

Pion Parton Distributions in the JAM Framework

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QNP2022, September 7th, 2022



Motivation

- QCD allows us to study the **structure of hadrons** in terms of **partons** (quarks, antiquarks, and gluons)
- Use **factorization theorems** to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

Game plan

What to do:

- **Define** a structure of hadrons in terms of quantum field theories
- **Identify** physical observables that can be factorized in theory with controllable approximations, or factorizable lattice QCD observables
- Perform **global QCD analysis** as structures are universal and are the same in all processes

Complicated Inverse Problem

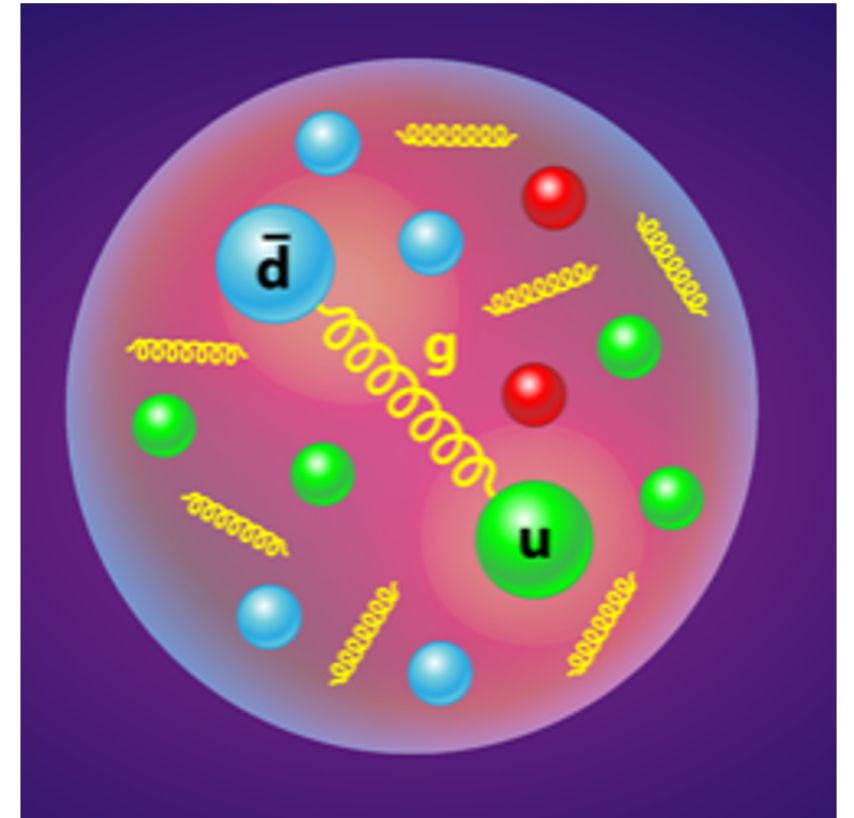
- Factorization theorems involve **convolutions** of **hard perturbatively calculable physics** and **non-perturbative objects**

$$\frac{d\sigma}{d\Omega} \propto \mathcal{H} \otimes f = \int_x^1 \frac{d\xi}{\xi} \mathcal{H} \left(\frac{x}{\xi} \right) f(\xi)$$

- Parametrize the **non-perturbative objects** and perform global fit

Pions

- Pion presents itself as a dichotomy
 1. It is the **Goldstone boson** associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
 2. Made up of **quark and antiquark constituents**



Large momentum fraction behavior

- Many theoretical papers have studied the behavior of the valence quark distribution as $x \rightarrow 1$ and
- Debate whether $q_v^\pi(x \rightarrow 1) \sim (1 - x)$ or $(1 - x)^2$

R. J. Holt and C. D. Roberts, *Rev. Mod. Phys.* **82**, 2991 (2010).

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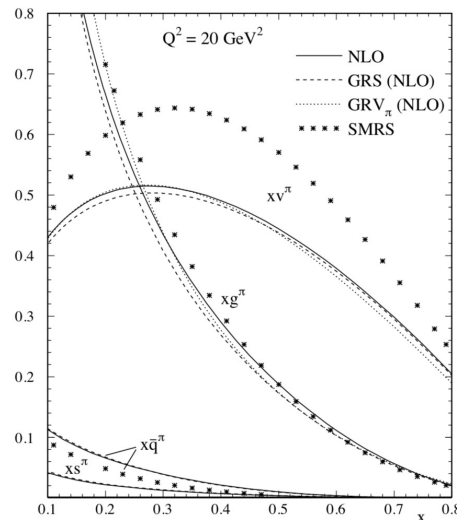
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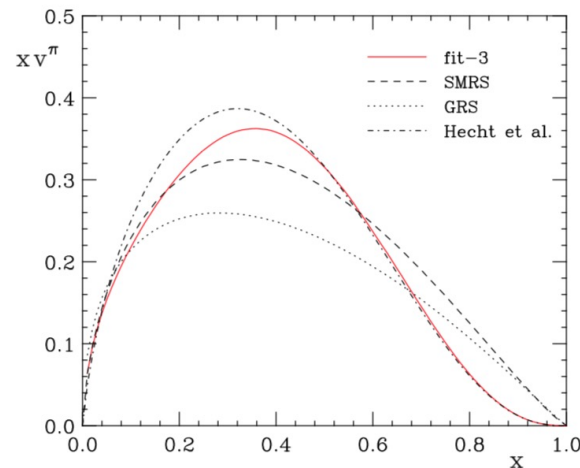
Pion structure in phenomenology

- Historically, pion distributions have been extracted from fixed target πA data
 - Drell-Yan (DY) $\pi A \rightarrow \mu^+ \mu^- X$
 - Prompt photon $\pi A \rightarrow \gamma X$

