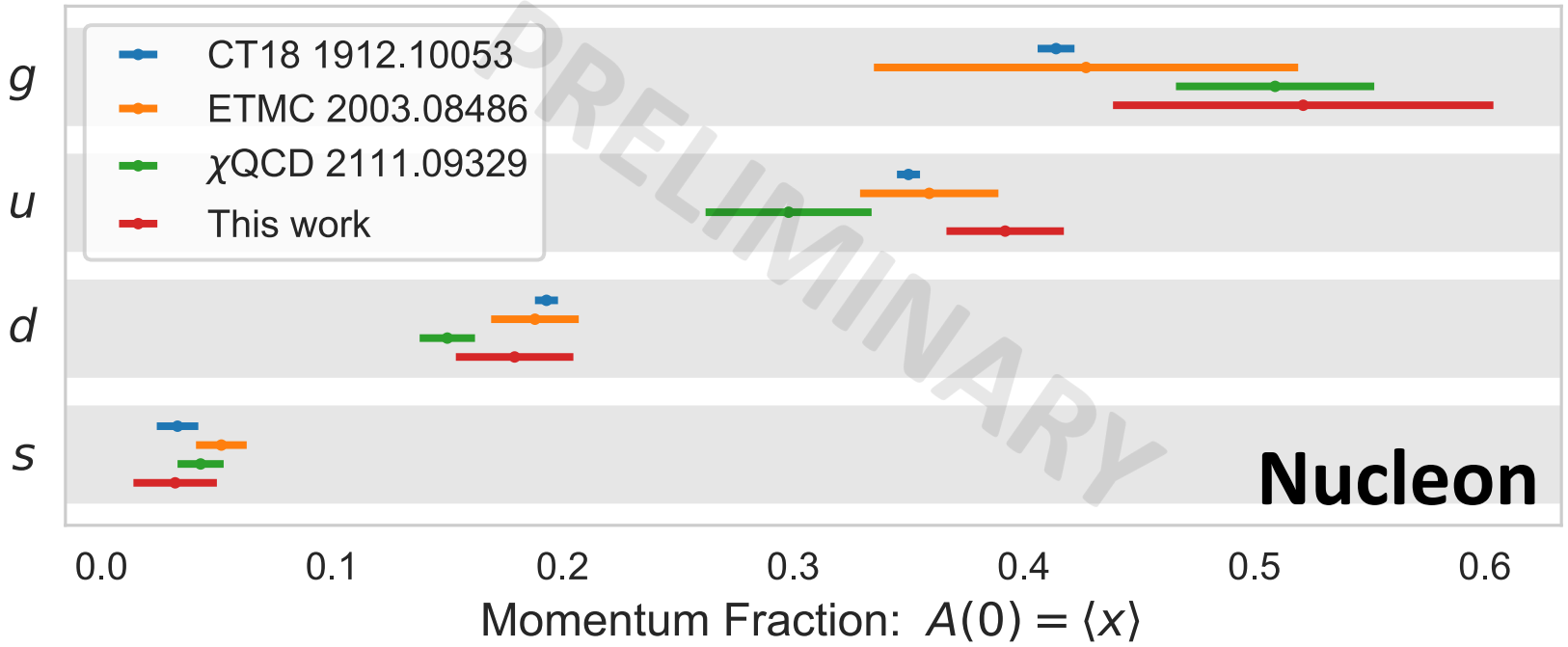


Momentum fractions vs. other lattice results

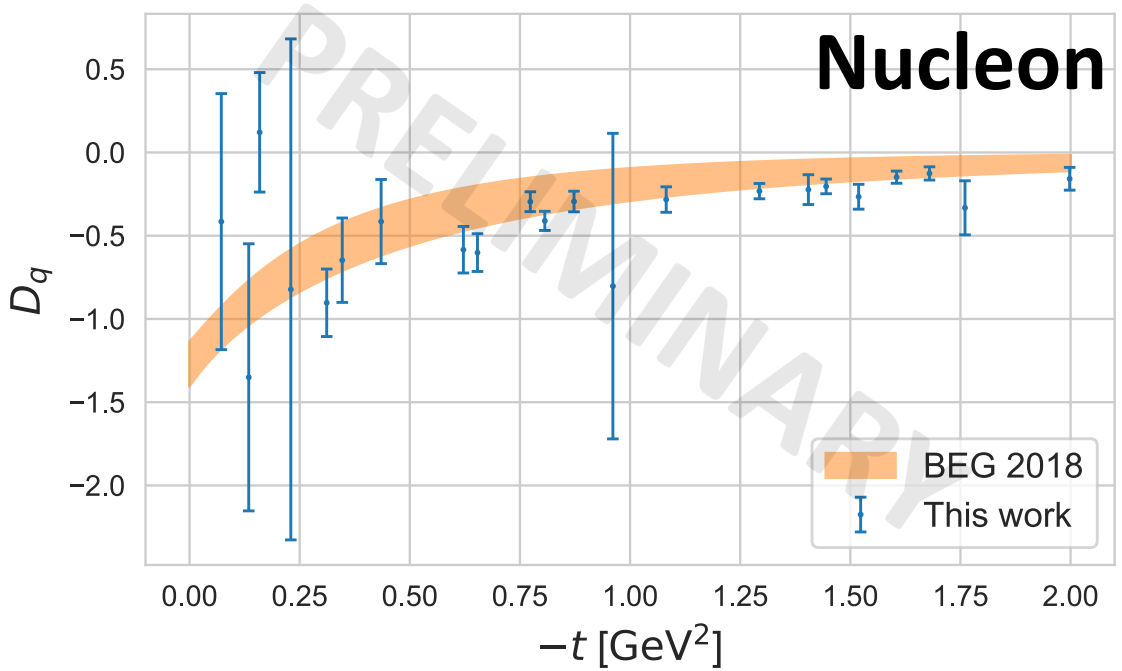
