Electron data for neutrino scattering cross sections

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Neutrino oscillations

\[ \mathcal{P}_{\alpha \rightarrow \beta} = \sin^2(2\theta)\sin^2 \left( \frac{\Delta m_{12}^2 L}{4E} \right) \]

source: www.dunescience.org
Aims & challenges

- very few events
- unknown $\nu$ energy
- scattering on nuclei: $^{12}\text{C}, ^{16}\text{O}, ^{40}\text{Ar}$...

- Measure CP-violation
- BSM physics

source: euscience.org.uk
Neutrino energy is reconstructed in each event

\[ \nu_{\mu} \rightarrow \mu^{-} + \pi^{+} + n \]
Nuclear response

\[ J_\mu = (\rho, \vec{j}) \mid \Psi \rangle \]

\[ \gamma, W^\pm, Z^0 \]

\[ \sigma \propto L^{\mu\nu} R_{\mu\nu} \]

lepton nuclear

tensor responses

\[ R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi \mid J_\mu^\dagger(q) \mid \Psi_f \rangle \langle \Psi_f \mid J_\nu(q) \mid \Psi \rangle \delta(E_0 + \omega - E_f) \]

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Electrons for neutrinos

\[
\frac{d\sigma}{d\omega dq} \bigg|_{\nu/\bar{\nu}} = \sigma_0 \left( v_{CC} R_{CC} + v_{CL} R_{CL} + v_{LL} R_{LL} + v_T R_T \pm v_{T'} R_{T'} \right)
\]

\[
\frac{d\sigma}{d\omega dq} \bigg|_{e} = \sigma_M \left( v_L R_L + v_T R_T \right)
\]

✓ much more precise data

✓ we can get access to \( R_L \) and \( R_T \) separately (Rosenbluth separation)

✓ experimental programs of electron scattering in JLab, MAMI, MESA
Ab initio nuclear theory

- Nuclear responses
- Spectral functions
- Optical potentials
...

Neutrinos challenge ab initio nuclear theory
Controllable approximations within ab initial nuclear theory
Ab initio nuclear theory for neutrinos

Nuclear Hamiltonian

\[ \mathcal{H} | \Psi \rangle = E | \Psi \rangle \]

\[
\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

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Ab initio nuclear theory for neutrinos

Nuclear Hamiltonian

\[ \mathcal{H} \left| \Psi \right\rangle = E \left| \Psi \right\rangle \]

Electroweak currents

\[ J^\mu = (\rho, \vec{j}) \]

Many-body method

\[ \mathcal{A} = \langle \Psi_m | J_\mu | \Psi_n \rangle \]
Coupled cluster method

Reference state (Hartree-Fock): \(|\Psi\rangle\)

Include correlations through \(e^T\) operator

\[
e^{-T\mathcal{H}e^T}|\Psi\rangle \equiv \tilde{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle
\]

Expansion: 
\[
T = \sum t^i_a a^+_a a^+_i + \sum t^{ij}_{ab} a^+_a a^+_b a^+_i a^+_j + \ldots
\]

\(\rightarrow\) coefficients obtained through coupled cluster equations
Coupled cluster method

✓ Controlled approximation through truncation in $T$

✓ Polynomial scaling with $A$ (predictions for $^{100}\text{Sn}$, $^{208}\text{Pb}$)

✓ Works most efficiently for doubly magic nuclei
Quasielastic response

- Momentum transfer ~hundreds MeV
- Upper limit for ab initio methods
- Important mechanism for T2HK, DUNE
- Role of final state interactions
- Role of 1-body and 2-body currents

First step: analyse the longitudinal response

\[ \frac{d\sigma}{d\omega dq} \bigg|_e = \sigma_M \left( \nu_L R_L + \nu_T R_T \right) \]

Charge operator \( \hat{\rho}(q) = \sum_{j=1}^{Z} e^{iqz_j} \)
Uncertainty band: inversion of LIT

\[ R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f) \]
Longitudinal response $^{40}\text{Ca}$

- Coupled cluster singles & doubles
- Two different chiral Hamiltonians
- Uncertainty from LIT inversion

First ab-initio results for many-body system of 40 nucleons

JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501
This allows to predict electron-nucleus cross-section

\[ \frac{d\sigma}{d\omega dq} \bigg|_e = \sigma_M \left( \nu_L R_L + \nu_T R_T \right) \]

- 2-body currents important for \(^4\text{He}\)
- more correlations needed?
- 2-body currents strength depends on nucleus?

\[ d\sigma d\omega dq \bigg|_e = \sigma_M \left( \nu_L R_L + \nu_T R_T \right) \]

Currently only 1-body current
Low/high energies

\[ \hat{H} |\psi_A\rangle = E |\psi_A\rangle \]
Many-body problem

Electroweak responses

\[ \langle \psi_f | \hat{j} | \psi_A\rangle \]

Impulse Approximation

Spectral function

Probability density of finding nucleon \((E, p)\) in ground state nucleus

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Spectral functions
from Coupled Cluster

\[ \sigma \propto |\mathcal{M}|^2 S(E, p) \]

Factorized interaction vertex
(relativistic, pion production...)

Spectral function - nuclear information

growing \( q \) momentum transfer \( \rightarrow \) final state interactions play minor role

Spectral function for neutrinos

- Comparison with T2K long baseline $\nu$ oscillation experiment
- CC0$\pi$ events
- Spectral function implemented into NuWro Monte Carlo generator

$\nu_\mu + ^{16}O \rightarrow \mu^- + X$


JES et al, in preparation (2022)
Outlook

- First results from the coupled cluster theory: on the way to obtain cross-section for neutrino scattering on medium-mass nuclei

- Role of 2-body currents and FSI for medium-mass nuclei

- Spectral functions (within Impulse Approximation):
  - Relativistic regime
  - Semi-inclusive processes
  - Further steps: 2-body spectral functions, accounting for FSI
Thank you for attention
Lorentz Integral Transform (LIT)

\[ S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^+ K(\mathcal{H} - E_0, \sigma) J_\nu | \Psi \rangle \]

Integral transform

\[ R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^+ | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f) \]

continuum spectrum

Lorentzian kernel:
\[ K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2} \]

\( S_{\mu\nu} \) has to be inverted to get access to \( R_{\mu\nu} \)
Aims & challenges

DUNE

T2HK

Position of the oscillation peak depends on energy reconstruction

DUNE aims at uncertainties < 1% meaning O(25 MeV) precision of energy reconstruction

Systematic errors should be small since statistics will be high.
Final state interactions

How to account for the FSI? Optical potential for the outgoing nucleon

JES et al, in preparation (2022)