New solution in the Non-linear BK equation and description of the experimental data

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Outline

- Current problem in theoretical description in high energy Scattering.
- Dipole scattering amplitudes: phenomenological input.
- Leading twist approximation: for nonlinear evolution in NLO
- Saturation scale and b dependence.
- Results of the fit: deep inelastic structure function F_2
- Conclusions.

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Proton Structure functions and DIS cross section

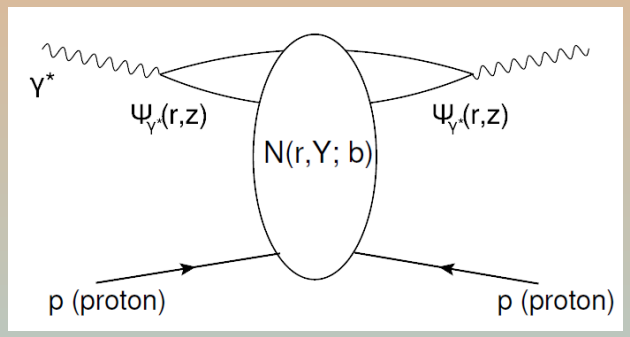
- Proton structure Function: F_2 and the Longitudinal F_L(Q, x)
- $Y = ln\ (1/x) \quad = ln\ s$

$$F_2(Q,Y) = \frac{Q^2}{4\pi^2\alpha_{\rm e.m.}}\{\sigma_T + \sigma_L\}$$

$$F_2^{cc}(Q,Y) = \frac{Q^2}{4\pi^2\alpha_{\rm e.m.}}\{\sigma_T^{cc} + \sigma_L^{cc}\}$$

$$F_L(Q,Y) = \frac{Q^2}{4\pi^2\alpha_{\rm e.m.}}\sigma_L$$

- Total cross section for DIS σ_T and σ_L



$$\sigma_{T,L} = 2\int d^2b\ N_{T,L}(Q,Y,;b)$$

$$N(Q,Y;b) = \int \frac{d^2r}{4\pi}\int_0^1 dz\,|\Psi_{\gamma^*}(Q,r,z)|^2\,N(r,Y;b)$$

- $\Psi_{L/T}^{\gamma*}(Q, r, z, m_f)$ Light front wave function
- $N(Q,Y;b)$, $N(r,Y;b)$, in the dipole Picture: dipole $\bar{q}q$-proton scattering Amplitude
- r dipole tranverse-size, b impact parameter
- z fraction of the light-front momenta of the virtual photon carried by the quark
- m_f mass of the quarks

- fit: deep inelastic structure function F_2

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$N(r,Y;b)$ in the dipole Picture: Scattering Amplitude

Solution: JIMWLK (Iancu, Jalilian-Marian, Kovner, Leonidov, McLerrran and Weigert) and large Nc limit BK equation (Balinky-Kovchegov)

$$\frac{\partial}{\partial Y} N\left(\boldsymbol{x}_{10}, \boldsymbol{b}, Y; R\right) = \bar{\alpha}_S \int \frac{d^2 \boldsymbol{x}_2}{2\,\pi} K\left(\boldsymbol{x}_{02}, \boldsymbol{x}_{12}; \boldsymbol{x}_{10}\right) \left(N\left(\boldsymbol{x}_{12}, \boldsymbol{b} - \frac{1}{2}\boldsymbol{x}_{20}, Y; R\right) + N\left(\boldsymbol{x}_{20}, \boldsymbol{b} - \frac{1}{2}\boldsymbol{x}_{12}, Y; R\right) - N\left(\boldsymbol{x}_{10}, \boldsymbol{b}, Y; R\right) - N\left(\boldsymbol{x}_{12}, \boldsymbol{b} - \frac{1}{2}\boldsymbol{x}_{20}, Y; R\right) N\left(\boldsymbol{x}_{20}, \boldsymbol{b} - \frac{1}{2}\boldsymbol{x}_{12}, Y; R\right) \right)$$

$$K^{\mathrm{LO}}\left(\boldsymbol{x}_{02}, \boldsymbol{x}_{12}; \boldsymbol{x}_{10}\right) = \frac{x_{10}^2}{x_{02}^2\, x_{12}^2}$$

