The new LHCb state $X(3960)$ seen in $D_s^+ D_s^-$ should be the same as the $X(3930)$ seen in $D^+ D^-$

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Outline

- Introduction and Motivation
- Formalism
- Results
- Summary and Conclusion

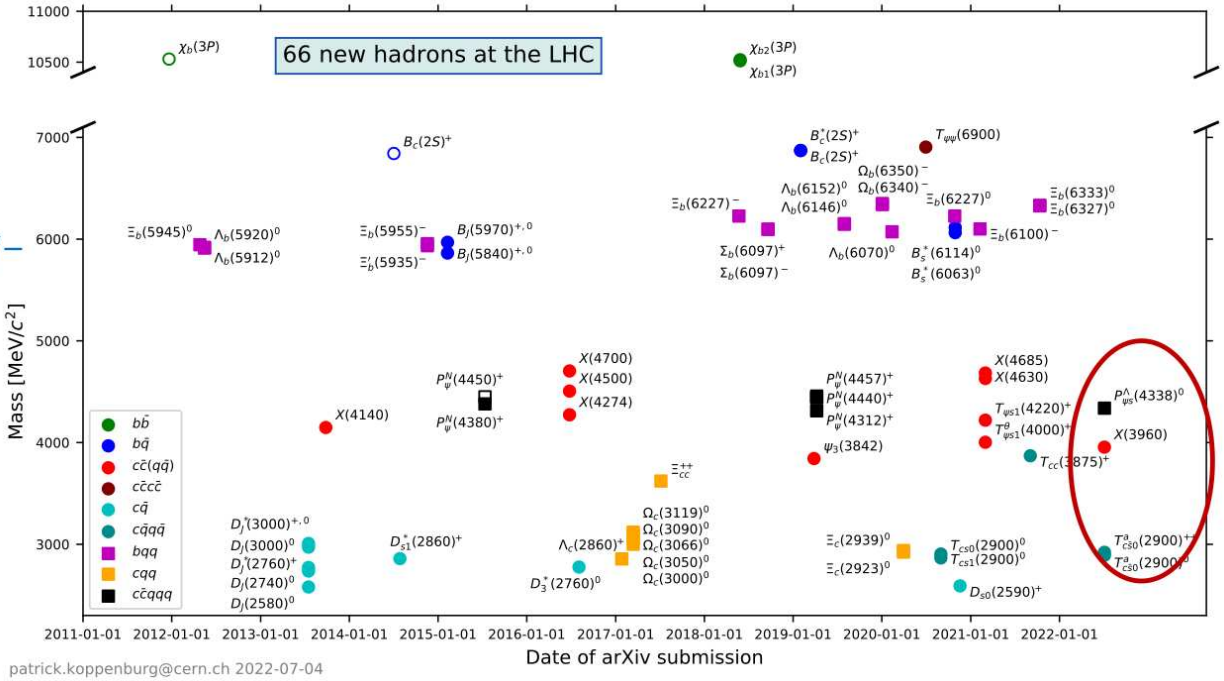
Chen Chen and Elisabetta Spadaro Norella, https://indico.cern.ch/event/1176505/ (5 July 2022)

New exotic members in Particle Zoo @ LHC

$P_{\psi s}^{\Lambda}(4338) \to J/\psi\Lambda$
- Near $\Xi_c^- D^+$ threshold
- In SU(3) flavor multiplets or molecular?

$T_{c\bar{s}0}^{a}(2900)^{++/0} \to D_s^+\pi^{+/-}$
- Isospin triplets
- Flavor partner of $T_{cs0}(2900)$?

