

$\cos 2\phi_t$ azimuthal asymmetry in back to back J/ψ and Jet production at EIC

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Outline

- Gluon TMDs
- $\cos 2\phi_t$ Azimuthal Asymmetry
- TMD parameterizations
- Results and discussions
- Conclusion

Gluon TMDs

		gluon pol.	
nucleon pol.		U	Circularly Linearly
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g, h_{1T}^{\perp g}$

At leading Twist

- TMD-PDF $f(x, k_t, Q^2)$
- The gluon correlator

$$\Phi^{\mu\nu}(x, q_T) = \int \frac{d\xi^- d^2\xi_T}{M_p(2\pi)^3} e^{iq \cdot \xi} \left\langle P | Tr[F^{+\mu}(0) U^{[C]} F^{+\nu}(\xi) U^{[C]}] | P \right\rangle \Big|_{\xi^+ = 0}$$

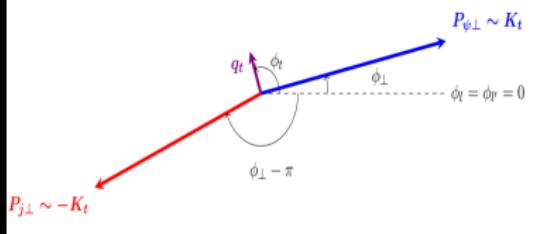
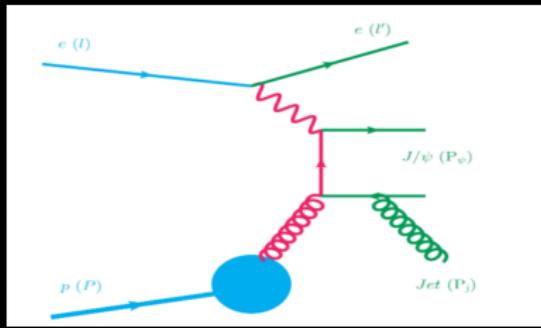
- Unpolarized proton

$$\Phi_g^{\nu\nu'}(x, q_T) = -\frac{1}{2x} \left\{ g_\perp^{\nu\nu'} f_1^g(x, q_T^2) - \left(\frac{q_T^\nu q_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'} \frac{q_T^2}{2M_p^2} \right) h_1^{\perp g}(x, q_T^2) \right\} \text{Mulders(2001)}$$

Linearly polarized gluon TMD $h_1^{\perp g}(x, q_T^2)$

- It can be probed in Semi Inclusive Deep Inelastic Scattering, Drell-Yan processes.
- By extracting $\cos 2\phi_t$ azimuthal asymmetry one can probe $h_1^{\perp g}(x, q_T^2)$.
- So far it has not been extracted from data.
- small-x region \rightarrow Weizsäcker Williams type or dipole distribution type with simple gauge link configuration ++ or -- and +- or -+
- It follows positivity bound, $\frac{q_T^2}{2M_p^2} |h_1^{\perp g}(x, q_T^2)| \leq f_1^g(x, q_T^2)$
Mulders (2001).

Kinematics



- In small-x $\rightarrow \gamma^* + g \rightarrow c\bar{c}(^{2S+1}L_J^{(1,8)}) + g$. D'Alesio (2019)
- virtual photon and Proton along $\pm z$ axis
- Leptonic plane \Rightarrow measuring azimuthal angles
- $Q^2 = -q^2$, $s = (P + l)^2$, $x_B = \frac{Q^2}{2P \cdot q}$, $y = \frac{P \cdot q}{P \cdot l}$, $z = \frac{P \cdot P_\psi}{P \cdot q}$.

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \quad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}$$

- back to back scattering \Rightarrow $|\mathbf{q}_t|^2 \ll |\mathbf{K}_t|^2 \sim M_\psi^2 \Rightarrow$ TMD factorization.
- ϕ_t and ϕ_{\perp}

Total cross-section

- In back to back lepto-production of J/ψ and jet \rightarrow TMD factorization is expected to follow - $\mathbf{q}_t^2 \ll W^2 = (q + p_g)^2$.

$$d\sigma = \frac{1}{2s} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_\psi}{2E_\psi (2\pi)^3} \frac{d^3 P_j}{2E_j (2\pi)^3} \int dx \ d^2 \mathbf{p}_T \ (2\pi)^4 \delta^4(q + p_g - P_j - P_\psi)$$
$$\frac{1}{Q^4} \Phi_g^{\nu\nu'}(x, \mathbf{p}_T) \otimes L^{\mu\mu'}(l, q) \ \mathcal{M}_{\mu\nu}^{g\gamma^* \rightarrow J/\psi \ g} \ \mathcal{M}_{\mu'\nu'}^{*g\gamma^* \rightarrow J/\psi \ g}$$

D'Alesio (2019)

NRQCD Factorization

The relative momentum $k^2 \ll M_c^2 \Rightarrow$ non relativistic approximation of QCD.

$$\mathcal{M}^{ab \rightarrow J/\psi} = \sum_n \mathcal{M}[ab \rightarrow c\bar{c}\left(^{2S+1}L_J^{(1,8)}\right)] \langle 0 | \mathcal{O}^{J/\psi}\left(^{2S+1}L_J^{(1,8)}\right) | 0 \rangle$$

- $\mathcal{M}[ab \rightarrow c\bar{c}\left(^{2S+1}L_J^{(1,8)}\right)]$: High energy perturbative part.
- $\langle 0 | \mathcal{O}^{J/\psi}\left(^{2S+1}L_J^{(1,8)}\right) | 0 \rangle$: Non perturbative Long Distance Matrix Element. $c\bar{c}\left(^{2S+1}L_J^{(1,8)}\right)$ to J/ψ

Bodwin, Braaten, Lepage (1994)

Matrix element for J/ψ and jet production

$$\begin{aligned} \mathcal{M}\left(\gamma^* g \rightarrow c\bar{c}[^{2S+1}L_J^{(1,8)}]g\right) &= \sum_{LL_z SS_z} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}) \langle LL_z; SS_z | JJ_z \rangle \\ &\times \text{Tr}[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \end{aligned}$$

