

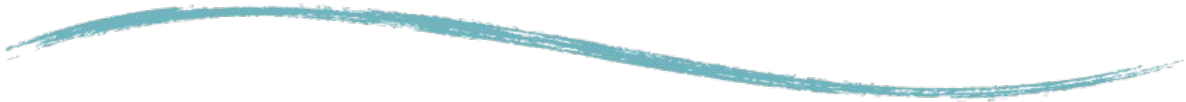
Color Memory and Novel Global Symmetries of Scattering Amplitudes

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Why do we find universal
phenomena?



Explanations for Universality

1. Infrared physics

Low-energy probes are **insensitive** to non-universal microscopic details.

2. Symmetries

Observables are **protected** from non-universal specifics of the system.

Are these related? If so, how?



A hint...

$$Q \sim \int d^3 \vec{x} \, \rho(\vec{x}) \sim \lim_{|\vec{p}| \rightarrow 0} \int d^3 \vec{x} \, e^{i\vec{p} \cdot \vec{x}} \rho(\vec{x})$$

Conserved charges involve long-wavelength data.



Why should we care?

Affords more precise control of when to expect observables to be universal and how to measure them.



Disclaimer: will not fully deliver today!

Today's application:

In the context of scattering problems, there is an **equivalence**

Universality in the
infrared



Universality due to
Symmetry

Crux of the equivalence

A blue wavy line drawn horizontally below the text "Crux of the equivalence".

Soft theorems
in quantum field theory



Conservation laws for
an infinite number of
(global) symmetries

Outline:

Soft theorems



Conservation laws

Gravity

- 1. Establish the equivalence**
 - Formally reinterpret soft theorems as statements of symmetry (conservation laws).
- 2. Identify physical interpretation of symmetry**
 - Compare with familiar conservation laws.
- 3. Determine an observable signature → “memory effect”**
 - Identify good observables for measuring conserved quantities → verify conservation law

Non-abelian gauge theory

1. Soft gluon theorem as a conservation law
2. Infinite-dimensional enhancement of color rotation symmetry
3. Color memory

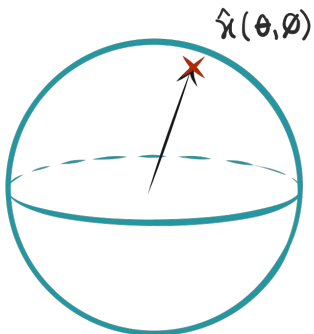
Weinberg's Soft Graviton Theorem

The scattering amplitude for the **emission of a soft graviton** has a **pole in the energy** of that graviton with a **universal residue**.

$$\lim_{\omega \rightarrow 0} \omega \langle \text{out} | a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n S_k(\hat{x}) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

Graviton momentum:

$$q^\mu = \omega \hat{q}^\mu = \omega (1, \hat{x}(\theta, \phi))$$



Soft factor: $S_k(\hat{x}) = \frac{\kappa}{2} \frac{\varepsilon_{\mu\nu}^+ p_k^\mu p_k^\nu}{\hat{q} \cdot p_k}$

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- **Infrared:** *low-energy graviton* probes long-distance properties of scattering process.
- **Universality:** soft factor only depends on momenta of external particles
 - *Insensitive* to precise particle content
 - *Insensitive* to most details of particle interactions
- **Symmetry?** exact relationship between scattering amplitudes

Symmetry in Scattering

- To rigorously establish the soft theorem as consequent of symmetry, must first recall how symmetries are realized in the context of scattering.

Symmetries of the Scattering Problem

- Transformations of initial and final states that *preserve the S -matrix*

$$\langle \text{out}' | \mathcal{S} | \text{in}' \rangle = \langle \text{out} | U^\dagger \mathcal{S} U | \text{in} \rangle = \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

Continuous symmetries:

$$U \sim 1 + iQ, \quad Q|\Psi\rangle \sim \delta|\Psi\rangle$$

$$\Rightarrow \langle \text{out} | [Q, \mathcal{S}] | \text{in} \rangle = 0$$


$\Rightarrow$ To establish soft theorems as statements of symmetry, simply need to show that they can be put in this form!

