T_{cc} and other exotic states with two open heavy quarks Methods to detect the J=1,2 partners of the $X_0(2900)$

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Local hidden gauge approach and chiral Lagrangians

Predictions for molecular D* K*bar and D*D* states in 2010

Methods to detect these states

Discussion to the light of the T_{cc} state

Predictions for D(s)(*)D(s)(*) and B(s)(*)B(s)(*) states

The new state X(3960) seen in $Ds^+ Ds^-$ is the same state X(3930) seen in D⁺D⁻



VP INTERACTION IN THE LOCAL HIDDEN GAUGE APPROACH Bando et al Phys Rep. 164

$$\begin{array}{c} \underbrace{V}{}, \underbrace{V}{}, \underbrace{V}{}, \underbrace{V}{}, \underbrace{\mathcal{L}_{VVV}}_{V} = ig\langle (V_{\mu}\partial_{\nu}V^{\mu} - \partial_{\nu}V_{\mu}V^{\mu})V^{\nu} \rangle & \text{Neglecting the k/M}_{\vee} \\ f = ig\langle V_{VV} = ig\langle (W_{\mu}\partial_{\nu}V^{\mu} - \partial_{\nu}V_{\mu}V^{\mu})K^{\nu}, f = 93 \text{ MeV} \rangle & \begin{aligned} \varepsilon_{1}(\mathbf{k}) = (0, 1, 0, 0) \\ \varepsilon_{2}(\mathbf{k}) = (0, 0, 1, 0) \\ \varepsilon_{2}(\mathbf{k}) = (0, 0, 1, 0) \\ \varepsilon_{3}(\mathbf{k}) = (|\mathbf{k}|, 0, 0, \omega_{\mathbf{k}})/m_{\mathbf{W}} \\ -it = -g(V^{\mu}\partial_{\nu}V_{\mu} - \partial_{\nu}V_{\mu}V^{\mu})_{ij}V^{\nu}_{ji}\frac{i}{q^{2} - M_{V}^{2}}V^{\nu'}_{lm}[P, \partial_{\nu'}P]_{ml} \\ \sum_{pol} \epsilon^{\nu}_{ji}\epsilon^{\nu'}_{lm} = \left(-g^{\nu\nu'} + \frac{q^{\nu}q^{\nu'}}{M_{V}^{2}}\right)\delta_{jl}\delta_{im} \\ -it = -i\frac{g^{2}}{M_{V}^{2}}\langle (V^{\mu}\partial_{\nu}V_{\mu} - \partial_{\nu}V_{\mu}V^{\mu})[P, \partial^{\nu}P]\rangle \end{array}$$

 $\mathcal{L} = -\frac{1}{4f^2} \langle [V^{\mu}, \partial_{\nu} V^{\mu}] [P, \partial^{\nu} P] \rangle \quad \text{Chiral Lagrangian of M. C. Birse, Z. Phys. A 355, 231 (1996)}$

New interpretation for the Ds2*(2573) and the prediction of novel exotic charmed mesons R. Molina, T. Branz, E. Oset, PHYSICAL REVIEW D 82, 014010 (2010)

State predicted of D* K*bar nature. This contains c s quarks and is exotic

The local hidden gauge for VV interaction has an extra contact term

$$\mathcal{L}_{\text{VVVV}} = \frac{1}{2}g^2 \langle [V_{\mu}, V_{\nu}] V^{\mu} V^{\nu} \rangle$$
Spin projection operators

$$V_{\mu} = \begin{pmatrix} \frac{\omega + \rho^{0}}{\sqrt{2}} & \rho^{+} & K^{*+} & \bar{D}^{*0} \\ \rho^{-} & \frac{\omega - \rho^{0}}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D^{*-}_{s} \\ D^{*0} & D^{*+} & D^{*+}_{s} & J/\psi \end{pmatrix}_{\mu} \qquad \mathcal{P}^{(0)} = \frac{1}{3} \epsilon_{\mu} \epsilon^{\mu} \epsilon_{\nu} \epsilon^{\nu} \\ \mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_{\mu} \epsilon_{\nu} \epsilon^{\mu} \epsilon^{\nu} - \epsilon_{\mu} \epsilon_{\nu} \epsilon^{\nu} \epsilon^{\mu}) \\ \mathcal{P}^{(2)} = \{ \frac{1}{2} (\epsilon_{\mu} \epsilon_{\nu} \epsilon^{\mu} \epsilon^{\nu} + \epsilon_{\mu} \epsilon_{\nu} \epsilon^{\nu} \epsilon^{\mu}) - \frac{1}{3} \epsilon_{\mu} \epsilon^{\mu} \epsilon_{\nu} \epsilon^{\nu} \epsilon^{\nu} \epsilon^{\nu} \right)$$

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TABLE XI. Amplitudes for C = 1, S = -1 and I = 0.