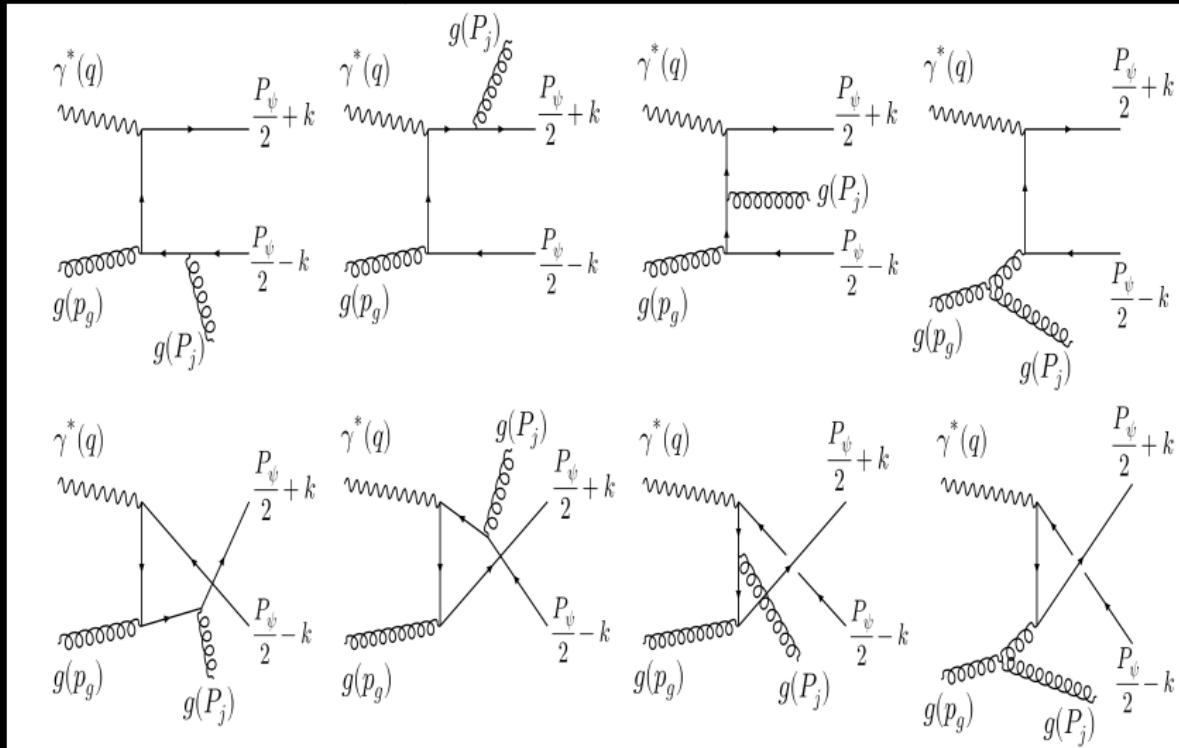
- $k^2 \ll M_c^2 \rightarrow \mathcal{M}(k) = \mathcal{M}(k)|_{k=0} + \dots$
- $\mathcal{M}(k)|_{k=0}$ - S wave scattering amplitude,
 $k\mathcal{M}'(k)|_{k=0}$ - P wave scattering amplitude
- $R_0(0), R'_1(0) \rightarrow \text{LDMEs D'Alesio (2019).}$

$$\begin{aligned} \langle 0 | \mathcal{O}_1^{J/\psi} ({}^{2S+1}S_J) | 0 \rangle &= \frac{N_c}{2\pi} (2J+1) |R_0(0)|^2 \\ \langle 0 | \mathcal{O}_8^{J/\psi} ({}^{2S+1}S_J) | 0 \rangle &= \frac{2}{\pi} (2J+1) |R_0(0)|^2 \\ \langle 0 | \mathcal{O}_8^{J/\psi} ({}^3P_J) | 0 \rangle &= \frac{2N_c}{\pi} (2J+1) |R'_1(0)|^2 \end{aligned}$$

Kishore, Mukherjee and Rajesh (2020)

$$\begin{aligned} \mathcal{M}[{}^{2S+1}S_J^{(1,8)}] &= \frac{1}{\sqrt{4\pi}} R_0(0) \text{Tr}[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \Big|_{k=0} \\ \mathcal{M}[{}^{2S+1}P_J^{(8)}] &= -i \sqrt{\frac{3}{4\pi}} R'_1(0) \sum_{LL_z SS_z} \varepsilon_{LL_z}^\alpha(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \\ &\quad \times \frac{\partial}{\partial k^\alpha} \text{Tr}[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \Big|_{k=0} \\ \mathcal{P}_{SS_z}(P_\psi, k) &= \sum_{s_1 s_2} \left\langle \frac{1}{2}s_1; \frac{1}{2}s_2 \middle| SS_z \right\rangle v\left(\frac{P_\psi}{2} - k, s_1\right) \bar{u}\left(\frac{P_\psi}{2} + k, s_2\right) \\ &= \frac{1}{4M_\psi^{3/2}} (-\not{p}_\psi + 2\not{k} + M_\psi) \Pi_{SS_z} (\not{p}_\psi + 2\not{k} + M_\psi) + \mathcal{O}(k^2) \end{aligned}$$

$$g\gamma^* \rightarrow J/\psi \ g$$



$\cos 2\phi_t$ Azimuthal Asymmetry

$$\begin{aligned} \frac{d\sigma}{dz dy dx_B d^2 q_t d^2 K_t} &= d\sigma^U + d\sigma^T \\ d\sigma^U &= \frac{1}{(2\pi)^4} \frac{1}{16sz(1-z)Q^4} (\mathbb{A}_0 + \mathbb{A}_1 \cos \phi_\perp + \mathbb{A}_2 \cos 2\phi_\perp) f_1^g(x, q_t^2) \\ d\sigma^T &= \frac{1}{(2\pi)^4} \frac{1}{16sz(1-z)Q^4} (\mathbb{B}_0 \cos 2\phi_t + \mathbb{B}_1 \cos(2\phi_t - \phi_\perp) + \mathbb{B}_2 \cos 2(\phi_t - \phi_\perp) \\ &\quad + \mathbb{B}_3 \cos(2\phi_t - 3\phi_\perp) + \mathbb{B}_4 \cos(2\phi_t - 4\phi_\perp)) \frac{q_t^2}{M_P^2} h_1^{\perp g}(x, q_t^2). \end{aligned}$$

