

# Towards the nature of exotic states from experimental line shapes

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QNP2022 - The 9th International Conference on Quarks and Nuclear Physics

September 07, 2022

in collaboration with

X. Dong, M. Du, A. Filin, F.-K. Guo, C. Hanhart, I. Matuschek, A. Nefediev, J. Nieves and Q. Wang

EPJA 57, 101 (2021)      and      PLB 833, 137290 (2022)

# Outline

- Intro about near-threshold exotic states
- Weinberg analysis of compositeness and its generalisations
  - Basics of the Weinberg's analysis for bound states
  - Evolution of poles and generalisation to virtual states and resonances
  - Extensions to coupled channels
- Applications to NN, X(3872) and Tcc+

# Evidence for Exotic States near thresholds

- Heavy-light sector:  $D_{s0}(2317)$ ,  $D_{s1}(2460)$ ,  $X_{0/1}(2900)$ , ...

- XYZ quarkonium-like states

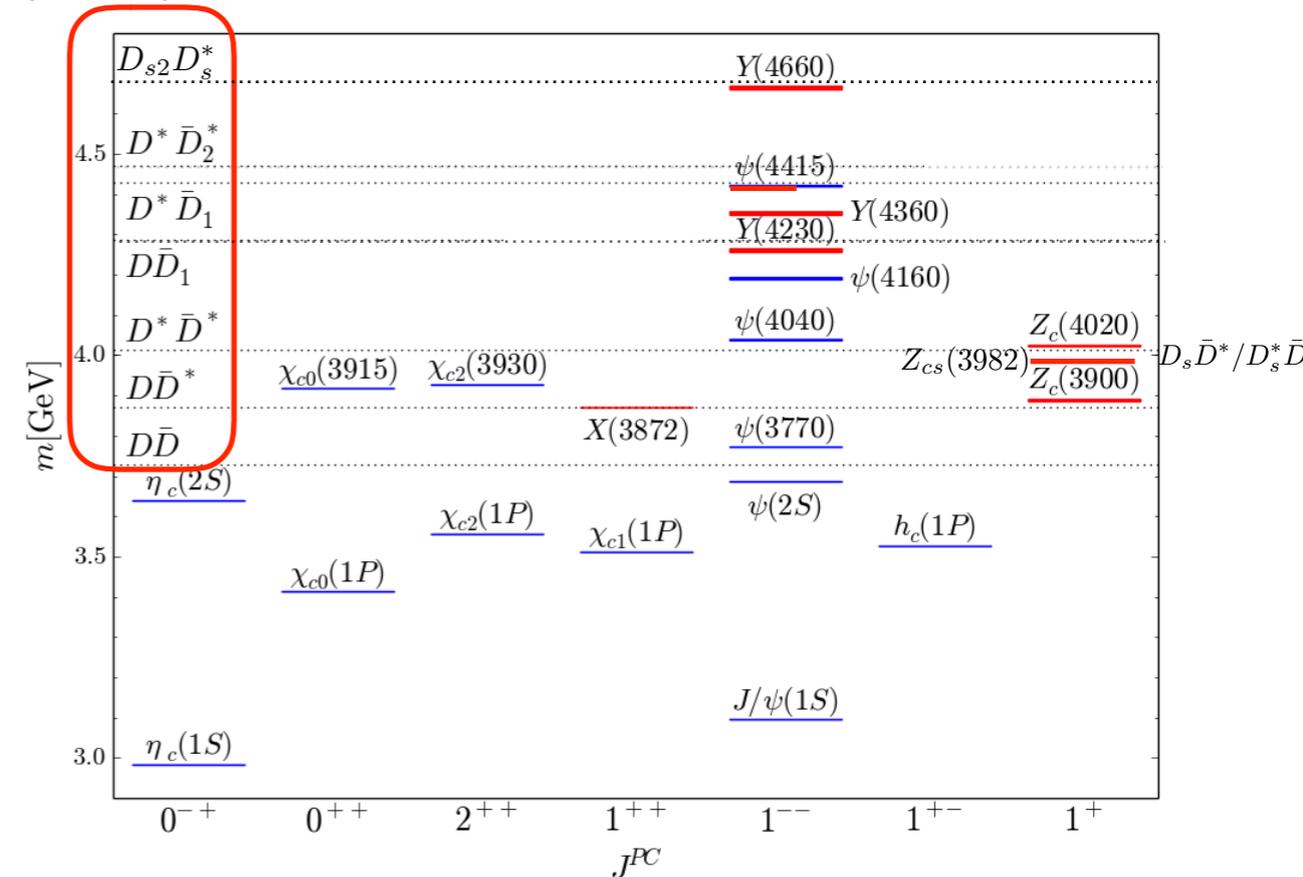
$C\bar{C}$ -sector:  $X(3872)$ ,  $X(3960)$ ,  $X(4020)$ , ...  
 $Z_c(3900)$ ,  $Z_c(4020)$ ,  $Z_{cs}(3982)$  ...  
 $Y(4230)$ ,  $Y(4360)$ ,  $Y(4660)$ , ...

Bottomonium sector:  $Z_b(10610)$ ,  $Z_b(10650)$

Fully charm tetraquark:  $X(6900)$

- Pentaquark states:  $P_c(4312)$ ,  $P_c(4440)$ ,  $P_c(4457)$ ,  $P_{cs}(4459)$ ,  $P_{cs}(4337)$

- $T_{cc+}$



How to discriminate?

Conventional

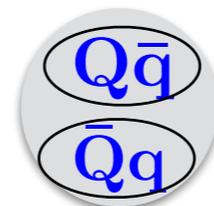


meson



baryon

Exotic

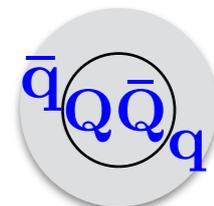


molecule

$R \gg 1\text{fm}$   
extended



tetraquark



hadroquarkonium

$R \sim 1\text{fm}$   
compact

Nature?