J	Amplitude	Contact	V exchange	~Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^{2}$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_*^*}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1+p_3).(p_2+p_4)$	$-9.9g^{2}$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1+p_3).(p_2+p_4)$	$-10.2g^{2}$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^{2}$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1 + p_3).(p_2 + p_4)$	$-15.9g^{2}$

TABLE XII. Amplitudes for C = 1, S = -1 and I = 1.

J	Amplitude	Contact	V exchange	~Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^{2}$	$\frac{\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_{*}}^2} + \frac{g^2}{2}(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2})(p_1 + p_3).(p_2 + p_4)}{(p_1 + p_3).(p_2 + p_4)}$	$9.7g^2$
1	$D^*\bar{K}^* \longrightarrow D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D^*}^2} + \frac{g^2}{2}(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2})(p_1+p_3).(p_2+p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^{2}$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{g^2}{2}\left(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2}\right)(p_1+p_3).(p_2+p_4)$	$15.7g^2$

$$T = (\hat{1} - VG)^{-1}V. \qquad G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_1^2 + i\epsilon} \frac{1}{(P - q)^2 - M_2^2 + i\epsilon}$$

Λ

G is regularized either with a cutoff in the three momentum or dimensional regularization, with qmax, or a subtraction constant α .

Decay terms, added to V and iterated in the Bethe Salpeter equation. Through its imaginary part they provide the decay to DKbar



$I[J^P]$	$\sqrt{s_{\text{pole}}}$ (MeV)	Model	Γ (MeV)
0[0+]	2848	A, $\Lambda = 1400 \text{ MeV}$	23
		A, $\Lambda = 1500 \text{ MeV}$	30
		B, $\Lambda = 1000 \text{ MeV}$	25
		B, $\Lambda = 1200 \text{ MeV}$	59
0[1+]	2839	Convolution	3
0[2+]	2733	A, $\Lambda = 1400 \text{ MeV}$	11
		A, $\Lambda = 1500 \text{ MeV}$	14
		B, $\Lambda = 1000 \text{ MeV}$	22
		B, $\Lambda = 1200 \text{ MeV}$	36

TABLE VI. C = 1; S = -1; I = 0. Mass and width for the states with J = 0 and 2.

R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 125, 242001 (2020)

R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 102, 112003 (2020)

 $X_{0}(2866): M = 2866 \pm 7 \text{ and } \Gamma = 57.2 \pm 12.9 \text{ MeV},$ $X_{1}(2900): M = 2904 \pm 5 \text{ and } \Gamma = 110.3 \pm 11.5 \text{ MeV}$ 6Decaying to DKbar The state predicted corresponds to the X_{0}(2866) Revision to the light of experimental results R. Molina, E. O. Phys.Lett.B 811 (2020) 135870



	state	Coupled channels	Γ[MeV]	M[MeV]	$I(J^P)$
	?	$D^*\bar{K}^*$	38	2775	0(2+)
No D Kbar decay	?	$D^*\bar{K}^*$	20	2861	0(1+)
No D* Khar decay	$X_0(2866)$	$D^*\bar{K}^*$	57	2866	0(0 ⁺)

7

The $\bar{B}^0 \rightarrow D^{*+}\bar{D}^{*0}K^-$ reaction to detect the $I = 0, J^P = 1^+$ partner of the $X_0(2866)$ L.R. Dai, R. Molina, E. O, Phys.Lett.B 832 (2022) 137219

Looking for the exotic $X_0(2866)$ and its $J^P = 1^+$ partner in the

 $\bar{B}^0 \to D^{(*)+} K^- K^{(*)0}$ reactions



Method to observe the $J^P = 2^+$ partner of the $X_0(2866)$ in the $B^+ \rightarrow D^+ D^- K^+$ reaction M. Bayar, E. Oset, PLB 833, 137364 (2022)







