$T_{cc}$ and other exotic states with two open heavy quarks

Methods to detect the J=1,2 partners of the $X_0(2900)$


IFIC, Universidad de Valencia and CSIC

Local hidden gauge approach and chiral Lagrangians

Predictions for molecular $D^* K^{*\bar{b}}$ and $D^*D^*$ states in 2010

Methods to detect these states

Discussion to the light of the $T_{cc}$ state

Predictions for $D(s)(*)D(s)(*)$ and $B(s)(*)B(s)(*)$ states

The new state $X(3960)$ seen in $D_s^+ D_s^-$ is the same state $X(3930)$ seen in $D^+D^-$. 
\[ \mathcal{L}_{VVV} = ig \langle (V_\mu \partial_\nu V^\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle \]

\[ g = M_V / 2f \ (M_V \approx 800 \text{ MeV}, \ f = 93 \text{ MeV}) \]

\[ \mathcal{L}_{VPP} = -ig \langle V_\mu [P, \partial_\mu P] \rangle \]

\[ -it = -g(V_\mu \partial_\nu V_\mu - \partial_\nu V_\mu V_\mu)_{ij} V_{ji} \frac{i}{q^2 - M_V^2} \frac{V_{\mu}^\mu}{V_{lm}[P, \partial_\nu P]_{ml}} \]

\[ \sum_{pol} \epsilon^\nu_{ji} \epsilon^\nu'_{lm} = \left( -g^{\nu\nu'} + \frac{q^\nu q^\nu'}{M_V^2} \right) \delta_{jl} \delta_{im} \]

\[ -it = -i \frac{g^2}{M_V^2} \langle (V_\mu \partial_\nu V_\mu - \partial_\nu V_\mu V_\mu)[P, \partial_\nu P] \rangle \]

\[ \mathcal{L} = -\frac{1}{4f^2} \langle [V_\mu, \partial_\nu V^\mu][P, \partial_\nu P] \rangle \]

Neglecting the \( k/M_V \)

\[ \epsilon_1(k) = (0, 1, 0, 0) \]

\[ \epsilon_2(k) = (0, 0, 1, 0) \]

\[ \epsilon_3(k) = (|k|, 0, 0, \omega_k)/m_w \]

State predicted of D* K*bar nature. This contains c s quarks and is exotic

The local hidden gauge for VV interaction has an extra contact term

\[ \mathcal{L}_{\text{VVVV}} = \frac{1}{2} g^2 \langle [V_\mu, V_\nu] V^\mu V^\nu \rangle \]

Spin projection operators

\[
V_\mu = \begin{pmatrix}
\frac{\omega + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\
\rho^- & \frac{\omega - \rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\
K^{*0} & \bar{K}^{*0} & \phi & D_s^{*-} \\
D^{*0} & D^{*+} & D_s^{*+} & J/\psi
\end{pmatrix}
\]

\[
\mathcal{P}^{(0)} = \frac{1}{3} \epsilon_\mu \epsilon_\nu \epsilon_\nu \epsilon_\nu \\
\mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon_\nu \epsilon_\nu - \epsilon_\mu \epsilon_\nu \epsilon_\nu \epsilon_\mu) \\
\mathcal{P}^{(2)} = \left\{ \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon_\nu \epsilon_\nu + \epsilon_\mu \epsilon_\nu \epsilon_\nu \epsilon_\mu) - \frac{1}{3} \epsilon_\mu \epsilon_\mu \epsilon_\nu \epsilon_\nu \right\}
\]
G is regularized either with a cutoff in the three momentum or dimensional regularization, with qmax, or a subtraction constant $\alpha$.

\[
T = (\hat{1} - VG)^{-1} V.
\]

\[
G_i = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - M_1^2 + i\epsilon} \frac{1}{(P - q)^2 - M_2^2 + i\epsilon}
\]
Decay terms, added to V and iterated in the Bethe Salpeter equation. Through its imaginary part they provide the decay to DKbar

\[ F(q^2) = e^{((q^0)^2 - |\tilde{q}|^2)/\Lambda^2} \]
# TABLE VI.

$C = 1; S = -1; I = 0$. Mass and width for the states with $J = 0$ and 2.

