Few-Neutron Systems in Pionless EFT

Sebastian König

9th International Conference on Quarks and Nuclear Physics

Virtual Talk, September 5, 2022

S. Dietz, H.-W. Hammer, SK, A. Schwenk, PRC **105** 064002 (2022)
SK et al., in preparation

Thanks...

...to my students and collaborators...

- **S. Dietz, H.-W. Hammer, A. Schwenk (TU Darmstadt)**
- H. Yu, **N. Yapa**, A. Andis, A. Taurence **(NCSU)**
- D. Lee (FRIB/MSU), K. Fossez (FSU)
- P. Klos, J. Lynn
- ...

...for support, funding, and computing time...

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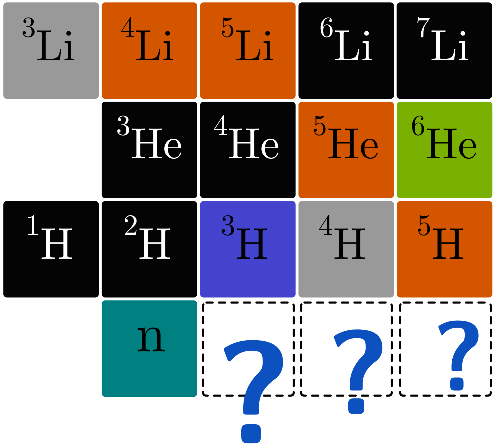
Few-neutron systems

Ongoing searches and speculations



- bound dineutron not excluded by pionless EFT

Kirscher + Phillips, PRC **84** 054004 (2011); SK et al., PLB **736** 208 (2014)

- new speculations about a three-neutron resonance...

Gandolfi et al., PRL **118** 232501 (2017)

- ...although excluded by previous work

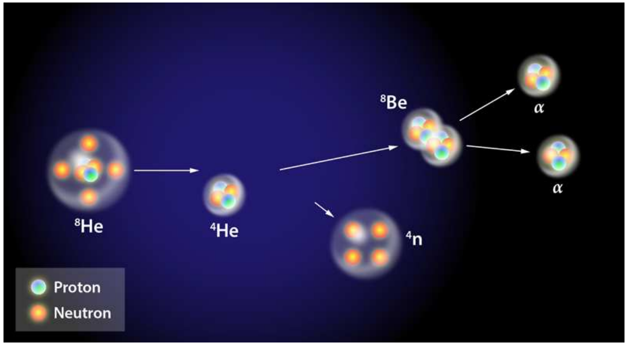
Lazauskas + Carbonell, PRC **71** 044004 (2005); Offermann + Glöckle, NPA **318** 138 (1979)

- **experimental evidence for tetraneutron resonance?**

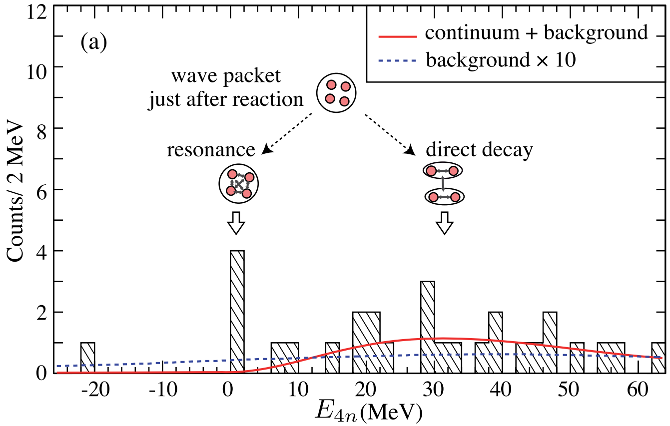
Kisamori et al., PRL **116** 052501 (2016), Duer et al., Nature **606** 678 (2022)

Tetraneutron situation

Observation at RIKEN (2016)



APS/Alan Stonebraker



Kisamori et al., PRL **116** 052501 (2016)

- double-charge exchange reaction
- **excess of near threshold events hints at possible resonance**
- **motivated follow-up experiment**

Tetraneutron situation

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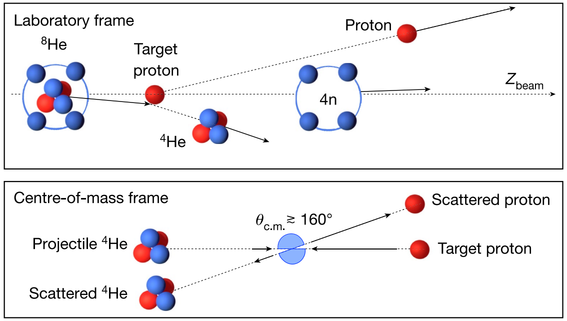
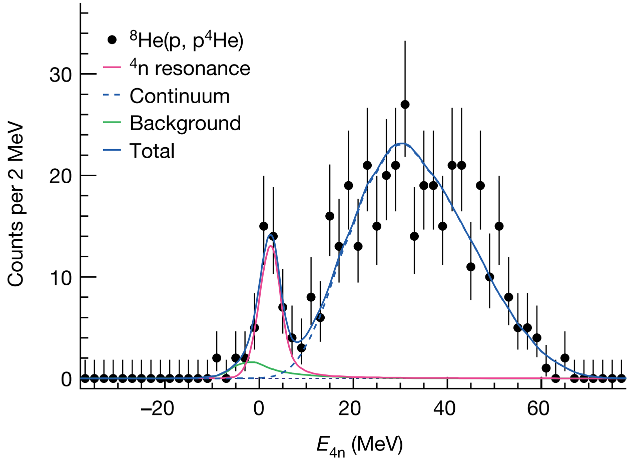


