Fluctuations and phases in baryonic matter

[LB, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

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Len Brandes
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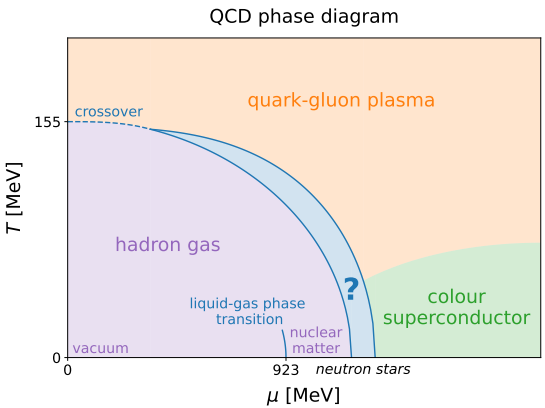
Technische Universität München

Motivation



- QCD vacuum characterized by confinement and spontaneously broken chiral symmetry
- Liquid-gas phase transition to nuclear matter at $\mu = 923\,\text{MeV}$
- At $\mu \sim 2.6\,\text{GeV}$ perturbative QCD results imply quark and gluon d.o.f. in color superconducting phase

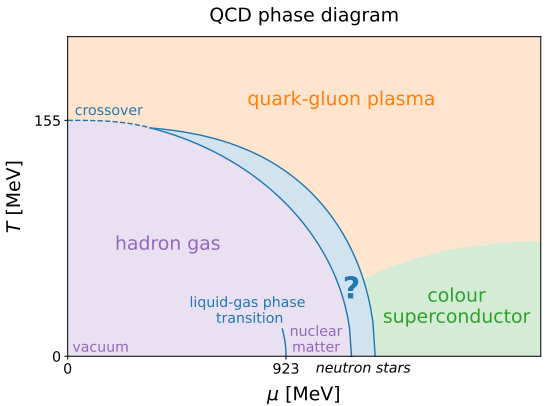
  [Alford, Rajagopal and Wilczek, Nucl. Phys. B 537 (1999)]

  → **Nature of transition from nuclear matter to color superconductor still unknown**

- Essential for neutron stars with central densities $n \sim 5 - 6\,n_0$

  [Legred et al., Phys. Rev. D 105 (2022)]

Motivation



- For large μ lattice QCD unavailable because of sign problem
- Chiral effective field theory only valid up to $n \lesssim 2\,n_0$

  [Holt, Rho and Weise, Phys. Rept. 621 (2016)]
- Chiral model calculations in mean-field approximation find **first-order phase transition** from spontaneously broken to restored chiral symmetry

  [Rößner et al., Nucl. Phys. A 814 (2008)]

→ Analyse impact of fluctuations beyond mean-field on possible chiral phase transition

[Drews and Weise, Prog. Part. Nucl. Phys. 93 (2017)]

Chiral nucleon-meson model

- Chiral theory of fermion doublet $\Psi = (p, n)$ [Floerchinger and Wetterich, Nucl. Phys. A 890–891 (2012)]

- Fermions interacting via chiral boson fields $\phi = (\sigma, \boldsymbol{\pi})$, with heavy scalar σ and pion $\boldsymbol{\pi}$

$$\mathscr{L} \;=\; \bar{\Psi}\left[\gamma_\mu \partial_\mu + g(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\Psi \;+\; \frac{1}{2}\left(\partial_\mu\sigma\partial_\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\right) \;+\; \mathscr{U}(\sigma,\boldsymbol{\pi}) \;+\; \Delta\mathscr{L}$$

- Boson self-interactions via expansion of chiral invariant, $\chi \equiv \frac{1}{2}\phi^\dagger\phi = \frac{1}{2}\left(\sigma^2 + \boldsymbol{\pi}^2\right)$, around vacuum expectation value χ_0 plus explicit symmetry breaking term

$$\mathscr{U}(\sigma,\boldsymbol{\pi}) \;=\; \sum_{n=1}^{4} \frac{a_n}{n!}\left(\chi - \chi_0\right)^n \;-\; m_\pi^2 f_\pi\left(\sigma - f_\pi\right)$$

