Proton spin structure at low x

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[RB, Hatta, Yuan] PLB 797 (2019) 134817

The proton spin puzzle

- The proton has spin 1/2.
- The proton is not an elementary particle
- Proton spin decomposition (sum rule)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L^q + J^g = \frac{1}{2}\Delta\Sigma + L^q + \Delta G + L^g$$

[Ji] [Jaffe, Manohar]

$\Delta\Sigma, \Delta G$ helicities, $L^{q,g}$ orbital angular momenta, $J^{q,g}$ total angular momenta

In the naive quark model, $\Delta\Sigma = 1$

The proton spin puzzle

In the naive quark model, $\Delta\Sigma = 1$

European Muon Collaboration (EMC), 1987:

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14$$

NLO QCD global analysis:

$$\Delta\Sigma \sim 0.25$$

The naive picture where the proton $1/2$ spin comes from 3 quark $1/2$ spins is far insufficient.

QCD approach to proton spin

QCD spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L^q + \Delta G + L^g$$

$\Delta\Sigma, \Delta G, L^q, L^g$ are moments of parton distributions.

E.g.

$$\Delta\Sigma = \sum_f \int_0^1 \mathrm{d}x \Delta q_f(x)$$

$\Delta q_f(x)$ is the polarized quark Parton Distribution Function for f quarks

QCD approach to proton spin

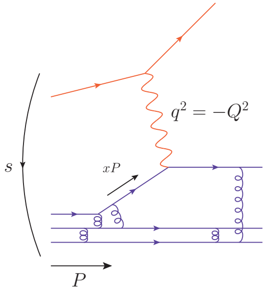
QCD spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L^q + \Delta G + L^g$$

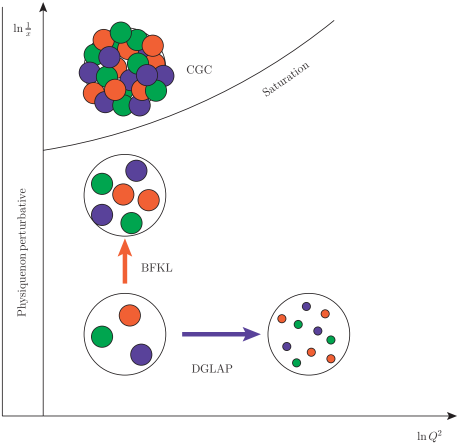
$\Delta\Sigma, \Delta G, L^q, L^g$ are moments of parton distributions.

- Need experimental constraints for parton distributions
- Need theoretical understanding of the integral's boundaries

Accessing the partonic content of hadrons with an electromagnetic probe

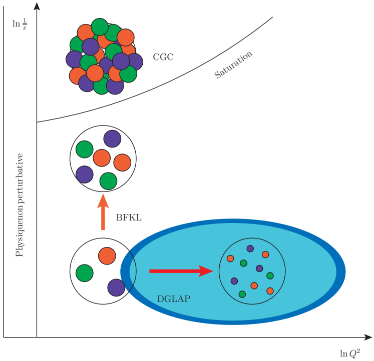


Electron-proton collision (parton model)



QCD at moderate $x_{\mathrm{Bj}} \sim Q^2/s$

Bjorken limit: $Q^2 \sim s$

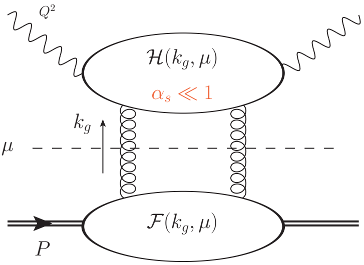


QCD at moderate x: QCD factorization

Factorization into:

- a hard part $\mathcal{H}$ computed with perturbative methods

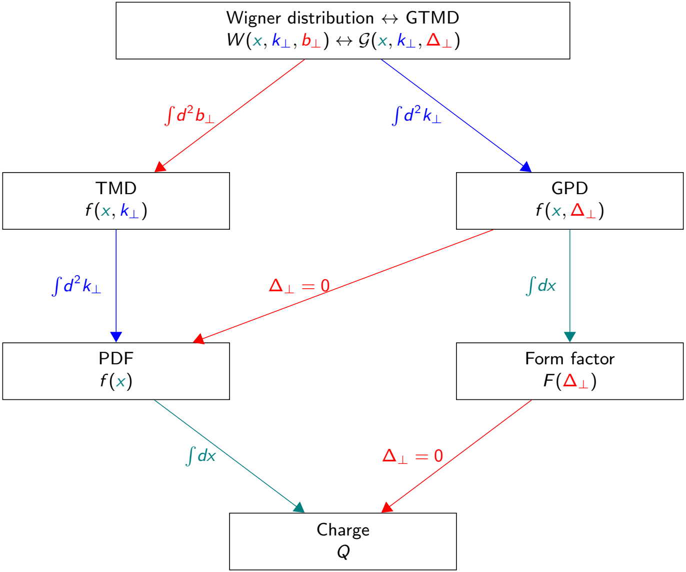
- a parton distribution $\mathcal{F}$ non-perturbative (constrained by experimental data or estimated with non-perturbative methods, e.g. lattice QCD)



- Twist expansion
  $\sigma = \sigma_0 + \frac{1}{Q}\sigma_1 + ...$
- Resummation of logarithms
  $\sigma_0 = \sum_n \left[A_n(\alpha_s \ln Q^2)^n + \alpha_s B_n(\alpha_s \ln Q^2)^n ...\right]$
- Cancellation of divergences $\Leftrightarrow$ Renormalization of the parton distribution $\mathcal{F}$

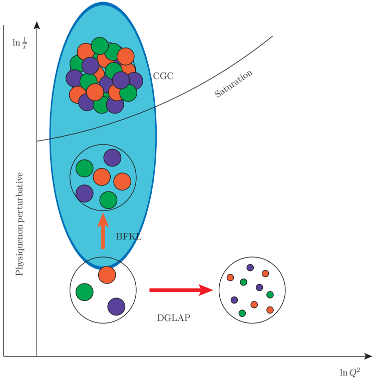
  E.g.: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution for a Parton Distribution Function

The family tree of parton distributions



QCD at small $x_{\mathrm{Bj}} \sim Q^2/s$

Regge limit: $Q^2 \ll s$



QCD at small x for non-spin observables:

- Eikonal expansion

  $$\sigma = \sigma_0 + x\sigma_1 + \ldots$$

- Resummation of logarithms

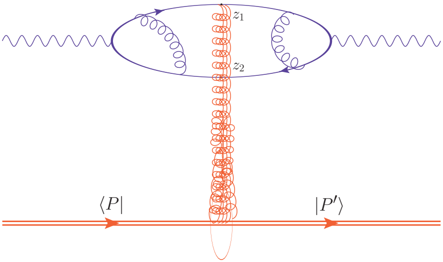
  $$\sigma_0 = \sum_n \left[A_n(\alpha_s \ln \tfrac{1}{x})^n + \alpha_s B_n(\alpha_s \ln \tfrac{1}{x})^n \ldots\right]$$

- Cancellation of divergences $\Leftrightarrow$ BFKL evolution (unsaturated), BK-JIMWLK evolution (saturated)

QCD at small x (non-spin): several approaches

Semi-classical effective theories (e.g. CGC)

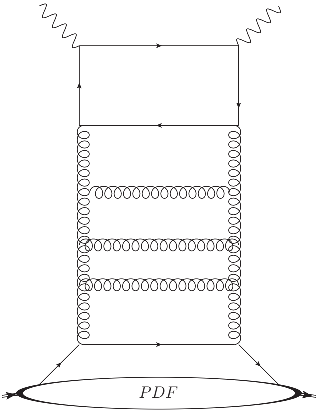
- Small-x "factorization" with Wilson line distributions
- Resum $(\alpha_s \ln \frac{1}{x})^n$ using evolution equations
- Takes all genuine higher twists into account
- Collinear sector for distributions at $x = 0$ needs fixing



QCD at small x (non-spin): several approaches

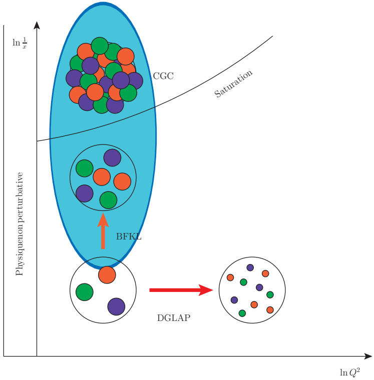
BFKL resummation

- Factorize the amplitude with QCD factorization
- "Improve" factorization by resumming $(\alpha_s \ln \frac{1}{x})^n$
- Relies on an ansatz for the proton
- Neglects genuine higher twists