Kisamori et al., PRL **116** 052501 (2016)



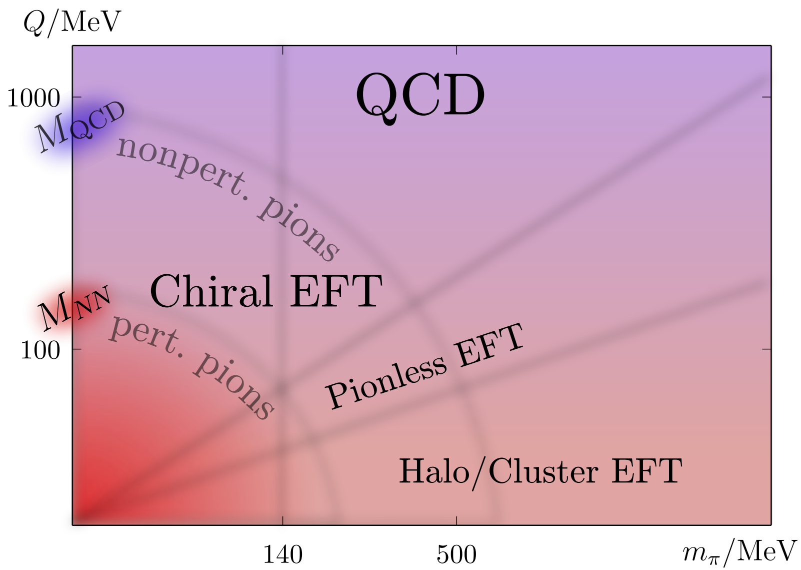
- knockout reaction: scattering ^{8}He beam off proton target
- **clear peak with resonance shape around 2 MeV**
- **theory suggests alternative explanations (phase space + FSI)**

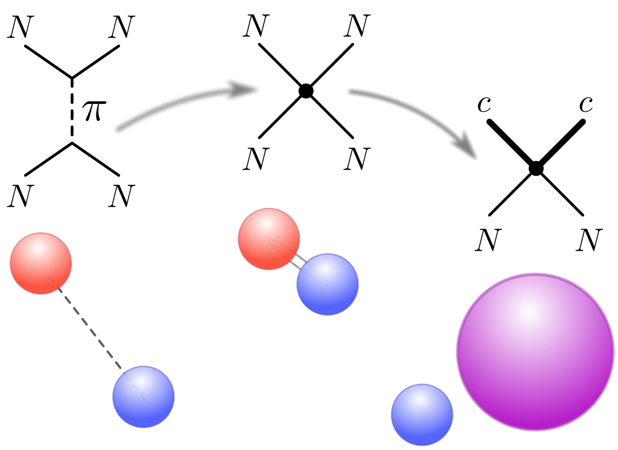
Higgins et al., PRC **103** 024004 (2021), Lazauskas et al., arXiv:2207.07575 [nucl-th]

Nuclear effective field theories

- choose **degrees of freedom** approriate to energy scale
- only restricted by **symmetry**, ordered by **power counting**

Hammer, SK, van Kolck, RMP **92** 025004 (2020)





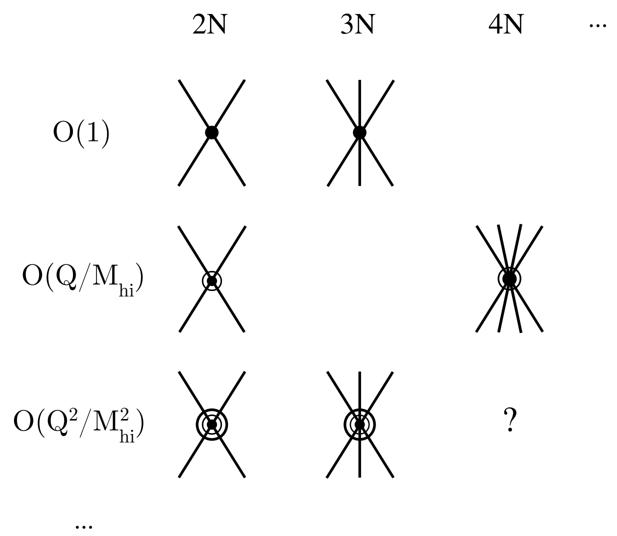
- degrees of freedom here: nucleons (and/or clusters thereof)
- even more effective d.o.f.: rotations, vibrations

Papenbrock, NPA **852** 36 (2011); ...

