

On the $Z_{CS}(3985)$ and $X(3960)$ states

Towards HQSS and $SU(3)$ multiplet descriptions



Miguel Albaladejo (IFIC)

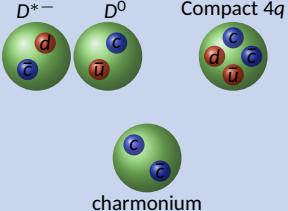
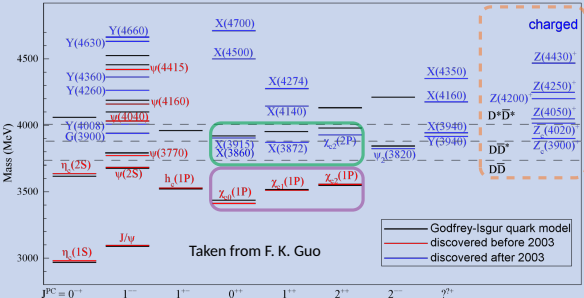
9th International Conference on Quarks and Nuclear Physics. Sept. 6, 2022



Outline

- 1 Introduction
- 2 Interactions
- 3 $X(3960)$
- 4 HQSS & $SU(3)$ multiplets
- 5 $Z_{cs}(3985)$ and $Z_c(3900)$

Quark model in the charmonium sector



- $\chi_{c1}(1P)$ well established, “very CQM model” state.
- $X(3872)$ discovered by Belle [PRL,91,262001('03)] (also 2003!)
 $J^{PC} = 1^{++}$ and $\Gamma \simeq 1$ MeV established by LHCb (e.g. [JHEP,08(2020),123]).
- $\chi_{c2}(2P)$ Not established. Influence of open thresholds? Is $X(3872)$ a molecular states?
- Z_c states have $I = 1$, clearly “tetraquarks” ($c\bar{c}u\bar{d}, \dots$)
- Many theoretical and lattice and experimental works: can't cite them properly here! (many references in [2207.08653])

HQSS and flavour SU(3) LO lagrangian

[Grinstein *et al.*, NP,B380('92); Alfiky *et al.*, PL,B640('06), ...]

- SU(3) light flavour symmetry:

$$H_a^{(Q)} \sim (Q\bar{u}, Q\bar{d}, Q\bar{s}) \sim (D^0, D^+, D_s^+)$$

- HQSS: $H_a^{(Q)} = \frac{1 + \not{v}}{2} (P_{a\mu}^{*(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5)$ (with $v \cdot P_a^{*(Q)} = 0$)

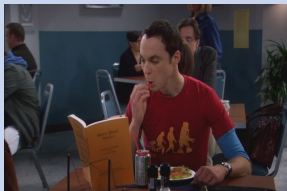
- LO lagrangian (S-wave contact interactions):

$$\begin{aligned} \mathcal{L}_{4H} = & \frac{1}{4} \text{Tr} [\bar{H}^{(Q)a} H_b^{(Q)} \gamma_\mu] \text{Tr} [H^{(\bar{Q})c} \bar{H}_d^{(\bar{Q})} \gamma^\mu] (F_A \delta_a^b \delta_c^d + F_A^\lambda \vec{\lambda}_a^b \cdot \vec{\lambda}_c^d) \\ & + \frac{1}{4} \text{Tr} [\bar{H}^{(Q)a} H_b^{(Q)} \gamma_\mu \gamma_5] \text{Tr} [H^{(\bar{Q})c} \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5] (F_B \delta_a^b \delta_c^d + F_B^\lambda \vec{\lambda}_a^b \cdot \vec{\lambda}_c^d), \end{aligned}$$

- **Only 4 constants**, many ways to write the constants.

$$C_{0a} = F_A + \frac{10F_A^\lambda}{3}, \quad C_{1a} = F_A - \frac{2}{3}F_A^\lambda,$$

$$C_{0b} = F_B + \frac{10F_B^\lambda}{3}, \quad C_{1b} = F_B - \frac{2}{3}F_B^\lambda.$$



$D_s^+ D_s^-$ interaction and $B^+ \rightarrow D_s^+ D_s^- K^+$ decay [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