- Short distance dynamics modeled by massive vector fields v_μ and $\boldsymbol{w}_\mu$

$$\Delta\mathscr{L} \;=\; -\Psi^\dagger\left[g_v\, v + g_w\, \tau_3\, w\right]\Psi \;-\; \frac{1}{2}m_v^2\left(v^2 + w^2\right)$$

Mean-field (MF) approximation

- Replace chiral fields by their expectation values $\langle\sigma\rangle$ and $\langle\boldsymbol{\pi}\rangle = 0$

- Introduce T and $\mu_{p/n}$ and determine the grand canonical potential

$$\Omega_{MF} \;=\; \Omega_F\big(T,\mu_p,\mu_n;\langle\sigma\rangle,v,w\big) \;+\; \mathscr{U}\big(\langle\sigma\rangle,\langle\boldsymbol{\pi}\rangle=0\big) \;-\; \frac{1}{2}m_v^2\Big(v^2+w^2\Big)$$

- Fermionic part with $E = \sqrt{p^2+M^2(\sigma)}$ and dynamical nucleon mass $M(\sigma) = g\langle\sigma\rangle$

$$\Omega_F \;=\; -2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\left[E \;+\; \frac{p^2}{3E}\sum_{r=\pm1}\frac{1}{1+e^{(E-r\bar{\mu}_i)/T}}\right] \qquad \text{with} \qquad \bar{\mu}_{p/n} \;=\; \mu_{p/n} \;-\; g_v v \;\mp\; g_w w$$

- Grand canonical potential evaluated at minimum yields thermodynamic observables

$$P \;=\; -\Omega_{MF} \qquad s \;=\; -\frac{\partial\Omega_{MF}}{\partial T} \qquad n_i \;=\; -\frac{\partial\Omega_{MF}}{\partial\mu_i} \qquad \varepsilon \;=\; -P \;+\; \sum_{i=p,n}\mu_i n_i \;+\; Ts$$

Extended mean-field (EMF)

- Diverging vacuum term in fermionic contribution

$$\delta\Omega_{vac} = -4\int \frac{d^3p}{(2\pi)^3}\, E$$

→ Neglected in standard mean-field analyses

- Can be computed via dimensional regularisation [Skokov et al., Phys. Rev. D 82 (2010)]

- **Extended mean-field (EMF)** includes vacuum contribution

$$\Omega_{EMF} \equiv \Omega_{MF} - \frac{(g\sigma)^4}{4\pi^2}\ln\frac{g\sigma}{\Lambda}$$

- Additional fluctuations beyond vacuum contribution (chiral boson and nucleon loops)

→ Include using non-perturbative **functional renormalization group (FRG)** approach

Functional renormalisation group (FRG)

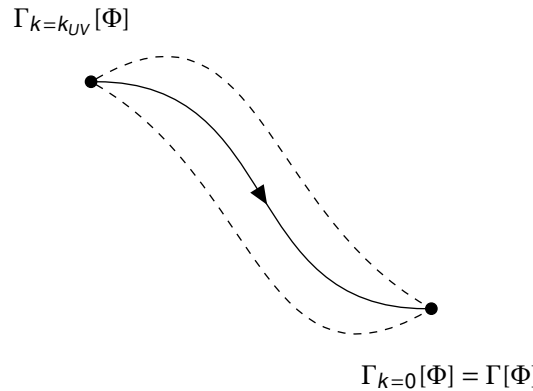
- Initialize scale-dependent effective action $\Gamma_k[\Phi]$ of chiral-nucleon meson model at $k_{UV} \sim 4\pi f_\pi$

- Evolution $k \to 0$ governed by Wetterich's flow equation
  [Wetterich, Phys. Lett. B 301 (1993)]

$$k\frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(2)}[\Phi]+R_k\right)^{-1}\right] = \frac{1}{2}$$

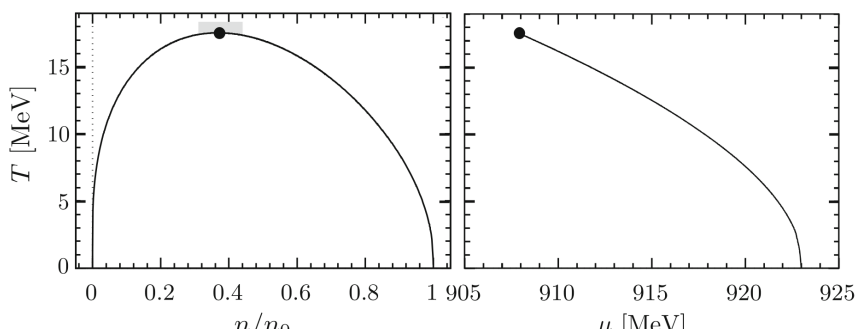
- $\Gamma_k[\Phi]$ contains all fluctuations with $p^2 \geq k^2$ through regulator $R_k(p)$