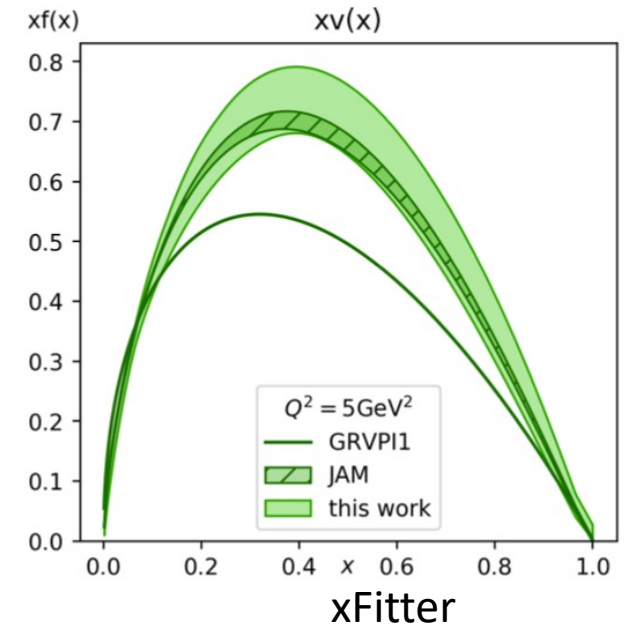
Owens
attempted to
use J/ψ
production



GRS, GRV, and SMRS

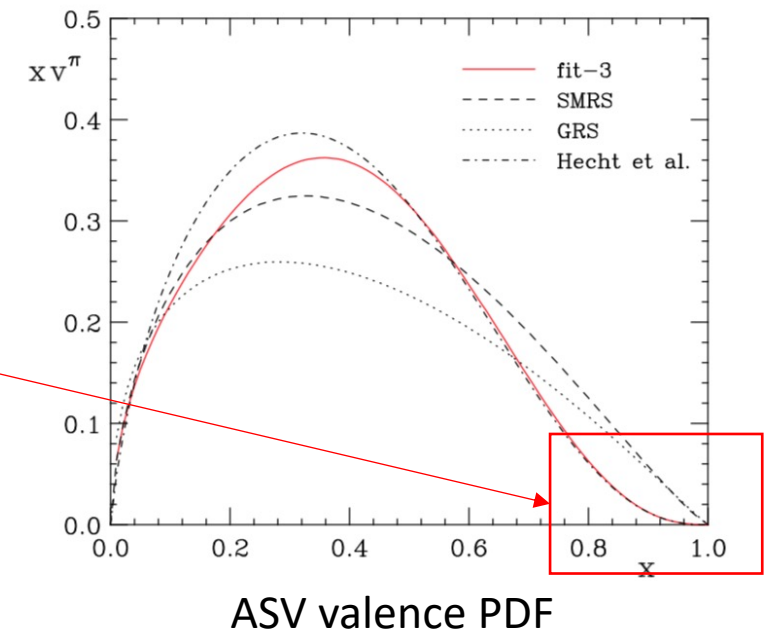


ASV valence PDF



Large- x_π behavior

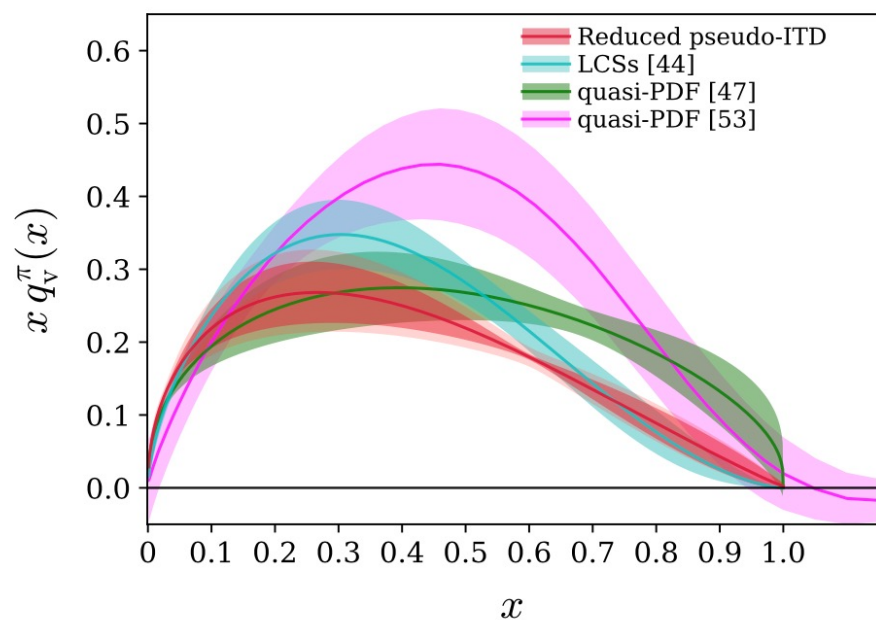
- Generally, the parametrization lends a behavior as $x \rightarrow 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a **fixed order analysis**, analyses find $\beta \approx 1$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with **threshold resummation**



Phys. Rev. Lett. **105**, 114023 (2011).

Lattice QCD Activity

- Simulations on the lattice have been done to investigate this structure



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Subset of pion lattice
QCD analyses

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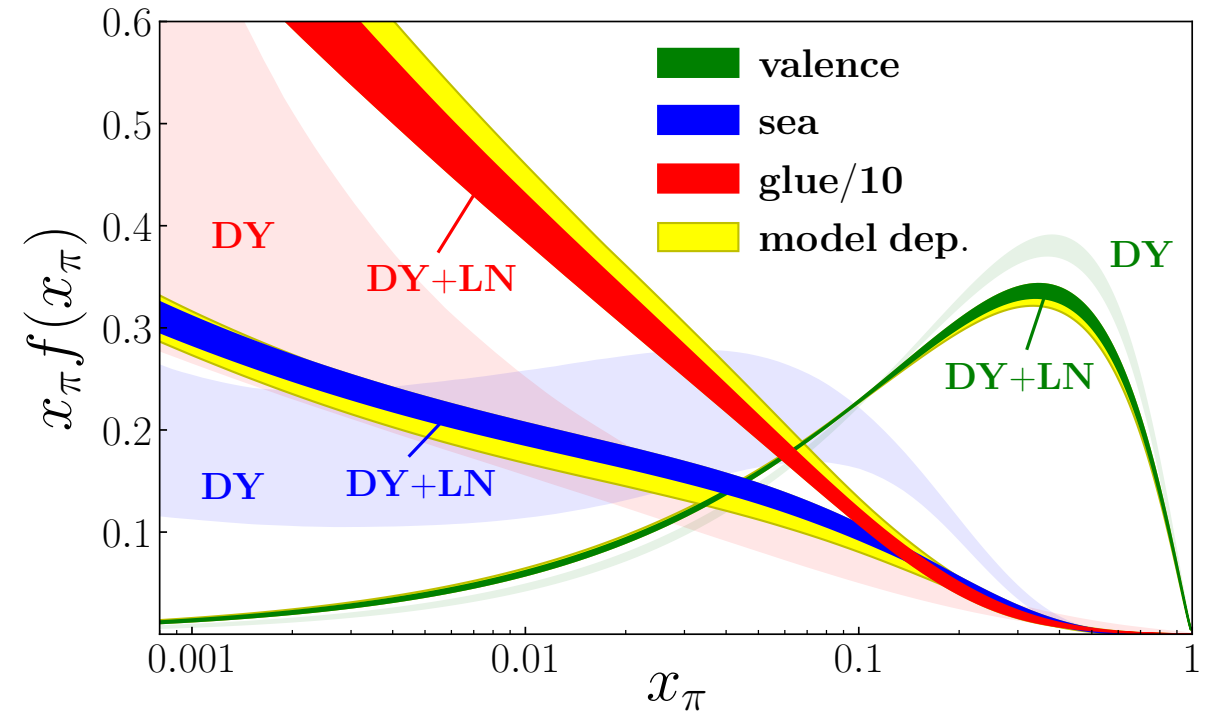
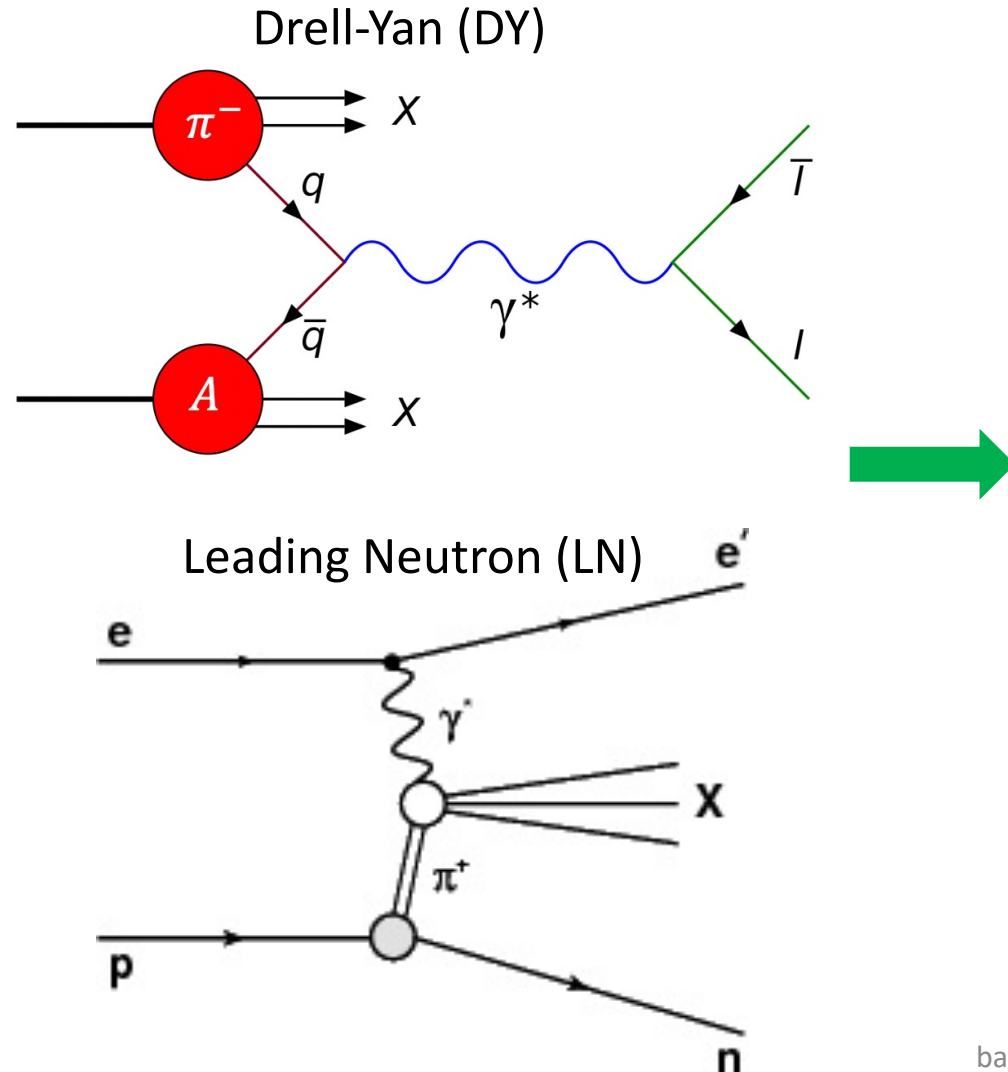
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Experiments to probe pion structure



