Functional Renormalization Group study of chiral phase transition at high density beyond the Local Potential Approximation

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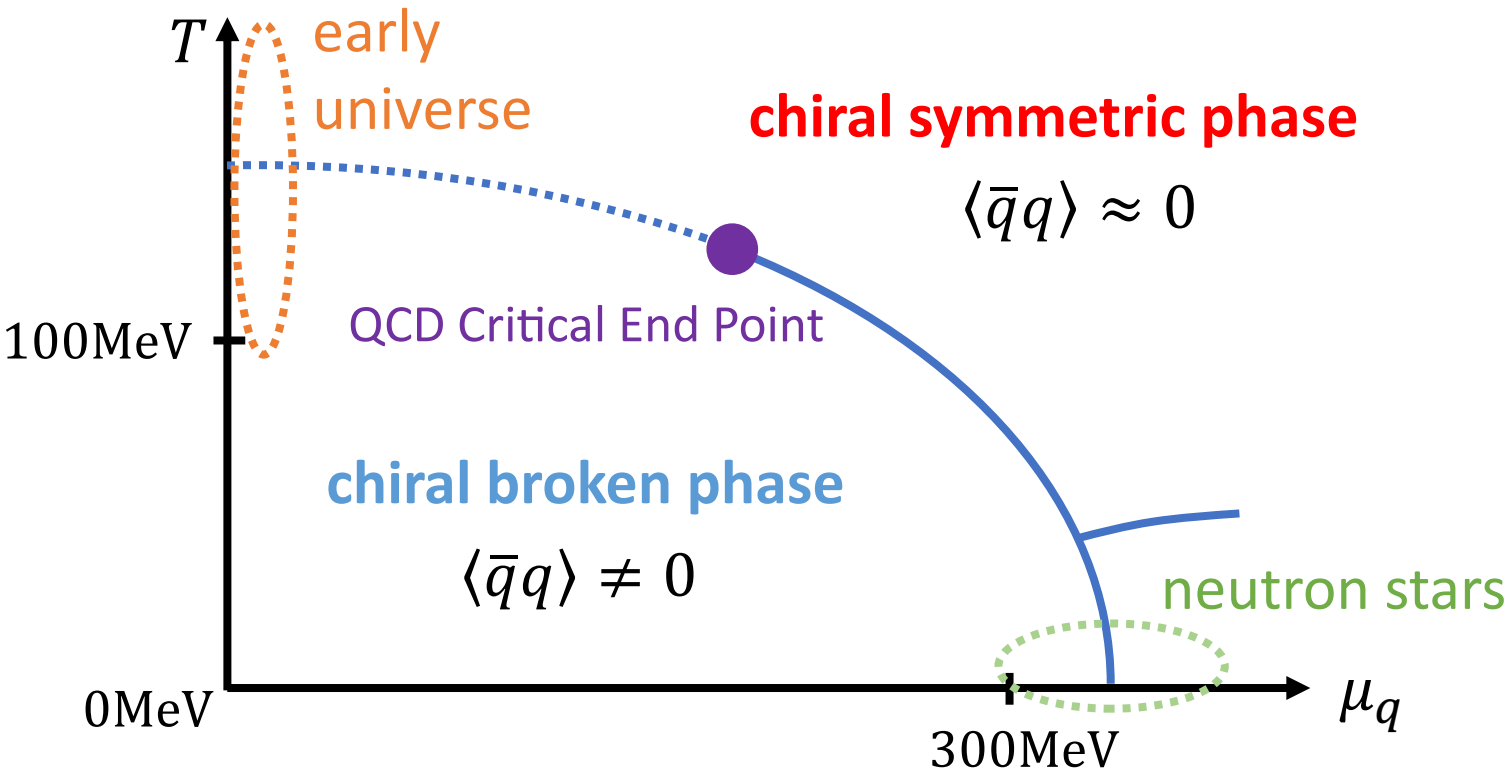
QCD phase diagram

describes phase structure of matter following the QCD dynamics.



We apply the Functional Renormalization Group to an effective model to take quantum fluctuations into account properly.

▶ previous studies

- Functional Renormalization Group Equation

C. Wetterich ,Phys.Lett B 301 (1993)

- FRG analyses at $\mu \neq 0$ and $T \neq 0$
  in the simplest approximation, "Local Potential Approximation"

B. -J. Schaefer and J. Wambach, Nucl. Phys. A. 757. (2005) 479-492

H. Zuang, D. Hou, T. Kojo and B. Qin, Phys. Rev. D 96. 114029 (2017), LPA+ω-meson

- FRG analyses at $\mu = 0$ and $T \neq 0$ beyond the LPA

J. Braun, Phys. Rev. D 81.016008 (2010)

A. J. Helmboldt, J. M. Pawlowski and N. Strodthoff, Phys. Rev. D 91.054010 (2015)

- FRG analyses at $\mu \neq 0$ and high T beyond the LPA

J. M. Pawlowski et al. , Phys. Rev. D 90.076002 (2014)

F. Rennecke et al. , Phys. Rev. D 96.016009 (2017)

We focus on the phase structure at $\mu \neq 0$ and zero T beyond the LPA.

