Photon and charmonium production in electron-positron annihilation at $\sqrt{s}=10.6$ GeV. in framework of Bethe-Salpeter equation

Shashank Bhatnagar
Chandigarh University, India

In collaboration with
Vaishali Guleria (Chandigarh University)

9th International Conference on Quarks and Nuclear Physics (Sept. 5-9, 2022)
Outline:

- Hadron production in electron-positron annihilation
- Bethe-Salpeter dynamics of composite hadron
  - $e^+e^- \rightarrow \gamma^* \rightarrow \gamma \chi_{c0,c1}(nP)$
  - $e^+e^- \rightarrow \gamma^* \rightarrow \gamma \eta_c(nS)$
- Summary
Hadron production in electron-positron annihilation

- There is significant discrepancy between the experimental measurements by BABAR and Belle collaborations of total cross sections of $e^+e^- \rightarrow J/\psi \eta_c ; J/\psi \chi_{cJ}$, and corresponding NRQCD predictions at energies $\sqrt{s}=10.6$ GeV.

- Measurement of cross sections for $e^+e^- \rightarrow J/\psi \eta_c$ carried out by Belle [PRL89(2002), PRD70(2004)] and BABAR [PRD72 (2005)] collaborations has led to rapid progress in theoretical description of heavy-quarkonium through NRQCD. Cross section measured by Belle was greater than the cross section calculated by Braaten and Lee [PRD67(2003); PRD72(2005)] by an order of magnitude.

- Incorporation of NLO corrections to strong coupling constant $\alpha_s$, were not large enough to bring theoretical results close to the Belle data. But incorporation of relativistic corrections by Bodwin, Lee, Yu [PRD77,(2008)], besides the NLO QCD corrections was found to resolve the discrepancy within errors.

- Also interference between QED and QCD tree-level diagrams also gave significant contributions[PRD98, 094001 (2018)]

- Also found that discrepancy between experimental results and theoretical predictions could be resolved by taking into account intrinsic motion of quarks inside hadron in Light-cone expansion method [Braguta et al., PRD72(2005)].

- Very recently measurements were made on the cross sections for the processes: $e^+e^- \rightarrow \gamma \rightarrow \gamma \eta_c$ (nS), and $e^+e^- \rightarrow \gamma \chi_{c0,c1}$ at $\sqrt{s}=10.52$ GeV., 10.58 GeV., and 10.867 GeV. at Belle [S.Jia et al., PRD 98(2018)].

- Also recent measurements on $e^+e^- \rightarrow \gamma \rightarrow \gamma \chi_{c0,c1,c2}$ at $\sqrt{s}=4\cdot 4.6$ GeV using integrated luminosity, 19.3 $fb^{-1}$ reported by BESIII reported no significant signal for these processes. [Ablikin et al., PRD104 (2021)]

Present work*: [S.Bhatnagar, V.Guleria, arxiv:2206.02229[hep-ph]]

- Calculated the cross sections for production of ground and excited charmonium states in \( e^+ e^- \rightarrow \gamma^* \rightarrow \gamma H \), at \( \sqrt{s}=10.6 \text{ GeV} \), through leading order (LO) (tree-level) diagrams, \( \sim O(\alpha^3_{em} \alpha_s^0) \) which proceed through exchange of a virtual photon in the framework of 4x4 Bethe-Salpeter equation (BSE) in colour singlet channel.

- \( H \) is a heavy quarkonium with \( C=+1 \), and can be S-wave spin singlet states, such as: \( \eta_c \ (nS) \), \( \eta_b \ (nS) \), or P-wave spin triplet states, such as \( \chi_{cJ} \) or \( \chi_{bj} \) (\( J=0,1,2 \)).
Features of BSE

- BSE is a dynamical equation based approach, with a firm base in field theory, and is a conventional approach in dealing with relativistic bound state problems.

- Interaction kernel, $K$ involves summation over all possible two-particle irreducible interactions between two constituents forming bound state.

- In practical calculations, we use Ladder Approximation- where kernel involves summation over an infinite ladder of quantum exchanges between two interacting particles forming bound state.

- BSE is quite general, and provides effective description of bound quark- anti-quark systems, through a suitable choice of input kernel for confinement.

Advantages of BSE approach:

- Fully relativistic approach, that incorporates the relativistic effect of quark spins and can also describe internal motion of constituent quarks within the hadron in a relativistically covariant manner.

- Due to above features, it provides a realistic description for analyzing hadrons as composite objects.