Note: quark/glue mixing not accounted for!

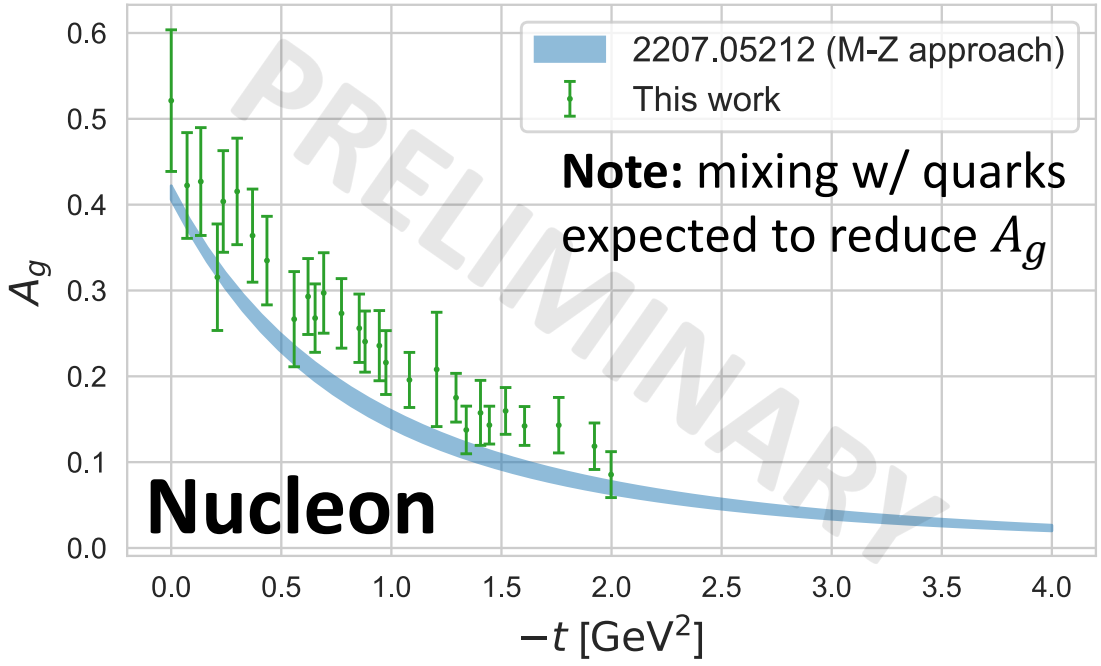
Note: other lattice studies also treated charm quark; not shown here