Scattering amplitude: $T(E) = \frac{V}{1 - VG(E)}$

$V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2}$

G loop functions, once-subtracted, $G(E_{th}) = G_{\Lambda}(E_{th})$

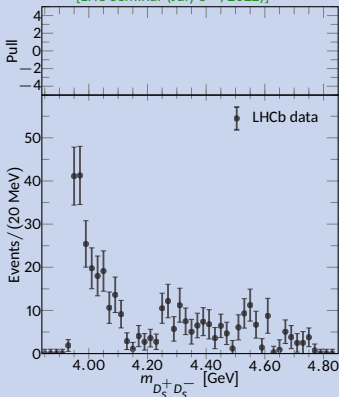
Simple production model:

$$T_B(E) = P + PG(E)T(E) = P \frac{1}{1 - VG(E)}$$

$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3} \frac{kp}{4m_B^2} |T_B(E)|^2$

[LHCb-PAPER-2022-018,019]

[LHC Seminar (July 5th, 2022)]



$D_s^+ D_s^-$ interaction and $B^+ \rightarrow D_s^+ D_s^- K^+$ decay [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- Scattering amplitude: $T(E) = \frac{V}{1 - VG(E)}$
 - $V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2}$
 - G loop functions, once-subtracted, $G(E_{th}) = G_{\Lambda}(E_{th})$

- Simple production model:

$$T_B(E) = P + PG(E)T(E) = P \frac{1}{1 - VG(E)}$$

$$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3} \frac{kp}{4m_B^2} |T_B(E)|^2$$

- Fit: two solutions (virtual or bound), in both:

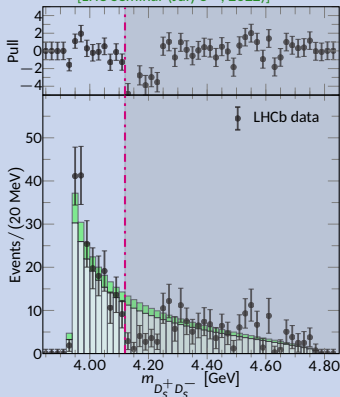
$$M_{X(3960)} = 3928(3) \text{ MeV}$$

$$2M_{D_s} - M_{X(3960)} = 8(3) \text{ MeV}$$

- LHCb: $M = 3956(5)(11) \text{ MeV}$, $\Gamma = 43(13)(7) \text{ MeV}$

- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8} \text{ MeV}$
(cf. also [Bayar, Feijoo, Oset, 2207.08490])

[LHCb-PAPER-2022-018,019]

[LHC Seminar (July 5th, 2022)]

	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1.0 \text{ GeV}$	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1.0 \text{ GeV}$
$C_{D_s^+ D_s^-} (\text{fm}^2)$	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

$D_s^+ D_s^-$ interaction and $B^+ \rightarrow D_s^+ D_s^- K^+$ decay [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- Scattering amplitude: $T(E) = \frac{V}{1 - VG(E)}$
 - $V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2}$
 - G loop functions, once-subtracted, $G(E_{th}) = G_\Lambda(E_{th})$

- Simple production model:

$$T_B(E) = P + PG(E)T(E) = P \frac{1}{1 - VG(E)}$$

$$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3} \frac{kp}{4m_B^2} |T_B(E)|^2$$

- Fit: two solutions (virtual or bound), in both:

$$M_{X(3960)} = 3928(3) \text{ MeV}$$

$$2M_{D_s} - M_{X(3960)} = 8(3) \text{ MeV}$$

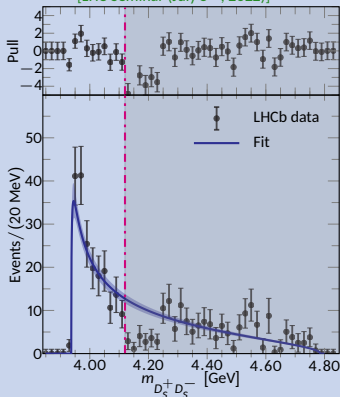
- LHCb: $M = 3956(5)(11) \text{ MeV}$, $\Gamma = 43(13)(7) \text{ MeV}$

- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8} \text{ MeV}$

(cf. also [Bayar, Feijoo, Oset, 2207.08490])

[LHCb-PAPER-2022-018,019]

[LHC Seminar (July 5th, 2022)]



	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1.0 \text{ GeV}$	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1.0 \text{ GeV}$
$C_{D_s^+ D_s^-} (\text{fm}^2)$	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