- It can be applied to study processes over a wide range of energies, and with little number of input parameters!!
**BSE under Covariant Instantaneous Ansatz**

- A quark-anti quark bound state system can be described by a Bethe-Salpeter equation (BSE),

\[
S_F^{-1}(p_1)\Psi(P, q)S_F^{-1}(-p_2) = i\int \frac{d^4q'}{(2\pi)^4} K(q, q')\Psi(P, q'),
\]

4D BSE \(\rightarrow\) 3D form using Covariant Instantaneous Ansatz, \(K(q, q') = K(\hat{q}, \hat{q}')\);

- \(\hat{q}_\mu = q_\mu - \frac{q.P}{p^2} P_\mu\) (Transverse to P), with \(\hat{q}.P=0\), and \(\sigma P_\mu = \frac{q.P}{p^2} P_\mu\) (Longitudinal to P),

- with 4D volume element, \(d^4q = d^3\hat{q}Md\sigma\)

\[
\Psi(P, q) = S_1(p_1)\Gamma(\hat{q})S_2(-p_2), \quad \Gamma(\hat{q}) = \int \frac{d^3\hat{q}'}{(2\pi)^3} K(\hat{q}, \hat{q}')\psi(\hat{q}').
\]

\[
|\hat{q}| = \sqrt{q^2 - (q.P)^2/P^2}
\]

is length of 3D vector, \(\hat{q}\) (a Lorentz-invariant variable)

**3D reduction of BSE under Covariant Instantaneous Ansatz leads to 3D Salpeter equations:**

\[
(M - \omega_1 - \omega_2)\psi^{++}(\hat{q}) = \Lambda_1^+(\hat{q})\Gamma(\hat{q})\Lambda_2^+(\hat{q})
\]

\[
(M + \omega_1 + \omega_2)\psi^{--}(\hat{q}) = -\Lambda_1^-(\hat{q})\Gamma(\hat{q})\Lambda_2^-(\hat{q})
\]

\[
\psi^{+-}(\hat{q}) = 0
\]

\[
\psi^{-+}(\hat{q}) = 0
\]
3D Salpeter equations used for determination of hadron mass spectrum

- A crucial role is played by variable, \( \hat{q} \), which is always orthogonal to \( P_\mu \).
- 3D Salpeter equations (used for mass spectral calculations), as well as the 4D hadron-quark vertex \( \Gamma(\hat{q}) \) (used for 4D transition amplitude calculations), have a common dependence on \( \hat{q}^2 \), whose most important property is \( \hat{q}^2 > 0 \) over the entire 4D space.

- Lorentz-invariant nature of \( \hat{q}^2 \) increases the applicability of this framework of BSE under CIA all the way from low energy spectra to high energy transition amplitudes.

- Most important aspect: Appearance of hadron-quark vertex \( \Gamma(\hat{q}) \) on RHS of Salpeter equations. This provides a vital dynamical link between 3D mass spectrum and the 4D transition amplitudes.
Recent calculations using BSE under Covariant instantaneous ansatz

- Calculated mass spectrum of ground and excited states of $0^{++}, 0^{-+}, 1^{--}, 1^{+-}, 1^{++}$ heavy-light ($q\bar{Q}$) quarkonia, with the perturbative incorporation of one-gluon exchange, through analytic solutions of mass spectral equations, that were derived through 3D Salpeter equations.

- Also derived the analytic structures of 3D forms of radial wave functions, $\phi(q\bar{q})$, through solutions mass spectral equations in approximate harmonic oscillator basis, which were in turn used to calculate the following transitions:
  
  - (a) Leptonic decays of $0^{-+}, 1^{--}, 1^{+-}, 1^{++}$ quarkonia,
  - (b) Two-photon decays of $0^{-+}$, and $0^{++}$ quarkonia,
  - (c) M1 radiative decays, $1^{--} \rightarrow 0^{-+}\gamma$, $0^{-+} \rightarrow 1^{--}\gamma$,
  - (d) E1 radiative decays, $1^{--} \rightarrow 0^{++}\gamma$, $0^{++} \rightarrow 1^{--}\gamma$, $1^{+-} \rightarrow 0^{-+}\gamma$, $0^{-+} \rightarrow 1^{+-}\gamma$,

E. Gebrehana, S. Bhatnagar, H. Negash, PRD100, 054034 (2019)
V. Guleria, S. Bhatnagar, IJTP 60, 3143 (2021)
S. Bhatnagar, E. Gebrehana, PRD102, 094024 (2020)
V. Guleria, E. Gebrehana, S. Bhatnagar, PRD104, 094045 (2021)
Cross section for $e^- e^+ \rightarrow \gamma^* \rightarrow \gamma \chi_{c0}$

Four diagrams at leading order contribute equally to this process.