- K^{LO} BFKL Kernel in the LO
  Balinsky, Fadin, Kuraev and Lipatov

- BFKL Kernel in the LO

$$N\left(r, b, Y; R\right) = \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{d\gamma}{2\,\pi\, i} e^{\omega(\bar{\alpha}_S, \gamma)\, Y}\, \phi_\gamma\left(\boldsymbol{r}, \boldsymbol{R}, \boldsymbol{b}\right)\, \phi_{\mathrm{in}}\left(\gamma, R\right)$$

$$\omega_{\mathrm{LO}}\left(\bar{\alpha}_S, \gamma\right) = \bar{\alpha}_S\, \chi^{LO}\left(\gamma\right) = \bar{\alpha}_S\left(2\psi\left(1\right) - \psi\left(\gamma\right) - \psi\left(1-\gamma\right)\right)$$

BK equation and NLO corrections

- NLO for the eigenvalues w_{NLO} **Fadin-Lipatov arXiv 9802290 Ciafaloni-Camici arXiv 9803389**

$$\omega_{\rm NLO}\left(\bar{\alpha}_S,\gamma\right) = \bar{\alpha}_S\,\chi^{LO}\left(\gamma\right) + \bar{\alpha}_S^2\,\chi^{NLO}\left(\gamma\right)$$

$\gamma \to 1$ *is singular*

$$\omega_{\rm NLO}\left(\bar{\alpha}_S,\gamma\right) = \bar{\alpha}_S\left(\chi_0\left(\omega_{\rm NLO},\gamma\right) + \omega_{\rm NLO}\frac{\chi_1\left(\omega_{\rm NLO},\gamma\right)}{\chi_0\left(\omega_{\rm NLO},\gamma\right)}\right)$$

resumation in high order correction

Ciafaloni, Colferai, Salam Stasto arXiv 0307188

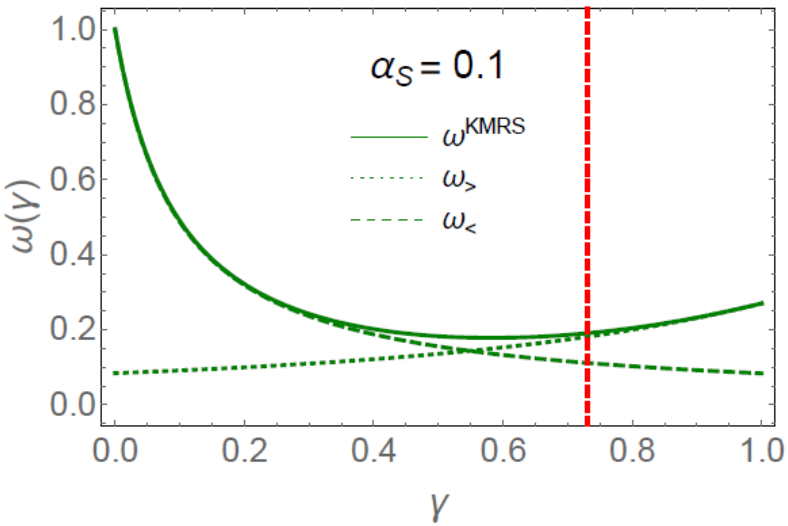
$$\chi_0\left(\omega,\gamma\right) = \chi^{LO}\left(\gamma\right) - \frac{1}{1-\gamma} + \frac{1}{1-\gamma+\omega}$$

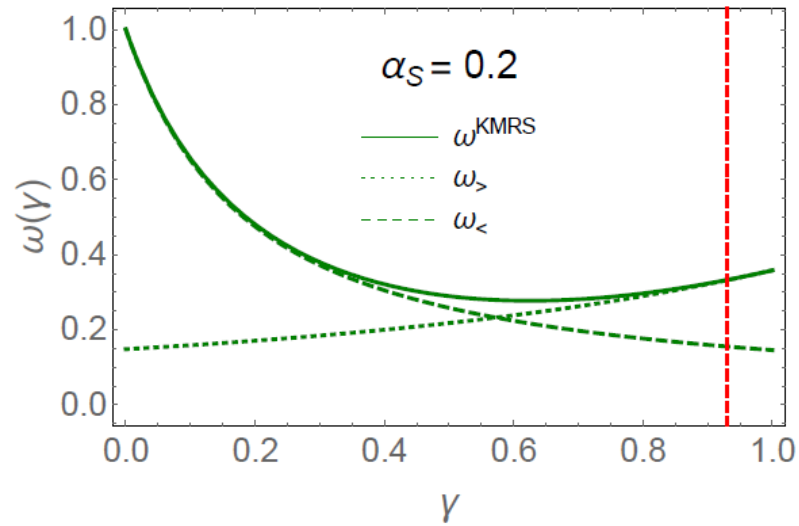
$$\chi_1\left(\omega,\gamma\right) = \chi^{NLO}\left(\gamma\right) + F\left(\frac{1}{1-\gamma} - \frac{1}{1-\gamma+\omega}\right) + \frac{A_T\left(\omega\right) - A_T\left(0\right)}{\gamma^2} + \frac{A_T\left(\omega\right) - b}{\left(1-\gamma+\omega\right)^2} - \frac{A_T\left(0\right) - b}{\left(1-\gamma\right)^2}$$