D'Alesio (2019)

$$\begin{aligned} \langle \cos 2\phi_t \rangle &\equiv A^{\cos 2\phi_t} = 2 \frac{\int d\phi_t d\phi_\perp \cos 2\phi_t d\sigma(\phi_t, \phi_\perp)}{\int d\phi_t d\phi_\perp d\sigma(\phi_t, \phi_\perp)} \\ \langle \cos 2\phi_t \rangle &\equiv A^{\cos 2\phi_t} = \frac{\int q_t dq_t \frac{q_t^2}{M_P^2} \mathbb{B}_0 h_1^{\perp g}(x, q_t^2)}{\int q_t dq_t \mathbb{A}_0 f_1^g(x, q_t^2)} \end{aligned}$$

For fixed kinematical variables and the saturation of positivity bound $\frac{q_t^2}{2M_P^2} |h_1^{\perp g}(x, q_t^2)| = f_1^g(x, q_t^2)$

$$A^{\cos 2\phi_t} \rightarrow U = \frac{2 * |\mathbb{B}_0|}{\mathbb{A}_0}$$

Gaussian Parameterization

- Drell-Yan and SIDIS \Rightarrow transverse momentum spectra \rightarrow roughly Gaussian in nature. Stefano Melis (2014)
- TMDs = Collinear pdf (x -dependent) \otimes transverse momentum dependent (q_t -dependent) part.
- The transverse momentum dependent part is Gaussian in nature.
D. Boer and C. Pisano (2012)

$$f_1^g(x, q_t^2) = f_1^g(x, Q) \frac{1}{\pi \langle q_t^2 \rangle} e^{-q_t^2 / \langle q_t^2 \rangle}$$

$$h_1^{\perp g}(x, q_t^2) = \frac{M_p^2 f_1^g(x, Q)}{\pi \langle q_t^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{q_t^2}{r \langle q_t^2 \rangle}}$$

$$\frac{q_t^2}{2M_p^2} |h_1^{\perp g}(x, q_t^2)| \leq f_1^g(x, q_t^2).$$

Spectator model

- Proton is treated as 2 particles, gluon and spectator particle.
- spectator particle has on-shell mass M_X (real and continuous variable)

$$F^g(x, q_t^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}^g(x, q_t^2; M_X)$$

$$\rho_X(M_X) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

$\{X\} \equiv \{A, B, a, b, C, D, \sigma\}$ are the free parameters of the model

$$\begin{aligned} \hat{f}_1^g(x, q_t^2; M_X) &= -\frac{1}{2} g^{ij} \left[\Phi^{ij}(x, q_t, S) + \Phi^{ij}(x, q_t, -S) \right] \\ &= \left[(2Mxg_1 - x(M + M_X)g_2)^2 [(M_X - M(1-x))^2 + q_t^2] \right. \\ &\quad \left. + 2q_t^2 (q_t^2 + xM_X^2) g_2^2 + 2q_t^2 M^2 (1-x) (4g_1^2 - xg_2^2) \right] \\ &\quad \times \left[(2\pi)^3 4xM^2 (L_X^2(0) + q_t^2)^2 \right]^{-1} \end{aligned}$$

$$\begin{aligned} \hat{h}_1^{\perp g}(x, q_t^2; M_X) &= \frac{M^2}{\varepsilon_t^{ij} \delta^{jm} (q_t^i q_t^m + g^{jm} q_t^2)} \varepsilon_t^{ln} \delta^{nr} [\Phi^{nr}(x, q_t, S) + \Phi^{nr}(x, q_t, -S)] \\ &= \left[4M^2 (1-x) g_1^2 + (L_X^2(0) + q_t^2) g_2^2 \right] \times \left[(2\pi)^3 \times (L_X^2(0) + q_t^2)^2 \right]^{-1} \end{aligned}$$

Spectator model

- $g_{1,2}(p^2)$ are model-dependent form factors

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2} = \kappa_{1,2} \frac{p^2 (1-x)^2}{(\mathbf{q}_t^2 + L_X^2(\Lambda_X^2))^2}$$

- $\kappa_{1,2}$ and Λ_X → normalization and cut-off parameters

$$p^2 = -\frac{\mathbf{q}_t^2 + L_X^2(0)}{1-x}$$

- $p \rightarrow$ gluon momentum

$$L_X^2(\Lambda_X^2) = x M_X^2 + (1-x) \Lambda_X^2 - x(1-x) M^2$$

TMD Evolution

- TMD evolution \rightarrow CSS evolution equation and RG evolution equation.
- Evolution in transverse momentum (q_t) and in the scattering scale (Q).
- TMD evolution \Rightarrow impact parameter space. [Aybat and Rogers \(2011\)](#)
- In CSS formalism general equation for TMD is given as,
- The coefficient function \Rightarrow series of α_s ,
- $b_t \ll 1/\Lambda_{QCD}$
 $e^{-\frac{1}{2}S_A(b_t^2, Q_f^2, Q_i^2)} \Rightarrow 1 - \frac{S_A(b_t^2, Q_f^2, Q_i^2)}{2}$
- DGLAP evolution from $Q_i = 2e^{-\gamma E}/b_t$ to $Q_f = \sqrt{M_\psi^2 + K_t^2}$ [D. Boer \(2020\)](#)

$$\hat{f}(x, b_t^2; Q_f^2) = \frac{1}{2\pi} \int d^2 q_t e^{iq_t \cdot b_t} f(x, q_t^2, Q_f^2).$$

$$\hat{f}(x, b_t^2, Q_f^2) = \frac{1}{2\pi} \sum_{p=q, \bar{q}, g} (C_{g/p} \otimes f_1^p)(x, Q_i^2) e^{-\frac{1}{2}S_A(b_t^2, Q_f^2, Q_i^2)} e^{-S_{np}(b_t^2, Q_f^2)}$$

$$C_{g/p}(x, Q_i) = \delta_{gp} \delta(1-x) + \sum_{k=1}^{\infty} \sum_{p=g, q, \bar{q}} C_{g/p}^k(x) \left(\frac{\alpha_s(Q_i)}{\pi} \right)^k.$$

$$f_1^g(x, Q_i^2) = f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi} (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} + \mathcal{O}(\alpha_s^2).$$

$$S_A(b_t^2, Q_f^2, Q_i^2) = \frac{C_A}{\pi} \alpha_s \left(\frac{1}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11 - 2n_f/C_A}{6} \log \frac{Q_f^2}{Q_i^2} \right).$$

