Photon and charmonium production in electron-positron annihilation at $\sqrt{s}$=10.6 GeV. in framework of Bethe-Salpeter equation

Shashank Bhatnagar

Chandigarh University, India

In collaboration with

Vaishali Guleria (Chandigarh University)

9th International Conference on Quarks and Nuclear Physics (Sept. 5-9, 2022)

Outline:

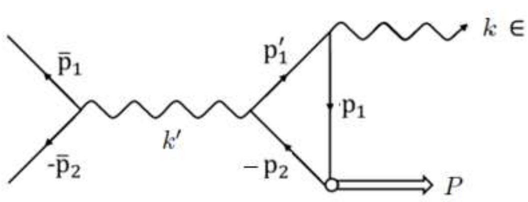
- Hadron production in electron-positron annihilation
- Bethe-Salpeter dynamics of composite hadron
- $e^+e^- \to \gamma^* \to \gamma\chi_{c0,c1}$(nP)
- $e^+e^- \to \gamma^* \to \gamma\eta_c$(nS)
- Summary

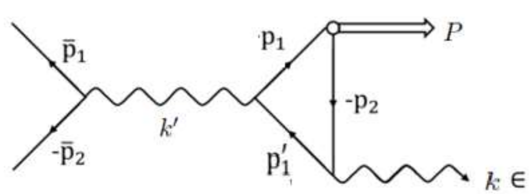
Hadron production in electron-positron annihilation

- There is significant discrepancy between the experimental measurements by BABAR and Belle collaborations of total cross sections of e^+e^--> $J/\psi\ \eta_c$; $J/\psi\ \chi_{cJ}$, and corresponding NRQCD predictions at energies $\sqrt{s}$=10.6 GeV.
- Measurement of cross sections for e^+e^--> $J/\psi\ \eta_c$ carried out by Belle [PRL89(2002), PRD70(2004)]and BABAR [PRD72 (2005)] collaborations has led to rapid progress in theoretical description of heavy-quarkonium through NRQCD. Cross section measured by Belle was greater than the cross section calculated by Braaten and lee [PRD67(2003); PRD72(2005)] by an order of magnitude.
- Incorporation of NLO corrections to strong coupling constant α_s, were not large enough to bring theoretical results close to the Belle data. But incorporation of relativistic corrections by Bodwin, lee, Yu [PRD77,(2008)], besides the NLO QCD corrections was found to resolve the discrepancy within errors.
- Also interference between QED and QCD tree-level diagrams also gave significant contributions[PRD98, 094001 (2018)]
- Also found that discrepancy between experimental results and theoretical predictions could be resolved by taking into account intrinsic motion of quarks inside hadron in Light-cone expansion method [Braguta et al.,PRD72(2005)].
- Very recently measurements were made on the cross sections for the processes: $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\ \eta_c$ (nS), and $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\chi_{c0,c1}$ at $\sqrt{s}$=10.52 GeV., 10.58 GeV., and 10.867GeV. at Belle [S.Jia et al., PRD 98(2018)].
- Also recent measurements on $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\chi_{c0,c1,c2}$ at $\sqrt{s}$=4- 4.6 GeV using integrated luminosity, 19.3 fb^{-1} reported by BESIII reported no significant signal for these processes. [Ablikin et al., PRD104 (2021)]
- Calculations of this process have been done using: NRQCD [Chung, Lee, Yu[PRD78(2008)], Light-cone[Braguta,PRD82 (2010)] , Bethe-Salpeter equation [Chang, Wang, Wu, arxiv:1005.4723 [hep-ph]]. Number of studies carried out.

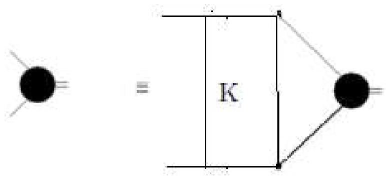
Present work*: [S.Bhatnagar, V.Guleria, arxiv:2206.02229[hep-ph]]

- Calculated the cross sections for production of ground and excited charmonium states in $e^{+}e^{-} \rightarrow \gamma^{*} \rightarrow \gamma\ H,$ at $\sqrt{s}$=10.6 GeV. through leading order (LO) (tree-level) diagrams, $\sim O(\alpha_{em}^{3}\,\alpha_{s}^{0})$ which proceed through exchange of a virtual photon in the framework of 4x4 Bethe-Salpeter equation (BSE) in colour singlet channel.
- H is a heavy quarkonium with C=+1, and can be S-wave spin singlet states, such as: η_c (nS), η_b (nS), or P-wave spin triplet states, such as χ_{cJ} or χ_{bj}(J=0,1,2).