<table>
<thead>
<tr>
<th>$I[J^P]$</th>
<th>$\sqrt{\Delta}$ pole (MeV)</th>
<th>Model</th>
<th>$\Gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0[$0^+$]</td>
<td>2848</td>
<td>A, $\Lambda = 1400$ MeV</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A, $\Lambda = 1500$ MeV</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B, $\Lambda = 1000$ MeV</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B, $\Lambda = 1200$ MeV</td>
<td>59</td>
</tr>
<tr>
<td>0[$1^+$]</td>
<td>2839</td>
<td>Convolution</td>
<td>3</td>
</tr>
<tr>
<td>0[$2^+$]</td>
<td>2733</td>
<td>A, $\Lambda = 1400$ MeV</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A, $\Lambda = 1500$ MeV</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B, $\Lambda = 1000$ MeV</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B, $\Lambda = 1200$ MeV</td>
<td>36</td>
</tr>
</tbody>
</table>


R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 102, 112003 (2020)

$X_0(2866) : M = 2866 \pm 7$ and $\Gamma = 57.2 \pm 12.9$ MeV,

$X_1(2900) : M = 2904 \pm 5$ and $\Gamma = 110.3 \pm 11.5$ MeV

Decaying to $D \bar{K}$

The state predicted corresponds to the $X_0(2866)$
Revision to the light of experimental results  

**Decay mode**

\[
\mathcal{L} = \frac{i G'}{\sqrt{2}} \varepsilon^{\mu \nu \alpha \beta} \langle \delta_{\mu \nu} V_\nu \delta_\alpha V_\beta P \rangle
\]

with \( G' = \frac{3 g'}{4 \pi^2 f} \); \( g' = -\frac{G_V m_\rho}{\sqrt{2} f^2} \), \( G_V \simeq 55 \text{ MeV} \),

\( q_{\text{max}} \) is chosen to fit the exact mass \( \Lambda \) to get the precise width \( \Gamma \) of \( X_0 \)

\[\begin{array}{cccc}
I(J^P) & M[\text{MeV}] & \Gamma[\text{MeV}] & \text{Coupled channels} & \text{state} \\
0(2^+) & 2775 & 38 & D^* \bar{K}^* & ? \\
0(1^+) & 2861 & 20 & D^* \bar{K}^* & ? \\
0(0^+) & 2866 & 57 & D^* \bar{K}^* & X_0(2866) \\
\end{array}\]

No D Kbar decay
No D* Kbar decay
The $\bar{B}^0 \to D^{*+}\bar{D}^{*0}K^-$ reaction to detect the $I = 0$, $J^P = 1^+$ partner of the $X_0(2866)$

L.R. Dai, R. Molina, E. O,

Looking for the exotic $X_0(2866)$ and its $J^P = 1^+$ partner in the

$\bar{B}^0 \to D^{(*)+}K^-K^{(*)0}$ reactions


Phys.Rev.D 105 (2022) 9, 096022
Method to observe the $J^P = 2^+$ partner of the $X_0(2866)$ in the
$B^+ \to D^+ D^- K^+$ reaction

\[
\frac{d\Gamma}{dM_{\text{inv}} d\Omega} = \frac{1}{(2\pi)^4} \frac{1}{8M_B^2} p_{D^-} \tilde{k} |t|^2
\]

\[
p_{D^-} = \frac{\lambda^{1/2}(M_B^2, m_D^2, M_{\text{inv}}^2)}{2M_B}
\]

\[
\tilde{k} \equiv q = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_D^2, m_K^2)}{2M_{\text{inv}}}
\]
\[ t = \alpha \frac{1}{G_{D^* \bar{K}^*}(M_{\text{inv}})} \frac{1}{\frac{M_{\text{inv}}^2 - M_{X_0}^2}{i M_{X_0} \Gamma_{X_0}}} Y_{00}(\cos \theta) \]

\[ + \beta \frac{1}{G_{D^* \bar{K}^*}(M_{\text{inv}})} \frac{1}{\frac{M_{\text{inv}}^2 - M_{X_2}^2}{i M_{X_2} \Gamma_{X_2}}} Y_{20}(\cos \theta) \]

\[ + \gamma \frac{1}{M_{\text{inv}}^2 - M_{X_1}^2 + i M_{X_1} \Gamma_{X_1}} Y_{10}(\cos \theta) \]

\[ = a Y_{00}(\cos \theta) + b Y_{20}(\cos \theta) + c Y_{10}(\cos \theta) \]

\[ a = \alpha \frac{1}{G_{D^* \bar{K}^*}(M_{\text{inv}})} \frac{1}{\frac{M_{\text{inv}}^2 - M_{X_0}^2}{i M_{X_0} \Gamma_{X_0}}} \]

\[ b = \beta \frac{1}{G_{D^* \bar{K}^*}(M_{\text{inv}})} \frac{1}{\frac{M_{\text{inv}}^2 - M_{X_2}^2}{i M_{X_2} \Gamma_{X_2}}} \]

\[ c = \gamma \frac{1}{M_{\text{inv}}^2 - M_{X_1}^2 + i M_{X_1} \Gamma_{X_1}} \]