- Model parameters adjusted to reproduce vacuum properties and nuclear phenomenology



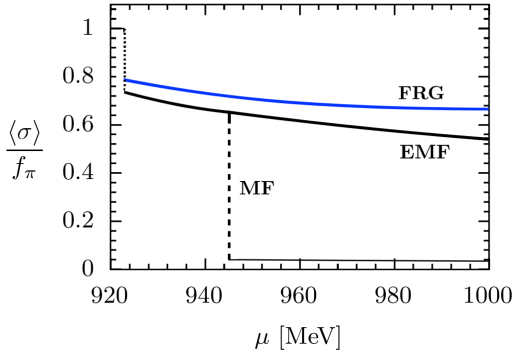
Nuclear thermodynamics

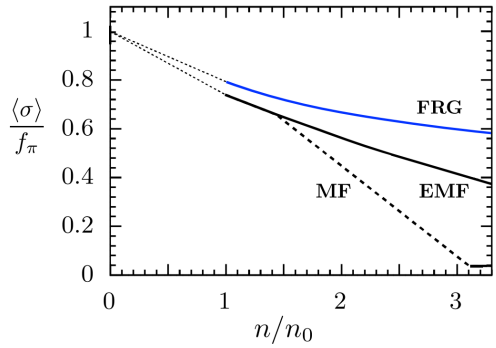
- $\langle\sigma\rangle_{vac} = f_\pi \simeq 93$ MeV
- $M_N = 939\,\text{MeV}$
- $E/A(n_0) = -16\,\text{MeV}$, $S(n_0) = 32\,\text{MeV}$
- Nuclear surface tension $\Sigma = 1.08(6)\,\text{MeV/fm}^2$, Landau mass $M_L^* = 0.7 - 0.8\,M_N$ and compression modulus $K = 240(20)\,\text{MeV}$
- **Nuclear liquid-gas phase transition** with empirical critical parameters [Elliot et al., Phys. Rev. C 87 (2013)]



Phase structure of symmetric nuclear matter

- $\langle\sigma\rangle$ compared to $\langle\sigma\rangle_{vac} = f_\pi$ serves as order parameter for chiral symmetry, because $M(\sigma) = g\langle\sigma\rangle$
- **Mean-field:** unphysical first-order phase transition to chirally restored phase $\langle\sigma\rangle = 0$ at $n \simeq 1.5\, n_0$
- **Extended mean-field:** vacuum contribution stabilizes order parameter
- **FRG:** further stabilization through additional fluctuations





Phase structure of pure neutron matter

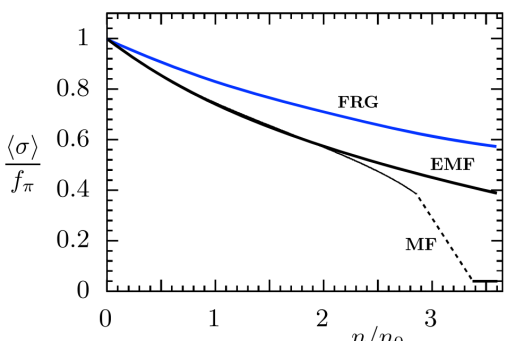
- Similar results for pure neutron matter

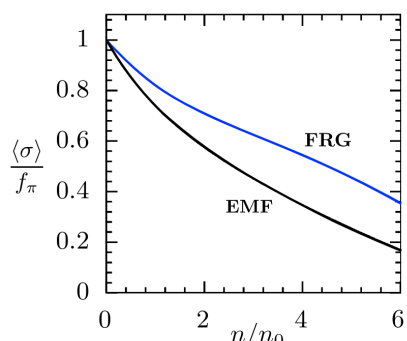
  → **First-order phase transition converted to smooth crossover at large densities**

- Model adjusted to low-density properties, potential expanded around $\chi_0 = 1/2\,\langle\sigma\rangle^2_{vac} = 1/2\,f_\pi^2$

  → For small $\langle\sigma\rangle/f_\pi$ model no longer applicable

- In FRG $\langle\sigma\rangle/f_\pi$ stays around 40% until $n \sim 6\,n_0$ (central densities in heavy neutron stars!)