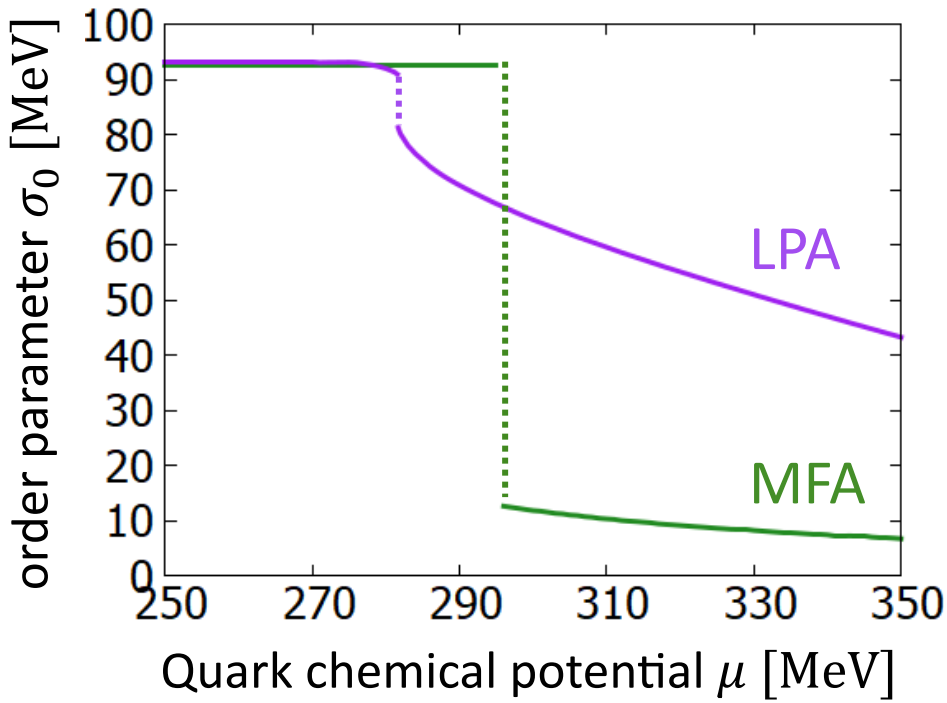
Results for chiral phase transitions in the 2-flavor Quark Meson model at $\mu \neq 0$ and $T = 0$

- FRG in the LPA

  B.-J. Schaefer and J. Wambach, Phys. Rev. D 75 (2007) 085015.
  L. Brandes, N. Kaiser and W. Weise, Eur. Phys. J. A (2021) 57:243

- Mean Field Approximation

  O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, Phys. Rev. C 64 (2001) 045202.



If we consider quantum fluctuations more properly than in the LPA, how does this change ? **Does this 1st phase transition remain ?**

Quark Meson model and FRG

▶ The 2-flavor Quark Meson model

$q = (u\,, d)$: quark field $\quad \phi^a = (\vec{\pi}, \sigma)$: meson field

($\phi^2 = \vec{\pi}^2 + \sigma^2$ … chiral invariant)

$$\mathcal{L} = \bar{q}\left[\, i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\,\right] q + \frac{1}{2}(\partial_\mu \phi)^2 - U(\phi^2) + c\sigma$$

$\langle\sigma\rangle$: order parameter for SU(2) chiral symmetry

FRG allows to get $\langle\sigma\rangle$ including effect of quantum fluctuations.

▶ Effective Action Γ for the Quark Meson model

$$\Gamma\left[\langle\xi\rangle_j\right] \equiv \underbrace{-\log\int \mathcal{D}\xi\, e^{-S[\xi]}}_{\text{Generating Functional}} + \int d^4x\, j^T \langle\xi\rangle_j$$

$\xi = (\sigma, \vec{\pi}, q, \bar{q}^T)^T,$

$j \cdots$ source of ξ

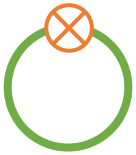
$$\int \mathcal{D}\xi \ldots = \prod_{p=0}^{\Lambda} \int d\xi(p) \ldots$$

introduce R_k into Γ to give a "mass" R_k to $\xi(p), |p| < k$ $\;(\Gamma \to \Gamma_k)$

▶ The exact Flow Equation for Γ_k

C. Wetterich, Phys. Lett B 301 (1993)

$$k\frac{\partial \Gamma_k}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\frac{1}{\Gamma_k^{(2)} + R_k}\right] = \frac{1}{2}$$



$$\left(\Gamma_k^{(2)} \equiv \frac{\delta^2}{\delta\langle\xi\rangle^2}\Gamma_k\right)$$

High Energy Scale ($k = \Lambda$)

$R_{k=\Lambda}(p)$ gives large masses to ξ

$\Gamma_{k=\Lambda} \approx \int d^4x\, \mathcal{L}$ **classical action**

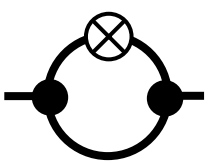
Flow

Low Energy Scale ($k \to 0$)

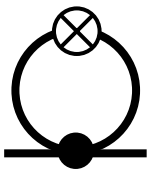
$R_{k=0}(p) = 0$ at any p

$\Gamma_{k=0} = \Gamma$ **quantum effective action**

$$k\frac{\partial \Gamma_k^{(2)}}{\partial k} =$$



$$-\frac{1}{2}$$



$\Gamma_k^{(3)}$ $\Gamma_k^{(3)}$ $\Gamma_k^{(4)}$

Problem : $\partial_k \Gamma_k^{(n)} = f_n\left(\Gamma_k^{(n+1)}, \Gamma_k^{(n+2)}\right)$

This Equation is not closed.