Features of BSE



$\psi(P,q) = S_F(p_1)\Gamma(q)S_F(\text{-}p_2)$ (4D BS wave function)

$$\psi(P,q) = S_F(p_1) i \int \frac{d^4q'}{(2\pi)^4} K(q,q')\psi(P,q')S_F(-p_2)$$

- BSE is a dynamical equation based approach, with a firm base in field theory, and is a conventional approach in dealing with relativistic bound state problems.
- Interaction kernel, K involves summation over all possible two-particle irreducible interactions between two constituents forming bound state.
- In practical calculations, we use Ladder Approximation- where kernel involves summation over an infinite ladder of quantum exchanges between two interacting particles forming bound state.
- BSE is quite general, and provides effective description of bound quark- anti-quark systems, through a suitable choice of input kernel for confinement.

Advantages of BSE approach:

- Fully relativistic approach, that incorporates the relativistic effect of quark spins and can also describe internal motion of constituent quarks within the hadron in a relativistically covariant manner.
- Due to above features, it provides a realistic description for analyzing hadrons as composite objects.
- It can be applied to study processes over a wide range of energies, and with little number of input parameters!!

BSE under Covariant Instantaneous Ansatz

- A quark-anti quark bound state system can be described by a Bethe-Salpeter equation (BSE),

$$S_F^{-1}(p_1)\Psi(P,q)S_F^{-1}(-p_2) = i\int \frac{d^4q'}{(2\pi)^4}K(q,q')\Psi(P,q'),$$

4D BSE → 3D form using Covariant Instantaneous Ansatz, $K(q,q') = K(\hat{q},\hat{q}')$;

- $\hat{q}_\mu$=q_μ - $\frac{q.P}{p^2}P_\mu$ (Transverse to P), with $\hat{q}$.P=0, and σP_μ= $\frac{q.P}{p^2}P_\mu$ (Longitudinal to P),
- with 4D volume element, $d^4q = d^3\hat{q}Md\sigma$

$$\Psi(P,q) = S_1(p_1)\Gamma(\hat{q})S_2(-p_2), \qquad \Gamma(\hat{q}) = \int \frac{d^3\hat{q}'}{(2\pi)^3}K(\hat{q},\hat{q}')\psi(\hat{q}').$$

$|\hat{q}| = \sqrt{q^2 - (q.P)^2/P^2}$ is length of 3D vector, $\hat{q}$ (a Lorentz-invariant variable)

3D reduction of BSE under Covariant Instantaneous Ansatz leads to 3D Salpeter equations:

$$(M - \omega_1 - \omega_2)\psi^{++}(\hat{q}) = \Lambda_1^+(\hat{q})\Gamma(\hat{q})\Lambda_2^+(\hat{q})$$
$$(M + \omega_1 + \omega_2)\psi^{--}(\hat{q}) = -\Lambda_1^-(\hat{q})\Gamma(\hat{q})\Lambda_2^-(\hat{q})$$
$$\psi^{+-}(\hat{q}) = 0$$
$$\psi^{-+}(\hat{q}) = 0$$

$$\psi^{\pm\pm}(\hat{q}) = \Lambda_1^\pm(\hat{q})\frac{\not{P}}{M}\psi(\hat{q})\frac{\not{P}}{M}\Lambda_1^\pm(\hat{q})$$

- 3D Salpeter equations used for determination of hadron mass spectrum

- A crucial role is played by variable, $\hat{q}$, which is always orthogonal to P_μ.

3D Salpeter equations (used for mass spectral calculations), as well as the 4D hadron-quark vertex $\Gamma(\hat{q})$(used for 4D transition amplitude calculations), have a common dependence on $\hat{q}^2$, whose most important property is $\hat{q}^2 > 0$ over the entire 4D space.

- Lorentz-invariant nature of $\hat{q}^2$ increases the applicability of this framework of BSE under CIA all the way from low energy spectra to high energy transition amplitudes.