Fixing all constants [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- $X(3960)$: fixes $C_{D_s \bar{D}_s} = (C_{0a} + C_{1a})/2$ (as previously seen)
- $Z_C(3900)$: fixes $V = C_{1a} - C_{1b}$
 - Assume virtual state $M = 3813_{-21}^{+28}$ MeV ([2201.08253; 1512.03638] from a fit to BESIII data)

- $X(3872)$: fixes $\begin{cases} C_{0X} = (C_{0a} + C_{0b})/2, \\ C_{1X} = (C_{1a} + C_{1b})/2 \end{cases}$

- Experimental information:

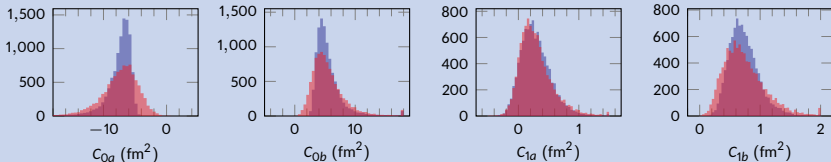
$$\begin{cases} \text{[LHCb, 2204.12597]} & R_{X(3872)}^{\text{exp}} = 0.29(4) \\ \text{[LHCb, PR,D 102,092005('20)]} & B_{X(3872)}^{\text{exp}} = [-150, 0] \text{ keV} \leftarrow M_{X(3872)}^{\text{exp}} = 3871.69_{-0.04}^{+0.00} {}_{-0.13}^{+0.05} \text{ MeV} \end{cases}$$

- Theoretically: [0911.4407; 1210.5431; 1504.00861]

$$V = \frac{1}{2} \begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}, \quad T = (\mathbb{I} - VG)^{-1}V.$$

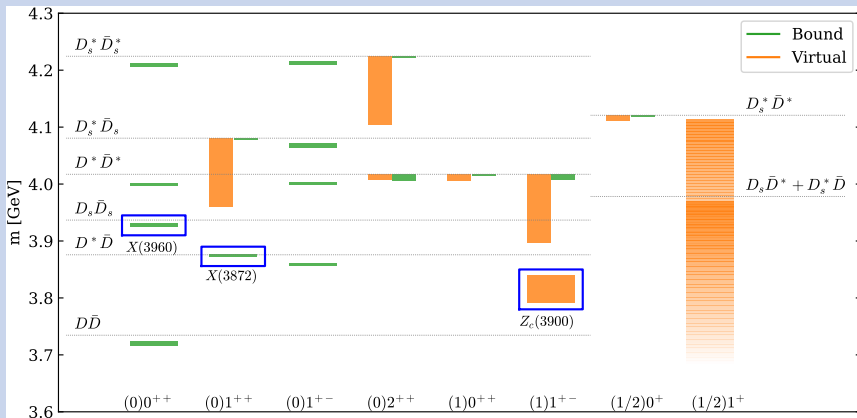
$$R_{X(3872)} = \frac{\hat{\Psi}_n - \hat{\Psi}_c}{\hat{\Psi}_n + \hat{\Psi}_c}, \quad \frac{\hat{\Psi}_n}{\hat{\Psi}_c} = \frac{1 - (2m_{D^+} + m_{D^{*-}}) G_2 (C_{0X} + C_{1X})}{(2m_{D^+} + m_{D^{*-}}) G_2 (C_{0X} - C_{1X})} = \frac{(2m_{D^0} m_{D^{*0}}) G_1 (C_{0X} - C_{1X})}{1 - (2m_{D^0} m_{D^{*0}}) G_1 (C_{0X} + C_{1X})},$$

- SU(3) and HQSS **breaking corrections** (30%) are taken into account for the LECs



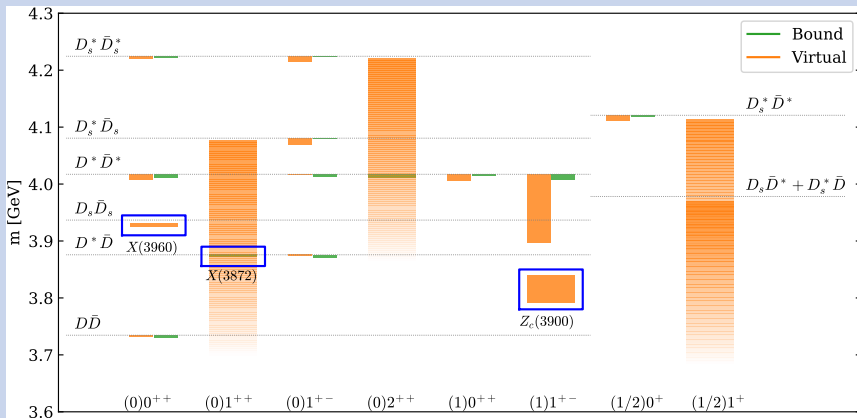
Predictions (complete multiplet): $X(3960)$ as bound [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

With the **four LECs** (C_{0a} , C_{0b} , C_{1a} , C_{1b}) fixed, poles can be looked for in every sector.