▶ Derivative Expansion (valid for low momentum)

G. R. Golner Phys. Rev. B 33 no. 11, (1986) 7863-7866

$$\Gamma_k = \int d^4x_E \left[Z_{q,k}(\bar{q},q,\phi^2)\cdot\bar{q}(\gamma_\mu\partial_\mu+\gamma^0\mu)q + g_k(\bar{q},q,\phi^2)\cdot\bar{q}(\sigma+i\gamma_5\vec{\tau}\cdot\vec{\pi})q \right.$$
$$\left. +\frac{1}{2}Z_{\phi,k}(\bar{q},q,\phi^2)\cdot(\partial_\mu\phi)^2+\frac{1}{8}Y_{\phi,k}(\bar{q},q,\phi^2)\cdot(\phi_a\partial_\mu\phi_a)^2+\ U_k(\phi^2)-c\sigma+\cdots\right]$$

Γ_k contains all possible terms that do not break SU(2) chiral symmetry.

- ignore higher order terms in derivatives

Flow Equation $\partial_k\Gamma_k = \int_0^\Lambda dp\ \partial_k R_k(p)\ \ldots \approx \int_0^k dp\ \ldots$

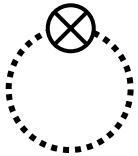
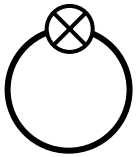
Only low momenta ($|p| < k$) contribute to Γ_k.

- ignore higher order terms in fields and flows of $Y_{\phi,k}$ and g_k

$$Z_k(\bar{q},q,\phi^2)\to Z_k, \qquad Y_{\phi,k}\to 0\ , g_k(\bar{q},q,\phi^2)\to g,$$

We consider $Z_{\phi,k}$ and $Z_{q,k}$, wavefunction renormalization constants.

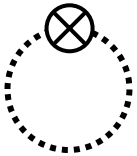
Local Potential Approximation (LPA)

Only U_k depends on k. $\dfrac{\partial U_k(\sigma)}{\partial k} = \dfrac{1}{2}$  $-$ 

To calculate this equation, we use the 3d optimized Regulator.

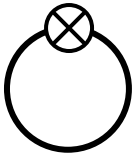
D.F. Litim , Phys. Rev. D 64, 105007 (2001)

<u>meson part</u>

$\dfrac{1}{2}$ 

$$= \frac{k^4}{12\pi^2}\left[\frac{1+2n_B(E_{\sigma,k})}{E_{\sigma,k}} + 3\,\frac{1+2n_B(E_{\pi,k})}{E_{\pi,k}}\right] \qquad E_{\sigma,k} = \sqrt{k^2 + \partial_\sigma^2 U_k} \qquad E_{\pi,k} = \sqrt{k^2 + \sigma^{-1}\partial_\sigma U_k}$$

<u>quark part</u>



$$= \frac{k^4}{12\pi^2} 4N_f N_c \frac{1 - n_F(E_{q,k} - \mu) - n_F(E_{q,k} + \mu)}{E_{q,k}} \qquad E_{q,k} = \sqrt{k^2 + (g\sigma)^2}$$

B. -J. Schaefer and J. Wambach, Nucl. Phys. A. 757. (2005) 479-492

It's not fully confirmed whether LPA is enough to describe the chiral phase transition.

We consider $Z_{q,k}$ and $Z_{\phi,k}$ as a next step of the LPA.

▶ previous studies beyond the LPA

$T \neq 0$ and $\mu = 0$ beyond the LPA

J. Braun, Phys. Rev. D 81.016008 (2010) $\cdots Z_{\phi,k}, Z_{q,k}$

A. J. Helmboldt et al. , Phys. Rev. D 91.054010 (2015) $\cdots Z_{\phi,k}(p), Y_{\phi,k}(p)$

$\rightarrow$ Z_k and $Y_{\phi,k}$ have little effect on finite T chiral phase transitions.

$T \neq 0$ and $\mu \neq 0$ beyond the LPA

J. M. Pawlowski et al. , Phys. Rev. D 90.076002 (2014) $\cdots Z_{\phi,k}, Z_{q,k}, g_k$

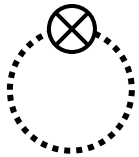
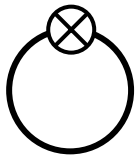
F. Rennecke et al. , Phys. Rev. D 96.016009 (2017) $\cdots Z_{\phi,k}, Z_{ud,k}, Z_{s,k}, g_{ud,k}, g_{s,k}$

U_k is expanded in powers of ϕ^2... $U_k = \sum_{n=0}^{N} \frac{a_{n,k}}{n!}(\phi^2 - \kappa)^n$

This method is not suitable for description of the 1st order phase transition.

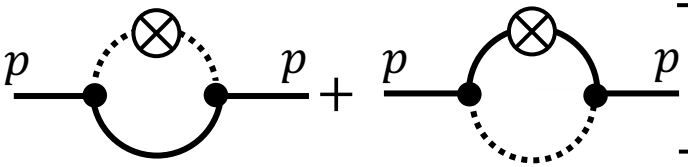
We use the Grid Method that can access full σ dependences of U_k.

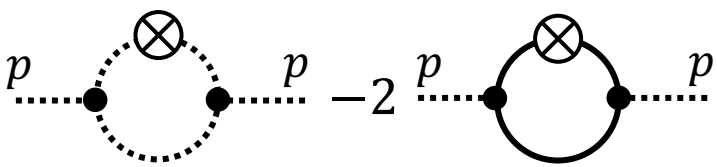
The FRG flow equation reduces to closed integro-differential equations for U_k, $Z_{q,k}$ and $Z_{\phi,k}$.