\( \theta \) is the angle between \( K^- \) and \( D^- \) en the \( K^- D^+ \) rest frame
We define the moments

\[ \frac{d \Gamma_i}{d M_{\text{inv}} d \tilde{\Omega}} = \int d \tilde{\Omega} \frac{d \Gamma}{d M_{\text{inv}} d \tilde{\Omega}} Y_{i0} \]

for \( i = 0, 1, 2, 3, 4 \)

\[ \frac{d \Gamma_0}{d M_{\text{inv}}} = FAC \left[ |a|^2 + |b|^2 + |c|^2 \right], \]

\[ \frac{d \Gamma_1}{d M_{\text{inv}}} = FAC \left[ 2 \text{Re}(ac^*) + \frac{2}{\sqrt{5}} 2 \text{Re}(bc^*) \right], \]

\[ \frac{d \Gamma_2}{d M_{\text{inv}}} = FAC \left[ \frac{2}{7} \sqrt{5} |b|^2 + \frac{2}{5} \sqrt{5} |c|^2 + 2 \text{Re}(ab^*) \right], \]

\[ \frac{d \Gamma_3}{d M_{\text{inv}}} = FAC \sqrt{\frac{15}{7}} \frac{3}{5} 2 \text{Re}(bc^*), \]

\[ \frac{d \Gamma_4}{d M_{\text{inv}}} = FAC \frac{6}{7} |b|^2, \]

\[ FAC = \frac{1}{\sqrt{4\pi}} \frac{1}{(2\pi)^4} \frac{1}{8M_B^2} p_{D- \tilde{k}}. \]
\[ FAC \ |b|^2 = \frac{7}{6} \frac{d \Gamma_4}{dM_{\text{inv}}}, \]

\[ FAC \ 2 \ Re(bc^*) = \frac{5}{3} \sqrt{\frac{7}{15}} \frac{d \Gamma_3}{dM_{\text{inv}}}, \]

\[ FAC \ 2 \ Re(ac^*) = \frac{d \Gamma_1}{dM_{\text{inv}}} - \frac{2}{3} \sqrt{\frac{7}{3}} \frac{d \Gamma_3}{dM_{\text{inv}}}, \]

\[ FAC \left( |a|^2 - \frac{\sqrt{5}}{2} 2 \ Re(ab^*) \right) = \frac{d \Gamma_0}{dM_{\text{inv}}} - \frac{\sqrt{5}}{2} \frac{d \Gamma_2}{dM_{\text{inv}}} - \frac{1}{3} \frac{d \Gamma_4}{dM_{\text{inv}}}. \]
α, β, γ, chosen to obtain the experimental data and X_0, X_1 signals. X_2 small to get little bump.
$\Gamma_3$ removes the $X_0$ and $X_1$ contributions and only shows the interference of $X_1$ and $X_2$. Magnifies the signal for $X_2$. 

\[
\frac{7}{6} \frac{d \Gamma_4}{d M_{\text{inv}}} \quad (2^+ \text{ state alone; solid blue line})
\]

\[
\frac{5}{3} \sqrt{\frac{7}{15}} \frac{d \Gamma_3}{d M_{\text{inv}}} \quad (\text{the interference of the } X_1 \text{ with the } X_2 \text{ resonances; black dashed line})
\]
INTERESTING: the 2$^+$ state in $D^-K^+$ predicted, plus the $X_1(2900)$ measured, gives a structure of 3,4 moments in remarkable agreement with experiment!

LHCb gets it from reflections of $D^+D^-$ resonances

Tim Gershon: data do not exclude 2$^+$ state but we will have to wait for a reanalysis with Run 3 data
Conclusions

--The $X_0(2900)$ as a $D^* K^*\bar{K}$ molecule implies the existence of two partner states with $J^P = 1^+, 2^+$.

--We propose to see the $1^+$ state with two reactions

\[
\overline{B}^0 \rightarrow D^{*}+\overline{D}^{*0}K^- \quad \overline{B}^0 \rightarrow D^{(*)}+K^-K^{(*)0}
\]

Looking in both cases to the $D^{**} K^-$ invariant mass. Estimates of signal to background indicate that the reactions are feasible, and the signals are sizable.

