Excited state hadrons in Dπ, DK scattering from lattice QCD

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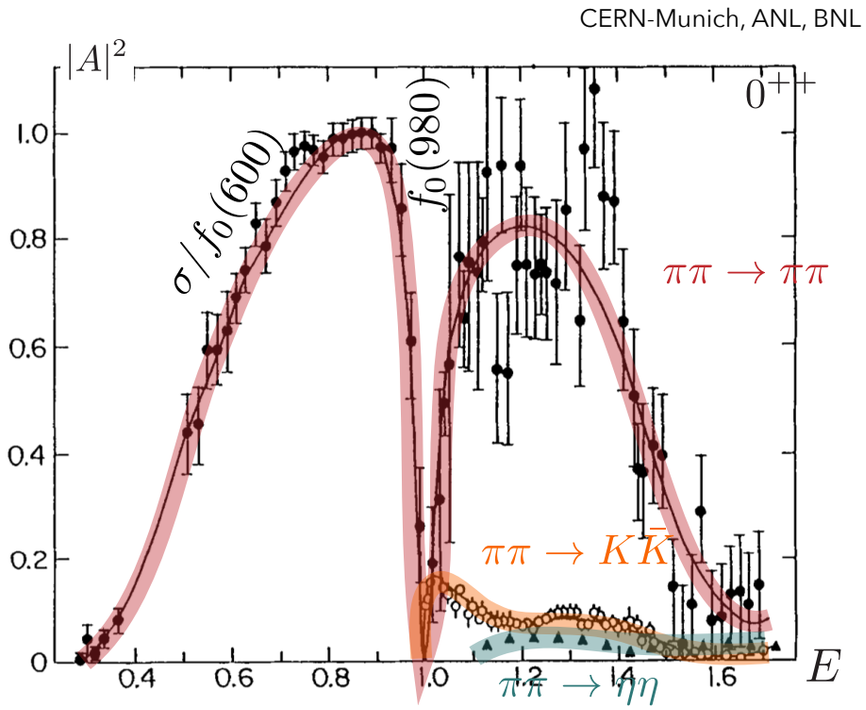
QNP2022
The 9th International Conference on Quarks and Nuclear Physics
8th September 2022

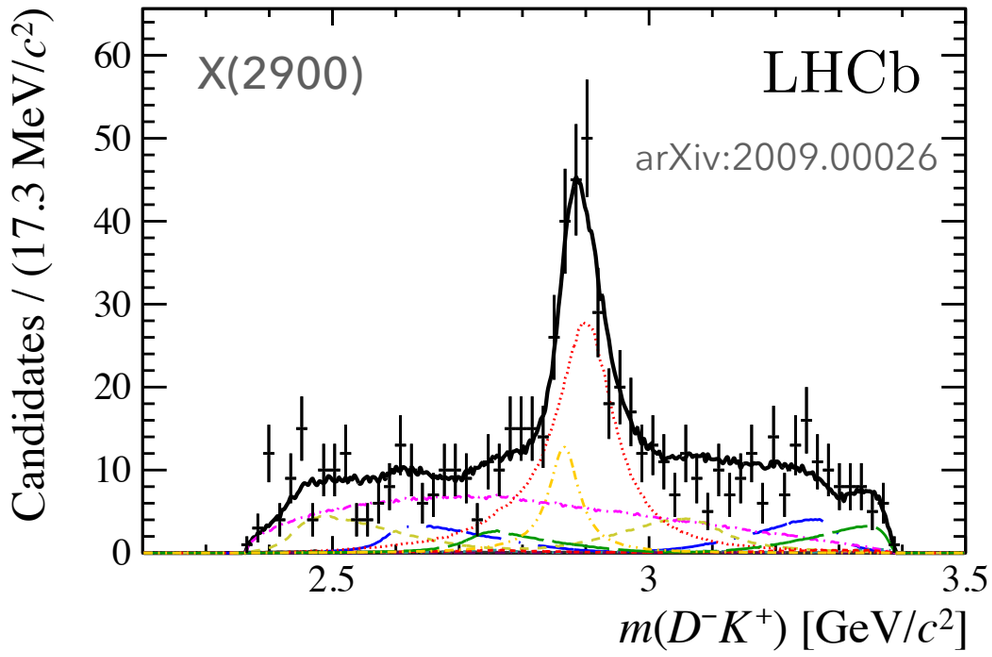
UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

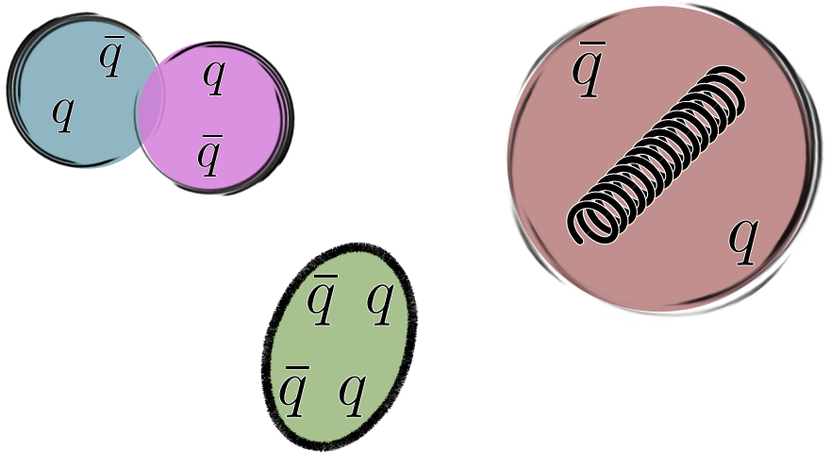
spectroscopy from first-principles is a hard problem



many puzzles - new and old



the quark model is a good guide for low-lying states



models are useful, but what does **QCD** say?

Lattice QCD provides a rigorous approach to hadron spectroscopy

- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude

tremendous progress in recent years

but not yet ready for precision comparisons

- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- small effects not considered in general:

  finite lattice spacing, isospin breaking, EM interactions

goal: what does **QCD** say about the excited hadron spectrum?

$J^P = 0^+$

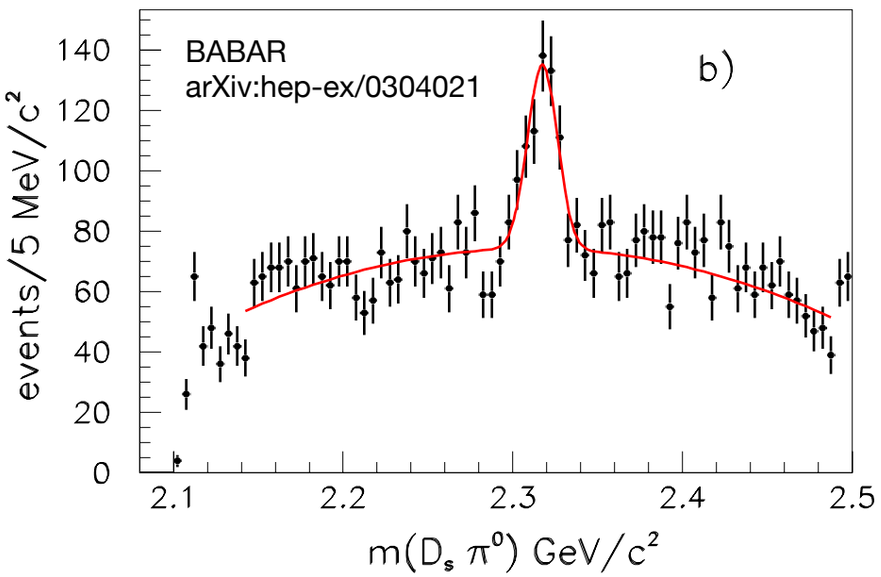
- $D_{s0}(2317)$ $c\bar{s}$
- $D_0^*(2300)$ $c\bar{l}$

compare : $J^P = 0^-$

- D_s m~1969 MeV
- D m~1870 MeV

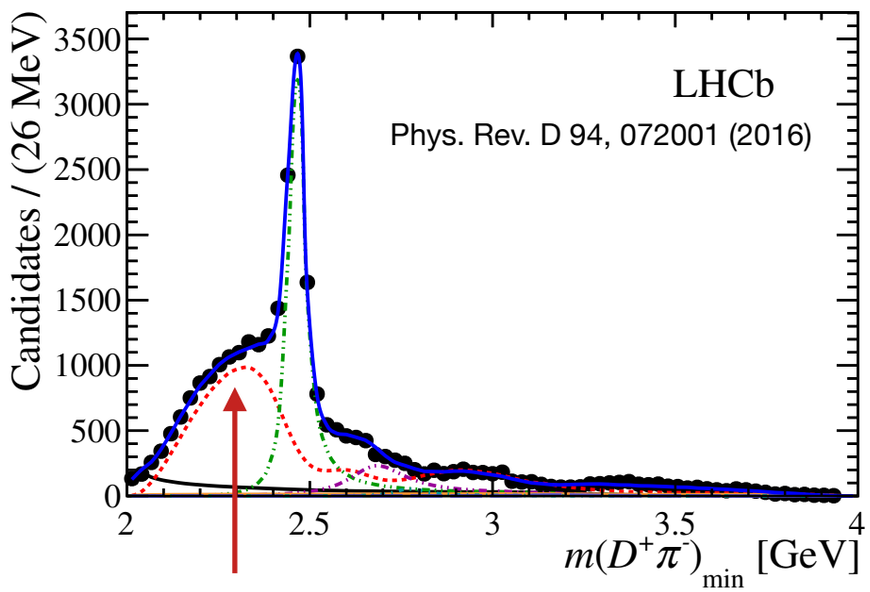
what is the mass ordering?
why are the masses so close?
why are the widths so different?

$D_{s0}(2317)$

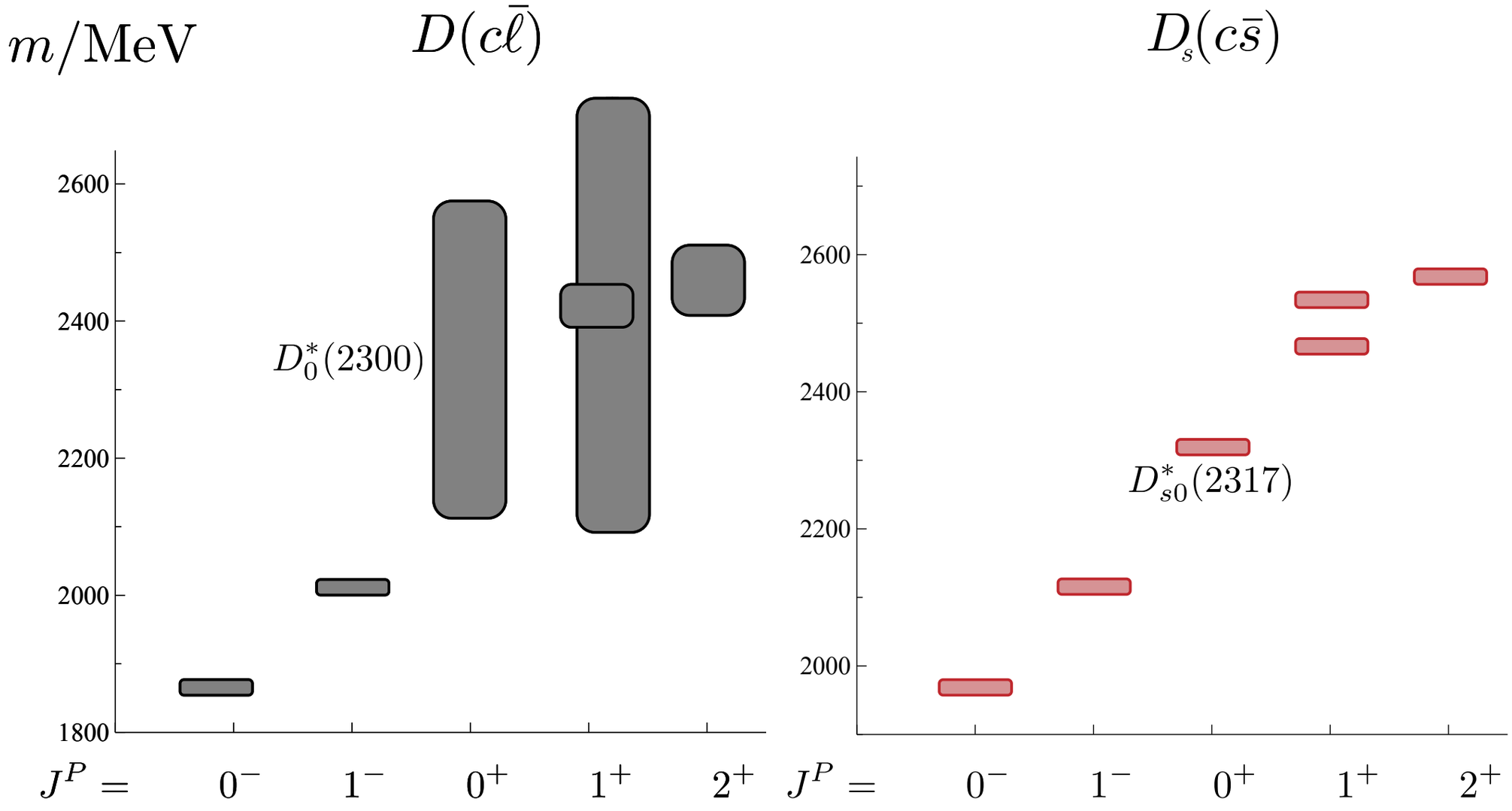


seen in an isospin breaking decay mode

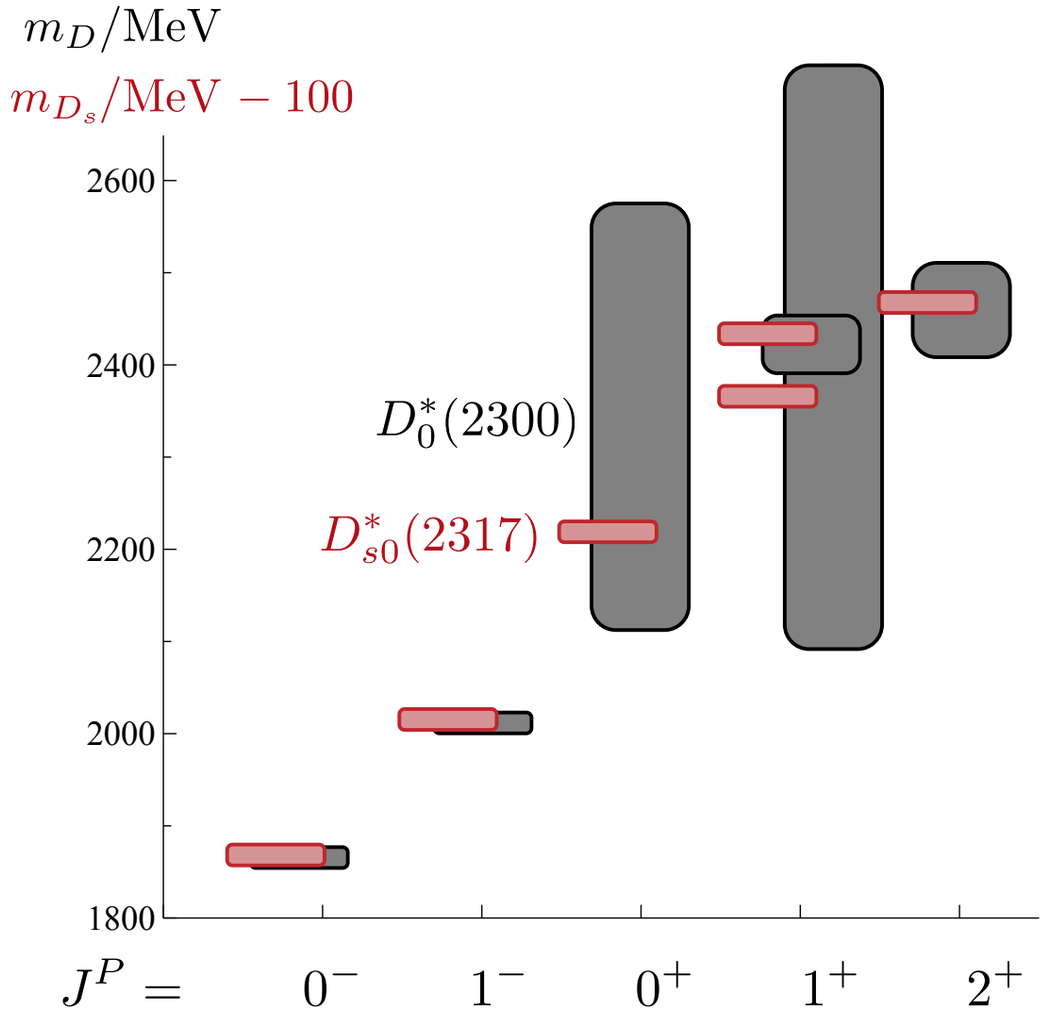
$D_0^*(2400)$



Dπ S-wave



[masses, widths from PDG]



