Exclusive production of charmonia, bottomonia and $B_c^+ B_c^-$ meson pairs

Marat Siddikov

In collaboration with Ivan Schmidt, Sebastian Andradé

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Exclusive quarkonia photoproduction


Single quarkonia photoproduction

- Clean probe of gluonic structure, complements inclusive data in phenomenological fits

**Color Dipole framework:**
  - probe dipole amplitude ($b$-dependence)

**Collinear factorization approach:**
  - probe (gluon) GPDs of the protons

- Heavy mass $m_Q \gg \Lambda_{QCD}$ is “natural” hard scale, wide region of applicability of perturbative treatment (up to photoproduction, $Q^2 \approx 0$)
  - Accessible both in $ep$ colliders and $pA, AA$ colliders in ultraperipheral kinematics

Photoproduction of quarkonia pairs

$$\gamma^{(*)} + p \rightarrow M_1 + M_2 + p, \quad M_1, M_2 = J/\psi, \eta_c, \chi_c, \ldots$$

- Can vary quantum numbers and kinematics of produced quarkonia in order to disentangle effects due to wave function from target-related effects
  ⇒ much more detailed information about the target (parton distributions, dipole amplitudes at $x \ll 1$)
- Cross-sections within reach of EIC, JLab@24 GeV, UPC@LHC, LHeC, FCC
Previous studies of meson pair production

▸ **Studies in Bjorken regime:**

\[ \gamma^{(*)} + p \rightarrow M_1 + M_2 + p, \quad M_1, M_2 = \pi^\pm, \pi^0 \ldots \]

Focused on light mesons
▶ mass \( M \ll Q \), twist expansion
▶ If \( (p_{M_1} + p_{M_2})^2 \) is small, contributions from feed-down channels \( (\rho \rightarrow \pi\pi, \ldots) \)
▶ Depend on 2-pion distribution amplitudes (poorly known as of now)

Not applicable when \( M \sim Q \gg \Lambda_{QCD} \)

▸ **Photoproduction of heavy quarkonia:**

[PRD 101, 034025; EPJC 49, 675; 73, 2335; 76, 103; 80, 806.]

▶ Focused on \( J/\psi J/\psi \) channel
▶ Is dominated by photon-photon fusion (C-parity)
▶ Extra photon \( \Rightarrow \) additional \( \mathcal{O}(\alpha_{em}^2) \)-suppression in cross-sections
▶ Might get contributions from diagrams with odd-eron in \( t \)-channel
Our suggestion: quarkonia pairs with opposite $C$-parity

- Wide range of possible quarkonia choices: $J/\psi \eta_c$, $J/\psi \chi_c$, $\Upsilon \eta_c$...
- Hard part is dominated by 2-gluon exchange in $t$-channel, $\Rightarrow$ significantly larger cross-section than for $J/\psi J/\psi$
- Two main production mechanisms, depending on heavy flavour content:

<table>
<thead>
<tr>
<th>Final state</th>
<th>Mechanism</th>
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<tbody>
<tr>
<td>2 charmonia or 2 bottomonia ($J/\psi \eta_c$, $\Upsilon \eta_b$, ...)</td>
<td>(A) ✓ (B) ✓</td>
</tr>
<tr>
<td>$B_c$ meson pairs ($B_c^+ B_c^-$), ($B_c^{*+} B_c^-$)</td>
<td>(A) ✓ (B) ✗</td>
</tr>
<tr>
<td>Charmonium+bottomonium ($J/\psi \eta_b$, $\Upsilon \eta_c$, ...)</td>
<td>(A) ✗ (B) ✓</td>
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**Kinematics and choice of framework**

**Our main focus: Electron Ion Collider**

- Typical values of variables $\xi, x_B$

\[ x_B \approx \frac{Q^2 + M_{12}^2}{Q^2 + W^2}, \quad \xi = \frac{x_B}{2 - x_B}. \]