-- The $2^+$ state might have been seen already in the moments of the $D K^+$ mass distribution. But to confirm it, a reanalysis must be done with Run 3 data.
Predictions were done for a $1^+ D^* D^*$ state

The interaction for $D^* D$ is the same since the contact term is zero for $1^+$

Thus, we predict a $D^* D$ bound state with mass with 141 MeV less, 3828 MeV. This overcounts the binding because $D^* D^*$ are identical particles, Bose enhancement, and $D^* D$ are not, but we should expect a $D^* D$ bound state in $1^+$

This state was found as the Tcc of the LHCb collaboration
The Tcc discovery by the LHCb collaboration

Spectra without correction by experimental resolution

\[ m_{\exp} = 3875.09 \text{ MeV} + \delta m_{\exp}, \]

\[ \delta m_{\exp} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV} \]

\[ \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV} \]

Spectra corrected by resolution and analyzed with a unitary amplitude

\[ \delta m_{\exp} = -360 \pm 40^{+4}_{-0} \text{ keV} \]

\[ \Gamma = 48 \pm 2^{+0}_{-14} \text{ keV} \]
\[\mathcal{L}_{VPP} = -ig \langle [P, \partial_\mu P] V^\mu \rangle,\]
\[\mathcal{L}_{VVV} = ig \langle (V^\nu \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu) V^\mu \rangle,\]
\[g = \frac{M_V}{2f}, \quad (M_V = 800 \text{ MeV}, \ f = 93 \text{ MeV}).\]

\[P = \begin{pmatrix}
\frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^- & \frac{\pi^+}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} & K^+ & \bar{D}^0 \\
\eta^-/\sqrt{3} + \eta'/\sqrt{6} & K^- & K^0 & D^- & \bar{D}^0 \\
K^- & \bar{K}^0 & \frac{K^0}{\sqrt{3}} + \sqrt{2/3} \eta' & D^- & \bar{D}^- \\
D^0 & D^- & D^+ & D_s^+ & \bar{D}_s^+ \\
\end{pmatrix} \quad V_\mu = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^- & \frac{\rho^+}{\sqrt{2}} & K^+ & \bar{D}^0 \\
\rho^- & \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^0 & D^- & \bar{D}^- \\
K^- & \bar{K}^0 & \frac{K^0}{\sqrt{3}} + \sqrt{2/3} \eta' & D^- & \bar{D}^- \\
D^0 & D^- & D^+ & D_s^+ & \bar{D}_s^+ \\
\end{pmatrix} \mu\]
There is attraction in $I=0$, repulsion in $I=1$, but due to different masses there is a bit of isospin breaking.
Convolution of the $G$ function:
Origin of the width.

Spectral function
Mass distribution

\[
\begin{align*}
\text{Im}[D(s_V)] &= \text{Im}\left(\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\right) \\
G(\sqrt{s}, M_k, m_k) &= \frac{(M_V + 2\Gamma_V)^2}{(M_V - 2\Gamma_V)^2} \int \frac{ds_V}{(M_V + 2\Gamma_V)^2} G(\sqrt{s}, \sqrt{s_V}, m_k) \times \text{Im}[D(s_V)] \\
G_1 &= i \int \frac{d^4q}{(2\pi)^4} \frac{M_1}{E_1(q) k^0 + p^0 - q^0 - E_1(q) + ie} \\
\Gamma_{D^{*+}}(M_{\text{inv}}) &= \Gamma(D^{*+}) \left(\frac{m_{D^{*+}}}{M_{\text{inv}}}\right)^2 \left[\frac{2}{3} \left(\frac{p_\pi}{p_{\pi,\text{on}}}\right)^3 + \frac{1}{3} \left(\frac{p'_\pi}{p'_{\pi,\text{on}}}\right)^3\right] \\
\Gamma_{D^{*0}}(M_{\text{inv}}) &= \Gamma(D^{*0}) \left(\frac{m_{D^{*0}}}{M_{\text{inv}}}\right)^2 \left[0.647 \left(\frac{p_\pi}{p_{\pi,\text{on}}}\right)^3 + 0.353\right] \\
\end{align*}
\]

where $p_\pi$ is the $\pi^+$ momentum in $D^{*+} \rightarrow D^0\pi^+$ decay $p'_\pi, p'_{\pi,\text{on}}$ are the same magnitudes for $D^{*+} \rightarrow D^+\pi^0$. 

\[\hat{D}^{*0'} \rightarrow D^0\pi^0 \quad D^{*0} \rightarrow D^0\gamma\]
With mass of experimental raw data

\[ \Gamma \approx 80 \text{ keV}. \]

With mass from unitary reanalysis of LHCb data, Mikhasenko

\[ \Gamma \approx 43 \text{ keV}. \]
Works along the molecular structure of Tcc


Xi-Zhe Ling, Ming-Zhu Liu, Li-Sheng Geng, En Wang, Ju-Jun Xie, Phys.Lett.B 826 (2022) 136897


Meng-Lin Du, Vadim Baru, Xiang-Kun Dong, Arseniy Filin, Feng-Kun Guo, Christoph Hanhart, Alexey Nefediev, Juan Nieves, Qian Wang