TMD Evolution

- suppress high b_t (non perturbative region).
- functional form [Scarpa\(2020\)](#),
- $A = 2.3 \text{ GeV}^2$.
- b_t range to $0.0 - 1.5 \text{ GeV}^{-1}$ [C.-P. Yuan \(2003\)](#) and [Scarpa\(2020\)](#).

$$S_{np} = \frac{A}{2} \log \left(\frac{Q_f}{Q_{np}} \right) b_c^2,$$

$$Q_{np} = 1 \text{ GeV},$$

$$b_c = \sqrt{b_t^2 + \left(\frac{2e^{-\gamma_E}}{Q_f} \right)^2}.$$

$$f_1^g(x, q_t^2) = \frac{1}{2\pi} \int_0^\infty b_t d b_t J_0(b_t q_t) \left\{ f_1^g(x, Q_f^2) \right. \\ \left. - \frac{\alpha_s}{2\pi} \left[\left(\frac{C_A}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6} \log \frac{Q_f^2}{Q_i^2} \right) f_1^g(x, Q_f^2) \right. \right. \\ \left. \left. + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2) \right] \right\} \times e^{-S_{np}(b_t^2)}.$$

$$\frac{q_t^2}{M_p^2} h_1^{\perp g(2)}(x, q_t^2) = \frac{\alpha_s}{\pi^2} \int_0^\infty d b_t b_t J_2(q_t b_t) \left[C_A \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^g(\hat{x}, Q_f^2) \right. \\ \left. + C_F \sum_{p=q, \bar{q}} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^p(\hat{x}, Q_f^2) \right] \times e^{-S_{np}(b_t^2)}.$$

Kinematical cuts

- results for NRQCD (CS and CO) and CSM (CS)
- $M_c = 1.3 \text{ GeV}$ and $M_\psi = 3.1 \text{ GeV}$
- $Q^2 = M_\psi^2 + K_t^2$, $\sqrt{s} = 140 \text{ GeV}$
- This kinematical region is accessible at EIC
- $z \rightarrow 1$ limit the final state gluon becomes soft.
- $z \rightarrow 0$ the final gluon will fragment to form J/ψ
- $0.1 < z < 0.9$

$\cos 2\phi_t$ Asymmetry in Gaussian Parameterization

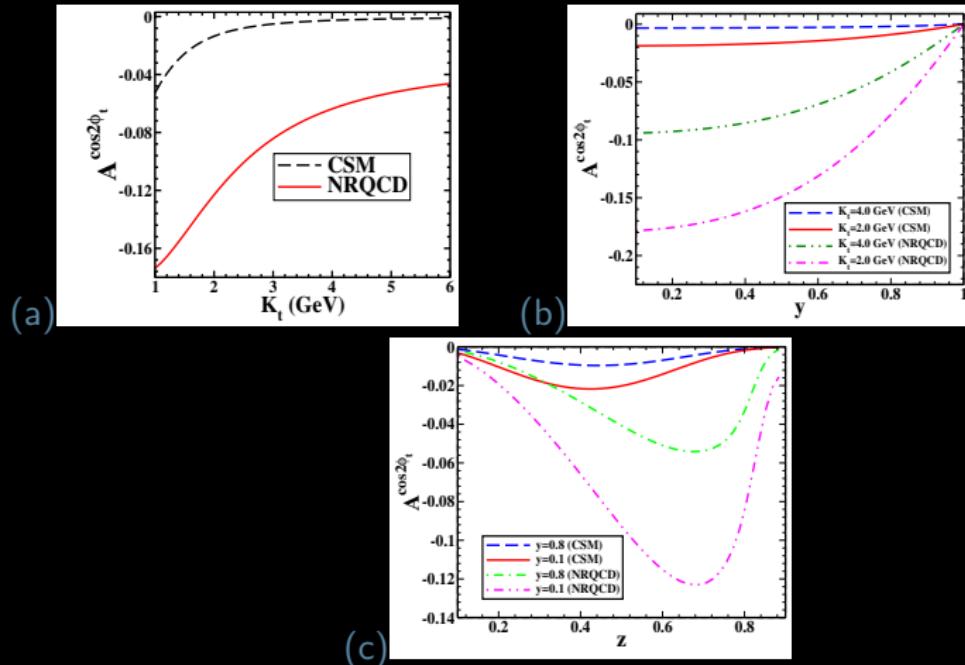


Figure 1: $\cos 2\phi_t$ asymmetry calculated using Gaussian parameterization of TMDs for $e + p \rightarrow e + J/\psi + Jet + X$ process, as functions of (a) K_t , (b) y and (c) z . We have used $\sqrt{s} = 140$ GeV. In (a) and (b) we have used $z = 0.7$. In (a) we have taken $0.1 \leq y \leq 1$ for the range of y integration and in (b) we have used fixed values of K_t . In (c) we have taken $K_t = 3$ GeV and fixed values of y . We have used CMSWZ set of LDMEs [Phys.Rev.D 106,034009](#).