① $\dfrac{\partial U_k(\sigma)}{\partial k} = \dfrac{1}{2}$  $-$ 

$\eta_k \equiv -k\partial_k \log Z_k$: anomalous dimension

$$= \frac{k^4}{12\pi^2}\left[\left(1-\frac{\eta_{\phi,k}}{5}\right)\left(\frac{1}{E_{\sigma,k}}+3\frac{1}{E_{\pi,k}}\right)-\left(1-\frac{\eta_{q,k}}{4}\right)4N_fN_c\frac{1-\theta\left(\mu-E_{q,k}\right)}{E_{q,k}}\right]$$

② $\dfrac{\partial Z_{q,k}}{\partial k} = \dfrac{-i}{4N_cN_f}\dfrac{\partial}{\partial|\vec{p}|^2}\mathrm{tr}\,\not{p}$ []$_{p_4,\vec{p}=0,\sigma=\sigma_{0,k}}$

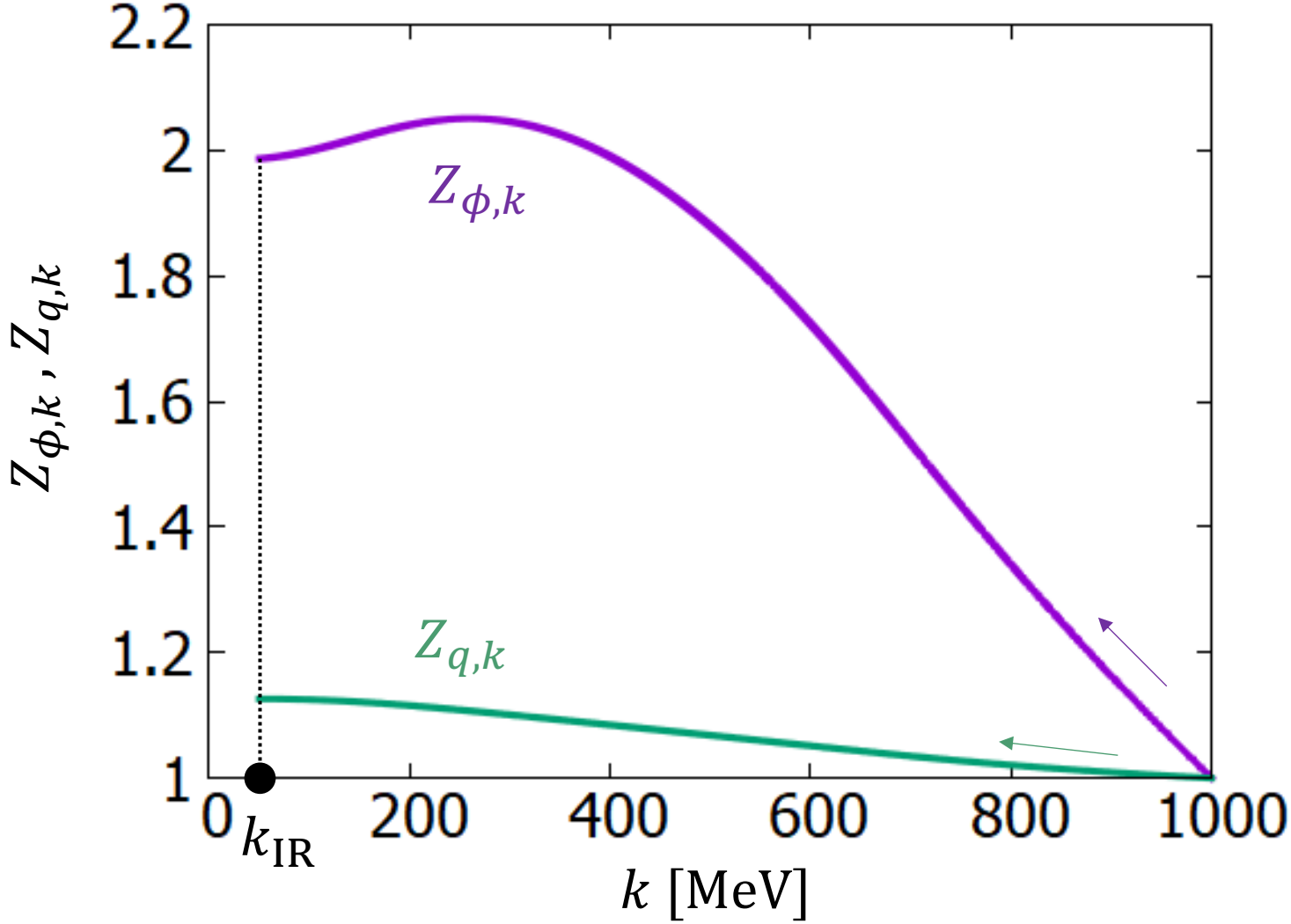
③ $\dfrac{\partial Z_{\phi,k}}{\partial k} = \dfrac{\partial}{\partial|\vec{p}|^2}$ []$_{p_4,\vec{p}=\vec{0},\sigma=\sigma_{0,k}}$

- Momenta of external lines : $p_\mu^\phi \to (0,\vec{0})$, $p_\mu^q \to (\pi T = 0,\vec{0})$
- σ dependence : $\sigma \to \sigma_{0,k} \equiv \underset{\sigma}{\mathrm{argmin}}[U_k(\sigma) - c\sigma]$