- In EIC kinematics, accessible $x_B, Q^2$ might vary significantly, depending on choice of electron-proton energy $E_e, E_p$(see left)
  - Values of $x_B, \xi \in (10^{-4}, 1)$
  - Dominant contribution (from spectrum of equivalent photons): $Q^2 \approx 0$ ($Q^2 \ll M_Q^2$)

**Choice of framework:**

- For $x_B, \xi \ll 1$ saturation effects are important, color dipole description is a natural choice.
  - Collinear factorization (and similar) unreliable due to large NLO corrections ($\sim \ln x_B$), lack of saturation effects
- For $x_B, \xi \gtrsim 0.01 \ldots 0.1$ collinear factorization is a natural choice
  - Eikonal-based description used in small-$x$ approaches might be unreliable
Evaluations in collinear factorization framework

**Bjorken regime** \((Q^2, p \cdot q \gg M_i^2, |t|)\)

Evaluation straightforward, amplitude:
\[
\mathcal{A} \sim \int dx \, F_g^{(a)}(x, \xi, t) \, C^{(a)}(x, \xi) ,
\]

\[
F_g^{(a)} = \left\{ H_g, E_g, \tilde{H}_g, \tilde{E}_g \right\}
\]

**Full expression for** \(C(x, \xi)\) **is too lengthy, has a structure**

\[
C(x, \xi) = \sum_{\pm} \frac{N_{\pm}(x, \xi)}{(x \mp \xi \pm i0)^n}, \quad n \lesssim 3
\]

where functions \(N_{\pm}(x, \xi)\) remain finite for \(x \to \pm \xi\)

- Behaviour near points \(x = \pm \xi\) signals potential breakdown of collinear factorization
  - Similar singularities observed for dijet production [PRL 128 (2022), 182002]
  - need to take into account transverse momenta [EPJC 74, 2725 (2014)]
    - Effective regularization: \(|k_\perp^2/Q^2|\)

**Caution:**

- Presence of large scale \(M_{12}^2 \gtrsim (M_{J/\psi} + M_{\eta_c})^2 \sim 36 \text{ GeV}^2\)
  - Cross-section is small, and decreases at least as \(\sim 1/Q^6\)
  - For \(Q^2 \gg M_{12}^2\) get extremely tiny numbers, "pure" Bjorken regime is difficult to achieve from experimental point of view
Evaluations in collinear factorization framework (II)

**Beyond Bjorken limit**

**Hierarchy of scales:**

\[ m_N, \sqrt{|t|}, |p_\perp| \ll Q, M_{J/\psi}, M_{\eta_c} \ll p^+, q^-, W_{\gamma p} \]

\[ \sim \Lambda^{-1} \approx 0 \quad \sim \Lambda^0 \quad \sim \Lambda^1 \]

\[ \alpha_s(m_c), \alpha_s(Q) \ll 1 \]

where \( \Lambda \to \infty \) is some hard scale. Keep only the leading order in \( \Lambda \) for simplicity

- Reasonable for EIC, LHeC, FCC-he kinematics
- **Subtle point:** in back-to-back kinematics no suppression if \( p_{J/\psi} \approx -p_{\eta_c} \):
  - * GPD is sensitive only to \( |t| \approx (p_{J/\psi} + p_{\eta_c})^2 \)
  - * \( p_\perp \) is disregarded in coef. functions in collinear factorization
    - Should be careful with observables which might get contributions from this region

**Structure function \( C(x, \xi) \):**

\[ C(x, \xi) = \sum_{\pm} \frac{\tilde{N}_\pm(x, \xi)}{x \mp \xi \pm i0} \]

where \( \tilde{N}_\pm(x, \xi) \) are finite for \( |x| < 1 \)

**Caution:** Evaluation done at LO.

- At higher orders (NLO, ...) large contributions due to BFKL logs.
  - Important if \( x_B \sim M_{12}^2/W^2 \ll 1 \),
  - Explicit evaluation of all those corrections is not feasible.