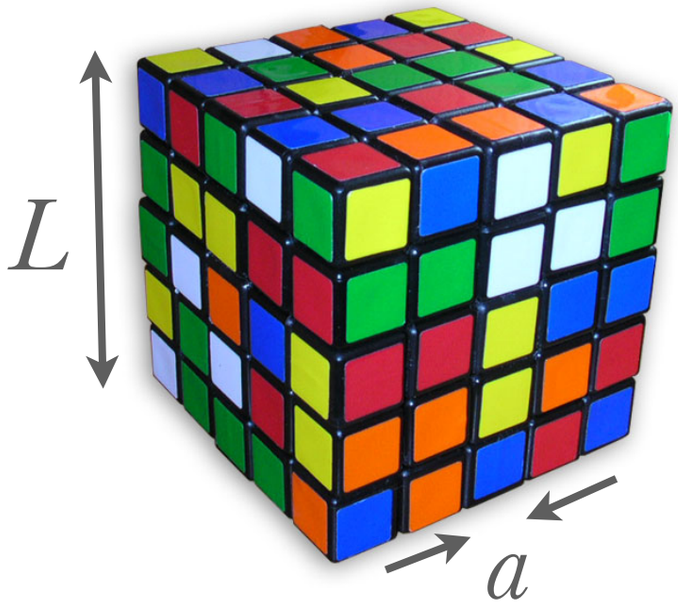
[masses, widths from PDG]

$D_0^*(2300)$ & $D_{s0}^*(2317)$

what is the mass ordering?

why are the masses so close?

why are the widths so different?



anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16,20,24 & 32
m_π = 391 & 239 MeV

rest and moving frames

operators used:

$\bar{\psi}\Gamma\overleftrightarrow{D}...\overleftrightarrow{D}\ \psi$ local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$

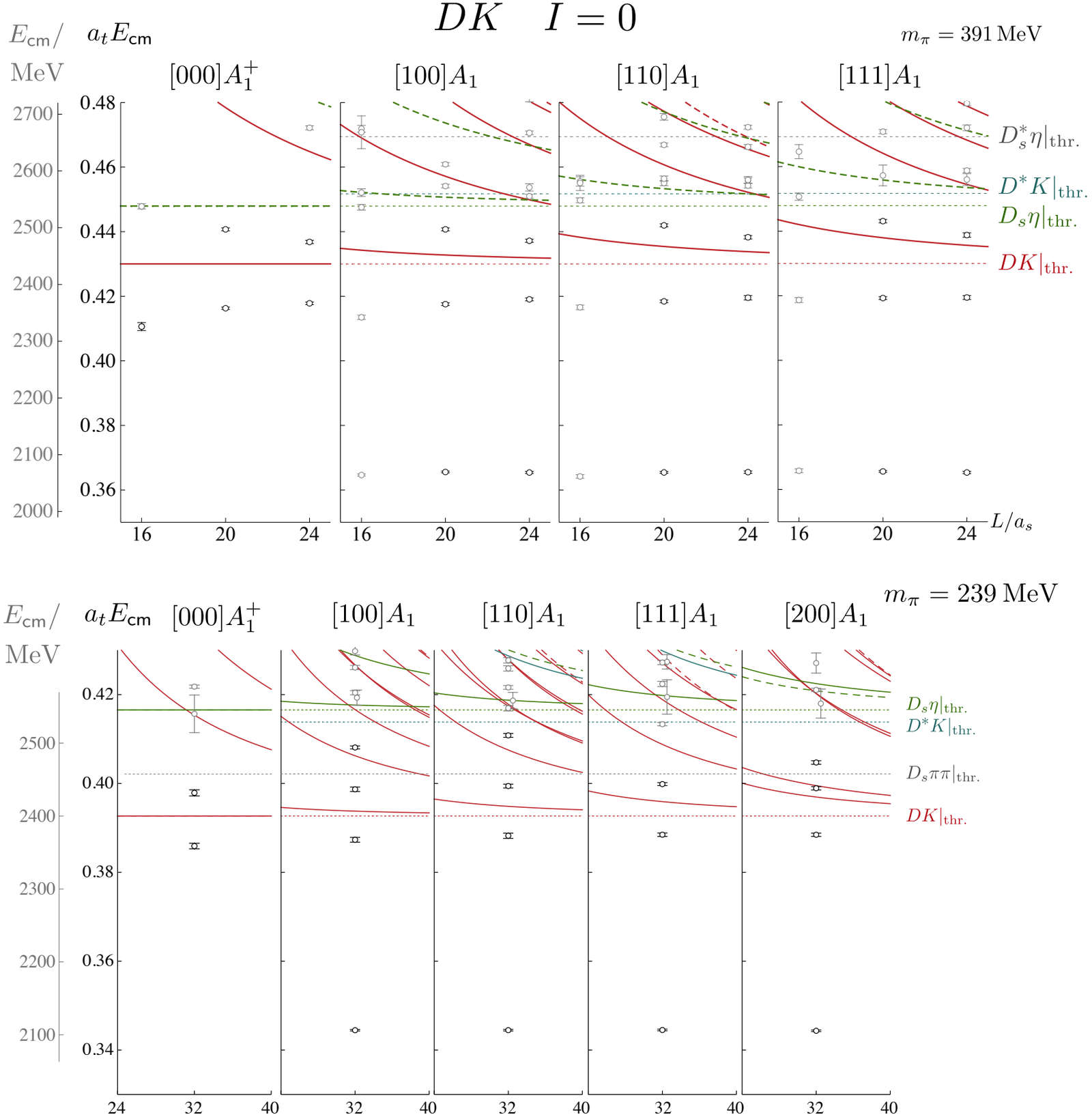
two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many wick contractions

- we compute a large correlation matrix
- then use GEVP to extract energies

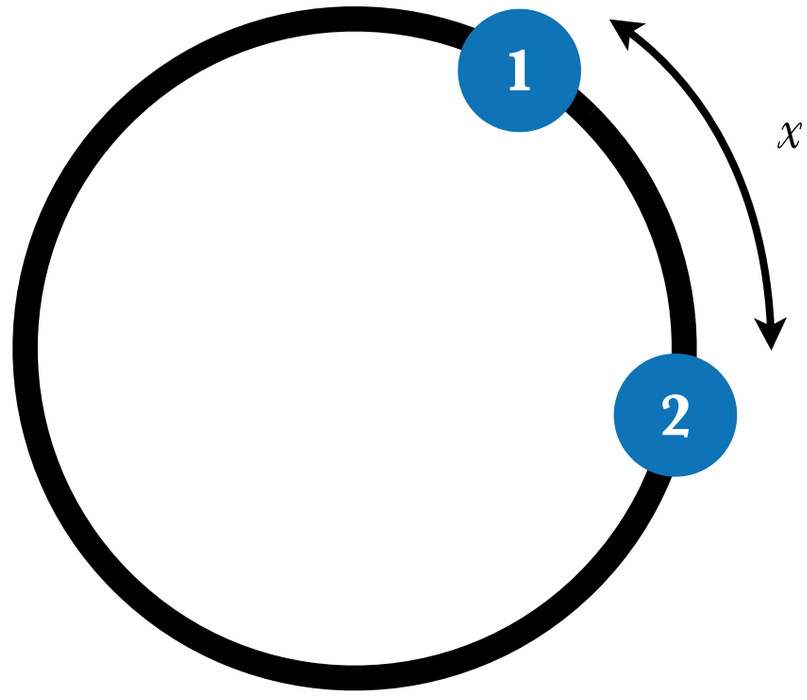

$DK \quad I = 0$
$m_\pi = 391\,\mathrm{MeV}$
$E_{\mathrm{cm}}/$ MeV
$a_t E_{\mathrm{cm}}$
$[000]A_1^+$
$[100]A_1$
$[110]A_1$
$[111]A_1$
$D_s^*\eta|_{\mathrm{thr.}}$
$D^*K|_{\mathrm{thr.}}$
$D_s\eta|_{\mathrm{thr.}}$
$DK|_{\mathrm{thr.}}$
L/a_s
$m_\pi = 239\,\mathrm{MeV}$
$[200]A_1$
$D_s\pi\pi|_{\mathrm{thr.}}$