(a)

$$\frac{d\Gamma}{dM_{\rm inv}d\tilde{\Omega}} = \frac{1}{(2\pi)^4} \frac{1}{8M_B^2} p_{D^-} \tilde{k} |t|^2$$
$$p_{D^-} = \frac{\lambda^{1/2}(M_B^2, m_D^2, M_{\rm inv}^2)}{2M_B}$$
$$\tilde{k} \equiv q = \frac{\lambda^{1/2}(M_{\rm inv}^2, m_D^2, m_K^2)}{2M_{\rm inv}}$$

$$\begin{split} t &= \alpha \ G_{D^*\bar{K}^*}(M_{\rm inv}) \frac{1}{M_{\rm inv}^2 - M_{X_0}^2 + i \ M_{X_0} \ \Gamma_{X_0}} Y_{00}(\cos\theta) \\ &+ \beta \ G_{D^*\bar{K}^*}(M_{\rm inv}) \frac{1}{M_{\rm inv}^2 - M_{X_2}^2 + i \ M_{X_2} \ \Gamma_{X_2}} Y_{20}(\cos\theta) \\ &+ \gamma \ \frac{1}{M_{\rm inv}^2 - M_{X_1}^2 + i \ M_{X_1} \ \Gamma_{X_1}} Y_{10}(\cos\theta) \\ &\equiv a \ Y_{00}(\cos\theta) + b \ Y_{20}(\cos\theta) + c \ Y_{10}(\cos\theta) \\ &= a \ Y_{00}(\cos\theta) + b \ Y_{20}(\cos\theta) + c \ Y_{10}(\cos\theta) \\ &a = \alpha \ G_{D^*\bar{K}^*}(M_{\rm inv}) \frac{1}{M_{\rm inv}^2 - M_{X_2}^2 + i \ M_{X_0} \ \Gamma_{X_0}} \\ &b = \beta \ G_{D^*\bar{K}^*}(M_{\rm inv}) \frac{1}{M_{\rm inv}^2 - M_{X_2}^2 + i \ M_{X_2} \ \Gamma_{X_2}} \\ &c = \gamma \ \frac{1}{M_{\rm inv}^2 - M_{X_1}^2 + i \ M_{X_1} \ \Gamma_{X_1}} \end{split}$$

 θ is the angle between K⁻ and D⁻ en the K⁻ D⁺ rest frame

 $|t|^2 = |a|^2 Y_{00}^2 + |b|^2 Y_{20}^2 + |c|^2 Y_{10}^2 + 2 \operatorname{Re}(ab^*) Y_{00} Y_{20}$ t= $a Y_{00}(\cos\theta) + b Y_{20}(\cos\theta) + c Y_{10}(\cos\theta)$ $+ 2 Re(ac^*) Y_{00} Y_{10} + 2 Re(bc^*) Y_{20} Y_{10}$

$$\frac{d\Gamma}{dM_{\text{inv}}d\tilde{\Omega}} = \frac{1}{(2\pi)^4} \frac{1}{8M_B^2} p_{D^-} \tilde{k} |t|^2 \qquad \begin{array}{l} \text{We define} \\ \text{the moments} \end{array} \quad \frac{d\Gamma_i}{dM_{\text{inv}}} = \int d\tilde{\Omega} \ \frac{d\Gamma}{dM_{\text{inv}}} \ d\tilde{\Omega} \ Y_{i0} \\ \text{for } i = 0, 1, 2, 3, 4 \end{array}$$

$$\frac{d \Gamma_0}{dM_{\rm inv}} = FAC \left[|a|^2 + |b|^2 + |c|^2 \right], \qquad \qquad \frac{d \Gamma_3}{dM_{\rm inv}} = FL$$

$$\frac{d \ \Gamma_3}{dM_{\rm inv}} = FAC \sqrt{\frac{15}{7}} \ \frac{3}{5} \ 2 \ Re(bc^*),$$

$$\frac{d\ \Gamma_1}{dM_{\rm inv}} = FAC \left[2\ Re(ac^*) + \frac{2}{\sqrt{5}}\ 2\ Re(bc^*) \right], \qquad \qquad \frac{d\ \Gamma_4}{dM_{\rm inv}} = FAC\ \frac{6}{7} |b|^2,$$

$$\frac{d \Gamma_2}{dM_{\rm inv}} = FAC \left[\frac{2}{7} \sqrt{5} \ |b|^2 + \frac{2}{5} \sqrt{5} \ |c|^2 + 2 \ Re(ab^*) \right]$$

$$\left. \int \frac{d \Gamma_4}{dM_{\rm inv}} = FAC \ \frac{6}{7} |b|^2,$$

$$FAC = \frac{1}{\sqrt{4\pi}} \frac{1}{(2\pi)^4} \frac{1}{8M_B^2} p_{D^-} \tilde{k}.$$