- most effective theory depends on energy scale and nucleus of interest
- **matching** different EFTs can leverage the reach of ab initio calculations

Pionless EFT

- only contact (zero-range) forces (plus electromagnetism)
- closely linked to universality for large scattering lengths
- excels at low energies, exact range of validity still an open question



Pionless EFT

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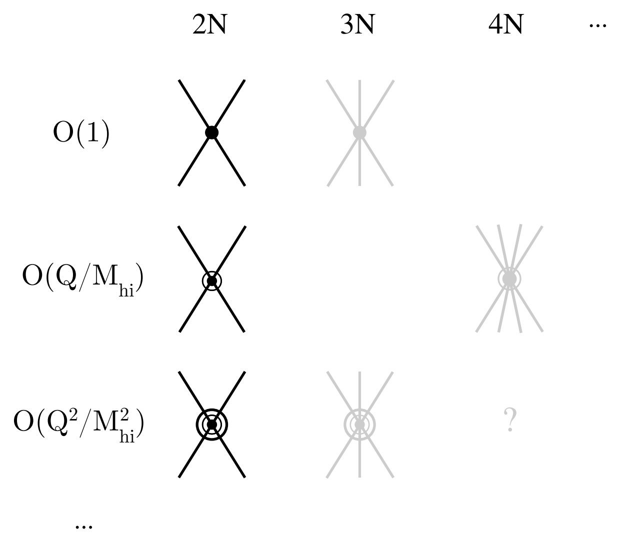
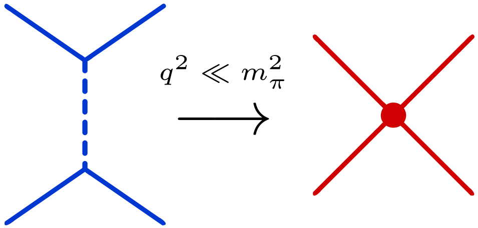
- closely linked to universality for large scattering lengths
- excels at low energies, exact range of validity still an open question
- no leading-order 3n force for pure neutron systems



Three-neutron system in pionless EFT

Pionless effective field theory

- at very low energies the nuclear force reduces to contact interactions
- not sensitive to short-range details such as pion exchange
- range corrections enter perturbatively via derivatives
  - power counting provides systematic expansion
- **naturally captures large nn scattering length**



$$V(p,p') = C_0(a_{nn};\Lambda)\, g(p)g(p') \;\;,\;\; g(p) = \mathrm{e}^{-p^2/\Lambda^2}$$

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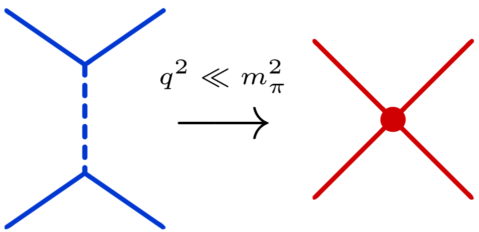


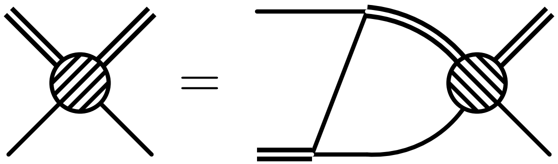
$$V(p,p') = C_0(a_{nn};\Lambda)\, g(p) g(p') \;\;,\;\; g(p) = \mathrm{e}^{-p^2/\Lambda^2}$$

Outline

- **search for resonance states with two direct methods**
  Dietz, SK et al., PRC **105** 064002 (2022)
  - **analytic continuation of Faddeev equation to complex plane**
    following Glöckle, PRC **18** 564 (1978); Afnan, Aust. J. Phys. **44** 201 (1991)
  - **finite-volume energy levels in very large boxes**
    following Klos, SK et al., PRC **98** 034004 (2018)
- **finite-volume eigenvector continuation**
  Yapa+König, PRC **106** 014309 (2022)
- **three-neutron scaling exponents**
  Dietz, SK et al., work in progress

Three-body equation

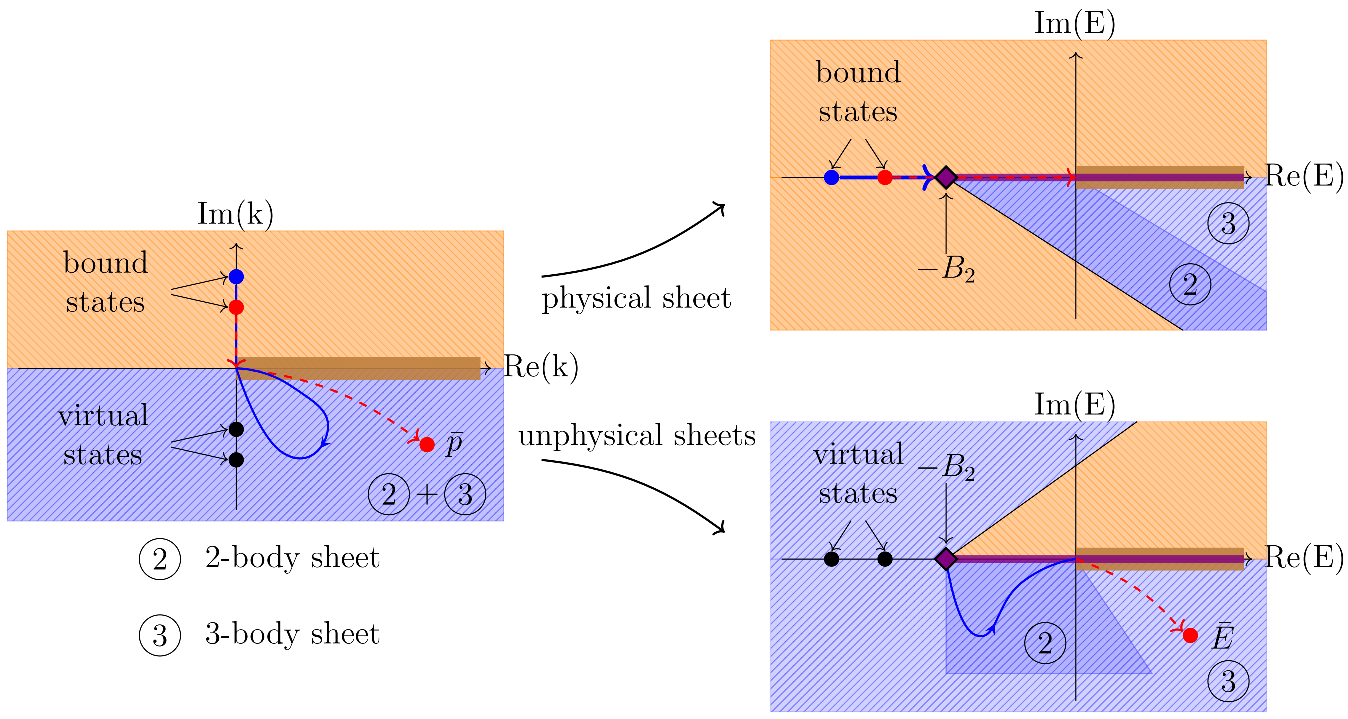
- **consider the Faddeev equation with separable interaction**