Phys.Rev.D 105 (2022) 1, 014024

Hong-Wei Ke, Xiao-Hai Liu, Xue-Qian Li, Eur.Phys.J.C 82 (2022) 2, 144


……..
**Prediction of new $T_{cc}$ states of $D^*D^*$ and $D_s^*D^*$ molecular nature**

L. R. Dai,1,2, * R. Molina,2, † and E. Oset2, ‡ 2110.15270

![Diagram](image)

**Table XVI.** Amplitudes for $C = 2$, $S = 0$, and $I = 0$.

<table>
<thead>
<tr>
<th>$J$</th>
<th>Amplitude</th>
<th>Contact</th>
<th>$V$ exchange</th>
<th>$\sim$Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>0</td>
<td>$0$</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>0</td>
<td>$\frac{1}{4}g^2(\frac{2}{m_{J/\psi}} + \frac{1}{m_{\omega}} - \frac{3}{m_{\eta}})(p_1 + p_4)(p_2 + p_3) + (p_1 + p_3)(p_2 + p_4)$</td>
<td>$-25.4g^2$</td>
</tr>
<tr>
<td>2</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>0</td>
<td>$0$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table XVII.** Amplitudes for $C = 2$, $S = 0$, and $I = 1$.

<table>
<thead>
<tr>
<th>$J$</th>
<th>Amplitude</th>
<th>Contact</th>
<th>$V$ exchange</th>
<th>$\sim$Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>$-4g^2$</td>
<td>$\frac{1}{4}g^2(\frac{2}{m_{J/\psi}} + \frac{1}{m_{\omega}} + \frac{1}{m_{\eta}})(p_1 + p_4)(p_2 + p_3) + (p_1 + p_3)(p_2 + p_4)$</td>
<td>$24.3g^2$</td>
</tr>
<tr>
<td>1</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>0</td>
<td>$0$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$D^<em>D^</em> \rightarrow D^<em>D^</em>$</td>
<td>$2g^2$</td>
<td>$\frac{1}{4}g^2(\frac{2}{m_{J/\psi}} + \frac{1}{m_{\omega}} + \frac{1}{m_{\eta}})(p_1 + p_4)(p_2 + p_3) + (p_1 + p_3)(p_2 + p_4)$</td>
<td>$30.3g^2$</td>
</tr>
</tbody>
</table>
### TABLE XVIII. Amplitudes for $C = 2$, $S = 1$, and $I = 1/2$. 

<table>
<thead>
<tr>
<th>$J$</th>
<th>Amplitude</th>
<th>Contact</th>
<th>$V$ exchange</th>
<th>$\sim$ Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$D_s^* D^* \rightarrow D_s^* D^*$</td>
<td>$-4g^2$</td>
<td>$\frac{g^2(p_1+p_4)(p_2+p_3)}{m_k^2} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{j/\phi}^2}$</td>
<td>$19.0g^2$</td>
</tr>
<tr>
<td>1</td>
<td>$D_s^* D^* \rightarrow D_s^* D^*$</td>
<td>0</td>
<td>$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_k^2} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{j/\phi}^2}$</td>
<td>$-19.5g^2$</td>
</tr>
<tr>
<td>2</td>
<td>$D_s^* D^* \rightarrow D_s^* D^*$</td>
<td>$2g^2$</td>
<td>$\frac{g^2(p_1+p_4)(p_2+p_3)}{m_k^2} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{j/\phi}^2}$</td>
<td>$25.0g^2$</td>
</tr>
</tbody>
</table>

### TABLE XIX. Amplitudes for $C = 2$, $S = 2$, and $I = 0$. 

<table>
<thead>
<tr>
<th>$J$</th>
<th>Amplitude</th>
<th>Contact</th>
<th>$V$ exchange</th>
<th>$\sim$ Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$D_s^* D_s^* \rightarrow D_s^* D_s^*$</td>
<td>$-4g^2$</td>
<td>$\frac{g^2}{2} \left( \frac{1}{m_{j/\phi}} + \frac{1}{m_\phi} \right) {(p_1 + p_4)(p_2 + p_3) + (p_1 + p_3)(p_2 + p_4)}$</td>
<td>$15.0g^2$</td>
</tr>
<tr>
<td>1</td>
<td>$D_s^* D_s^* \rightarrow D_s^* D_s^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$D_s^* D_s^* \rightarrow D_s^* D_s^*$</td>
<td>$2g^2$</td>
<td>$\frac{g^2}{2} \left( \frac{1}{m_{j/\phi}} + \frac{1}{m_\phi} \right) {(p_1 + p_4)(p_2 + p_3) + (p_1 + p_3)(p_2 + p_4)}$</td>
<td>$21.0g^2$</td>
</tr>
</tbody>
</table>
1. $D^*D^* \to D^*D$ decay

$$|D^*D^*, I = 0\rangle = -\frac{1}{\sqrt{2}} |D^{*+}D^{*0} - D^{*0}D^{*+}\rangle$$

$$\mathcal{L}_{VPP} = -i g \langle [\mathbf{P}, \partial_\mu \mathbf{P}] V^\mu \rangle$$