Khoze, Martin, Rymskim and Stirling: arXiv 0406135

$$\omega^{\rm KMRS} = \bar{\alpha}_S\left(1-\omega^{\rm KMRS}\right)\left(\frac{1}{\gamma} + \frac{1}{1-\gamma+\omega^{\rm KMRS}} + \underbrace{\left(2\psi(1) - \psi\left(2-\gamma\right) - \psi\left(1+\gamma\right)\right)}_{\text{high twist contributions}}\right)$$

Effects of the NLO in BFKL





$$\omega \equiv \omega_> = \frac{1}{2}\left(-(1-\gamma+\bar{\alpha}_S) + \sqrt{4\,\bar{\alpha}_S + (1-\gamma+\bar{\alpha}_S)^2}\right) \qquad \gamma \to 1$$

$$\gamma \to 0 \qquad \gamma = \frac{\bar{\alpha}_S}{1+\bar{\alpha}_S}\,\frac{1}{\omega}$$

$$\bar{\gamma} = 1 - \gamma_{cr} = \frac{1}{2} + 2\bar{\alpha}_S$$

$\gamma \to 1$ BFKL

$\gamma \to 0$ BK saturation region

Resummed anomalous dimensions in NLO

$$N(\xi, Y; R) = \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{d\gamma}{2\pi i} e^{\omega(\bar{\alpha}_S, \gamma) Y + \gamma \xi} \phi_{\rm in}(\gamma, R)$$