Soft Theorems Imply Symmetries

- **Goal:** write soft theorem in the form $\langle \text{out} | [Q, \mathcal{S}] | \text{in} \rangle = 0$.

$$\lim_{\omega \rightarrow 0} \omega \langle \text{out} | a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n S_k(\hat{x}) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

$$S_k(\hat{x}) = \frac{\kappa}{2} \frac{\varepsilon_{\mu\nu}^+ p_k^\mu p_k^\nu}{\hat{q} \cdot p_k}$$

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- Regard soft factor S_k as eigenvalue of single particle state under operator Q_H

$$S_k(\hat{x}) |p_k\rangle = Q_H(\hat{x}) |p_k\rangle = -i \delta_{(\hat{x})} |p_k\rangle$$

- RHS gives total transformation of all single particle states under $\delta_{(\hat{x})}$

$$\sum_{k=1}^n S_k \langle \text{out} | \mathcal{S} | \text{in} \rangle = \langle \text{out} | [Q_H, \mathcal{S}] | \text{in} \rangle$$

- Soft theorem implies that the $\mathcal{S}$ -matrix is invariant under $\delta_{(\hat{x})}$ provided that *a soft particle is added*.

Soft Theorems Imply Symmetries

- **Goal:** write soft theorem in the form $\langle \text{out} | [Q, \mathcal{S}] | \text{in} \rangle = 0$.

$$\lim_{\omega \rightarrow 0} \omega \langle \text{out} | a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n S_k(\hat{x}) \langle \text{out} | \mathcal{S} | \text{in} \rangle \quad S_k(\hat{x}) = \frac{\kappa}{2} \frac{\varepsilon_{\mu\nu}^+ p_k^\mu p_k^\nu}{\hat{q} \cdot p_k}$$

- Denote operator which **adds soft particles** Q_S

$$Q_S(\hat{x}) \sim - \lim_{\omega \rightarrow 0} \omega \left[a_+(\omega \hat{x}) + a_-^\dagger(\omega \hat{x}) \right]$$

- Then, the LHS can be written as

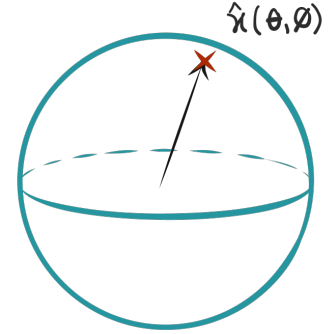
$$\lim_{\omega \rightarrow 0} \langle \text{out} | \omega a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = - \langle \text{out} | [Q_S, \mathcal{S}] | \text{in} \rangle$$

- Soft theorem can be rearranged to take the form

$$\langle \text{out} | [Q, \mathcal{S}] | \text{in} \rangle = 0, \quad Q = Q_H + Q_S$$

Summary

$$\lim_{\omega \rightarrow 0} \omega \langle \text{out} | a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n S_k(\hat{x}) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$



$$S_k(\hat{x}) = \frac{\kappa}{2} \frac{\varepsilon_{\mu\nu}^+ p_k^\mu p_k^\nu}{\hat{q} \cdot p_k}$$

- Soft theorem can be put in the form $\langle \text{out} | [Q, \mathcal{S}] | \text{in} \rangle = 0$.
- Implies that the $\mathcal{S}$ -matrix is **invariant under symmetry** generated by Q .
- Q is parametrized by $\hat{x}$

$\Rightarrow$ Soft theorems imply the $\mathcal{S}$ -matrix is invariant under an **infinite** number of **symmetries**, one for each point on the sphere!

Outline:

Soft theorems



Conservation laws

Gravity

✓ Establish equivalence

- Formally reinterpret soft theorems as statements of symmetry (conservation laws).

2. Identify physical interpretation of symmetry

- Compare with familiar conservation laws.

3. Determine an observable signature $\rightarrow$ “memory effect”

- Identify good observables for measuring conserved quantities $\rightarrow$ verify conservation law.

Physical Interpretation of the Symmetries

- To facilitate an interpretation, parametrize charges by **functions** $f(\theta, \phi)$ rather than **points** $\hat{x}(\theta, \phi)$.