- Assume there is a bound state.  
Its full wave function is

$$|\Psi\rangle = \begin{pmatrix} \lambda |\psi_0\rangle \\ \chi(\vec{p}) |h_1 h_2\rangle_{\vec{p}} \end{pmatrix} \quad \begin{array}{l} \text{— compact part} \\ \text{— continuum part} \end{array}$$

$$|\langle\psi_0|\Psi\rangle|^2 = \lambda^2 \quad \text{— probability to find a compact component}$$

$$\langle\Psi|\Psi\rangle = 1 \implies X = 1 - \lambda^2 \quad \text{— probability to find a molecular component}$$

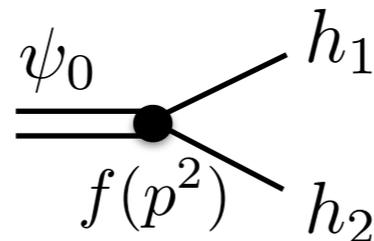
- Interaction Hamiltonian:

$$\hat{\mathbf{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{hh}^0 \end{pmatrix} \quad \hat{\mathbf{H}}|\Psi\rangle = E|\Psi\rangle$$

Then, continuum w.f. reads

$$\chi(p) = \lambda \frac{f(p^2)}{E - p^2/(2\mu)}$$

with the transition vertex



$$f(p^2) = \langle\psi_0|\hat{V}|h_1 h_2\rangle_{\vec{p}}$$

Note:  $\hat{H}_{hh}^0$  is a free Hamiltonian

for details see Weinberg (1963), V.B. et al (2010)

# From probability to effective coupling

Normalization:  $1 = \langle \Psi | \Psi \rangle = \lambda^2 \left( 1 + \int \frac{d^3 p}{(2\pi)^3} \frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2} \right)$

Model-independent extraction of  $\lambda^2$  is possible only in the weak binding limit for S-wave interaction

$$\int \frac{d^3 p}{(2\pi)^3} \frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2} = 4\mu^2 \pi^2 \frac{g_0^2}{\gamma} \left[ 1 + \mathcal{O}\left(\frac{\gamma}{\beta}\right) \right]$$

$f(p^2) \approx g_0^2 \left( 1 + \mathcal{O}\left(\frac{\gamma}{\beta}\right) \right)$   
 $f^2(-\gamma^2) = g_0^2$  —bare coupling  
 $\gamma = \sqrt{2\mu E_B}$  —binding momentum  
 $\frac{1}{\beta}$  — range of forces

Relation between  $\lambda^2$  and measurable coupling of a state with a hadronic channel:

$$g_0^2 = \frac{2\pi\gamma}{\mu^2} \frac{1 - \lambda^2}{\lambda^2} \implies g_R^2 = g_0^2 \lambda^2 = \frac{2\pi\gamma}{\mu^2} (1 - \lambda^2) \equiv \frac{2\pi\gamma}{\mu^2} X$$

Remarks:

1) for S-waves  $X = 1 - \lambda^2$  scales with  $\gamma$  — unitarity-driven leading term

2) for P-waves  $X$  scales with  $\beta \Rightarrow$  No model-independent Conc possible

# One-channel scattering amplitude

Weinberg 1963-65

- Matching of the scattering amplitude to ERE:

$$T(E) = \frac{g_0^2}{E + E_B + \frac{g_0^2 \mu}{2\pi} (ik + \gamma)} \quad \Longrightarrow \quad T(E) = -\frac{2\pi}{\mu} \frac{1}{1/a + (r/2)k^2 - ik}$$

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Physical coupling and the ERE parameters via  $\lambda^2$

$$a = -2 \frac{1 - \lambda^2}{2 - \lambda^2} \frac{1}{\gamma} + \mathcal{O}(1/\beta) \quad r = -\frac{\lambda^2}{1 - \lambda^2} \frac{1}{\gamma} + \mathcal{O}(1/\beta) \quad g_R^2 = \frac{2\pi\gamma}{\mu^2} (1 - \lambda^2) + \mathcal{O}(1/\beta)$$

$a < 0$  — bound state      $\gamma = \sqrt{2\mu E_B}$

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$a < 0$  — bound state      $\gamma = \sqrt{2\mu E_B}$

$X = 1 - \lambda^2$  — probability to find a molecular component

$$X = 1 - \lambda^2 = \sqrt{\frac{1}{1 + 2r/a}}$$

If  $|a| \gg |r|$ ,  $r \sim 1/\beta \implies X \rightarrow 1 \implies$  **Molecule**

If  $|a| \ll |r|$ ,  $r < 0 \implies X \rightarrow 0 \implies$  **Compact state**

- Same information can be inferred from pole counting

Morgan 1992

# Weinberg compositeness: Applicability range

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## These results are valid only

– for bound states in S waves

– up to range corrections: Weak binding  $\gamma \ll \beta \sim 1/m_{\text{ex}} \implies O(\gamma/\beta)$  small

Beyond weak binding  $\gamma \sim \beta \implies O(\gamma/\beta) \sim 1$  100%

For insights on range effects, see Albaladejo, Nieves 2022, Li et al. 2022, Song et al 2022, ...

