Recent $\not{\pi}$EFT studies of Λ and $\Lambda\Lambda$ few-body hypernuclei

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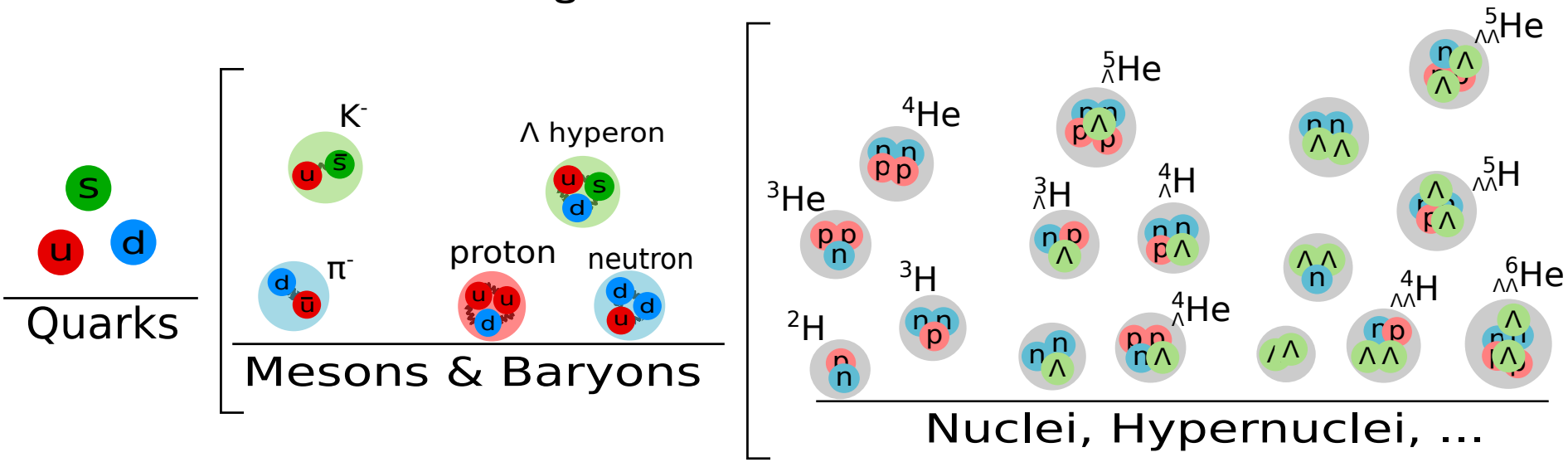
Why few-body hypernuclei ?

Interactions of hadrons :

- currently described by QCD

At low and intermediate energies ...



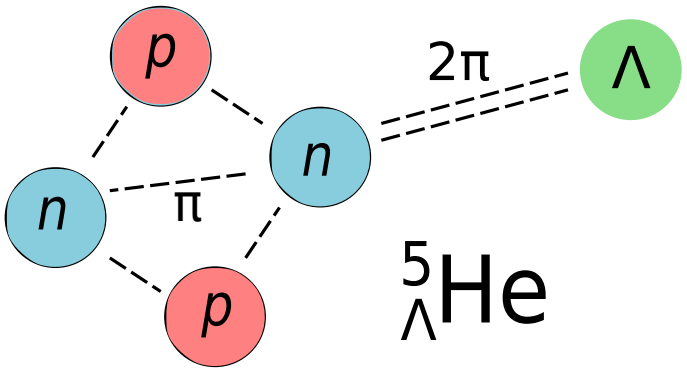
- **QCD is notoriously difficult to solve in this energy regime !**
  $\rightarrow$ lattice QCD and effective field theories (EFTs)

Observed properties of few-body hypernuclei $\longleftrightarrow$ **Precise few-body methods** $\longleftrightarrow$ **Underlying interaction models**

Hypernuclei

Where do we stand ?

- experimentaly observed more than 30 Λ-hypernuclei
- three well-established $\Lambda\Lambda$-hypernuclei
- scarce ΛN and no $\Lambda\Lambda$ scattering data



$\rightarrow$ difficult to fix parameters of interaction models,
many parameters and few data points $\rightarrow$ large uncertainties

What do we do ?

$\rightarrow$ we build low-energy EFT without π ($\not\pi$EFT) employing both scattering lengths and s-shell hypernuclear data (3-body NNN, ΛNN, and $\Lambda\Lambda N$ interaction)