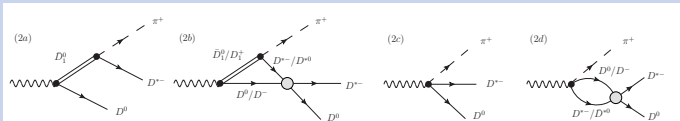
Predictions (complete multiplet): $X(3960)$ as virtual [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

With the **four LECs** (C_{0a} , C_{0b} , C_{1a} , C_{1b}) fixed, poles can be looked for in every sector.



A closer look to $Z_C(3900)$ and $Z_{CS}(3985)$ [Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\langle D\bar{D}^*; 1(1^{+-}) | \hat{T} | D\bar{D}^*; 1(1^{+-}) \rangle = \langle D_S\bar{D}^*; \frac{1}{2}(1^+) | \hat{T} | D_S\bar{D}^*; \frac{1}{2}(1^+) \rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach by:
 - ① Coupled channels: $\begin{cases} l = 1 & \left(\frac{1}{\sqrt{2}} [D\bar{D}^* - D^*\bar{D}], J/\psi \pi \right) \\ l = \frac{1}{2} & \left(\frac{1}{\sqrt{2}} [D_S\bar{D}^* - D_S^*\bar{D}], J/\psi K \right) \end{cases}$ (also necessary for $e^+e^- \rightarrow J/\psi \pi \pi$ data)
 - ② Energy dependence: $C_{1Z} \rightarrow C_{1Z} + b \frac{s - E_{\text{th}}^2}{2E_{\text{th}}}$
- Production mechanism for $Y \rightarrow D^0 \bar{D}^{*-} \pi^+$



$$|\overline{\mathcal{A}}_2(s, t)|^2 = \left| \frac{1}{t - m_{D_1}^2} + l(s)T_{22}(s) \right|^2 q_\pi^4(s) + \frac{1}{2} \left| E_\pi(s) \frac{h_S}{h_D} \left[\frac{1}{t - m_{D_1}^2} + l(s)T_{22}(s) \right] + \beta [1 + G_2(s)T_{22}(s)] \right|^2$$

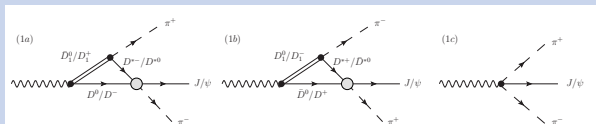
A closer look to $Z_C(3900)$ and $Z_{CS}(3985)$ [Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\langle D\bar{D}^*; 1(1^{+-}) | \hat{T} | D\bar{D}^*; 1(1^{+-}) \rangle = \langle D_S\bar{D}^*; \frac{1}{2}(1^+) | \hat{T} | D_S\bar{D}^*; \frac{1}{2}(1^+) \rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach by:

- 1 Coupled channels: $\begin{cases} l = 1 & \left(\frac{1}{\sqrt{2}} [D\bar{D}^* - D^*\bar{D}], J/\psi \pi \right) \\ l = \frac{1}{2} & \left(\frac{1}{\sqrt{2}} [D_S\bar{D}^* - D_S^*\bar{D}], J/\psi K \right) \end{cases}$ (also necessary for $e^+e^- \rightarrow J/\psi \pi \pi$ data)

- 2 Energy dependence: $C_{1Z} \rightarrow C_{1Z} + b \frac{s - E_{\text{th}}^2}{2E_{\text{th}}}$

