Unveiling Nucleon 3D Chiral-Odd Structure with Jet Axes

Wai Kin Lai

University of California, Los Angeles
South China Normal University


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Motivation

- 3D structures of proton were studied typically using
  - semi-inclusive DIS/Drell-Yan
    - [Mulders, Tangerman (1996), Brodsky, Hwang, Schmidt (2002), Bacchetta et al. (2007), Bacchetta et al., MAP Collaboration (2022)]
  - jet production/hadron in jet
    - [Kang, Metz, Qiu, Zhou (2011), Liu, Ringer, Vodelsang, Yuan (2019), Kang, Lee, Shao, Zhao (2021)]
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- Jet was thought to be able to probe only a subset of TMD PDFs (4 out of 8 at leading twist).

- We will demonstrate how all TMD PDFs at leading twist can be accessed by jets by including the T-odd jet function.

T-odd jet function was introduced by [X. Liu and H. Xing (2021)]
Consider $l + p(P, S) \rightarrow l' + J(P_J) + X$ at EIC

- A lot of statistics at small $p_T$ in the forward region.

- Focus on the region $\Lambda_{QCD} \sim p_T \ll Q$.
  This is unlike LHC, for which only jets with $p_T \gg \Lambda_{QCD}$ are of interest.

- Still get jets if we use jet algorithms that involve energy (e.g. spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]) instead of $k_T$.
  Low $p_T$ ($\sim \Lambda_{QCD}$) and low $Q^2$ ($\sim 5 - 100 \text{ GeV}^2$) is not a problem.
Role of jet axis definition

- Probes of TMD PDFs amount to measuring $q_T$ of the virtual photon w.r.t. two pre-defined axes:
  - Drell-Yan: Two nucleon beams define two axes.
  - SIDIS: Nucleon beam and momentum of tagged hadron define two axes.
- In DIS, with a specific recombination scheme, a jet axis can be defined for a given jet. Once the axis is defined, we can forget about the fact that it’s a jet. We thus get a nucleon beam axis and a jet axis, w.r.t. which $q_T$ of the virtual photon can be defined. Therefore, jet probes of nucleon structure in DIS are as differential as SIDIS or Drell-Yan.
For $Q \gg |q_T|$ and $Q \gg \Lambda_{QCD}$, factorization from SCET:

$$\sigma = H \otimes \Phi \otimes J$$

$H$: hard function, $\Phi$: TMD PDFs, $J$: TMD jet functions (JFs)

$[\text{Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)}]$

$$\Phi^{ij}(x, p_T) = \int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{i p \cdot y} \langle P | \bar{\chi}^j_n (0) \chi^i_n (y) | P \rangle |_{y^+ = 0}$$

$$J^{ij}(z, k_T) = \frac{1}{2z} \sum_X \int \frac{dy^+ d^2 y_T}{(2\pi)^3} e^{i k \cdot y} \langle 0 | \chi^i_n (y) | JX \rangle \langle JX | \bar{\chi}^j_n (0) | 0 \rangle |_{y^- = 0}$$

GNS frame: ($P_{J\perp} = -z q_T$)
AZIMUTHAL ASYMMETRIES

TMD PDFs and TMD JFs encoded in azimuthal asymmetries (leading power in $1/Q$):

\[
\frac{d\sigma}{dxdydzd\phi J dP_{J,\perp}^2} = \frac{\alpha^2}{xyQ^2} \left\{ \left( 1 - y + \frac{y^2}{2} \right) F_{UU,T} + (1 - y) \cos(2\phi J) F_{UU}^{\cos(2\phi J)} \right.
\]

\[
+ S_\parallel (1 - y) \sin(2\phi J) F_{UL}^{\sin(2\phi J)} + S_\parallel \lambda e y \left( 1 - \frac{y}{2} \right) F_{LL}
\]

\[
+ |S_\perp| \left[ \left( 1 - y + \frac{y^2}{2} \right) \sin(\phi J - \phi_S) F_{UT,T}^{\sin(\phi J - \phi_S)} + (1 - y) \sin(\phi J + \phi_S) F_{UT}^{\sin(\phi J + \phi_S)} \right]
\]

\[
+ (1 - y) \sin(3\phi J - \phi_S) F_{UT}^{\sin(3\phi J - \phi_S)} \right] + |S_\perp| \lambda e y \left( 1 - \frac{y}{2} \right) \cos(\phi J - \phi_S) F_{LT}^{\cos(\phi J - \phi_S)} \right\}
\]