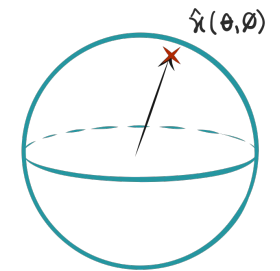
$$Q[f] = \int d^2 \hat{x} f(\hat{x}) \mathcal{D}_{(\hat{x})} Q(\hat{x})$$

- Obtain charges with **local** action

$$-i\delta_{(\hat{x})}|p_k\rangle = Q_H(\hat{x})|p_k\rangle = S_k(\hat{x})|p_k\rangle$$



$$-i\delta_f|p_k\rangle = Q_H[f]|p_k\rangle = \omega_k f(\hat{x}_k)|p_k\rangle$$



$$p_k^\mu = \omega_k(1, \hat{x}_k)$$

Physical Interpretation of the Symmetries

$$-i\delta_f |p_k\rangle = Q_H[f] |p_k\rangle = \omega_k f(\hat{x}_k) |p_k\rangle$$


- When $f = 1$, find total energy

$$-i\delta_{f=1} |p_k\rangle = \omega_k |p_k\rangle$$

$Q[f = 1]$ generates
ordinary time translations

- Generic $f = f(\theta, \phi)$:

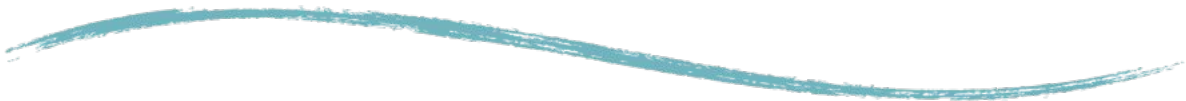
$$-i\delta_f |p_k\rangle = \omega_k f(\hat{x}_k) |p_k\rangle$$

f parametrizes *amount* of
translation at every angle

Arbitrary $f \Rightarrow$ identify symmetry as **independent** time translations at **every angle**.



Why *infinitely* many?



To gain intuition, let's reevaluate the transformations that observers could
possibly perform on the system...

Why *infinitely* many?

In scattering, *only* observe **initial** & **final states**

Interactions between
particles are negligible

Observers at different angles must be sufficiently far
apart.

Formally implemented by assuming observers
populate a sphere of infinite radius

“The Celestial Sphere”

Observers at different angles are *out of causal
contact* with one another.



The Celestial Sphere

Why *infinitely* many?

Observers at different angles are *out of causal contact* with one another.



Cannot synchronize transformations across different points on the celestial sphere



Independent time translational symmetry at each point



Infinite-dimensional enhancement of the translation group

Supertranslations

[Bondi, Metzner, van der Burg & Sachs, 1962]



The Celestial Sphere

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How do we know when a system admits a
symmetry?



We observe that it respects a **conservation
law**!

(Noether's theorem)

Constraints from Supertranslations

- Recall conserved charge:

$$Q = \underbrace{Q_H}_{\text{blue}} + \underbrace{Q_S}_{\text{red}}$$

$$Q_H \sim \sum_k \omega_k f(\hat{x}_k)$$


Q_H characterizes **local** energy
flux at every angle

$$Q_S(\hat{x}) \sim - \lim_{\omega \rightarrow 0} \omega \left[a_+(\omega \hat{x}) + a_-^\dagger(\omega \hat{x}) \right]$$


How do we measure Q_S ?

Observable Signatures of Supertranslations

- Rewrite soft charge in terms of position-space field:

$$Q_S(\theta, \phi) \sim \lim_{\omega \rightarrow 0} \omega [a(\omega \hat{x}) + a^\dagger(\omega \hat{x})] \sim \lim_{\omega \rightarrow 0} \int dt \, \varepsilon^{\mu\nu} e^{i\omega t} \partial_t h_{\mu\nu}^{\text{rad}}$$

Q_S captures the **zero-frequency component** of the **radiative gravitational field**.

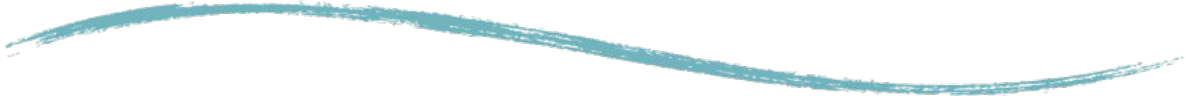
- But also...

$$Q_S(\theta, \phi) \sim \varepsilon^{\mu\nu} \underbrace{\Delta h_{\mu\nu}}_{h_{\mu\nu}(t_f) - h_{\mu\nu}(t_i)}$$

$$\left(\begin{array}{c} \text{Follows from} \\ \int d\omega \frac{e^{i\omega t}}{\omega} \sim \Theta(t) \end{array} \right)$$

Q_S captures a **permanent net shift** in the **asymptotic metric**.