- Most important aspect: Appearance of hadron-quark vertex $\Gamma(\hat{q})$ on RHS of Salpeter equations. This provides a vital dynamical link between 3D mass spectrum and the 4D transition amplitudes.

Recent calculations using BSE under Covariant instantaneous ansatz

- Calculated mass spectrum of ground and excited states of $0^{++}, 0^{-+}, 1^{--}, 1^{+-}, 1^{++}$ heavy-light ($q\bar{Q}$) quarkonia, with the perturbative incorporation of one-gluon exchange, through analytic solutions of mass spectral equations, that were derived through 3D Salpeter equations.
- Also derived the analytic structures of 3D forms of radial wave functions, $\phi(\hat{q})$, through solutions mass spectral equations in approximate harmonic oscillator basis, which were in turn used to calculate the following transitions:
- (a) Leptonic decays of $0^{-+}, 1^{--}, 1^{+-}, 1^{++}$ quarkonia,
- (b) Two-photon decays of 0^{-+}, and 0^{++}quarkonia,
- (c) M1 radiative decays, $1^{--} \to 0^{-+}\gamma,\ 0^{-+} \to 1^{--}\gamma$,
- (d) E1 radiative decays, $1^{--} \to 0^{++}\gamma,\ \ 0^{++} \to 1^{--}\gamma,\ \ 1^{+-} \to 0^{-+}\gamma,\ 0^{-+} \to 1^{+-}\gamma$,

E.Gebrehana, S.Bhatnagar, H.Negash, PRD100, 054034 (2019)

S.Bhatnagar, L. Alemu, PRD97, 034021 (2018)

V.Guleria, S.Bhatnagar, IJTP 60, 3143 (2021)

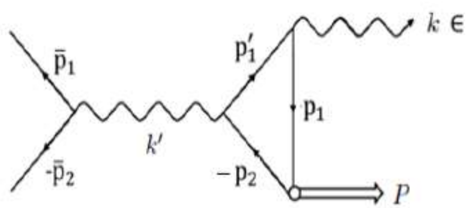
S.Bhatnagar, E.Gebrehana, PRD102, 094024 (2020)

V.Guleria, E.Gebrehana, S.Bhatnagar, PRD104, 094045 (2021)

Cross section for $e^{-}e^{+} \rightarrow \gamma^{*} \rightarrow \gamma\chi_{c0}$

- Four diagrams at leading order contribute equally to this process.

$$M^1_{fi} = -iee_Q^2[\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)]\frac{-1}{s}\int\frac{d^4q}{(2\pi)^4}Tr[\bar{\Psi}_S(P,q)\not{\epsilon}^{\lambda'}S_F(p_1')\gamma_\mu].$$



$$\bar{\Psi}_S(P,q) = S_F(-p_2)\bar{\Gamma}_S(\hat{q})S_F(p_1).$$

4D adjoint BS wave function

$$\psi(P,q) = f_1(q,P) - i\not{P}f_2(q,P) - i\not{q}f_3(q,P) - [\not{P},\not{q}]f_4(q,P),$$

4D BS wave function of scalar meson written as superposition of various Dirac structures [1]

$$\psi(\hat{q}) = Mf_1(\hat{q}) - i\not{P}f_2(\hat{q}) - i\hat{\not{q}}f_3(\hat{q}) - \frac{2\not{P}\hat{\not{q}}}{M}f_4(\hat{q}).$$

3D form of BS wave function of dimensionality, M under Covariant Instantaneous Ansatz

1. C. H. L. Smith, Ann. Phys. (N.Y.) 53, 521 (1969),
2. S.Bhatnagar, S-Y.Li, JPG32, 949 (2006).
3. S.Bhatnagar, J.Mahecha, Y.Mangesha, PRD90, 014034 (2014)

$\Psi_S(\hat{q}) = [M - i\not{P}]\phi_S(\hat{q})$ 3D BS wave function of scalar quarkonium with leading Dirac structures.