$$FAC |b|^{2} = \frac{7}{6} \frac{d \Gamma_{4}}{dM_{\text{inv}}}, \qquad J=2$$

$$FAC 2 Re(bc^{*}) = \frac{5}{3} \sqrt{\frac{7}{15}} \frac{d \Gamma_{3}}{dM_{\text{inv}}}, \qquad \text{Inter J=2,1}$$

$$FAC 2 Re(ac^{*}) = \frac{d \Gamma_{1}}{dM_{\text{inv}}} - \frac{2}{3} \sqrt{\frac{7}{3}} \frac{d \Gamma_{3}}{dM_{\text{inv}}}, \qquad \text{Inter J=0,1}$$

$$FAC \left[|a|^{2} - \frac{\sqrt{5}}{2} 2 Re(ab^{*}) \right] = \frac{d \Gamma_{0}}{dM_{\text{inv}}} - \frac{\sqrt{5}}{2} \frac{d \Gamma_{2}}{dM_{\text{inv}}} - \frac{1}{3} \frac{d \Gamma_{4}}{dM_{\text{inv}}}. \qquad \text{Inter J=0,2 plus J=0}$$



 α , β , γ , chosen to obtain the experimental data and X₀, X₁ signals. X₂ small to get little bump

M_{inv} (D⁺ K⁻) [MeV]

520

INTERESTING: the 2^+ state in D⁻K⁺ predicted, plus the X₁(2900) measured, gives a structure of 3,4 moments in remarkable agreement with experiment!

LHCb gets it from reflections of D⁺D⁻ resonances

Tim Gershon: data do not exclude 2⁺ state but we will have to wait for a reanalysis with Run 3 data

Conclusions

--The X₀(2900) as a D* K*bar molecule implies the existence of two partner states with $J^P = 1^+$, 2^+ .

--We propose to see the 1⁺ state with two reactions

$$\bar{B}^0 \to D^{*+} \bar{D}^{*0} K^ \bar{B}^0 \to D^{(*)+} K^- K^{(*)0}$$

Looking in both cases to the $D^{*+} K^{-}$ invariant mass. Estimates of signal to background indicate that the reactions are feasible, and the signals are sizable.

-- The 2⁺ state might have been seen already in the moments of the D⁻K⁺ mass distribution. But to confirm it, a reanalysis must be done with Run 3 data.

In R. Molina, T. Branz, E. Oset, PHYSICAL REVIEW D 82, 014010 (2010)

Predictions were done for a 1^+ D* D* state

TABLE IVpositions, a	<i>C</i> = 2; <i>S</i> = 0; <i>I</i> = 0. Quantum and couplings g_i in units of MeV. Her	numbers, pole e $\alpha = -1.4$.
$I[J^P]$	$\sqrt{s_{\text{pole}}}$ (MeV)	$g_{D^*D^*}$
0[1+]	3969	16 825

The interaction for D*D is the same since the contact term is zero for 1+

Thus, we predict a D*D bound state with mass with 141 MeV less, 3828 MeV. This overcounts the binding because D*D* are identical particles, Bose enhancement, and D* D are not, but we should expect a D*D bound state in 1+

This state was found as the Tcc of the LHCb collaboration

The Tcc discovery by the LHCb collaboration

Spectra without correction by experimental resolution $m_{exp} = 3875.09 \text{ MeV} + \delta m_{exp},$

$$\delta m_{\rm exp} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}. \ \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}.$$

Spectra corrected by resolution and analyzed with a unitary amplitude

10 $\delta m_{\rm exp} = -360 \pm 40^{+4}_{-0} \text{ keV}, \qquad \Gamma = 48 \pm 2^{+0}_{-14} \text{ keV}.$

A. Feijoo, W.H. Liang, Eulogio Oset, Phys.Rev.D 104 (2021) 11, 114015

$$\begin{aligned} \mathcal{L}_{VPP} &= -ig \, \langle [P, \partial_{\mu} P] V^{\mu} \rangle, \\ \mathcal{L}_{VVV} &= ig \, \langle (V^{\nu} \partial_{\mu} V_{\nu} - \partial_{\mu} V^{\nu} V_{\nu}) V^{\mu} \rangle, \\ g &= \frac{M_V}{2 \, f}, \ (M_V = 800 \text{ MeV}, \ f = 93 \text{ MeV}). \end{aligned}$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^{0}}{\sqrt{2}} & \pi^{+} & K^{+} & \bar{D}^{0} \\ \pi^{-} & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^{0}}{\sqrt{2}} & K^{0} & D^{-} \\ K^{-} & \bar{K}^{0} & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D^{-}_{s} \\ D^{0} & D^{+} & D^{+}_{s} & \eta_{c} \end{pmatrix} \qquad V_{\mu} = \begin{pmatrix} \frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+} & \bar{D}^{*0} \\ \rho^{-} & -\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D^{*-}_{s} \\ D^{*0} & D^{*+} & D^{*+}_{s} & J/\psi \end{pmatrix}_{\mu}$$