$$\xi = \ln\left(r^2 Q_s^2 (Y = 0; \boldsymbol{b}, \boldsymbol{R})\right).$$

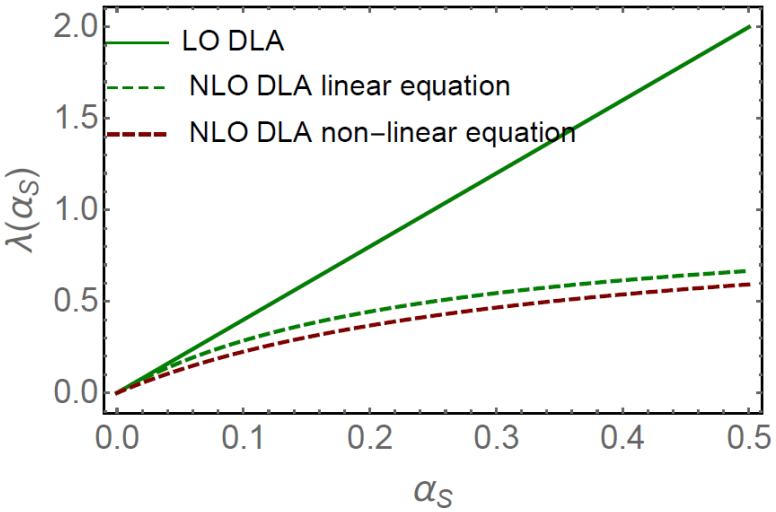


Fig. 5-a

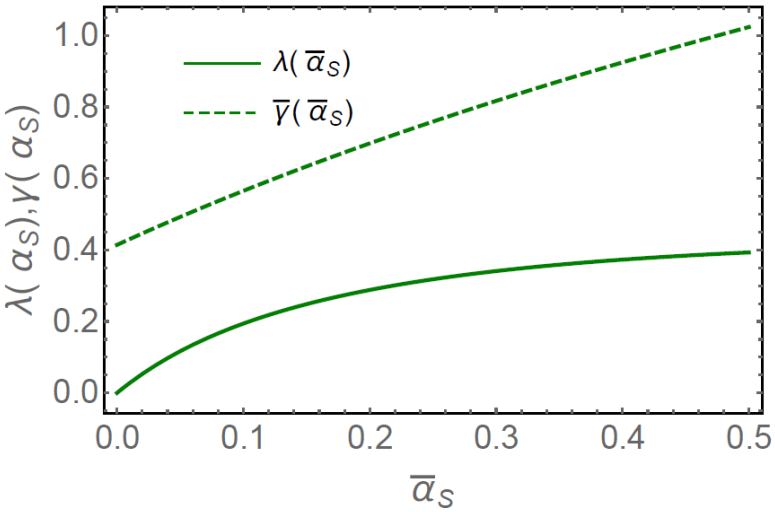


Fig. 5-b

$$\lambda = \frac{1}{2}\bar{\alpha}_S \frac{1 + \sqrt{2}}{\sqrt{2} - 1 + \frac{1}{2}\bar{\alpha}_S\left(1 + \sqrt{2}\right)}$$

$$\bar{\gamma} = \sqrt{2} - 1 + \frac{1}{2}\bar{\alpha}_S\left(1 + \sqrt{2}\right)$$

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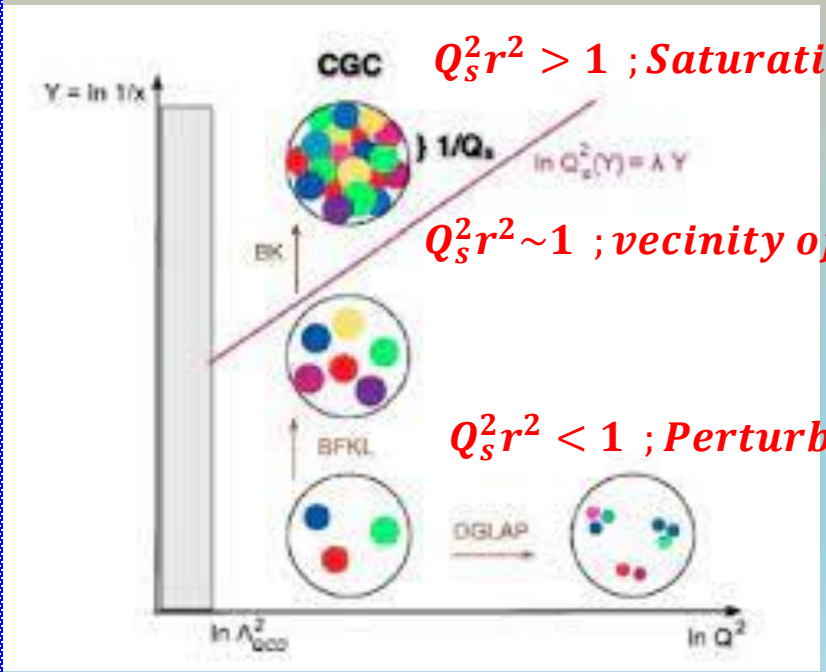
Scattering amplitude *N(r,Y; b)*

$$\sigma_{T,L} = 2\int d^2b\, N_{T,L}\left(Q,Y,;b\right)$$

$$N\left(Q,Y;b\right) = \int \frac{d^2r}{4\pi}\int_0^1 dz\,|\Psi_{\gamma^*}\left(Q,r,z\right)|^2\, N\left(r,Y;b\right)$$

- $\Psi_{L/T}^{\gamma^*}$ (Q, r, z, m) Light front wave function
- *N(r,Y; b)* --> saturation condition
- Three differents Kinematics region



$Q_s^2 r^2 > 1$; *Saturation region BK Equation*

$Q_s^2 r^2 \sim 1$; *vecinity of Saturation region*

$Q_s^2 r^2 < 1$; *Perturbative region BFKL Equation*

Scattering Amplitud

- $Q_s^2 r^2 < 1$ **(Perturbative region)**

$$\frac{\partial^2}{\partial Y\,\partial \xi'} N(\xi', Y) + \frac{\partial}{\partial Y} N(\xi', Y) = \frac{1}{2}\bar{\alpha}_S\, N(\xi', Y) - \frac{1}{2}\bar{\alpha}_S \frac{\partial}{\partial \xi'} N(\xi', Y)$$

- $Q_s^2 r^2 \sim 1$ **(Vicinity of saturation scale)**

$$N(r, \eta; b) = N_0 \left(Q_s^2(Y, b)\, r^2\right)^{\bar{\gamma}}$$

- $Q_s^2 r^2 \gg 1$ **(Saturation region)**

$$N(z) = a\Big(1 - \exp(-\Omega(z))\Big) + (1 - a)\frac{\Omega(z)}{1 + \Omega(z)}$$