- **Photoproduction @very high energy:** \( \xi \approx M_{J/\psi}^2/W_{\gamma p}^2 \approx 0 \), probe GPD moments (form factors) \( R_n(t) = \int dx x^n H_g(x, 0, t), \quad n \gtrsim -1 \)
Results in collinear factorization

- Use Kroll-Goloskokov GPD for gluons
- In $ep \rightarrow ep\eta_c J/\psi$ cross-section there is $\sim 1/Q^2$ from leptonic part, so cross-section is dominated by small-$Q$ photons
  - Consider instead cross-section of $\gamma^* p \rightarrow p\eta_c J/\psi$ subprocess:
    - Consider normalized ratio
      \[ R_Q \equiv \frac{d\sigma(Q, \ldots)/d\Omega}{d\sigma(Q = 0, \ldots)/d\Omega} \]
    - Values of $d\sigma$ depends on $y_a, p_{a\perp}$
    - For $R_Q$ dependence largely cancels

- The $Q^2$-dependence is controlled by
  \[ M_{12} = \sqrt{(p_{J/\psi} + p_{\eta_c})^2} \gtrsim (M_{J/\psi} + M_{\eta_c}) \]
  - Very mild dependence for $Q^2 \lesssim M_{12}^2$
  - $R_Q(Q) \sim 1/Q^6$ for $Q^2 \gg M_{12}^2$
Results in collinear factorization (II)

\( p_T \) dependence for \( ep \rightarrow ep \eta_c J/\psi \):

- Exponential suppression at large-\( p_T \)
  - due to implemented \( t \)-dependence in gluon GPD \( H_g(x, \xi, t) \)

\[ H_g(x, \xi, t) \sim e^{B(x)t} \]

- recall that \( t \sim -4p_{\perp}^2 + ... \)

Rapidity dependence for \( ep \rightarrow ep \eta_c J/\psi \):

- Peaked at central rapidities \((y_1, y_2 \sim (-1, 0))\)
  - At negative rapidities \( x_B, \xi \) increase; so gluon GPD decreases
  - At positive rapidities suppression due to leptonic factor, elasticity \( y = q \cdot p/k \cdot p = \nu/E_e \) approaches unity
Framework for evaluations (II)

- **Color Dipole approach**
  - Valid at high energies \((\text{small-}x, \xi \ll 1)\)
  - Based on eikonal picture in the target rest frame
  - Replaces individual partons \(\Rightarrow\) parton showers
  - Interaction with target is described by (universal) dipole amplitude, which:
    - satisfies Balitsky-Kovchegov equation
    - effectively resums the fan-like diagrams as shown in the Figure

- Interactions of shower with heavy quarks is still suppressed by \(\alpha_s \left(m_Q\right)\):

  Eikonal picture: factorize amplitude into WFs and dipole amplitudes in config. space
  - WFs from potential models
Color dipole vs. collinear factorization approach

Use Kroll-Goloskokov GPD for gluons, \( b \)-CGC for color dipole amplitude

\[ E_{e \times E_p} = 18 \times 275 \text{ (GeV)} \]

\[ x_B \text{ large, } y = \nu / E_e \]

At positive \( y_1, y_2 > 0 \):
- the shape of rapidity dependence is similar
- numerically predictions for cross-section differ by a factor of 2.

At negative \( y_1, y_2 < 0 \):
- the shape of rapidity dependence is completely different
- approach the kinematics \( x_B \gtrsim 10^{-2} \), where dipole model is less reliable.
Analysis of cross-sections

- $J/\psi \eta_c$ has the largest cross-section, dominated by contributions of “type-A” diagrams, “type-B” is strongly suppressed.
  ⇒ Important for understanding processes, which get contributions only from “type-B”.
- Strong $p_T$-dependence $\sim 1/p_T^n$. Dominant contribution from $p_T \lesssim 1$ GeV.