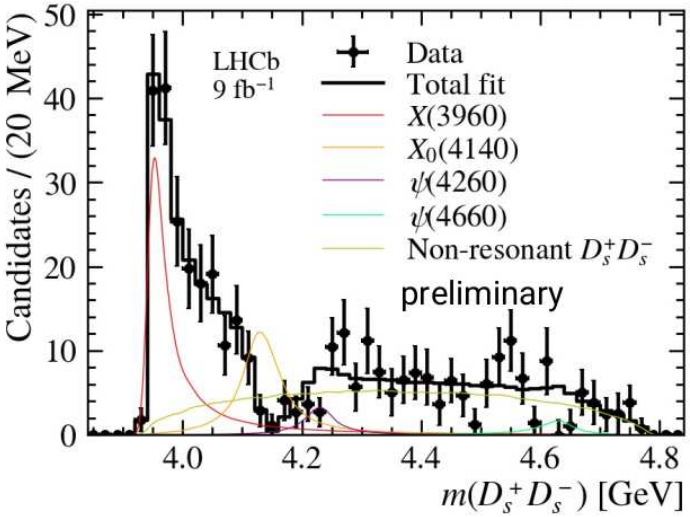
$X(3960) \to D_s^+ D_s^-$
- Near $D_s^+ D_s^-$ threshold
- Exotic candidate; need more studies



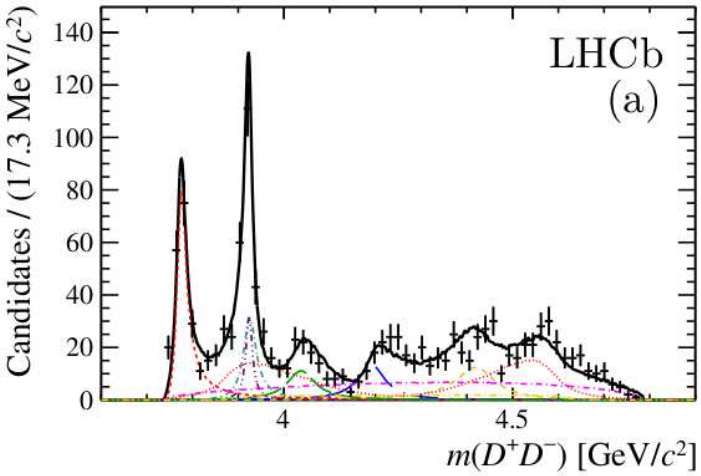
Introduction and Motivation

(a) The $D_s^+ D_s^-$ mass distribution of the $B^+ \to D_s^+ D_s^- K^+$ decay (Chen Chen and Elisabetta Spadaro Norella, https://indico.cern.ch/event/1176505/)

(b) The $D^+ D^-$ mass distribution of the $B^+ \to D^+ D^- K^+$ decay (R. Aaij et al. [LHCb], Phys. Rev. D 102 (2020), 112003.)



(a)



(b)

$J^{PC} = 0^{++}$; $M_0 = 3955 \pm 6 \pm 11\ MeV$; $\Gamma_0 = 48 \pm 17 \pm 10\ MeV$

$J^{PC} = 0^{++}$; $M_0' = 3924 \pm 2\ MeV$; $\Gamma_0' = 17 \pm 5\ MeV$

$(D_s^+ D_s^-)_{threshold} = 3937 MeV$

$(D^+ D^-)_{threshold} = 3739 MeV$

Introduction and Motivation

- $D\bar{D}$ and $D_s^+ D_s^-$ with lighter coupled channels:
  $\Rightarrow$ a $D\bar{D}$ bound state was found
  $\Rightarrow$ no bound state was found close to the $D_s^+ D_s^-$ threshold

  D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D 76 (2007), 074016. C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D 87 (2013) no.7, 076006.
- The QCD lattice result:
  $\Rightarrow$ a 0^{++} bound state coupling strongly to $D_s^+ D_s^-$ $\Rightarrow$ weakly to $D^+ D^-$ is found below the $D_s^+ D_s^-$ threshold.