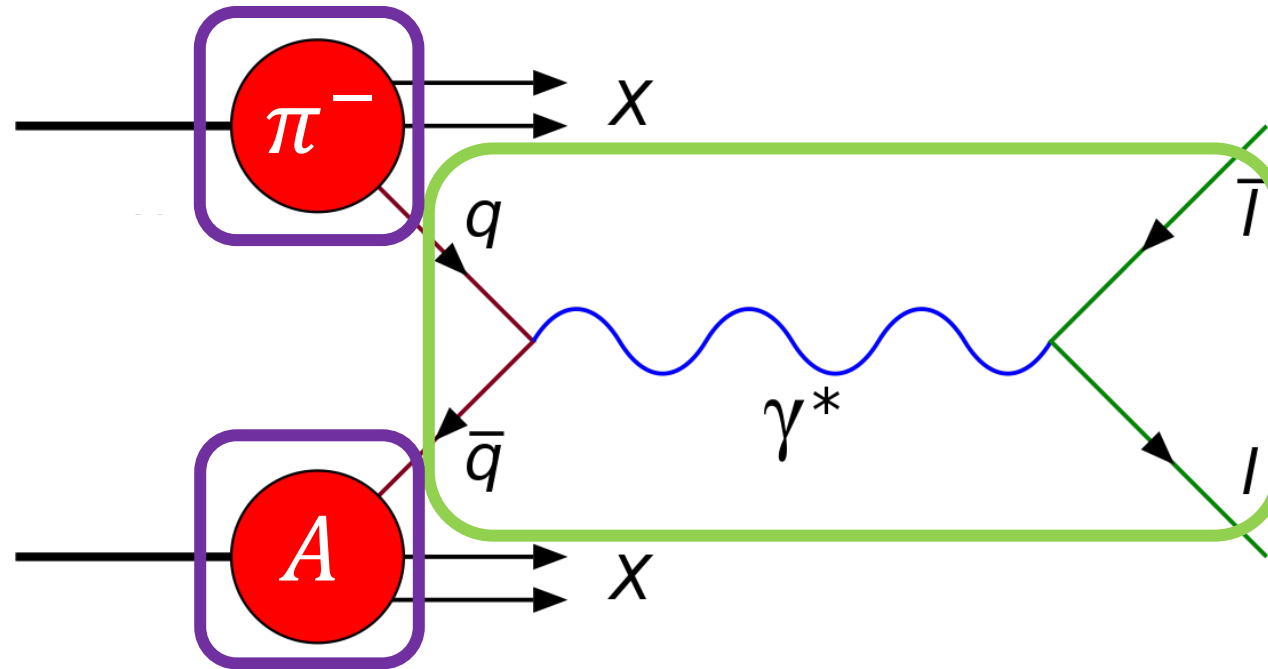
PHYSICAL REVIEW LETTERS **121**, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

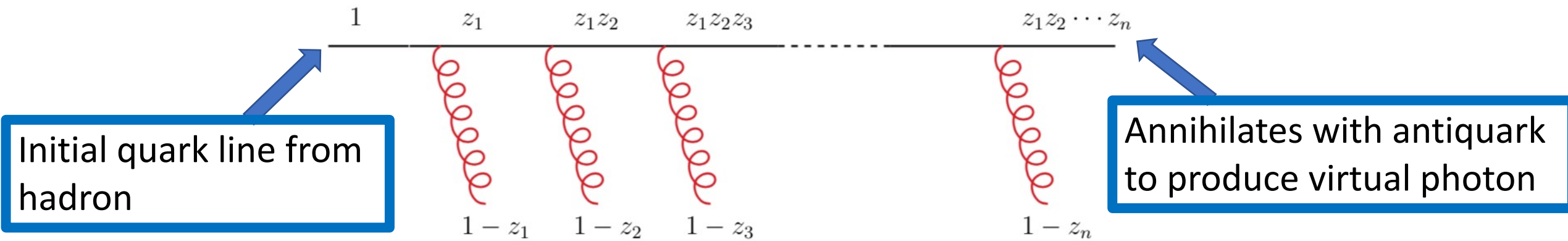
P. C. Barry,¹ N. Sato,² W. Melnitchouk,³ and Chueng-Ryong Ji¹

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Threshold Resummation



Significant contributions to cross section occur in **soft gluon emissions** and follow the pattern

$$d\hat{\sigma}_{N^k LO}^{q\bar{q}} \propto \alpha_S^k \frac{\ln^{2k-1}(1-z)}{1-z} + \dots$$

Methods of resummation – Mellin-Fourier

- Threshold resummation is done in conjugate space

$$\sigma_{\text{MF}}(N, M) \equiv \int_0^1 d\tau \tau^{N-1} \int_{\log \sqrt{\tau}}^{\log \frac{1}{\sqrt{\tau}}} dY e^{iMY} \frac{d^2\sigma}{d\tau dY},$$

Two choices occur when isolating the hard part

$$\hat{\sigma}_{\text{MF}}(N, M) = \int_0^1 dz z^{N-1} \cos\left(\frac{M}{2} \log z\right) \frac{d^2\hat{\sigma}}{d\tau dY}(z)$$

Keep cosine intact –
“cosine” method

Keep the first order term in
the expansion – $\cos\left(\frac{M}{2} \log z\right) \approx 1$
“expansion” method

Method of resummation – double Mellin

- Alternatively, perform a **double Mellin** transform

$$\sigma_{\text{DM}}(N, M) \equiv \int_0^1 dx_{\pi}^0 (x_{\pi}^0)^{N-1} \int_0^1 dx_A^0 (x_A^0)^{M-1} \frac{d^2\sigma}{d\tau dY}.$$

where $x_{\pi}^0 = \sqrt{\tau}e^Y, \quad x_A^0 = \sqrt{\tau}e^{-Y}$

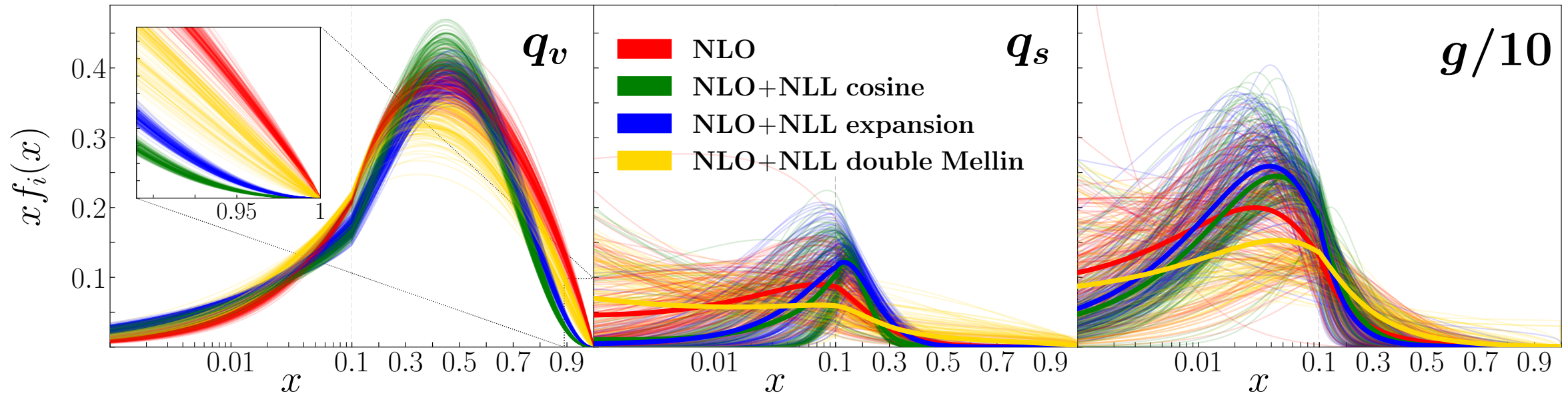
- **Double Mellin transform** is theoretically cleaner and sums up terms appropriately

Including threshold resummation in DY - Resulting PDFs

PHYSICAL REVIEW LETTERS **127**, 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

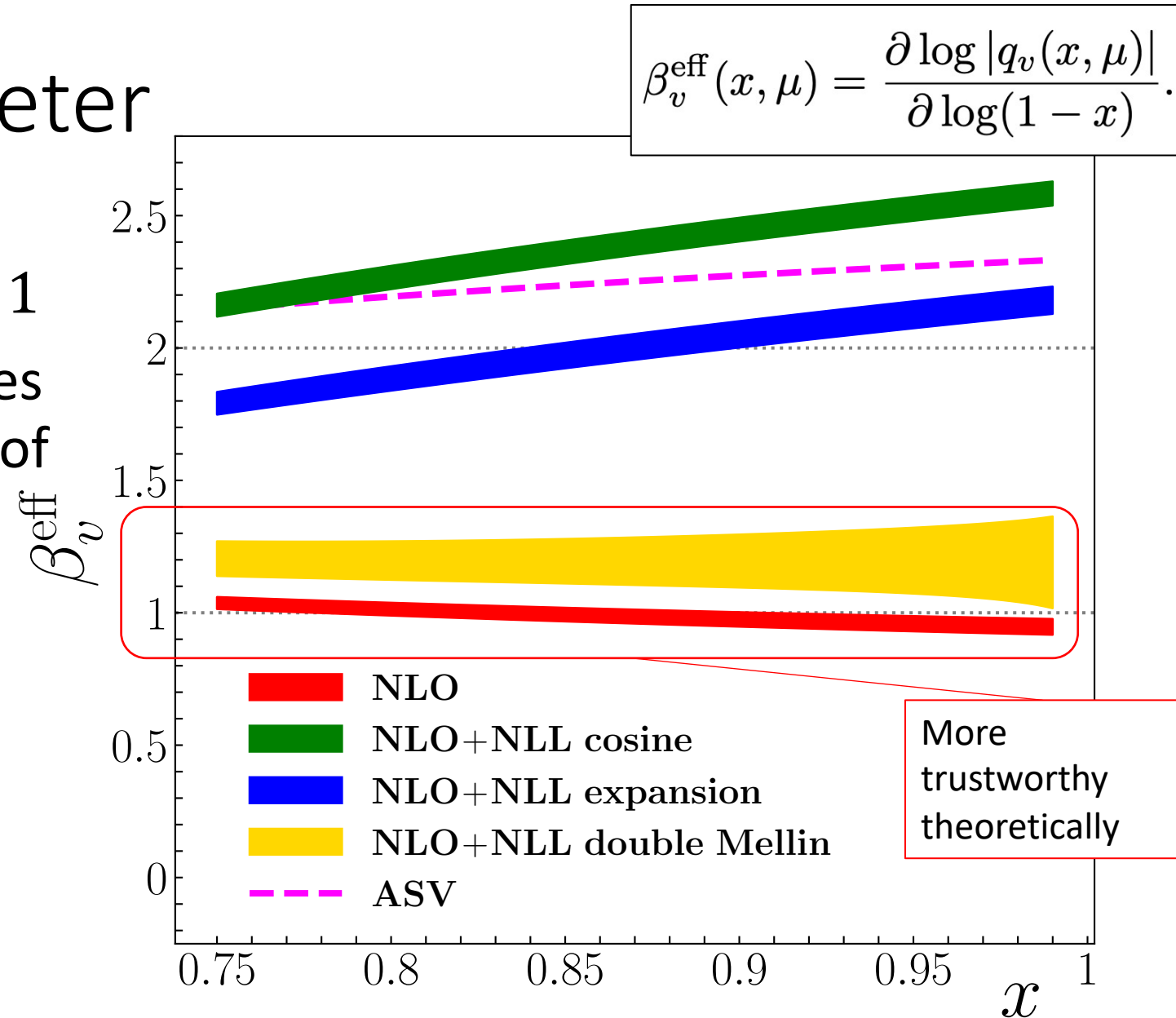
P. C. Barry¹, Chueng-Ryong Ji², N. Sato¹, and W. Melnitchouk¹



