Oscillating behavior in neutron and proton form factors in the time-like region

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### Nucleon Charge and Magnetic Distributions

**Space-like FFs are real**

-\( q^2 < 0 \)

**Time-Like FFs are complex**

-\( q^2 > 0 \)

**Unphysical region**

-\( p + p \leftrightarrow e^+ + e^- \)

**Asymptotics**

- **QCD**
- **Analyticity**

**GE = GM**

-\( G_E(0) = 1 \)
-\( G_M(0) = \mu \)

**QNP2022, 6-IX-2022**

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\[
\begin{align*}
e+p & \rightarrow e+p \\
p+p & \leftrightarrow e^-+e^+\pi^0 \\
0 & \quad q^2 = 4m_p^2 \\
\text{GE} & = \text{GM}
\end{align*}
\]
Nucleon Charge and Magnetic Distributions

What about Radiative Corrections?

$G_E(0) = 1$

$G_M(0) = \mu_p$

$q^2 < 0$

$q^2 > 0$

$e + p \rightarrow e + p$

$0 \quad q^2 = 4m_p^2$

$GE = GM$

$p + \bar{p} \leftrightarrow e^+ + e^-$

Time-Like

FFs are complex

Asymptotics

- QCD

Unphysical region

real

What about Radiative Corrections?
The Time-like Region

Expected QCD scaling \( (q^2)^2 \)

\[
|F_{\text{scaling}}(q^2)| = \frac{A}{(q^2)^2 \log^2(q^2/\Lambda^2)}
\]

\[
A = \frac{3\beta q^2\sigma}{2\pi\alpha^2 \left(2 + \frac{1}{\tau}\right)}
\]

\[
|F_{T3}(q^2)| = \frac{A}{(1 - q^2/m_1^2)(2 - q^2/m_2^2)}
\]


\[ e^+ + e^- \rightarrow \bar{p} + p, \]
\[ \bar{p} + p \rightarrow e^+ + e^- \]
The Time-like Region

Expected QCD scaling \((q^2)^2\)

\[
\frac{\mathcal{A}}{(q^2)^2 \left[ \log^2 \left( \frac{q^2}{\Lambda^2} \right) + \pi^2 \right]},
\]

\[
\frac{\mathcal{A}}{\left(1 + \frac{q^2}{m_a^2}\right) \left[1 - \frac{q^2}{0.71}\right]^2},
\]

\[|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1 - \frac{q^2}{m_1^2})(2 - \frac{q^2}{m_2^2})}.\]

\[e^+ + e^- \rightarrow \bar{p} + p,\]

\[\bar{p} + p \rightarrow e^+ + e^-\]

\[GE=GM\]

\[BABAR\]

Oscillations in $e^+e^- \rightarrow p\bar{p}$

- BaBar and BESIII data on the proton time-like effective form factor show a systematic sinusoidal modulation in terms of the $p-\bar{p}$ relative 3-momentum in the near-threshold region.

- $\sim 10\%$ size oscillations on the top of a regular background (dipole x monopole)

- The periodicity and the simple shape of the oscillations point to an interference of 2 mechanisms: scales 0.1-0.2 / $\sim$1 fm.

- The hadronic matter is distributed in non-trivial way.

- High order radiative corrections are applied (structure functions method)

\[
F_p^\text{fit}(s) = F_{3p}(s) + F_{\text{osc}}(p(s))
\]

\[
F_{3p}(s) = \frac{F_0}{\left(1 + \frac{s}{m_a^2}\right)\left(1 - \frac{s}{m_0^2}\right)}^2
\]

\[
F_{\text{osc}}(p(s)) = Ae^{-Bp} \cos(Cp + D).
\]

Cross section from $e^+e^- \rightarrow p\bar{p}$

\[ \sigma = 0.87 \pm 0.02 \text{ nb.} \]

- Novosibirsk 38pt
  \[ 1.9<2E<4.5 \]
  \[ PLB794,64 (2019) \]

- BaBar 85pt
  \[ 1.9<2E<4.5 \]
  \[ PRD87,092005 (2013) \]

- ISR-ISR-SA 30pt
  \[ 2<2E<3.6 \]
  \[ PRD99,092002 (2019) \]

- ISR-Scan 22pt
  \[ 2<2E<3.1 \]
  \[ PRL124,042001 (2020) \]
Generalized Form Factor

\[ F_p(s)^2 = \frac{2\tau|G_M(s)|^2 + |G_E(s)|^2}{2\tau + 1} \]