$\cos 2\phi_t$ Asymmetry in Spectator model

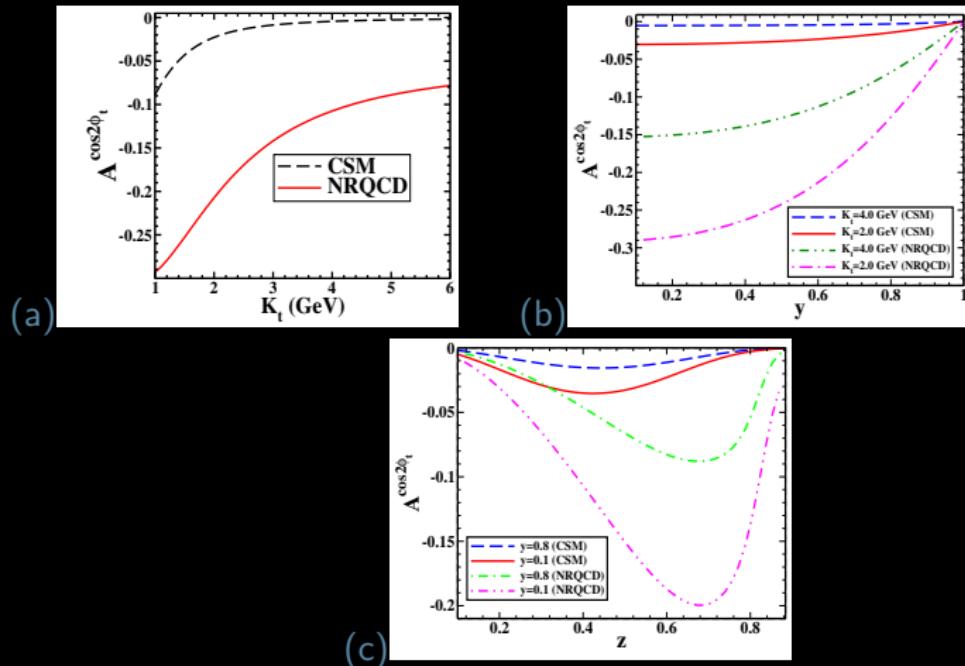


Figure 2: $\cos 2\phi_t$ asymmetry calculated in the spectator model for $e + p \rightarrow e + J/\psi + \text{Jet} + X$ process, in both NRQCD and in the CS; as functions of (a) K_t , (b) y and (c) z . We have used $\sqrt{s} = 140$ GeV. In (a) and (b) we have used $z = 0.7$. In (a) we have taken $0.1 \leq y \leq 1$ for the range of y integration and in (b) we have used fixed values of K_t . In (c) we have taken $K_t = 3$ GeV and fixed values of y . We have used CMSWZ set of LDMEs. [Phys.Rev.D 106,034009](#).

$\cos 2\phi_t$ Asymmetry in TMD Evolution

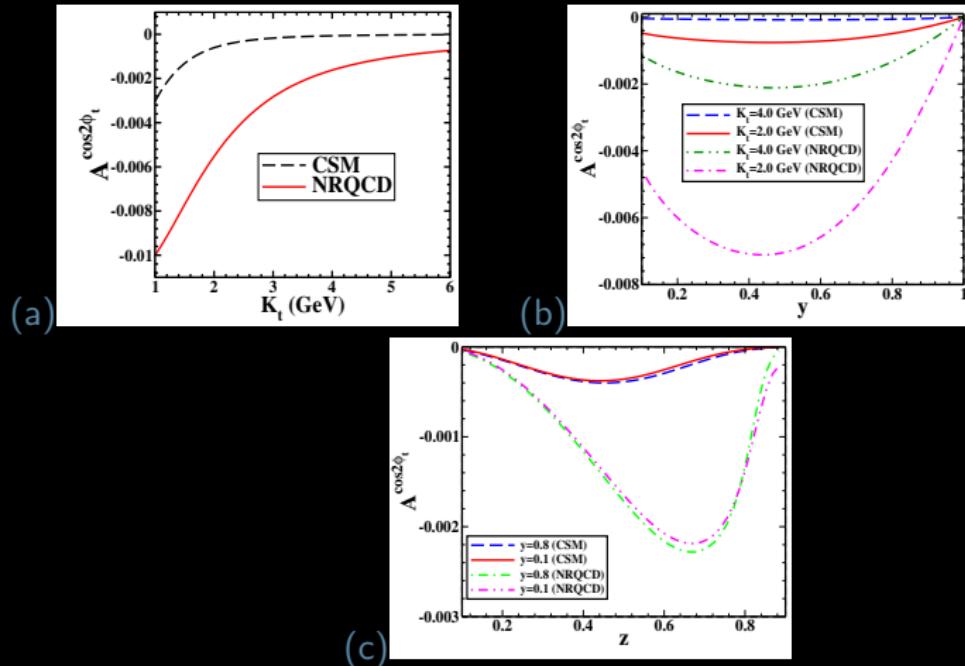


Figure 3: $\cos 2\phi_t$ asymmetry calculated in TMD evolution approach for $e + p \rightarrow e + J/\psi + Jet + X$, in both NRQCD and in the CS; as functions of (a) K_t , (b) y and (c) z . We have used $\sqrt{s} = 140$ GeV. In (a) and (b) we have used $z = 0.7$. In (a) we have taken $0.1 \leq y \leq 1$ for the range of y integration and in (b) we have used fixed values of K_t . In (c) we have taken $K_t = 3$ GeV and fixed values of y . We have used CMSWZ set of LDMEs. [Phys.Rev.D 106,034009](#) .

