Structure of Low-Lying Baryon Resonances

Derek Leinweber

In collaboration with: Curtis Abell, Liam Hockley, Waseem Kamleh,
Yan Li, Zhan-Wei Liu, Finn Stokes, Tony Thomas, Jia-Jun Wu

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

THE UNIVERSITY of ADELAIDE

Outline

- Hamiltonian Effective Field Theory (HEFT)
  - Nonperturbative extension of chiral effective field theory aimed at resonance physics.
  - Incorporates the Lüscher formalism.
  - Connects scattering observables to the finite-volume spectrum of lattice QCD.
- Δ Resonance: introduce HEFT and illustrate the constraints provided by Lüscher.
  - Discuss the role of lattice QCD results in constraining the description of multi-channel scattering processes.
- Odd-parity $N^*(1535)$ and $N^*(1650)$ Resonances:
  - Knowledge of eigenstate composition can be used to understand lattice QCD results.
- Roper $N(1440)$ Resonance:
  - Lattice QCD results constrain the HEFT description of experimental data.
  - Provides deep insight into the nature of the Roper resonance.
- Conclusions

Hamiltonian Effective Field Theory (HEFT)

J. M. M. Hall, *et al.* [CSSM], Phys. Rev. D **87** (2013) 094510 [arXiv:1303.4157 [hep-lat]]
C. D. Abell, D. B. L., A. W. Thomas and J. J. Wu, Phys. Rev. D **106** (2022) 034506 [arXiv:2110.14113 [hep-lat]]

- An extension of chiral effective field theory incorporating the Lüscher formalism
  - Linking the energy levels observed in finite volume to scattering observables.

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- Fitting resonance phase-shift data and inelasticities,
  - Predictions of the finite-volume spectrum are made.
- The eigenvectors of the Hamiltonian provide insight into the composition of the energy eigenstates.
  - Insight is similar to that provided by correlation-matrix eigenvectors in Lattice QCD.

Infinite Volume Model

- The rest-frame Hamiltonian has the form $H = H_0 + H_I$, with

$$H_0 = \sum_{B_0} |B_0\rangle\, m_{B_0}\, \langle B_0| + \sum_{\alpha} \int d^3k\, |\alpha(\boldsymbol{k})\rangle\, \omega_\alpha(\boldsymbol{k})\, \langle \alpha(\boldsymbol{k})| \,,$$

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- $|B_0\rangle$ denotes a quark-model-like basis state.
- $|\alpha(\boldsymbol{k})\rangle$ designates a two-particle non-interacting basis-state channel with energy

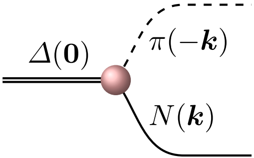
$$\omega_\alpha(\boldsymbol{k}) = \omega_{\alpha_M}(\boldsymbol{k}) + \omega_{\alpha_B}(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + m_{\alpha_M}^2} + \sqrt{\boldsymbol{k}^2 + m_{\alpha_B}^2}\,,$$

for $M =$ Meson, $B =$ Baryon.

Infinite Volume Model

- The interaction Hamiltonian includes two parts, $H_I = g + v$.
- $1 \to 2$ particle vertex

$$g = \sum_{\alpha,\, B_0} \int d^3k \, \left\{ |\alpha(\boldsymbol{k})\rangle \, G^{\dagger}_{\alpha,B_0}(k) \, \langle B_0| + h.c. \right\} \, ,$$



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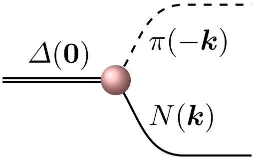
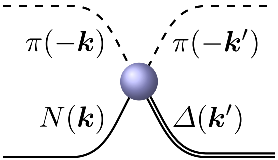


- $2 \to 2$ particle vertex

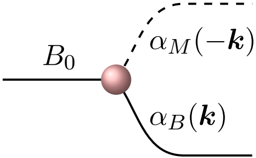
$$v = \sum_{\alpha,\beta} \int d^3k \; d^3k' \, |\alpha(\boldsymbol{k})\rangle \, V^S_{\alpha,\beta}(k,k') \, \langle \beta(\boldsymbol{k}')| \, .$$



S-wave vertex interactions

- S-wave vertex interactions between the one baryon and two-particle meson-baryon channels for e.g. $N^*(1535)$ or $\Lambda^*(1405)$ cases take the form

$$G_{\alpha,B_0}(k) = g_{B_0\alpha}\,\frac{\sqrt{3}}{2\,\pi\,f_\pi}\,\sqrt{\omega_{\alpha_M}(k)}\,u(k,\Lambda)\,,$$



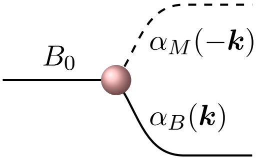
with regulator

$$u(k,\Lambda) = \left(1+\frac{k^2}{\Lambda^2}\right)^{-2}\,, \qquad \text{and fixed } \Lambda \sim 0.8 \to 1.0 \text{ GeV.}$$

P-wave and higher vertex interactions

- P-wave vertex interactions between the one baryon and two-particle meson-baryon channels for e.g. for the $\Delta(1232)$ or $N^*(1440)$ case cases take the form

$$G_{\alpha,B_0}(k) = g_{B_0\alpha}\,\frac{1}{4\pi^2}\left(\frac{k}{f_\pi}\right)^{l_\alpha}\frac{u(k,\Lambda)}{\sqrt{\omega_{\alpha_M}(k)}}\,,$$



where l_α is the orbital angular momentum in channel α.

Two-to-two particle interactions

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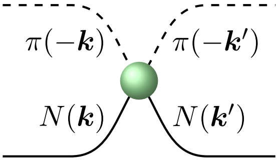
- For the S_{11} πN channel

$$V^{S}_{\pi N,\pi N}\,(\,k,\;k'\,) = v_{\pi N,\pi N}\,\frac{3}{4\pi^2\,f_\pi^2}\,\tilde{u}_{\pi N}(k,\varLambda)\,\tilde{u}_{\pi N}(k',\varLambda)$$



where the scattering potential gains a low energy enhancement via

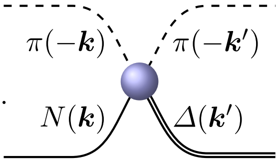
$$\tilde{u}_{\pi N}(k,\varLambda) = u(k,\varLambda)\,\frac{m_\pi^{\text{phys}} + \omega_\pi(k)}{\omega_\pi(k)}$$

and $u(k,\varLambda)$ takes the dipole form.