$F$'s: convolutions of TMD PDFs and TMD JFs.
$F$'s: accessible by traditional jet function
$F$'s: inaccessible by traditional jet function
TMD PDFs at leading twist

\[ \Phi = \frac{1}{2} \left\{ f_1 \eta - f_1^T \frac{\epsilon_{\alpha\beta} p_T^\alpha S_T^\beta}{M} \eta + \left( S_L g_1 L - \frac{p_T \cdot S_T}{M} g_1 T \right) \gamma_5 \eta \right. \]
\[ + h_{1T} \left[ S_T^\gamma, \eta \right] \gamma_5 \frac{2}{2} + \left( S_L h_{1L}^\dagger - \frac{p_T \cdot S_T}{M} h_{1T}^\perp \right) \left[ p_T, \eta \right] \gamma_5 \frac{2M}{2M} + i h_{1}^\perp \left[ p_T, \eta \right] \frac{2M}{2M} \right\} \]

<table>
<thead>
<tr>
<th>hadron</th>
<th>quark</th>
<th>unpolarized</th>
<th>longitudinal</th>
<th>transverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>( f_1 )</td>
<td>( g_1 L )</td>
<td>( h_{1}^\perp ) (Boer-Mulders)</td>
<td></td>
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<tr>
<td>( L )</td>
<td>( f_{1L}^\perp ) (Sivers)</td>
<td>( g_1 T )</td>
<td>( h_{1L}^\perp )</td>
<td></td>
</tr>
<tr>
<td>( T )</td>
<td>( f_{1T}^\perp )</td>
<td>( g_1 T )</td>
<td>( h_{1T}, h_{1T}^\perp ) (transversity)</td>
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</tr>
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</table>

[Angeles-Martinez et al. (2015)]

- 8 TMD PDFs at leading twist, functions of \( x \) and \( p_T^2 \)
- 3 functions \( f_1, g_1 L, h_{1T} \) survive after \( p_T \) integration giving collinear PDFs
- T-even: \( f_1, g_1 L, g_1 T, h_{1T}, h_{1L}^\perp, h_{1T}^\perp \)
  T-odd: \( f_{1T}^\perp, h_{1}^\perp \)
- Chiral-even (accessible by traditional jet function): \( f_1, f_{1T}^\perp, g_1 L, g_1 T \)
  Chiral-odd (inaccessible by traditional jet function): \( h_{1}^\perp, h_{1L}^\perp, h_{1T}, h_{1T}^\perp \)
Traditionally, only jets with high $p_T \gg \Lambda_{QCD}$ were of interest. Production of high-$p_T$ jets is perturbative. Since massless perturbative QCD is chiral-symmetric, only chiral-even (and T-even) jet functions appear.

At low $p_T \sim \Lambda_{QCD}$, the jet is sensitive to nonperturbative physics. In particular, spontaneous chiral symmetry breaking leads to a nonzero chiral-odd (and T-odd) jet function when the jet axis is different from the direction of the fragmenting parton. (This is similar to Collins effect in fragmentation functions of hadrons [Collins (2002)].) [Liu and Xing (2021)]

$$J(z, k_T) = J_1(z, k_T) \frac{\vec{\gamma}}{2} + iJ_T(z, k_T) \frac{k_T \vec{\gamma}}{2}$$

- $J_1$: chiral-even, T-even, traditional jet function
- $J_T$: chiral-odd, T-odd, encodes correlations of quark transverse spin with quark transverse momentum (analogue of Collins function)
Advantages of T-odd jet function

- **Universality**
  Like the T-even $J_1$, T-odd $J_T$ is process independent.

- **Flexibility**
  Flexibility of choosing jet recombination scheme and hence the jet axis
  ⇒ Adjust sensitivity to different nonperturbative contributions
  ⇒ Provide opportunity to “film” the QCD nonperturbative dynamics, if one continuously change the axis from one to another.

- **High predictive power**
  - *Perturbative predictability*. Since a jet contains many hadrons, the jet function has more perturbatively calculable degrees of freedom than the fragmentation function. For instance, in the WTA scheme, for $R \sim \mathcal{O}(1) \gg |q_T|/E_J$, the $z$-dependence in the jet function is completely determined:

  $$J(z, k_T, R) = \delta(1-z)J(k_T) + \mathcal{O}\left(\frac{k_T^2}{E_J^2 R^2}\right)$$

  [Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]

- *Nonperturbative predictability*. Similar to the study in [Becher and Bell (2014)], $J_T$ can be factorized into a product of a perturbative coefficient and a nonperturbative factor. The nonperturbative factor has an operator definition [Vladimirov (2020)], and as a vacuum matrix element can be calculated on the lattice. This is unlike the TMD fragmentation function, which is an operator element of $|h + X\rangle$. 
Probing Transversity

\[ A^{\sin(\phi_J + \phi_S)}(|P_J|) = \frac{2}{|S|} \int d\sigma \sin(\phi_J + \phi_S) = \frac{\langle \epsilon F_{UT}^{\sin(\phi_J + \phi_S)} \rangle}{\langle F_{UU,T} \rangle} \]

- \( F_{UT}^{\sin(\phi_J + \phi_S)} \sim h_1 \otimes J_T \), probes transversity \( h_1 \equiv h_{1T}^T + \frac{p_T^2}{2M^2} h_{1T}^T \)

- We simulate using Pythia 8.2+StringSpinner [Kerbizi, Loennblad (2021)], with jet charge [Kang, Liu, Mantry, Shao (2020)] measured to enhance flavor separation (not mandatory), with EIC kinematics.