$$F(q) = -\frac{1}{2}\int \mathrm{d}q' q'^2 \int_{-1}^{1} \mathrm{d}x \, g(\pi_1) G_0(E;\pi_2,q') \\ g(\pi_2) P_1(x) \tau\left(E - \frac{3}{4}q'^2\right) F(q')$$

- effective two-body equation structure
- written here for three neutrons with $J^\pi = \frac{1}{2}^+$ or $\frac{3}{2}^+$ (degenerate)
  - very similar form (plus three-body force) for three bosons
- energies for which a solution exists correspond to S-matrix poles

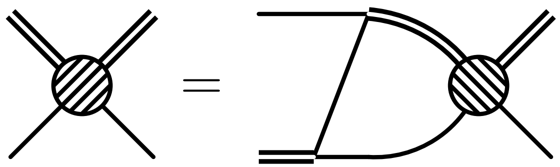
S-matrix pole trajectories



- bound and virtual states move in the complex plane as the interaction is varied
- these trajectories can be followed by solving the equations on the second sheet

Three-body equation

- **consider the Faddeev equation with separable interaction**



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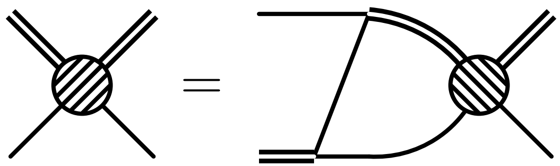

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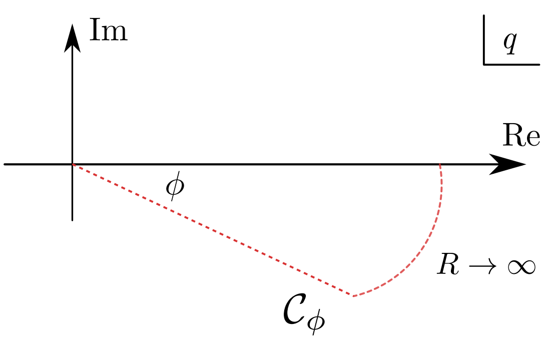
Analytic continuation

- rotate the integration contour: $q \to q\mathrm{e}^{-\mathrm{i}\phi}$
  - this exposes lower right quadrant

  Afnan, Aust. J. Phys. **44** 201 (1991)
- possible to rotate back and pick up a residue
  - leads to modified effective interaction