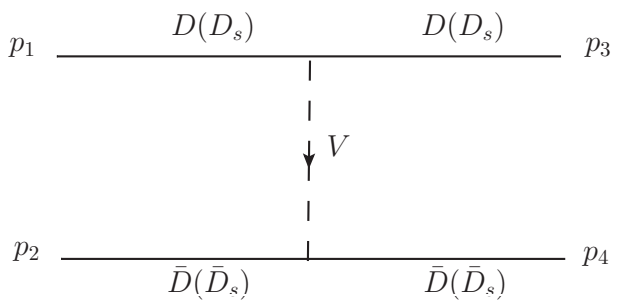
  S. Prelovsek, S. Collins, D. Mohler, M. Padmanath and S. Piemonte, JHEP 06 (2021), 035.
- If the $X_0(3930)$ state coupled both to $D^+ D^-$ and $D_s^+ D_s^-$, that state would necessarily produce an enhancement close to threshold in the $D_s^+ D_s^-$ mass distribution.
- Could explain the experimental observation without the need to introduce an extra resonance?

Formalism

The coupled channels $D\bar{D}$, $I = 0$ and $D_s^+ D_s^-$
$(D^+, -D^0)$ $(\bar{D}^0, D^-)$

$$(D\bar{D},\ I = 0) = \frac{1}{\sqrt{2}}(D^+ D^- + D^0 \bar{D}^0);\ D_s^+ D_s^-$$

Dynamics of $D(D_s) \to \bar{D}(\bar{D}_s)$ interaction due to vector exchange:



the VPP ($V =$ vector, $P =$pseudosclar) vertex

$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\nu P] V^\mu \rangle \tag{1}$$

$$g = \frac{M_V}{2f}\ (M_V \simeq 800\ MeV, f = 93\ MeV)$$

where P and V are the $q_i \bar{q}_j$ matrices written in terms of P and V mesons

Formalism

pseudoscalar (P) and vector (V) mesons:

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \tag{2}$$

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} & \bar{D}^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}, \tag{3}$$

$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\nu P] V^\mu \rangle$$

Formalism

The interaction potential:

$$V_{ij} = -B_{ij} g^2 (p_1 + p_3)(p_2 + p_4) \tag{4}$$

with

$$B = \begin{pmatrix} \frac{1}{2}\left(\frac{3}{M_\rho^2} + \frac{1}{M_\omega^2} + \frac{2}{M_{J/\Psi}^2}\right) & \sqrt{2}\frac{1}{M_{K^*}^2} \\ \sqrt{2}\frac{1}{M_{K^*}^2} & \left(\frac{1}{M_\phi^2} + \frac{1}{M_{J/\Psi}^2}\right) \end{pmatrix}. \tag{5}$$

$(p_1 + p_3)(p_2 + p_4)$, projected over S-wave

$$(p_1 + p_3)(p_2 + p_4) \to \frac{1}{2}\left[3s - (m_1^2 + m_2^2 + m_3^2 + m_4^2) - \frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2)\right] \tag{6}$$