current mass term

Results

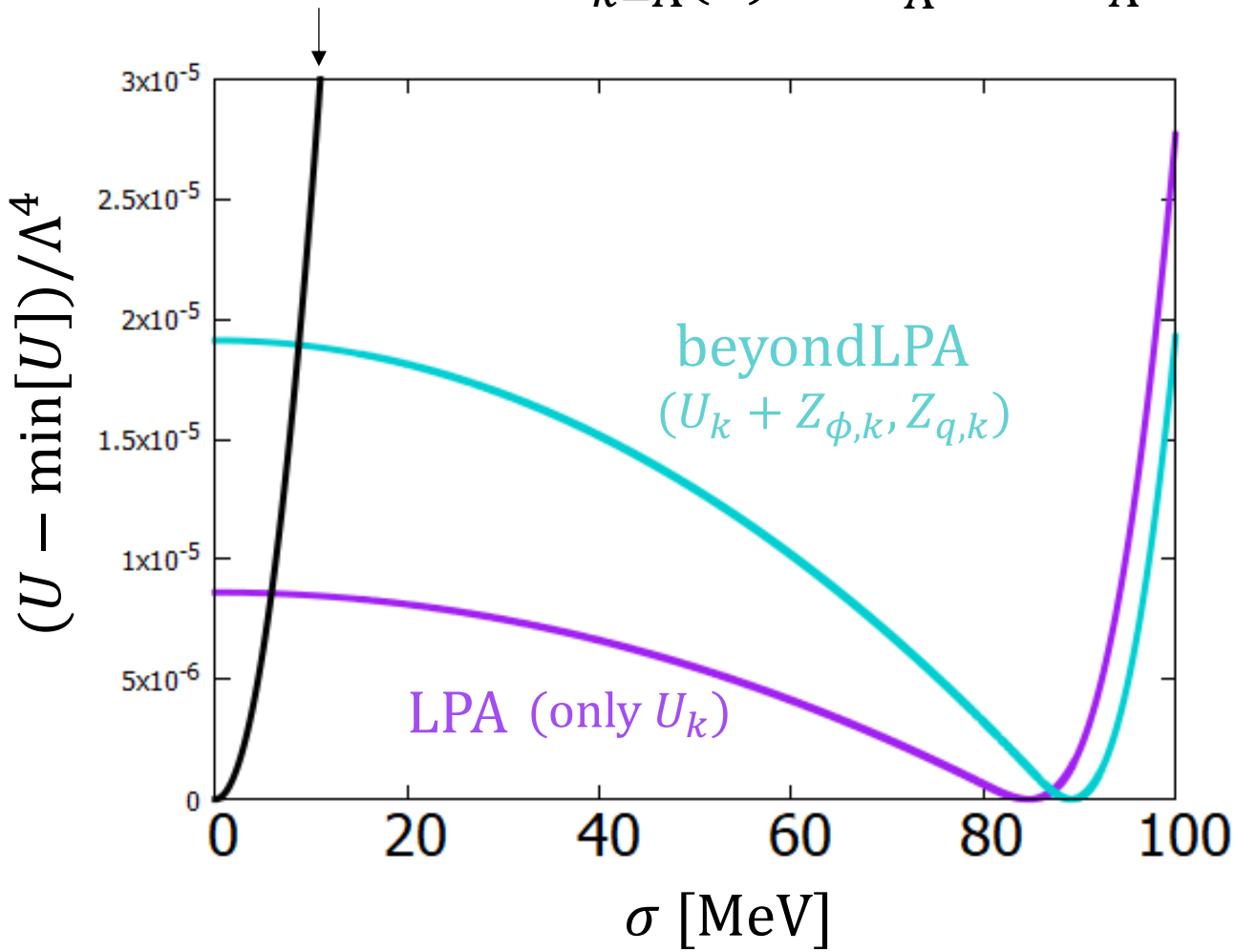
▶ Flows of $Z_{\phi,k}$ and $Z_{q,k}$ at $T, \mu = 0$



calculate up to a sufficiently small k ($k_{\mathrm{IR}} = 50\mathrm{MeV}$)

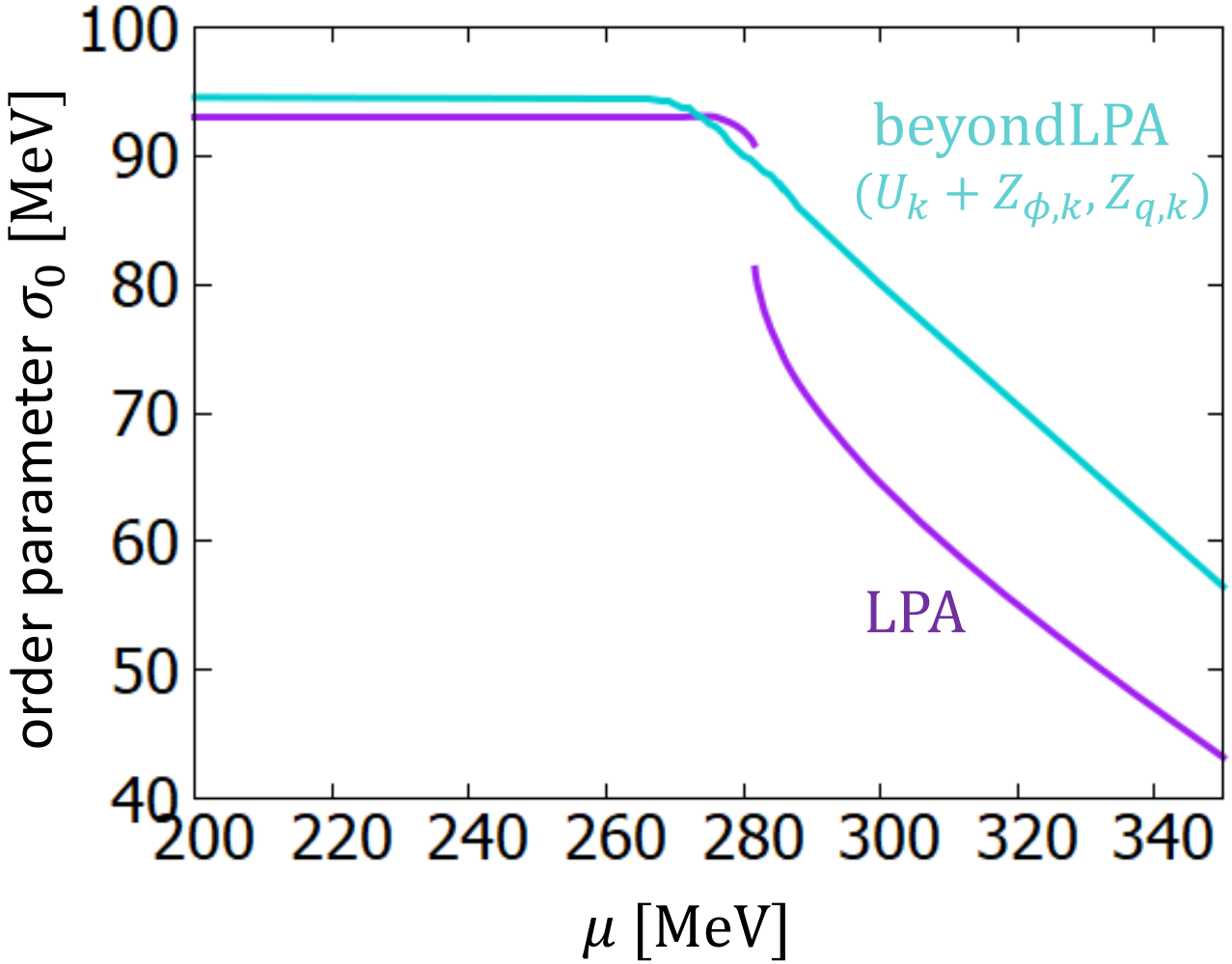
▶ Flow of the $U_k(\sigma)$ at $T, \mu = 0$