– if the ERE is valid at low energies: no CDD zeros near the threshold

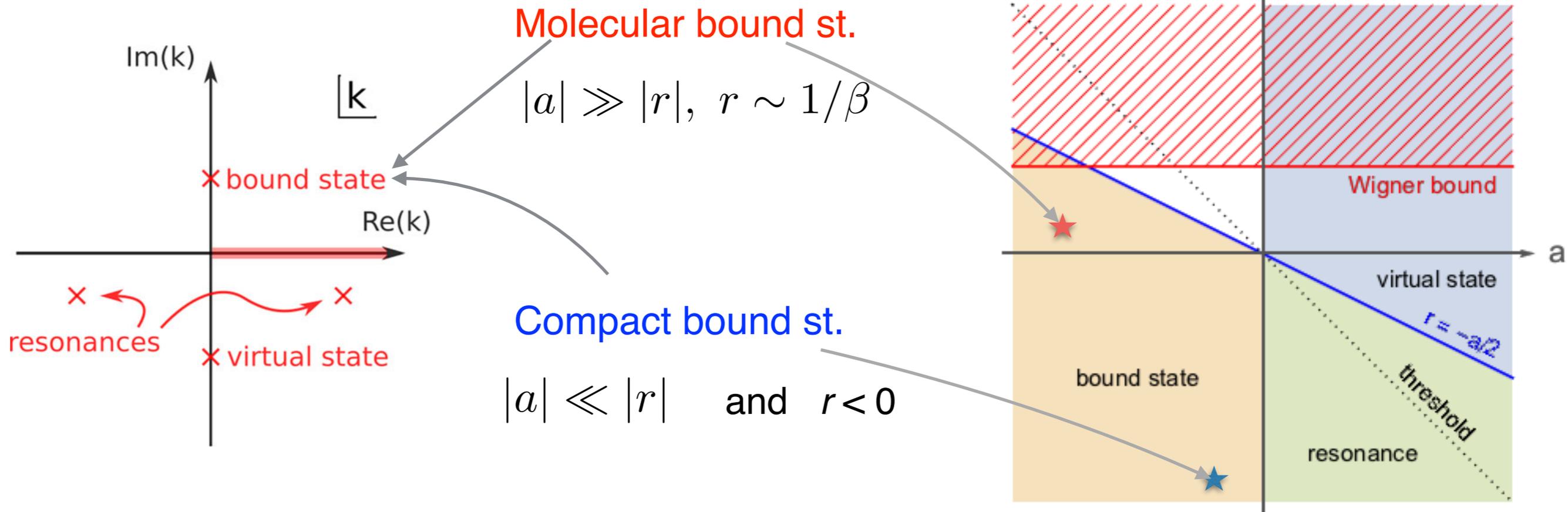
Otherwise, generalisations are needed – see VB et al. 2010, Kang, Oller 2017, Oller 2018

– for stable states: deuteron

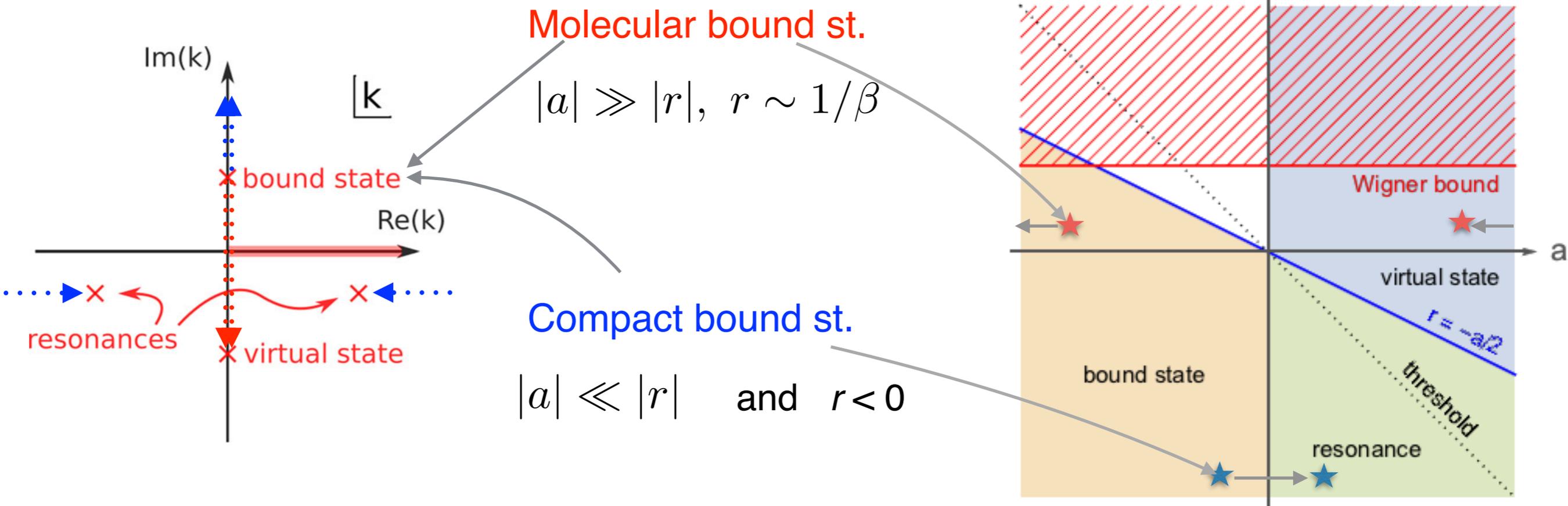
Generalisations to narrow unstable states with remote inelastic thr. – see VB et al. 2004

- Many extensions mostly for resonances by Jido, Kamai, Nieves, Oller, Oset, Sekihara, ... review Kamai and Hyodo 2017