Rapidity sign:
- Positive in direction of $e/\gamma^\star$
- Negative in direction of proton

- For $ep$ suppression at very forward direction due to leptonic prefactor
- For other colliders (FCC-he, LHeC) predictions are similar
Azimuthal dependence

- Can study dependence on azimuthal angle $\phi$ between $p^J/\psi$ and $p^{\eta_c}$

**Motivation:**
- might be used to study dependence on angle $\varphi$ between color dipole orientation and impact parameter

- Early phenomenological parametrizations ($b$-CGC, $b$-Sat): no $\varphi$-dep.,
  \[ N = N(x, |r|, |b|) \]

- Might be incompatible with JIMWLK [PRD 99 (2019) 7, 074004]

**Modern expectations:**
- Dependence is mild:
  \[ N(x, |r|, |b|) = v_0 (1 + v_2 \cos 2\varphi) \]
- $v_0$ is taken from conventional parametrizations
- $v_2 \ll 1$ is a small phenomenological parameter
- $v_2$ might be accessed in dijet production

[PRD 99 (2019) 7, 074004]
Azimuthal dependence

- The dependence on $\phi$ has a peak at $\phi = \pi$ (back-to-back kinematics):

$$EIC_{Ep=275 \text{ GeV}} \quad y_1 = y_2 = 0 \quad v_2 = 0 \quad v_2 = 0.05 \quad v_2 = 0.1 \quad \pi/4 \quad \pi/2 \quad 3\pi/4 \quad \pi$$

- Minimizes momentum transfer to proton at fixed $|p_{J/\psi}^\perp|$ and $|p_{\eta_c}^\perp|$:

$$|t| \approx |p_{J/\psi}^\perp|^2 + |p_{\eta_c}^\perp|^2 - 2|p_{J/\psi}^\perp||p_{\eta_c}^\perp| \cos \phi$$

- $R(\phi)$ is cross-section normalized to 1 at peak ($\phi = \pi$) to eliminate rapidity dependent factors, integrated over $|p_{J/\psi}^\perp|$, $|p_{\eta_c}^\perp|$ to improve sensitivity

- Changes $\gtrsim$ an order of magnitude, whereas $v_2$ is just a few % effect

- Suggestion: study $G(\phi) = \sqrt{R(\phi)R(\pi - \phi)}$ instead of $R(\phi)$
  - At small $|t|$ cross-section $d\sigma \sim e^{Bt}$, so $\phi$-dependence due to $t(\phi)$ (almost) cancels.
  $\Rightarrow$ Sensitivity to $v_2$ is much stronger
Predictions for quarkonia with $b$-mesons

- Bottomonia-bottomonia pairs (e.g. $\Upsilon(1S)\eta_b$) are similar to all-charm; numerically have much smaller cross-section

- Mixed pairs are more interesting, probe subsets of diagrams:

  \[ B_c^+ B_c^- \text{ might be produced only via mechanism } (A): \]

  \[ \begin{array}{c}
    \gamma^* \\
    M_1 \\
    \psi(\gamma)
  \end{array} \]

  \[ \begin{array}{c}
    M_2 \\
    \bar{Q}Q \bar{Q}Q
  \end{array} \]

  \[ p \quad p \]

  - $C$-parity does NOT give restrictions for internal quantum numbers, so can study both scalar ($B_c$) and vector ($B_c^*$) quarkonia in different combinations

\[ EIC \quad (se_{ep}=141 \text{ GeV}) \quad E_{e}\times E_p=18\times275 \text{ GeV} \quad y_1=y_2=0 \]

\[ d\sigma/dy_1 dp_1 \quad d\sigma/dy_2 dp_2 \quad d\phi, \text{ pb}/\text{GeV}^4 \]
Predictions for quarkonia with $b$-mesons (II)

- Charmonia-Bottomonia pairs are produced only via mechanism ($B$):

- Results for cross-sections:

- Qualitatively similar behaviour for $p_T$, $\phi$, $y$-dependence.
- Suppression with mass $\sim (\Lambda/\mu_1)^{2n} (\Lambda/\mu_2)^{2n}$, where $\mu_i$ is the reduced mass of the $\bar{Q}Q$ pair in mesons $M_1, M_2$
  - For $\Upsilon(1S)\eta_c$ cross-section is much smaller than for $B_c^+B_c^-$ since it gets contribution only from “small” type-$B$ diagrams
Studies in ultraperipheral $pp$ and $pA$ collisions @LHC