1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

$$\mathcal{Z}_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

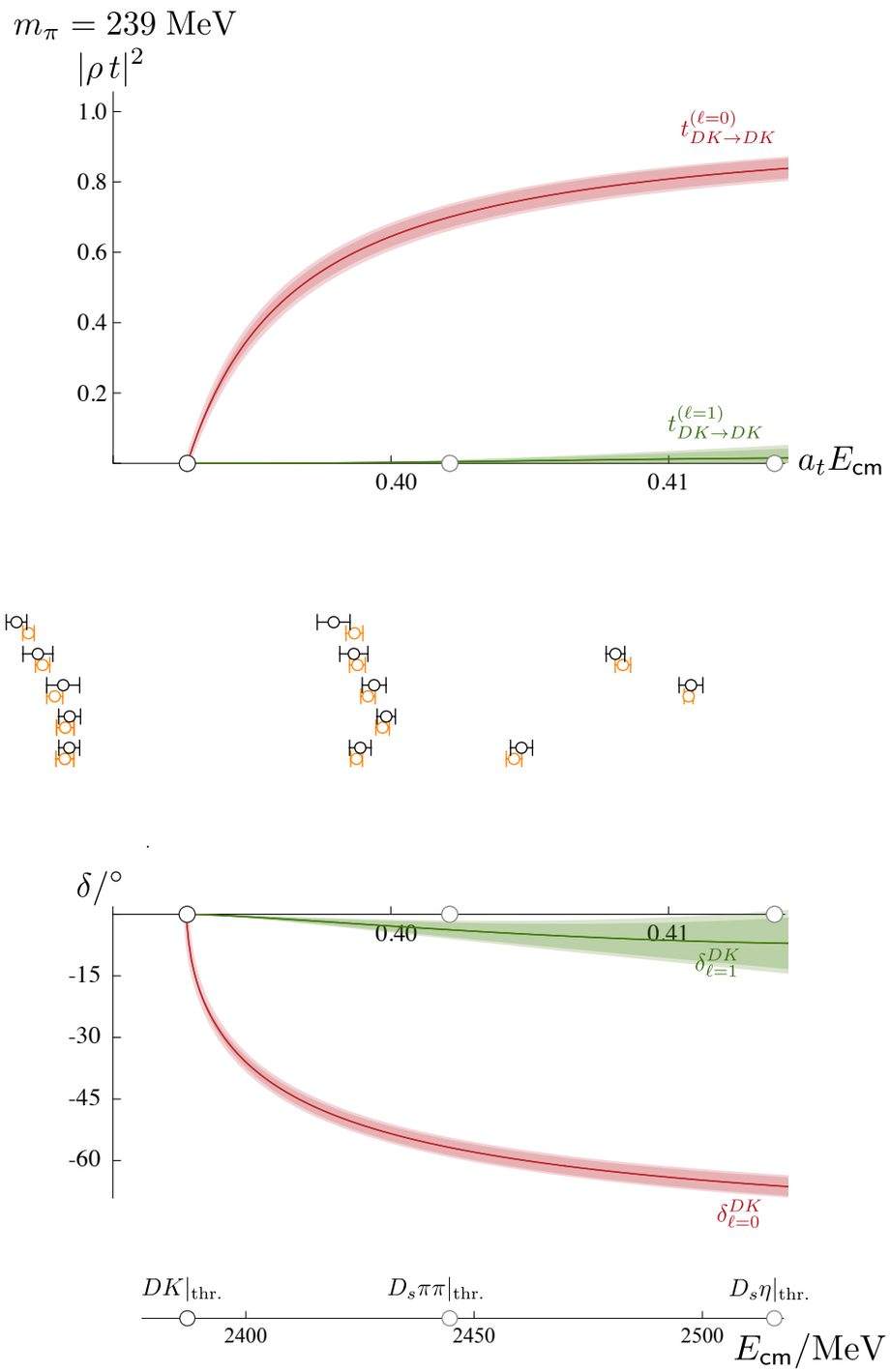
Lüscher 1986, 1991

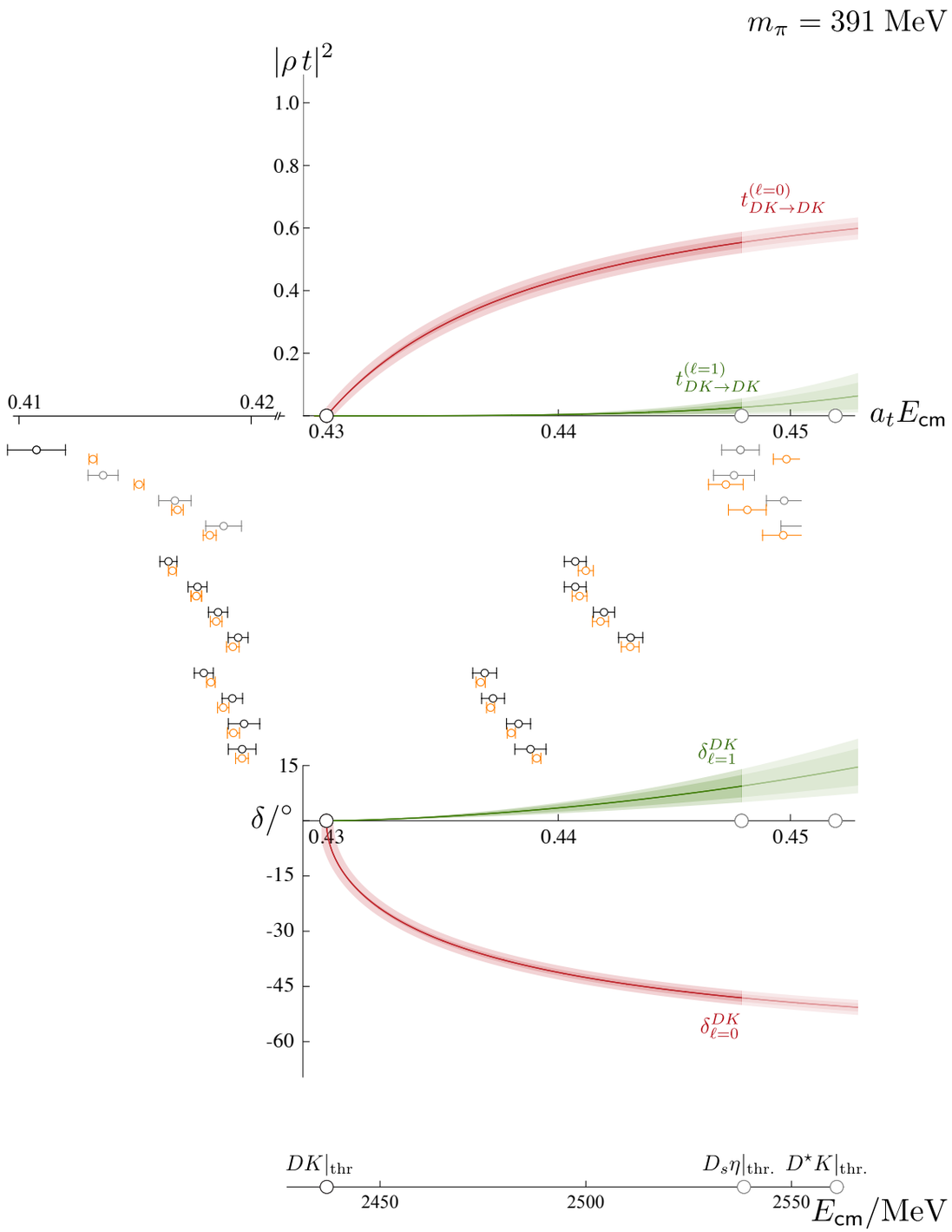
generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

G. Cheung et al (HadSpec), JHEP 02 (2021) 100 arXiv: 2008.06432

$D_{s0}(2317)$



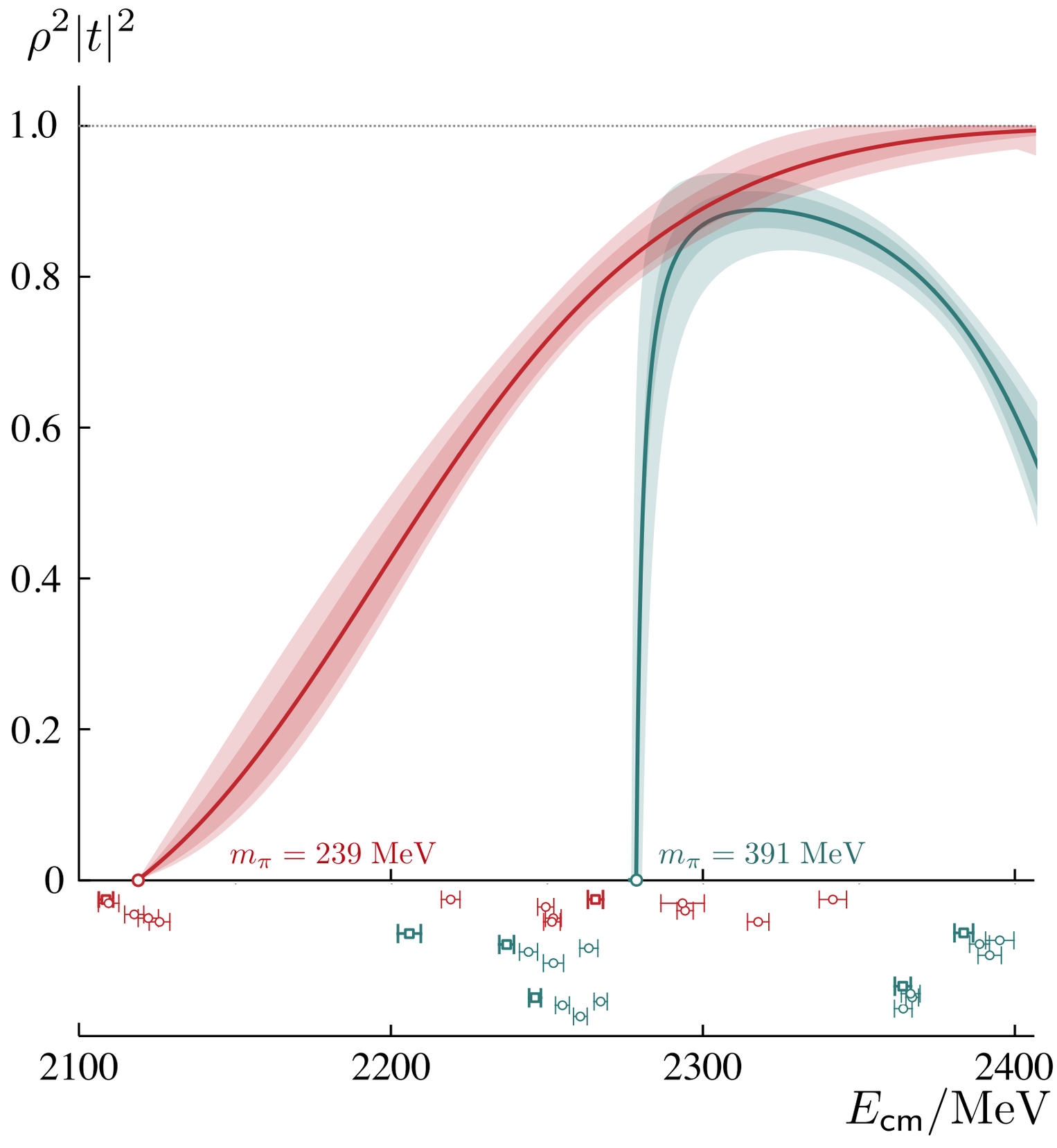


bound states in DK amplitude at both masses

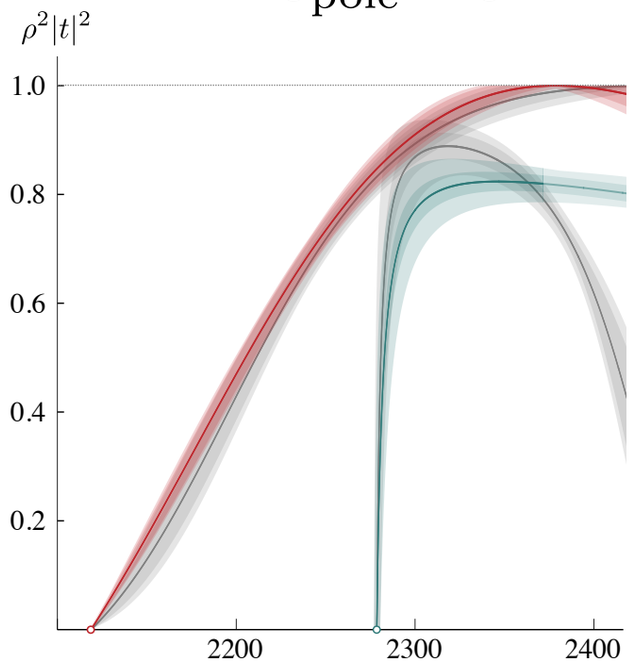
similar couplings c~1400 MeV

L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973

D_0*(2300)



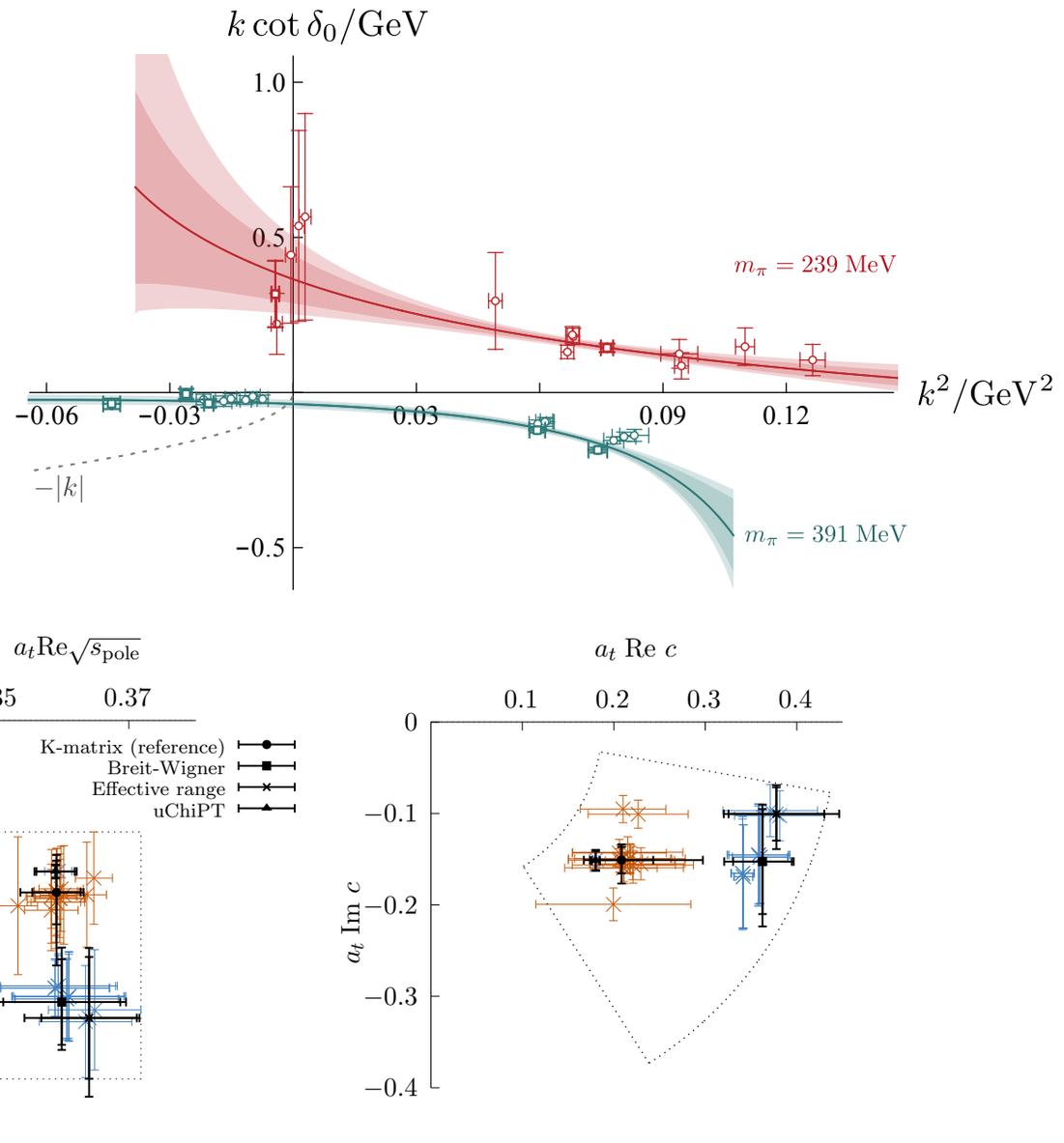
$$t \sim \frac{c^2}{s_{\text{pole}} - s} \qquad \sqrt{s_{\text{pole}}} = m \pm \frac{i}{2}\Gamma$$

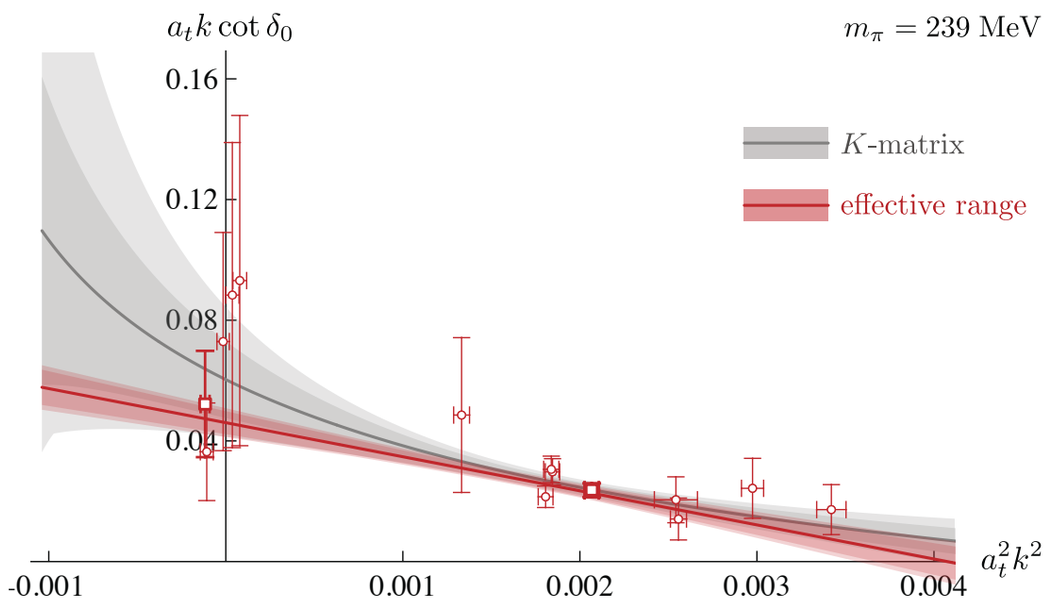


Breit-Wigner $m_\pi = 239$ MeV
Breit-Wigner $m_\pi = 391$ MeV
K-matrix

m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\text{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\text{Re}\sqrt{s_0}$/MeV	2189(72)	2275(1)
-2 $\text{Im}\sqrt{s_0}$/MeV	510(97)	-
$\lvert c\rvert$/MeV	2391(411)	826(133)