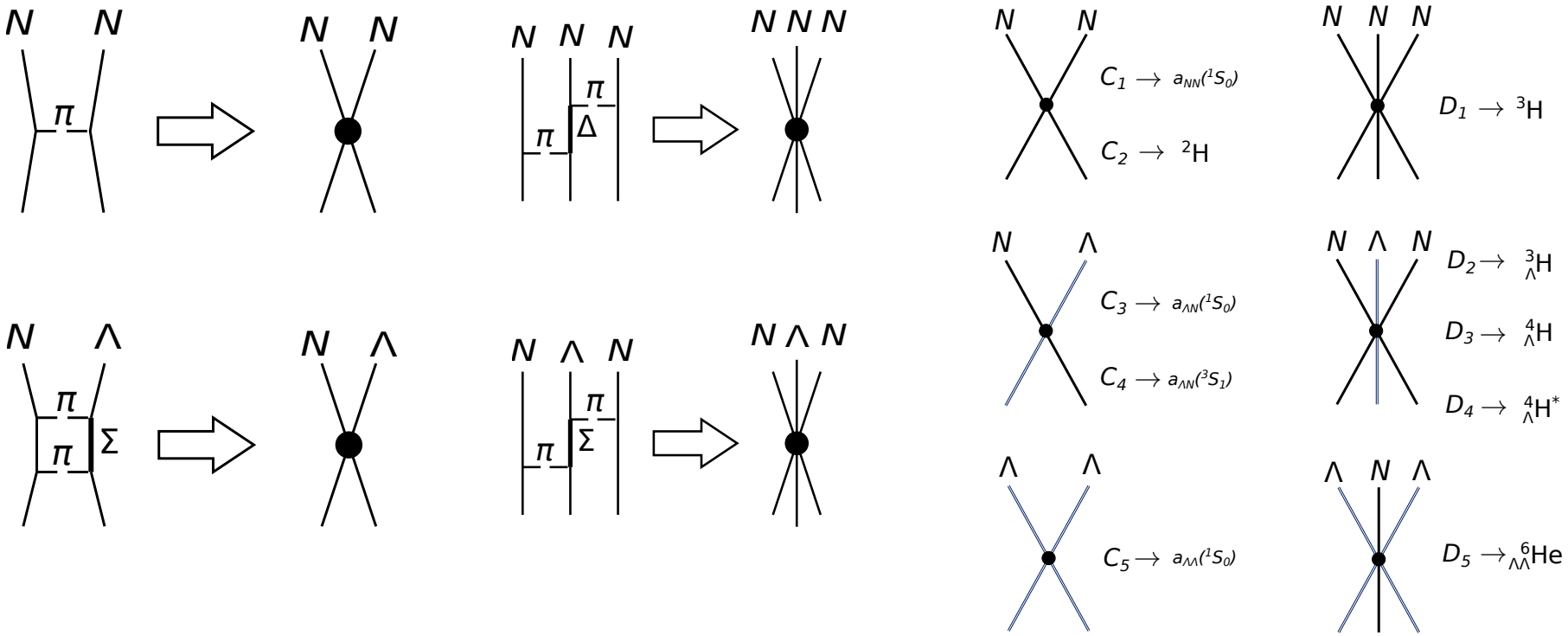
Hyper(nuclear) $\not\pi$EFT

Hamiltonian :

$$H_\lambda^{(\mathrm{LO})} = T_\mathrm{k} + V_2 + V_3 \qquad V_2 = \sum_{I,S} C_\lambda^{I,S} \sum_{i<j} \mathcal{P}_{ij}^{I,S} \delta_\lambda(\mathbf{r}_{ij})$$

$$V_3 = \sum_{I,S,\alpha} D_{\lambda,\alpha}^{I,S} \sum_{i<j<k} \mathcal{Q}_{ijk}^{I,S,\alpha} \sum_{\mathrm{cyc}} \delta_\lambda(\mathbf{r}_{ij})\delta_\lambda(\mathbf{r}_{jk})$$

Contact terms (minimal amount of parameters) $\rightarrow$ constrained by exp. data



$\rightarrow$ prediction of $\Lambda\mathrm{nn}$, $\Lambda\Lambda\mathrm{n}$, $\Lambda\Lambda\mathrm{nn}$, ${}^3_\Lambda\mathrm{H}^*$, ${}^5_\Lambda\mathrm{He}$, ${}^4_{\Lambda\Lambda}\mathrm{H}$, ${}^5_{\Lambda\Lambda}\mathrm{H}$, ${}^5_{\Lambda\Lambda}\mathrm{He}$

YN scattering data

- cross-section datapoints for $p_{\rm lab} \gtrsim 100$ MeV
  - 12 d.p. for $\Lambda + p \to \Lambda + p$
  - 22 d.p. for $\Sigma^- + p \to \Lambda + n$, $\Sigma^+ + p \to \Sigma^+ + p$, $\Sigma^- + p \to \Sigma^- + p$, and $\Sigma^- + p \to \Sigma^0 + n$
- no information regarding spin-depence

- **Alexander et al.** (PR173, 1452, 1968)
  $a_{\Lambda N}(^1S_0)$=-1.8 fm
  $a_{\Lambda N}(^3S_1)$=-1.6 fm
- **Sechi-Zorn et al.** (PR175, 1735, 1968)
  $0 > a_{\Lambda N}(^1S_0) > -9.0$ fm
  $-0.8 > a_{\Lambda N}(^3S_1) > -3.2$ fm

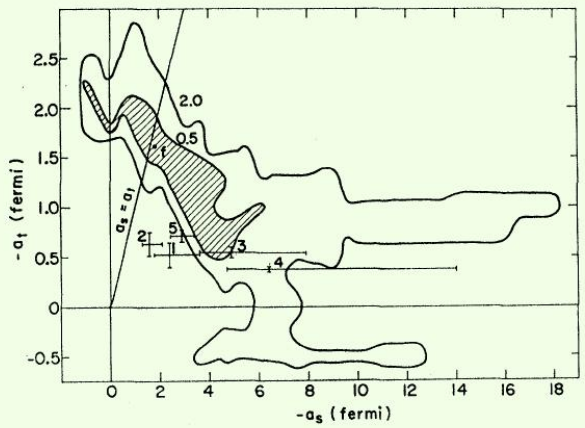


FIG. 9. Mapping of the likelihood function L in the a_s-a_t plane for the four-parameter fit. The shaded area includes all points with likelihood values above $L_{\max}$/exp0.5, where $L_{\max}$ is the value of the best fit (point f). The external smooth curve encloses likelihood values lying above $L_{\max}$/exp2.0. Points 1–5 represent scattering lengths derived from early hypernuclei calculations.

ΛN and $\Lambda\Lambda$ scattering lengths

ΛN scattering lengths (Rev. Mod. Phys. 88, 035004, 2016)

	$a_{\Lambda N}(^1S_0)$ [fm]	$a_{\Lambda N}(^3S_1)$ [fm]
NSC89	-2.79	-1.36
NSC97e	-2.17	-1.84
NSC97f	-2.60	-1.71
ESC08c	-2.54	-1.72
Jülich '04	-2.56	-1.66
χEFT(LO)	-1.91	-1.23
χEFT(NLO)	-2.91	-1.54
Alexander	-1.80	-1.60

$\Lambda\Lambda$ scattering length

	$a_{\Lambda\Lambda}(^1S_0)$ [fm]	
$^{12}C(K^-, K^+)\Lambda\Lambda X$	-1.2(6)	(Phys. Rev. C 85, 015204, 2012)
HAL QCD	-0.81 $\pm 0.23^{+0.00}_{-0.13}$	(Nucl. Phys. A 998, 121737, 2020)
χEFT(LO; 600)	-1.52	(Phys. Lett. B 653, 29, 2007)
χEFT(NLO; 600)	-0.66	(Nucl. Phys. A 954, 273, 2016)
$\Lambda\Lambda$ correlations; STAR	$-0.79^{+0.29}_{-1.13}$	(Phys. Rev. C 91, 024916, 2016)
		(Phys. Rev. Lett. 114, 022301, 2015)