$D^{\ast +}D^0, D^{\ast 0}D^+$ the 1, 2 channels, the interaction that we obtain is

12

to different masses there is a bit of isospin breaking

Convolution of the G function: Origin of the width. Spectral function Mass distribution

bution
$$\operatorname{Im}[D(s_V)] = \operatorname{Im}\left(\frac{1}{s_V - M_V^2 + iM_V\Gamma}\right)$$

$$G(\sqrt{s}, M_k, m_k) = \frac{\int\limits_{(M_V - 2\Gamma_V)^2}^{(M_V + 2\Gamma_V)^2} ds_V G(\sqrt{s}, \sqrt{s_V}, m_k) \times \operatorname{Im}[D(s_V)]}{\int\limits_{(M_V - 2\Gamma_V)^2}^{(M_V + 2\Gamma_V)^2} ds_V \operatorname{Im}[D(s_V)]}$$

$$G_{l} = i \int \frac{d^{4}q}{(2\pi)^{4}} \frac{M_{l}}{E_{l}(\mathbf{q})} \frac{1}{k^{0} + p^{0} - q^{0} - E_{l}(\mathbf{q}) + i\epsilon}$$

$$\Gamma_{D^{*+}}(M_{\rm inv}) = \Gamma(D^{*+}) \left(\frac{m_{D^{*+}}}{M_{\rm inv}}\right)^2 \cdot \left[\frac{2}{3} \left(\frac{p_{\pi}}{p_{\pi,\rm on}}\right)^3 + \frac{1}{3} \left(\frac{p'_{\pi}}{p'_{\pi,\rm on}}\right)^3\right]$$

where p_{π} is the π^+ momentum in $D^{*+} \to D^0 \pi^+$ decay $p'_{\pi}, p'_{\pi,\text{on}}$ are the same magnitudes for $D^{*+} \to D^+ \pi^0$.

$$\Gamma_{D^{*0}}(M_{\rm inv}) = \Gamma(D^{*0}) \left(\frac{m_{D^{*0}}}{M_{\rm inv}}\right)^2 \cdot \left[0.647 \left(\frac{p_{\pi}}{p_{\pi,\rm on}}\right)^3 + 0.353\right]$$
$$\dot{D}^{*0} \rightarrow D^0 \pi^0 \qquad D^{*0} \rightarrow D^0 \gamma$$

With mass from unitary reanalysis of LHCb data, Mikhasenko

 $- \overset{*}{D}^{+} \overset{*}{D}^{0}$ thresh. - $\overset{*}{D}^{0} \overset{*}{D}^{+}$ thresh.

3879

3880

3878

Works along the molecular structure of Tcc

L. Meng, G. J. Wang, B. Wang and S. L. Zhu, Phys. Rev. D 104, 051502 (2021)

Xi-Zhe Ling, Ming-Zhu Liu, Li-Sheng Geng, En Wang, Ju-Jun Xie, Phys.Lett.B 826 (2022) 136897

M. Albaladejo, arXiv:2110.02944 [hep-ph]

Meng-Lin Du, Vadim Baru, Xiang-Kun Dong, Arseniy Filin, Feng-Kun Guo, Christoph Hanhart, Alexey Nefediev, Juan Nieves, Qian Wang Phys.Rev.D 105 (2022) 1, 014024

Hong-Wei Ke, Xiao-Hai Liu, Xue-Qian Li, Eur.Phys.J.C 82 (2022) 2, 144

Xiang-Kun Dong, Feng-Kun Guo, Bing-Song Zou, Commun. Theor. Phys. 73 (2021) 12, 125201

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Prediction of new T_{cc} states of D^*D^* and $D^*_sD^*$ molecular nature

TABLE XVII. Amplitudes for C = 2, S = 0, and I = 1.