initial condition $U_{k=\Lambda}(\sigma) = m_\Lambda^2 \sigma^2 + \lambda_\Lambda \sigma^4$



order parameter : $\sigma_0(T, \mu) \equiv \sigma_{0,k_{\mathrm{IR}}} = \underset{\sigma}{\mathrm{argmin}}\left[U_{k_{\mathrm{IR}}}(\sigma) - c\sigma\right]$

▶ chiral phase transitions at $T = 0$ and high μ in LPA and LPA $+Z_{\phi,k}, Z_{q,k}$

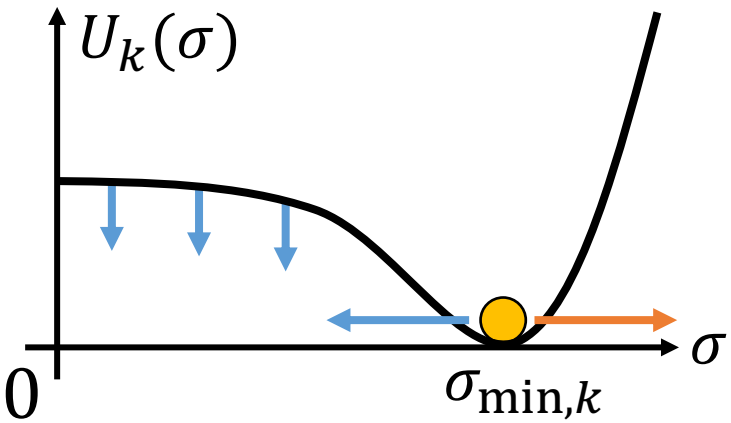


The 1st order phase transition becomes **weak** in the LPA $+Z_{\phi,k}, Z_{q,k}$.

▶ The anomalous dimensions effect on the potential

$$(T,\mu=0)\ \frac{\partial U_k(\sigma)}{\partial k}=\frac{k^4}{12\pi^2}\left[\left(1-\frac{\eta_{\phi,k}}{5}\right)\left(\frac{1}{E_{\sigma,k}}+3\frac{1}{E_{\pi,k}}\right)-\left(1-\frac{\eta_{q,k}}{4}\right)4N_fN_c\frac{1}{E_{q,k}}\right]$$

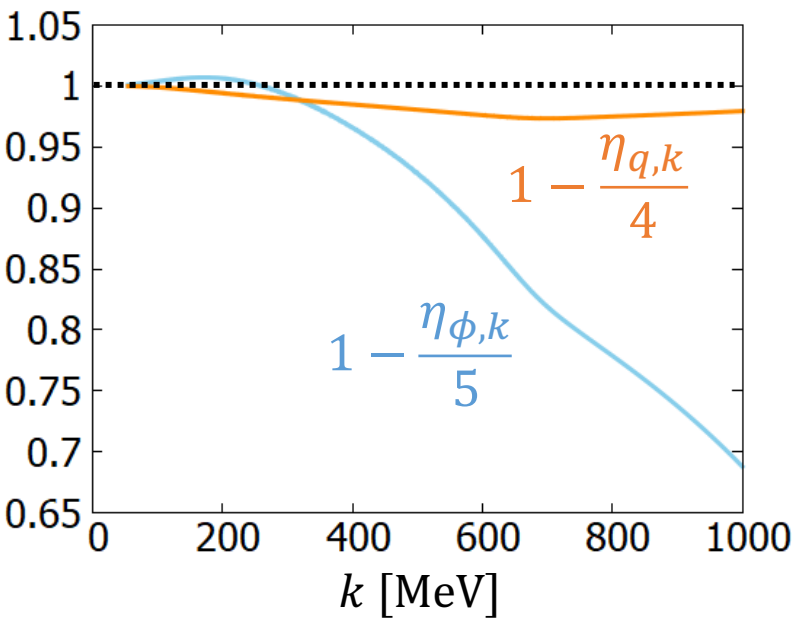
meson term　　quark term



$$\sigma_{\mathrm{min},k}\equiv\underset{\sigma}{\mathrm{argmin}}\,U_k(\sigma)$$

The meson term reduces $\sigma_{\mathrm{min},k}$.