Outline of this talk

Nature of the $\Lambda\mathrm{nn}$ $(\mathrm{J}^\pi = 1/2^+, \mathrm{I} = 1)$ and ${}^3_\Lambda\mathrm{H}^*$ $(J^\pi = 3/2^+, I = 0)$ states
M. Schäfer, B. Bazak, N. Barnea, and J. Mareš
(Phys. Lett. B 808, 135614, 2020; Phys. Rev. C 103, 025204, 2021)

The onset of ΛΛ hypernuclear binding
L. Contessi, M. Schäfer, N. Barnea, A. Gal, and J. Mareš
(Phys. Lett. B 797, 134893, 2019)

In-medium Λ isospin impurity from charge symmetry breaking in the ${}^4_\Lambda\mathrm{H} - {}^4_\Lambda\mathrm{He}$ mirror hypernuclei
M. Schäfer, N. Barnea, and A. Gal
(arXiv:2202.07460 [nucl-th], accepted in Phys. Rev. C letters, 2022)

Hypernucler trios ${}^{3}_{\Lambda}$H, ${}^{3}_{\Lambda}$H*, Λnn - physical motivation

${}^{3}_{\Lambda}$H$(1/2^{+})$

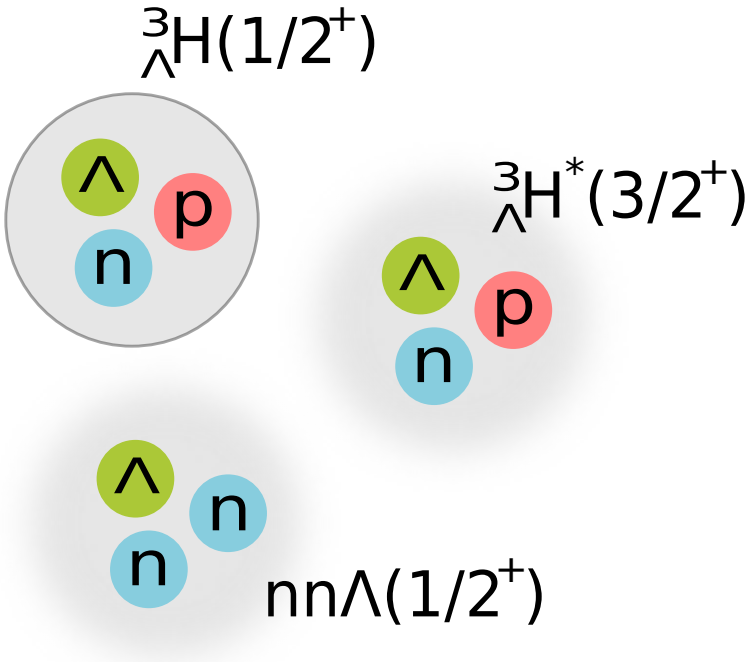
- lightest bound hypernucleus
- currently no experimental consensus on its B_{Λ}
- constraint in ΛN interaction models

${}^{3}_{\Lambda}$H$^{*}(3/2^{+})$

- no experimental evidence
- strict constraint on ΛN $S = 1$ interaction
- JLab C12-19-002 proposal

Λnn$(1/2^{+})$

- experiment (HypHI)
- JLab E12-17-003 experiment
- valuable source of Λn interaction
- structure of neutron-rich Λ-hypernuclei



Λnn and ${}^3_\Lambda$H* - early work

- **R. H. Dalitz, B. W. Downs** (PR110, 958, 1958; PR111, 967, 1958; PR114, 593, 1959)
  $\rightarrow$ first calculation, variational approach, unbound Λnn

- **H. Garcilazo** (J. Phys. G: Nucl. Phys. 13, 63, 1987)
  $\rightarrow$ Faddeev equations, separable potentials, unbound Λnn

- **K. Miyagawa et al.** (PRC51, 2905, 1995)
  $\rightarrow$ Faddeev equations, realistic Nijmegen interaction, unbound Λnn and ${}^3_\Lambda$H*

- **H. Garcilazo et al.** (PRC75, 034002, 2007; PRC76, 034001, 2007)
  $\rightarrow$ Faddeev equations, Chiral Quark Model ($N\Lambda - N\Sigma$ coupling, tensor force)
  $\rightarrow$ unbound Λnn
  $\rightarrow$ constraints on $a_{\Lambda N}^{S=0}$, $a_{\Lambda N}^{S=1}$ from ${}^3_\Lambda$H, unbound ${}^3_\Lambda$H*, and Λp data

- **V. B. Belyaev et al.** (NPA803, 210, 2008)
  $\rightarrow$ first resonance calculation, 3-body Jost function, phenomenological potential
  $\rightarrow$ Λnn pole just above/below the threshold, large widths

Λnn and ${}^{3}_{\Lambda}$H* - current status

- **HypHI Collaboration** (PRC88, 041001(R), 2013)
  → suggestion of bound Λnn, ^{6}Li $+{}^{12}$ C @ 2A GeV