Spin physics at small $x_{\rm Bj} \sim Q^2/s$

Regge limit: $Q^2 \ll s$



QCD at small x for spin observables:

- Eikonal expansion

  $\sigma = \sigma_0 + x\sigma_1 + \ldots$ The first order cancels

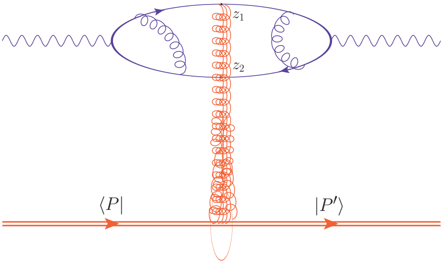
- Resummation of double logarithms

  $\sigma_1 = \sum_n \left[A_n(\alpha_s \ln^2 \frac{1}{x})^n + \alpha_s B_n(\alpha_s \ln^2 \frac{1}{x})^n \ldots\right]$

Spin at small x: several approaches

Semi-classical effective description: KPS approach
[Kovchegov, Pitonyak, Sievert]

- Small-x "factorization" with polarized Wilson line distributions
- Resum $(\alpha_s \ln^2 \frac{1}{x})^n$ using evolution equations
- Takes all genuine higher twists into account
- Collinear sector for distributions at $x = 0$ needs fixing

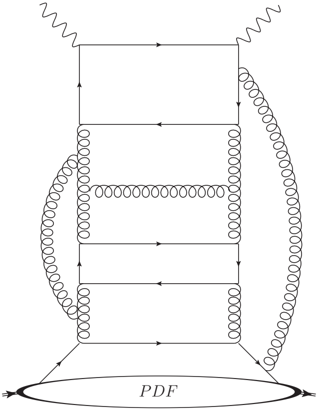


Some Wilson lines are subeikonal

Spin at small x: several approaches

BFKL-like resummation: BER approach
[Bartels, Ermolaev, Ryskin]

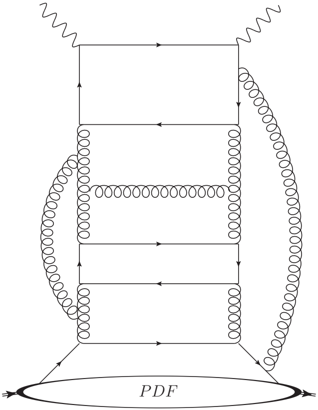
- Factorize the amplitude with QCD factorization
- "Improve" factorization by resumming $(\alpha_s \ln^2 \frac{1}{x})^n$
- Relies on an ansatz for the proton
- Neglects genuine higher twists



BER in a nutshell

BER approach to the polarized DIS structure function g_1

- Infrared cutoff scale μ^2
- Double logs: lifetime ordering
  $\beta_i / k_{i\perp}^2 \gg \beta_{i+1} / k_{i+1\perp}^2,$
  $\beta_i \gg \beta_{i+1}, \quad k_{i\perp}^2 \gg \mu^2$
- No ordering in $k_\perp$ Very tough "ladder" to resum!

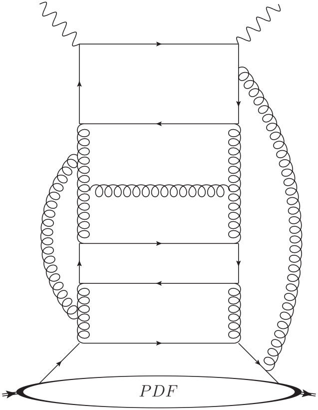


BER in a nutshell

BER approach to the polarized DIS structure function g_1

InfraRed Evolution Equations
(IREE)
[Kirschner, Lipatov]

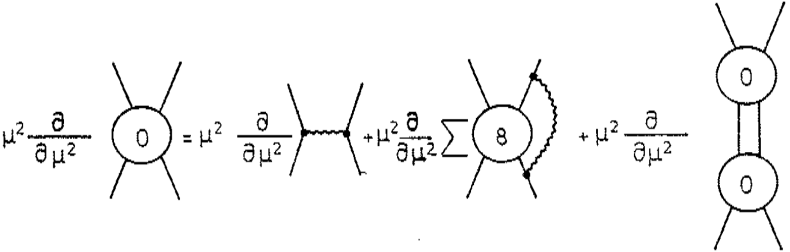
Provide recursive relations to resum the "ladder"