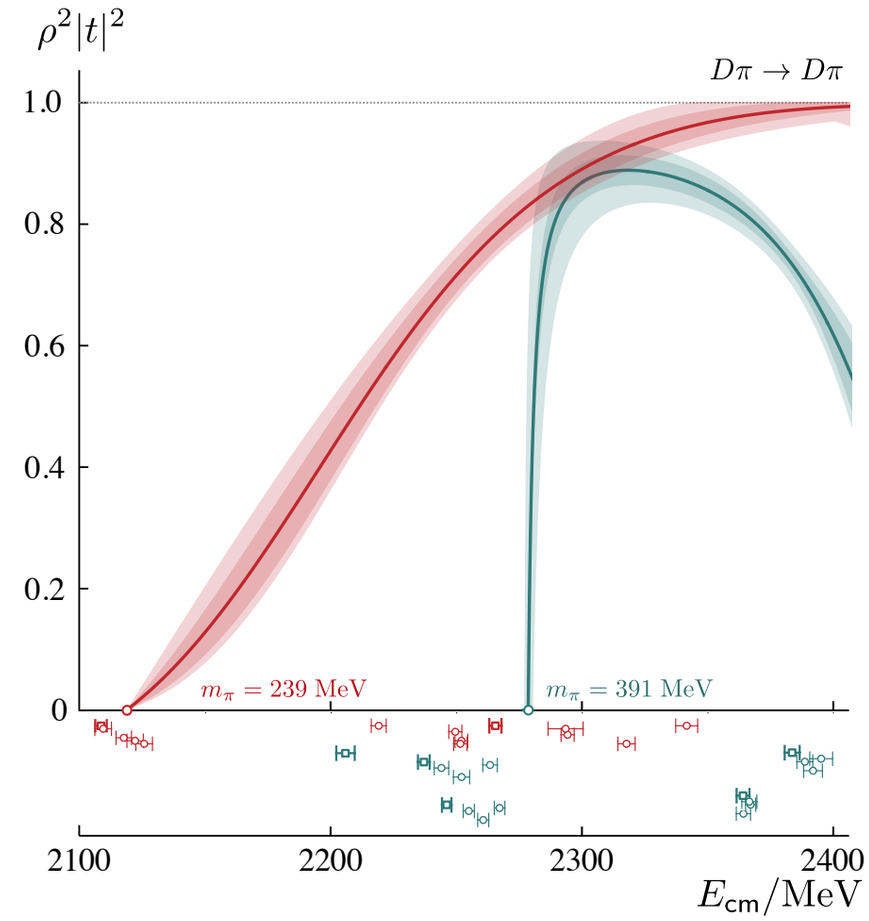
L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973

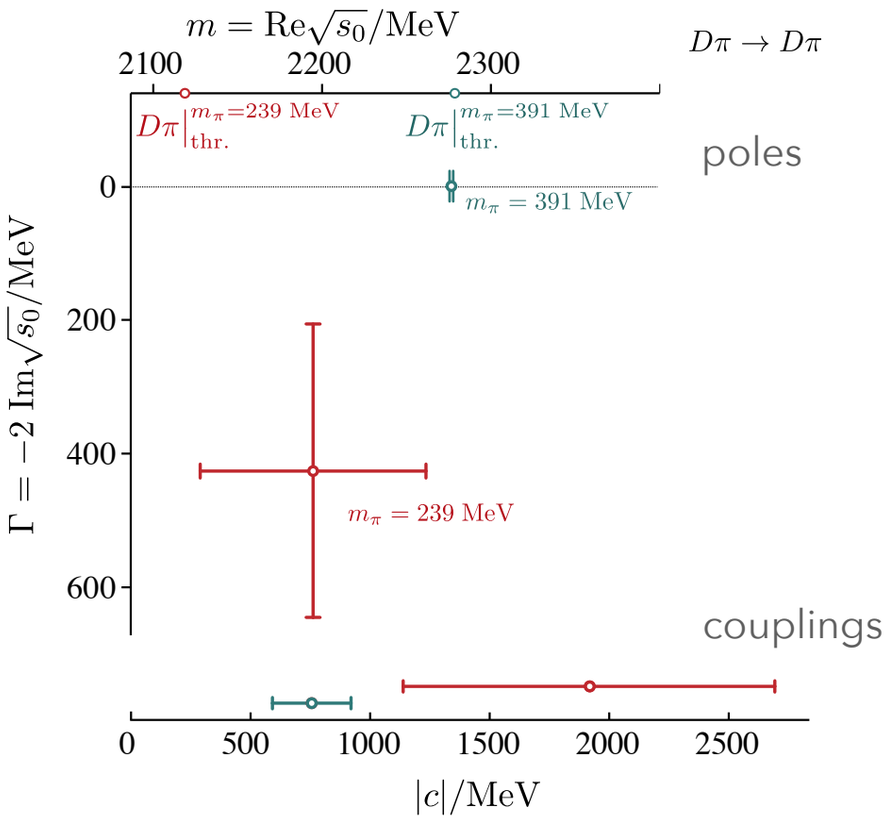




simple parameterisations work well over this narrow range:
effective range, Breit-Wigner, elastic K-matrix

simplification in a lattice calcs vs experiment:
pure s-channel elastic scattering
no heavy hadron production process to parameterise

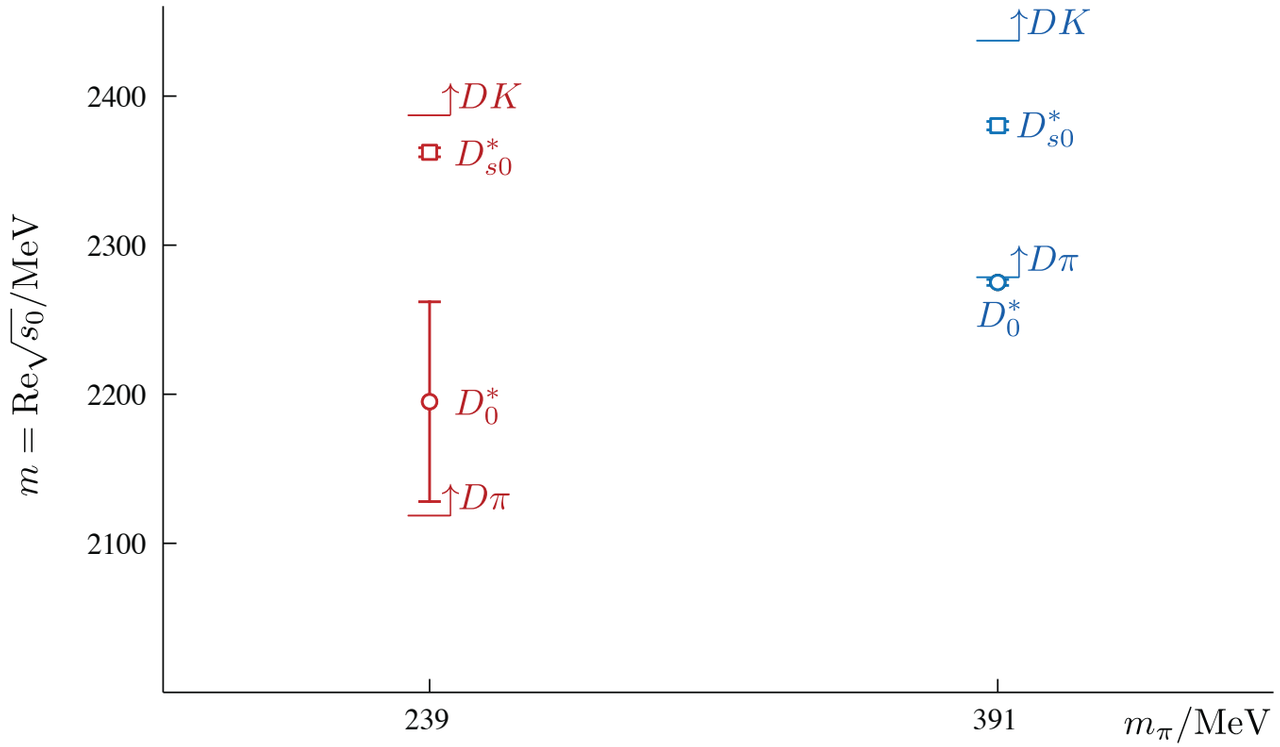




L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973

$$t \sim \frac{c^2}{s_{\mathrm{pole}} - s} \qquad \sqrt{s_{\mathrm{pole}}} = m \pm \frac{i}{2}\Gamma$$

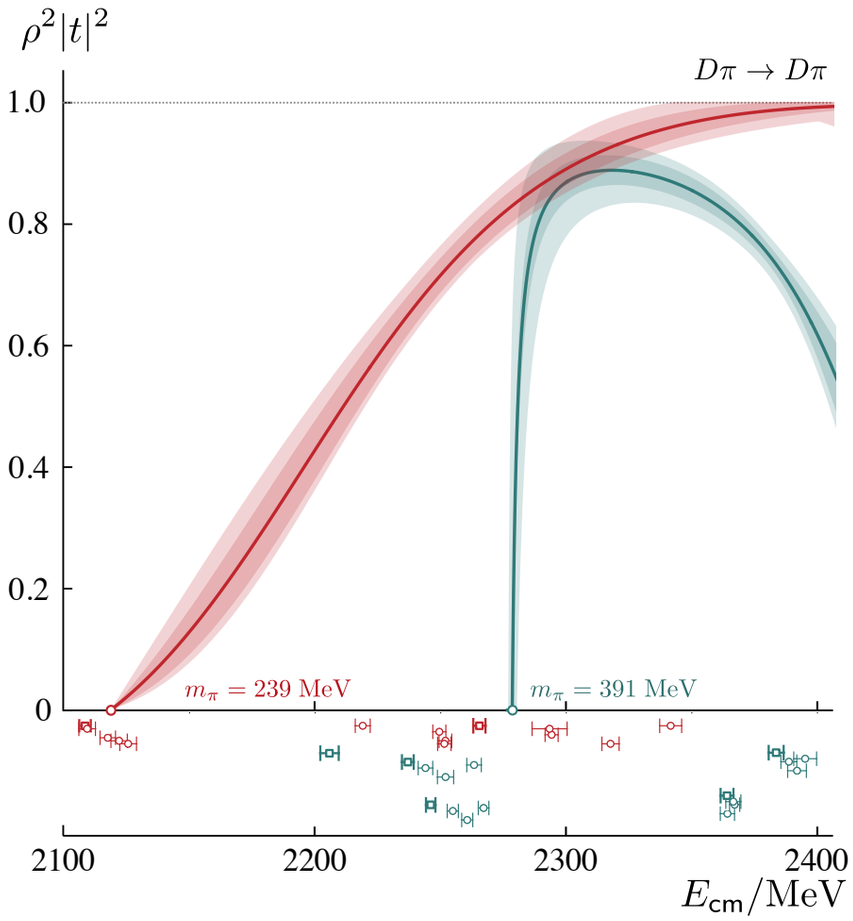
suggestive of a much lighter D_0^* compared with the D_{s0}^*

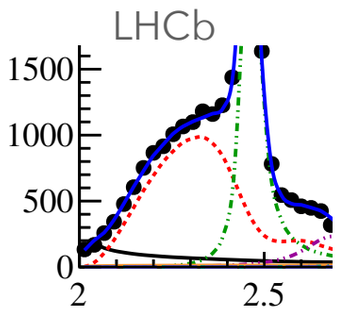


natural mass ordering: given light, strange constituents

likely hypothesis: D_0^* pole position is lower, m~2100-2200 MeV ?

see also LHCb data+ChiPT+unitarity: Du et al, PRL 126, 192001





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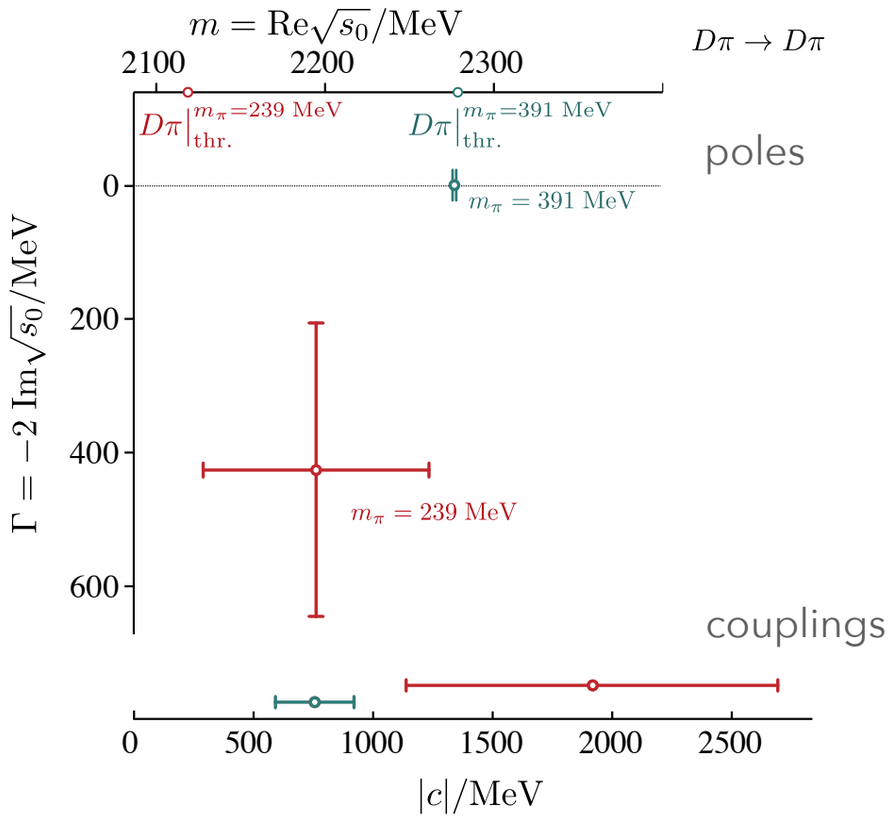
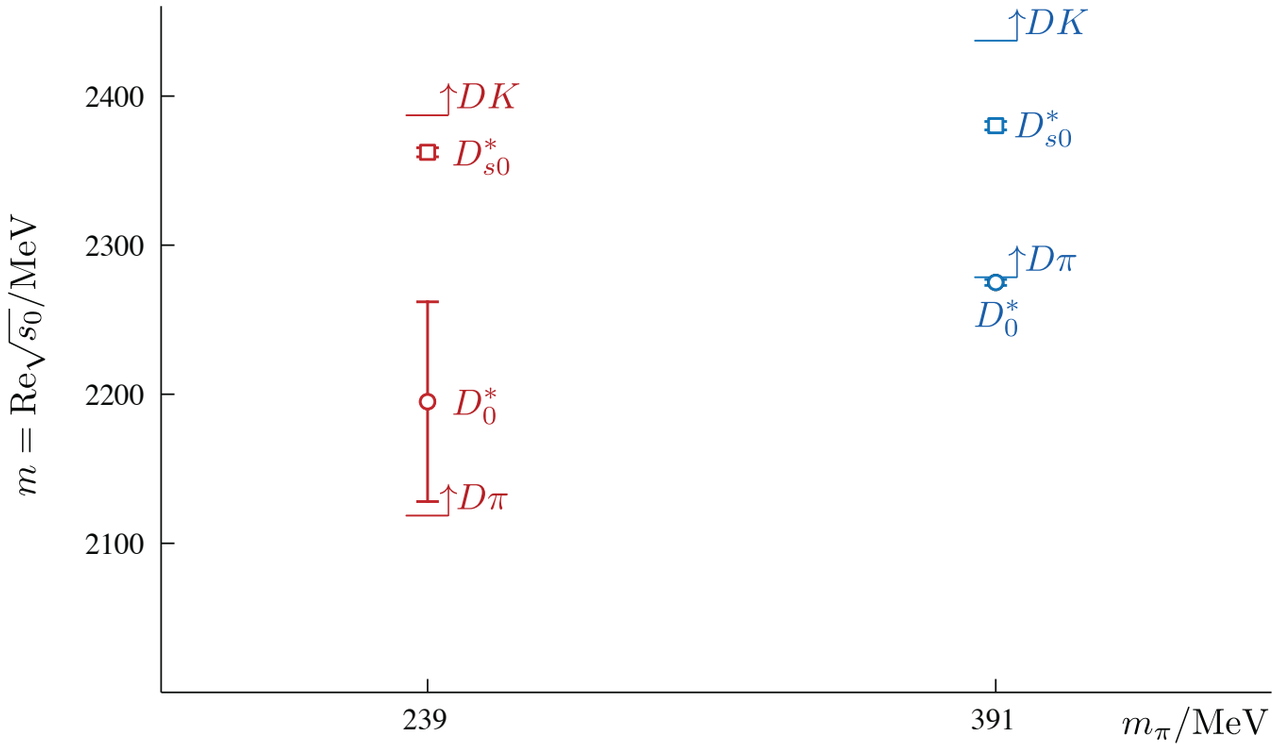
$$t \sim \frac{c^2}{s_{\rm pole} - s} \qquad \sqrt{s_{\rm pole}} = m \pm \frac{i}{2}\Gamma$$