Are there observables that are sensitive to
a permanent shift in the asymptotic
metric?



Soft Charges and Vacuum Transitions

- Revisit Q_S from yet another angle:

$$Q_S(\hat{x}) \sim - \lim_{\omega \rightarrow 0} \omega \left[a_+(\omega \hat{x}) + a_-^\dagger(\omega \hat{x}) \right]$$

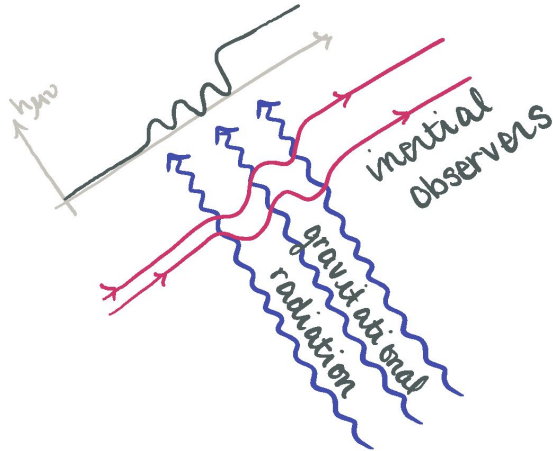


Q_S carries zero energy

- Acting with Q_S does not change the energy of a state.
- In particular, Q_S maps a **vacuum** state **to** a **vacuum** state.
- Suggests objects **charged under the symmetry** might be sensitive to **vacuum transitions**
 - In a theory of gravity, consider a **pair of inertial detectors**.

Gravitational Memory

Permanent shift in metric induces permanent change in relative displacement between inertial observers



Geodesic Deviation Equation

$$\Delta s^i \sim \Delta h^{ij} s_j$$

$$\sim \lim_{\omega \rightarrow 0} \int dt e^{i\omega t} \partial_t h^{ij} s_j$$

[Zel'dovich & Polnarev 1974]

NATURE VOL. 327 14 MAY 1987

Gravitational-wave bursts with memory and experimental prospects

Vladimir B. Braginsky* & Kip S. Thorne†

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{\text{TT}}$ is equal to the 'transverse, traceless (TT) part'³⁶ of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{\text{TT}}$ has the following form:

$$\delta h_{ij}^{\text{TT}} = \delta \left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k} \cdot \mathbf{P}^A} \right)^{\text{TT}}$$

Conservation law relating change in asymptotic metric to soft factor $\leftrightarrow Q_H \leftrightarrow$ local energy flux

Gravitational Memory & Supertranslations

Relative displacement of inertial observers:

$$\begin{aligned}\Delta s^i &\sim \Delta h^{ij} s_j \\ &\sim \lim_{\omega \rightarrow 0} \int dt e^{i\omega t} \partial_t h^{ij} s_j\end{aligned}$$

- Measures net shift in asymptotic metric!
 - How to measure soft radiation!
- ⇒ Tile the celestial sphere with inertial observers and measure the change in their relative displacements.



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Non-abelian gauge theory

1. Soft gluon theorem as a conservation law
2. Infinite-dimensional enhancement of color rotation symmetry
3. Color memory

Novel Global Symmetries in Gauge Theory

Tree-level Soft Gluon Theorem:

$$\lim_{\omega \rightarrow 0} \omega \langle \text{out} | a_+^a(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n g T_k^a \frac{\varepsilon^+ \cdot p_k}{\hat{q} \cdot p_k} \langle \text{out} | \mathcal{S} | \text{in} \rangle$$
$$= - \langle \text{out} | [Q_S^a(\hat{x}), \mathcal{S}] | \text{in} \rangle = \langle \text{out} | [Q_H^a(\hat{x}), \mathcal{S}] | \text{in} \rangle$$

- Soft theorem can be put in the form

$$\langle \text{out} | [Q(\hat{x}), \mathcal{S}] | \text{in} \rangle = 0, \quad Q = Q_H + Q_S$$

- Implies that the $\mathcal{S}$ -matrix is **invariant under symmetry** generated by Q , parametrized by $\hat{x}$.