Fit PRL114,232301 (2015)
6-parameter Fit

Constant \( \sigma \)

\[ F_0 = \frac{A}{(1 + \frac{q^2}{m_a^2})[1 - \frac{q^2}{0.71}]^2}, \]

\[ F_{osc}(p) \equiv A \exp(-Bp) \cos(Cp + D). \]
Form Factor Ratio $R=|GE|/|GM|$ 

- Precise data from BESIII
- Dip at $|q^2|\sim 5.8$ GeV$^2$
- Comparison with SL (Jlab-GEp data) – fitted by a monopole
- Oscillations on top of a monopole: from GE or GM?

$$F_R(\omega(s)) = \frac{1}{1 + \omega^2/r_0} \left[ 1 + r_1 e^{-r_2\omega} \sin(r_3\omega) \right], \quad \omega = \sqrt{s - 2m_p},$$
Sachs form factors: $|G_E|$, $|G_M|$

From the fit on $F_p$ and the fit on $R$, the Sachs FFs (moduli) can be reconstructed

$$|G_E(s)| = F_p(s) \sqrt{\frac{1 + 2\tau}{R^2(s) + 2\tau / R^2(s)}}$$

$$|G_M(s)| = F_p(s) \sqrt{\frac{1 + 2\tau}{R^2(s) + 2\tau}}.$$ 

Threshold constrain $R=1$ for $\tau=1$

The fit gives: $|G_E| = |G_M| = 0.48$
Neutron time-like form factor

M. Ablikim et al. (BESIII Collaboration), Nature Phys. 17, 1200 (2021)

\[ R \text{ (nn/pp)} < 1 \]

Same 6 parameter fit
Simultaneous fit p & n
Same parameters but \( \Delta \phi = 125^\circ \pm 12^\circ \)

- Interfering amplitudes?

- I=0,1 channel mixing?
  X. Cao, J.-P. Dai, and H. Lenske
  PRD 105 (2022) L071503

- Resonances?
  H. Lin, H.-W. Hammer, and U.-G. Meissner,
  P.R.L. 128, 052002 (2022)
Proton & Neutron

Similar 6-parameter fit for p & n with a different phase

*M. Ablikim et al. (BESIII Collaboration), Nature Phys. 17, 1200 (2021)*

- Depends on background
- Gap between the points
- Include Novosibirsk data
**New structures in the proton-antiproton system**

I. T. Lorenz, H.-W. Hammer, and Ulf-G. Meiβner

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**First flavorless vector meson**

**FSI-meson exchange**

**N*-excitation (Δ)**

- Combined fit to SL and TL data, n & p (prior to BESIII)
- NΔ and ΔΔ thresholds (cusp effects)
- Dispersion relations
- Enhancement of FFs in the unphysical region

**Selected data**

**Limited range for the fit**
Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

Xu Cao,1,2,* Jian-Ping Dai3,† and Horst Lenske4,‡

- No multimeson rescattering processes
- Competing isospin-clean vector meson intermediate states $\phi(2170) \ (I=0)$ and $\rho(2150) \ (I=1)$
- Sinusoidal modulation
- Energy dependent relative phase
- Related to the imaginary part of FFs

\[
\frac{|I_1^{\text{rsd}} + I_0^{\text{rsd}}|}{|I_1^{\text{rsd}} - I_0^{\text{rsd}}|} = \frac{A_p}{A_n} = 0.88 \pm 0.35
\]

Balanced isospin content
Not depending on energy

Limited range for the fit
10 years ago a generalization of FFs in SL and TL was proposed

\[ F(q^2) = \int d^4x e^{iq\cdot x} \rho(x), \quad q\cdot x = q_0 t - \vec{q} \cdot \vec{x} \]

\[ \rho(x) = \rho(\vec{x}, t) \]

SL photon ‘sees’ a charge density

TL photon can NOT test a space distribution

How to connect and understand the amplitudes?