suggestive of a much lighter D_0^* compared with the D_{s0}^*



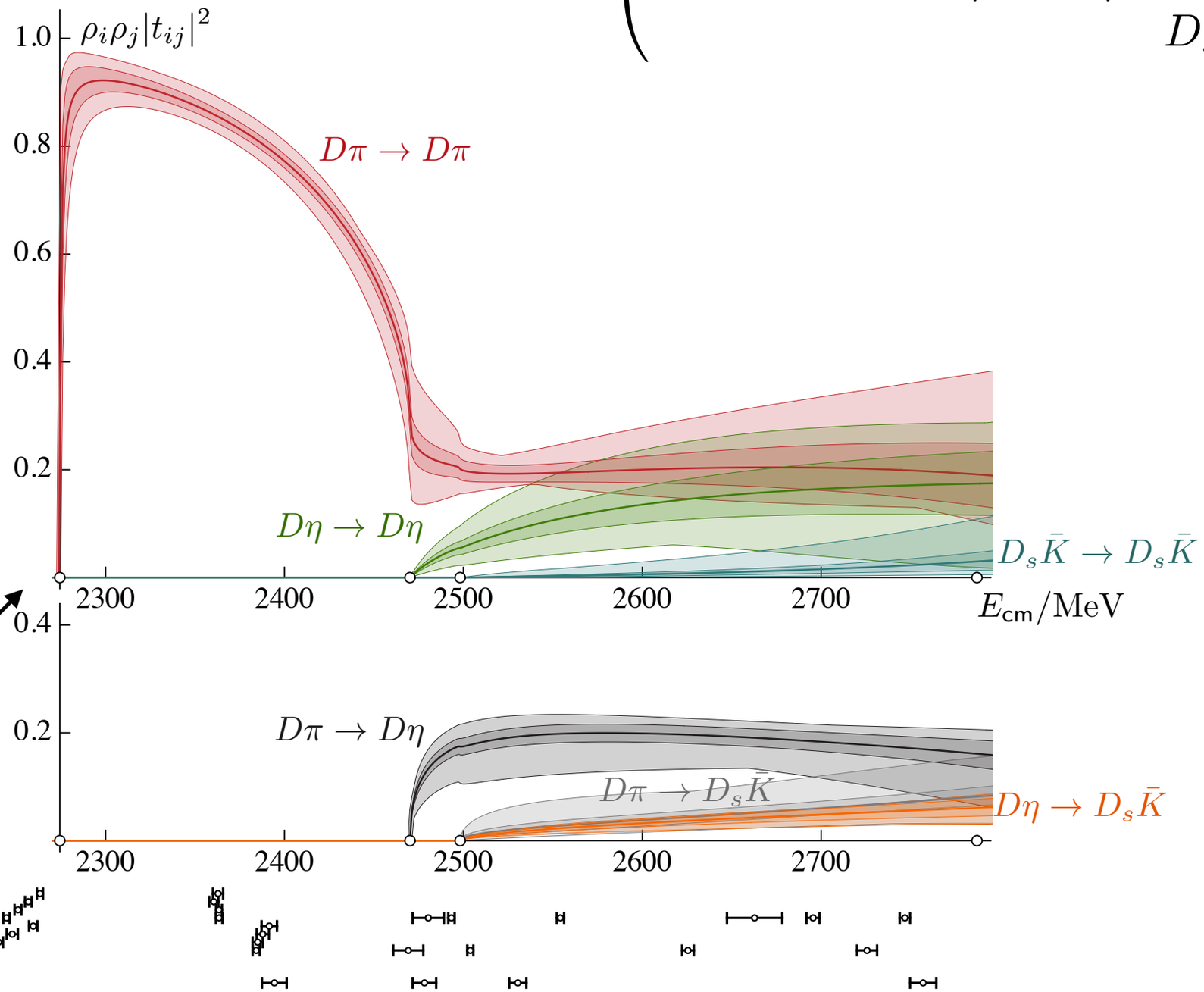
natural mass ordering: given light, strange constituents

likely hypothesis: D_0^* pole position is lower, m~2100-2200 MeV ?

see also LHCb data+ChiPT+unitarity: Du et al, PRL 126, 192001

$m_\pi = 391\,\mathrm{MeV}$

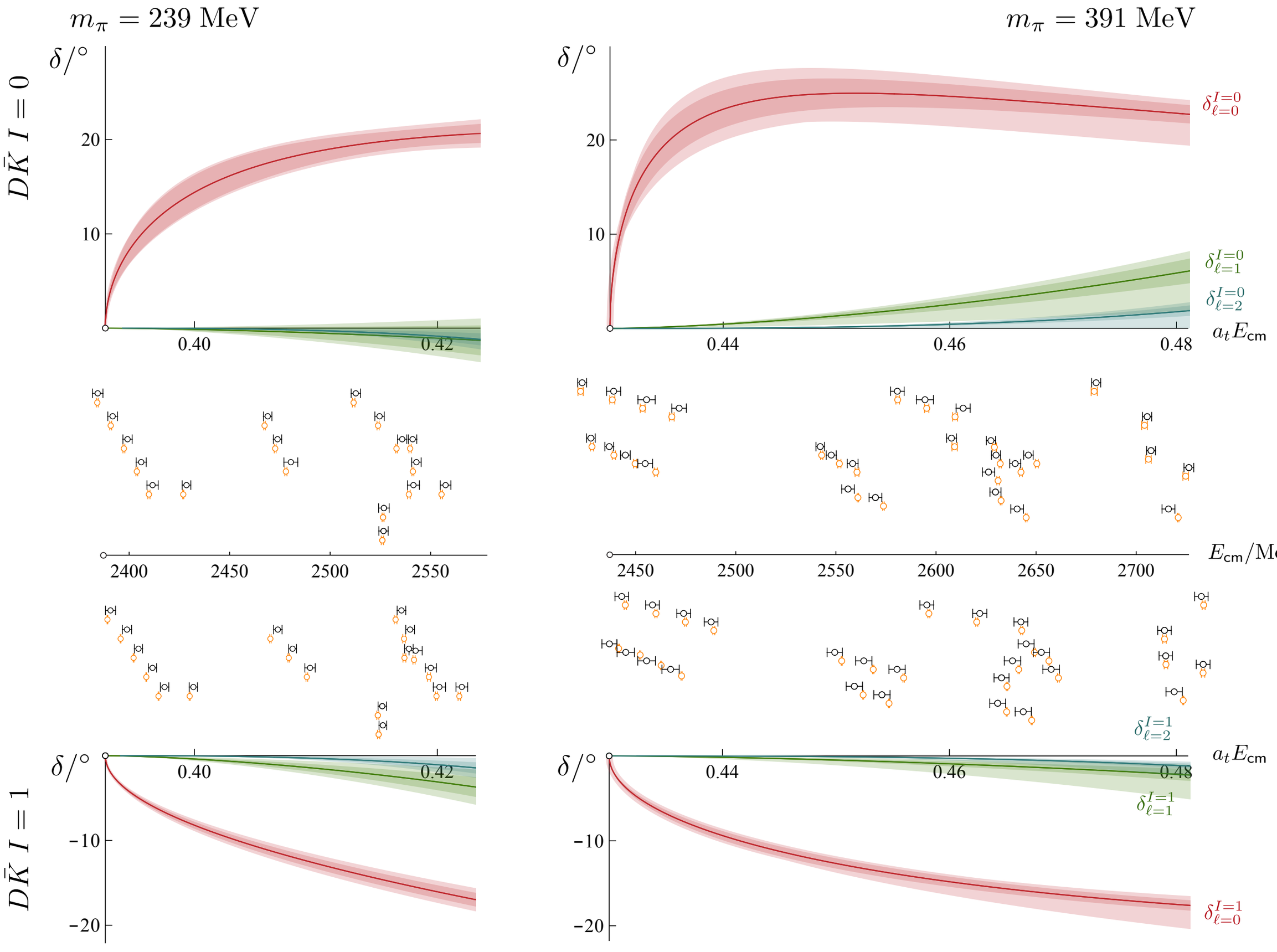
$$\boldsymbol{t} = \begin{pmatrix} D\pi \to D\pi & D\pi \to D\eta & D\pi \to D_s\bar{K} \\ & D\eta \to D\eta & D\eta \to D_s\bar{K} \\ & & D_s\bar{K} \to D_s\bar{K} \end{pmatrix}$$



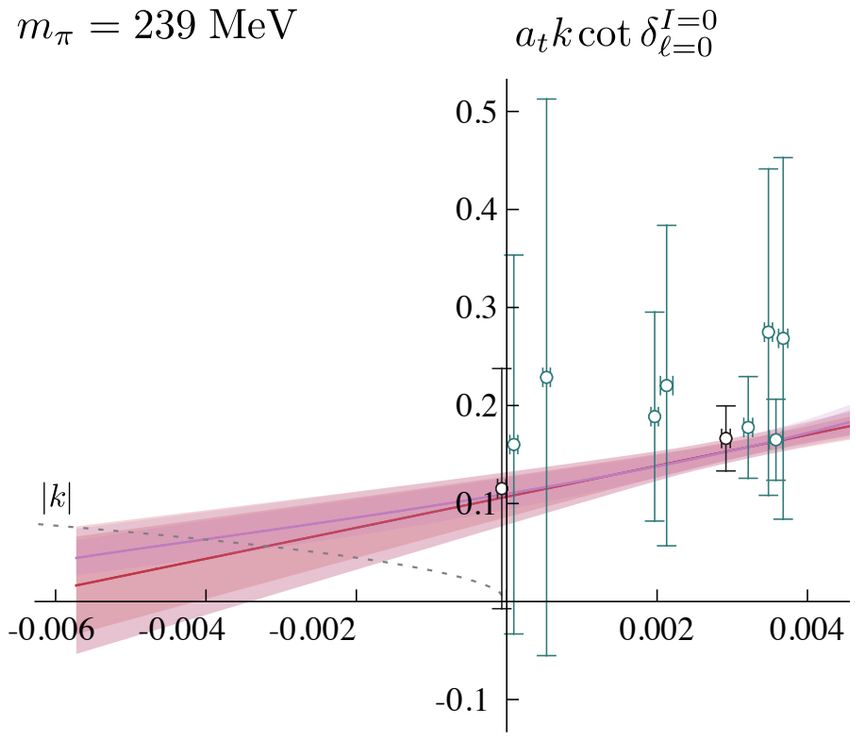
bound state D_0^*

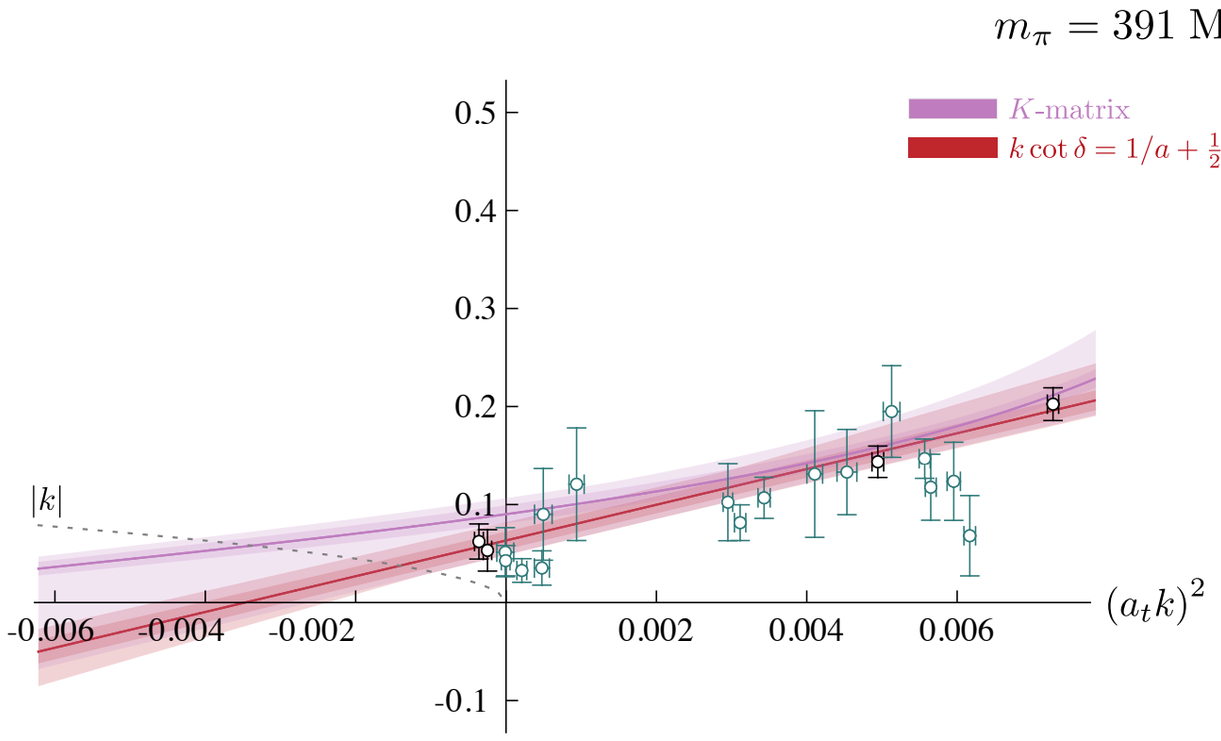
relatively sharp features, possibly a second pole