\[ M_{fi}^{\lambda} = -i e e_Q^2 \sum_{s_1} \left( \bar{u}^{(s_2)}(\vec{p}_2) \gamma_\mu u^{(s_1)}(\vec{p}_1) \right) \frac{-1}{s} \int \frac{d^4q}{(2\pi)^4} \text{Tr}[\Psi_S(P,q)\phi^\lambda S_F(p'_1)\gamma_\mu]. \]

4D adjoint BS wave function

4D BS wave function of scalar meson written as superposition of various Dirac structures [1]

3D form of BS wave function of dimensionality, $M$ under Covariant Instantaneous Ansatz

\[ \Psi_S(P,q) = S_F(-p_2)\Gamma_S(q)S_F(p_1) \]

\[ \psi(P,q) = f_1(q,P) - i\not{P} f_2(q,P) - i\not{\phi} f_3(q,P) - [\not{P}, \not{\phi}] f_4(q,P). \]

\[ \psi(\hat{q}) = M f_1(\hat{q}) - i\not{P} f_2(\hat{q}) - i\not{\phi} f_3(\hat{q}) - \frac{2\not{P}\not{\phi}}{M} f_4(\hat{q}). \]

\[ \Psi_S(\hat{q}) = [M - i \hat{P}] \phi_S(\hat{q}) \] 3D BS wave function of scalar quarkonium with leading Dirac structures.

\[ \phi_s \] derived analytically through analytic solutions of mass spectral equations of scalar quarkonia in approximate harmonic oscillator basis with expressions:

\[ \phi_S(1P, \hat{q}) = \sqrt{\frac{2}{3}} \frac{1}{\beta_s^{3/2}} \frac{1}{\beta_s^{3/2}} q e^{-\frac{\hat{q}^2}{2 \beta_s^2}} \]
\[ \phi_S(2P, \hat{q}) = \sqrt{\frac{5}{3}} \frac{1}{\beta_s^{3/2}} \frac{1}{\beta_s^{3/2}} (1 - \frac{2 \hat{q}^2}{5 \beta_s^2}) e^{-\frac{\hat{q}^2}{2 \beta_s^2}} \]

\[ M_{Jt}^1 = -i e^2 [s^{(2)}(p_2) \gamma_\mu u^{(s1)}(p_1)] \frac{1}{s} (\frac{1}{M^2}) (\frac{1}{M^2}) \times \]
\[ \int \frac{d^3 \hat{q}}{(2\pi)^3} \int \frac{Md\sigma}{2\pi i} \text{Tr} \left[ \left( - \frac{\overline{\Psi}^+(\hat{q})(M - 2\omega)}{\sigma - (\frac{1}{2} + \frac{\alpha}{M})[\sigma - (\frac{1}{2} + \frac{\alpha}{M})]} \right) - \frac{\overline{\Psi}^-(\hat{q})(M + 2\omega)}{\sigma - (\frac{1}{2} - \frac{\alpha}{M})[\sigma - (\frac{1}{2} + \frac{\alpha}{M})]} \right] \times \]
\[ \frac{\alpha}{[\sigma - (\frac{1}{2} - \frac{2E^2}{M^2} + \frac{1}{M} \sqrt{\omega^2 + 4E^4/M^2})][\sigma - (\frac{1}{2} - \frac{2E^2}{M^2} - \frac{1}{M} \sqrt{\omega^2 + 4E^4/M^2})]} \gamma_\mu \].
The poles of the propagators, $S_F(p_1), S_F(-p_2),$ and $S_F(p'_1)$ with corresponding pole positions in complex $\sigma - plane$:

\[
\sigma_1^\pm = -\frac{1}{2} \mp \frac{\omega}{M} \pm i\epsilon \\
\sigma_2^\pm = \frac{1}{2} \mp \frac{\omega}{M} \pm i\epsilon \\
\beta^\pm = \left(-\frac{1}{2} - \frac{2E^2}{M^2}\right) \pm \frac{1}{M} \sqrt{\omega^2 + \frac{4E^4}{M^2}} \pm i\epsilon
\]