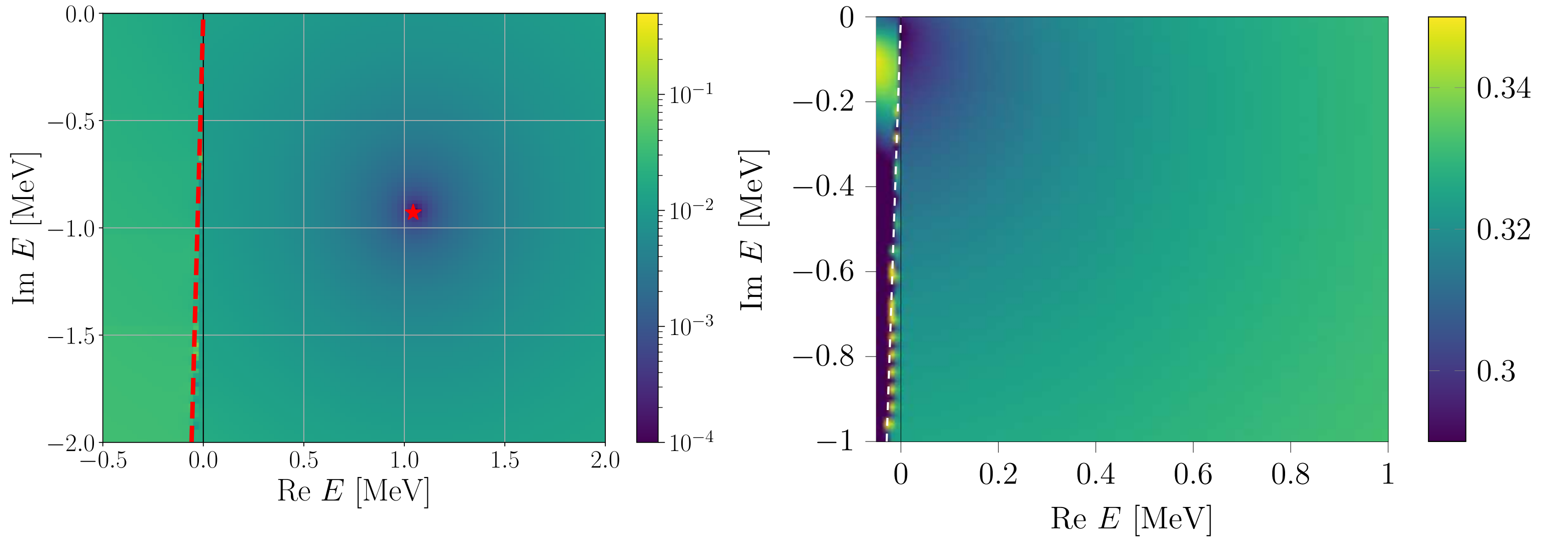
  Glöckle, PRC **18** 564 (1978)



Neutrons vs. bosons

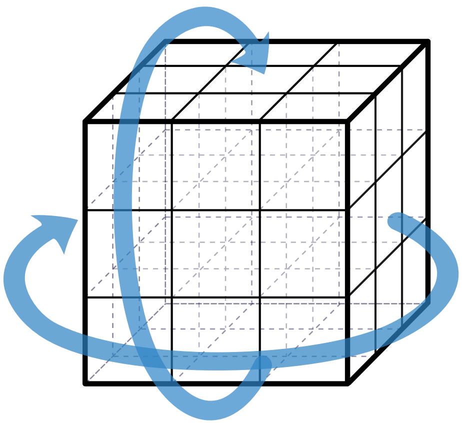
- for three bosons we can follow the resonance trajectory of an Efimov state
  - consistent with previous work Bringas et al., PRA **69** 040702 (2004); Deltuva, PRC **102** 034003 (2020)
- for three neutrons, we can reproduce Glöckle's Yamaguchi model Glöckle, PRC **18** 564 (1978)
  - generates a $3n$ resonance with deep $2n$ bound state
- **no sign of a three-neutron resonance for physical nn scattering length**
  - consistent with related work

Lazauskas + Carbonell, PRC **71** 044004 (2005); Deltuva + Lazauskas, PRL **123** 069201 (2019)

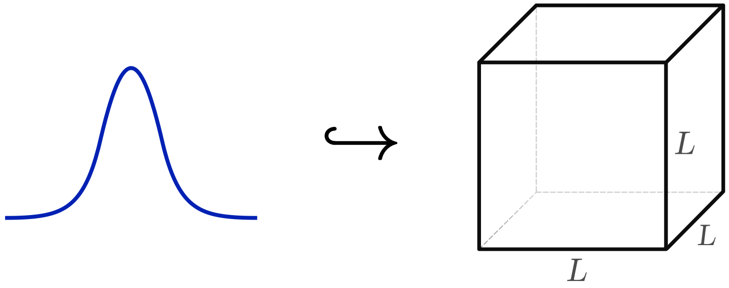


Dietz, SK et al., PRC **105** 064002 (2022)

Finite periodic boxes



- physical system enclosed in finite volume (box)
- typically used: periodic boundary conditions
- **leads to volume-dependent energies**



Lüscher formalism

- physical properties encoded in the volume-dependent energy levels
- infinite-volume S-matrix governs discrete finite-volume spectrum
- **finite volume used as theoretical tool**

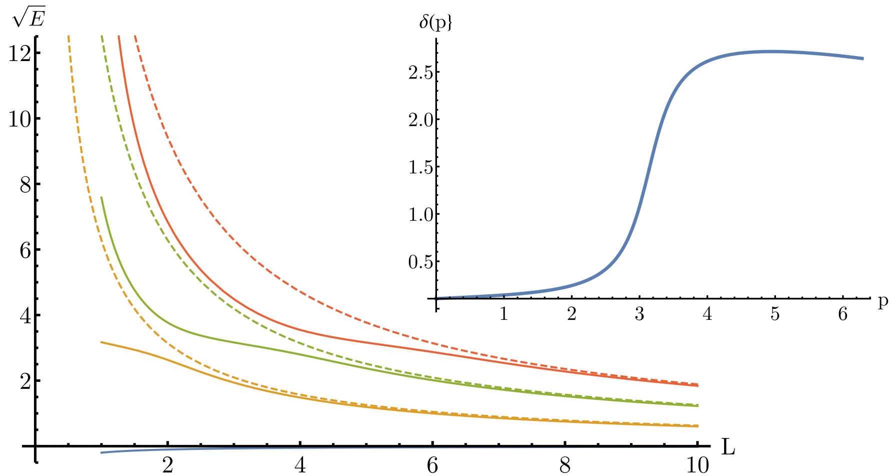
Lüscher, Commun. Math. Phys. **104** 177 (1986); ...