- Large x behavior of q_v **highly sensitive** to method of resummation

Effective β_v parameter

- $q_v(x) \sim (1-x)^{\beta_v^{\text{eff}}}$ as $x \rightarrow 1$
- Threshold resummation does not give universal behavior of β_v^{eff}
- **NLO** and **double Mellin** give $\beta_v^{\text{eff}} \approx 1$ – theoretically cleaner
- **Cosine** and **Expansion** give $\beta_v^{\text{eff}} > 2$



Including lattice QCD data from HadStruc

- Can lattice QCD simulations further discriminate between NLO and the double Mellin methods?

PHYSICAL REVIEW D **105**, 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry¹, C. Egerer¹, J. Karpie², W. Melnitchouk¹, C. Monahan^{1,3}, K. Orginos^{1,3},
Jian-Wei Qiu^{1,3}, D. Richards¹, N. Sato¹, R. S. Sufian^{1,3} and S. Zafeiropoulos⁴

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

How to relate PDFs with lattice observables?

- Make use of good lattice cross sections and appropriate matching coefficients

$$\begin{aligned}\Sigma_{n/h}(\nu, z^2) &\equiv \langle h(p) | T\{\mathcal{O}_n(z)\} | h(p) \rangle \\ &= \sum_i f_{i/h}(x, \mu^2) \otimes C_{n/i}(x\nu, z^2, \mu^2) \\ &\quad + \mathcal{O}(z^2 \Lambda_{\text{QCD}}^2)\end{aligned}$$

- Structure just like experimental cross sections – good for global analysis

Fitting the Data and Systematic Corrections

Valence quark
distribution in pion

Wilson coefficients
for matching

$$\text{Re } \mathfrak{M}(\nu, z^2) = \int_0^1 dx \, q_v(x, \mu_{\text{lat}}) \mathcal{C}^{\text{Rp-ITD}}(x\nu, z^2, \mu_{\text{lat}}) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi(L-z)} F_1(\nu) + \dots$$

Integration lower bound is 0

Systematic corrections to parametrize

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi(L-z)} F_1(\nu)$: finite volume corrections

Other potential
systematic
corrections the data
is not sensitive to

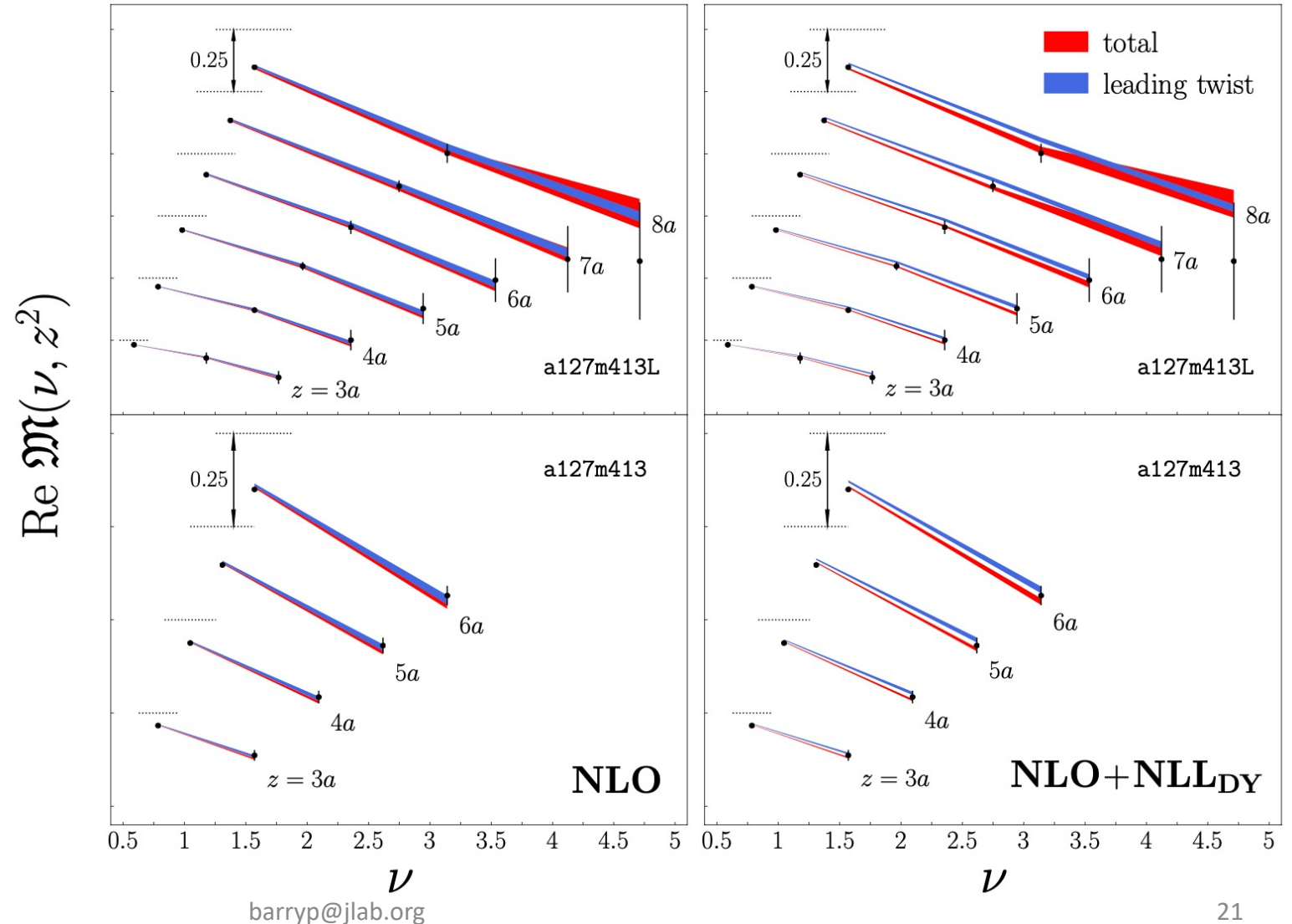
Goodness of fit

- Scenario A:
experimental data
alone
- Scenario B:
experimental + lattice,
no systematics
- Scenario C:
experimental + lattice,
with systematics

Process	Experiment	N_{dat}	Scenario A		Scenario B		Scenario C	
			NLO	+NLL _{DY}	NLO	+NLL _{DY}	NLO	+NLL _{DY}
			$\bar{\chi}^2$		$\bar{\chi}^2$		$\bar{\chi}^2$	
DY	E615	61	0.84	0.82	0.83	0.82	0.84	0.82
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	0.52	0.55
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	0.78	0.87
LN	H1	58	0.36	0.35	0.39	0.39	0.37	0.37
	ZEUS	50	1.56	1.48	1.62	1.69	1.58	1.60
Rp-ITD	a127m413L	18	–	–	1.04	1.06	1.04	1.06
	a127m413	8	–	–	1.98	2.63	1.14	1.42
Total		251	0.82	0.80	0.89	0.92	0.85	0.87