BER in a nutshell

BER approach to the polarized DIS structure function g_1

Relations from the IREE [Diagrams from BER]



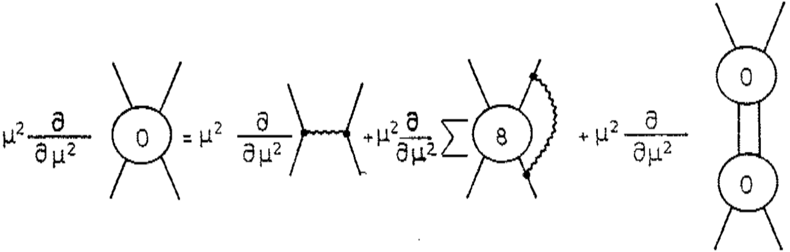
3 cases for color singlet splitting functions

- Born diagram
- The lowest $k_\perp$ is carried by a gluon: Bremstrahlung [Gribov's theorem]
- The lowest $k_\perp$ is carried by a quark

BER in a nutshell

BER approach to the polarized DIS structure function g_1

Relations from the IREE [Diagrams from BER]



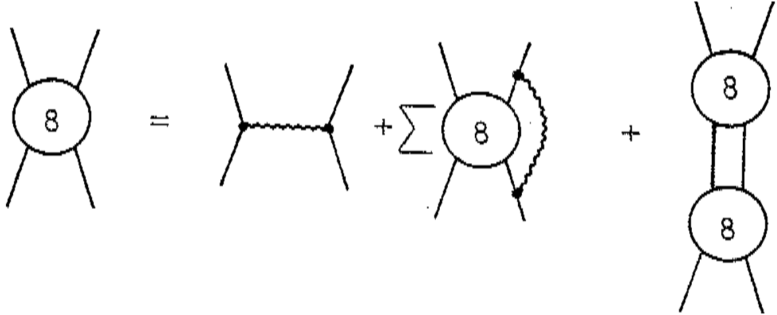
3 cases for color singlet functions

The singlet recursion relations involve a color octet function

BER in a nutshell

BER approach to the polarized DIS structure function g_1

Relations from the IREE [Diagrams from BER]



3 cases for color octet functions

The octet recursion relation is a closed relation

Extending BER

Application of BER to $\mathbf{S} = (\Delta\Sigma, \Delta G, L^q, L^g)$
[RB, Hatta, Yuan]

Can we get the low x behavior of $\mathbf{S}(x, Q^2)$ from the IREE?

Extending BER

IREE in Mellin space for the proton spin

For $\mathbf{S}(x, Q^2) = \int \frac{\mathrm{d}\omega}{2\pi i} x^{-\omega} \mathbf{S}^\omega(Q^2)$, we have:

$$\frac{\partial}{\partial \ln Q^2} \mathbf{S}^\omega(Q^2) = \frac{1}{8\pi^2} F_0 \mathbf{S}^\omega(Q^2)$$

F_0 splitting kernel for spin densities at small x.

Generalizes BER which is for $(\Delta\Sigma, \Delta G)$

Extending BER

IREE in Mellin space for the proton spin

Singlet

$$F_0 = \frac{g^2}{\omega} M_0 - \frac{g^2}{2\pi^2\omega} F_8 G_0 + \frac{1}{8\pi^2\omega} F_0^2$$

Octet

$$F_8 = \frac{g^2}{\omega} M_8 + \frac{g^2 C_A}{8\pi^2\omega} \frac{\mathrm{d}}{\mathrm{d}\omega} F_8 + \frac{1}{8\pi^2\omega} F_8^2$$

Explicit matrices [RB, Hatta, Yuan]

Extending BER

IREE in Mellin space for the proton spin

Rq: up to three loops, the last two columns of the 4x4 matrix $M_0^2 - 4M_8 G_0$ are null.
Assuming this holds to all orders, we find:

$$F_0 = \frac{g^2}{\omega} M_0 + \begin{pmatrix} a_1 & a_2 & 0 & 0 \\ b_1 & b_2 & 0 & 0 \\ -a_1 & -a_2 & 0 & 0 \\ -2b_1 & -2b_2 & 0 & 0, \end{pmatrix}$$

with a_1, a_2, b_1, b_2 components of the 2x2 BER solution.