Further results

- **Similar behaviour in chiral quark-meson models** (with Polyakov loop)

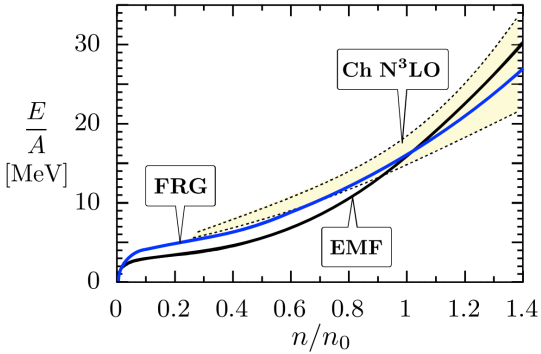
  → Chiral restoration seen in mean-field approximation avoided by vacuum fluctuations and in FRG

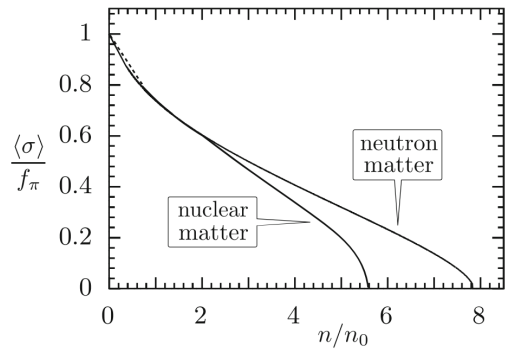
  [Zacchi and Schaffner-Bielich, Phys. Rev. D 97 (2018)] [Gupta and Tiwari, Phys. Rev. D 85 (2012)]

- Good agreement with pure neutron matter E/A from ChEFT calculations (left)

  [Drischler, Hebeler and Schwenk, Phys. Rev. Lett. 122 (2019)]

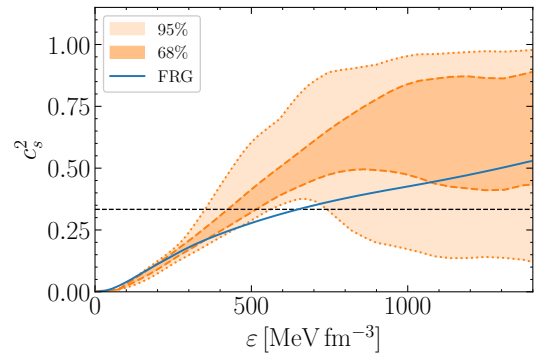
- In chiral limit $m_\pi \to 0$ crossover turns into second-order phase transition (right)





Neutron stars

- Recent multimessenger measurements:
  - General relativistic Shapiro time delays
  - NICER X-ray measurements
  - Gravitational waves from binary neutron star mergers
- Use **Bayesian inference** to constrain speed of sound $c_s^2 = \partial P / \partial \varepsilon$ inside neutron stars
  [LB, Weise and Kaiser, arXiv:2208.03026 (2022)]
- Strong first-order phase transition inside neutron stars $M \leq 2\,M_\odot$ unlikely, crossover still possible
- EoS based solely on nuclear degrees of freedom cannot be ruled out



Summary

- Chiral nucleon-meson model reproduces empirical nuclear properties including liquid-gas phase transition
- **Mean-field approximation**: chiral first-order phase transition at unphysically low densities
- **Extended mean-field**: includes fermionic vacuum contribution

  → Chiral symmetry remains spontaneously broken up to higher densities
- **Functional renormalisation group**: additional fluctuations provide an even stronger stabilization against chiral restoration
- Similar results for chiral quark-meson models

  → **Fluctuations convert chiral first-order phase transition into smooth crossover at high baryon densities** $n \geq 6\, n_0$
- Based on multimessenger measurements strong first-order phase transition unlikely in neutron stars with $M \leq 2\, M_\odot$