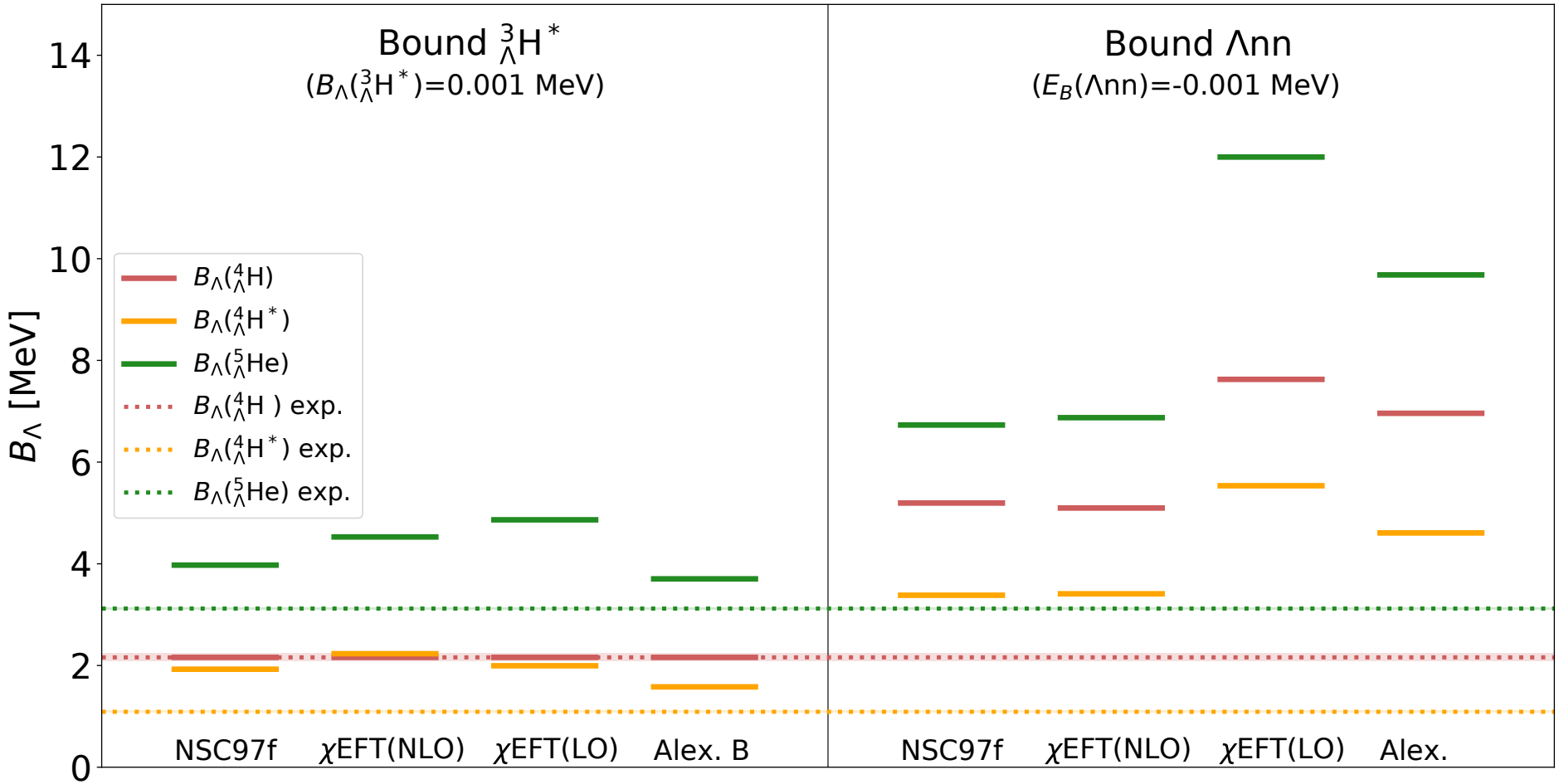
- **E. Hiyama et al.** (PRC89, 061302(R), 2014)
  → YN model equivalent to NSC97f; changing ${}^{3}V^{T}_{N\Lambda-N\Sigma}$, ${}^{0}V_{NN}$ to bind Λnn
  → nonexistence of bound Λnn (${}^{3}_{\Lambda}$H, ${}^{3}_{\Lambda}$H*, ${}^{4}_{\Lambda}$H, ^{3}H)

- **A. Gal, H. Garcilazo** (PLB736, 93, 2014)
  → Faddeev equations, separable potentials
  → nonexistence of bound Λnn ($\sigma_{\Lambda p}$, ${}^{3}_{\Lambda}$H, and ${}^{4}_{\Lambda}$H exc. energy)

- **I. R. Afnan, B. F. Gibson** (PRC92, 054608, 2015)
  → Faddeev equations, Λnn resonance calculations, separable potentials
  → subthreshold (non-physical) Λnn resonance

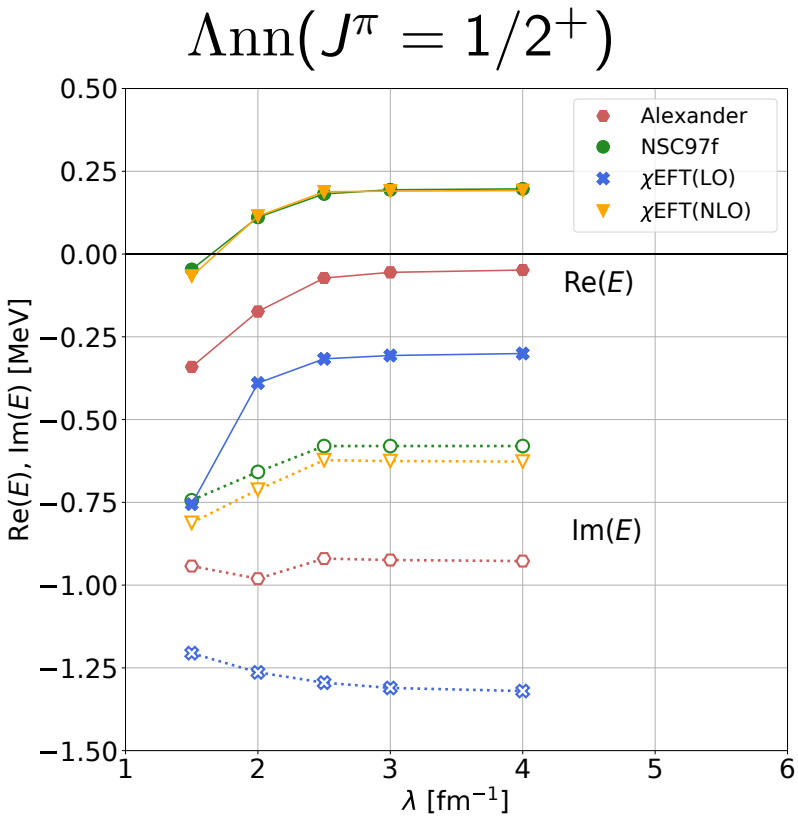
- **JLab E12-17-003 Experiment** (PTEP92 2022, 013D01, 2022)
  → ^{3}H$(e, e'K^{+})\Lambda nn$
  → No significant structures observed