Amplitude, $M_{fi}$ is finally expressed as,

\[
M_{fi} = i[v^{(s2)}(p_2)\gamma^\mu u^{(s1)}(p_1)] \left[ \beta_1 \epsilon_{\mu} + \beta_2 (I, \epsilon') P_{\mu} + \beta_3 (I, \epsilon') k_{\mu} \right]
\]

\[
\beta_1 = \frac{8eQ^2 N_S}{M^4 s} \int \frac{d^3 \hat{q}}{(2\pi)^3} \phi_{S}(\hat{q})(\alpha_1 + \alpha_4 \hat{q}^2);
\beta_2 = \frac{8eQ^2 N_S}{M^4 s} \int \frac{d^3 \hat{q}}{(2\pi)^3} \phi_{S}(\hat{q}) |\hat{q}| \alpha_2;
\beta_3 = \frac{8eQ^2 N_S}{M^4 s} \int \frac{d^3 \hat{q}}{(2\pi)^3} \phi_{S}(\hat{q}) |\hat{q}| \alpha_3,
\]

\[
M_{\mu} = \langle \gamma \chi c_0 | J_{\mu} | 0 \rangle \quad \text{(Amplitude for } \gamma^* \rightarrow \gamma \chi c_0)\]

e.m. gauge invariance demands, $k_{\mu} M_{\mu} = 0$.

\[
\Rightarrow \beta_2 = 0
\]

\[
\alpha_1 = -4X_2(P, k) + 2X_2 M^2 - 4X_3 \hat{q}^2 + 4M^2 Y_2 + 4X_1 m;
\alpha_2 = (2X_3 + 4X_2 + 4Y_3 - 4X_4 m)
\alpha_3 = 4X_3,
\alpha_4 = 8X_3,
\]
\[ |\bar{M}_{fi}|^2 = \left[ 2\beta_1^2(-s + 3m_e^2) - \frac{1}{2}\beta_3^2 s^2(1 - \cos^2\theta) \right] \]

\[ \sigma = \frac{1}{32\pi^2 s^{3/2}} |\bar{P}^\prime| \int d\Omega' |\bar{M}_{fi}|^2 \] (total cross section for \( e^- e^+ \rightarrow \gamma^* \rightarrow \gamma\chi_{c0} \) in center-of-mass frame)

with \[ |\bar{P}^\prime| = \frac{s-M^2}{\sqrt{s}} \]

\[ \sigma(e^+e^- \rightarrow \gamma\chi_{c0}(1P)) = 1.352 \text{ fb} \quad (\beta_1 = -5.51 \times 10^{-6} \text{ GeV}^{-1}, \quad \beta_3 = -2.054 \times 10^{-6} \text{ GeV}^{-2}) \]

Exp. = \(-20.0_{-111.0}^{+122.3} \text{ fb} \) [S. Jia et al (Belle Collaboration), PRD98 (2018)]

\[ \sigma(e^+e^- \rightarrow \gamma\chi_{c0}(2P)) = 0.602 \text{ fb} \quad (\beta_1 = -1.664 \times 10^{-6} \text{ GeV}^{-3}, \quad \beta_3 = 3.686 \times 10^{-6} \text{ GeV}^{-3}) \]

Other Models:

\[ \sigma(e^+e^- \rightarrow \gamma\chi_{c0}(1P)) = 3.11_{-0.94}^{+1.05} \text{ fb} \] [Chung, JHEP 09, 195 (2021)]

\[ \sigma(e^+e^- \rightarrow \gamma\chi_{c0}(1P)) = 1.885 \text{ fb} \] [Chen, Wu, Sun, Wang, Shen, PRD88, 074021 (2013)]
Cross section for $e^-e^+ \to \gamma^* \to \gamma \chi_{c1}$ [SB, VG, arxiv:2206.02229[hep-ph]]

\[
M_{f_i}^1 = -ie e_Q \bar{u}^{(s_2)}(\vec{p}_2) \gamma_\mu u^{(s_1)}(\vec{p}_1) \frac{-1}{s} \int \frac{d^4q}{(2\pi)^4} Tr[\bar{\Psi}_A(P,q)\gamma_\mu S_F(p_1') \gamma_\mu],
\]

\[
M_{f_i} = [\bar{u}^{(s_2)}(p_2) \gamma_\mu u^{(s_1)}(p_1)] [g_2 \epsilon_{\mu\nu\alpha\beta} \epsilon_\nu^\lambda \epsilon_\alpha^\lambda k_\beta + g_3 (I, \epsilon) \epsilon_{\mu\nu\alpha\beta} P_\nu \epsilon_\alpha^\lambda k_\beta].
\]

\[
g_1 = \frac{8e^2 Q N_A}{M^4 s} \int \frac{d^3\hat{q}}{(2\pi)^3} \phi_A(\hat{q}) (\frac{1}{2} \Theta_1 + \Omega_1 + \Theta_2 m + \Theta_3 \hat{q}^2),
\]

\[
g_2 = \frac{8e^2 Q N_A}{M^4 s} \int \frac{d^3\hat{q}}{(2\pi)^3} \phi_A(\hat{q}) \Theta_1,
\]