Plus $p3 \leftarrow p4$

$$\mathcal{L}_{VVP} = \frac{G'}{\sqrt{2}} e^{\mu \nu \alpha \beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta \mathbf{P} \rangle$$
\[-i t = 4 \frac{9}{2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2E_{D^*}(q)} \frac{i}{p_1^0 - q^0 - E_{D^*}(q) + i\varepsilon} \frac{1}{2E_D(q)} \times \frac{q^0 - m^2_D + i\varepsilon}{p_2^0 + q^0 - E_D(q) + i\varepsilon (p_2^0 - p_4 + q)^2 - m^2_{\pi} + i\varepsilon} q^4 \]

\[\text{Im} \frac{1}{x+i\varepsilon} = -i\pi\delta(x)\]

\[\text{Im} V_{\text{box}} = -6 \frac{1}{8\pi} \frac{1}{\sqrt{s}} q^5 E_{D^*}^2 (\sqrt{2}g)^2 \left( \frac{G'}{2} \right)^2 \left( \frac{1}{(p_2^0 - E_D(q))^2 - q^2 - m^2_{\pi}} \right) F^4(q) F_{HQ} \]

\[q = \frac{\lambda^{1/2}(s, m^2_{D^*}, m^2_{D})}{2\sqrt{s}} ; \quad E_{D^*} = \frac{\sqrt{s}}{2} \quad F(q) = e^{(q^0)^2 - q^2}/\Lambda^2 \quad q^0 = p_1^0 - E_{D^*}(q) \quad F_{HQ} = \left( \frac{m_{D^*}}{m_K} \right)^2 \]

\[\Lambda = 1200 \text{MeV} \]
\[ \left| T_{D^* D \to D^* D^*} \right|^2 \]

\begin{align*}
\text{\( q_{\text{max}} = 450 \text{ MeV} \)} & & \text{\( q_{\text{max}} = 420 \text{ MeV} \)} \\
M_{D^* D^*} & = 4014.08 \text{ MeV} & M_{D^* D^*} & = 4015.54 \text{ MeV} \\
B_{D^* D^*} & = 3.23 \text{ MeV} & B_{D^* D^*} & = 1.56 \text{ MeV} \\
\Gamma_{D^* D^*} & = 2.3 \text{ MeV} & \Gamma_{D^* D^*} & = 1.5 \text{ MeV} \\
M_{D_s^+ D^*} & = 4122.46 \text{ MeV (cusp)} & M_{D_s^+ D^*} & = 4122.46 \text{ MeV (cusp)} \\
\Gamma_{D_s^+ D^*} & = 70 - 100 \text{ KeV} & \Gamma_{D_s^+ D^*} & = 70 - 100 \text{ KeV} 
\end{align*}
Masses and widths of the exotic molecular $B(s)^*(s)$ states


*Phys.Rev.D* 105 (2022) 7, 074017

$$G(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_B^2 + i\varepsilon (P - q)^2 - m_{B^*}^2 + i\sqrt{(P - q)^2} \Gamma_{B^*} ((P - q)^2)}$$

$$\Gamma_{B^*}(s') = \Gamma_{B^*}(m_{B^*}^2) \frac{m_{B^*}^2}{s'} \left( \frac{p_{\gamma}(s')}{p_{\gamma}(m_{B^*}^2)} \right)^3 \Theta(\sqrt{s'} - m_B)$$

After $q^0$ integration

$$G(s) \simeq \int_0^{q_{\text{max}}} dq \frac{q^2}{4\pi^2} \frac{\omega_B + \omega_{B^*}}{\omega_B \omega_{B^*}} \frac{1}{\sqrt{s + \omega_B + \omega_{B^*}}}$$

$$\times \frac{1}{\sqrt{s - \omega_B - \omega_{B^*}} + i \frac{\sqrt{s}}{2\omega_{B^*}} \Gamma_{B^*}(s')}$$
TABLE V. States of $J^P = 1^+$ obtained from different configurations. The binding $B$ is referred to the closest threshold.