$\Rightarrow$ Soft gluon theorem implies the $\mathcal{S}$ -matrix is invariant under an **infinite** number of **symmetries**, one for each point on the sphere!

Physical Interpretation of the Symmetries

$$-i\delta_\varepsilon|p_k\rangle = Q_H[\varepsilon]|p_k\rangle = g\varepsilon(\hat{x}_k)|p_k\rangle$$

Parametrize by (Lie algebra-valued) **functions** $\varepsilon(\theta, \phi) = T^a \varepsilon^a(\theta, \phi)$, rather than **points** $\hat{x}(\theta, \phi)$

- When $\varepsilon = T^a$, find total color charge

$$-i\delta_{\varepsilon=T^a}|p_k\rangle = gT_k^a|p_k\rangle$$

$Q[\varepsilon = T^a]$ generates *ordinary* color rotations

- Generic $\varepsilon = T^a \varepsilon^a(\theta, \phi)$

$$-i\delta_\varepsilon|p_k\rangle = gT_k^a \varepsilon^a(\hat{x}_k)|p_k\rangle$$

ε parametrizes *amount* of color rotation at every angle

Arbitrary $\varepsilon \Rightarrow$ identify symmetry as **independent** color rotations at **every angle**.

Verifying the Conservation Law

- Recall in gravity, subtlety involved measuring Q_S

$$Q_S(\hat{x}) \sim \lim_{\omega \rightarrow 0} \omega [a^a(\omega \hat{x}) + a^{a\dagger}(\omega \hat{x})] \sim \lim_{\omega \rightarrow 0} \int dt \, \varepsilon^\mu e^{i\omega t} \partial_t A_\mu^{a \text{ rad}} \sim \varepsilon^\mu \Delta A_\mu^a$$

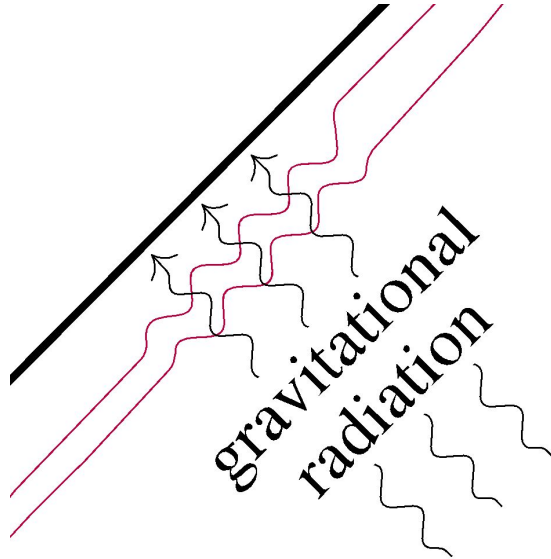

Q_S captures the **zero-frequency component** of the **radiative gluon field** or equivalently, a **permanent net shift** in the **radiative gluon field**.

- Recall the resolution was to study the effect on objects **charged under the symmetry**
 - In gauge theory, consider **test quarks**

Color Memory by Analogy

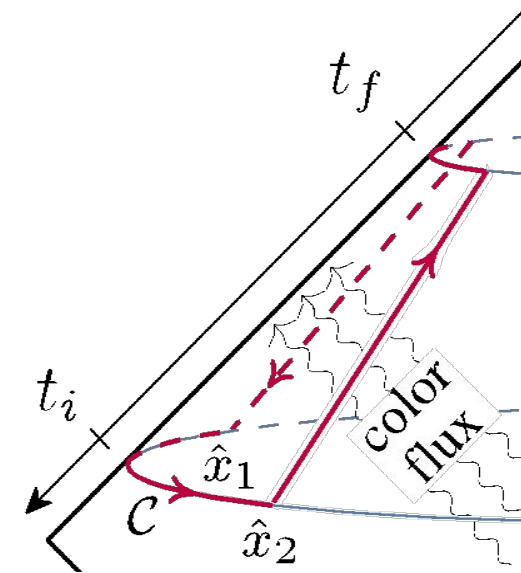
Gravitational Memory

Permanent shift in the **metric** induces permanent shift in the **relative displacement** of a pair of **inertial observers**.



Color Memory

Permanent shift in the **gluon field** induces permanent shift in the **relative color** of a pair of **test quarks**.