Two-to-two particle interactions

- For P-wave scattering in the $\Delta(1232)$ or $N^*(1440)$ channels

$$V^S_{\alpha,\beta}\,(\,k,\,k'\,) = v_{\alpha,\beta}\,\frac{1}{4\pi^2\,f_\pi^2}\,\frac{k}{\omega_{\alpha_M}(k)}\,\frac{k'}{\omega_{\beta_M}(k')}\,u(k,\Lambda)\,u(k',\Lambda)\,.$$



Infinite-Volume scattering amplitude

- The T-matrices for two particle scattering are obtained by solving the coupled-channel integral equations

$$T_{\alpha,\beta}(k,k';E) = \tilde{V}_{\alpha,\beta}(k,k';E) + \sum_{\gamma} \int q^2 dq \, \frac{\tilde{V}_{\alpha,\gamma}(k,q;E)\, T_{\gamma,\beta}(q,k';E)}{E - \omega_\gamma(q) + i\epsilon} \, .$$

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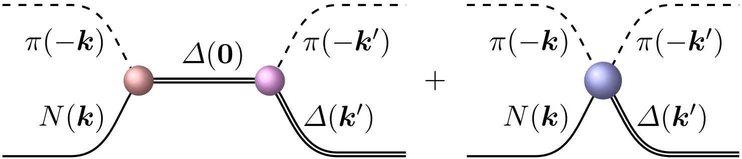
- The coupled-channel potential is readily calculated from the interaction Hamiltonian

$$\tilde{V}_{\alpha,\beta}(k,k') = \sum_{B_0} \frac{G^{\dagger}_{\alpha,B_0}(k)\, G_{\beta,B_0}(k')}{E - m_{B_0}} + V^{S}_{\alpha,\beta}(k,k') \,,$$



Infinite-Volume scattering matrix

- The S-matrix is related to the T-matrix by

$$S_{\alpha,\beta}(E) = 1 - 2i\,\sqrt{\rho_\alpha(E)\,\rho_\beta(E)}\;T_{\alpha,\beta}(k_{\alpha\,\mathrm{cm}}, k_{\beta\,\mathrm{cm}}; E)\,,$$

with

$$\rho_\alpha(E) = \pi\,\frac{\omega_{\alpha_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\alpha_B}(k_{\alpha\,\mathrm{cm}})}{E}\,k_{\alpha\,\mathrm{cm}}\,,$$

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- The cross section $\sigma_{\alpha\,,\beta}$ for the process $\alpha \to \beta$ is

$$\sigma_{\alpha\,,\beta} = \frac{4\pi^3\,k_{\alpha\,\mathrm{cm}}\,\omega_{\alpha_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\alpha_B}(k_{\alpha\,\mathrm{cm}})\,\omega_{\beta_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\beta_B}(k_{\alpha\,\mathrm{cm}})}{E^2\,k_{\beta\,\mathrm{cm}}}\;|T_{\alpha,\beta}(k_{\alpha\,\mathrm{cm}}, k_{\beta\,\mathrm{cm}}; E)|^2\,.$$

πN phase shift and inelasticity

- The S-matrix is related to the T-matrix by

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\begin{aligned}
S_{\pi N,\pi N}(E) &= 1 - 2i\pi \frac{\omega_\pi(k_{\rm cm})\,\omega_N(k_{\rm cm})}{E}\, k_{\rm cm}\; T_{\pi N,\pi N}(k_{\rm cm}, k_{\rm cm}; E)\,, \\
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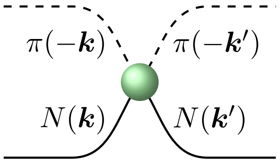

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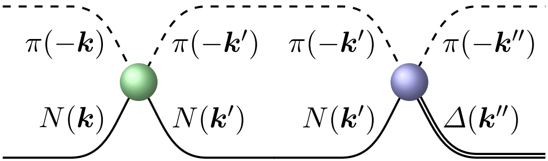

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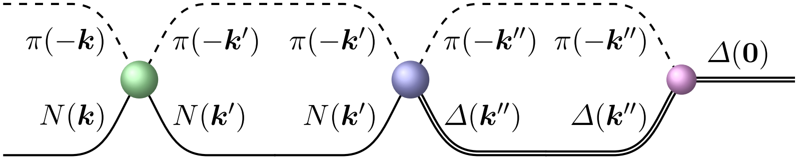

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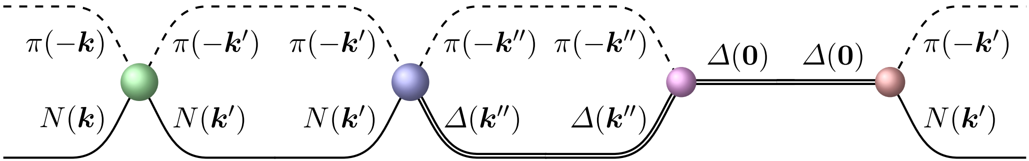
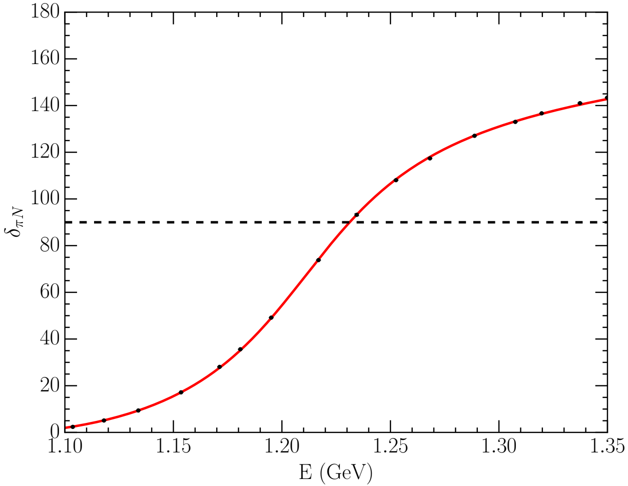


P-wave πN phase shifts in the Δ channel - 1 πN channel



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with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.

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with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.
- The non-interacting Hamiltonian takes the form

$$H_0 = \begin{pmatrix} m_{B_0} & & 0 & & & 0 & & \cdots \\ & \omega_{\pi N}(k_0) & & & & & & \\ 0 & & \ddots & & & 0 & & \cdots \\ & & & \omega_{\pi\Delta}(k_0) & & & & \\ & & & & \omega_{\pi N}(k_1) & & & \\ 0 & & 0 & & & \ddots & & \cdots \\ & & & & & & \omega_{\pi\Delta}(k_1) & \\ \vdots & & \vdots & & & \vdots & & \ddots \end{pmatrix}.$$

$1 \rightarrow 2$ particle interaction terms

- $1 \rightarrow 2$ particle interaction terms sit in the first row and column.

$$H_I = \begin{pmatrix} 0 & g_{\pi N}(k_0) & \cdots & g_{\pi \Delta}(k_0) & g_{\pi N}(k_1) & \cdots & g_{\pi \Delta}(k_1) \cdots \\ g_{\pi N}(k_0) & 0 & \cdots & & & & \\ \vdots & \vdots & 0 & & & & \\ & & & \ddots & & & \\ g_{\pi \Delta}(k_0) & & & & & & \\ g_{\pi N}(k_1) & & & & & & \\ \vdots & & & & & & \\ g_{\pi \Delta}(k_1) & & & & & & \\ \vdots & & & & & & \end{pmatrix} .$$

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- $2 \rightarrow 2$ particle interaction terms fill out the rest of the Hamiltonian matrix.