Finite-volume resonance signatures

Lüscher formalism

- finite volume $\rightarrow$ discrete energy levels $\rightarrow p\cot\delta_0(p) = \frac{1}{\pi L} S(E(L))$ $\rightarrow$ phase shift
- **resonance contribution $\leftrightarrow$ avoided level crossing**

Lüscher, NPB **354** 531 (1991); ...
Wiese, NPB (Proc. Suppl.) **9** 609 (1989); ...



- spectrum signature **carries over to few-body systems**
  - need considerable range of volumes for such studies!

Klos, SK et al., PRC **98** 034004 (2018)

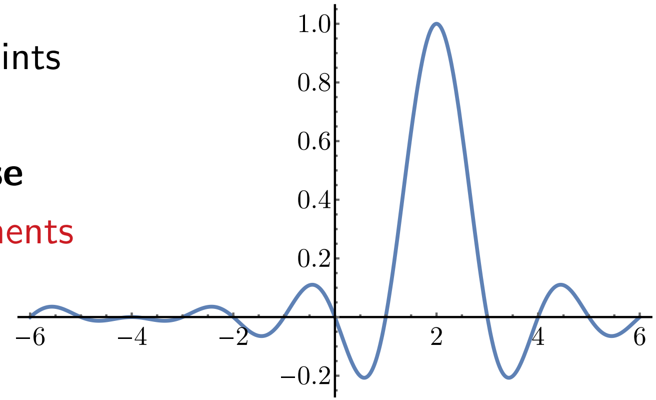
Discrete variable representation

Need calculation of several few-body energy levels

- **use a Discrete Variable Representation (DVR)**

well established in quantum chemistry, suggested for nuclear physics by Bulgac+Forbes, PRC **87** 051301 (2013)

- basis functions localized at grid points
- potential energy matrix diagonal
- **kinetic energy matrix very sparse**
  - precalculate only 1D matrix elements



- periodic boundary condistions ↔ plane waves as starting point
- **efficient implementation for large-scale calculations**
  - handle arbitrary number of particles (and spatial dimensions)
  - numerical framework scales from laptop to HPC clusters Klos, SK et al., PRC **98** 034004 (2018)
  - recent extensions: GPU acceleration, separable interactions

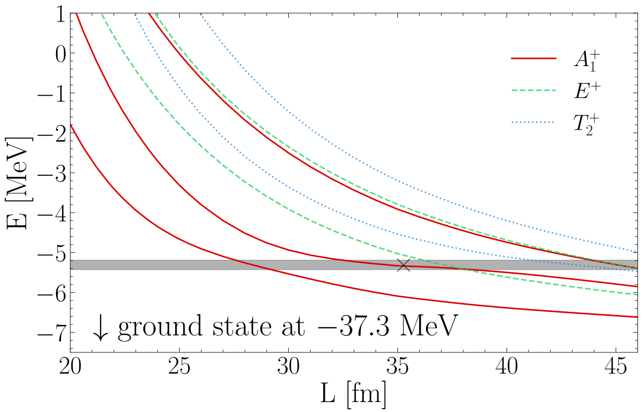
Dietz, SK et al., PRC **105** 064002 (2022)

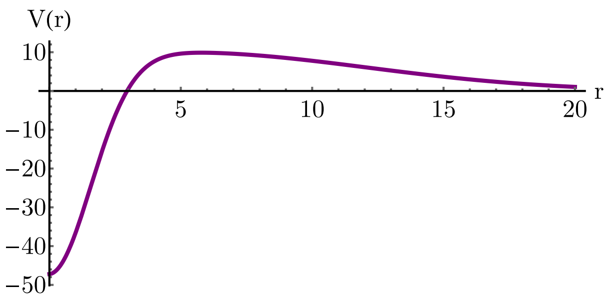
Benchmark calculation

Study established three-body resonance from literature

Fedorov et al., Few-Body Syst. **33** 153 (2003); Blandon et al., PRA **75** 042508 (2007)

- three bosons with mass $m = 939.0$ MeV, potential = sum of two Gaussians
- **three-body resonance at**
  - $-5.31 - i0.12$ MeV (Blandon et al.)
  - $-5.96 - i0.40$ MeV (Fedorov et al.) (note: potential S-wave projected!)