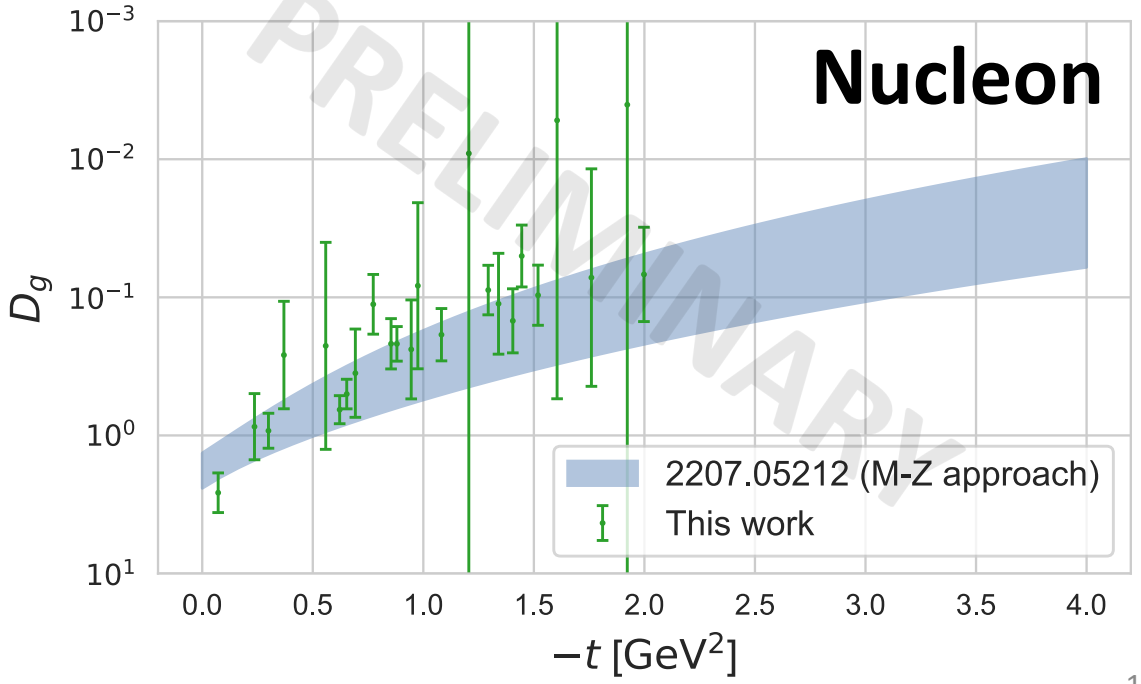
Check the sum rule:
$\sum_{u,d,s,g}\langle x\rangle = 1.12(10)$



Versus experimental results







- Experimental quark extraction from DVCS [Burkert Elouadrhiri Girod 2018]
- Experimental glue extraction from J/Ψ photoproduction [JLab HallC 2207.05212]

  Note: unpublished at present

  Note': strong method dependence

Energy, pressure, and shear force densities

1. Parametrize $T_{\mu\nu}(t)$ with GFFs
2. Fourier transform $T_{\mu\nu}(t) \to T_{\mu\nu}(r)$
3. Identify e.g.

$$T_{\mu\nu}(r) = \begin{bmatrix} T_{tt}(r) & \\ & T_{ij}(r) \end{bmatrix} = \begin{bmatrix} \epsilon(r) & \\ & \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r) \end{bmatrix}$$