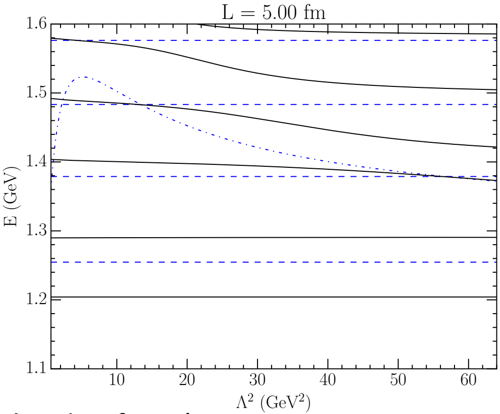
Finite Volume Eigenmode Solution

- Standard Lapack routines provide eigenmode solutions of

$$\langle\, i \,|\, H \,|\, j \,\rangle\, \langle\, j \,|\, E_\alpha \,\rangle = E_\alpha \,\langle\, i \,|\, E_\alpha \,\rangle\,,$$

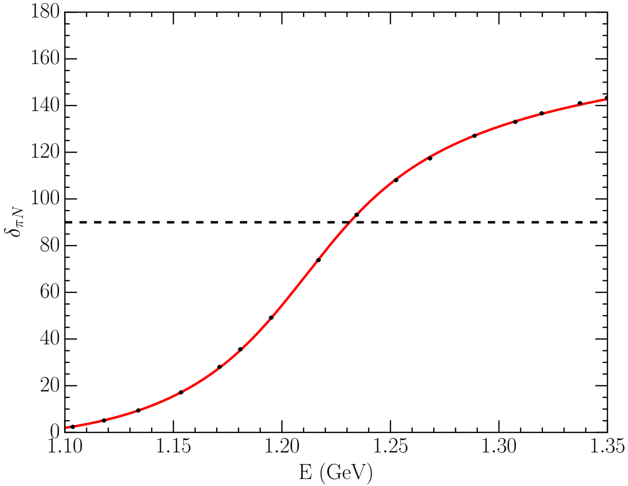
  - where $|\, i \,\rangle$ and $|\, j \,\rangle$ are the non-interacting basis states,
  - E_α is the energy eigenvalue, and
  - $\langle\, i \,|\, E_\alpha \,\rangle$ is the eigenvector of the
  - Hamiltonian matrix $\langle\, i \,|\, H \,|\, j \,\rangle$.

Energy eigenstates on an $L = 5$ fm lattice for different regulators



- Incorporation of the Lüscher formalism ensures energy eigenstates below 1.35 GeV are model independent.

P-wave πN phase shifts in the Δ channel - 1 πN channel



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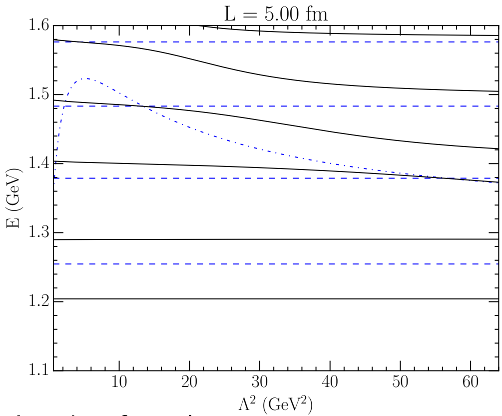

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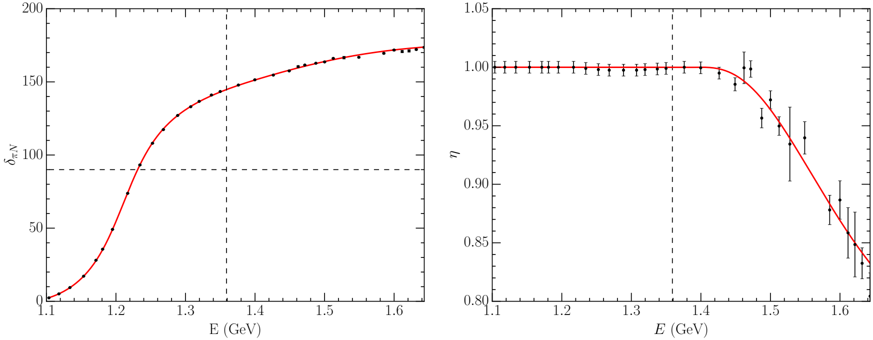
P-wave πN scattering in the Δ channel - 2 channel πN and $\pi\Delta$



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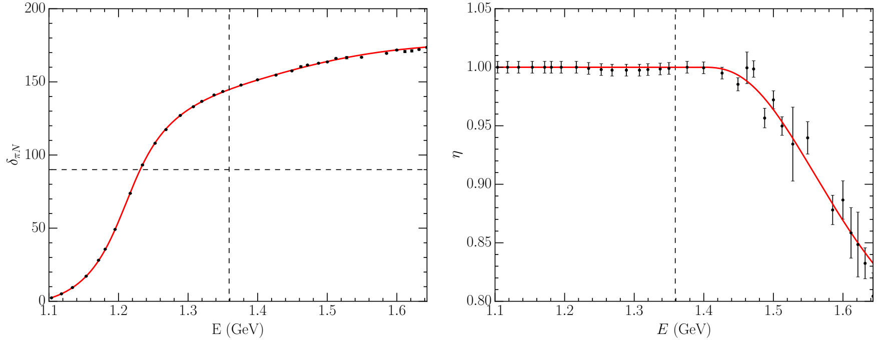

- Anticipate regulator independence to 1.7 GeV.