Formalism

Bethe-Salpeter equation

$$T = [1 - VG]^{-1}\, V$$

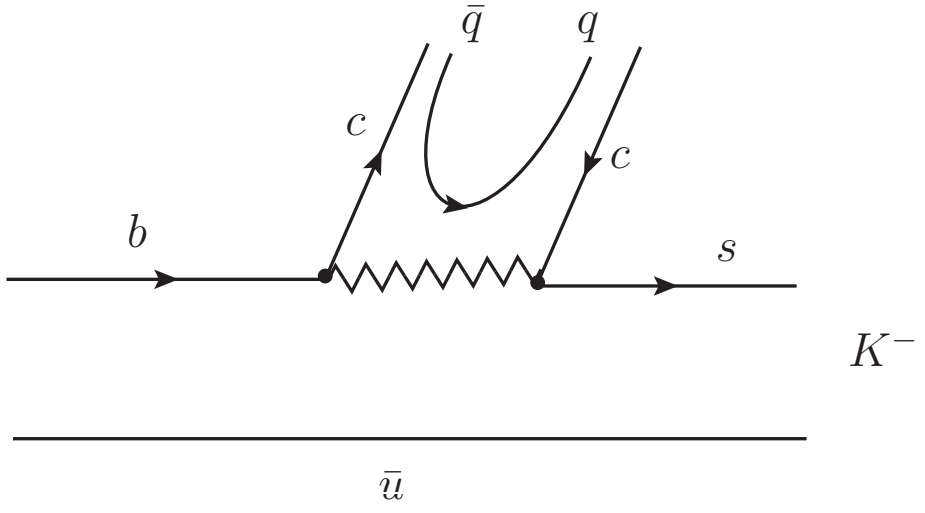
Two meson loop function

$$\widehat{G}_i(\sqrt{s}) = \frac{1}{16\pi^2}\Big(a_i + Log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} Log\frac{m_2^2}{m_1^2} + \frac{q_i}{\sqrt{s}}\Big(Log\frac{s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}{-s + m_2^2 - m_1^2 + 2q_i\sqrt{s}} + Log\frac{s + m_2^2 - m_1^2 + 2q_i\sqrt{s}}{-s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}\Big)\Big)$$

Formalism

The D^+D^- and $D_s^+D_s^-$ production in $B^- \to D^+D^-K^-$ and $B^- \to D_s^+D_s^-K^-$

B^- decay via internal emission at the quark level and hadronization



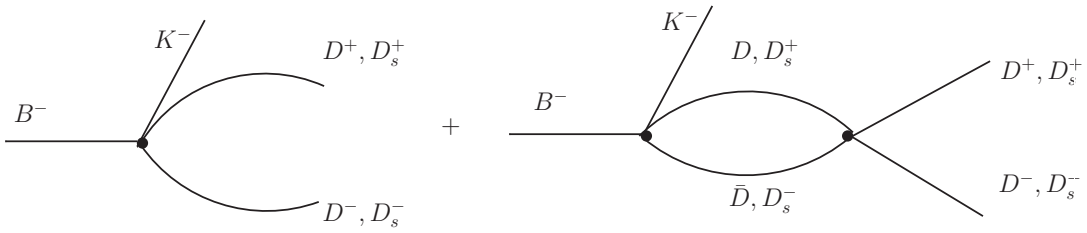
The $c\bar{c}$ pair is hadronized and we have

$$c\bar{c} \to \sum_i c\bar{q}_i q_i \bar{c} \to \sum_i P_{4i}P_{i4} = D^0\bar{D}^0 + D^+D^- + D_s^+D_s^-$$
$$= \sqrt{2}D\bar{D} + D_s^+D_s^- \qquad (7)$$

where we have eliminated $\eta_c\eta_c$ which plays no role here.

Formalism: The D^+D^- and $D_s^+D_s^-$ production

Production and propagation of the D^+D^- and $D_s^+D_s^-$ components through final state interaction



$$\tilde{t}_{D^+D^-} = C\left(1 + G_{D\bar{D}}(M_{inv})T_{D\bar{D},D\bar{D}}(M_{inv}) + \frac{1}{\sqrt{2}}G_{D_s\bar{D}_s}(M_{inv})T_{D_s\bar{D}_s,D\bar{D}}(M_{inv})\right)$$

$$\tilde{t}_{D_s^+D_s^-} = C\left(1 + \sqrt{2}G_{D\bar{D}}(M_{inv})T_{D\bar{D},D_s^+D_s^-}(M_{inv}) + G_{D_s\bar{D}_s}(M_{inv})T_{D_s^+D_s^-,D_s^+D_s^-}(M_{inv})\right)$$