- Production mechanism for $Y \rightarrow J/\psi \pi^+ \pi^-$



$$\overline{|\mathcal{A}_1(s, t)|^2} = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3 \cos^2 \theta - 1}{2} [\tau(s)\tau(t)^* + \tau(s)^*\tau(t)] q_\pi^2(s) q_\pi^2(t) \\ + \frac{1}{2} \left\{ |\tau'(s)|^2 E_\pi^2(s) + |\tau'(t)|^2 E_\pi^2(t) + [\tau'(s)^*\tau'(t) + \tau'(s)\tau'(t)^*] E_\pi(s) E_\pi(t) \right\}$$

$$\tau(s) = \sqrt{2l(s)} T_{12}(s) + \alpha \quad \tau'(s) = \frac{hs}{h_D} \sqrt{2l(s)} T_{12}(s)$$

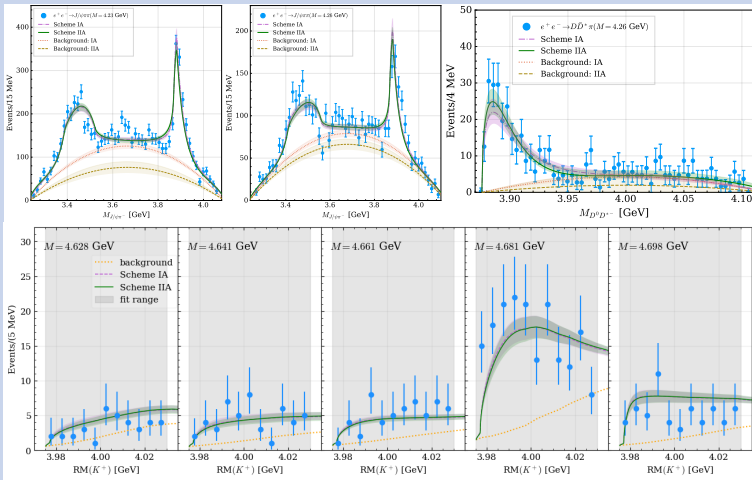


Fit to data [Du, MA, Guo, Nieves, PR,D105,074018('22)]

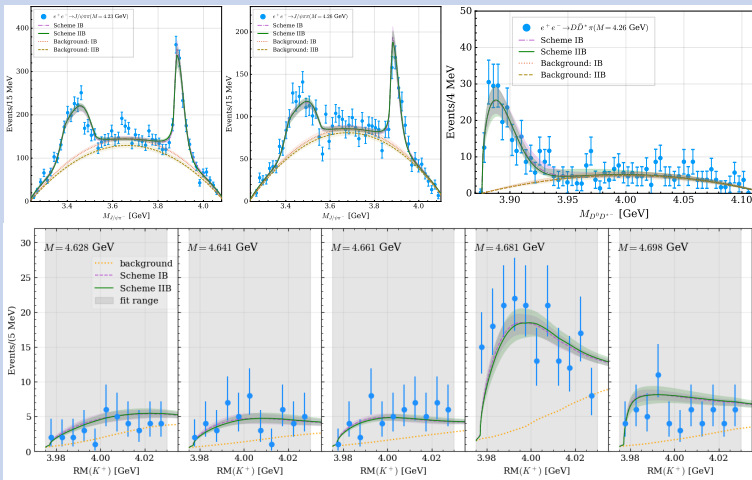
- Fitted data:
 - $J/\psi\pi^-$ distribution in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ [BESIII, PRL,119('17)]
 - D^0D^{*-} distribution in $e^+e^- \rightarrow D^0D^{*-}\pi^+$ [BESIII,PR, D92('15)]
 - $e^+e^- \rightarrow (D^{*0}D_s^- + D^0D_s^{*-})K^+$ [BESIII, PRL,126('21)]
- Some production/background/normalization constants are also fitted (not shown here)
- Four schemes:
 - A or B: $b = 0$ or free
 - I or II: $h_s = 0$ or not (D- or S- and D-waves)

Scheme	$D_1D^*\pi$	$a_2(\mu)$	χ^2/dof	$C_{12} [\text{fm}^2]$	$C_Z [\text{fm}^2]$	$b [\text{fm}^3]$
IA	D	-2.5	1.62	0.005(1)	-0.226(10)	0*
		-3.0	1.62	0.005(1)	-0.177(6)	0*
IIA	S+D	-2.5	1.83	0.006(1)	-0.217(10)	0*
		-3.0	1.83	0.006(1)	-0.171(6)	0*
IB	D	-2.5	1.24	0.007(4)	-0.222(6)	-0.447(44)
		-3.0	1.21	0.008(1)	-0.177(4)	-0.255(30)
IIB	S+D	-2.5	1.37	0.005(1)	-0.203(7)	-0.473(45)
		-3.0	1.27	0.005(1)	-0.171(5)	-0.270(30)