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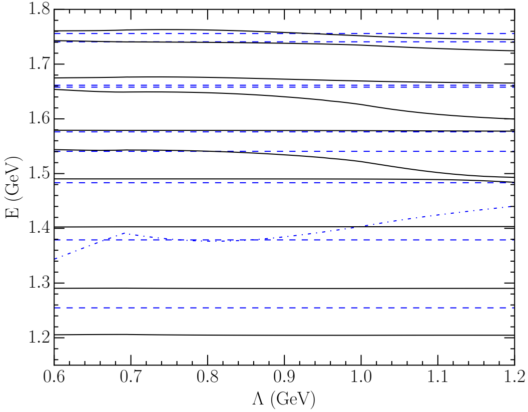

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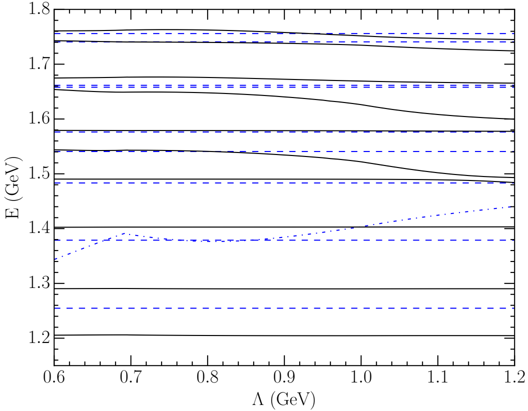
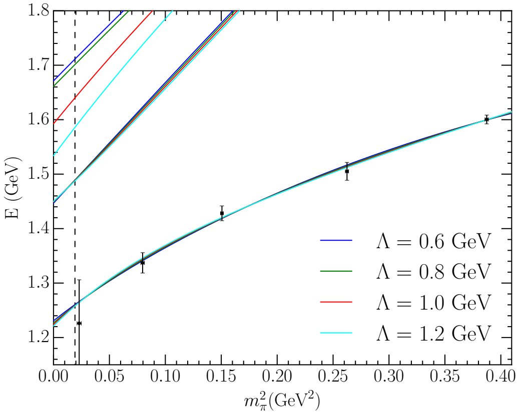


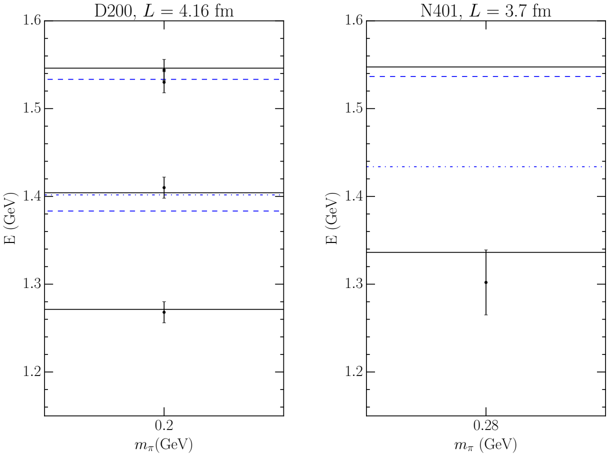
- πN scattering data alone is insufficient to uniquely constrain the Hamiltonian.

Mass dependence of energy eigenstates - Fit to PACS-CS Δ masses



- Lattice QCD results can constrain the Hamiltonian description of experimental data.

CLS Consortium finite-volume lattice energies of Δ-channel excitations

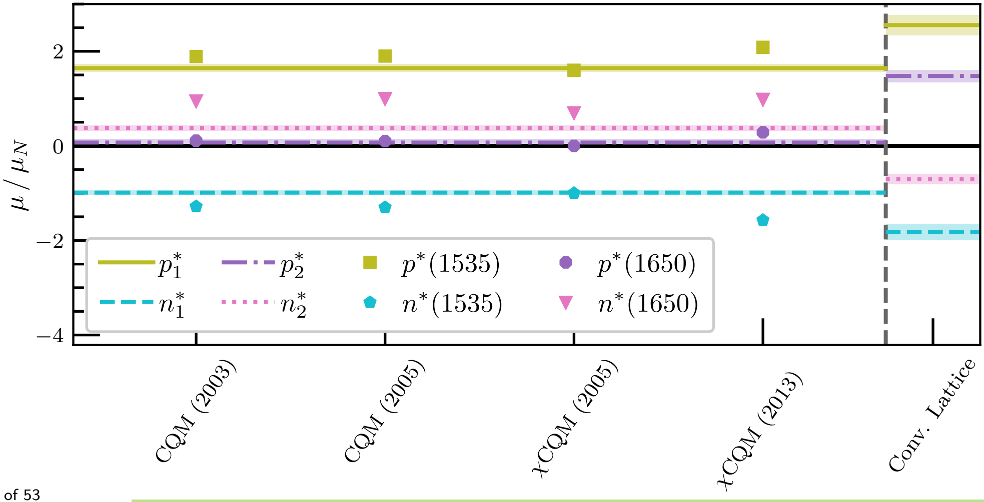


- C. Morningstar, *et al.* PoS **LATTICE2021** (2022), 170 [arXiv:2111.07755 [hep-lat]].
- C. W. Andersen, J. Bulava, B. Hörz and C. Morningstar, Phys. Rev. D **97** (2018) no.1, 014506 [arXiv:1710.01557 [hep-lat]].

Examination of low-lying odd-parity nucleon resonances

- Motivated by lattice QCD calculations of the electromagnetic form factors of the two low-lying odd-parity states.
- Parity-expanded variational analysis (PEVA) removes opposite-parity contaminants.
- Confirms quark model predictions for N^* magnetic moments.
- F. M. Stokes, W. Kamleh and D. B. L., Phys. Rev. D **102** (2020) 014507 [arXiv:1907.00177 [hep-lat]].

N^* Magnetic Moments and the constituent quark model



Model Calculation References

- CQM (2003)

  W.-T. Chiang, S. N. Yang, M. Vanderhaeghen, and D. Drechsel, Magnetic dipole moment of the S 11 (1535) from the $\gamma p \to \gamma\eta p$ reaction, Nucl. Phys. **A723**, 205 (2003), nucl-th/0211061

- χCQM (2005)

  J. Liu, J. He, and Y. Dong, Magnetic moments of negative-parity low-lying nucleon resonances in quark models, Phys. Rev. **D71**, 094004 (2005).

- χCQM (2013)

  N. Sharma, A. Martinez Torres, K. Khemchandani, and H. Dahiya, Magnetic moments of the low-lying $1/2^-$ octet baryon resonances, Eur. Phys. J. **A49**, 11 (2013), arXiv:1207.3311

Examination of low-lying odd-parity nucleon resonances

- Motivated by lattice QCD calculations of the electromagnetic form factors of the two low-lying odd-parity states
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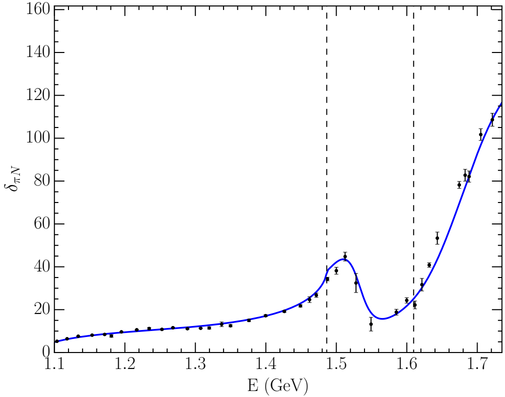
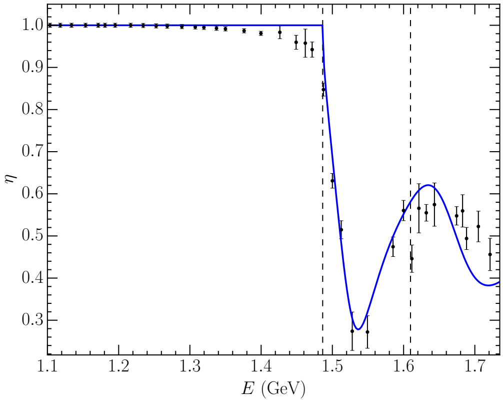
  - Pole positions agree with PDG.