J	Amplitude	Contact	V exchange	~Total
0	$D^*D^* \rightarrow D^*D^*$	$-4g^{2}$	$\frac{1}{4}g^{2}(\frac{2}{m_{J/\psi}^{2}} + \frac{1}{m_{\omega}^{2}} + \frac{1}{m_{\rho}^{2}})\{(p_{1} + p_{4}).(p_{2} + p_{3}) + (p_{1} + p_{3}).(p_{2} + p_{4})\}$	$24.3g^2$
1	$D^*D^* \rightarrow D^*D^*$	0	0	0
2	$D^*D^* \to D^*D^*$	$2g^2$	$\frac{1}{4}g^{2}(\frac{2}{m_{J/\psi}^{2}} + \frac{1}{m_{\omega}^{2}} + \frac{1}{m_{\rho}^{2}})\{(p_{1} + p_{4}).(p_{2} + p_{3}) + (p_{1} + p_{3}).(p_{2} + p_{4})\}$	$30.3g^2$

J	Amplitude	Contact	V exchange	~Total
0	$D_s^*D^* \rightarrow D_s^*D^*$	$-4g^{2}$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{\nu^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{J/\psi^2}}$	$19.0g^{2}$
1	$D_s^*D^* \rightarrow D_s^*D^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{\nu^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{I/\psi^2}}$	$-19.5g^{2}$
2	$D_s^*D^* \rightarrow D_s^*D^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{K^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{J/\psi^2}}$	$25.0g^2$

TABLE XVIII. Amplitudes for C = 2, S = 1, and I = 1/2.

TABLE XIX. Amplitudes for C = 2, S = 2, and I = 0.

J	Amplitude	Contact	V exchange	~Total
0	$D_s^*D_s^* \to D_s^*D_s^*$	$-4g^{2}$	$\frac{g^2}{2}\left(\frac{1}{m_{J/\psi}^2} + \frac{1}{m_{\phi}^2}\right)\left\{(p_1 + p_4).(p_2 + p_3) + (p_1 + p_3).(p_2 + p_4)\right\}$	$15.0g^{2}$
1	$D_s^*D_s^* \longrightarrow D_s^*D_s^*$	0	0	0
2	$D_s^* D_s^* \to D_s^* D_s^*$	$2g^2$	$\frac{g^2}{2}(\frac{1}{m_{J/\psi}^2} + \frac{1}{m_{\phi}^2})\{(p_1 + p_4).(p_2 + p_3) + (p_1 + p_3).(p_2 + p_4)\}$	$21.0g^2$

$$\begin{split} -i\,t &= 4\frac{9}{2}\frac{1}{3}\int \frac{d^4q}{(2\pi)^4}\frac{1}{2E_{D^*}(\boldsymbol{q})}\frac{i}{p_1^0 - q^0 - E_{D^*}(\boldsymbol{q}) + i\epsilon}\frac{1}{2E_D(\boldsymbol{q})} \\ &\times \frac{i}{p_2^0 + q^0 - E_D(\boldsymbol{q}) + i\epsilon}\frac{i}{q^2 - m_\pi^2 + i\epsilon}\frac{i}{(p_2 - p_4 + q)^2 - m_\pi^2 + i\epsilon}\,\boldsymbol{q}^4 \end{split} \qquad \qquad Im\frac{1}{x + i\epsilon} = -i\pi\delta(x)$$

$$ImV_{\text{box}} = -6\frac{1}{8\pi} \frac{1}{\sqrt{s}} q^5 E_{D^*}^2 (\sqrt{2}g)^2 \left(\frac{G'}{2}\right)^2 \left(\frac{1}{(p_2^0 - E_D(\boldsymbol{q}))^2 - \boldsymbol{q}^2 - m_\pi^2}\right)^2 F^4(q) F_{HQ}$$

$$q = \frac{\lambda^{1/2}(s, m_{D^*}^2, m_D^2)}{2\sqrt{s}}; \qquad E_{D^*} = \frac{\sqrt{s}}{2} \qquad F(q) = e^{((q^0)^2 - q^2)/\Lambda^2} \qquad q^0 = p_1^0 - E_{D^*}(q) \qquad F_{HQ} = \left(\frac{m_{D^*}}{m_{K^*}}\right)^2$$

 $\Lambda = 1200~{\rm MeV}$

Masses and widths of the exotic molecular $B_{(s)}^{(*)}$ B_(s)^(*) states L.~R.~Dai, E.~Oset, A.~Feijoo, R.~Molina, L.~Roca, A.~M.~Torres and K.~P.~Khemchandani

$$Phys.Rev.D \ 105 \ (2022) \ 7, \ 074017$$

$$P-q \qquad \gamma$$

$$P-q \qquad \gamma$$

$$B^* \qquad B^* \qquad B^$$

Phys Rev D 105 (2022) 7 074017

TABLE V. States of $J^P = 1^+$ obtained from different configurations. The binding B is referred to the closest threshold.