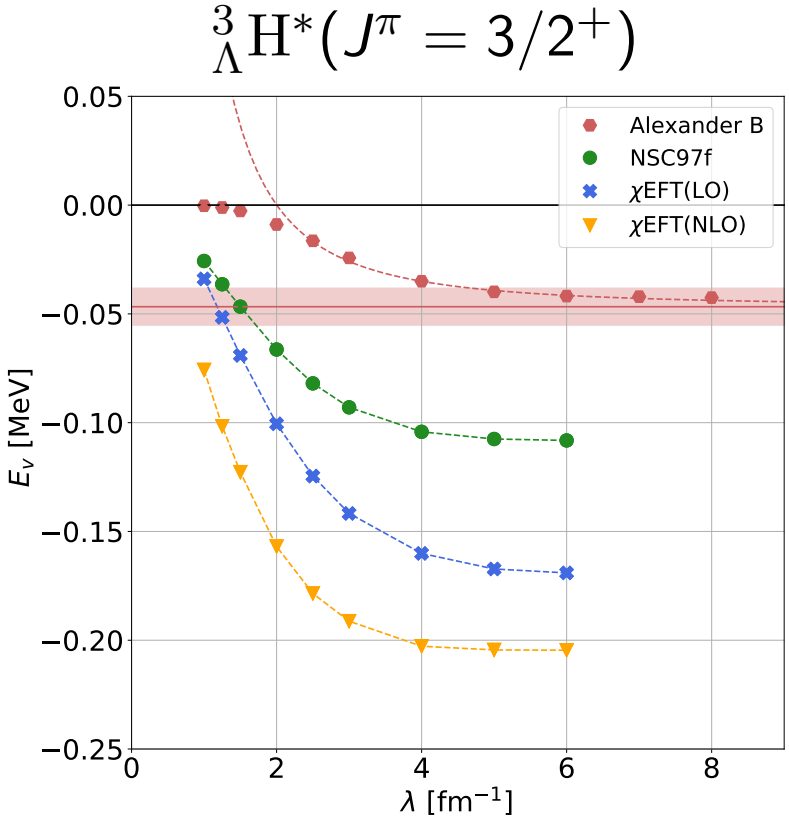
Implications of just bound Λnn and ${}^3_\Lambda\mathrm{H}^*$ ($\lambda = 6$ fm^{-1})



- $B_\Lambda({}^3_\Lambda\mathrm{H})$ is used to fix three-body force in $I, S = 0, 1/2$ channel and remains unaffected

Λnn system and ${}^3_\Lambda$H$^*(J^\pi = 3/2^+)$ excited state

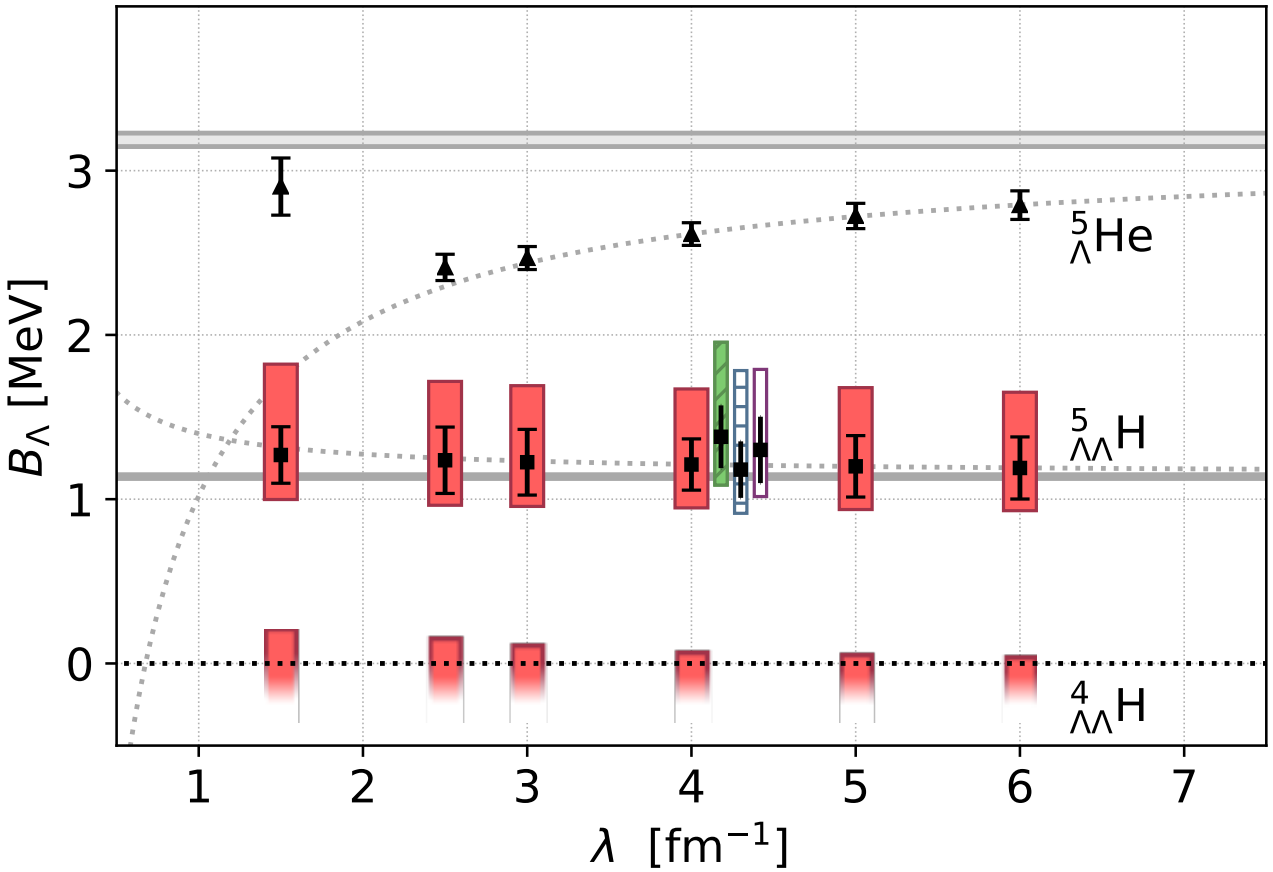




Λnn **predicted as a near-threshold resonance**
$\rightarrow$ large width $1.16 \leq \Gamma \leq 2.00$ MeV