# Extensions beyond bound states



# Extensions beyond bound states

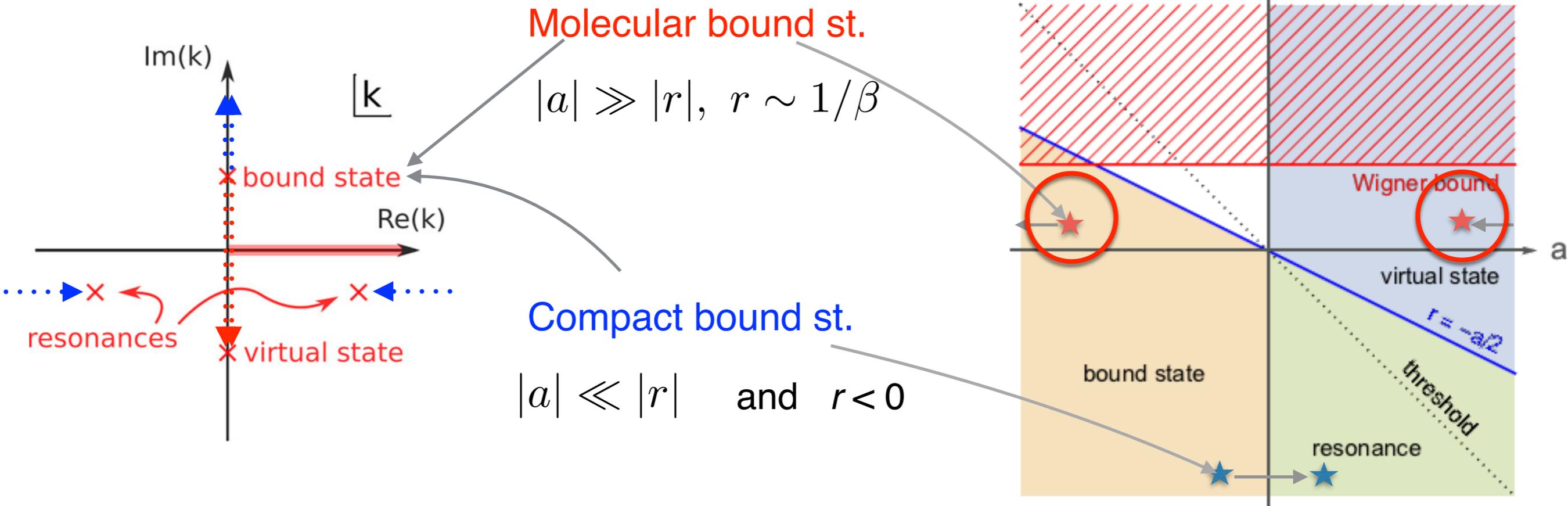


- Evolution of poles when some QCD parameter is varied → Extensions beyond bound states

**Molecular pole:**  $k_1 = -\frac{i}{a} \left[ 1 + \mathcal{O}\left(\frac{r}{a}\right) \right]$  if sc. length changes sign → virtual state  
 $|a| \gg |r|$

**Compact pole:**  $k_{1,2} = \pm i \sqrt{\frac{2}{ar}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$  if sc. length changes sign → turns to a resonance  
 $|a| \ll |r|$

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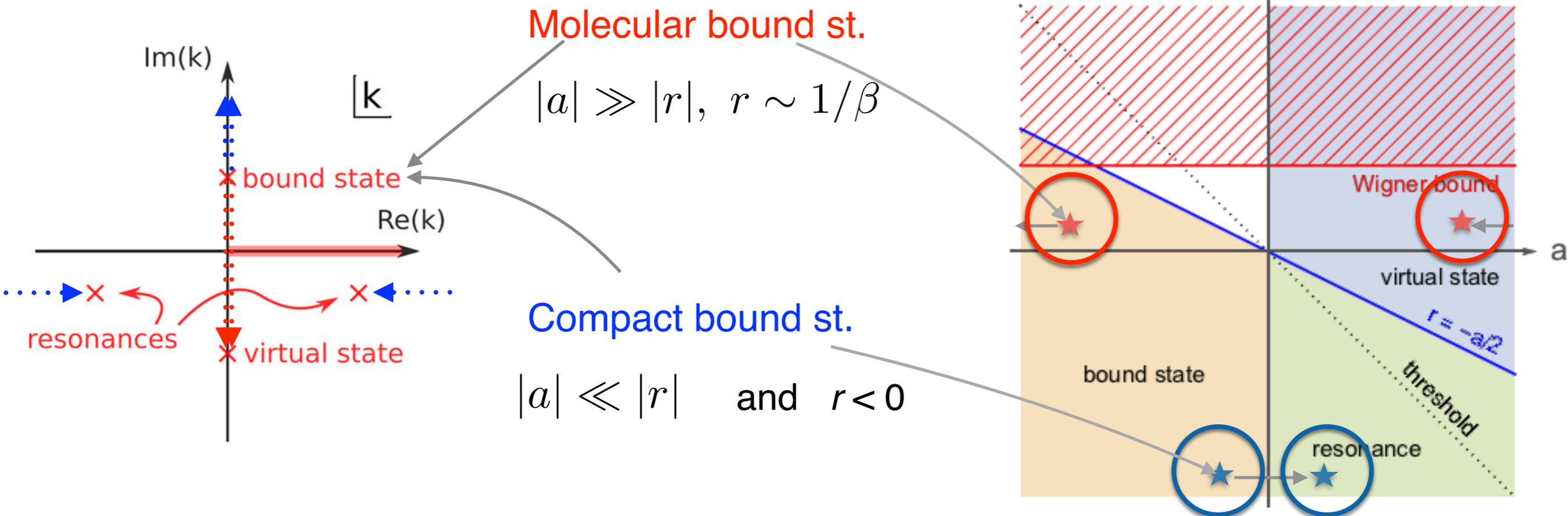
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Near thr. molecules

**Compact pole:**  $k_{1,2} = \pm i \sqrt{\frac{2}{ar}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$  if sc. length changes sign → turns to a resonance