Ultraperipheral collisions:
- Impact parameter $b > R_A + R_B$
- Proceed via exchange of quasireal photon ($Q^2 < 1 \text{GeV}^2$)
- Nuclear targets:
  - Enhancement by $\sim Z$ (atomic number) in amplitude, $\sim Z^2$ in cross-section
  - Due to nuclear form factor $Q^2 \lesssim 1/R_A^2 \lesssim 0.1 \text{GeV}^2$
- Feasibility demonstrated at RHIC, LHC

- Qualitatively the same behavior as for $ep$ (see our publication for more details)
Our mechanism \((J/\psi \eta_c)\) vs. \(J/\psi J/\psi\) production

- \(J/\psi J/\psi\) proceeds via \(\gamma\gamma \rightarrow J/\psi J/\psi\) sub-process, extra suppression \(\sim \mathcal{O}(\alpha_{em}^2)\)
- Extra photon \(\Rightarrow\) additional \(\mathcal{O}(\alpha_{em}^2)\)-suppression in cross-sections

- Comparison with predictions from [PRD 101 (2020) no.3, 034025]:

- Cross-section of suggested mechanism is larger by 2 orders of magnitude (not by factor \(\sim \mathcal{O}(\alpha_{em}^{-2}) \sim 10^4\) as naively expected)
We suggest to study exclusive production of opposite C-parity quarkonia (\(J/\psi \eta_c, J/\psi \chi_c\)) as a complementary source of information about the gluonic field of the target.

- Access to GPD \(H_g\) in collinear factorization approach.
- Access to dipole amplitude in color dipole approach.

The cross-section are large enough for experimental studies, at least for charmonia pairs.

- Bottomonia and \(B_c^+ B_c^-\) pairs have smaller cross-sections, but are also theoretically interesting.
- The quarkonia pairs are produced predominantly with small and oppositely directed transverse momenta (\(|p_\perp| \lesssim 1\, \text{GeV}\)), small rapidity difference.
Summary

- We suggest to study exclusive production of opposite C-parity quarkonia ($J/\psi \eta_c$, $J/\psi \chi_c$...) as a complementary source of information about the gluonic field of the target
  - Access to GPD $H_g$ in collinear factorization approach
  - Access to dipole amplitude in color dipole approach

- The cross-section are large enough for experimental studies, at least for charmonia pairs
  - Bottomonia and $B_c^+ B_c^-$ pairs have smaller cross-sections, but are also theoretically interesting
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Thank You for your attention!
Appendix I: Amplitude in color dipole picture

Eikonal picture:

- \( t \)-channel interactions \( \Rightarrow \) multiplicative factors in config. space

\( \Rightarrow \) The amplitude factorizes into wave functions with a linear combination of color singlet dipole amplitudes

\[
\mathcal{A} \sim \prod_{s=1}^{4} \left( \int d\alpha_s d^2 r_s \right) \sum_{ijklm} \psi_{M_1} (\alpha_{ij}, r_i - r_j) \psi_{M_2} (\alpha_{kl}, r_k - r_\ell) \otimes c_m N (x, r_m, b_m) \psi^{(\gamma)}_{QQQQ} (r_1, r_2, r_3, r_4) e^{i \left( p_T^{(1)} \cdot r_{ij} + p_T^{(2)} \cdot r_{k\ell} \right)}
\]

where \( r_m, b_m \) are some linear combinations of \( r_1...r_4 \), and \( c_m \) are color factors

\( \Rightarrow \) The wave function \( \psi^{(\gamma)}_{QQQQ} \) is evaluated perturbatively, since \( \alpha_s (m_c) \ll 1 \)

\( \Rightarrow \) The quarkonia WFs \( \psi_{M_1}, \psi_{M_2} \) are evaluated in potential models; comparable with LC-Gauss for \( J/\psi \) wave function

\( \Rightarrow \) In general results are close to each other, discrepancy \( \sim \alpha_s (m_c) \sim 1/3 \) (see left)