Scattering Amplitud

$Q_s^2 r^2 \gg 1$ (Saturation region)

$$N(z) = a\left(1 - \exp(-\Omega(z))\right) + (1-a)\frac{\Omega(z)}{1+\Omega(z)}$$

$$\Omega = \Omega_0\left\{\cosh\left(\sqrt{\sigma}\,(\xi_s+\xi)\right) + \frac{\bar{\gamma}}{\sqrt{\sigma}}\sinh\left(\sqrt{\sigma}\,(\xi_s+\xi)\right)\right\}$$

$$\xi_s = \ln\left(Q_s^2(Y)/Q_s^2(Y=0;\boldsymbol{b},\boldsymbol{R})\right) = \lambda Y \qquad \xi = \ln\left(r^2 Q_0^2\right)$$

For z >>1 Levin-Tuchin Solution

$$N(Y,\xi) = 1 - \exp(-\Omega(Y,\xi))$$

$$\Omega(z) = \frac{\sigma}{2}z^2 + \text{Const}$$

$$N(z) = 1 - \text{Const}\, e^{-\frac{\sigma}{2}z^2}$$

Geometric Scaling and Saturation

- $Q_s^2 r^2 \sim 1$ **(Vicinity of saturation scale)**

$$N(r,\eta;b) = N_0 \left(Q_s^2(Y,b)\, r^2\right)^{\bar{\gamma}}$$

N→ No parameter free

$$N(\xi,Y;R) = \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\gamma}{2\pi i} e^{\omega(\bar{\alpha}_S,\gamma)Y + \gamma\xi}\, \phi_{\rm in}(\gamma,R)$$

$$\xi = \ln\left(r^2 Q_s^2(Y=0;\boldsymbol{b},\boldsymbol{R})\right).$$

- Saturation condition → linear BFKL equation

$$v_{\rm group} = \frac{d\omega(\gamma)}{d\gamma} = v_{\rm phase} = \frac{\omega(\gamma)}{\gamma}$$

- Saddle Point condition with saturation $Q_s^2 r^2 \sim 1$

- $\frac{\omega(\bar{\gamma})}{\bar{\gamma}} Y - \xi_s = 0 \rightarrow \xi_s = \ln(Q_S^2/Q_0^2) = \frac{\omega(\bar{\gamma})}{\bar{\gamma}} Y \rightarrow Q_S^2(\mathrm{Y},\mathrm{b}) = Q_S^2(0,\mathrm{b}) Exp\left(\frac{\omega(\bar{\gamma})}{\bar{\gamma}} Y\right)$

$Q_S^2(\mathrm{Y},\mathrm{b}) = Q_S^2(0,b) Exp(\lambda(\alpha)\, Y)$ Qo, $\lambda(\alpha)$ free parameters

Two approaches: saturation scale and impact parameter impact

A. Kovner and U.A. Wiedemann, *Phys. Rev.* D **66** (2002) 051502; *Phys. Rev.* D **66** (2002) 034031; *Phys. Lett.* B **551** (2003) 311.

- $Q_S^2(\mathrm{Y},\mathrm{b}) = Q_S^2(0,b)Exp(\lambda\, Y)$

- **We introduced exponential suppression which follows from Froissart theorem.**

$$Q_s^{(1)2}(Y,b) = Q_s^{(1)2}(Y=0,b=0)\; e^{-m\,b}\; e^{\lambda\,Y} = Q_0^2\, e^{-m\,b}\, e^{\lambda\,Y}$$

m, $\lambda(\alpha)$ free parameters

- **Different approach summing pion loops in Qs. (Gotsman-Levin 2020)**

$$2.\quad Q_s^{(2)2}(Y,b) = Q_s^{(2)2}(Y=0,b=0)\; e^{-\frac{3}{4}\mathcal{Z}}\; e^{\lambda\,Y} = Q_0^2\; e^{\lambda\,Y}\; e^{-\frac{3}{4}\mathcal{Z}}$$

$$\mathcal{Z} = \left(\frac{b^4}{4\alpha'^2_{\mathrm{eff}}\,Y}\right)^{1/3}$$

α'_{eff} free parameters

Restriction of kinematics region and parameters of our fit.