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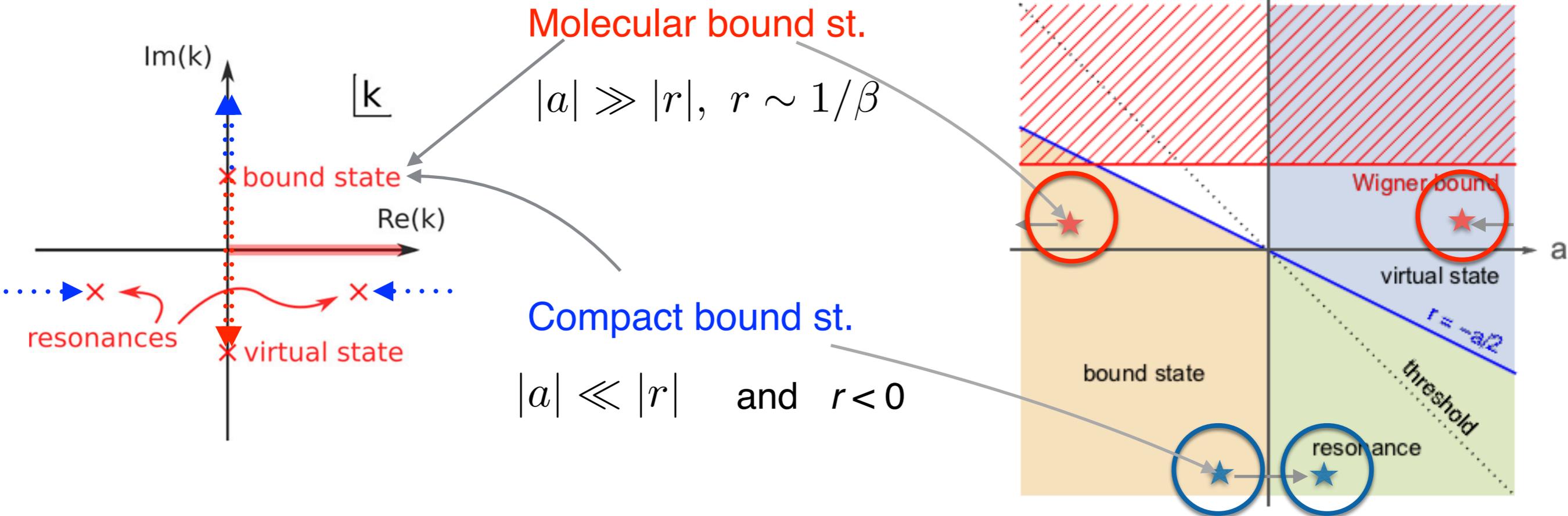
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Near thr. compact states

# Extensions beyond bound states



- Evolution of poles when some QCD parameter is varied → Extensions beyond bound states

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$$|a| \ll |r|$$

Near thr. compact states

$$X_W = 1 - \lambda^2 = \sqrt{\frac{1}{1 + 2r/a}}$$

⇒

$$\bar{X} = \sqrt{\frac{1}{1 + |2r/a|}}$$

both cases

subsumed here

- $\bar{X}$  allows one to test compositeness for bound/virtual states and resonances 9

# Ex: proton-neutron bound and virtual states Matuschek, VB, Guo, Hanhart EPJA 2021

## Deuteron

- $a = -5.41 \text{ fm}$   
 $r = +1.75 \text{ fm}$   $\implies$  **large  $a$ :**  $|a| \gg |r|$   
 $r \sim O(1/M_\pi)$

$\implies$  **Clear molecule**

- **But**  $X = 1 - \lambda^2 = \sqrt{\frac{1}{1 + 2r/a}} \simeq 1.7 \gg 1$

–  $X$  was derived in the zero-range approximation and has **a pole** when  $r/a$  is **negative**

- **Meanwhile,**  $\bar{X} = \sqrt{\frac{1}{1 + |2r/a|}} \approx 0.8$

## 1S0 pn virtual state

- $a = 23.74 \text{ fm}$   
 $r = +2.75 \text{ fm}$   $\implies$  **large  $a$ :**  $|a| \gg |r|$   
 $r \sim O(1/M_\pi)$

Dumbrajs et al 1983

$\implies$  **Clear molecule**

– both  $a$  and  $r$  changed the sign  $\implies$  **no pole**

- $X = \bar{X} \approx 0.9$

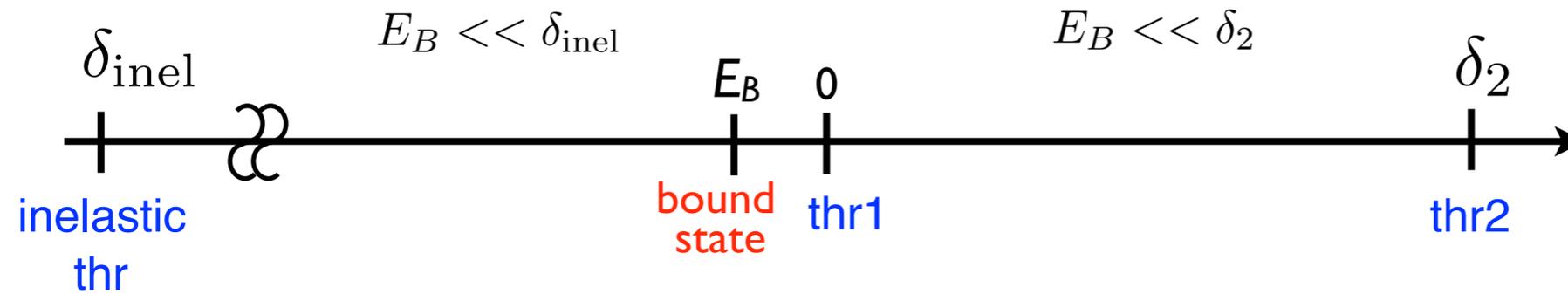
$\bar{X} \simeq 1$ , as expected for a molecule up to the range corrections!

# Large and negative effective range = compact state???

- Using recent data by LHCb for  $X(3872) \rightarrow J/\Psi \pi \pi$  Esposito et al. PRD 105 (2022) L031503  
extracted  $r_{D^0 \bar{D}^{*0}} = -5.34 \text{ fm} \implies X(3872)$  is a compact state!
- Conclusion is criticised by VB, Dong, Du, Filin, Guo, Hanhart, Nefediev, Nieves, Wang PLB 833 (2022) 137290
- The conditions for a compact state are  $|a| \ll |r|$  and  $r < 0$   
Not fulfilled for the  $X(3872)$ :  $\mathcal{R} a_{D^0 \bar{D}^{*0}} = -28.6 \text{ fm} \implies |a| \gg |r|$
- Coupled-channel dynamics greatly affects the value of the effective range!