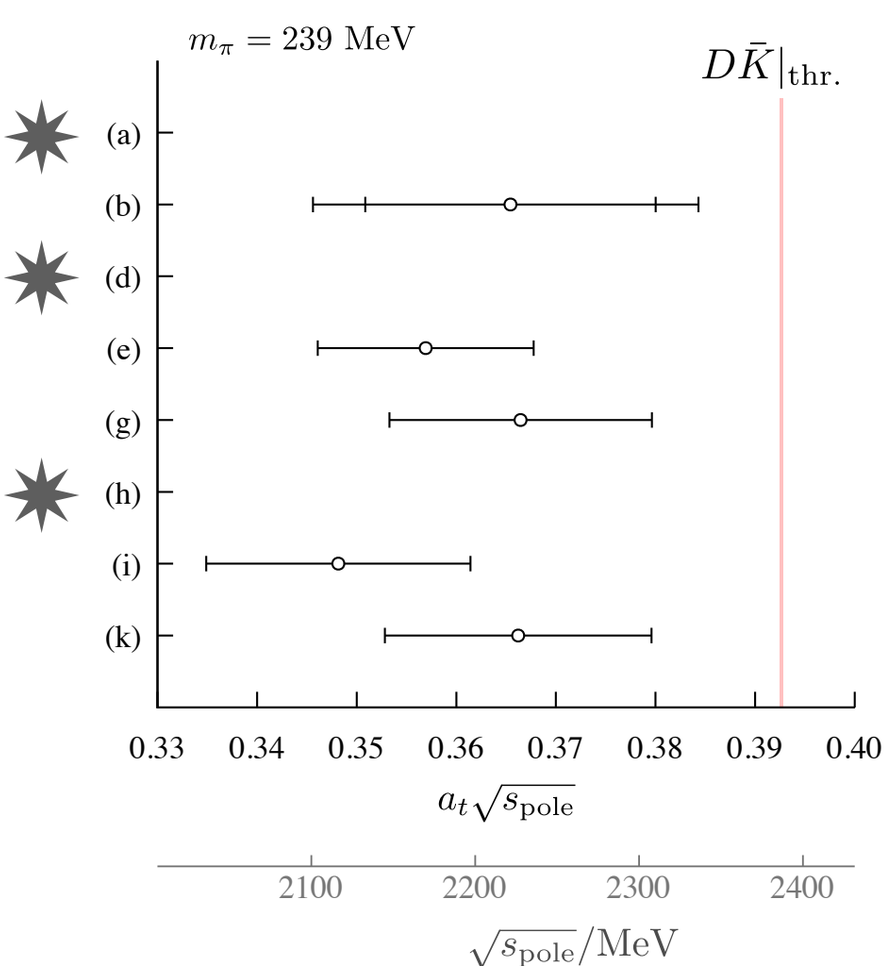
G. Moir et al (HadSpec), JHEP 10 (2016) 011

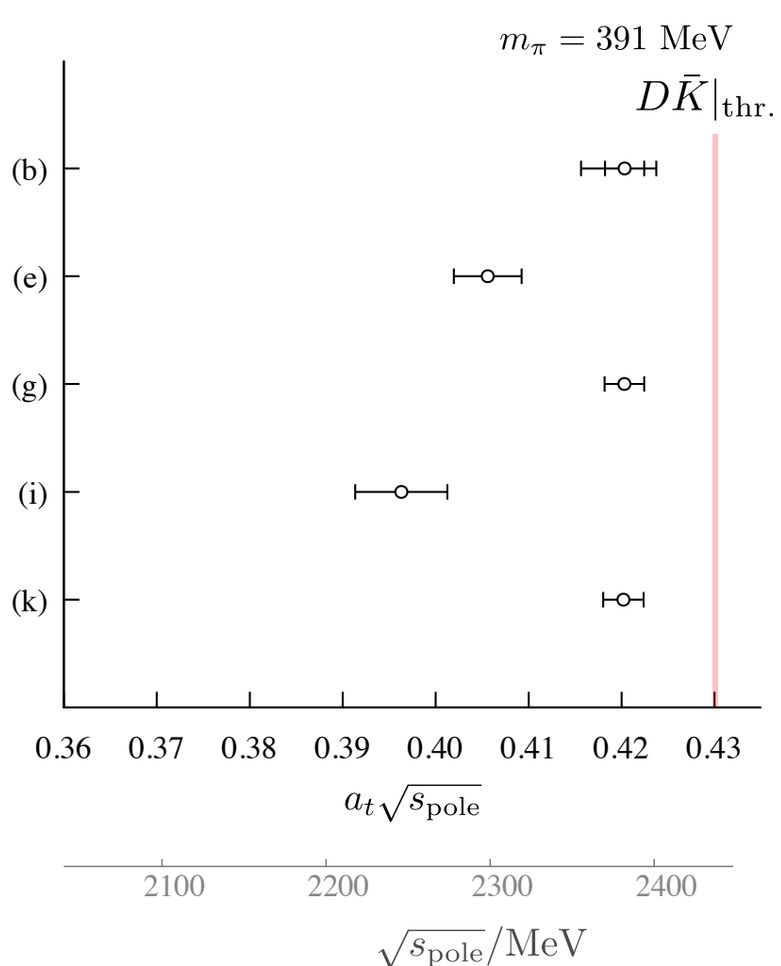

$m_\pi = 239$ MeV
$m_\pi = 391$ MeV
$D\bar{K}$ $I = 0$
$D\bar{K}$ $I = 1$
$\delta/^\circ$
20
10
−10
−20
0.40
0.42
0.44
0.46
0.48
$a_t E_{\mathrm{cm}}$
2400
2450
2500
2550
2600
2650
2700
$E_{\mathrm{cm}}/\mathrm{MeV}$
$\delta^{I=0}_{\ell=0}$
$\delta^{I=0}_{\ell=1}$
$\delta^{I=0}_{\ell=2}$
$\delta^{I=1}_{\ell=2}$
$\delta^{I=1}_{\ell=1}$
$\delta^{I=1}_{\ell=0}$

SU(3)F symmetry - DKbar I=0 poles









poles not found in every parameterisation

$$\underset{(D,D_s \quad \pi,K,\sim\eta)}{\overline{\mathbf{3}} \otimes \mathbf{8}} \rightarrow \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \underset{(D,D_s \quad \sim\eta')}{\overline{\mathbf{3}} \otimes \mathbf{1}} \rightarrow \overline{\mathbf{3}}.$$

$(I=0)$ DK-$D_s\eta$: $\overline{\mathbf{3}} \oplus \overline{\mathbf{15}}$

$(I=1)$ DK-$D_s\pi$: $\mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=\frac{1}{2})$ D_sK, $(I=1)$ $D\bar{K}$, $(I=\frac{3}{2})$ $D\pi$: $\overline{\mathbf{15}}$

$(I=\frac{1}{2})$ $D\pi$-$D\eta$-$D_s\bar{K}$: $\overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=0)$ $D\bar{K}$: $\mathbf{6}$

bound state in the $\overline{3}$

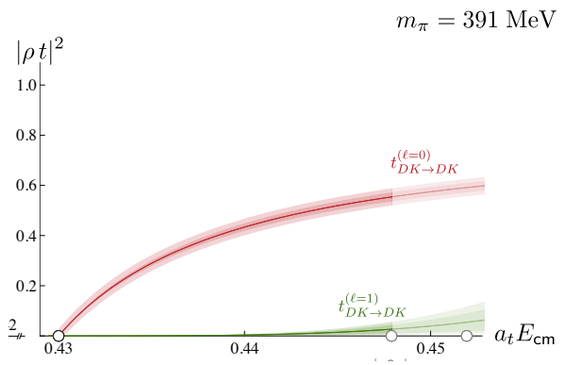
attraction, possibly a virtual state in the 6 (D$\boldsymbol{\pi}$ I=1/2 and DKbar I=0)

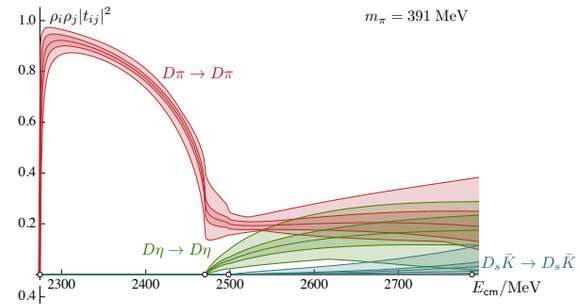
repulsion in 15: e.g. I=3/2 D$\boldsymbol{\pi}$

[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); PR D98 014510 (2018) (1801.10122); EPJ C79, 13 (2019) (1811.05585); arXiv:2106.15391]

$$\overline{\mathbf{3}} \otimes \mathbf{8} \to \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \overline{\mathbf{3}} \otimes \mathbf{1} \to \overline{\mathbf{3}}.$$

$(D, D_s \quad \pi, K, \sim \eta)$ $(D, D_s \quad \sim \eta')$

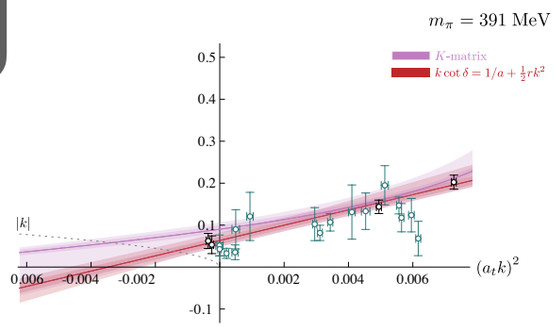




$(I=0)$ DK-$D_s\eta$: $\overline{\mathbf{3}} \oplus \overline{\mathbf{15}}$ $(I=\frac{1}{2})$ $D\pi$-$D\eta$-$D_s\bar{K}$: $\overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=1)$ DK-$D_s\pi$: $\mathbf{6} \oplus \overline{\mathbf{15}}$ $(I=0)$ $D\bar{K}$: $\mathbf{6}$

$(I=\frac{1}{2})$ D_sK, $(I=1)$ $D\bar{K}$, $(I=\frac{3}{2})$ $D\pi$: $\overline{\mathbf{15}}$

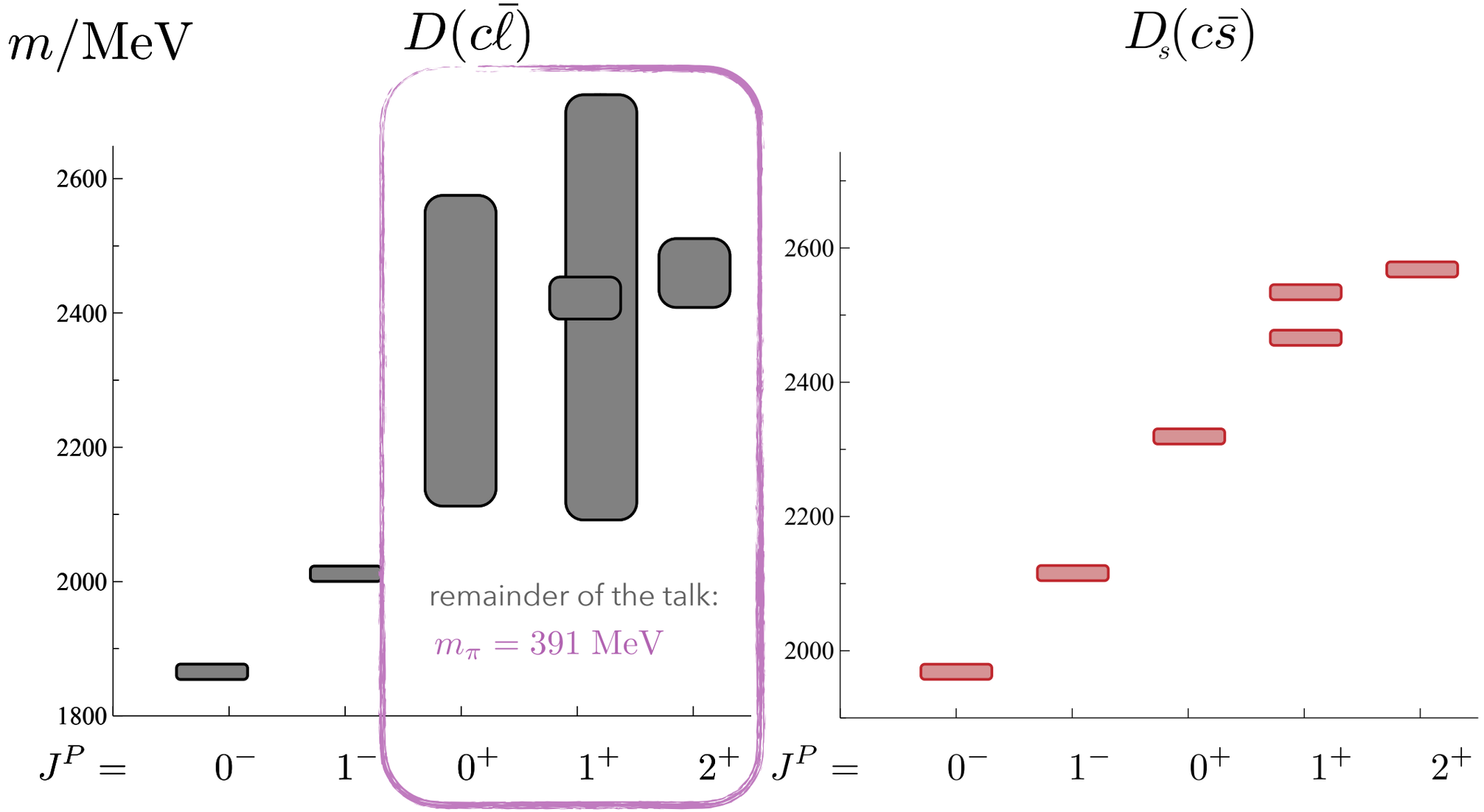


bound state in the 3

attraction, possibly a virtual state in the 6 (Dπ I=1/2 and DKbar I=0)