Photon-Charge coupling

...access projections of $F(q^2)$

**SL**: Fourier transform of a stationary charge and current distribution  (*Breit frame*)

**TL**: Amplitude for creating *charge-anticharge pairs* at time $t$  (*CMS frame*)

Charge distribution: distribution in time of $\gamma^* \rightarrow$ *charge-anticharge vertices*

The simplest picture: $qq$ pair + compact di-quark

Resolved

Unresolved representation

It is generally assumed that the nucleon is composed by 3 valence quarks and a neutral sea of $q\bar{q}$ pairs

Nucleon: antisymmetric state of colored quarks

$$|p > \sim \epsilon_{ijk} |u^i u^j d^k >$$

$$|n > \sim \epsilon_{ijk} |u^i d^j d^k >$$

Main assumption of the Abu3 model:

Does not hold in the spatial center of the nucleon: the center of the nucleon is *electrically neutral*, due to the strong gluonic field

Inner region: gluonic condensate of clusters with randomly oriented chromo-magnetic field (Vainshtein, 1982)

The color quantum number of quarks does not play any role, due to stochastic averaging. Pauli principle applies.
Additional suppression for the scalar part due to colorless internal region: “charge screening in a plasma”:

$$\Delta \phi = -4\pi e \sum Z_i n_i, \quad n_i = n_{i0} \exp\left[-\frac{Z_i e \phi}{kT}\right]$$

Neutrality condition: $$\sum Z_i n_{i0} = 0$$

$$\Delta \phi - \chi^2 \phi = 0, \quad \phi = \frac{e^{-\chi r}}{r}, \quad \chi^2 = \frac{4\pi e^2 Z_i^2 n_{i0}}{kT}$$

Additional suppression (Fourier transform)

$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu} \left(1 + \frac{Q^2}{q_1^2}\right)^{-1}$$

Time-like region

Antisymmetric state of colored quarks

The vacuum state transfers all the released energy to a state of matter consisting at least of 6 massless valence quarks, a set of gluons, sea of $\bar{q}q$ with $q_0 > 2M_p$, $J=1$, dimensions $\hbar/(2M_p) \sim 0.1$ fm.

- uu (dd) quarks are repulsed from the inner region
- The 3\textsuperscript{rd} quark $u$ ($p$) or $d$ ($n$) is attracted by one of the identical quarks, forming a compact di-quark: competition between attraction force and stochastic force of the gluon field
- The color state is restored: the ‘point-like’ hadron expands and cools down: the current quarks and antiquarks absorb gluons and transform into constituent quarks
TL - np-correlation : 3 steps

Experimental points at the same $P_L$
Proton values calculated from the 6-parameter fit

1) pQCD applies
2) di-quark phase charge redistributed
3) The hadron is formed

Quark pairs created by quantum vacuum fluctuations: *all quark flavors are equally probable*, but, due to Heisenberg principle, the associated time depends on the energy (baryon mass).

Conclusions

• BESIII new data on TL n & p FFs, their ratio and first determination of individual proton TL FFs (|G_E| and |G_M|) show a very rich structure probably related to the complex nature of FFs.

• Origin of oscillatory phenomena:
  Di-quark as a necessary step towards hadron creation?

• Main features of the SL and TL FFs data qualitatively explained by the Arbutz model:
  ➢ The monopole-like decrease of the FF ratio
  ➢ The formation of a di-quark component in the nucleon
  ➢ The npΛ correlation

• Predicts
  • similarities between n&p, SL & TL, non zero crossing in SL

→ Deepen the quantitative aspects
Thank you for your attention
**SL- the most precise ruler**

### $Q^2$-Monopole fit

$$\mathcal{R} = \mu_p \frac{G_{Ep}}{G_{Mp}} = \left(1 + \frac{Q^2}{m_r^2}\right)^{-1}.$$

### $Q^2$ -linear fit

- **Zero crossing?**
- **Prediction: NO**

The photon ‘sees’ the neutral, screened region $G_{Ep} \approx 0$ for $r < 0.1 \text{ fm}$

$$r \ [\text{fm}] = \lambda = \frac{\hbar c}{\sqrt{Q^2}} = 0.197 [\text{GeV fm}] / \sqrt{Q^2} [\text{GeV}],$$
SL Form Factors Ratio

Large $Q^2$-> Small $r$

\[ R = \mu_p \frac{G_{Ep}}{G_{M\mu}} = \left(1 + \frac{Q^2}{m_r^2}\right)^{-1}. \]

$Q^2 (<0.15 \text{ GeV}^2)$ – linear fit

$Q^2$-Monopole fit

$r [\text{fm}] = \lambda = \frac{\hbar c}{\sqrt{Q^2}} = 0.197 \ [\text{GeV \ fm}] / \sqrt{Q^2} [\text{GeV}], \]
SL - the most precise ruler

\[ G_{Ep} \approx 0 \]

\[ Q^2 (<0.15 \text{ GeV}^2) \text{-linear fit} \]

Zero crossing?