Phase shift and inelasticity for the low-lying odd-parity spin-1/2 nucleon resonances





- WI08 single-energy data from SAID.
- Vertical lines indicate the opening of the ηN and $K\Lambda$ thresholds.

Finite Volume Eigenmode Solution

- Standard Lapack routines provide eigenmode solutions of

$$\langle\, i \,|\, H \,|\, j \,\rangle \,\langle\, j \,|\, E_\alpha \,\rangle = E_\alpha \,\langle\, i \,|\, E_\alpha \,\rangle \,.$$

- Eigenvector $\langle\, i \,|\, E_\alpha \,\rangle$ describes the composition of the eigenstate $|\, E_\alpha \,\rangle$ in terms of the basis states $|\, i \,\rangle$ with

$$|\, i \,\rangle = |\, B_0 \,\rangle, \quad |\, \pi N, k_0 \,\rangle, \quad |\, \pi N, k_1 \,\rangle, \quad \cdots |\, \eta N, k_0 \,\rangle, \quad |\, \eta N, k_1 \,\rangle, \quad \cdots \,.$$

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- The overlap of the bare basis state $|\, B_0 \,\rangle$ with eigenstate $|\, E_\alpha \,\rangle$,

$$\langle\, B_0 \,|\, E_\alpha \,\rangle \, ,$$

is of particular interest,

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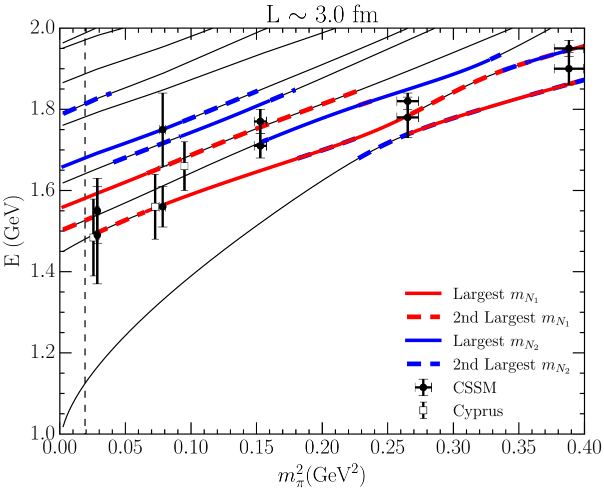
$$\overline{\chi}(0)\,|\Omega\rangle \simeq |B_0\rangle\ ,$$

- Thus, element $\langle\, B_0 \,|\, E_\alpha \,\rangle$ of the eigenvector governs the likelihood of observing eigenstate $|\, E_\alpha \,\rangle$.

Finite-volume $L = 3$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



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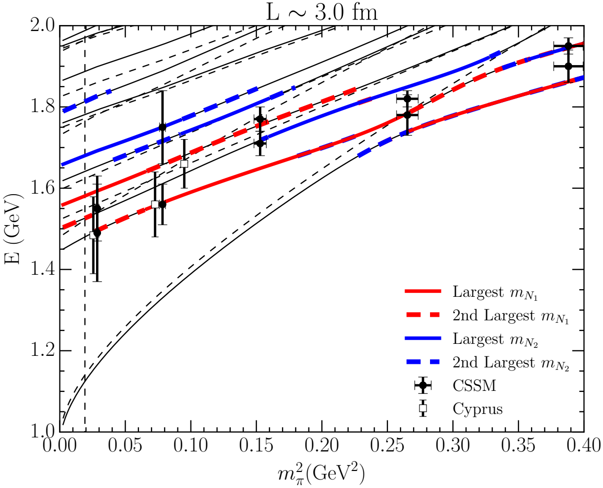
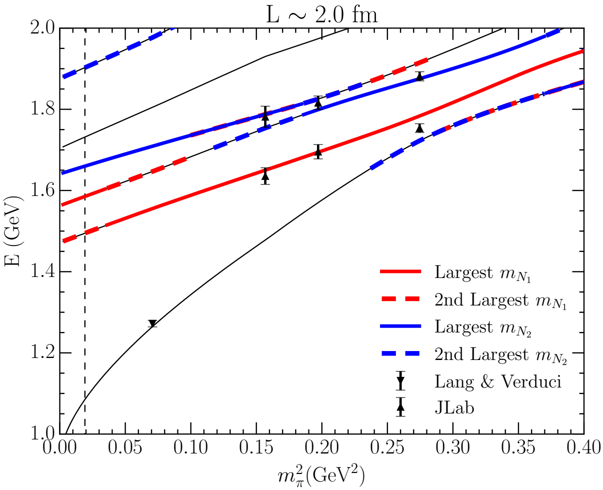


Finite-volume $L = 2$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



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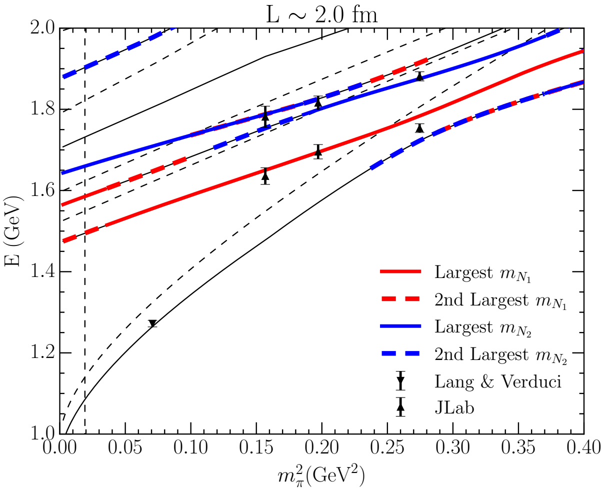
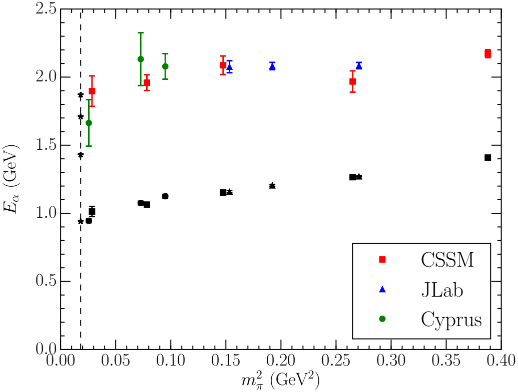


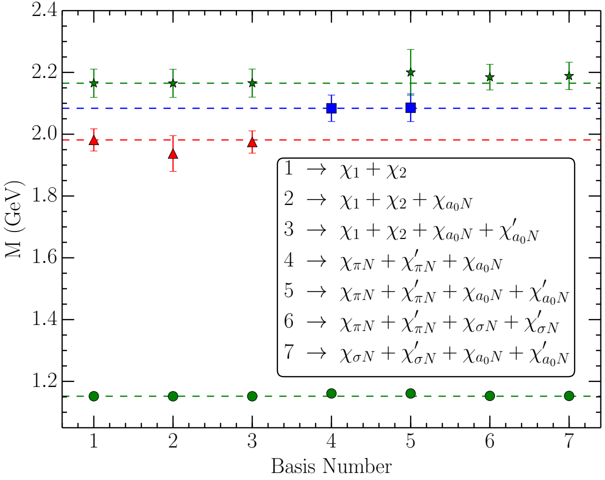
Where is the Roper resonance?