# Eff. range in a coupled-channel case

Matuschek, VB, Guo, Hanhart  
EPJA 2021



Flatte, PLB 63, 224 (1976)  
VB et al., PLB 586, 53 (2004)

- In a coupled-channel case the amplitude to be used in fits to data reads

$$f_{ab}(E) = -\frac{1}{2} \frac{g_a g_b}{E - E_f + \frac{i}{2} (g_1^2 k_1 + g_2^2 k_2 + \Gamma_{\text{inel}}(E))}$$

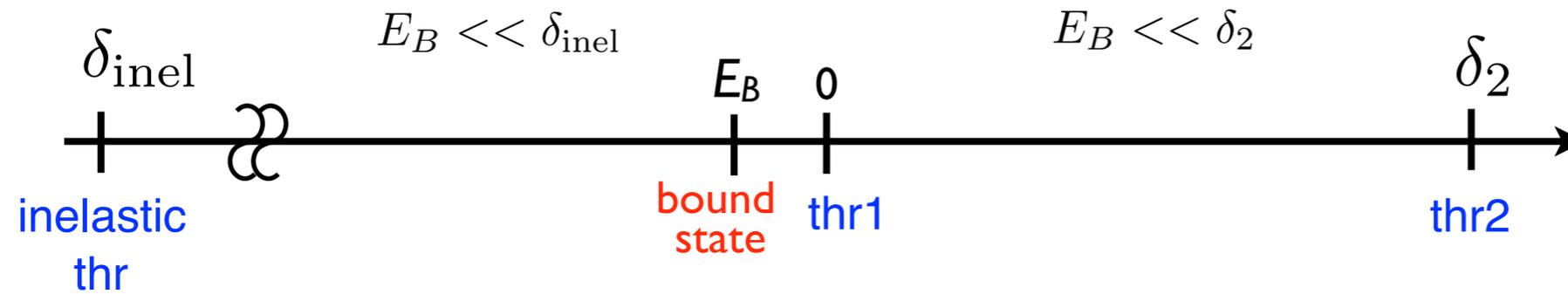
$$k_2 = i \sqrt{2\mu_2 \left( \delta_2 - \frac{k_1^2}{2\mu_1} \right)} \stackrel{E_B \ll \delta_2}{=} i \sqrt{2\mu_2 \delta_2} - \frac{i}{2} \sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} k_1^2 + \mathcal{O} \left( \frac{k_1^4}{\mu_1^2 \delta_2^2} \right),$$

$$r = -\frac{2}{\mu_1 g_1^2} + \Delta r \quad \Delta r = -\frac{g_2^2}{g_1^2} \sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} \quad \delta_2 = m_{\text{thr2}} - m_{\text{thr1}}$$

Term from second channel

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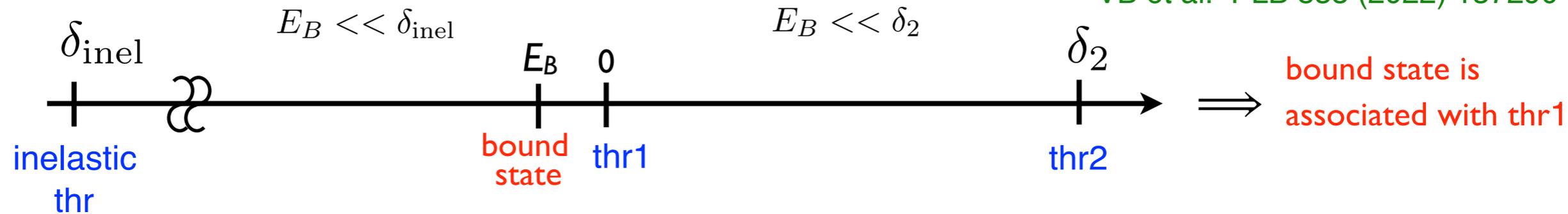
## Examples:

- $X(3872)$  as  $D^0 \bar{D}^{*0} / D^\pm \bar{D}^{*\mp}$        $\delta_2 = 8 \text{ MeV}$        $\Delta r \simeq -1.6 \text{ fm}$
- $T_{cc}^+$  as  $D^0 D^{*+} / D^+ D^{*0}$        $\delta_2 = 1.4 \text{ MeV}$        $\Delta r \simeq -3.8 \text{ fm}$

- This is a hadronic coupled-channel effect  $\implies$  unrelated with a possible compact component!

# Compositeness in a coupled-channel case

VB et al. PLB 833 (2022) 137290



- Scattering parameters in a coupled-channel case

$$a^{-1} = -\frac{1}{g_1^2} \left( \gamma_1^2 / \mu_1 + g_1^2 \gamma_1 + g_2^2 (\gamma_2 - \sqrt{2\mu_2 \delta_2}) + i\Gamma_{\text{inel.}} \right) \quad r = -\frac{2}{\mu_1 g_1^2} - \frac{g_2^2}{g_1^2} \sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}}$$

- Subtract from  $a^{-1}$  and  $r$  all coupled-channel terms related with the thr2 and inelastic channels

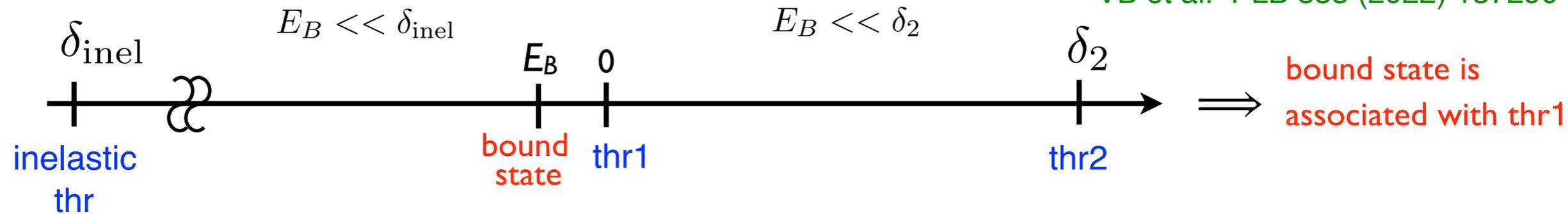
$\implies$  Scattering parameters from channel 1 to be used for estimating compositeness

$$a^{-1} = -\gamma_1 \left( 1 + \frac{\gamma_1}{\mu_1 g_1^2} \right) \quad r = -\frac{2}{\mu_1 g_1^2}$$

simple and straightforward even for exp. analyses

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- Alternative methods  $\implies$  same results up to range corrections