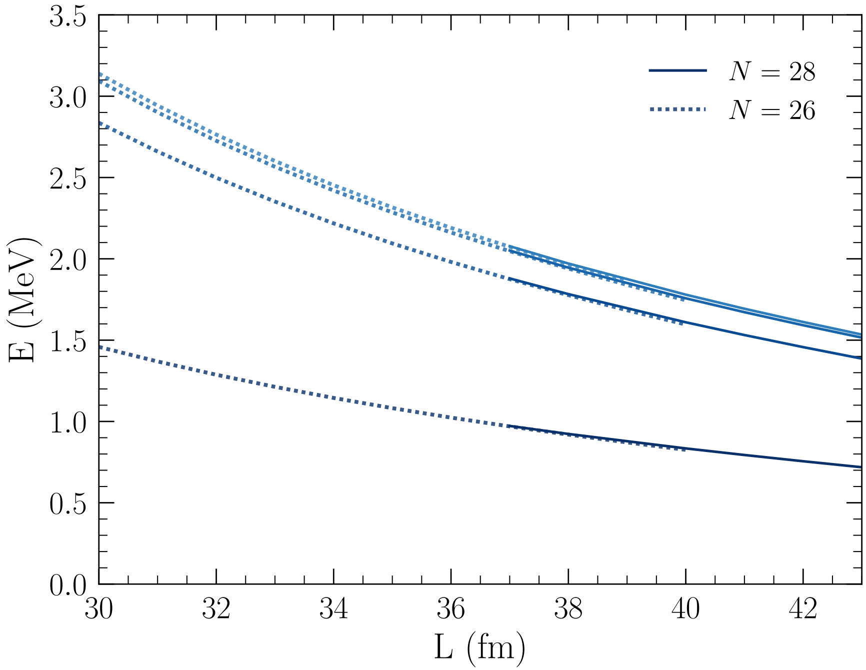
- **fit inflection point(s) to extract resonance energy:** $E_R = -5.32(1)$ MeV

Klos, SK et al., PRC **98** 034004 (2018)

Three-neutron energy levels

Physical n-n scattering length $a_{nn} = -18.9\ \mathrm{fm}$

- interacting levels with positive parity, $S_z = 1/2$



- good convergence up to very large boxes ✓
- **no sign of a three-neutron resonance**

Dietz, SK et al., PRC **105** 064002 (2022)

Three-neutron energy levels

Positive n-n scattering length $a_{nn} = +18.9\ \mathrm{fm}$

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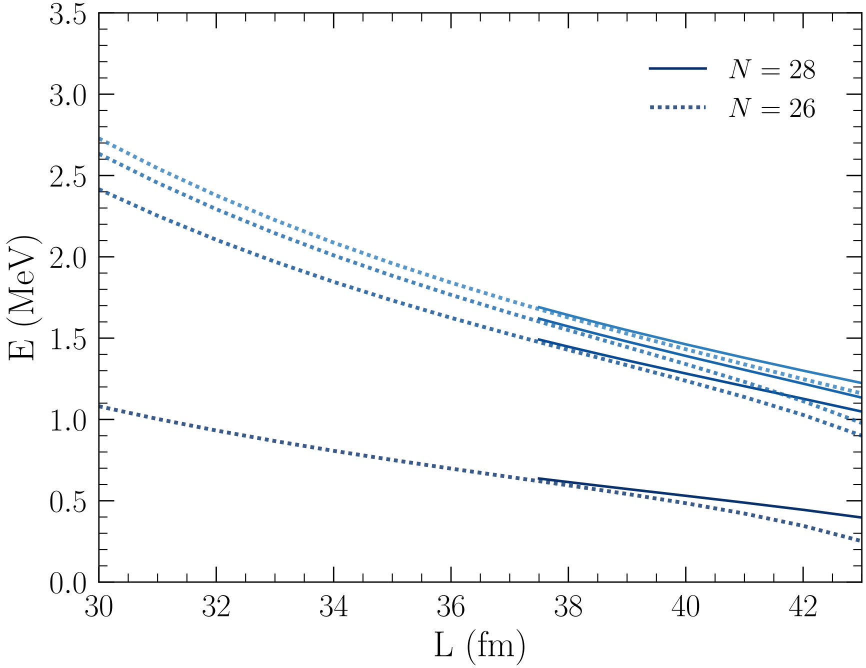

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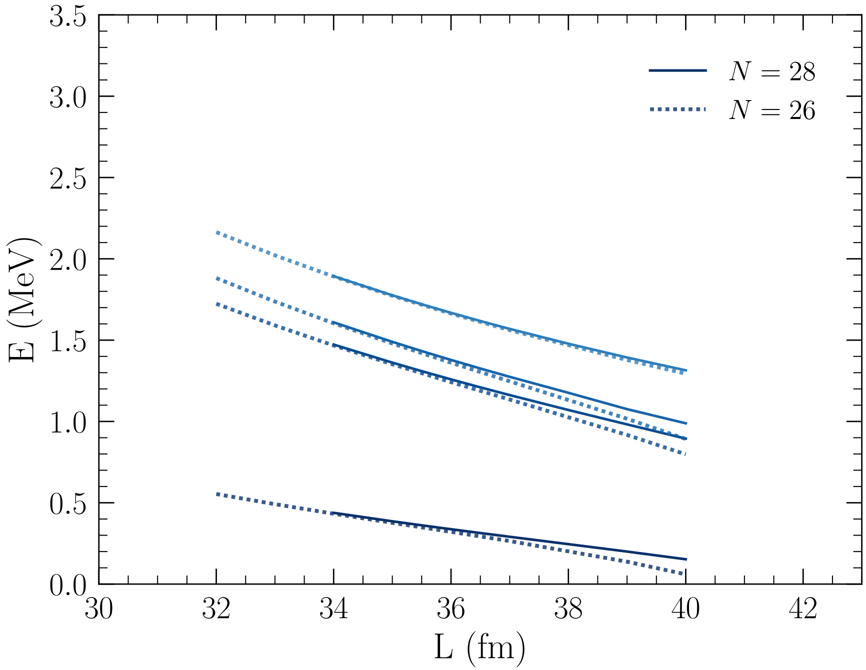
Dietz, SK et al., PRC **105** 064002 (2022)