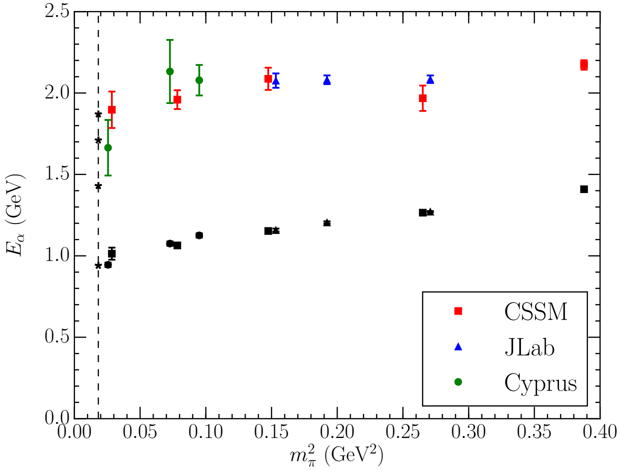
- CSSM: Z. W. Liu, *et al.* [CSSM], Phys. Rev. D **95**, 034034 (2017) arXiv:1607.04536 [nucl-th]
- Cyprus: C. Alexandrou, *et al.* (AMIAS), Phys. Rev. D **91**, 014506 (2015) arXiv:1411.6765 [hep-lat]
- JLab: R. G. Edwards, *et al.* [HSC] Phys. Rev. D **84**, 074508 (2011) [arXiv:1104.5152 [hep-ph]].

Search for low-lying lattice QCD eigenstates in the Roper regime

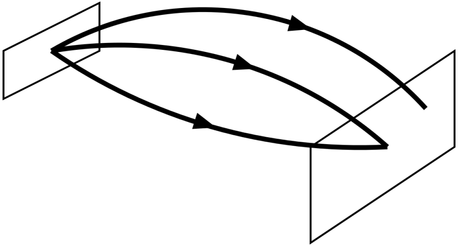
A. L. Kiratidis, *et al.*, [CSSM] Phys. Rev. D **95**, no. 7, 074507 (2017) [arXiv:1608.03051 [hep-lat]].



Have we seen the $2s$ excitation of the quark model?

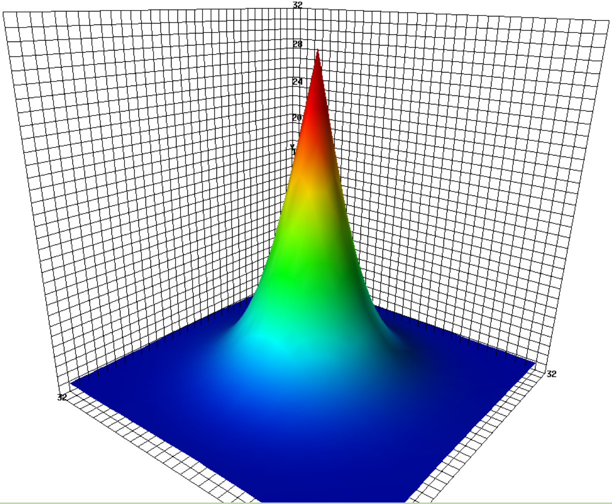


Landau-Gauge Wave functions from the Lattice

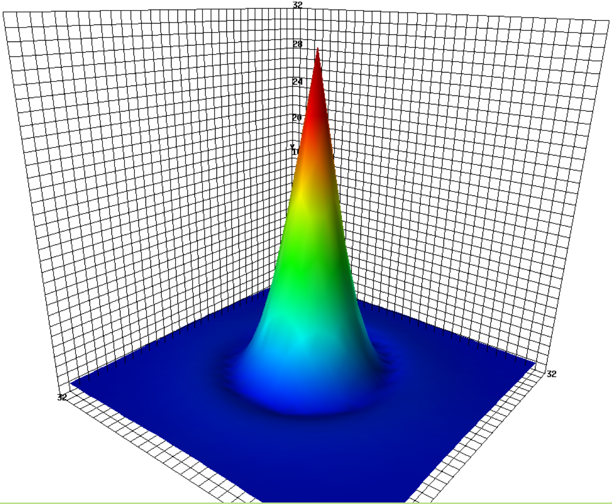


- Measure the *overlap* of the annihilation operator with the state as a function of the quark positions.

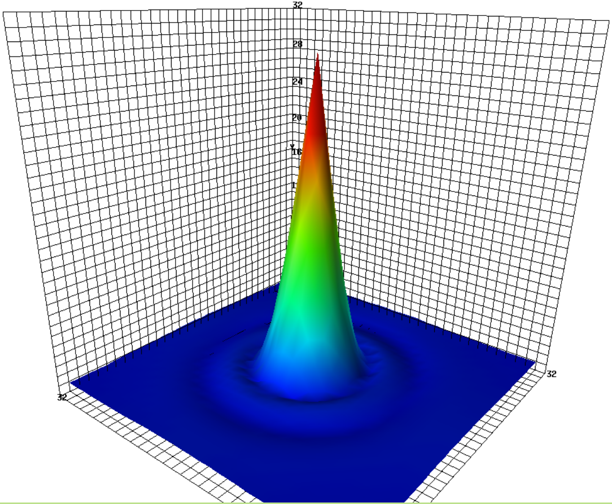
d-quark probability density in ground state proton [CSSM]



d-quark probability density in 1st excited state of proton [CSSM]