→ Spatial densities

energy $\epsilon(r)$

pressure $p(r)$

shear forces $s(r)$

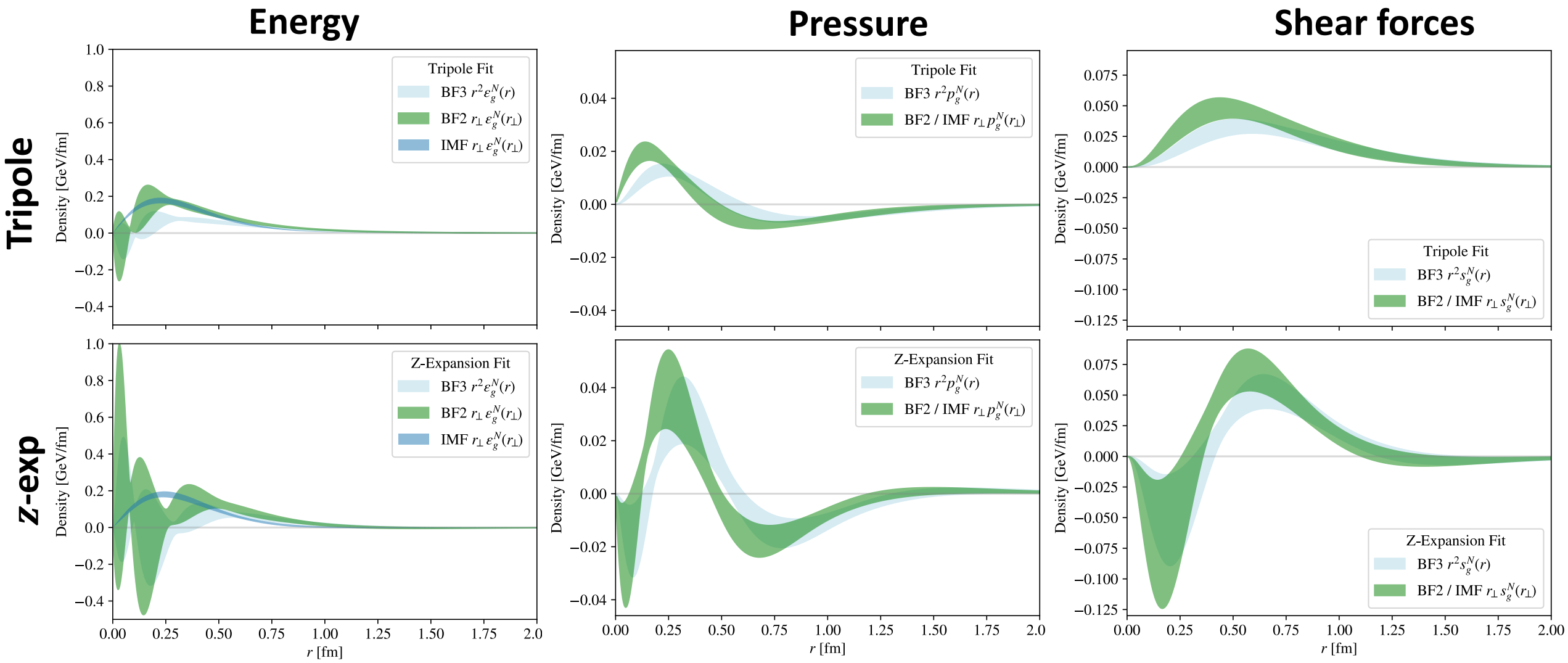
Note: frame dependent

$$T^{\mu\nu}_{i,\mathrm{BF3}}(r) = \int \frac{d^3\Delta e^{-i\mathbf{\Delta}\cdot\mathbf{r}}}{2P^0(2\pi)^3} \langle h(p,s)| T^{\mu\nu}_i |h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{BF2}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)| T^{\mu\nu}_i |h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{IMF}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)| T^{\mu\nu}_i |h(p',s')\rangle\big|^{P_z\to\infty}_{\mathbf{P}\cdot\mathbf{\Delta}=0}$$