Fit to data (scheme A) [Du, MA, Guo, Nieves, PR,D105,074018('22)]



Fit to data (scheme B) [Du, MA, Guo, Nieves, PR,D105,074018('22)]



$Z_c^{(*)}$ and $Z_{cs}^{(*)}$ poles [Du, MA, Guo, Nieves, PR,D105,074018('22)]

	$D_1 D^* \pi$	$a_2(\mu)$	Z_c [MeV]		Z_{cs} [MeV]		Z_c^* [MeV]		Z_{cs}^* [MeV]	
			Mass	$\Gamma/2$	Mass	$\Gamma/2$	Mass	$\Gamma/2$	Mass	$\Gamma/2$
IA	D	-2.5	3813^{+21}_{-28}	vir.	3920^{+18}_{-26}	vir.	3962^{+19}_{-25}	vir.	4069^{+12}_{-16}	vir.
		-3.0	3812^{+22}_{-26}	vir.	3924^{+19}_{-23}	vir.	3967^{+19}_{-22}	vir.	4078^{+17}_{-13}	vir.
IIA	S+D	-2.5	3799^{+24}_{-33}	vir.	3907^{+22}_{-31}	vir.	3949^{+22}_{-30}	vir.	4057^{+20}_{-28}	vir.
		-3.0	3798^{+25}_{-31}	vir.	3911^{+17}_{-27}	vir.	3955^{+22}_{-27}	vir.	4067^{+19}_{-25}	vir.
IB	D	-2.5	3897^{+4}_{-4}	37^{+8}_{-6}	3996^{+4}_{-4}	37^{+8}_{-6}	4035^{+4}_{-4}	37^{+8}_{-6}	4137^{+4}_{-4}	36^{+7}_{-6}
		-3.0	3898^{+5}_{-5}	38^{+10}_{-7}	3996^{+5}_{-6}	35^{+9}_{-6}	4035^{+4}_{-5}	34^{+9}_{-6}	4136^{+5}_{-6}	33^{+8}_{-6}
IIB	S+D	-2.5	3902^{+6}_{-6}	38^{+9}_{-6}	4002^{+6}_{-6}	38^{+9}_{-7}	4042^{+5}_{-5}	38^{+9}_{-7}	4144^{+5}_{-6}	37^{+9}_{-7}
		-3.0	3902^{+5}_{-5}	37^{+9}_{-6}	4000^{+5}_{-6}	35^{+8}_{-7}	4039^{+5}_{-6}	35^{+8}_{-6}	4140^{+5}_{-6}	33^{+8}_{-6}

- Z_c exp.:
 $M = 3881.2(4.2)(52.7)$ MeV
 $\Gamma/2 = 25.9(2.3)(18.0)$ MeV

- Z_{cs} exp.:
 $M = 3982.5(2.6)(2.1)$ MeV
 $\Gamma/2 = 6.4(2.6)(1.5)$ MeV

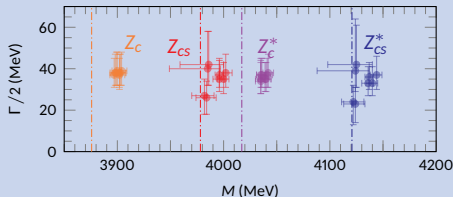
- Both schemes (A and B)

- Including $SU(3)$ breaking effects

- [Yang et al., PR,D103('21): Z_{cs} and distribution]

- [Ikeno, Molina, Oset, PL,B814('21)] (Z_{cs} threshold effect)

- [JPAC, PL,B772('17)] Several possibilities for Z_c





Summary and conclusions

- We have considered $D_{(s)}^{(*)} \bar{D}_{(s)}^{(*)}$ interactions with **HQSS** and **SU(3) light-flavour symmetry**.
- The $X(3960)$ structure in LHCb data on $B^+ \rightarrow D_s^+ D_s^- K^+$ can be explained with a **bound or virtual** state.
- The experimental information coming from $X(3960)$, $X(3872)$, and (a virtual) $Z_C(3900)$ allows to fix the **four constants** appearing in the LO lagrangian.
- Predictions are made based on these constants for **multiplet partners** of these states in other sectors
- Considering a generalization of the interactions, the BESIII data for the Z_C and Z_{CS} states can be well reproduced, being **Z_C and Z_{CS} flavour partners**.