${}^3_\Lambda$H* **obtained as a near-threshold virtual state**
$\rightarrow$ enhanced s-wave $\Lambda + {}^2$H phaseshifts in $J^\pi = 3/2^+$ channel

The onset of ΛΛ hypernuclear binding



Decisive role of 3-body ΛNN and $\Lambda\Lambda N$ forces

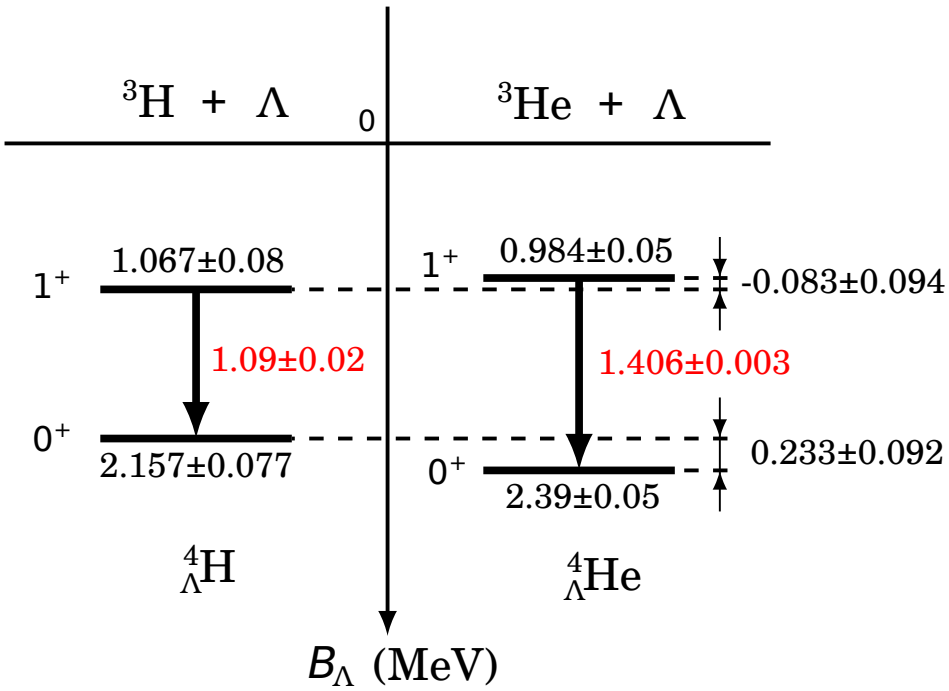
→ neutral $\Lambda\Lambda$n and $\Lambda\Lambda nn$ systems far from being bound

→ $^{4}_{\Lambda\Lambda}$H on the verge of binding (bound for ΛΛ strength equivalent to ΛN)

→ robust binding for $^{5}_{\Lambda\Lambda}$H hypernucleus $B_\Lambda(^{5}_{\Lambda\Lambda}\mathrm{H};\infty)=1.14\pm0.01^{+0.44}_{-0.26}$ MeV

Charge symmetry breaking in ${}^{4}_{\Lambda}\mathrm{H}/{}^{4}_{\Lambda}\mathrm{He}$

- $B_\Lambda({}^{4}_{\Lambda}\mathrm{H};\ 0^+)$ measurement at MAMI (Nucl. Phys. A, 954, 149, 2016)
- $B_\Lambda({}^{4}_{\Lambda}\mathrm{He};\ 0^+)$ measurement (emulsion) (Nucl. Phys. A 754, 3c, 2005)
- $E_\gamma({}^{4}_{\Lambda}\mathrm{H};\ 1^+ \to 0^+)$, $E_\gamma({}^{4}_{\Lambda}\mathrm{He};\ 1^+ \to 0^+)$ γ-ray energies (J-PARC) (Phys. Rev. Lett., 115, 222501, 2015)



Sizable CSB splitting in 0^+ ground states, while small in 1^+ excited states.

Theoretical works

- **R. H. Dalitz and F. von Hippel** (Phys. Lett. 10, 153, 1964)
  → CSB OPE contribution by allowing $\Lambda - \Sigma^0$ mixing in $SU(3)_f$

$$g_{\Lambda\Lambda\pi} = 2\mathcal{A}^{(0)}_{I=1} g_{\Lambda\Sigma\pi}; \qquad \mathcal{A}^{(0)}_{I=1} = -\frac{\langle \Sigma^0 | \, \delta M \, | \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} = -0.0148(6)$$

- **A. Gal** (Phys. Let. B 744, 352, 2015)
  → generalization of DvH

  $\langle N\Lambda | \, V^{CSB}_{\Lambda N} \, | N\Lambda \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \tau_{N_z} \langle N\Lambda | \, V \, | N\Sigma \rangle$

  → $\Delta B_\Lambda(0^+_{\mathrm{g.s.}}) \approx 240$ keV $\qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) \approx 35$ keV

- **D. Gazda and A. Gal** (PPRL116, 122501, 2016; NPA 954, 161, 2016)
  → generalized DvH; LO χEFT YN interaction; NSCM

  → $\Delta B_\Lambda(0^+_{\mathrm{g.s.}}) \approx 180 \pm 130$ keV $\qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) \approx -200 \pm 30$ keV

- **J. Haidenbauer et al.** (Few-Body Syst. 62, 105, 2021)
  → talk of Hoai Le

Hypernuclear CSB within $\not\pi$EFT

Charge Symmetric (CS) LO $\not\pi$EFT

Nuclear :

$$V_{NN} = \sum_S C^S_{NN}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}\mathbf{r}_{12}^2}$$

$$V_{NNN} = D^{1/2\ 1/2}_{\lambda}\mathcal{Q}^{1/2\ 1/2}\sum_{\text{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{12}^2+\mathbf{r}_{23}^2)}$$