Results: nucleon densities at 450 MeV

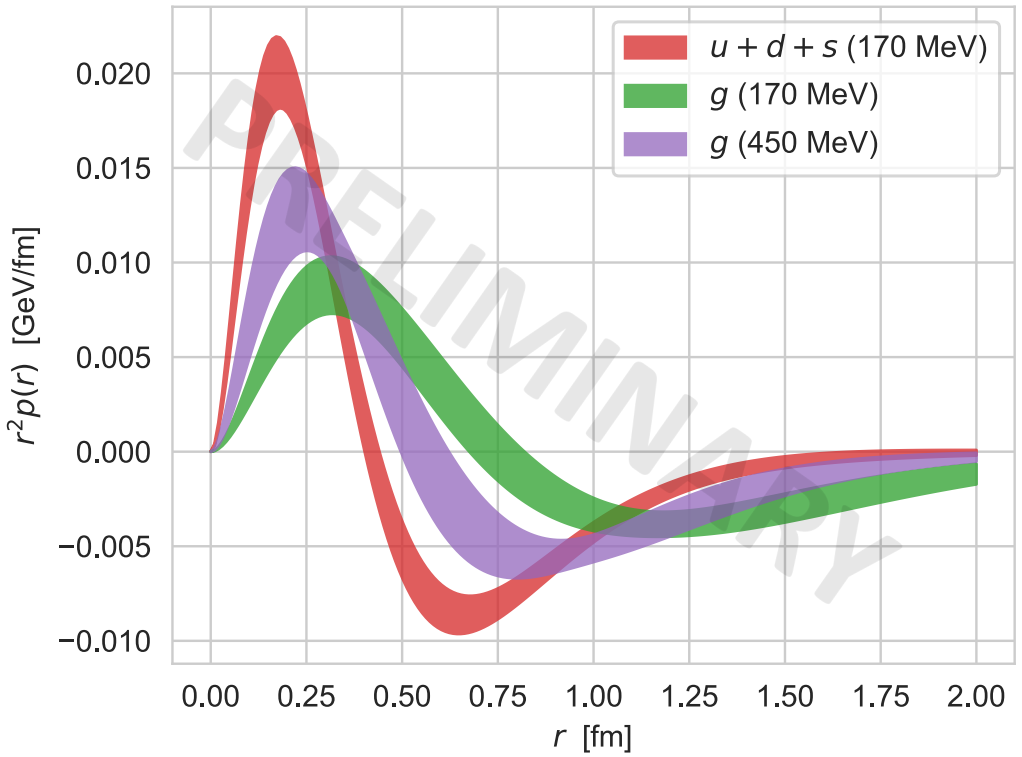


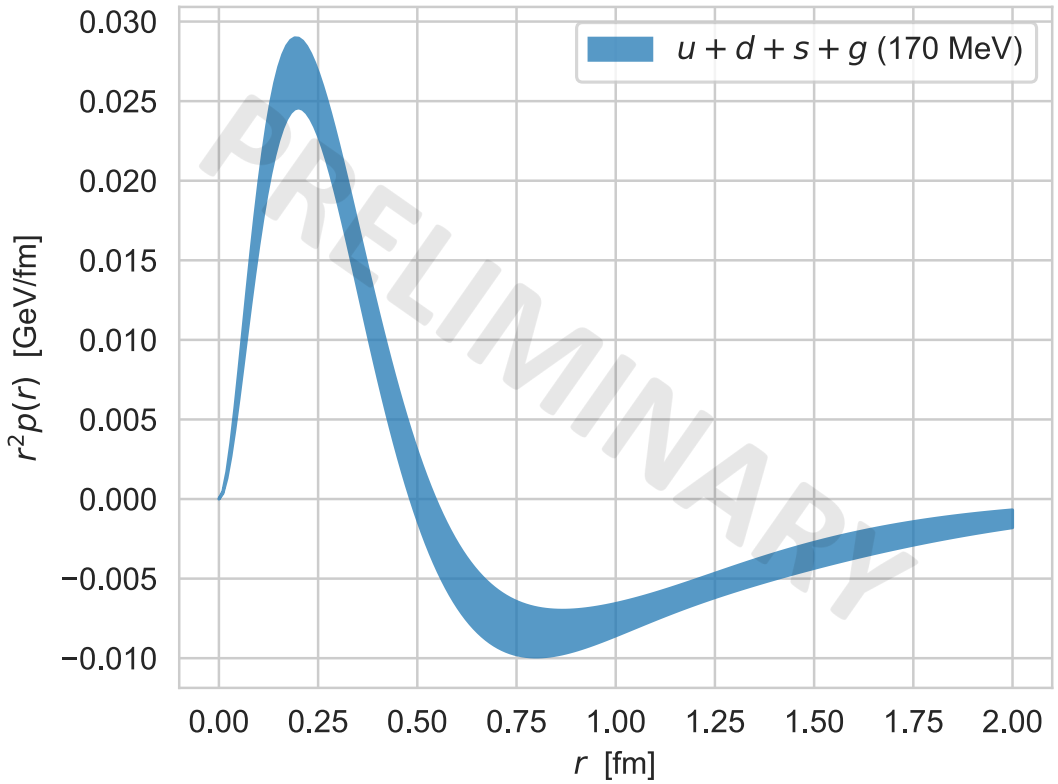
Preliminary results: pressures

…for 3d Breit frame kinematics, tripole model

Note: likely strong model dependence, as with 450 MeV results

Note: quark/glue mixing not accounted for!