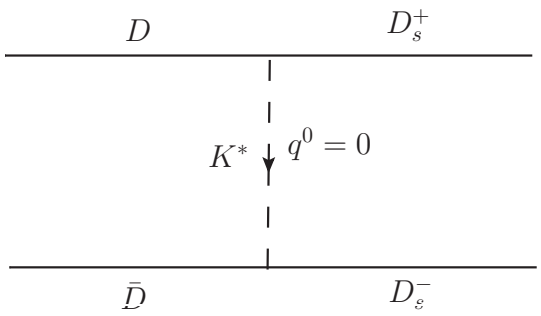
Results:

$$T = [1 - VG]^{-1}\, V, \qquad V_{ij} = -B_{ij} g^2 (p_1 + p_3)(p_2 + p_4)$$

$$B = \begin{pmatrix} \frac{1}{2}\left(\frac{3}{M_\rho^2} + \frac{1}{M_\omega^2} + \frac{2}{M_{J/\psi}^2}\right) & \sqrt{2}\frac{1}{M_{K^*}^2} \\ \sqrt{2}\frac{1}{M_{K^*}^2} & \left(\frac{1}{M_\phi^2} + \frac{1}{M_{J/\psi}^2}\right) \end{pmatrix}.$$

- If we remove non diagonal term $\Rightarrow$ two states appear
- If we keep that term $\rightarrow$ the state coupling to $D_s^+ D_s^-$ disappears
- Reducing by 0.7 the $\frac{1}{M_{K^*}^2}$ term two states appear

The reduction of the $D\bar{D} \rightarrow D_s^+ D_s^-$ transition is natural



$\sqrt{\vec{q}^2 + m_D^2} = m_{D_s}^2$ at threshold $\Rightarrow q^2 = m_{D_s}^2 - m_D^2$

Reduction: $m_{K^*}^2/(m_{D_s}^2 + m_{K^*}^2 - m_D^2) \simeq 0.68$

- $a_{D_s\bar{D}_s} = -1.58$; $\alpha = 0.7$ to get approximately $X_0(3930)$
  $M_0' = 3924 \pm 2\ MeV$; $\Gamma_0' = 17 \pm 5\ MeV$ (LHCb, PRD102(2020)112003.)
- $a_{D\bar{D}} = -1$ to get a $D\bar{D}$ bound state around 3700 MeV

Table: Masses, widths and the couplings $|g_i|$.

	M [MeV]	Γ [MeV]	$\lvert g_{\bar{D}D}\rvert$ [MeV]	$\lvert g_{\bar{D}_s D_s}\rvert$ [MeV]
Pole I	3699	–	14516	5897
Pole II ($X_0(3930)$)	3936	11	2858	9076

Results: The Invarinant mass distributions

$$\frac{d\,\Gamma}{dM_{\rm inv}(D^+D^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_B^2}\,p_{K^-}\,\tilde{p}_{D^+}\,|\tilde{t}_{D^+D^-}|^2$$

$$p_{K^-} = \frac{\lambda^{1/2}(M^2_{B^-}, m^2_{K^-}, M^2_{inv}(D^+D^-))}{2M_{B^-}}$$

$$\tilde{p}_{D^+} = \frac{\lambda^{1/2}(M^2_{inv}(D^+D^-), m^2_{D^+}, m^2_{D^-})}{2M_{inv}(D^+D^-)}$$