ϕ_s derived analytically through analytic solutions of mass spectral equations of scalar quarkonia in approximate harmonic oscillator basis with expressions:

$$\phi_S(1P,\hat{q}) = \sqrt{\frac{2}{3}}\frac{1}{\pi^{3/4}}\frac{1}{\beta_S^{5/2}}\hat{q}e^{-\frac{\hat{q}^2}{2\beta_S^2}},$$

$$\phi_S(2P,\hat{q}) = \sqrt{\frac{5}{3}}\frac{1}{\pi^{3/4}}\frac{1}{\beta_S^{5/2}}\hat{q}(1-\frac{2\hat{q}^2}{5\beta_S^2})e^{-\frac{\hat{q}^2}{2\beta_S^2}}$$

$$S_F(p_1) = \frac{\Lambda_1^+(\hat{q})}{\eta_1} + \frac{\Lambda_1^-(\hat{q})}{\eta_2}$$

$$S_F(-p_2) = \frac{-\Lambda_2^+(\hat{q})}{\eta_3} + \frac{-\Lambda_2^-(\hat{q})}{\eta_4};$$

$$\eta_{1,2} = M\sigma + \frac{1}{2}M \mp \omega_1;$$

$$\eta_{3,4} = -M\sigma + \frac{1}{2}M \mp \omega_2,$$

$$M^1_{fi} = -iee_Q^2[\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)]\frac{-1}{s}(\frac{-1}{M^2})(\frac{-1}{M^2}) \times$$
$$\int\frac{d^3\hat{q}}{(2\pi)^3}\int\frac{Md\sigma}{2\pi i}Tr\Big[\Big(-\frac{\bar{\Psi}^{++}(\hat{q})(M-2\omega)}{[\sigma-(-\frac{1}{2}+\frac{\omega}{M})][\sigma-(\frac{1}{2}-\frac{\omega}{M})]} - \frac{\bar{\Psi}^{--}(\hat{q})(M+2\omega)}{[\sigma-(-\frac{1}{2}-\frac{\omega}{M})][\sigma-(\frac{1}{2}+\frac{\omega}{M})]}\Big) \times$$
$$\not{\epsilon}'\frac{-i(\not{k}+\frac{1}{2}\not{P}+\not{\hat{q}}+\sigma\not{P})+m}{[\sigma-(-\frac{1}{2}-\frac{2E^2}{M^2}+\frac{1}{M}\sqrt{\omega^2+\frac{4E^4}{M^2}})][\sigma-(-\frac{1}{2}-\frac{2E^2}{M^2}-\frac{1}{M}\sqrt{\omega^2+\frac{4E^4}{M^2}})]}\gamma_\mu\Big].$$

$$S_F(p_1') = \frac{-i(\not{k}+\frac{1}{2}\not{P}+\not{\hat{q}}+\sigma\not{P})+m}{-M^2\sigma^2-(4E^2+M^2)\sigma-\alpha}$$

$$\alpha = \hat{q}^2+m^2-2E^2-\frac{M^2}{4}.$$

1. S.Bhatnagar, V.Guleria, arxiv: 2206.02229[hep-ph]
2. V.Guleria, E.Gebrehana, S.Bhatnagar, PRD104, 094045 (2021)

- The poles of the propagators, $S_F(p_1)$, $S_F(-p_2)$, and $S_F(p'_1)$ with corresponding pole positions in complex $\sigma - plane$:

$$\sigma_1^\pm = -\frac{1}{2} \mp \frac{\omega}{M} \pm i\epsilon$$
$$\sigma_2^\pm = \frac{1}{2} \mp \frac{\omega}{M} \pm i\epsilon$$
$$\beta^\pm = (-\frac{1}{2} - \frac{2E^2}{M^2}) \mp \frac{1}{M}\sqrt{\omega^2 + \frac{4E^4}{M^2}} \pm i\epsilon.$$

Amplitude, M_{fi} is finally expressed as,

$$M_{fi} = i[\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)]\left[\beta_1\epsilon'_\mu + \beta_2(I.\epsilon')P_\mu + \beta_3(I.\epsilon')k_\mu\right];$$
$$\beta_1 = \frac{8ee_Q^2 N_S}{M^4 s}\int \frac{d^3\hat{q}}{(2\pi)^3}\phi_S(\hat{q})(\alpha_1 + \alpha_4\hat{q}^2);$$
$$\beta_2 = \frac{8ee_Q^2 N_S}{M^4 s}\int \frac{d^3\hat{q}}{(2\pi)^3}\phi_S(\hat{q})|\hat{q}|\alpha_2;$$
$$\beta_3 = \frac{8ee_Q^2 N_S}{M^4 s}\int \frac{d^3\hat{q}}{(2\pi)^3}\phi_S(\hat{q})|\hat{q}|\alpha_3,$$

$$M_{fi} = i[\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)]M_\mu;$$
$$M_\mu = \beta_1\epsilon'_\mu + \beta_2(I.\epsilon')P_\mu + \beta_3(I.\epsilon')k_\mu,$$