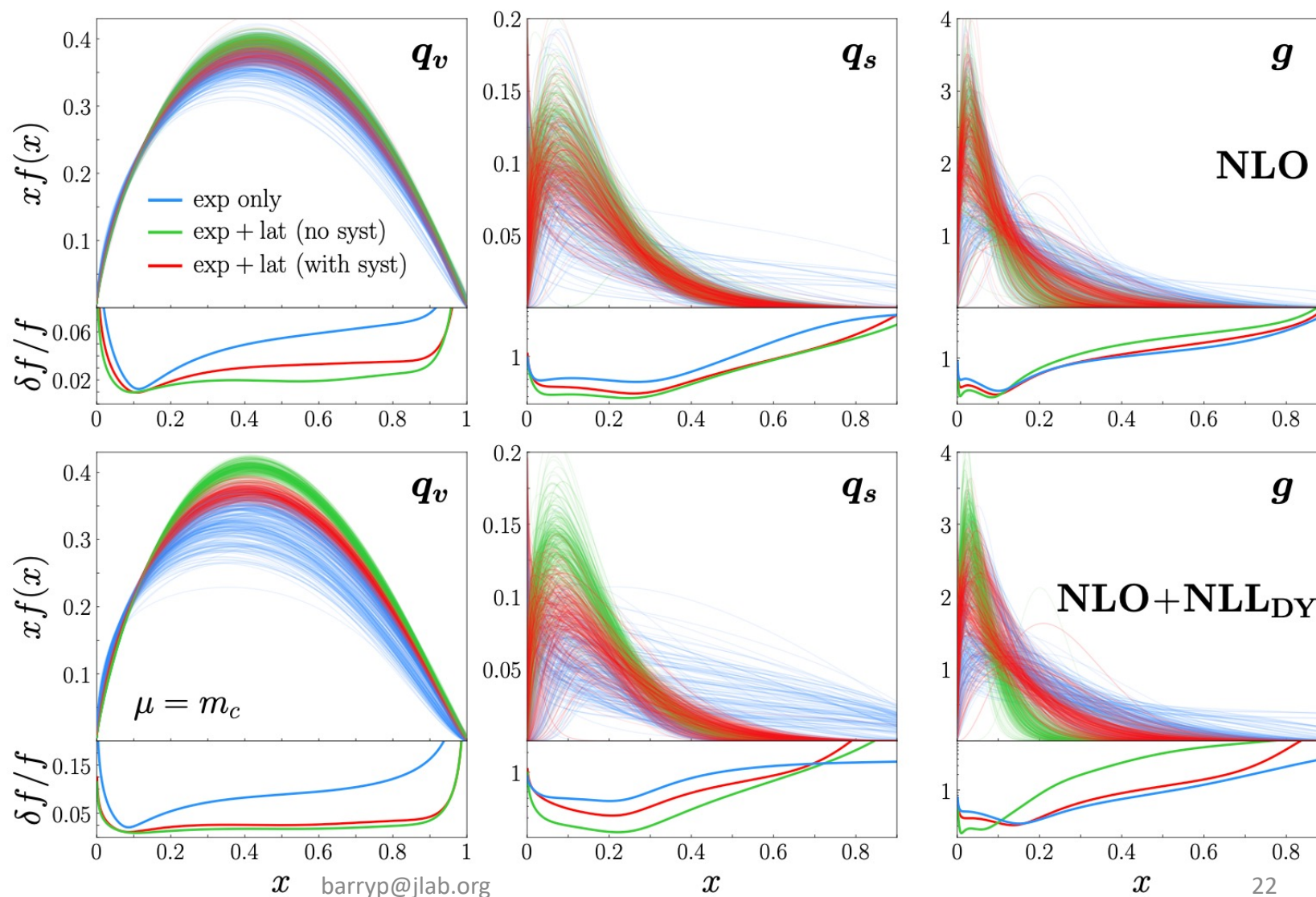
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

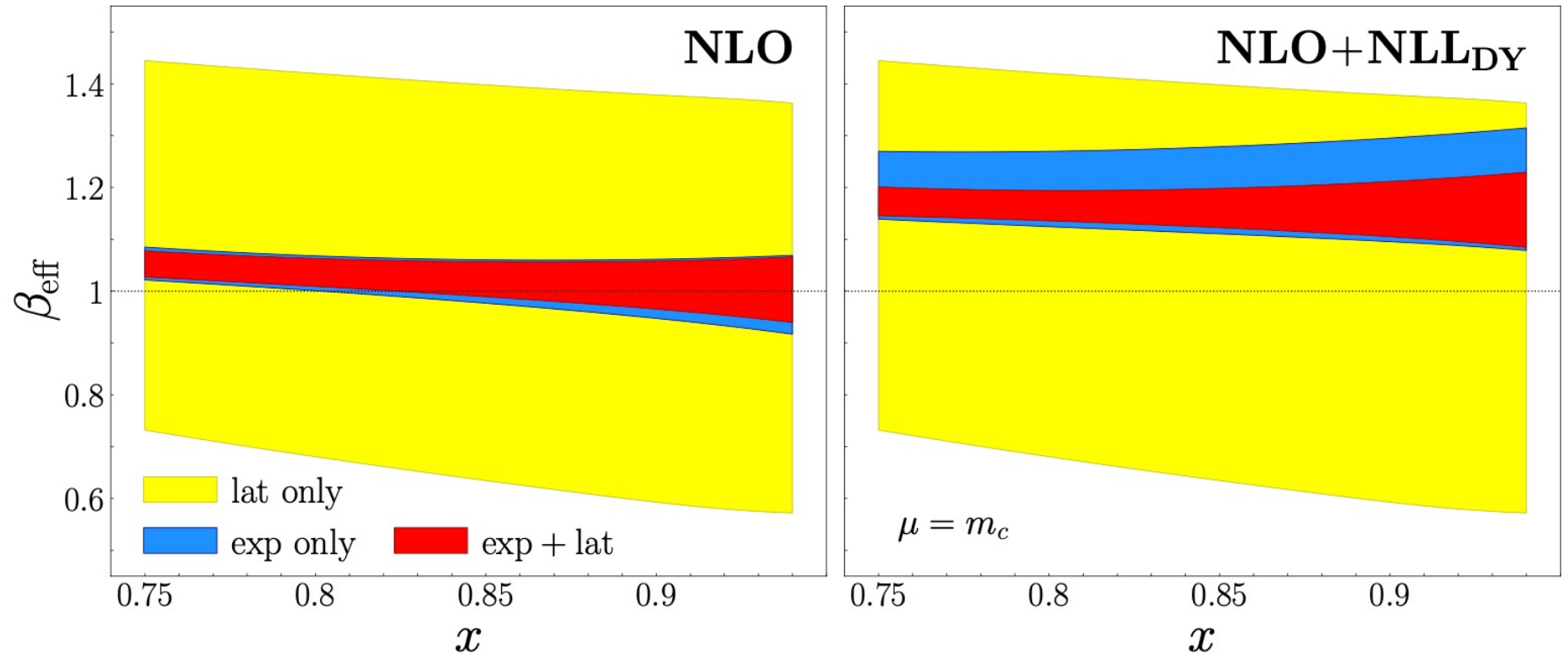
- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL_{DY} changes a lot – unstable under new data



Effective β from $(1-x)^{\beta_{\text{eff}}}$

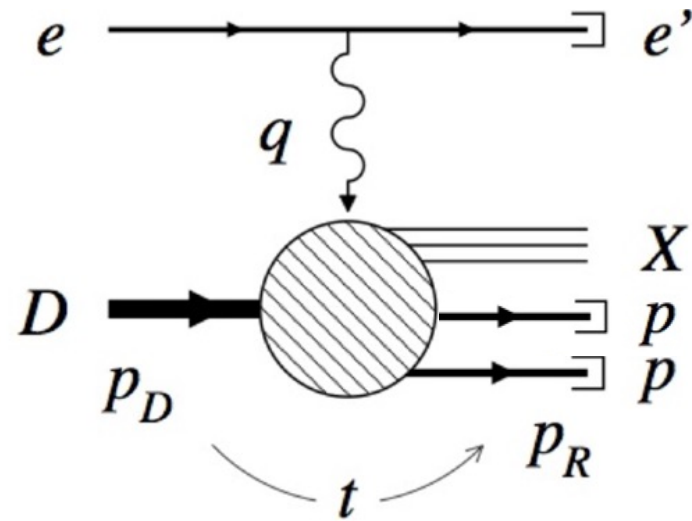
$$\beta_{\text{eff}}(x, \mu) = \frac{\partial \log |q_v(x, \mu)|}{\partial \log(1-x)}$$