Hypernuclear :

$$V_{\Lambda N} = \sum_S C^S_{\Lambda N}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}\mathbf{r}_{12}^2}$$

$$V_{\Lambda NN} = \sum_{IS} D^{IS}_{\Lambda NN}(\lambda)\mathcal{Q}^{IS}\sum_{\text{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{12}^2+\mathbf{r}_{23}^2)}$$

CSB in ΛN interaction

$$C^S_{\Lambda N}\mathcal{P}^S \rightarrow (C^S_{\Lambda p}\frac{1+\tau_{Nz}}{2} + C^S_{\Lambda n}\frac{1-\tau_{Nz}}{2})\mathcal{P}^S$$

$$C^S_{\Lambda N} = \frac{1}{2}(C^S_{\Lambda p} + C^S_{\Lambda n}), \qquad \delta C^S_{\Lambda N} = \frac{1}{2}(C^S_{\Lambda p} - C^S_{\Lambda n})$$

$$V_{\Lambda N} = \overbrace{\sum_S C^S_{\Lambda N}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}\mathbf{r}_{12}^2}}^{\text{part of LO CS } \not\pi\text{EFT}} + \overbrace{\sum_S \delta C^S_{\Lambda N}(\lambda)\mathcal{P}^S \tau_{N_z} e^{-\frac{\lambda^2}{4}\mathbf{r}_{12}^2}}^{\text{perturbative CSB}}$$

Fitting CSB LECs

→ perturbatively
→ two experimental constraints

$$\Delta B_\Lambda(0^+_{\mathrm{g.s.}}) = 233 \pm 92 \text{ keV} \qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) = -83 \pm 94 \text{ keV}$$

System of two linear equation for $\delta C^0_{\Lambda N}$ and $\delta C^1_{\Lambda N}$:

$$2\ \delta C^0_{\Lambda N}\ \Delta V^0_{\Lambda N;\ 0^+} + 2\ \delta C^1_{\Lambda N}\ \Delta V^1_{\Lambda N;\ 0^+} = \Delta B_\Lambda(0^+_{\mathrm{g.s.}})$$
$$2\ \delta C^0_{\Lambda N}\ \Delta V^0_{\Lambda N;\ 1^+} + 2\ \delta C^1_{\Lambda N}\ \Delta V^1_{\Lambda N;\ 1^+} = \Delta B_\Lambda(1^+_{\mathrm{exc.}})$$

where

$$\Delta V^S_{\Lambda N;\ J^\pi} = \underbrace{\langle {}^4_\Lambda\mathrm{H}; J^\pi | \tau_{Nz} \mathcal{P}_S \delta_\lambda(\Lambda N) | {}^4_\Lambda\mathrm{H}; J^\pi \rangle}_{\text{CS LO } \not\pi\text{EFT wave function}}$$

In-medium Λ isospin impurity

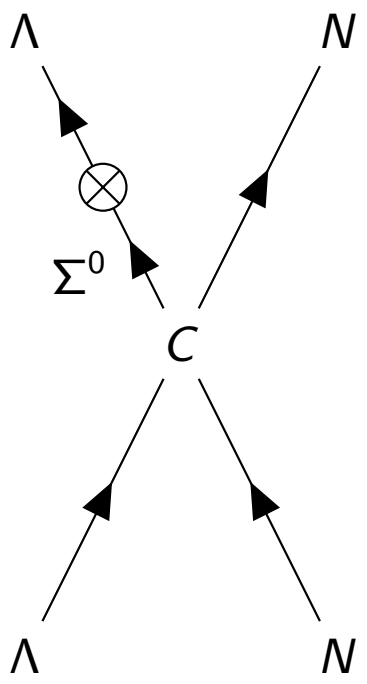
DvH ansatz :

(A. Gal, Phys. Lett. B 744, 352, 2015)

$$\langle \Lambda N | V_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \langle \Sigma N | V_{\rm CS} | \Lambda N \rangle \, \tau_{Nz}$$

$$\downarrow$$

$$\delta C^S_{\Lambda N} = -\frac{2}{\sqrt{3}} \mathcal{A}^S_{I=1} C^S_{\Lambda N;\Sigma N}$$



SU(3)$_f$ symmetry:

(C.B. Dover, H. Feshbach, Ann. Phys. (NY) 198, 321, 1990)

$$\left.\begin{aligned} C^0_{\Lambda N,\Sigma N} &= -3(C^0_{NN} - C^0_{\Lambda N}) \\ C^1_{\Lambda N,\Sigma N} &= \quad (C^1_{NN} - C^1_{\Lambda N}) \end{aligned}\right\} \longrightarrow \begin{aligned} -\mathcal{A}^0_{I=1} &= (\sqrt{3}/2)\delta C^0_{\Lambda N}/[-3(C^0_{NN} - C^0_{\Lambda N})] \\ -\mathcal{A}^1_{I=1} &= (\sqrt{3}/2)\delta C^1_{\Lambda N}/[(C^1_{NN} - C^1_{\Lambda N})] \end{aligned}$$

In-medium Λ isospin impurity

→ considering more precise $\Delta E_\gamma = 316 \pm 20$ keV

Relation between CSB LECs and ΔE_γ :