The quark term increases $\sigma_{\mathrm{min},k}$.



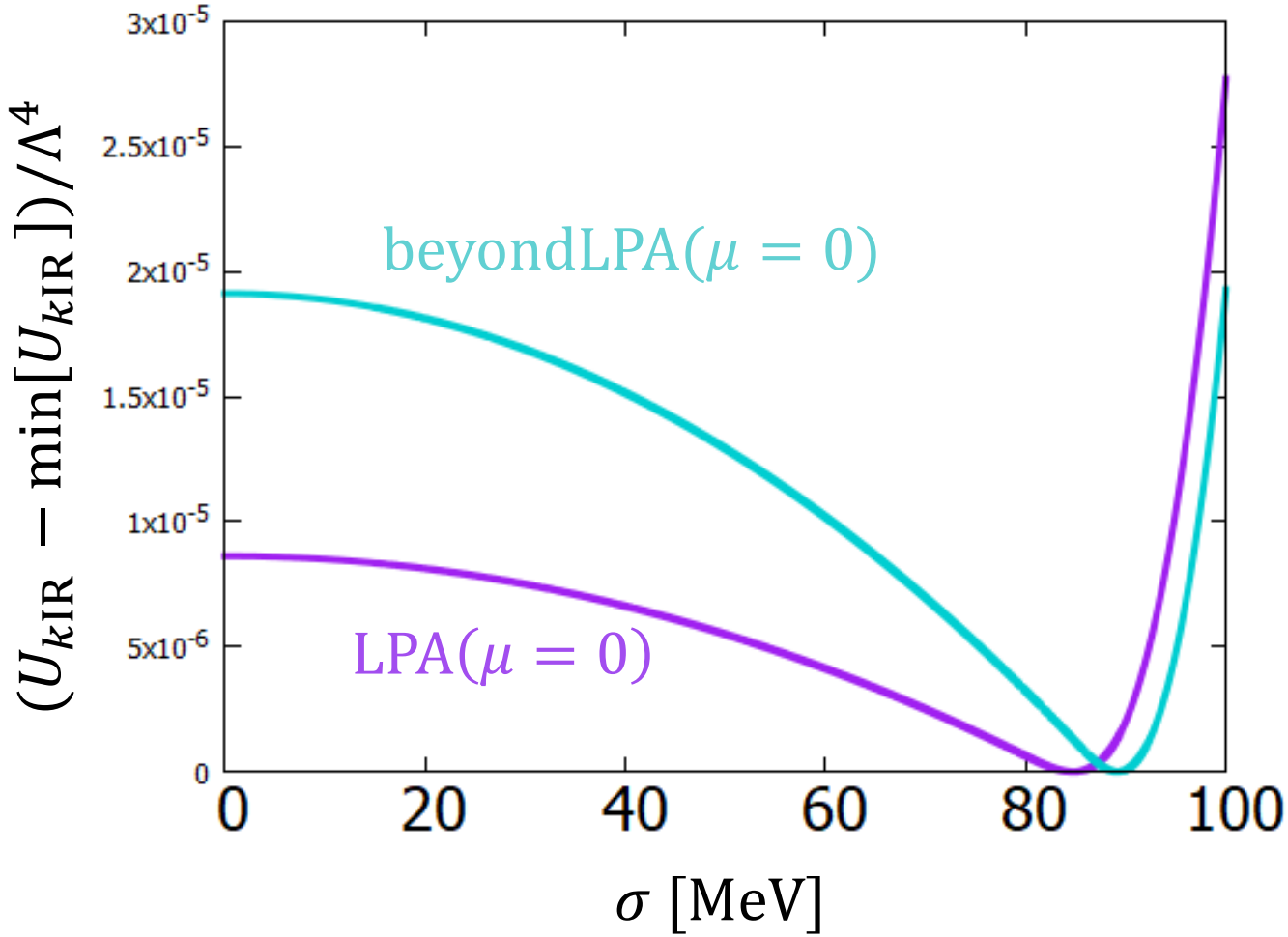
$$\eta_{\phi,k}=-k\partial_k\log Z_{\phi,k}$$

$$\eta_{q,k}=-k\partial_k\log Z_{q,k}$$

The meson effect is suppressed more strongly.

beyond the LPA

- The potential in the range of small σ becomes larger.
- $\sigma_{\mathrm{min},k\mathrm{IR}}$ is shifted toward larger value.