States	$M ({ m MeV})$	$B~({ m MeV})$	Γ
$B^*B \ (I=0)$	10583	21	14 eV
$B_s^*B - B^*B_s \ (I = \frac{1}{2})$	10681	11	$45 \mathrm{eV}$
$B^*B^*\ (I=0)$	10630	19	$8 { m MeV}$
$B_s^*B^*~(I=\frac{1}{2})$	10728	12	$0.5 { m MeV}$

The X(3960) seen in $D_s^+ D_s^-$ as the X(3930) state seen in $D^+ D^-$ M. Bayar,^{1,2,*} A. Feijoo,^{2,†} and E. Oset^{2,‡} 2207.08490

The LHCb finds a peak in the Ds+ Ds- invariant $B^+ \rightarrow D_s^+ D_s^- K^+$ decay. mass of the reaction

And associate it to a new state

$$J^{PC} = 0^{++}$$
; $M_0 = 3955 \pm 6 \pm 11 \ MeV$; $\Gamma_0 = 48 \pm 17 \pm 10 \ MeV$

We argue that this is not a new state but the one already observed in D⁺D⁻ at 3930 MeV

The D⁺ D⁻ , Ds⁺ Ds⁻ interaction together with lighter coupled channels was studied in

D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D 76 (2007), 074016.

C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D 87 (2013) no.7, 076006.

But only one bound state of D Dbar in I=0 was obtained

But in QCD Lattice a state a state coupling to Ds⁺ Ds⁻ was found S. Prelovsek, S. Collins, D. Mohler, M. Padmanath and S. Piemonte, JHEP 06 (2021), 035.

It also couples to D⁺D⁻ but weakly.

CAN ONE RECONCILE THESE FACTS?

D Dbar and Ds⁺ Ds⁻ interaction revisited

$$(p_1 + p_3)(p_2 + p_4) \rightarrow \frac{1}{2} \left[3s - (m_1^2 + m_2^2 + m_3^2 + m_4^2) - \frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2) \right]$$

If we remove non diagonal term \rightarrow two states appear

 $T = \left[1 - VG\right]^{-1}V$

If we keep that term \rightarrow the state coupling to Ds+ Ds- disappears

Reducing by 0.7 the 1/MK² term two states appear

Fine tuning the interaction

In K* exchange one has the interaction

 $1/(q^{02}-q^2-MK^{*2})$, $q^0=0$, q=613 MeV at Ds⁺ Ds⁻ threshold

1/(q⁰²-q²-MK^{*2})/(1/MK^{*2})=0.67. With this natural reduction TWO STATES APPEAR

TABLE I. Masses and widths of the poles dynamically generated by the model, as well as, the corresponding modulus of the couplings $|g_i|$.

	$M [{ m MeV}]$	$\Gamma \ [{\rm MeV}]$	$ g_{\bar{D}D} $ [MeV]	$ g_{\bar{D}_s D_s} $ [MeV]
Pole I	3699	—	14516	5897
Pole II $(X_0(3930))$	3936	11	2858	9076

 $B^- \rightarrow D^+ D^- K^-$ and $D^+_s D^-_s K^-$ Relationship between D⁺D⁻ and Ds⁺ Ds⁻ production in $\begin{array}{c} \text{The } c\bar{c} \text{ pair is hadronized} \\ \hline \end{array} \\ \xrightarrow{s} \quad c\bar{c} \rightarrow \sum_{i} c\bar{q}_{i}q_{i}\bar{c} \rightarrow \sum_{i} P_{4i}P_{i4} = D^{0}\bar{D}^{0} + D^{+}D^{-} + D^{+}_{s}D^{-}_{s} = \sqrt{2}D\bar{D} + D^{+}_{s}D^{-}_{s} \end{array}$ **Final state interaction** \overline{u} D^{+}, D^{+}_{s} B^{-}

$$\tilde{t}_{D^+D^-} = C\left(1 + G_{D\bar{D}}(M_{inv})T_{D\bar{D},D\bar{D}}(M_{inv}) + \frac{1}{\sqrt{2}}G_{D_s\bar{D}_s}(M_{inv})T_{D_s\bar{D}_s,D\bar{D}}(M_{inv})\right)$$