– First go to isospin limit, then estimate compositeness:

Solution of the dynamical problem is needed to find the pole and the sc.length

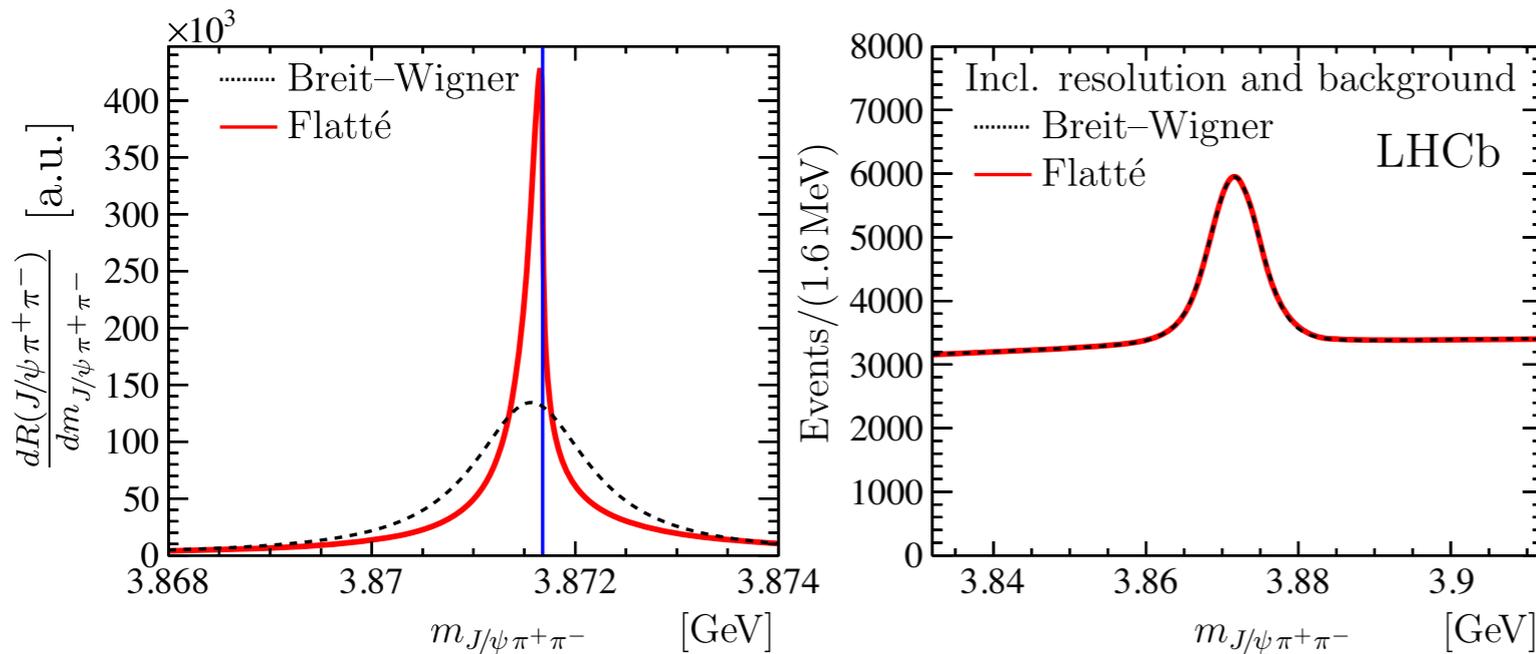
– Derivative of the self-energy loops in each channel:

$$X_i = g_i^2 \frac{d\Sigma_i(M_{\text{pole}})}{dM_{\text{pole}}^2}$$

Calculation is needed

# Nature of the X(3872) from LHCb data

- Recent LHCb study of  $X(3872) \rightarrow J/\psi \pi \pi$  [LHCb, PRD 102 092005 \(2020\)](#)



The signal from the threshold is smeared by the energy resolution

⇒ bad sensitivity to  $g^2$   
and therefore to the effective range

- Nevertheless  $g_1^2 = 0.108 \pm 0.003$  was extracted ⇒ should be treated as a lower bound

- ERE parameters [VB, Dong, Du, Filin, Guo, Hanhart, Nefediev, Nieves, Wang PLB 833 \(2022\) 137290](#)

lower bound

$$\Rightarrow r = -\frac{2}{\mu_1 g_1^2} - \sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} = (-3.8 - 1.6) \text{ fm} \Rightarrow r_{D^0 \bar{D}^{*0}} \in [-3.8, 0)$$