Color Memory by Analogy

Gravitational Memory

Permanent shift in the **metric** induces permanent shift in the **relative displacement** of a pair of **inertial observers**.

Geodesic Deviation Equation

$$\begin{aligned}\Delta s^i &\sim \Delta h^{ij} s_j \\ &\sim \lim_{\omega \rightarrow 0} \int dt e^{i\omega t} \partial_t h^{ij} s_j\end{aligned}$$


Color Memory

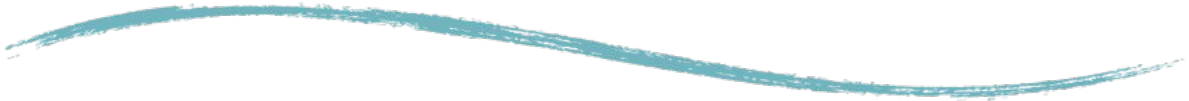
Permanent shift in the **gluon field** induces permanent shift in the **relative color** of a pair of **test quarks**.

Wong's Equation

$$\frac{dx^\mu}{d\tau} (\partial_\mu q - ig A_\mu q) = 0$$

$$\mathcal{W}_C = \mathcal{P} \exp \left(ig \oint_C A \right)$$


Is color memory accessible by experiment?



Obvious objection: confinement obstructs our ability to access long-wavelength modes of the gluonic field by experiment...

Color Memory in Experiment

- There exists a regime in scattering in which the physics is dominated by a classical color memory effect.

Regge limit

$$\begin{array}{ll} t \text{ fixed,} & s \rightarrow \infty \\ \text{(momentum transfer)} & \text{(center of mass energy)} \end{array}$$

- Heavy ion scattering in the Regge limit is argued to admit a classical description.

“Color Glass Condensate”
(Effective Field Theory)

DIS in the Regge Limit

- For concreteness, consider deep inelastic scattering in the Regge limit.

Two-stage Process:

1. Photon fluctuates into quark dipole



Pair of test quarks

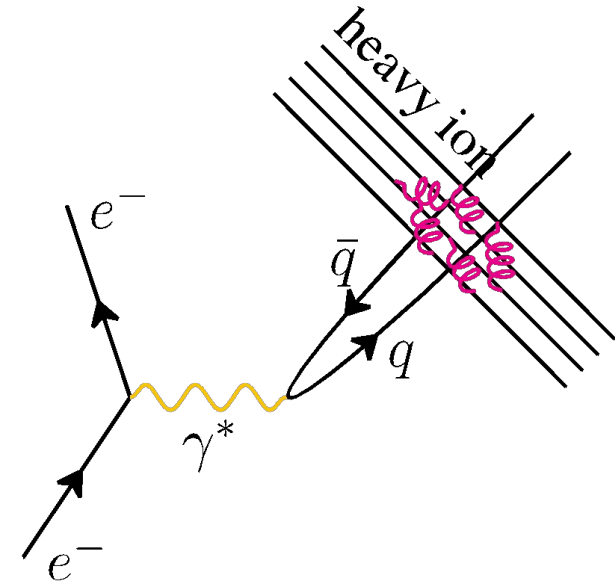
2. Quark dipole interacts with heavy ion



*Color shockwave
sourcing permanent
shift in gluon field*

Inclusive DIS virtual photon-heavy ion cross section:

$$\sigma_{\gamma^* \text{ion}}(x, Q^2) = \int_0^1 dz \int d^2 \vec{r} \underbrace{|\Psi_{\gamma^* \rightarrow q\bar{q}}(z, \vec{r}, Q)|^2}_{\text{QED}} \underbrace{\sigma_{\text{dipole}}(x, \vec{r})}_{\text{QCD}}$$