\[
g_3 = \frac{8e^2 Q N_A}{M^4 s} \int \frac{d^3\hat{q}}{(2\pi)^3} \phi_A(\hat{q}) |\hat{q}| \Theta_3,
\]

\[
\sigma(e^-e^+ \to \gamma^* \to \gamma \chi_{c1}(1P)) = 7.389 \text{ fb}^* (g_2 = -4.72 \times 10^{-6} \text{ GeV}^{-2}, \ g_3 = 2.914 \times 10^{-8} \text{ GeV}^{-3})
\]

\[
\sigma(e^+ \to \gamma^* \to \gamma \chi_{c1}(2P)) = 4.143 \text{ fb}^* (g_2 = 4.023 \times 10^{-6} \text{ GeV}^{-2}, \ g_3 = -7.976 \times 10^{-8} \text{ GeV}^{-3})
\]

\[
\sigma(e^-e^+ \to \gamma^* \to \gamma \chi_{c1}) = 17.3^{+4.2}_{-3.9} \text{ fb}, \text{ Belle, PRD98,092015(2018)}
\]

\[
\sigma(e^-e^+ \to \gamma^* \to \gamma \chi_{c1}) = 10.9^{+3.7}_{-3.4} \text{ fb}, \text{ Sang, Chen, PRD81,034028 (2010)}
\]

Transition matrix element $M_\mu$ of e.m. current from $\gamma^* \to \gamma \chi_{c1}$ simplifies to a general form required by Lorentz covariance.
Cross section for $e^-e^+ \rightarrow \gamma^* \rightarrow \gamma \eta_c$

\[
M_{fi} = \left[ \tilde{v}^a_2(p_2) \gamma_{\mu} u^{s_1}(p_1) \right] \beta \epsilon_{\mu \nu \alpha \beta} P_{\nu} X'_{\alpha} k_{\beta};
\]
\[
\beta = \frac{8e^2 Q N_P}{M^4 s} \int \frac{d^3 \tilde{q}}{(2\pi)^3} X'_{2} \phi_{P}(\tilde{q});
\]
\[
X'_{2} = -b_{1}'(M - 2\omega)I_{1}' + b_{2}'(M + 2\omega)I_{1}''.
\]
\[
|\tilde{M}_{fi}|^2 = \frac{1}{4} \beta^2 [16m_c^2 M^4 + 4m_e^2 s^2 + s^3(1 + \cos^2 \theta) - M^2 (16m_c^2 s + s^2(1 + 3\cos^2 \theta))]
\]
\[
\sigma = \frac{1}{32\pi^2 s^{3/2}} |\tilde{P}| \int d\Omega' |\tilde{M}_{fi}|^2
\]

\[
\sigma(e^-e^+ \rightarrow \gamma^* \rightarrow \gamma \eta_c) = 45.841 \text{fb} \ [\text{Bhatnagar, Guleria, arxiv:2206.02229[hep-ph]}]
\]
\[
\sigma(e^-e^+ \rightarrow \gamma^* \rightarrow \gamma \eta_c) = 11.3^{+7.0}_{-6.6} \text{fb} \ [\text{Belle, PRD98,092015 (2018)}]
\]

Other models:
\[
\sigma(e^-e^+ \rightarrow \gamma^* \rightarrow \gamma \eta_c) = 26.2 \text{ fb} \ [\text{Li, Feng, Ma, JHEP2020 (2020)}]
\]
\[
\sigma(e^-e^+ \rightarrow \gamma^* \rightarrow \gamma \eta_c) = 41.6 \text{ fb} \ [\text{Braguta, PRD82,074009 (2010)}]
\]
Summary

- Calculated cross sections for $e^- e^+ \rightarrow \gamma^* \rightarrow \gamma\eta_c$, and $e^- e^+ \rightarrow \gamma^* \rightarrow \gamma\chi_{c0,c1}$ in the framework of 4 x 4 BSE under covariant instantaneous ansatz at 10.6 GeV, using leading order $\sim O(\alpha^3_{em} \alpha_s^0)$ diagrams in QCD.

- Reduction of amplitude to 3D form done using Covariant Instantaneous Ansatz, due to which the amplitude retains Lorentz-covariance, and is expressed in a general form in terms of form factors.

- Our results of cross sections with leading order diagrams provide a sizable contribution, which might be due to the BSE being a fully relativistic approach that incorporates: (i) Relativistic effect of quark spins, and (ii) can describe internal motion of quarks within the hadron in a relativistic covariant manner.

- However there is a wide variation in results of various models for cross sections for each of the process studied.

- Further calculations are being done on incorporations of NLO QCD corrections, and double charmonium production processes.