Three-neutron energy levels

Positive n-n scattering length $a_{nn} = +10.0\ \mathrm{fm}$

- interacting levels with positive parity, $S_z = 1/2$



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Dietz, SK et al., PRC **105** 064002 (2022)

Finite-volume eigenvector continuation

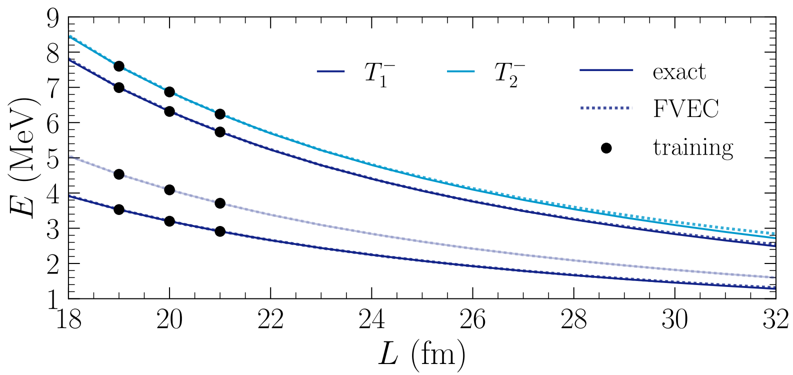
- parametric dependence of Hamiltonian $H(c)$ traces only small subspace
- this can be exploited to construct a powerful extrapolation method called eigenvector continuation

  Frame et al., PRL **121** 032501 (2018)
- special case of "reduced basis method" (RBM)

  Bonila et al., arXiv:2203.05282; Melendez et al., arXiv:2203.05528

- method extended to handle parametric dependence in model space directly
  - **enables highly efficient volume extrapolation**

  Yapa+König, PRC **106** 014309 (2022)



- total number of training data: $3 \times 8 = 24$ (partly covering cubic group multiplets)
- **four-neutron finite-volume resonance search enabled by FVEC!**

  SK et al., work in progress

Conclusion

Summary

- studied three neutrons in pionless EFT with two different methods:
  - analytic continuation of Faddeev equation
  - energy levels in periodic fininte volumes
- possible to reproduce three-boson resonances
- no indication for three-neutron resonance with large nn scattering length
  - consistent with previous work
- finite-volume eigenvector continuation enables studies of larger system

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Outlook

- four-neutron calculations in finite volume
- extension of DVR method to handle nuclear halo systems ($\rightarrow$ Halo EFT)
- few-neutron systems as unparticles
  - final-state spectrum governed by conformal symmetry
  - study universal scaling exponents in point production amplitudes

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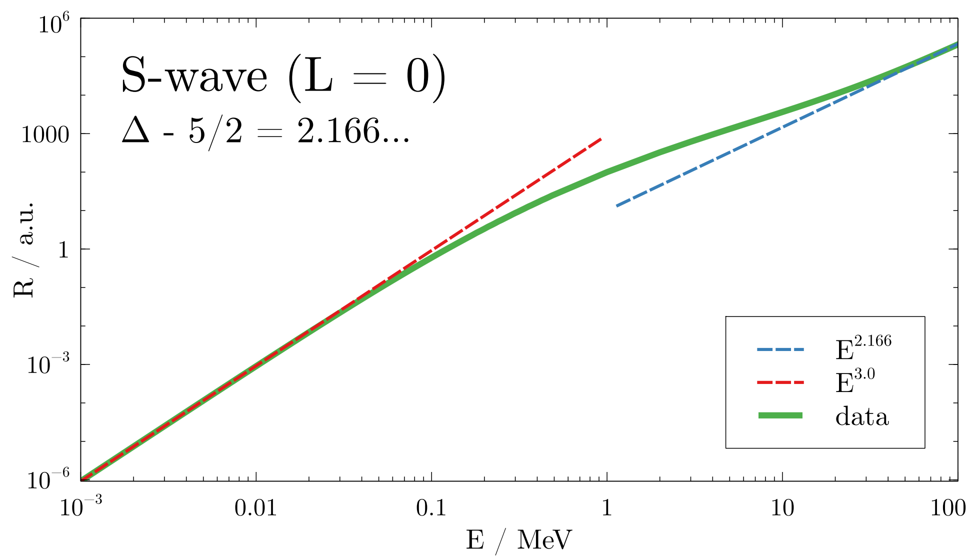
...and to you, for your attention!

Few-neutron point production

- assume that final-state neutrons in experiments are created effectively in a point
- possible to write down Faddeev equation for production amplitude
  - same interaction kernel as shown previously in this talk
- spectrum governed by conformal symmetry in universal regime: $\dfrac{1}{ma^2} \ll E \ll \dfrac{1}{mr^2}$
- cross section $\dfrac{\mathrm{d}\sigma}{\mathrm{d}E} \sim R(E) \sim E^{\Delta-5/2}$

Hammer+Son, Proc. Natl. Acad. Sci. 118, e2108716118 (2021)

  - scaling dimension Δ depends on partial wave: 4.666 $(L=0)$, 4.273 $(L=1)$, ...



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