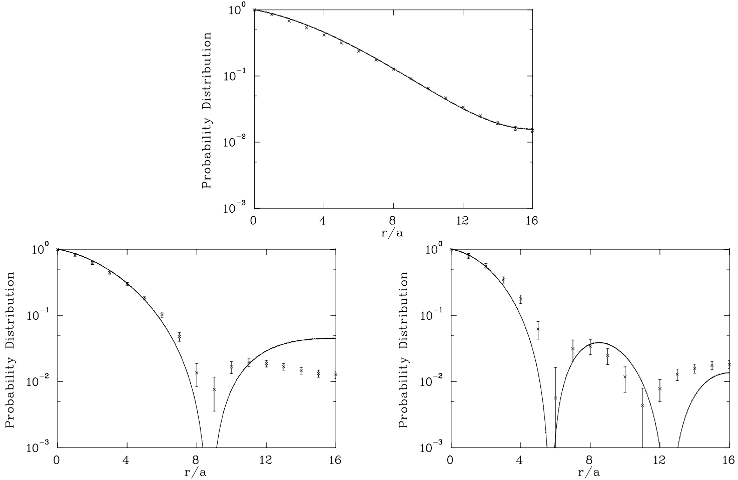


d-quark probability density in $N = 3$ excited state of proton [CSSM]

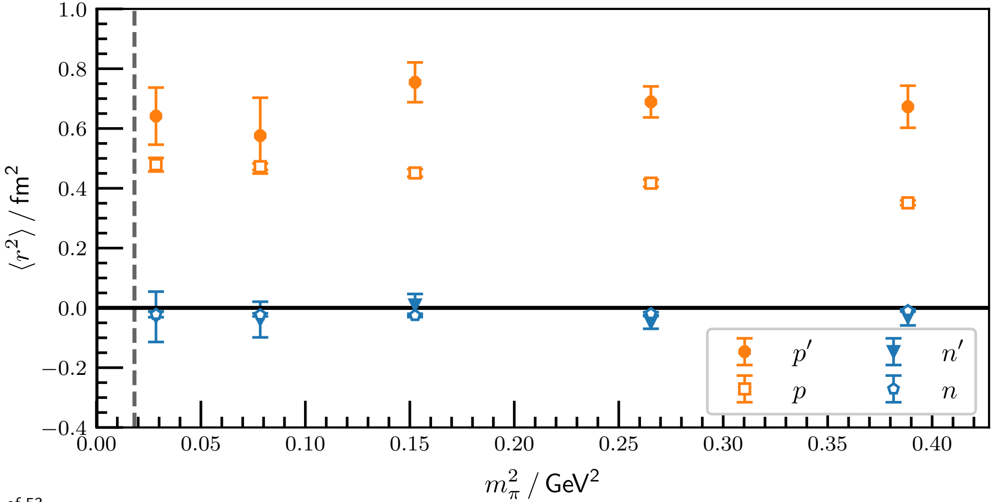


Comparison with the Simple Quark Model [CSSM]

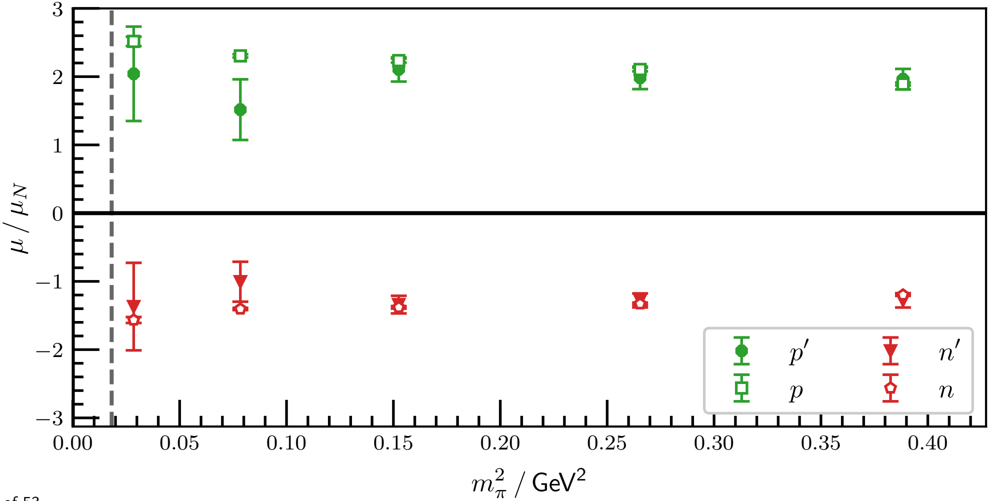
D. S. Roberts, W. Kamleh and D. B. Leinweber, Phys. Lett. B **725**, 164 (2013) [arXiv:1304.0325 [hep-lat]].



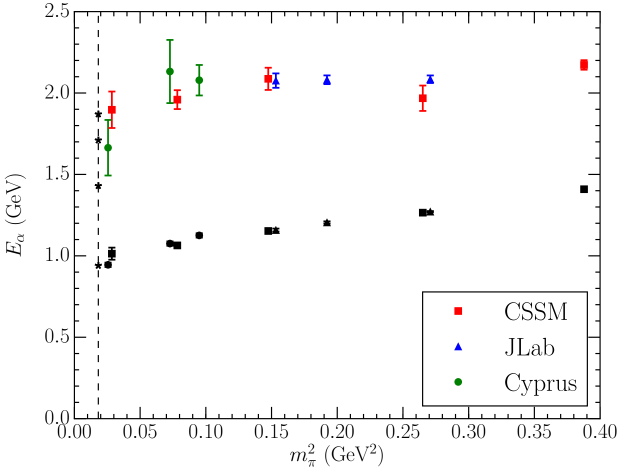
First positive-parity excitation: Charge Radii



First positive-parity excitation: Magnetic moments

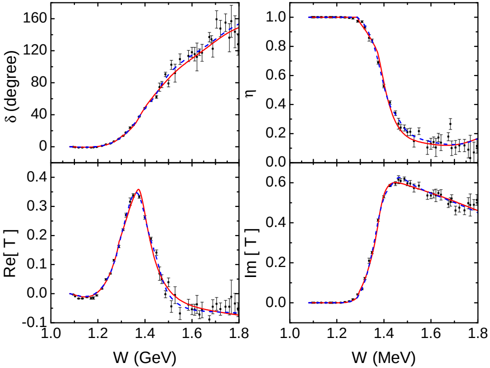


Are these lattice results consistent with the Roper Resonance?



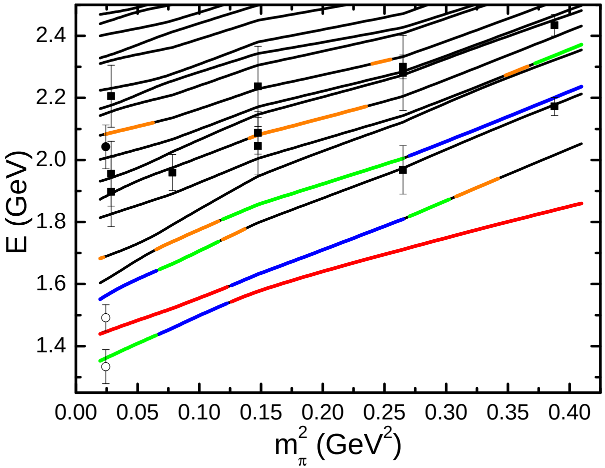
Positive-parity Nucleon Spectrum: Bare Roper Case with $m_0 = 1.7$ GeV

- Consider πN, $\pi\Delta$ and σN channels, dressing a bare state.
- Fit to phase shift and inelasticity. (dashed blue curve)



- Fit yields two poles in the region of the PDG estimate $1365 \pm 15 - i\, 95 \pm 15$ MeV.