Upperbound

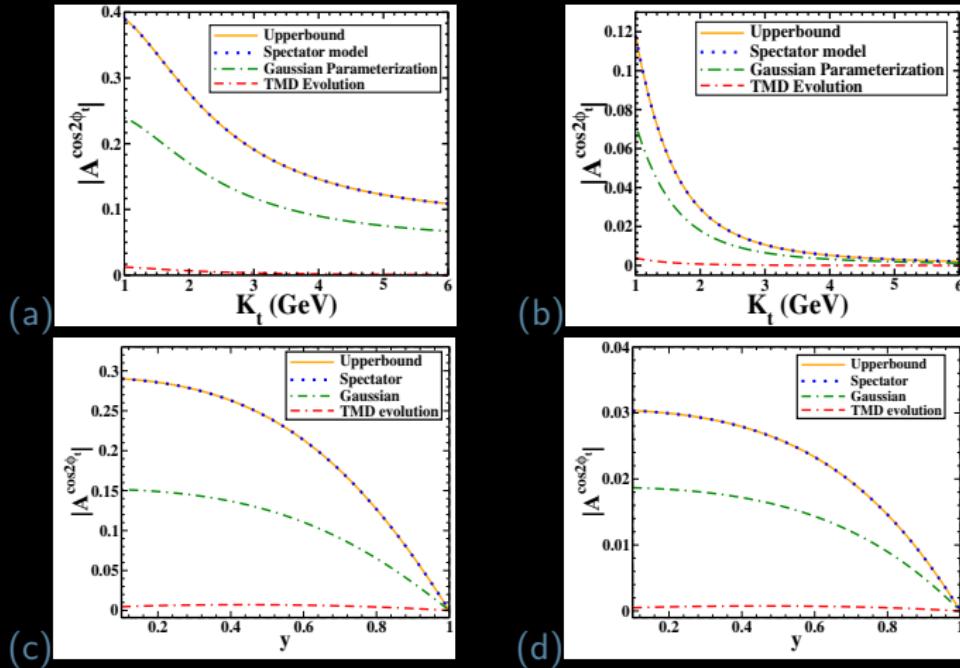


Figure 4: Upper bound of the asymmetry compared with the absolute values of $A^{\cos 2\phi_t}$ calculated in spectator model, Gaussian model and TMD evolution, respectively, for $e^- + P \rightarrow e^- + J/\psi + \text{Jet} + X$ at $\sqrt{s} = 140$. Left panel shows the asymmetry in NRQCD and right panel in CS. We have taken $y = 0.3$ in the upper panel ((a) and (b)) and $K_t = 2$ GeV in the lower panel ((c) and (d)). [Phys.Rev.D 106,034009](#).

Comparison of two LDME sets

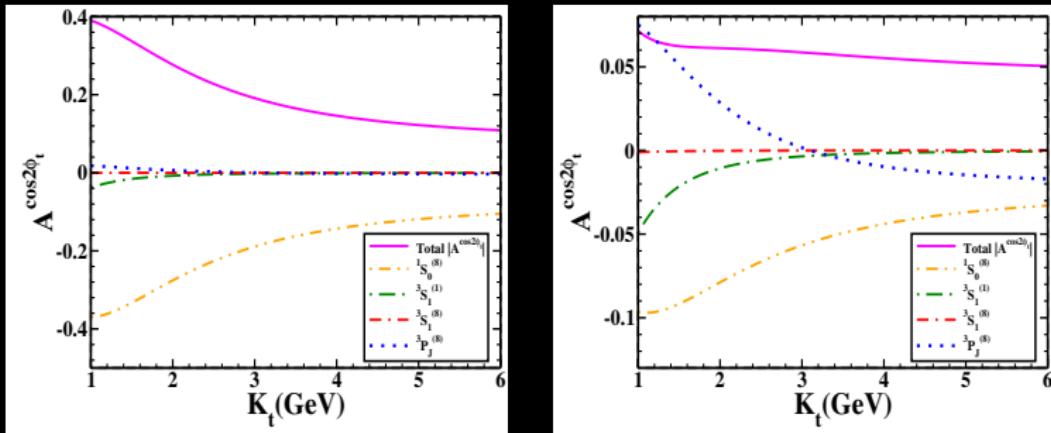


Figure 5: Contribution to the upper bound of the asymmetry coming from individual states, as a function of K_t at $\sqrt{s} = 140$ GeV, $Q = \sqrt{M_\psi^2 + K_t^2}$ and $y = 0.3$; (a) using the CMSWZ set of LDMEs (Chao (2012)) and (b) using the SV set of LDMEs (Sharma (2013)) Phys.Rev.D 106,034009 .

Conclusion

- NRQCD $\Rightarrow J/\psi$ and Jet production rate
- parameterized gluon TMDs \rightarrow Gaussian, Spectator, TMD Evolution.
- $\cos 2\phi_t$ Asymmetry: TMD evolution < Gaussian < Spectator \leq Upperbound.
- These kinematical regions will be accessible at EIC
- These results will help us to extract the Linearly polarized gluon TMD

Thank you

Coefficient functions and perturbative Sudakov factor

- P_{gg} and $P_{g\bar{q}}$ are leading order splitting functions which are given as,

$$P_{gg}(\hat{x}) = 2C_A \left[\frac{\hat{x}}{(1-\hat{x})_+} + \frac{1-\hat{x}}{\hat{x}} + \hat{x}(1-\hat{x}) \right] + \delta(1-\hat{x}) \frac{11C_A - 4n_f T_R}{6},$$
$$P_{gq}(\hat{x}) = P_{g\bar{q}}(\hat{x}) = C_F \frac{1 + (1-\hat{x})^2}{\hat{x}} .$$

$$C_F = (N_c^2 - 1)/2N_c \text{ and } T_R = 1/2.$$

- In P_{gg} , the first term involves the plus prescription and thus avoids an infrared divergence because of $(1-\hat{x})$ in denominator. The plus prescription is given as,

$$\int_y^1 dz \frac{G(z)}{(1-z)_+} = \int_y^1 dz \frac{G(z) - G(1)}{1-z} - G(1) \log \left(\frac{1}{1-z} \right).$$

- The \otimes symbol denotes convolution of the two quantities;

$$(P_{gg} \otimes f_1^g)(x, Q^2) = \int_x^1 \frac{d\hat{x}}{\hat{x}} P(\hat{x}, Q^2) f\left(\frac{x}{\hat{x}}, Q^2\right).$$

- After substituting all this, we end up with the equation for the unpolarized gluon TMD in parameter space

Coefficient functions and perturbative Sudakov factor

$$\hat{f}_1^g(x, b_t^2; Q_f^2) = \frac{1}{2\pi} \left\{ f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi} \left[\left(\frac{C_A}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6} \log \frac{Q_f^2}{Q_i^2} \right) f_1^g(x, Q_f^2) \right. \right. \\ \left. \left. + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2) \right] \right\}.$$