- Use the spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]:

\[ d_{ij} = \min(E_i^{-2}, E_j^{-2}) \frac{1 - \cos \theta_{ij}}{1 - \cos R}, \quad d_{iB} = E_i^{-2} \]

(Conventional anti-\( k_T \) algorithms using \( k_T \) instead of \( E \) not good for low-\( p_T \) jets)

- Change the jet axis from one to another (WTA → E-scheme), “film” nonperturbative physics.

**WTA scheme:**

\[ \hat{n}_T = \begin{cases} \hat{n}_1, & \text{if } E_1 > E_2 \\ \hat{n}_2, & \text{if } E_2 > E_1 \end{cases} \]

\[ k_{\perp} \rightarrow \hat{n}_J \]

\[ E_e = 275 \text{ GeV}, E_p = 10 \text{ GeV} \]

\[ Q^2 > 100 \text{ GeV}^2, 0.1 < y < 0.9 \]

\[ 0.15 < x < 0.3, z > 0.2 \]
Probing transversity

\[ A^{\sin(\phi_J + \phi_S)}(|P_{J\perp}|) = \frac{2}{|S_{\perp}|} \int d\sigma \sin(\phi_J + \phi_S) = \frac{\langle \epsilon F_{UT}^{\sin(\phi_J + \phi_S)} \rangle}{\epsilon(F_{UU}, T)} \]

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  (Conventional anti-\( k_T \) algorithms using \( k_T \) instead of \( E \) not good for low-\( p_T \) jets)

- Change the jet axis from one to another (WTA \( \rightarrow \) E-scheme), “film” nonperturbative physics.

**E-scheme:**

\[ k_r = k_1 + k_2 \]

**Diagram:**

\[ e+p \rightarrow e+J^{\text{E-scheme}}+X \text{ (EIC)} \]

\[ E_e = 275 \text{ GeV}, \ E_p = 10 \text{ GeV} \]

- \( Q_J > 0.25 \)
- \( Q_J < -0.25 \)

\[ Q^2 > 100 \text{ GeV}^2, \ 0.1 < y < 0.9 \]

\[ 0.15 < x < 0.3, \ z > 0.2 \]
Probing Transversity

\[ e+p \rightarrow e+J_{WTA} + X \text{ (EIC)} \]

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- Data points: from Pythia simulations
- Lines: from factorization formula (including evolution via Sudakov factors, normalization of \( J_1 \) fixed by jet charge bins, \( k_T \)-dependence of \( J_1 \) and \( J_T \) from pion FFs, ratio of normalization of \( J_T \) to that of \( J_1 \) set equal to that for pions)
- Asymmetries at lower \( \sqrt{s} \) are generally larger, owing to the perturbative Sudakov factor.
**Probing Boer-Mulders function**

\[
A^{\cos(2\phi_J)}(|P_{J\perp}|) = \frac{2}{\int d\sigma \epsilon} \int d\sigma \cos(2\phi_J) = \frac{\langle \epsilon F_{UU}^{\cos(2\phi_J)} \rangle}{\overline{\epsilon} \langle F_{UU,T} \rangle}
\]

\(F_{UU}^{\cos(2\phi_J)} \sim h_1^\perp \otimes J_T\), probes Boer-Mulders function

Predictions from factorization formula:

\(e+p \rightarrow e+J_{\text{WTA}}+X\) (EIC)

\(e+p \rightarrow e+J_{\text{WTA}}+X\) (EICC)

\(E_e=275\text{ GeV, } E_p=10\text{ GeV}\)

\(Q^2>100\text{ GeV}^2, 0.1<y<0.9, 0.15<x<0.3\)

\(E_e=3.5\text{ GeV, } E_p=20\text{ GeV}\)

\(Q^2>5\text{ GeV}^2, 0.1<y<0.9, 0.15<x<0.3\)
Comparison with HERMES SIDIS data

**e+p→e+π+X (HERMES)**

- $E_e=27.6$ GeV, $E_p=M_p$
- $Q^2 > 1$ GeV$^2$, $0.1 < y < 0.95$
- $0.023 < x < 0.4$
- $0.2 < z < 0.7$

**e+p→e+π+X (HERMES)**

- $E_e=27.6$ GeV, $E_p=M_p$
- $Q^2 > 1$ GeV$^2$, $0.3 < y < 0.85$
- $0.2 < z < 0.75$