Solution

Solution for the proton spin structure

$$\mathbf{S}(x, Q^2) = \int \frac{\mathrm{d}\omega}{2\pi i} x^{-\omega} \left(\frac{Q^2}{\mu^2} \right)^{\frac{F_0}{8\pi^2}} \mathbf{S}^\omega(\mu^2)$$

Diagonalizing F_0 at the BER saddle point

$\omega = \omega_s = 3.45\sqrt{\frac{\alpha_s N_c}{2\pi}}$

$$\boxed{\begin{gathered} \Delta G(x) \simeq -2.29 \Delta\Sigma(x) \sim x^{-1.01} \\ L_q(x) \simeq -\Delta\Sigma(x), \quad L_g \simeq -2\Delta G(x) \end{gathered}}$$

Compatible with 3-loop collinear splitting functions
Compatible with exact QCD operator relations for L^q, L^g.

Conclusions

Conclusions

- The proton spin decomposition involves integrals of parton distributions
- The boundaries of the integrals must be constrained by theory
- By extending the BER framework, we computed the proton spin structure within Double Logarithmic accuracy at small x
- Our results are compatible with 3-loop splitting functions and exact QCD relations

Limitations

- Our framework restricts itself to leading genuine twist $\Rightarrow$ no saturation effects
- The KPS framework takes saturation into account
- Corrected KPS is now compatible with BER [Cougoulic, Kovchegov, Tarasov, Tawabutr]
- Exciting results from the Wordline formalism for g_1 as well [Tarasov, Venugopalan]

Backup

Up to genuine higher twists,

$$L_q(x) = x \int_x^1 \frac{\mathrm{d}x'}{x'} \left[H_q(x') + E_q(x') - \frac{1}{x'} \Delta\Sigma(x') \right]$$

$$L_g(x) = x \int_x^1 \frac{\mathrm{d}x'}{x'} \left[H_g(x') + E_g(x') - 2\frac{1}{x'} \Delta G(x') \right]$$

In Mellin space,

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} L_q^\omega \\ L_g^\omega \end{pmatrix} = \begin{pmatrix} \gamma_{qq}^{\omega+1} & \gamma_{qg}^{\omega+1} \\ \gamma_{gq}^{\omega+1} & \gamma_{gg}^{\omega+1} \end{pmatrix}\begin{pmatrix} L_q^\omega \\ L_g^\omega \end{pmatrix}$$
$$+ \frac{1}{\omega+1}\begin{pmatrix} \gamma_{qq}^{\omega+1} - \Delta\gamma_{qq}^{\omega} & 2\gamma_{qg}^{\omega+1} - \Delta\gamma_{qg}^{\omega} \\ \gamma_{gq}^{\omega+1} - 2\Delta\gamma_{gq}^{\omega} & 2\gamma_{gg}^{\omega+1} - 2\Delta\gamma_{gg}^{\omega} \end{pmatrix}\begin{pmatrix} \Delta\Sigma^\omega \\ \Delta G^\omega \end{pmatrix} + \mathrm{g.h.t} \tag{1}$$

γ_{ij}: single logarithm
$\Delta\gamma_{ij}$: double logarithm

DLA, leading genuine twist:

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} L_q^\omega \\ L_g^\omega \end{pmatrix} = \frac{1}{\omega+1}\begin{pmatrix} -\Delta\gamma_{qq}^\omega & -\Delta\gamma_{qg}^\omega \\ -2\Delta\gamma_{gq}^\omega & -2\Delta\gamma_{gg}^\omega \end{pmatrix}\begin{pmatrix} \Delta\Sigma^\omega \\ \Delta G^\omega \end{pmatrix} + \mathrm{g.h.t}$$

Then,

$$L_q(x) \simeq -x\int_x^1 \frac{\mathrm{d}x'}{x'^2}\Delta\Sigma(x')$$

$$L_g(x) = -2x\int_x^1 \frac{\mathrm{d}x'}{x'^2}\Delta G(x')$$

If $\Delta\Sigma \sim x^{-\alpha_q}, \Delta G \sim x^{-\alpha_g}$, then

$$L_q \simeq -\frac{1}{1+\alpha_q}L_q, \quad L_g \simeq -\frac{2}{1+\alpha_g}\Delta G$$