Prediction: NO zero crossing

The photon ‘sees’ the neutral, screened, central region
TL- the most precise clock

\[ t \text{ [sec]} = \frac{\hbar c}{q_0} = \frac{0.0658 \cdot 10^{-23} \text{[GeV s]}}{q_0 \text{[GeV]}} \]

10\(^{-23}\) s is the time for light to cross a proton

Di-quark phase dominant at \( t \sim 0.02-0.03 \) [10\(^{-23}\) s]
Fourier Transform


\[
F_0(p) \equiv \int d^3 \vec{r} \exp(i \vec{p} \cdot \vec{r}) M_0(r)
\]

\[
F(p) = F_0(p) + F_{osc}(p) \equiv \int d^3 \vec{r} \exp(i \vec{p} \cdot \vec{r}) M(r).
\]

- **Rescattering processes**
- **Large imaginary part**
- **Related to the time evolution of the charge density?**
  

- **Consequences for the SL region?**
- **Data from BESIII, expected from PANDA**

\[
p: \text{relative momentum}
\]

\[
r: \text{distance between the center of the forming hadrons}
\]

\[
(p, r) \text{ conjugate variables, } r \leftrightarrow t
\]

\[
F_0 = \frac{\mathcal{A}}{(1 + q^2/m_a^2) [1 - q^2/0.71]^2},
\]

\[
F_{osc}(p) = A \exp(-Bp) \cos(Cp + D).
\]
Proton radius

Data from Mainz, PRC 90, 015206 (2014)

Large $r \rightarrow$ Small $Q^2$
Proton radius

Data from Mainz, CLAS…

How can a photon with wavelength \( \sim 15 \text{ fm} \) distinguish between a proton size of 0.84 or 0.87 fm?

\[ G_{Ep} \]

Large \( r \) \( \rightarrow \) Small \( Q^2 \)
Symmetry Relations (annihilation)

- Differential cross section at complementary angles:

The SUM cancels the $2\gamma$ contribution:

\[
\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) = 2 \frac{d\sigma^{\text{Born}}}{d\Omega}(\theta)
\]

The DIFFERENCE enhances the $2\gamma$ contribution:

\[
\frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) = 4N \left[ (1 + x^2) \text{Re} G_M \Delta G_M^* + \frac{1 - x^2}{\tau} \text{Re} G_E \Delta G_E^* + \sqrt{\tau (\tau - 1)} x (1 - x^2) \text{Re} \left( \frac{1}{\tau} G_E - G_M \right) F_3^* \right]
\]

\[\tau = \frac{q^2}{4m^2}, \quad x = \cos \theta\]
Neutron time-like form factor

M. Ablikim et al. (BESIII Collaboration), Nature Phys. 17, 1200 (2021)

Cross section \( e^+ e^- \rightarrow n\bar{n} \)

Form factor

(a) \( \sigma_B \) (pb) vs. \( \sqrt{s} \) (GeV)

(b) Form factor vs. \( \sqrt{s} \) (GeV)
As in SL region:
- Dependence on $q^2$ contained in FFs
- Even dependence on $\cos^2 \theta$ (1$\gamma$ exchange)
- No dependence on sign of FFs
- Enhancement of magnetic term

but TL form factors are complex!
Unpolarized cross section

The cross section for $p + p \rightarrow e^+ + e^-$ (1 $\gamma$-exchange):

$$\frac{d\sigma}{d(\cos \theta)} = \frac{\pi \alpha^2}{8m^2 \sqrt{\tau - 1}} \left[ \tau |G_M|^2 (1 + \cos^2 \theta) + |G_E|^2 \sin^2 \theta \right]$$

$\theta$: angle between $e^-$ and $\bar{p}$ in cms.

Two Photon Exchange:

- Induces four new terms
- Odd function of $\theta$
- Does not contribute at $\theta=90^\circ$

$D = (1 + \cos^2 \theta)(|G_M|^2 + 2 Re G_M \Delta G_M^*) + \frac{1}{\tau} \sin^2 \theta(|G_E|^2 + 2 Re G_E \Delta G_E^*) + 2 \sqrt{\tau(\tau - 1)} \cos \theta \sin^2 \theta Re(\frac{1}{\tau} G_E - G_M) F_3^*$