repulsion in 15: e.g. I=3/2 Dπ

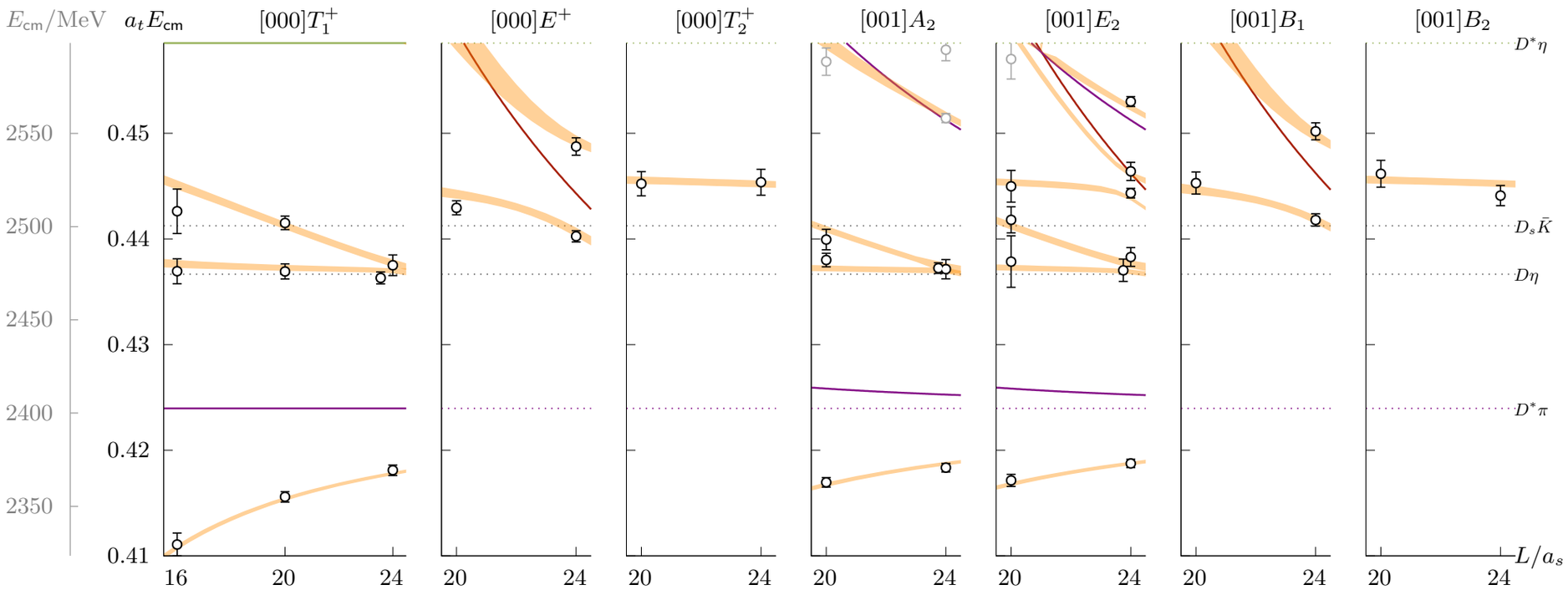
[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); PR D98 014510 (2018) (1801.10122); EPJ C79, 13 (2019) (1811.05585); arXiv:2106.15391]

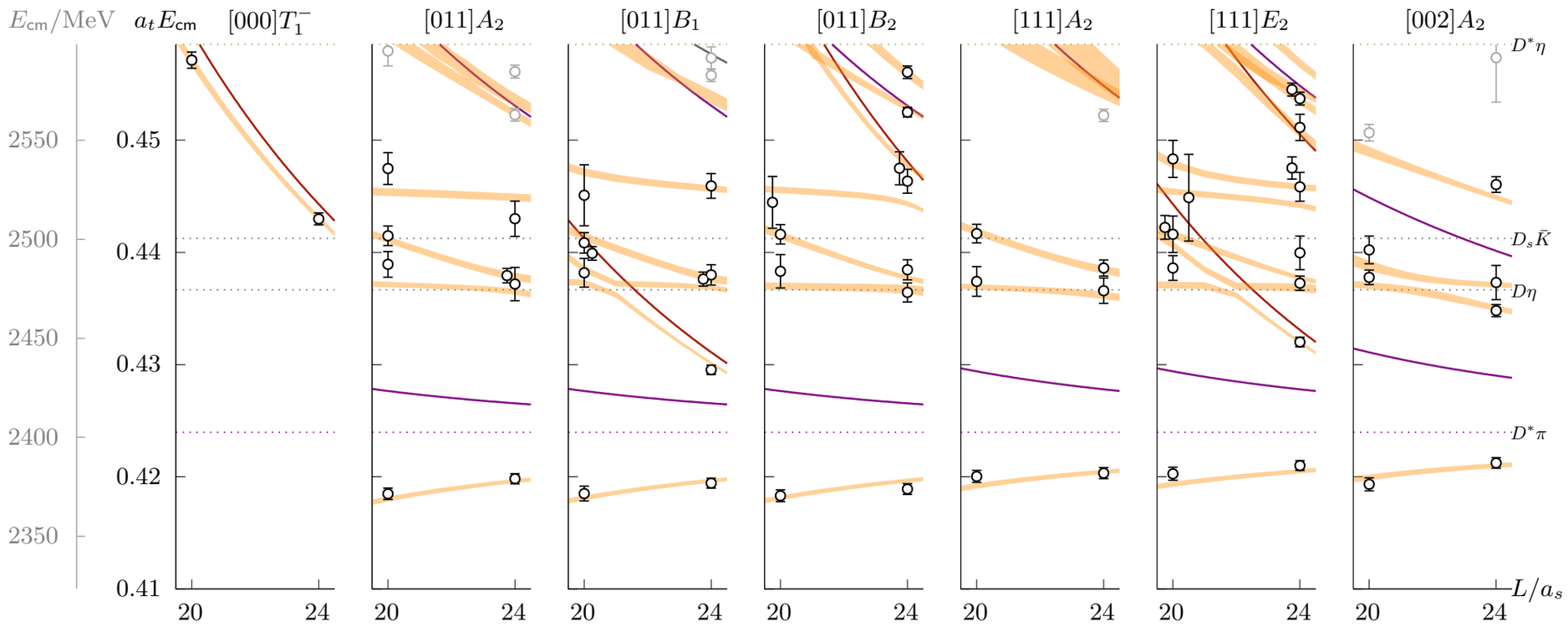


[masses, widths from PDG]

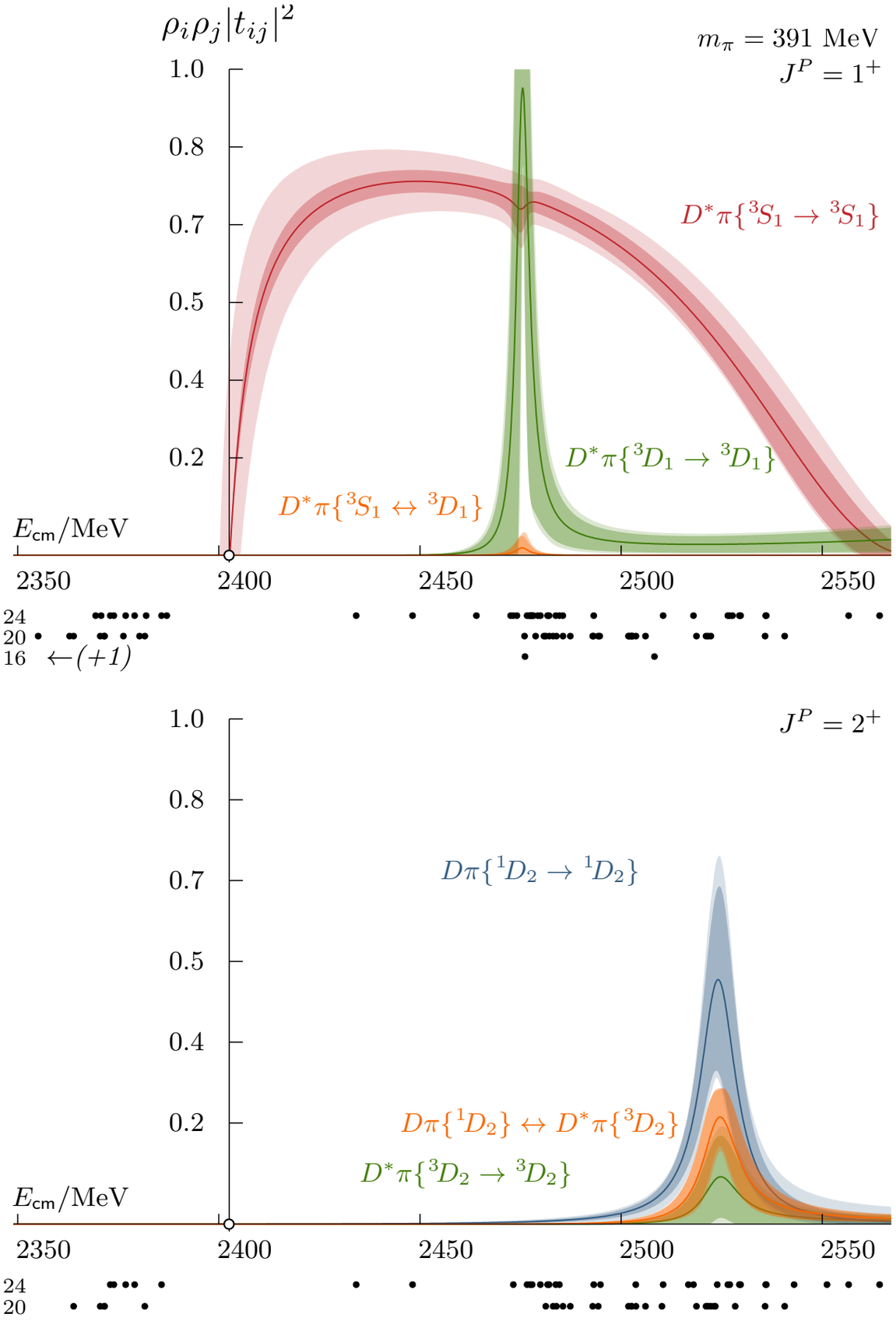
D*π scattering

$m_\pi = 391$ MeV



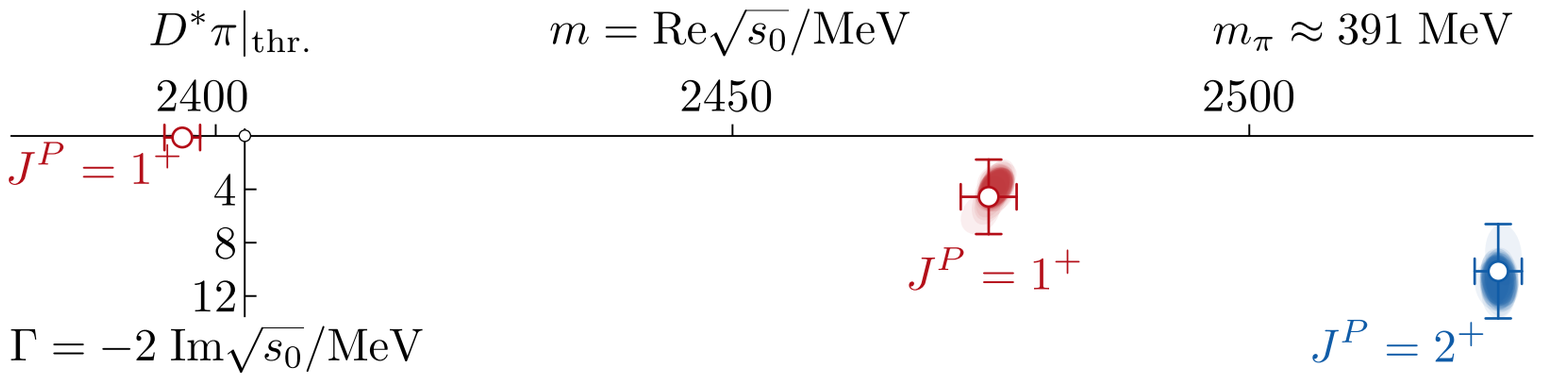


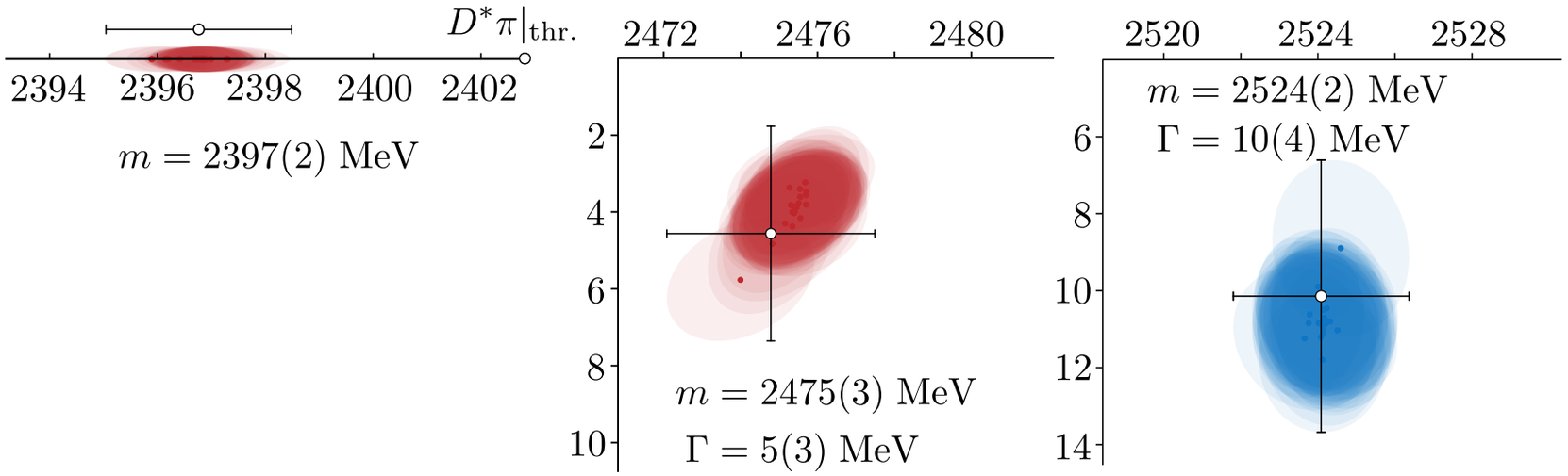
N. Lang and DW (HadSpec)
arXiv: 2205.05026