$M_\mu = < \gamma\chi_{c0}|J_\mu|0>$ (Amplitude for $\gamma^* \to \gamma\chi_{c0}$)

e.m. gauge invariance demands, $k_\mu M_\mu = 0$.

=> $\beta_2 = 0$

$$\alpha_1 = -4X_2(P.k) + 2X_2M^2 - 4X_3\hat{q}^2 + 4M^2Y_2 + 4X_1m;$$
$$\alpha_2 = (2X_3 + 4X_2 + 4Y_3 - 4X_4m)$$
$$\alpha_3 = 4X_3,$$
$$\alpha_4 = 8X_3,$$

$$|\bar{M}_{fi}|^2 = \left[2\beta_1^2(-s+3m_e^2) - \frac{1}{2}\beta_3^2 s^2(1-cos^2\theta)\right]$$

$$\sigma = \frac{1}{32\pi^2 s^{3/2}}|\vec{P}'| \int d\Omega' |\bar{M}_{fi}|^2$$ (total cross section for $e^- e^+ \to \gamma^* \to \gamma \chi_{c0}$ in center-of-mass frame)

with $|\vec{P}'| = \frac{s-M^2}{\sqrt{s}}$

$\sigma(e^+e^- \to \gamma\chi_{c0}(1P))$=1.352fb $(\beta_1 = -5.51 \times 10^{-6}\ GeV^{-1},\quad \beta_3 = -2.054 \times 10^{-6} GeV^{-2})$

Exp.=-20.0$^{+122.3}_{-111.0}$ fb [S. Jia et al (Belle Collaboration), PRD98 (2018)]

$\sigma(e^+e^- \to \gamma\chi_{c0}(2P))$=0.602fb $(\beta_1 = -1.664 \times 10^{-6}\ GeV^{-3},\quad \beta_3 = 3.686 \times 10^{-6}\ GeV^{-3})$

Other Models:

$\sigma(e^+e^- \to \gamma\chi_{c0}(1P))$=3.11$^{+1.05}_{-0.94}$ fb [Chung, JHEP 09, 195 (2021)]

$\sigma(e^+e^- \to \gamma\chi_{c0}(1P)) = 1.885\ fb$ [Chen, Wu, Sun, Wang, Shen, PRD88,074021 (2013)]

Cross section for $e^-e^+ \to \gamma^* \to \gamma\chi_{c1}$ [SB, VG, arxiv:2206.02229[hep-ph]]

$$M^1_{fi} = -iee_Q^2[\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)]\frac{-1}{s}\int\frac{d^4q}{(2\pi)^4}Tr[\bar{\Psi}_A(P,q)\not{\epsilon}S_F(p_1')\gamma_\mu],$$

$$M_{fi} = [\bar{v}^{(s2)}(\bar{p}_2)\gamma_\mu u^{(s1)}(\bar{p}_1)] < 0, \chi_{c1}|J_\mu|0>$$

Transition matrix element M_μ of e.m. current from $\gamma^* \to \gamma\chi_{c1}$ simplifies to a general form required by Lorentz covariance.

$$M_{fi} = [\bar{v}^{s_2}(p_2)\gamma_\mu u^{s_1}(p_1)][g_2\epsilon_{\mu\nu\alpha\beta}\epsilon_\nu^\lambda\epsilon_\alpha^{\lambda'}k_\beta + g_3(I.\epsilon)\epsilon_{\mu\nu\alpha\beta}P_\nu\epsilon_\alpha^{\lambda'}k_\beta].$$

$$g_1 = \frac{8ee_Q^2N_A}{M^4s}\int\frac{d^3\hat{q}}{(2\pi)^3}\phi_A(\hat{q})(\frac{1}{2}\Theta_1 + \Omega_1 + \Theta_2 m + \Theta_3\hat{q}^2),$$