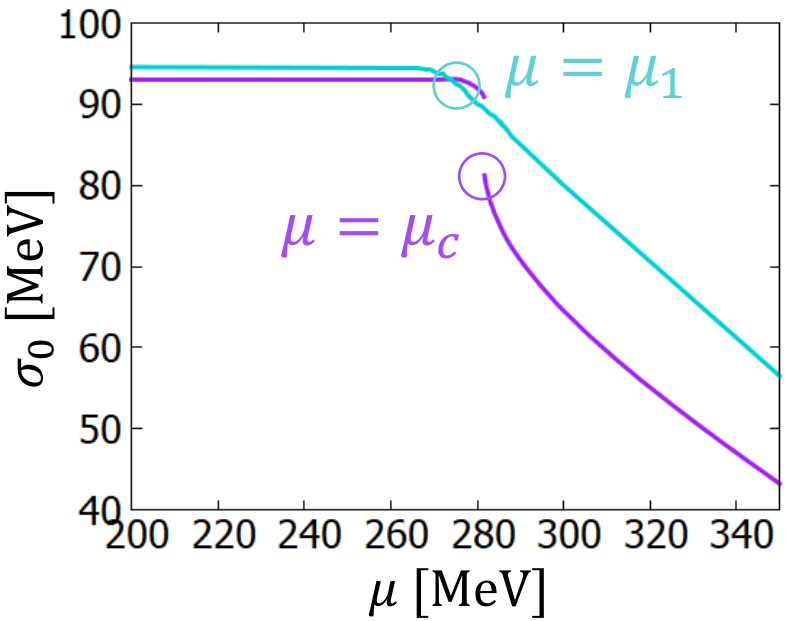


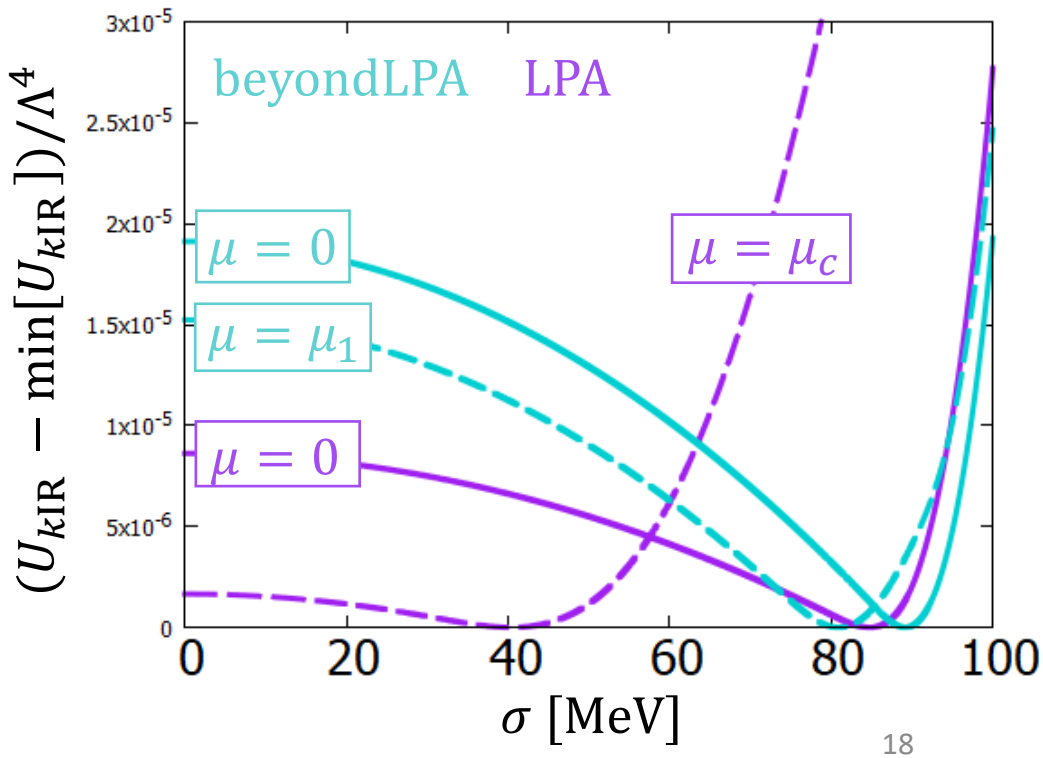
▶Why does the 1st order phase transition become weaker?

$$\frac{\partial U_k(\sigma)}{\partial k} = \frac{k^4}{12\pi^2}\left[\left(1-\frac{\eta_{\phi,k}}{5}\right)\left(\frac{1}{E_{\sigma,k}}+3\frac{1}{E_{\pi,k}}\right)-\left(1-\frac{\eta_{q,k}}{4}\right)4N_fN_c\frac{1-\theta(\mu-E_{q,k})}{E_{q,k}}\right]$$

LPA $\theta(\mu - E_{q,k})$ term reduces the potential and change $\sigma_0(\mu)$ discontinuously at a transition point μ_c.

beyondLPA Due to $\eta_{q,k}$, $\theta(\mu - E_{q,k})$ effect becomes weak.



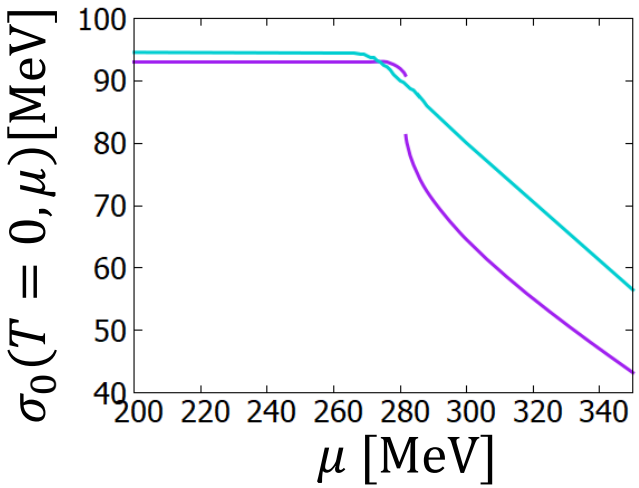


▶ Summary FRG analysis of the 2-flavor Quark-Meson model

<u>Approximation</u>

- LPA ... Only the potential U_k depends on k.
- beyondLPA ... $U_k + Z_{\phi,k}, Z_{q,k}$

<u>Result</u> The 1st order phase transition becomes weak.



η_ks suppress the $\theta(\mu - E_{q,k})$ effect.

It's important to consider Z_k at finite μ.

▶ Outlook

consider the flow of $Y_{\phi,k}, g_k$ and the dependence of ϕ^2

(not negligible flows in terms of the derivative expansion)