Color Memory in DIS

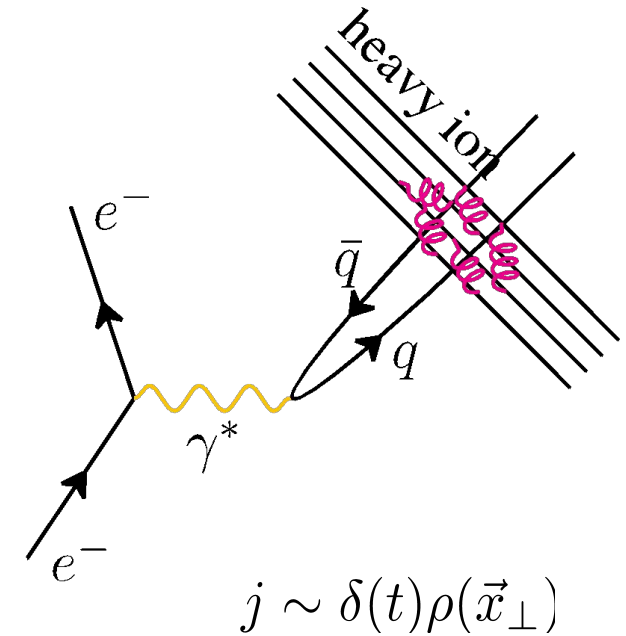
- The QCD process is dominated by a color memory effect
[Ball, **MP**, Raclariu, Strominger & Venugopalan, hep-ph/1805.12224]

QCD process cross section

$$\sigma_{\text{dipole}}(x, \vec{r}_{\perp}) = 2 \int d\vec{b}_{\perp} [1 - \langle \text{Re } \mathcal{S}(\vec{x}_{1\perp}, \vec{x}_{2\perp}) \rangle]$$

$$\mathcal{S}(\vec{x}_{1\perp}, \vec{x}_{2\perp}) = \frac{1}{N_c} \text{Tr} \left[\mathcal{P} \exp \left(ig \oint_c A \right) \right]$$

Color memory
(Relative color rotation)



$$\partial_i \Delta A_i \sim \partial_i \lim_{\omega \rightarrow 0} \omega [a + a^\dagger] \sim \int_{t_i}^{t_f} dt j$$

Identify low-frequency mode of the gluon field as responsible for non-trivial dipole scattering.

Observables in the Color Glass Condensate

- Non-trivial physics of the heavy ion is captured by averaging over **an ensemble** of sources.

$$\langle \mathcal{O} \rangle = \int [\mathcal{D}\rho] W_{\Lambda^+}[\rho] \mathcal{O}[\rho] \quad j \sim \delta(t) \rho(\vec{x}_\perp)$$

- ⇒ Ensemble of shockwave profiles
- ⇒ Ensemble of “permanent shift” field config.

⇒ Generates **systematic dependence** on transverse momentum scale a.k.a. **“saturation scale”**.

Characteristic distance for large color fluctuations
CGC predicts systematic dependence of observables on this scale.

$$Q_S^2 \sim A^{1/3}$$

While the full low-frequency field configuration is not captured by observables, can experimentally verify whether physics is governed by the saturation scale: **characteristic** (transverse) **distance** for **fluctuations** in the **low-frequency mode**.



Summary

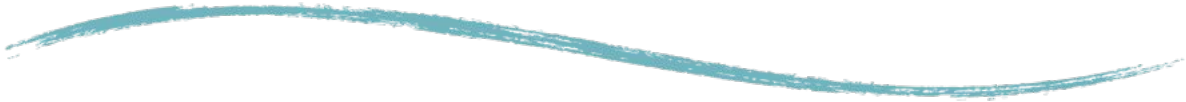
- Presented formal argument that **soft theorems** imply the S -matrix is **invariant** under an **infinite-dimensional symmetry**
- Identified symmetry as **enhancement** of **translations** and **color rotations** in theories of **gravity** and **gauge**, respectively.
- Established “**memory effects**” as observables with **simple dependence** on associated **conserved quantities**,
 - Necessary to verify conservation laws, a natural observable signature of symmetry
- Described context in which a **classical color memory** is relevant to **experiment**



Outlook

- Additional infinite-dimensional symmetries in gauge and gravitational theories from other known soft theorems:
 - **Subleading soft theorems**
 - Soft theorems with **fermionic soft particles**
- Exploit developments concerning **universal features** of generic theories of gauge and gravity to
 1. Determine the theory of **quantum gravity**
 2. Identify/design **robust observables** for particle colliders

Thank You!



Saturation Scale Effects

- The saturation scale is predicted to control the transverse momentum kick received by a probe quark propagating through a target.
- Larger saturation scales lead to larger momentum kicks.
- E.g. predicts diminished likelihood that hadron pairs in deuteron-gold collisions are azimuthally correlated back-to-back as compared with those in proton-proton collisions