$$g_2 = \frac{8ee_Q^2N_A}{M^4s}\int\frac{d^3\hat{q}}{(2\pi)^3}\phi_A(\hat{q})\Theta_1,$$

$$g_3 = \frac{8ee_Q^2N_A}{M^4s}\int\frac{d^3\hat{q}}{(2\pi)^3}\phi_A(\hat{q})|\hat{q}|\Theta_3,$$

$$\sigma = \frac{1}{32\pi^2 s^{3/2}}|\vec{P'}|\int d\Omega'|\bar{M}_{fi}|^2,$$

$\sigma(e^-e^+ \to \gamma^* \to \gamma\chi_{c1}$(1P))=7.389 fb* (g2=-4.72 x 10^{-6} GeV^{-2}, g3=2.914 x10^{-8} GeV^{-3})

$\sigma(e^+ \to \gamma^* \to \gamma\chi_{c1}$(2P))=4.143 fb* (g2=4.023 x 10^{-6} GeV^{-2}, g3=-7.976 x10^{-8} GeV^{-3})

$\sigma(e^-e^+ \to \gamma^* \to \gamma\chi_{c1}$)=$17.3^{+4.2}_{-3.9}$ fb, Belle, PRD98,092015(2018)

$\sigma(e^-e^+ \to \gamma^* \to \gamma\chi_{c1}$)=$10.9^{+3.7}_{-3.4}$fb, Sang, Chen, PRD81,034028 (2010)

Cross section for $e^-e^+ \to \gamma^* \to \gamma\eta_c$

$$M_{fi} = [\bar{v}^{s_2}(p_2)\gamma_\mu u^{s_1}(p_1)]\beta\epsilon_{\mu\nu\alpha\beta}P_\nu\epsilon_\alpha^{\lambda'}k_\beta;$$
$$\beta = \frac{8ee_Q^2 N_P}{M^4 s}\int\frac{d^3\hat{q}}{(2\pi)^3}X_2'\phi_P(\hat{q});$$
$$X_2' = -b_1'(M-2\omega)I_1' + b_2'(M+2\omega)I_1''.$$

β absorbs entire momentum integration in M_{fi}

$$|\bar{M}_{fi}|^2 = \frac{1}{4}\beta^2[16m_e^2M^4 + 4m_e^2s^2 + s^3(1+cos^2\theta) - M^2(16m_e^2 s + s^2(1+3cos^2\theta))]$$

$$\sigma = \frac{1}{32\pi^2 s^{3/2}}|\vec{P}'|\int d\Omega'|\bar{M}_{fi}|^2$$

$\sigma(e^-e^+ \to \gamma^* \to \gamma\eta_c)$=45.841fb [Bhatnagar, Guleria, arxiv:2206.02229[hep-ph]
$\sigma(e^-e^+ \to \gamma^* \to \gamma\eta_c)$=$11.3^{+7.0}_{-6.6}$fb [Belle, PRD98,092015 (2018)]

Other models:
$\sigma(e^-e^+ \to \gamma^* \to \gamma\eta_c)$=26.2 fb [Li, Feng, Ma, JHEP2020 (2020)]
$\sigma(e^-e^+ \to \gamma^* \to \gamma\eta_c)$=41.6 fb [Braguta, PRD82,074009 (2010)]

Summary

- Calculated cross sections for $e^- e^+ \to \gamma^* \to \gamma\eta_c$, and $e^- e^+ \to \gamma^* \to \gamma\chi_{c0,c1}$ in the framework of 4 x 4 BSE under covariant instantaneous ansatz at 10.6 GeV. using leading order $\sim O(\alpha_{em}^3\, \alpha_s^0)$ diagrams in QCD.
- Reduction of amplitude to 3D form done using Covariant Instantaneous Ansatz, due to which the amplitude retains Lorentz-covariance, and is expressed in a general form in terms of form factors.
- Our results of cross sections with leading order diagrams provide a sizable contribution, which might be due to the BSE being a fully relativistic approach that incorporates: (i) Relativistic effect of quark spins, and (ii) can describe internal motion of quarks within the hadron in a relativistic covariant manner.
- However there is a wide variation in results of various models for cross sections for each of the process studied.
- Further calculations are being done on incorporations of NLO QCD corrections, and double charmonium production processes.