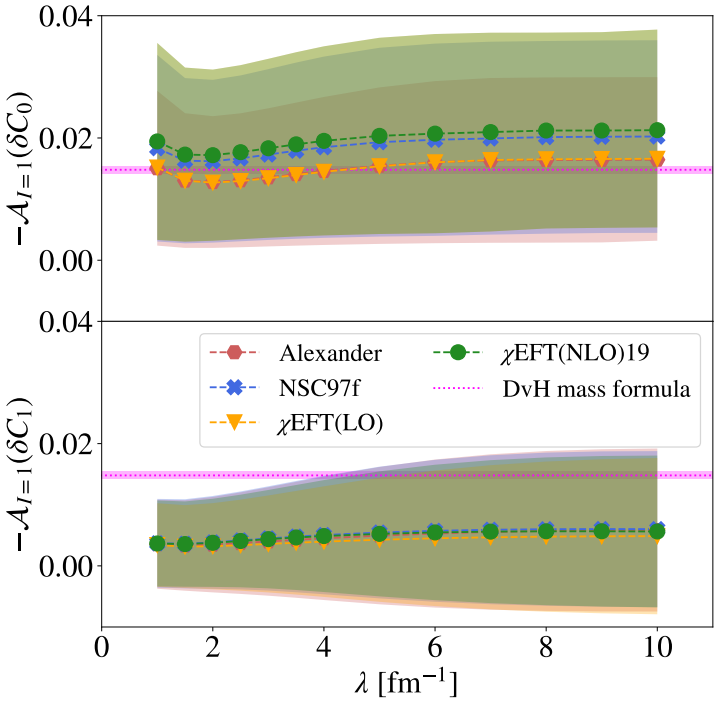
$$2\,\delta C^0_{\Lambda N}\left[\Delta V^0_{\Lambda N;\ 0^+} - \Delta V^0_{\Lambda N;\ 1^+}\right] + 2\,\delta C^1_{\Lambda N}\left[\Delta V^1_{\Lambda N;\ 0^+} - \Delta V^1_{\Lambda N;\ 1^+}\right] = \Delta E_\gamma$$

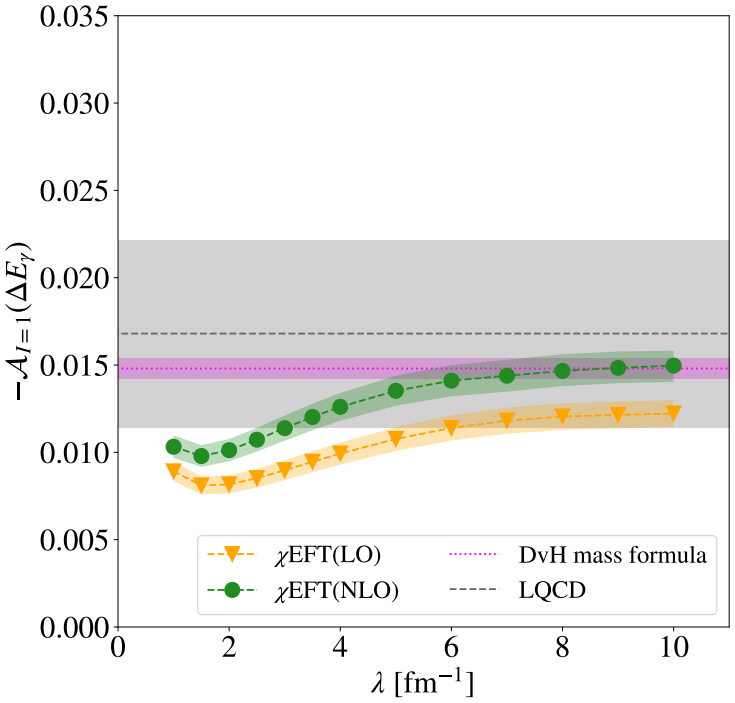
→ assuming DvH ansatz, SU(3)$_f$ symmetry, and $\mathcal{A}^0_{I=1} = \mathcal{A}^1_{I=1}$

Relation between $I = 1$ admixture amplitude and ΔE_γ :

$$-\mathcal{A}_{I=1} = \frac{\sqrt{3}}{2}\Delta E_\gamma \Big(-6(C^0_{NN} - C^0_{\Lambda N})[\Delta V^0_{\Lambda N;\ 0^+} - \Delta V^0_{\Lambda N;\ 1^+}] + 2\,(C^1_{NN} - C^1_{\Lambda N})[\Delta V^1_{\Lambda N;\ 0^+} - \Delta V^1_{\Lambda N;\ 1^+}]\Big)^{-1}$$

In-medium Λ isospin impurity





Method/Input	B	$-\mathcal{A}_{I=1}$
SU(3)$_{\mathrm{f}}$ (Phys. Lett 10, 153, 1964)	1	0.0148 ± 0.0006
LQCD (Phys. Rev. D 101, 034517, 2020)	1	0.0168 ± 0.0054
$\not\pi$EFT (LO)/[χEFT(LO); $\Lambda \to \infty$]	4	0.0139 ± 0.0013
$\not\pi$EFT (LO)/[χEFT(NLO); $\Lambda \to \infty$]	4	0.0168 ± 0.0014

Conclusions

$\rightarrow$ comprehensive study of $\Lambda\mathrm{nn}$, ${}^{3}_{\Lambda}\mathrm{H}^{*}$, ${}^{4}_{\Lambda\Lambda}\mathrm{H}$, ${}^{5}_{\Lambda\Lambda}\mathrm{H}$ systems and CSB within LO $\not\pi$EFT

Hypernuclear trios $\Lambda\mathrm{nn}(1/2^{+})$ & ${}^{3}_{\Lambda}\mathrm{H}^{*}(3/2^{+})$

- question of experimentally observable $\Lambda\mathrm{nn}$ resonance (physical Riemann sheet)
- ${}^{3}_{\Lambda}\mathrm{H}^{*}(3/2^{+})$ virtual state

${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$ & ${}^{5}_{\Lambda\Lambda}\mathrm{H}(1/2^{+})$

- ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$ on the verge of binding
- ${}^{5}_{\Lambda\Lambda}\mathrm{H}$ particle stable taking into account both theoretical and experimental uncertainties

Charge symmetry breaking in ${}^{4}_{\Lambda}\mathrm{H}/{}^{4}_{\Lambda}\mathrm{He}$

- extraction of in-medium Λ isospin impurity $\mathcal{A}_{I=1}$; all cases in agreement with free-space LQCD prediction and in most cases with free-space DvH value
- using $\mathcal{A}^{(0)}_{I=1}$ DvH value the procedure can be applied in reverse thus predicting experimental CSB in ${}^{4}_{\Lambda}\mathrm{H}/{}^{4}_{\Lambda}\mathrm{He}$