$$\mathcal{R} a_{D^0 \bar{D}^{*0}} = -28.6 \text{ fm}$$

$$|a| \gg |r|$$

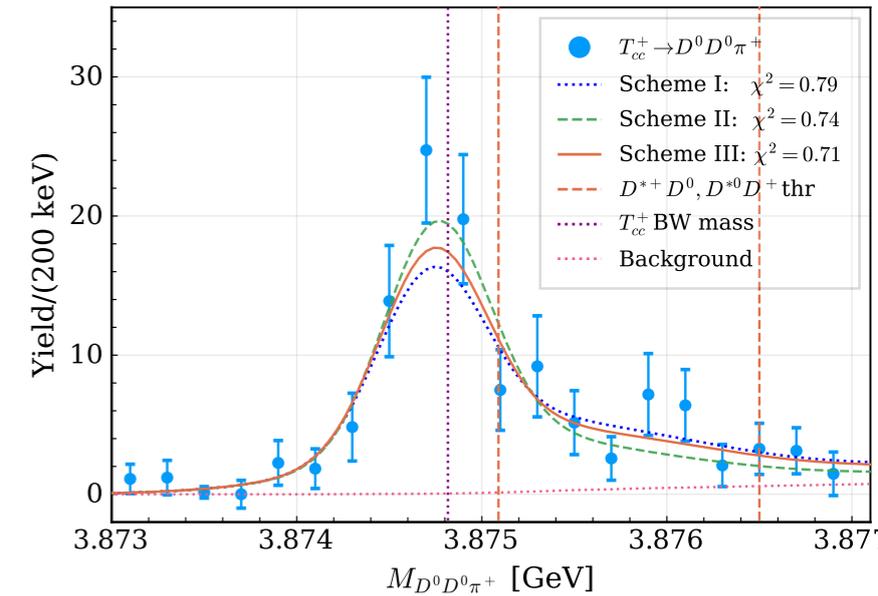
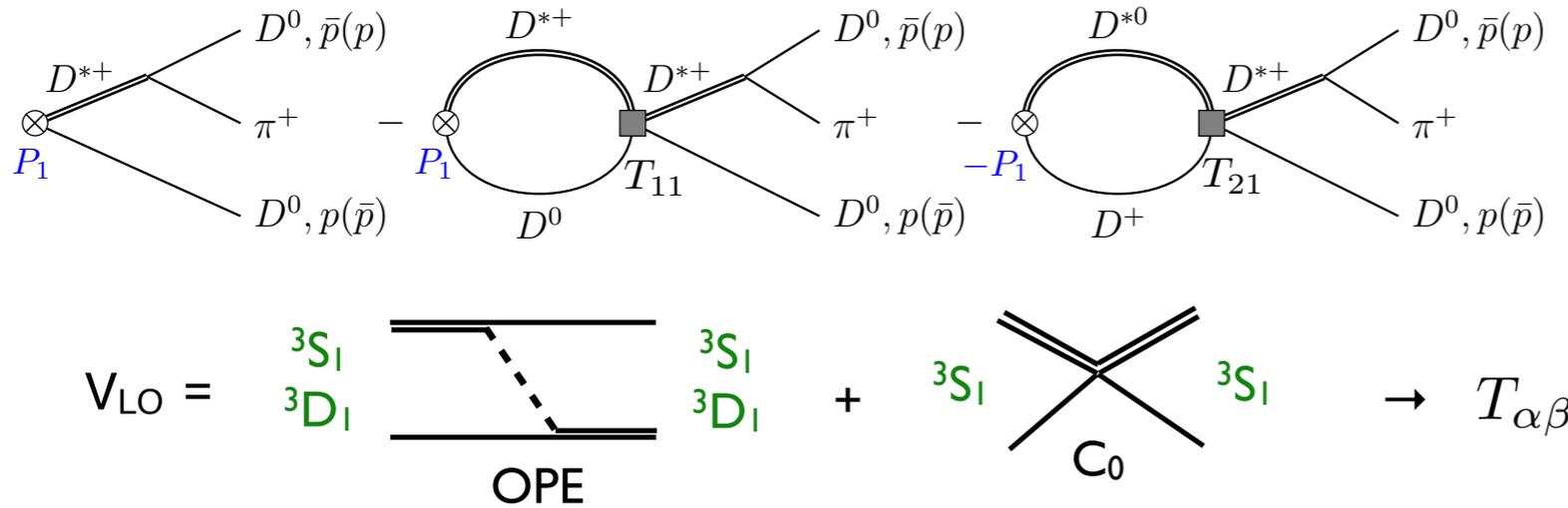
$$\Rightarrow \bar{X} > 0.9 \quad \text{predominantly molecular!}$$

# Application to $T_{cc}^+$

$\chi$ EFT analysis of the LHCb data

Du et al. PRD 105 (2022) 1, 014024

data: LHCb Nat.Com. 13 (2022) 1, 3351



Scattering amplitude  
in the 1st channel:

$$T_{D^{*+}D^0 \rightarrow D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}} \left( \frac{1}{a_0} + \frac{1}{2}r_0 k^2 - ik + \mathcal{O}(k^4) \right)^{-1}$$

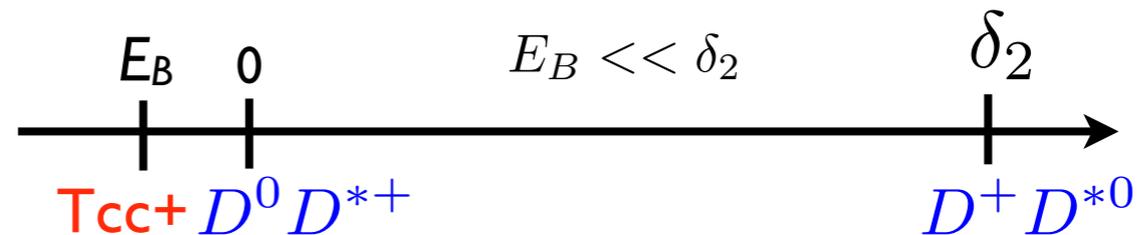
$$r'_0 = r_0 - \Delta r$$

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} \simeq -3.8 \text{ fm}$$

Term from 2nd channel

$$\delta_2 = m_{\text{thr}2} - m_{\text{thr}1}$$

Eff. range in the  
1st channel



$a_0$ [fm]	$r_0$ [fm]	$r'_0$ [fm]	$\bar{X}$
$\begin{pmatrix} -6.72^{+0.36} \\ -0.45 \\ \pm 0.27 \end{pmatrix} - i \begin{pmatrix} 0.10^{+0.03} \\ -0.03 \\ \pm 0.03 \end{pmatrix}$	$-2.40 \pm 0.01$ $\pm 0.85$	$1.38 \pm 0.01$ $\pm 0.85$	$0.84 \pm 0.01$ $\pm 0.06$

$T_{cc}^+$  is consistent with  
a pure molecule!

# Summary

Weinberg analysis & extensions: right tool for testing nature

⇒ model independent in the weak binding limit

⇒ uncertainty quantification as  $O(\gamma/\beta)$

—Analyticity of the pole trajectories when *sc. length* and *eff. range* change sign ⇒

Extensions to virtual states and resonances are straightforward

—Large and negative *eff. range* may come from coupled channels

⇒ not necessarily a compact state

—A simple method to estimate compositeness in a coupled-channel case if  $E_B \ll \delta$

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Thank You!