<table>
<thead>
<tr>
<th>States</th>
<th>$M$ (MeV)</th>
<th>$B$ (MeV)</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^*B$ ($I = 0$)</td>
<td>10583</td>
<td>21</td>
<td>14 eV</td>
</tr>
<tr>
<td>$B_s^*B - B^*B_s$ ($I = \frac{1}{2}$)</td>
<td>10681</td>
<td>11</td>
<td>45 eV</td>
</tr>
<tr>
<td>$B^<em>B^</em>$ ($I = 0$)</td>
<td>10630</td>
<td>19</td>
<td>8 MeV</td>
</tr>
<tr>
<td>$B_s^<em>B^</em>$ ($I = \frac{1}{2}$)</td>
<td>10728</td>
<td>12</td>
<td>0.5 MeV</td>
</tr>
</tbody>
</table>
The $X(3960)$ seen in $D_s^+D_s^-$ as the $X(3930)$ state seen in $D^+D^-$

M. Bayar,$^{1,2,*}$ A. Feijoo,$^{2,†}$ and E. Oset$^{2,‡}$

And associate it to a new state

The LHCb finds a peak in the $D_s^+D_s^-$ invariant mass of the reaction

$$B^+ \rightarrow D_s^+D_s^-K^+$$

decay.

And associate it to a new state

$$J^{PC} = 0^{++} \ ; \ M_0 = 3955 \pm 6 \pm 11 \ \text{MeV} \ ; \ \Gamma_0 = 48 \pm 17 \pm 10 \ \text{MeV}$$

We argue that this is not a new state but the one already observed in $D^+D^-$ at 3930 MeV
The $D^+ D^-$, $Ds^+ Ds^-$ interaction together with lighter coupled channels was studied in


But only one bound state of $D \bar{D}$ in $I=0$ was obtained.

But in QCD Lattice a state coupling to $Ds^+ Ds^-$ was found

S. Prelovsek, S. Collins, D. Mohler, M. Padmanath and S. Piemonte, JHEP 06 (2021), 035.

It also couples to $D^+ D^-$ but weakly.

CAN ONE RECONCILE THESE FACTS?
D Dbar and Ds+ Ds- interaction revisited

If we remove non diagonal term → two states appear

If we keep that term → the state coupling to Ds+ Ds- disappears

Reducing by 0.7 the $1/MK^2$ term two states appear

\[ (p_1 + p_3)(p_2 + p_4) \rightarrow \frac{1}{2} \left[ 3s - (m_1^2 + m_2^2 + m_3^2 + m_4^2) - \frac{1}{s} (m_1^2 - m_2^2)(m_3^2 - m_4^2) \right] \]

If we remove non diagonal term → two states appear

If we keep that term → the state coupling to Ds+ Ds- disappears

\[ T = [1 - VG]^{-1} V \]

Reducing by 0.7 the $1/MK^2$ term two states appear
Fine tuning the interaction

In K* exchange one has the interaction

\[ \frac{1}{(q^0^2 - q^2 - M_{K^*}^2)} , \quad q^0 = 0, \quad q = 613 \text{ MeV at Ds}^+ \text{ Ds}^- \text{ threshold} \]

\[ \frac{1}{(q^0^2 - q^2 - M_{K^*}^2)} / (1/M_{K^*}^2) = 0.67. \] With this natural reduction TWO STATES APPEAR

TABLE I. Masses and widths of the poles dynamically generated by the model, as well as, the corresponding modulus of the couplings |\(g_i\)|.

| Pole I | M [MeV] | \(\Gamma\) [MeV] | \(|g_{\bar{D}D}|\) [MeV] | \(|g_{\bar{D}sD_s}|\) [MeV] |
|--------|---------|-----------------|------------------|------------------|
| Pole I | 3699    | -               | 14516            | 5897             |
| Pole II \((X_0(3930))\) | 3936 | 11              | 2858             | 9076             |
Relationship between $D^+D^-$ and $Ds^+Ds^-$ production in $B^- \rightarrow D^+D^-K^-$ and $Ds^+Ds^-K^-$

The $c\bar{c}$ pair is hadronized

$$c\bar{c} \rightarrow \sum_i c\bar{q}_i q_i \bar{c} \rightarrow \sum_i P_{4i} P_{4i} = D^0\bar{D}^0 + D^+D^- + D_s^+D_s^- = \sqrt{2}D\bar{D} + D_s^+D_s^-$$