$$0.85\,GeV^2 \leq Q^2 \leq 27\,GeV^2$$

$$x \leq 0.01$$

	Dipole amplitude					Wave function				$\chi^2/d.o.f.$
Set	$\bar{\alpha}_S$	N_0	$Q_0^2(GeV^2)$	m(GeV)	$\alpha_{eff}(GeV^{-2})$	m_u(MeV)	m_d(MeV)	m_s(MeV)	m_c(GeV)	$0.85 \leq Q^2 \leq 27\ GeV^2$
1	0.091	0.236	0.998	0.612	n/a	140	140	140	1.4	124.9/133 = 0.93
2	0.20	0.25	1.00	0.551	n/a	140	140	140	1.4	61.99/66 =0.93
3	0.096	0.448	0.921	0.840	n/a	2.3	4.8	95	1.4	117.2/133 = 0.88
4	0.20	0.343	0.999	1.300	n/a	2.3	4.8	95	1.4	91.74/66= 1.39
5	0.038	0.599	1.284	n/a	0.216	140	140	140	1.4	175/133 = 1.31
6	0.043	0.565	1.429	n/a	0.149	2.3	4.8	95	1.4	143.7/133=1.08

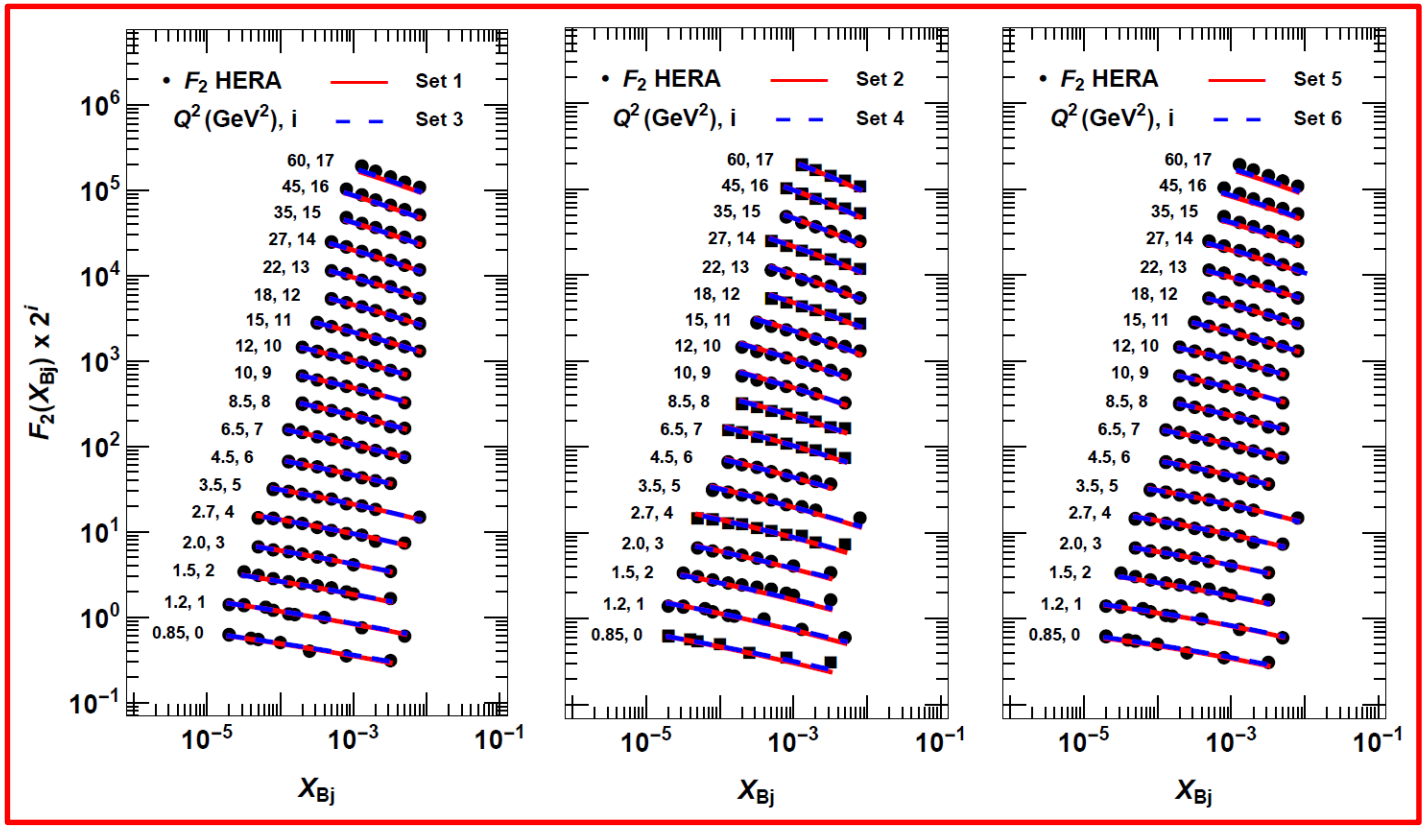
Quark mass:

Set 1 and 2 is with ligh quark masses 140 MeV

Set 3 and for is for currents masses

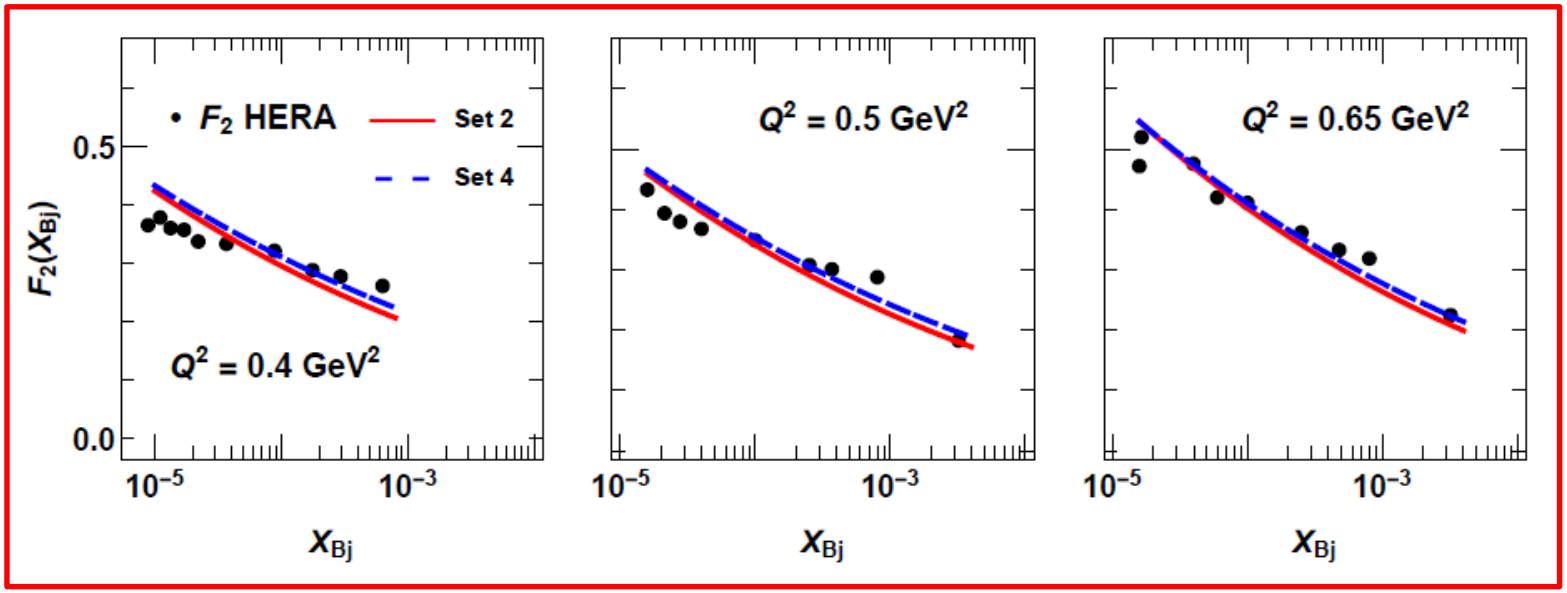
Mass of the c quark 1.4 GeV and we need to modify $x \rightarrow x_c = \left(1 + 4\,\frac{m_c^2}{Q^2}\right)x$

Confronting the model with experimental data: F_2 vs x_{B_j}



[arXiv:0911.0884 [hep-ex]]

Confronting the model with experimental data: F_2 vs x_{B_j}, (at low Q^2)