Calculations
from QCD do
not predict
 $\beta_{\text{eff}} = 2$



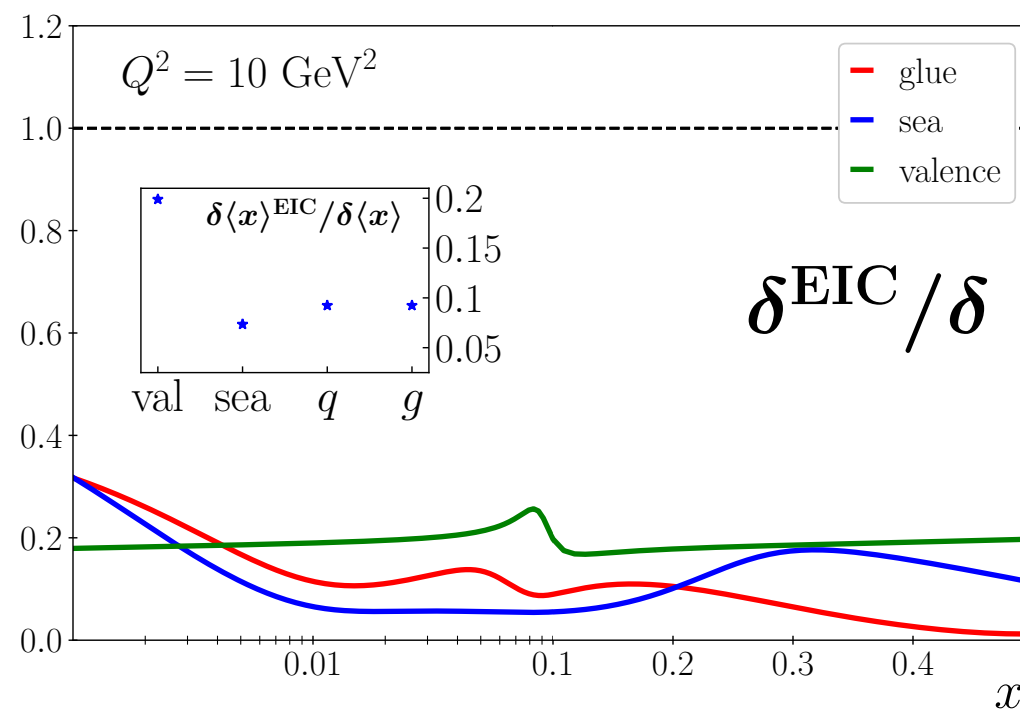
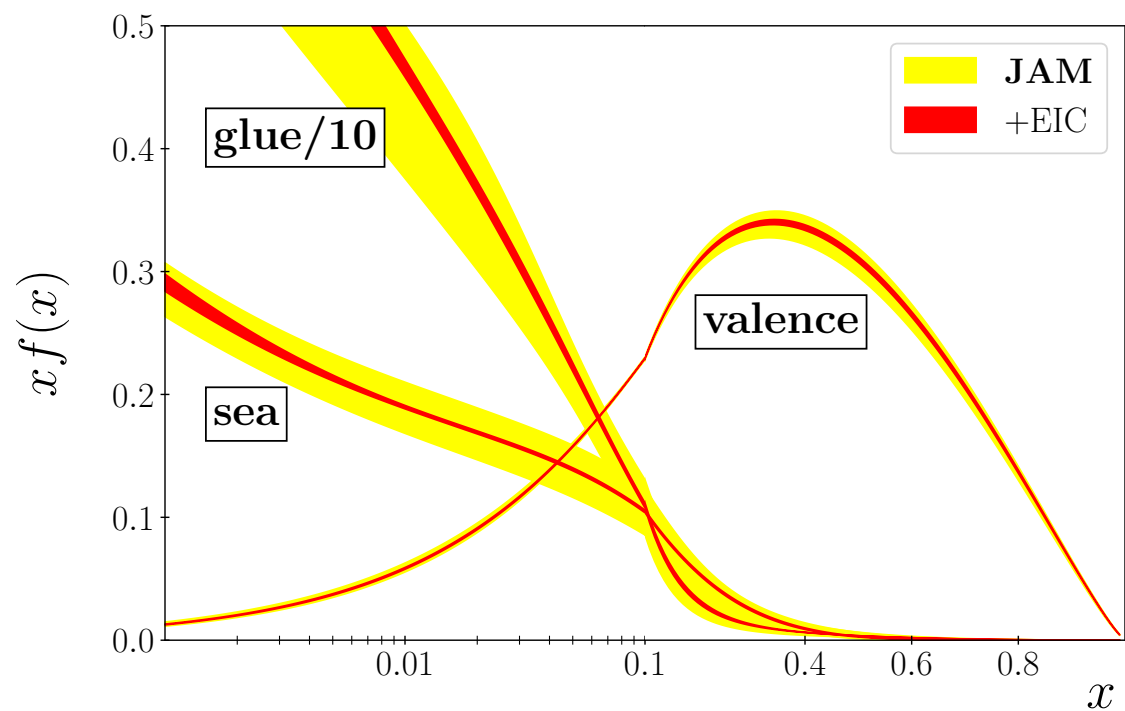
Future Experiments

- **TDIS** experiment at 12 GeV upgrade from **JLab**, which will tag a proton in coincidence with a spectator proton
- Gives **leading proton observable**, complementary to LN, but with a fixed target experiment instead of collider
- **Electron-Ion collider** can measure LN
- Proposed **COMPASS++/AMBER** also give π -induced **DY** data
- Both π^+ and π^- beams on carbon and tungsten targets



EIC Impact on Pion PDFs

- Statistical uncertainties are small compared with HERA because of larger luminosity – systematics dominate
- $s = 5400 \text{ GeV}^2$, 1.2% systematic uncertainty, integrated $\mathcal{L} = 100\text{fb}^{-1}$

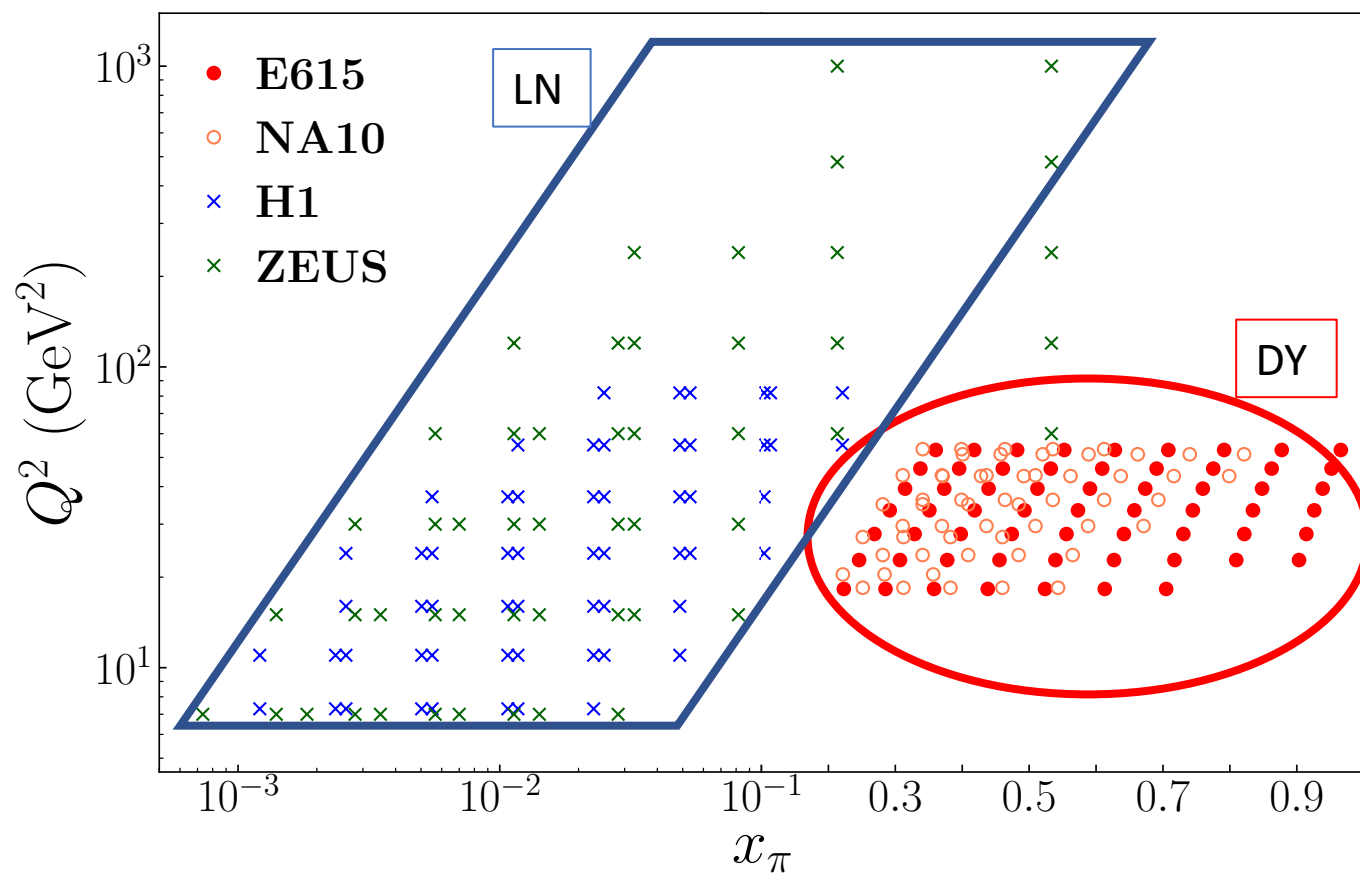


Conclusions

- Behavior of large- x valence distribution with double Mellin threshold resummation $q_v(x \rightarrow 1) \propto (1 - x)^{\sim 1.2}$
- The complementarity between lattice and experimental data sheds light on the pion PDF itself as well as systematics associated with the lattice
- Future experimental and lattice data are needed to further pin down large- x behavior of the valence quark distribution
- Other processes such as p_T -dependent cross sections are sensitive to PDFs

Backup Slides

Datasets -- Kinematics



Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p | \bar{\psi}(0) \gamma^0 \mathcal{W}(z; 0) \psi(z) | p \rangle$$