 D^{-}, D^{-}_{a}

 $\tilde{t}_{D_s^+ D_s^-} = C \left(1 + \sqrt{2} G_{D\bar{D}}(M_{inv}) T_{D\bar{D}, D_s^+ D_s^-}(M_{inv}) + G_{D_s\bar{D}_s}(M_{inv}) T_{D_s^+ D_s^-, D_s^+ D_s^-}(M_{inv}) \right)$

 \overline{D}, D_{s}^{-}

 D^{-}, D_{s}^{--}

The second pole couples both to Ds⁺ Ds⁻ and D⁺D⁻ and produces the peak at 3930 MeV and an enhacement at the Ds⁺Ds⁻ threshold compatible with the LHCb data. THERE IS NO NEED TO INVOKE A NEW STATE at 3955 MeV !!

Conclusions

- -- We have made predictions for the interaction of D* D and found a bound state with mass and width in agreement with the Tcc state
- -- With the same regulator for the loops we study the D* D* interaction and find a bound state with I=0 and $J^P = 1^+$, with binding and width of the order of 1 MeV, much bigger than for the T_{cc} state. It decays to D* D.
- -- We also make predictions for states of B(*) B(*) type or B(*)s B(*) and find four bound states all with J^P=1⁺
- -- We give arguments to support the idea that the X(3930) and X(3960) states are actually the same state, showing in D⁺ D⁻ and Ds⁺ Ds⁻, respectively.

Conclusions

In the recently observed states in the LHCb, Belle, Babar, BesIII, there are many states which qualify as dynamically generated from the interaction of hadron components: molecular states

Many of these states were predicted before. The experiment has served to fine tune some parameters which allow to make more refined predictions for other states not yet found.

The chiral unitary approach in the SU(3) sector has proved to be quite accurate to study the interaction of hadrons and eventually find poles in the t-matrix that correspond to states

The local hidden gauge aproach, with the exchange of vector mesons, is equivalent to the chiral unitary approach in SU(3). An extension of the LHGA has been done to the charm and bottom sectors, which respects heavy quark symmetry and turns out rather accurate interpreting results and making predictions.

More predictions have been made. We hope that they can be tested in the near future.

Attention must also be payed to hybrids of q qbar or qqq and molecular components. J. Nieves, F. K. Guo, David Rodriguez Entem

TABLE II. The pole position of the T_{cc}^+ relative to the $D^{*+}D^0$ threshold and the Riemann sheet (RS) where the pole is located in each scheme (see the text for details). The errors are statistical propagated from fitting to the LHCb data while the uncertainties from the cutoff variation are well within the errors quoted here.

Scheme	Ι	II	III
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$ (RS-I)	$-333^{+41}_{-36} - i(18 \pm 1)$ (RS-II)	$-356^{+39}_{-38} - i(28 \pm 1)$ (RS-II)

Unveiling the $K_1(1270)$ double-pole structure in the $\overline{B} \rightarrow J/\psi\rho\overline{K}$ and $\overline{B} \rightarrow J/\psi\overline{K}^*\pi$ decays PHYSICAL REVIEW D 103, 116019 (2021)

J. M. Dias,^{*} G. Toledo⁽¹⁾,^{1,†} L. Roca,^{2,‡} and E. Oset^{3,§}

How much is the compositeness of a bound state constrained by a and r_0 ? The role of the interaction range

Arxiv 2201.04414

Jing Song,^{1,2,*} L.R.Dai,^{3,2,†} and E.Oset^{2,‡}

$$V_{\text{eff}} = V_0 + \beta(s - s_0) \qquad T(s) = \frac{1}{[V_0 + \beta(s - s_0)]^{-1} - G(s)} \qquad T(s) = \frac{1}{[\frac{1}{G(s_0)} + \beta(s - s_0)]^{-1} - G(s)}$$

$$G_l = \int_{|q| < q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{w_1(q) + w_2(q)}{2w_1(q)w_2(q)} \frac{1}{s - (w_1(q) + w_2(q))^2 + i\epsilon}$$

$$P_2 = 1 - P_1 = Z = -g^2 G(s_0)^2 \beta \qquad \qquad g^2 = \frac{1}{-G(s_0)^2 \beta - \frac{\partial G}{\partial s}|_{s_0}}$$

$$8\pi\sqrt{s}\left\{\left[\frac{1}{G(s_0)} + \beta(s-s_0)\right]^{-1} - \operatorname{Re}G(s)\right\} \approx \frac{1}{a} - \frac{1}{2}r_0k^2$$