Final state interaction

$$\tilde{t}_{D^+D^-} = C \left( 1 + G_{D\bar{D}}(M_{inv}) T_{D\bar{D},D\bar{D}}(M_{inv}) + \frac{1}{\sqrt{2}} G_{Ds\bar{D}s}(M_{inv}) T_{Ds\bar{D}s,\bar{D}\bar{D}}(M_{inv}) \right)$$

$$\tilde{t}_{D_s^+D_s^-} = C \left( 1 + \sqrt{2} G_{D\bar{D}}(M_{inv}) T_{D\bar{D},D_s^+D_s^-}(M_{inv}) + G_{Ds\bar{D}s}(M_{inv}) T_{Ds\bar{D}s,D_s^+D_s^-}(M_{inv}) \right)$$
The second pole couples both to $D_s^+ D_s^-$ and $D^+ D^-$ and produces the peak at 3930 MeV and an enhancement at the $D_s^+ D_s^-$ threshold compatible with the LHCb data. **THERE IS NO NEED TO INVOKE A NEW STATE at 3955 MeV!!**
Conclusions

-- We have made predictions for the interaction of D* D and found a bound state with mass and width in agreement with the Tcc state.

-- With the same regulator for the loops we study the D* D* interaction and find a bound state with I=0 and J^P= 1^+, with binding and width of the order of 1 MeV, much bigger than for the T_{cc} state. It decays to D* D.

-- We also make predictions for states of B(*) B(*) type or B(*)s B(*) and find four bound states all with J^P=1^+

-- We give arguments to support the idea that the X(3930) and X(3960) states are actually the same state, showing in D^+ D^- and D_{s}^+ D_{s}^- , respectively.
Conclusions

In the recently observed states in the LHCb, Belle, Babar, BesIII, there are many states which qualify as dynamically generated from the interaction of hadron components: molecular states.

Many of these states were predicted before. The experiment has served to fine tune some parameters which allow to make more refined predictions for other states not yet found.

The chiral unitary approach in the SU(3) sector has proved to be quite accurate to study the interaction of hadrons and eventually find poles in the t-matrix that correspond to states.

The local hidden gauge approach, with the exchange of vector mesons, is equivalent to the chiral unitary approach in SU(3). An extension of the LHGA has been done to the charm and bottom sectors, which respects heavy quark symmetry and turns out rather accurate interpreting results and making predictions.

More predictions have been made. We hope that they can be tested in the near future.

Attention must also be payed to hybrids of q qbar or qqq and molecular components. J. Nieves, F. K. Guo, David Rodriguez Entem ….
TABLE II. The pole position of the $T_{cc}^{+}$ relative to the $D^{*+}D^{0}$ threshold and the Riemann sheet (RS) where the pole is located in each scheme (see the text for details). The errors are statistical propagated from fitting to the LHCb data while the uncertainties from the cutoff variation are well within the errors quoted here.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole [keV]</td>
<td>$-368^{+43}_{-42} - i(37 \pm 0)$ (RS-I)</td>
<td>$-333^{+41}_{-36} - i(18 \pm 1)$ (RS-II)</td>
<td>$-356^{+39}_{-38} - i(28 \pm 1)$ (RS-II)</td>
</tr>
</tbody>
</table>
Unveiling the $K_1(1270)$ double-pole structure in the $\bar{B} \to J/\psi \rho \bar{K}$ and $\bar{B} \to J/\psi \bar{K}^* \pi$ decays


J. M. Dias, G. Toledo, L. Roca, and E. Oset

(a) $\bar{K}^*$ $\pi$ $\rho$ $V$

(b) $\bar{K}^*$ $K^*$ $K^*$ $V$
How much is the compositeness of a bound state constrained by $a$ and $r_0$? The role of the interaction range

Jing Song,$^{1,2,*}$ L.R. Dai,$^{3,2,†}$ and E. Oset$^{2,‡}$

\begin{align*}
V_{\text{eff}} &= V_0 + \beta(s - s_0) \\
T(s) &= \frac{1}{[V_0 + \beta(s - s_0)]^{-1} - G(s)} \\
T(s) &= \frac{1}{\left[\frac{1}{G(s_0)} + \beta(s - s_0)\right]^{-1} - G(s)} \\
G_1 &= \int_{|q| < q_{\text{max}}} \frac{d^3q}{(2\pi)^3} \frac{w_1(q) + w_2(q)}{2w_1(q)w_2(q)} \frac{1}{s - (w_1(q) + w_2(q))^2 + i\epsilon} \\
P_2 &= 1 - P_1 = Z = -g^2G(s_0)^2\beta \\
g^2 &= \frac{1}{-G(s_0)^2\beta - \frac{\partial G}{\partial s}|_{s_0}} \\
8\pi \sqrt{s} \left\{ \frac{1}{G(s_0)} + \beta(s - s_0) \right\}^{-1} - \text{Re}G(s) &\approx \frac{1}{a} - \frac{1}{2}r_0 k^2
\end{align*}