Quark and antiquark fields

Gauge link

"Ioffe time"

$$\nu = p \cdot z$$

$$z = (0, 0, 0, z_3)$$

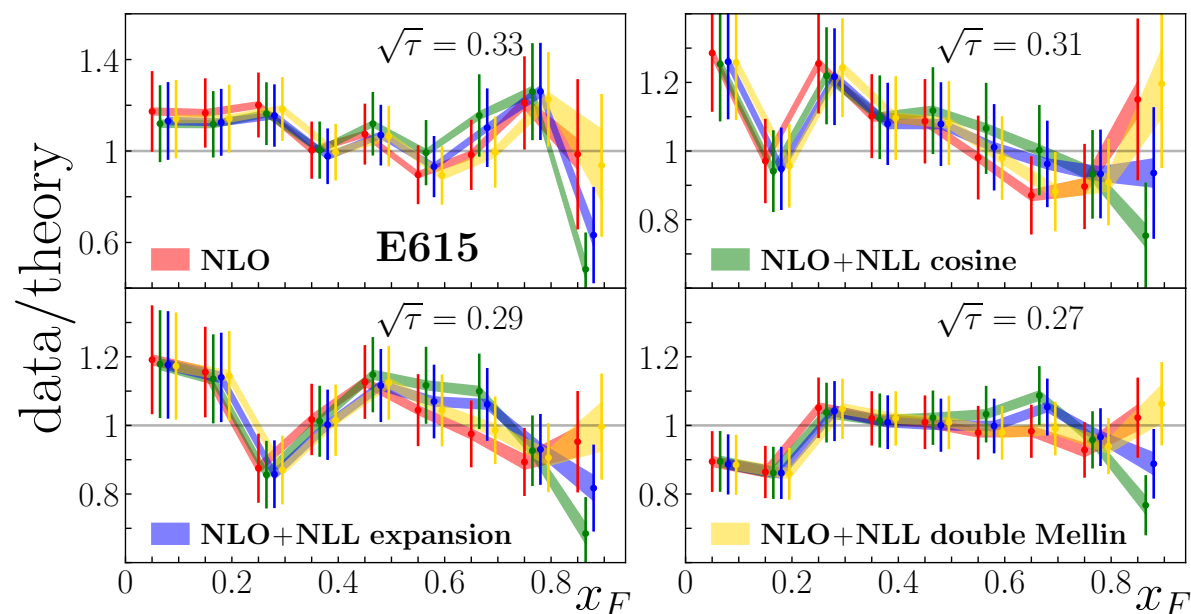
Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

Ratio cancels UV divergences

Data and theory comparison

- **Cosine** method tends to overpredict the data at very large x_F
- **Double Mellin** method is qualitatively very similar to **NLO**



Current data do not distinguish between NLO and NLO+NLL

Method	χ^2/npts
NLO	0.85
NLO+NLL cosine	1.29
NLO+NLL expansion	0.95
NLO+NLL double Mellin	0.80

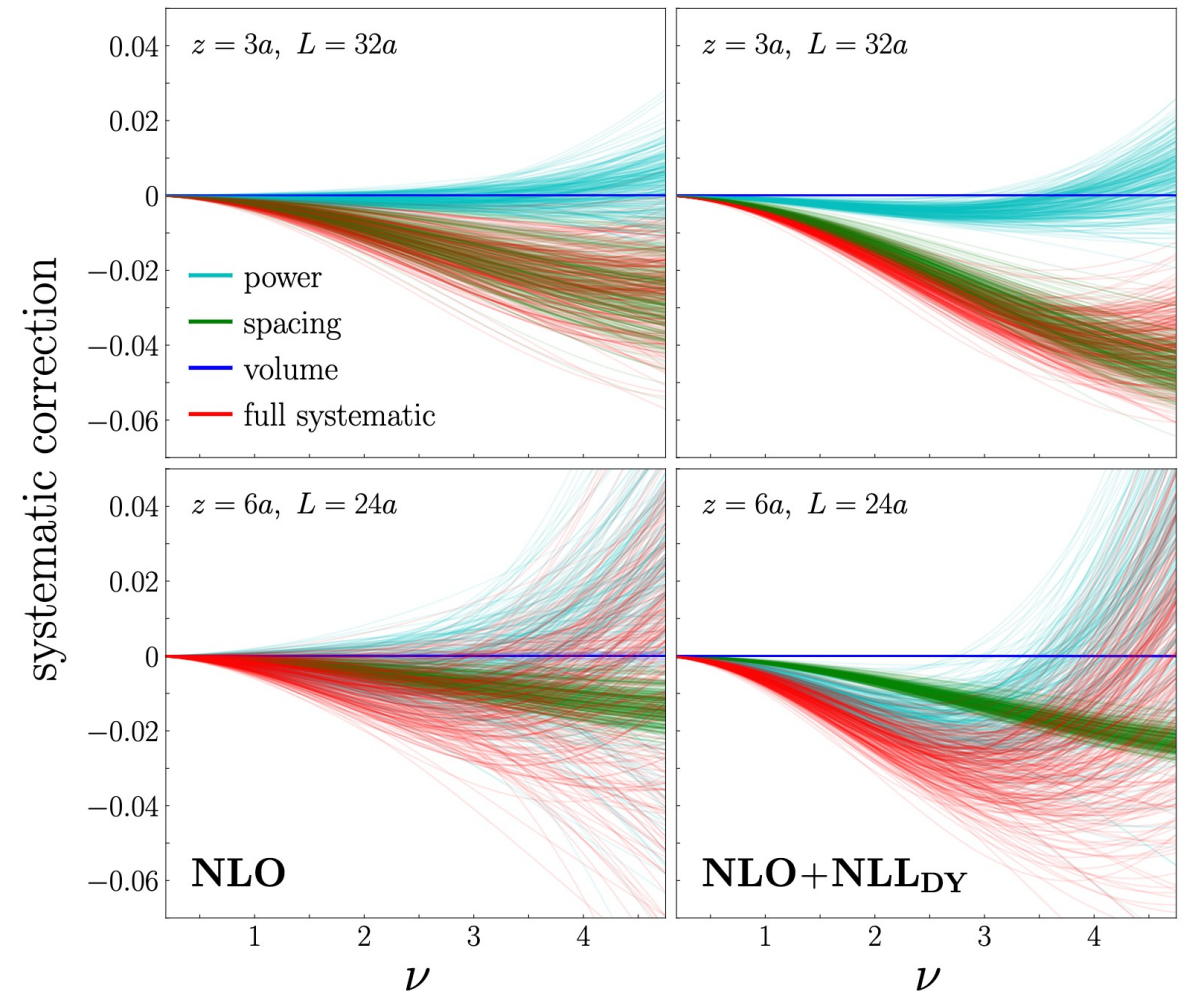
Slightly disfavored

Quantifying individual systematic corrections on the lattice

- Breaking down by the 3 systematics

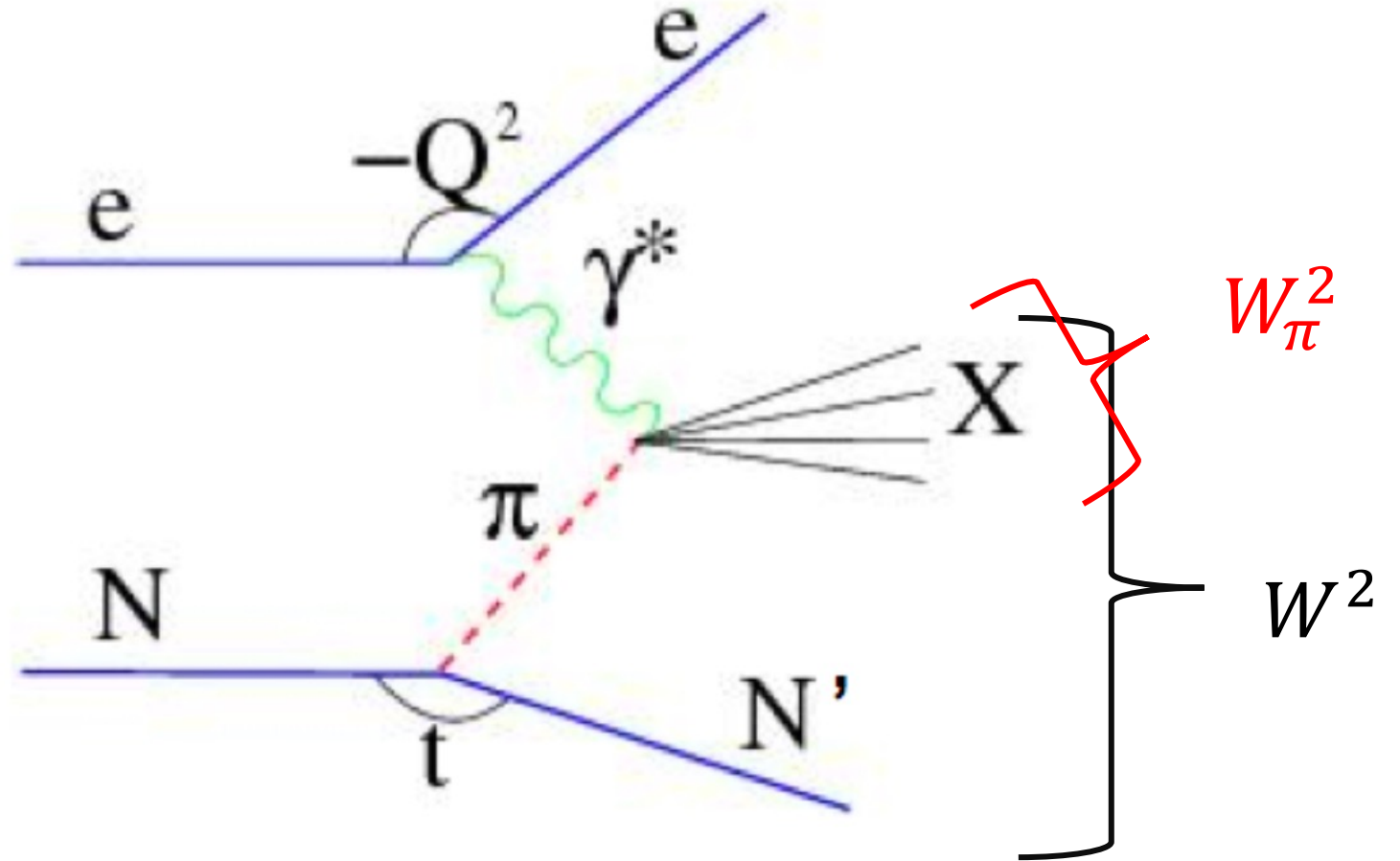
$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi(L-z)} F_1(\nu)$$