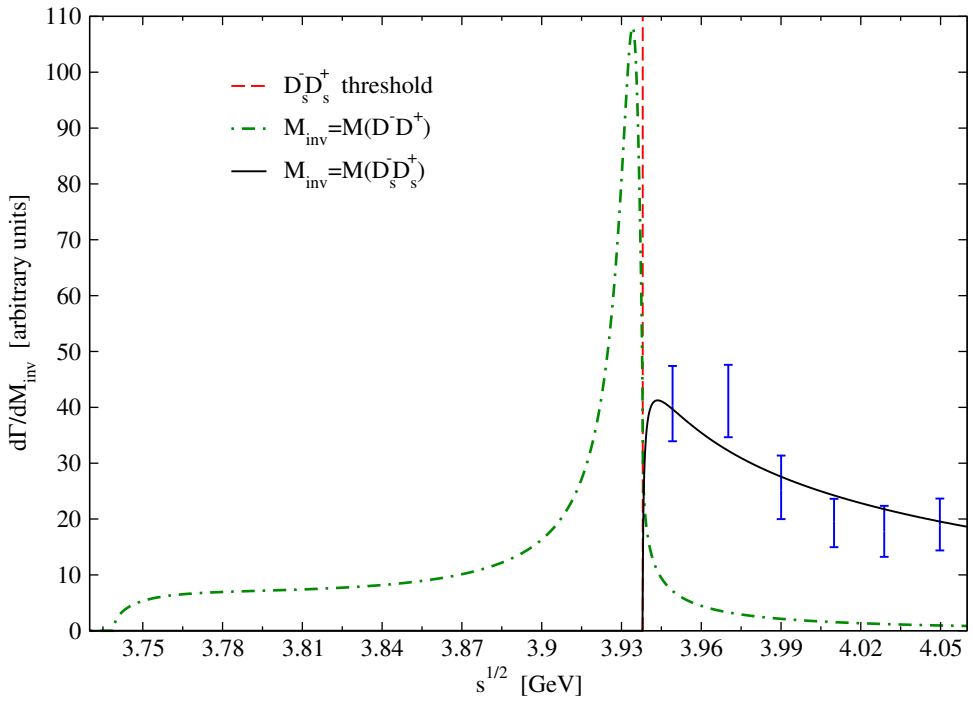
$$\frac{d\,\Gamma}{dM_{\rm inv}(D_s^+D_s^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_B^2}\,p_{K^-}\,\tilde{p}_{D_s^+}\,|\tilde{t}_{D_s^+D_s^-}|^2$$

$$\tilde{p}_{D_s^+} = \frac{\lambda^{1/2}(M^2_{inv}(D_s^+D_s^-), m^2_{D_s^+}, m^2_{D_s^-})}{2M_{inv}(D_s^+D_s^-)}$$

$p_{K^-} \Rightarrow$ the K^- momentum in the B^- rest frame

$\tilde{p}_{D^+}(\tilde{p}_{D_s^+}) \Rightarrow$ the D^+ or D_s^+ momenta in the D^+D^- $(D_s^+D_s^-)$ rest frame.

Results: $d\Gamma/dM_{\rm inv}(D^+D^-)$ and $d\Gamma/dM_{\rm inv}(D_s^+D_s^-)$ of $B^- \to D^+D^-K^-$ and $B^- \to D_s^+D_s^-K^-$ decays



The experimental points are taken from (C. Chen and E. S. Norella, ttps://indico.cern.ch/event/1176505/)

⇒ The second pole couples both to $D\bar{D}$ and $D_s^+D_s^-$

⇒ produces the peak at 3930 MeV and an enhacement at the $D_s^+D_s^-$ threshold compatible with the LHCb

⇒ THERE IS NO NEED TO INVOKE A NEW $X_0(3960)$ STATE !!

Summary and Conclusion

- The D^+D^- and $D_s^+D_s^-$ mass distributions in the $B^- \to D^+D^-K^-$ and $B^- \to D_s^+D_s^-K^-$ decays
- A $D_s^+D_s^-$ bound state appears $\Rightarrow$ can be associated to the $X_0(3930)$
  $\Rightarrow$ coupling strongly to $D_s\bar{D}_s$ and more weakly to $D\bar{D}$
  $\Rightarrow$ produces an enhancement in the $D_s^+D_s^-$ mass distribution close to threshold with a shape in agreement with experiment
- There is no need to invoke a new $X_0(3960)$ state
- The experimental observation is due to the presence of the $X_0(3930)$.

THANK YOU FOR YOUR ATTENTION