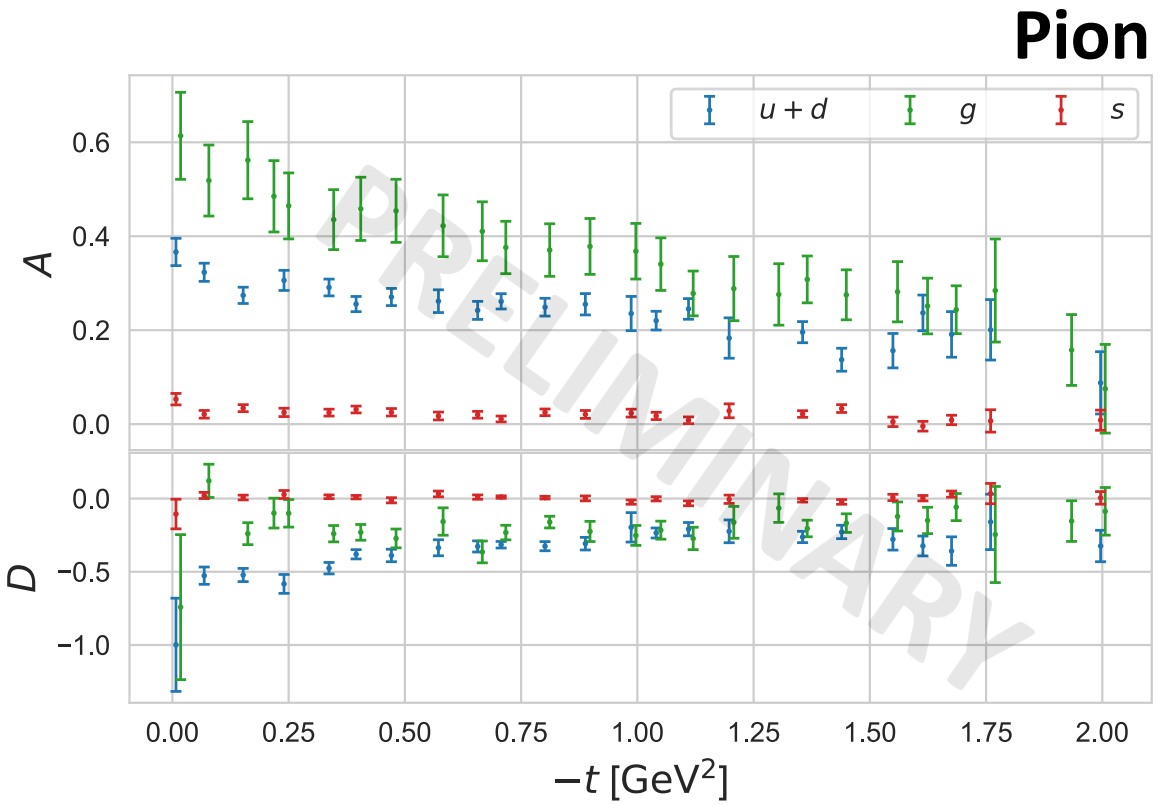


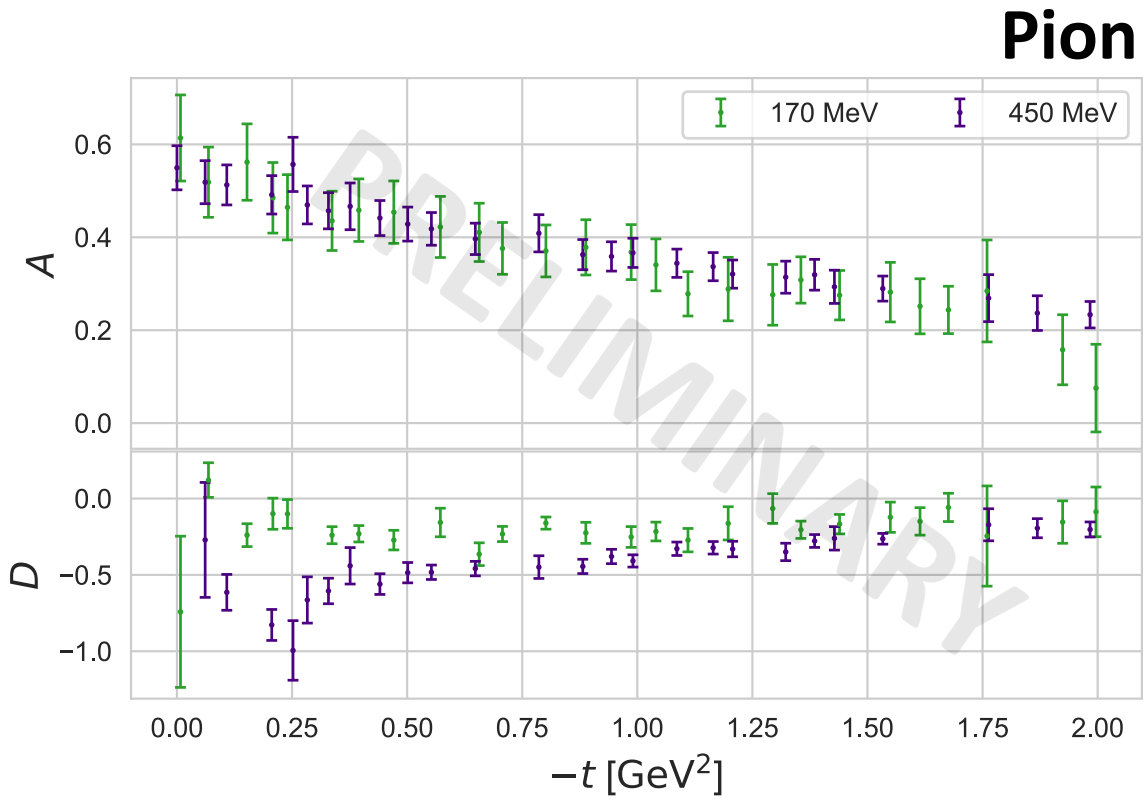


Other states

Also calculating pion GFFs

Previous results: glue GFFs for ρ meson, Δ baryon at 450 MeV [Pefkou DH Shanahan 2022]





Outlook

Preliminary results already compare well w/ other lattice, experiment

First calculation of complete set of N GFFs on a single ensemble

(Away from forward limit!)

When complete: *physical* densities of energy, pressure, shear forces

Opportunities for synergy

Lattice: incorporate GFFs into quasi/pseudo extraction?

Experiment: incorporate GFFs into global fits

→ **need full control over uncertainties!**