- Dominance of power or spacing corrections depends on z
- Finite volume corrections don't matter



Sullivan process and W_π^2

- Impose kinematic cuts on experimental data
- Such as lower limit on the totally *inclusive* W^2
- At small Q^2 , we must be careful of the pion resonances as a function W_π^2



Critiques suggested $(1 - x)^2$ is a fact of QCD

$$u^\pi(x; \zeta) \stackrel{x \lesssim 1}{\sim} (1 - x)^{\beta = 2 + \gamma(\zeta)}$$

T1: If QCD describes the pion, then at any scale for which an analysis of data using known techniques is valid, the form extracted for the pion's valence-quark DF **must behave** as $(1 - x)^\beta$, $\beta > 2$, on $x \gtrsim 0.9$ [10, 59, 73, 74].

the associated disagreement with Eq. (27) requires explanation; and these are the only possibilities: [a] the dM scheme is incomplete, omitting or misrepresenting some aspect or aspects of the hard processes involved; [b] (some of) the data being considered in the analysis are not a true expression of a quality intrinsic to the pion; or [c] QCD, as it is currently understood, is not the theory of strong interactions.

- T1: There is **no proof** of this in QCD
- [a] The double Mellin method is more rigorous than Mellin-Fourier
- [b] We carefully apply factorization; lattice QCD data prefer a linear falloff; there is no evidence to suggest these data are wrong
- [c] There is no indication to insinuate QCD is not the theory of strong interactions

Ezawa

Wide-Angle Scattering in Softened Field Theory.

Z. F. EZAWA

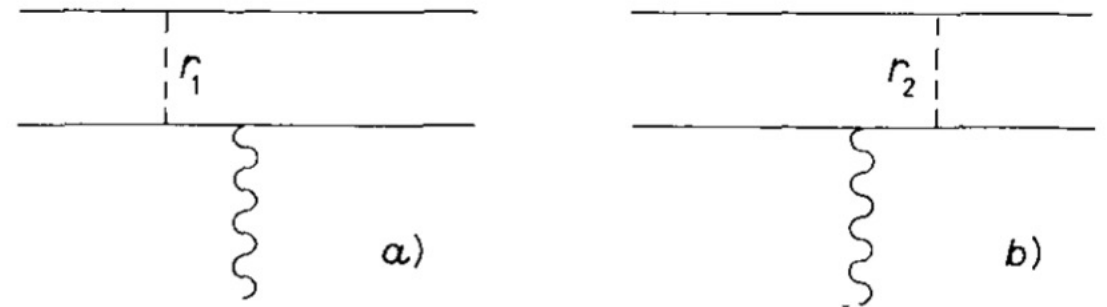
*Department of Applied Mathematics and Theoretical Physics
University of Cambridge - Cambridge*

(ricevuto il 25 Marzo 1974)

Not QCD

Summary. — The picture of Brodsky and Farrar for scattering processes at large transverse momentum is formulated in softened field theory. A modest softening of the quark-quark-gluon vertex is introduced to suppress unwanted logarithms in the formalism. It is shown that the electromagnetic form factors of the proton and the pion yield asymptotically behaviours which agree with the result of simple dimensional counting. The threshold behaviours of the deep inelastic structure functions are calculated for the proton and the pion to give $\sim(1-\omega)^3$ and $\sim(1-\omega)^2$, respectively. Thus the Drell-Yan-West relation holds in the case of the proton target but is violated in the case of the pion target. It is also proved that the asymptotic behaviours of wide-angle elastic $\pi\pi$ and pp scattering naively predicted by dimensional counting and conjectured by Brodsky and Farrar on the basis of simple Born diagrams are actually the next-to-leading-order terms. The highest-order terms come from a certain set of diagrams that Landshoff studied.

- No explicit proof of nonperturbative $q_v^\pi(x \rightarrow 1) \sim (1-x)^2$
- Assumes one hard gluon exchange dominance

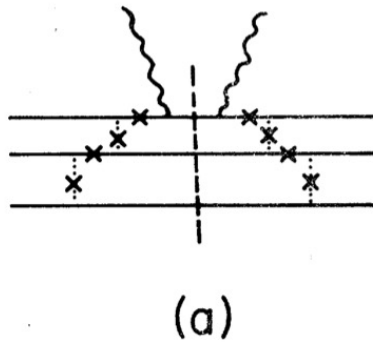


Farrar and Jackson

Pion and Nucleon Structure Functions near $x = 1$ *

Glennys R. Farrar† and Darrell R. Jackson
California Institute of Technology, Pasadena, California 91125
 (Received 4 August 1975)

- Assumption made that the below diagram dominates the structure



In a colored-quark and vector-gluon model of hadrons we show that a quark carrying nearly all the momentum of a nucleon ($x \approx 1$) must have the same helicity as the nucleon; consequently $\nu W_2^{\pi}/\nu W_2^p \rightarrow \frac{3}{7}$ as $x \rightarrow 1$, not $\frac{2}{3}$ as might naively have been expected. Furthermore as $x \rightarrow 1$, $\nu W_2^{\pi} \sim (1-x)^2$ and $(\sigma_L/\sigma_T)^{\pi} \sim \mu^2 Q^{-2} (1-x)^{-2} + O(g^2)$; the resulting angular dependence for $e^+e^- \rightarrow h^{\pm} + X$ is consistent with present data and has a distinctive form which can be easily tested when better data are available.

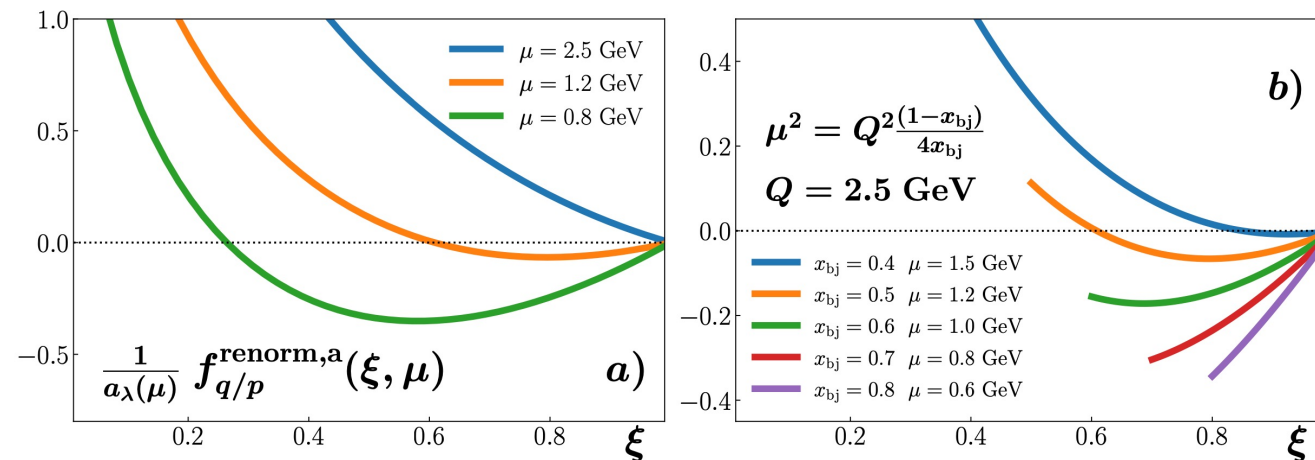
Assumption

go from the normal to “exceptional” (one quark having large p^2) wave functions. We assume that (a) the normal wave function is sufficiently damped at large p^2 's that the convolution is dominated by the region in which the p^2 's of the incoming quarks are finite, and (b) the spin and

- This is a *perturbative* assumption – we cannot say that higher order terms or soft gluons do not contribute to the *nonperturbative* structure of the hadron in QCD
- First principles QCD does not *prove* this behavior for PDF

Not necessary to have $(1 - x)^\beta$ behavior

- A recent work by Collins, Rogers, and Sato proved that $\overline{\text{MS}}$ PDFs were not necessarily positive as long as *cross section was positive*.



Phys. Rev. D **105**, 076010 (2022).

- PDFs do not have to have a large- x behavior associated with the counting rules

QCD does not fail if $\beta_v^\pi \neq 2$

- The perturbative expansion performed in Ezawa and Farrar & Jackson does not capture nonperturbative effects
- Like in threshold resummation, the buildup of very **soft gluon** exchanges between quark states may be non-negligible contributions to the perturbation
- When $(1 - x) \rightarrow 0$, the **light front zero mode** could play a non-trivial role, which cannot be calculated perturbatively