- Now we convert the TMD from parameter space to transverse momentum space using

$$\hat{f}^{(n)}(x, b_t^2) \equiv \frac{2\pi n!}{M^{2n}} \int_0^\infty d q_t q_t \left(\frac{q_t}{b_t} \right)^n J_n(q_t b_t) f(x, q_t^2)$$

[Tom Van Daal \(2016\)](#)

- Final expression for TMD evolution of unpolarized gluon TMD is given as,

$$f_1^g(x, q_t^2) = \frac{1}{2\pi} \int_0^\infty b_t db_t J_0(b_t q_t) \left\{ f_1^g(x, Q_f^2) \right. \\ \left. - \frac{\alpha_s}{2\pi} \left[\left(\frac{C_A}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6} \log \frac{Q_f^2}{Q_i^2} \right) f_1^g(x, Q_f^2) \right. \right. \\ \left. \left. + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2) \right] \right\} \times e^{-S_{np}(b_t^2)}.$$

Coefficient functions and perturbative Sudakov factor

- The $C_{g/p}^{(0)}$ coefficient is 0 for $h_1^{\perp g}(x, b_t^2)$. D. Boer (2020)
- $C_{g/p}^{(1)}$ for $h_1^{\perp g}(x, q_t^2)$ are given as $C_{g/g}^{(1)} = C_A \left(\frac{\hat{x}}{x} - 1 \right)$ and $C_{g/p}^{(1)} = C_F \left(\frac{\hat{x}}{x} - 1 \right)$
- using the $C_{g/p}^{(1)}$ and general equation for TMDs using CS evolution equation we end up with final equation for $h_1^{\perp g}(x, q_t^2)$

$$\frac{q_t^2}{M_p^2} h_1^{\perp g(2)}(x, q_t^2) = \frac{\alpha_s}{\pi^2} \int_0^\infty d b_t b_t J_2(q_t b_t) \left[C_A \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^g(\hat{x}, Q_f^2) + C_F \sum_{p=q,\bar{q}} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^p(\hat{x}, Q_f^2) \right] \times e^{-S_{np}(b_t^2)}.$$

Matrix element for J/ψ and jet production

$$\begin{aligned} \mathcal{M}\left(\gamma^* g \rightarrow Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g\right) &= \sum_{L_z S_z} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}) \langle LL_z; SS_z | JJ_z \rangle \\ &\times Tr[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \end{aligned}$$

- expand $\mathcal{M}\left(\gamma^* g \rightarrow Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g\right)$ as series expansion of k
- zeroth order term is S wave scattering amplitude
first order terms in k is P wave scattering amplitude

$$\begin{aligned} \mathcal{M}[^{2S+1}S_J^{(1,8)}] &= \frac{1}{\sqrt{4\pi}} R_0(0) Tr[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \Big|_{k=0} \\ \mathcal{M}[^{2S+1}P_J^{(8)}] &= -i \sqrt{\frac{3}{4\pi}} R'_1(0) \sum_{L_z S_z} \varepsilon_{L_z}^\alpha(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \\ &\times \frac{\partial}{\partial k^\alpha} Tr[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \Big|_{k=0} \end{aligned}$$

S and P amplitude

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon_{S_z}^{\alpha}(P_\psi) \varepsilon_{L_z}^{\beta}(P_\psi) = \sqrt{\frac{1}{3}} \left(g^{\alpha\beta} - \frac{P_\psi^\alpha P_\psi^\beta}{M_\psi^2} \right),$$

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon_{S_z}^{\alpha}(P_\psi) \varepsilon_{L_z}^{\beta}(P_\psi) = -\frac{i}{M_\psi} \sqrt{\frac{1}{2}} \epsilon_{\delta\zeta\xi\varrho} g^{\xi\alpha} g^{\varrho\beta} P_\psi^\delta \varepsilon_{J_z}^\zeta(P_\psi),$$

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon_{S_z}^{\alpha}(P_\psi) \varepsilon_{L_z}^{\beta}(P_\psi) = \varepsilon_{J_z}^{\alpha\beta}(P_\psi).$$

$$\varepsilon_{J_z}^{\alpha}(P_\psi) P_{\psi\alpha} = 0,$$

$$\sum_{J_z} \varepsilon_{J_z}^{\alpha}(P_\psi) \varepsilon_{J_z}^{*\beta}(P_\psi) = \left(-g^{\alpha\beta} + \frac{P_\psi^\alpha P_\psi^\beta}{M_\psi^2} \right) = Q^{\alpha\beta}.$$

The $\varepsilon_{J_z}^{\alpha\beta}(P_\psi)$ is polarization tensor corresponding to $J = 2$ which is symmetric in the Lorentz indices and follows the relations ,

$$\varepsilon_{J_z}^{\alpha\beta}(P_\psi) = \varepsilon_{J_z}^{\beta\alpha}(P_\psi) \quad \varepsilon_{J_z\alpha}^{\alpha}(P_\psi) = 0 \quad \varepsilon_{J_z}^{\alpha}(P_\psi) P_{\psi\alpha} = 0$$

$$\varepsilon_{J_z}^{\alpha\beta}(P_\psi) \varepsilon_{J_z}^{*\mu\nu}(P_\psi) = \frac{1}{2} [Q^{\alpha\mu} Q^{\beta\nu} + Q^{\alpha\nu} Q^{\beta\mu}] - \frac{1}{3} [Q^{\alpha\beta} Q^{\mu\nu}]$$

D. Boer and Pisano (2012)

S and P amplitude

S – Wave amplitudes

$$\mathcal{M}[{}^3S_1^{(1)}](P_\psi, p_g) = \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) \frac{\delta_{ab}}{2\sqrt{N_c}} Tr \left[\sum_{i=1}^3 O_i(0) (\not{P}_\psi + M_\psi) \not{\epsilon}_{S_z} \right],$$

$$\mathcal{M}[{}^3S_1^{(8)}](P_\psi, p_g) = \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) \frac{\sqrt{2}}{2} d_{abc} Tr \left[\sum_{i=1}^3 O_i(0) (\not{P}_\psi + M_\psi) \not{\epsilon}_{S_z} \right]$$

$$\begin{aligned} \mathcal{M}[{}^1S_0^{(8)}](P_\psi, p_g) = & \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) i \frac{\sqrt{2}}{2} f_{abc} Tr \left[(O_1(0) - O_2(0) - O_3(0) + 2O_4(0)) \right. \\ & \left. (\not{P}_\psi + M_\psi) \gamma_5 \right] \end{aligned}$$