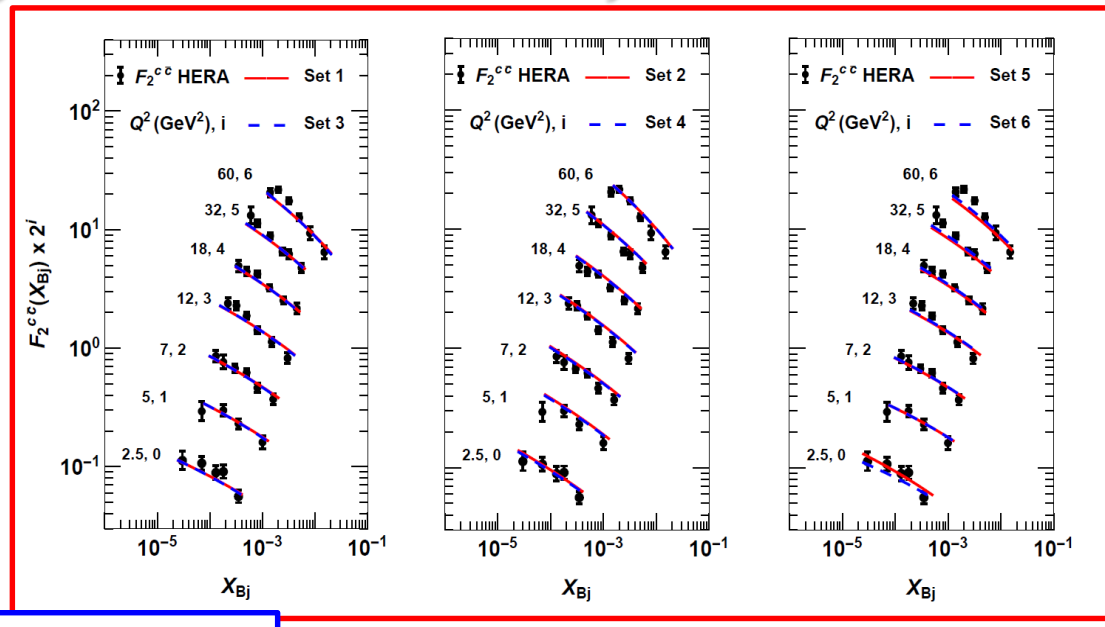
Combined Measurement and QCD Analysis of the Inclusive e ± p Scattering Cross Sections at HERA

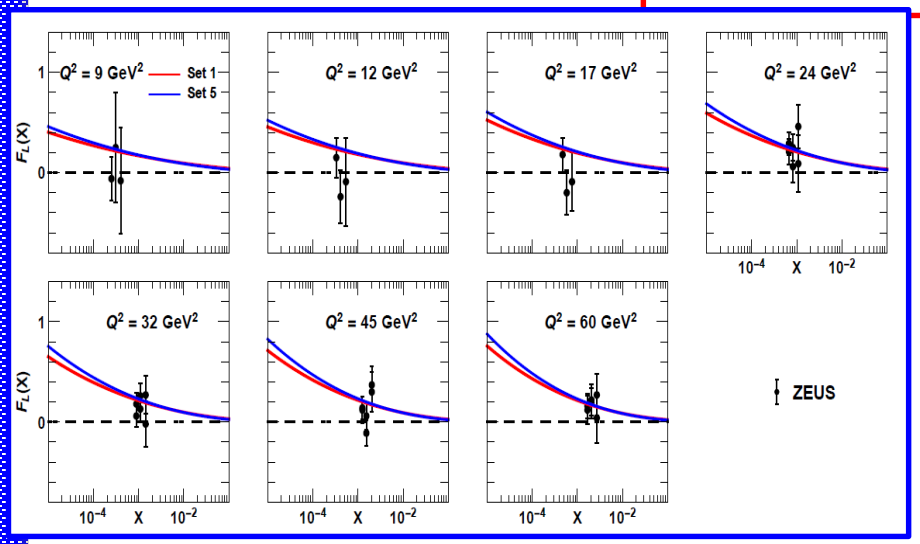
[arXiv:0911.0884 [hep-ex]]

Confronting the model with experimental data: F_L vs x_{B_j} - $F_2^{c\bar{c}}$ vs x_{B_j}

arXiv:1211.1182 [hep-ex]]

Combination and QCD Analysis of Charm Production Cross Section Measurements in Deep-Inelastic ep Scattering at HERA





arXiv:0911.0884 [hep-ex]]

Summary

- We treated the non-linear evolution equation including NLO corrections.
- We have found numerical solution and analytical approximation to BK equation.
- Two approaches in Saturation momentum, in accordance with Froissart theorem.
- Our fits describe quite well the experimental data of DIS.

THANK YOU