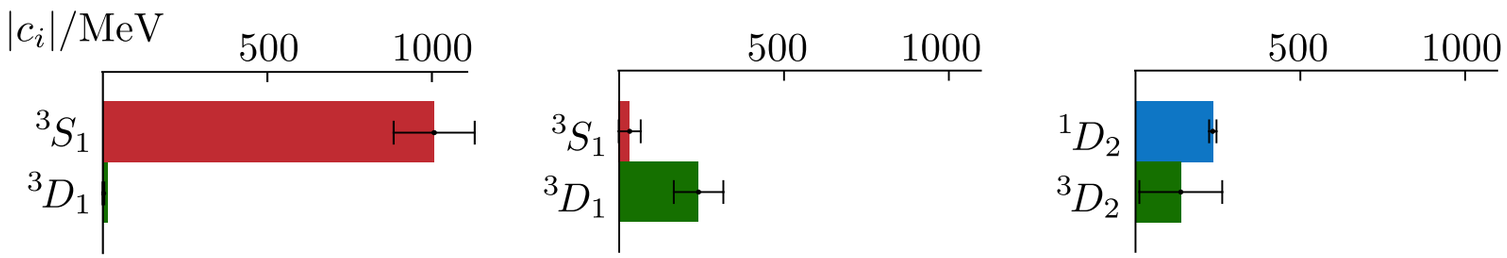


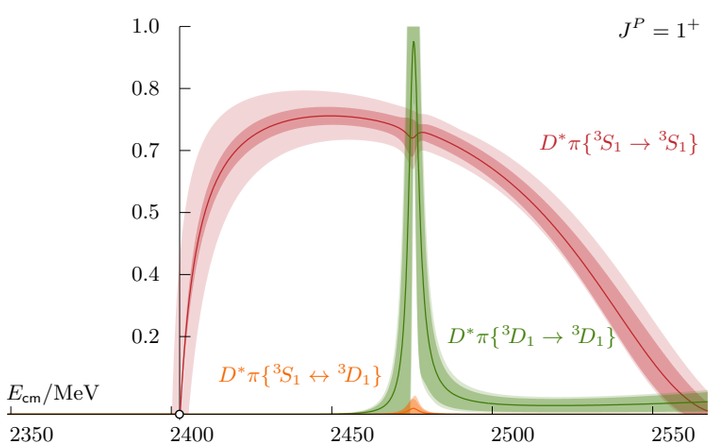
$$\boldsymbol{t} = \begin{pmatrix} D^*\pi\{^3S_1\} \to D^*\pi\{^3S_1\} & D^*\pi\{^3S_1\} \to D^*\pi\{^3D_1\} \\ & D^*\pi\{^3D_1\} \to D^*\pi\{^3D_1\} \end{pmatrix}$$

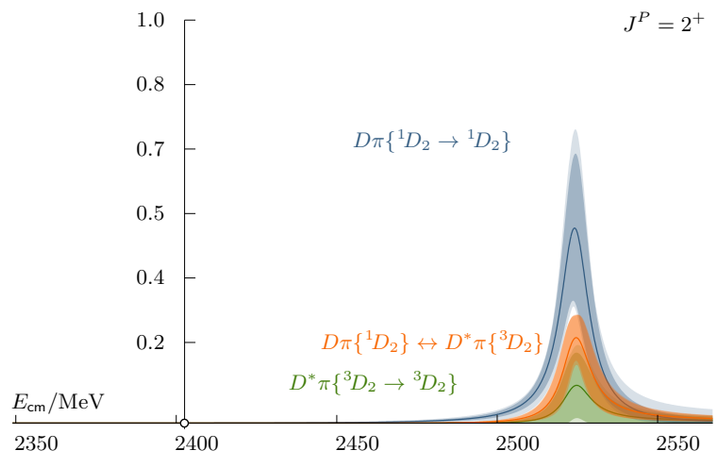
$$\boldsymbol{t} = \begin{pmatrix} D\pi\{^1D_2\} \to D\pi\{^1D_2\} & D\pi\{^1D_2\} \to D^*\pi\{^3D_2\} \\ & D^*\pi\{^3D_2\} \to D^*\pi\{^3D_2\} \end{pmatrix}$$


$D^*\pi|_{\text{thr.}}$
$m = \text{Re}\sqrt{s_0}/\text{MeV}$
$m_\pi \approx 391$ MeV
2400
2450
2500
$J^P = 1^+$
$J^P = 1^+$
$J^P = 2^+$
4
8
12
$\Gamma = -2\,\text{Im}\sqrt{s_0}/\text{MeV}$


$D^*\pi|_{\text{thr.}}$
2394
2396
2398
2400
2402
$m = 2397(2)$ MeV
2472
2476
2480
2
4
6
8
10
$m = 2475(3)$ MeV
$\Gamma = 5(3)$ MeV
2520
2524
2528
$m = 2524(2)$ MeV
$\Gamma = 10(4)$ MeV
6
8
10
12
14


$|c_i|/\text{MeV}$
500
1000
$^3S_1$
$^3D_1$
500
1000
$^3S_1$
$^3D_1$
500
1000
$^1D_2$
$^3D_2$


$J^P = 1^+$
1.0
0.8
0.7
0.5
0.4
0.2
$D^*\pi\{^3S_1 \to {}^3S_1\}$
$D^*\pi\{^3D_1 \to {}^3D_1\}$
$D^*\pi\{^3S_1 \leftrightarrow {}^3D_1\}$
E_{cm}/MeV
2350
2400
2450
2500
2550


$J^P = 2^+$
1.0
0.8
0.7
0.5
0.4
0.2
$D\pi\{^1D_2 \to {}^1D_2\}$
$D\pi\{^1D_2\} \leftrightarrow D^*\pi\{^3D_2\}$
$D^*\pi\{^3D_2 \to {}^3D_2\}$
E_{cm}/MeV
2350
2400
2450
2500
2550

Rosner: Comments Nucl.Part.Phys. 16 (1986) 3, 109-13,
Isgur, Wise: PRL 66 (1991) 1130-1133

in the limit of an infinitely massive quark,
the spin of the heavy quark cannot be perturbed by QCD interactions

states can be arranged into two doublets depending on heavy-quark spin

charm quark mass is still large compared with the scale of QCD interactions

suggests decoupled S and D-wave amps

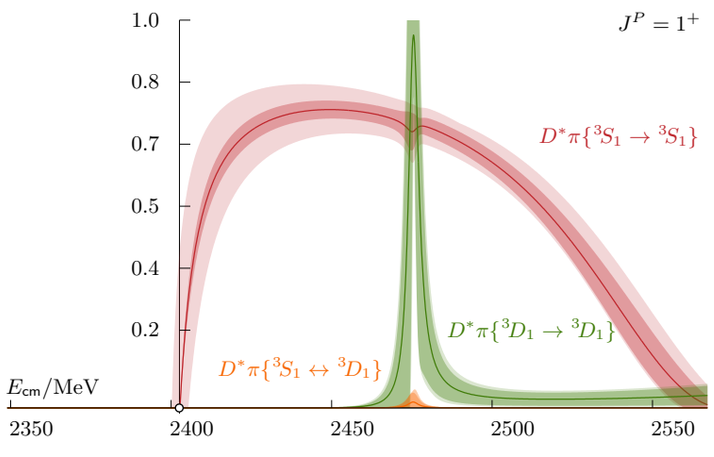
$$\vec{S} = \vec{s}_Q + \vec{s}_\ell$$

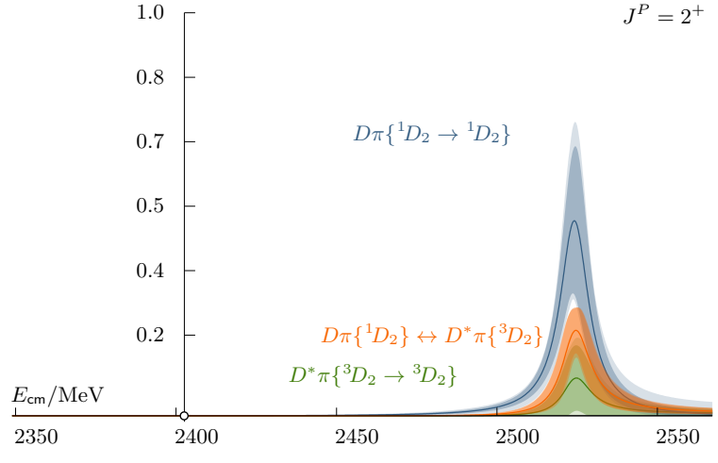
$$s_\ell = \frac{1}{2} \qquad \begin{pmatrix} D_0^* \\ D_1 \end{pmatrix}$$

decays via S-wave

$$s_\ell = \frac{3}{2} \qquad \begin{pmatrix} D_1' \\ D_2 \end{pmatrix}$$

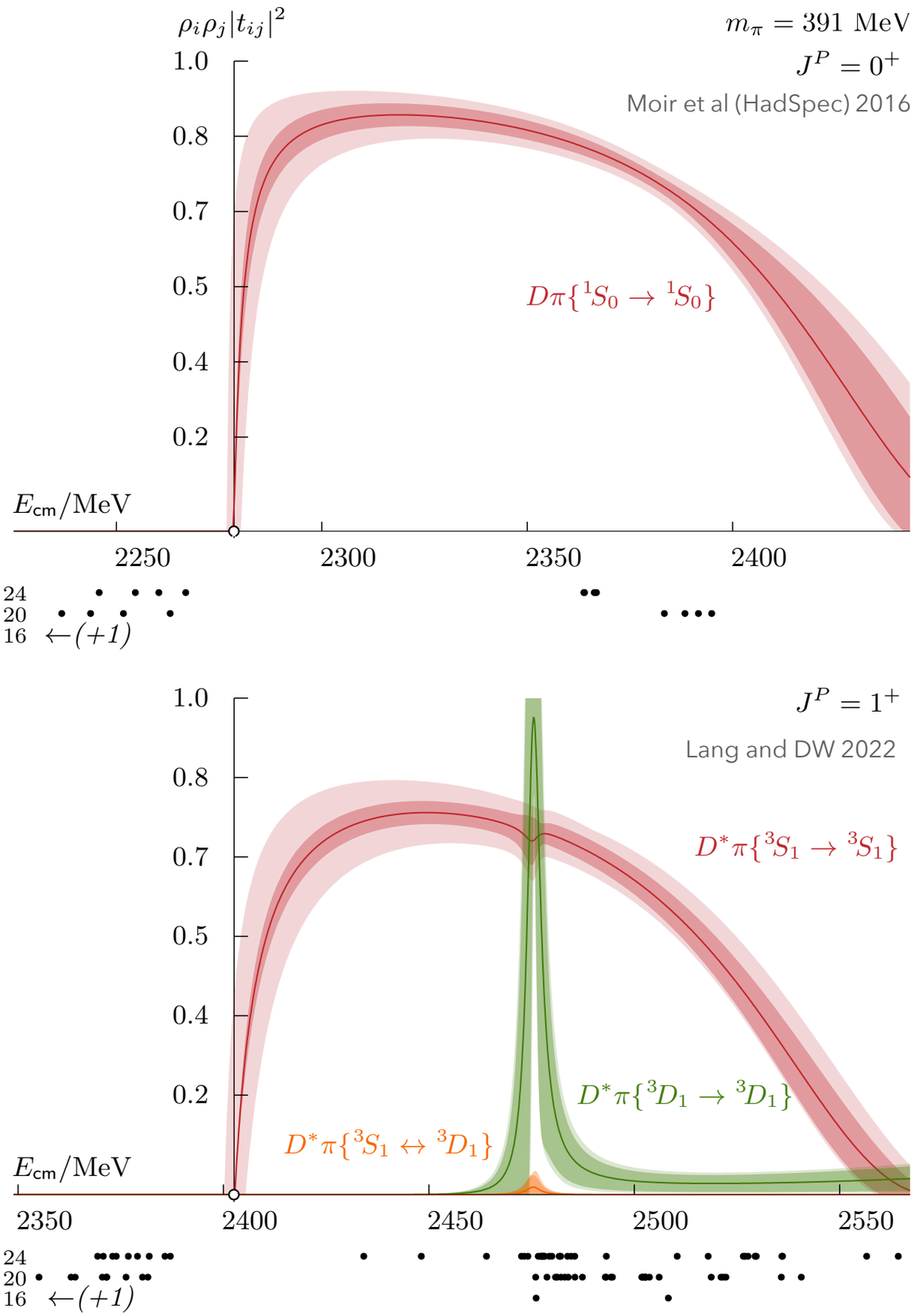
decays via D-wave





ratios of couplings can also be estimated

$$\begin{pmatrix} D_0^* \\ D_1 \end{pmatrix}$$



${}^3S_1,\ {}^1S_0$ *amplitudes* are very similar

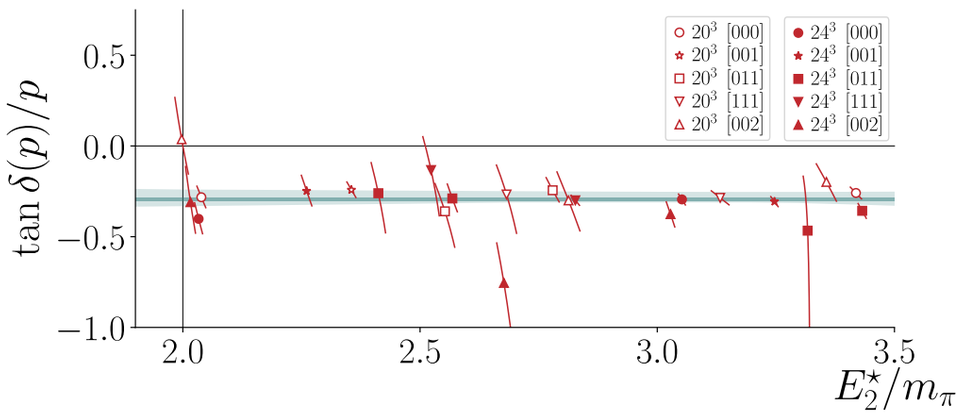
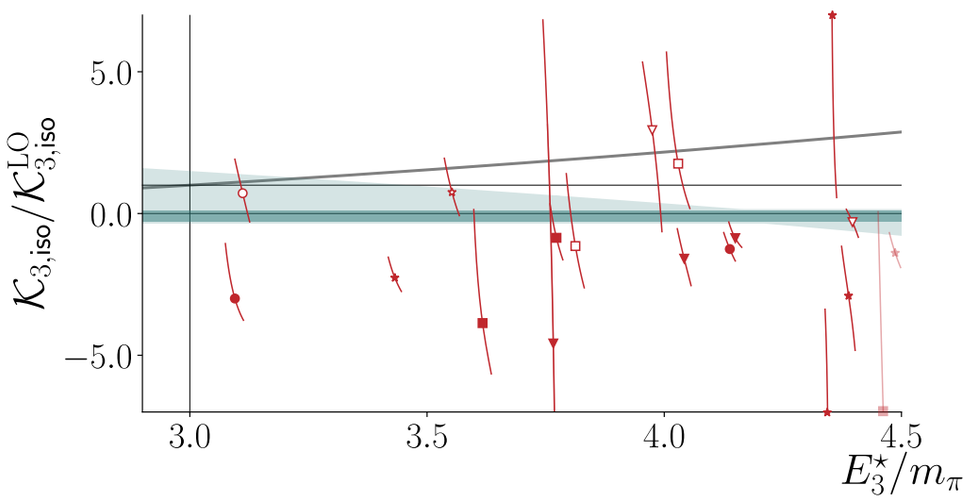
suggestive of a lightest D_1 with a pole mass below 2400 MeV for physical quark masses

Lattice QCD provides a **first-principles** tool to do **hadron spectroscopy**

D and Ds systems
- readily accessible in lattice QCD calculations
- useful place to compare lattice with experiment & other theoretical approaches

These methods are widely applicable
- coupled-channel scattering
- baryons
- charmonium, b-quarks
- form factors, radiative transitions (incl. resonances)

…

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances

M. T. Hansen et al (HadSpec), PRL 126 (2021) 012001, arXiv:2009.04931