P – Wave amplitudes

$$\begin{aligned} \mathcal{M}[{}^3P_J^{(8)}](P_\psi, p_g) = & \frac{\sqrt{2}}{2} f_{abc} \sqrt{\frac{3}{4\pi}} R'_1(0) \sum_{L_z S_z} \varepsilon_{L_z}^\alpha(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \\ & Tr[(O_{1\alpha}(0) + O_{6\alpha}(0) - O_{3\alpha}(0) + 2O_{4\alpha}(0)) \mathcal{P}_{SS_z}(0) \\ & + (O_1(0) + O_6(0) - O_3(0) + 2O_4(0)) \mathcal{P}_{SS_z\alpha}(0)]. \end{aligned}$$

Kishore, Mukherjee and Rajesh (2020)

Total differential cross-section

Integrated over the final momenta,

$$\frac{d^3\ell'}{(2\pi)^3 2E_{\ell'}} = \frac{dQ^2 dy}{16\pi^2}.$$

$$\frac{d^3P_\psi}{(2\pi)^3 2E_\psi} = \frac{dz d^2P_{\psi\perp}}{(2\pi)^3 2z}, \quad \frac{d^3P_j}{(2\pi)^3 2E_j} = \frac{d\bar{z} d^2P_{j\perp}}{(2\pi)^3 2\bar{z}}$$

The four-momentum delta function

$$\delta^4(q + p_g - P_\psi - P_j) = \frac{2}{ys} \delta(1 - z - \bar{z}) \delta\left(x - \frac{\bar{z}(M^2 + P_{\psi\perp}^2) + zP_{j\perp}^2 + z\bar{z}Q^2}{z(1-z)ys}\right) \delta^2(\mathbf{p} - \mathbf{P}_{j\perp} - \mathbf{P}_{\psi\perp}),$$

Back to back J/ψ and jet in transverse plane

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \quad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}$$

LDMEs

	$\langle 0 \mathcal{O}_8^{J/\psi} ({}^1S_0) 0 \rangle$	$\langle 0 \mathcal{O}_8^{J/\psi} ({}^3S_1) 0 \rangle$	$\langle 0 \mathcal{O}_1^{J/\psi} ({}^3S_1) 0 \rangle$	$\langle 0 \mathcal{O}_8^{J/\psi} ({}^3P_0) 0 \rangle / m_c^2$
Ref. Sharma (2013)	1.8 ± 0.87 $\times 10^{-2} \text{GeV}^3$	0.13 ± 0.13 $\times 10^{-2} \text{GeV}^3$	1.2×10^2 $\times 10^{-2} \text{GeV}^3$	1.8 ± 0.87 $\times 10^{-2} \text{GeV}^3$
Ref. Chao (2012)	8.9 ± 0.98 $\times 10^{-2} \text{GeV}^3$	0.30 ± 0.12 $\times 10^{-2} \text{GeV}^3$	1.2×10^2 $\times 10^{-2} \text{GeV}^3$	0.56 ± 0.21 $\times 10^{-2} \text{GeV}^3$

Spectator model

Parameter	Replica 11	Parameter	Replica 11
A	6.0	κ_2 (GeV 2)	0.414
a	0.78	σ (GeV)	0.50
b	1.38	Λ_X (GeV)	0.448
C	346	κ_1 (GeV 2)	1.46
D (GeV)	0.548		

all 4-momenta

$$p_g^\mu = xP^\mu + (p_g \cdot P + M^2 x)n^\mu + \mathbf{p}_T^\mu \approx xP^\mu + \mathbf{p}_T^\mu,$$

$$P_\psi^\mu = \frac{\mathbf{P}_{\psi\perp}^2 + M_\psi^2}{2zP \cdot q} P^\mu + z(P \cdot q)n^\mu + \mathbf{P}_{\psi\perp}^\mu,$$

$$P_j^\mu = \frac{\mathbf{P}_{J\perp}^2}{2(1-z)P \cdot q} P^\mu + (1-z)(P \cdot q)n^\mu + \mathbf{P}_{J\perp}^\mu,$$

$$q^\mu = -x_B n_-^\mu + \frac{Q^2}{2x_B} n_+^\mu \approx -x_B P^\mu + (P \cdot q)n_+^\mu,$$

$$l^\mu = \frac{(1-y)x_B}{y} P^\mu + \frac{(P \cdot q)}{y} n^\mu + \frac{\sqrt{1-y}}{y} Q \hat{l}_\perp^\mu$$

$$l'^\mu = l^\mu - q^\mu$$

$$Q^2 = -q^2, \quad s = (P + l)^2$$

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_\psi}{P \cdot q}$$

Quarkonium Production models

- Color Singlet Model:

The initial quark pair should be in same spin, orbital and color state as that of final particle. **Braaten, Fleming, Yuan (1996)**

- Color Evaporation model:

Probability of formation of quarkonium is independent of initial quark pair. **Amundson, Eboli, Georges, Halzen (1997)**

- NRQCD:

Initial quark pair can be color singlet or color octet.
Bodwin, Braaten, Lepage (1994)

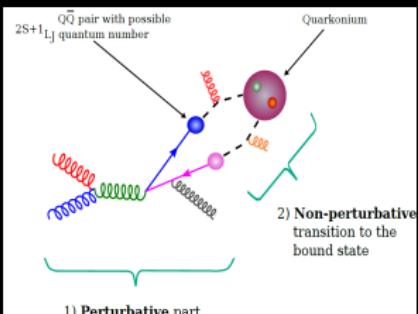


Image: Thesis Rajesh