A comparison of the data with theoretical predictions for the angular distributions of the scattered electron ($e$) and the produced pion ($\pi$) in the reaction $e+p→e+\pi+X$. The plots show the dependence of $A \sin(\phi_J + \phi_S)$ (left) and $A \cos(2\phi_J)$ (right) on the transverse momentum $P_{J\perp}$ for different $Q^2$ and $y$ ranges.
We give prediction on azimuthal asymmetry in $e^+e^- \rightarrow J + X$ at $\sqrt{s} = \sqrt{110}$ GeV (Belle, BaBar), 91.2, 165, 240 GeV (CEPC) with WTA scheme:

$$A = 2 \int d\cos \theta \frac{d\phi_1}{\pi} \cos(2\phi_1) A^{J_1J_2}$$

$$A^{J_1J_2} = 1 + \cos(2\phi_1) \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{F_T}{F_U}$$

$$F_T \sim J_T \otimes J_T$$
With spherically-invariant jet algorithms, we can study jets at low $p_T$ ($\sim \Lambda_{\text{QCD}}$), e.g., at EIC.

Specification of a jet axis makes jet probes of TMD PDFs in DIS as differential as SIDIS or Drell-Yan.

Using the T-odd jet function, together with the traditional T-even one, we can probe all 8 TMD PDFs at leading twist using jets.

T-odd jet function has the advantages of universality, flexibility, and high predictive power.

We have shown that the T-odd jet function gives rise to sizable azimuthal asymmetries at EIC, which help probe the chiral-odd TMD PDFs, such as the quark transversity and the Boer-Mulders function, which the traditional jet function is unable to access.

T-odd jet function provides new input to the global analysis of nonperturbative proton structure.
Thank you.
Backup slides
TMD PDFs and TMD JFs encoded in azimuthal asymmetries:

\[
\frac{d\sigma}{dxdydzd\phi_JdP^2_{J\perp}} = \frac{\alpha^2}{xyQ^2} \left\{ \left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1 - y) \cos(2\phi_J) F_{UU,\cos}^{2\phi_J} \right. \\
+ S_{\parallel}(1 - y) \sin(2\phi_J) F_{UL,\sin}^{2\phi_J} + S_{\parallel} \lambda_e y \left(1 - \frac{y}{2}\right) F_{LL} \\
+ |S_{\perp}| \left[ \left(1 - y + \frac{y^2}{2}\right) \sin(\phi_J - \phi_S) F_{UT,T}^{\sin(\phi_J - \phi_S)} + (1 - y) \sin(\phi_J + \phi_S) F_{UT}^{\sin(\phi_J + \phi_S)} \\
+ (1 - y) \sin(3\phi_J - \phi_S) F_{UT}^{\sin(3\phi_J - \phi_S)} \right] + |S_{\perp}| \lambda_e y \left(1 - \frac{y}{2}\right) \cos(\phi_J - \phi_S) F_{LT}^{\cos(\phi_J - \phi_S)} \right. \\}
The $F$'s are convolutions of TMD PDFs and TMD JFs:

$$C[wfJ] \equiv x \sum_a e_q^2 \int d^2 p_T \int d^2 k_T \delta^{(2)}(p_T + q_T - k_T) w(p_T, k_T) f(x, p_T^2) J(z, k_T^2)$$

$$F_{UU,T} = C[f_1 J_1], \quad F_{LL} = C[g_1 L J_1]$$

$$F_{UT,T}^{\sin(\phi J - \phi S)} = C \left[ -\frac{\hat{h} \cdot p_T}{M} f_1^T J_1 \right], \quad F_{LT}^{\cos(\phi J - \phi S)} = C \left[ \frac{\hat{h} \cdot p_T}{M} g_1 T J_1 \right],$$

$$F_{UU}^{\cos(2\phi J)} = C \left[ \frac{(2\hat{h} \cdot k_T)(\hat{h} \cdot p_T) - k_T \cdot p_T}{M} h_1^T J_T \right]$$

$$F_{UL}^{\sin(2\phi J)} = C \left[ \frac{(2\hat{h} \cdot k_T)(\hat{h} \cdot p_T) - k_T \cdot p_T}{M} h_1 L J_T \right]$$

$$F_{UT}^{\sin(\phi J + \phi S)} = C \left[ -\hat{h} \cdot k_T h_1 J_T \right]$$

$$F_{UT}^{\sin(3\phi J - \phi S)} = C \left[ \frac{2(\hat{h} \cdot p_T)(p_T \cdot k_T) + p_T^2(\hat{h} \cdot k_T) - 4(\hat{h} \cdot p_T)^2(\hat{h} \cdot k_T)}{2M^2} h_1^T J_T \right]$$

where $\hat{h} \equiv P_{J\perp}/|P_{J\perp}|$ and $h_1 \equiv h_1 T + \frac{p_T^2}{2M^2} h_1^T$. 