1.7 GeV Bare Roper: Hamiltonian Model N' Spectrum



Positive-parity Nucleon Spectrum: Bare Roper Case with $m_0 = 2.0$ GeV

J. j. Wu, *et al.* [CSSM], arXiv:1703.10715 [nucl-th]

- Consider πN, $\pi \Delta$ and σN channels, dressing a bare state.
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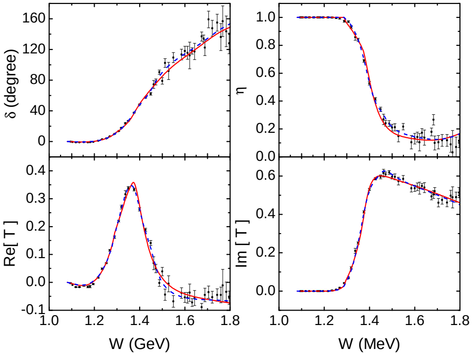
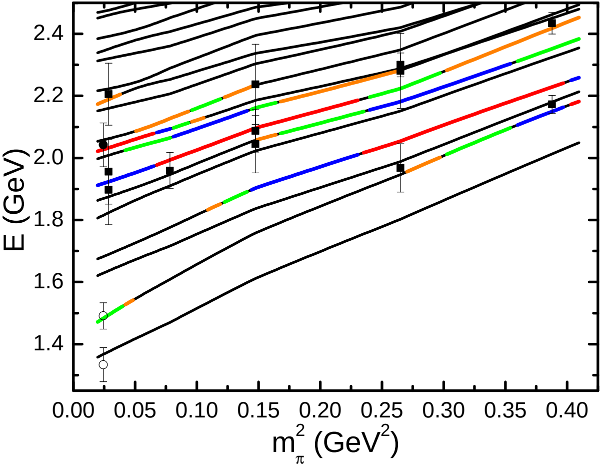


- Fit yields a pole at $1393 - i\,167$ MeV $\sim$ PDG estimate $1365 \pm 15 - i\,95 \pm 15$ MeV.

2.0 GeV Bare Roper: Hamiltonian Model N' Spectrum



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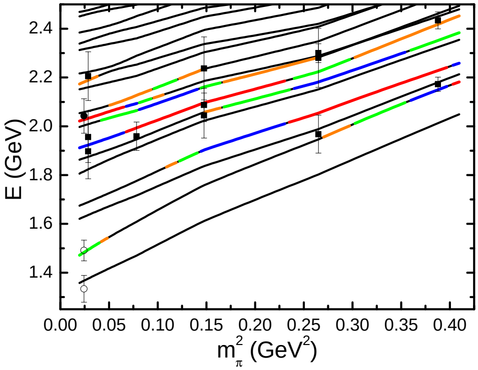
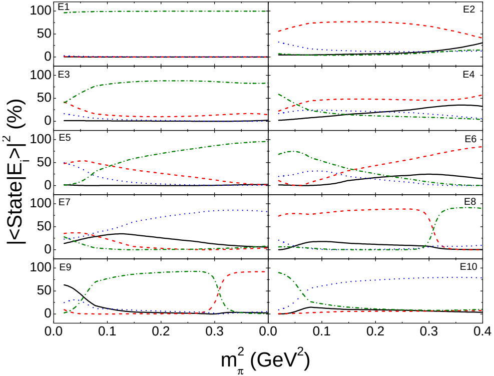
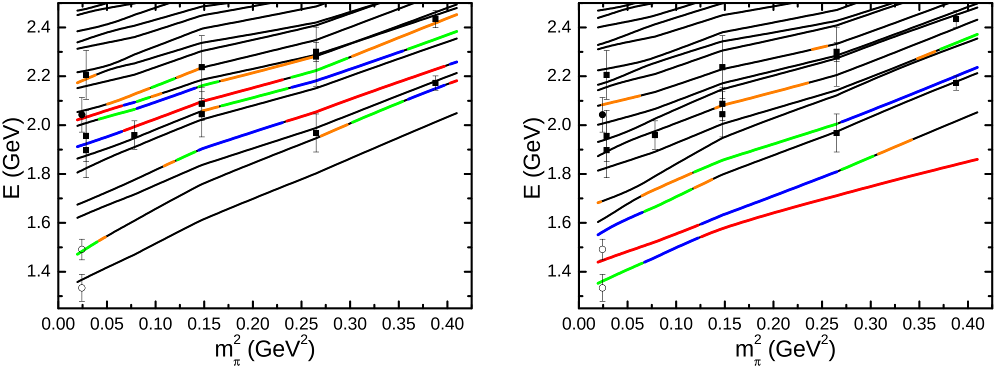




πN, $\pi\Delta$ and σN channels, dressing a bare state.

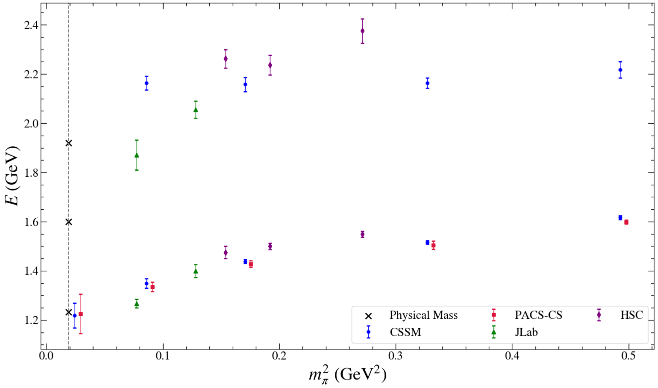
C. B. Lang, L. Leskovec, M. Padmanath and S. Prelovsek, Phys. Rev. D **95**, no. 1, 014510 (2017) [arXiv:1610.01422 [hep-lat]].

Two different descriptions of the Roper resonance



(left) Resonance generated by strong rescattering in meson-baryon channels.
(right) Meson dressings of a quark-model like core.

Δ-baryon spectrum from lattice QCD



HSC: J. Bulava, *et al.*, Phys. Rev. D **82** (2010) 014507 [arXiv:1004.5072 [hep-lat]].

JLab: T. Khan, D. Richards and F. Winter, Phys. Rev. D **104** (2021) 034503 [arXiv:2010.03052 [hep-lat]].

PACS-CS: S. Aoki *et al.* [PACS-CS], Phys. Rev. D **79** (2009) 034503 [arXiv:0807.1661 [hep-lat]].

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