On the $Z_{cs}(3985)$ and $X(3960)$ states

Towards HQSS and $SU(3)$ multiplet descriptions

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Outline

1. Introduction
2. Interactions
3. $X(3960)$
4. HQSS & $SU(3)$ multiplets
5. $Z_{cS}(3985)$ and $Z_c(3900)$
Quark model in the charmonium sector

- $\chi_{cJ}(1P)$ well established, “very CQM model” state.
- $X(3872)$ discovered by Belle [PRL, 91, 262001 (2003)] (also 2003!)
  $J^{PC} = 1^{++}$ and $\Gamma \simeq 1$ MeV established by LHCb (e.g. [JHEP, 08(2020), 123]).
- $\chi_{cJ}(2P)$ Not established. Influence of open thresholds? Is $X(3872)$ a molecular states?
- $Z_c$ states have $I = 1$, clearly “tetraquarks” ($c\bar{c}u\bar{d}$, …)
- Many theoretical and lattice and experimental works: can’t cite them properly here! (many references in [2207.08653])
The HQSS and flavour $SU(3)$ LO lagrangian

$SU(3)$ light flavour symmetry:

$$H_a^{(Q)} \sim (Q\bar{u}, Q\bar{d}, Q\bar{s}) \sim (D^0, D^+, D^+_s)$$

HQSS: $H_a^{(Q)} = \frac{1 + \gamma^\mu}{2} \left( P_{a\mu}^{*(Q)} \gamma^\mu - P_{a\mu}^{(Q)} \gamma_5 \right)$ (with $v \cdot P_{a\mu}^{*(Q)} = 0$)

LO lagrangian (S-wave contact interactions):

$$\mathcal{L}_{4H} = \frac{1}{4} \mathrm{Tr} \left[ \bar{H}^{(Q)a} a H^{(Q)b} \gamma_\mu \right] \mathrm{Tr} \left[ H^{(\bar{Q})c} \bar{H}^{(Q)d} \gamma^\mu \right] \left( F_A \delta_a^b \delta_c^d + F_A^\lambda \bar{\lambda}_a^b \cdot \bar{\lambda}_c^d \right)$$

$$+ \frac{1}{4} \mathrm{Tr} \left[ \bar{H}^{(Q)a} a H^{(Q)b} \gamma_\mu \gamma_5 \right] \mathrm{Tr} \left[ H^{(\bar{Q})c} \bar{H}^{(Q)d} \gamma^\mu \gamma_5 \right] \left( F_B \delta_a^b \delta_c^d + F_B^\lambda \bar{\lambda}_a^b \cdot \bar{\lambda}_c^d \right),$$

Only 4 constants, many ways to write the constants.

$$C_{0a} = F_A + \frac{10 F_A^\lambda}{3}, \quad C_{1a} = F_A - \frac{2}{3} F_A^\lambda,$$

$$C_{0b} = F_B + \frac{10 F_B^\lambda}{3}, \quad C_{1b} = F_B - \frac{2}{3} F_B^\lambda.$$
**D^+_s D^-_s** interaction and \( B^+ \rightarrow D^+_s D^-_s K^+ \) decay \([\text{Ji, Dong, MA, Du, Guo, Nieves, 2207.08563}]\)

- Scattering amplitude: \( T(E) = \frac{V}{1 - V G(E)} \)
  
  - \( V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2} \)
  
  - \( G \) loop functions, once-subtracted, \( G(E_{th}) = G_{\Lambda}(E_{th}) \)
- Simple production model:
  
  \[
  T_B(E) = P + P G(E) T(E) = P \frac{1}{1 - V G(E)}
  \]

\[
\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3} \frac{k p}{4m_B^2} |T_B(E)|^2
\]

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**Figure:**

- Pull graph
- LHCb data

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**Equation:**

- \( \frac{d\Gamma}{dE} \) vs. \( m_{D^+_s D^-_s} \)

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**References:**

- \([\text{LHCb-PAPER-2022-018,019}]\)
- \([\text{LHC Seminar (July 5th, 2022)}]\)
**D^+_s D^-_s** interaction and **B^+ \rightarrow D^+_s D^-_s K^+** decay [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- Scattering amplitude: \( T(E) = \frac{V}{1 - V G(E)} \)
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- Simple production model:
  \( T_B(E) = P + P G(E) T(E) = P \frac{1}{1 - V G(E)} \)

- Fit: two solutions (virtual or bound), in both:
  \( M_{X(3960)} = 3928(3) \) MeV
  \( 2M_{D_s} - M_{X(3960)} = 8(3) \) MeV

- LHCb: \( M = 3956(5)(11) \) MeV, \( \Gamma = 43(13)(7) \) MeV

[Prelovsek et al., JHEP 06,035('20)]: Bound state \( B = 6.2_{-3.8}^{+2.0} \) MeV

(cf. also [Bayar, Feijoo, Oset, 2207.08490])

### Pull

<table>
<thead>
<tr>
<th>( \Lambda ) (GeV)</th>
<th>Vir. (S-I)</th>
<th>Bou. (S-II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>(-0.74_{-0.04}^{+0.04} )</td>
<td>(-0.46_{-0.02}^{+0.02} )</td>
</tr>
<tr>
<td>1.0</td>
<td>(-3.36_{-1.02}^{+0.56} )</td>
<td>(-0.91_{-0.06}^{+0.05} )</td>
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**$D_s^+ D_s^-$ interaction and $B^+ \rightarrow D_s^+ D_s^- K^+$ decay** [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- Scattering amplitude: $T(E) = \frac{V}{1 - V G(E)}$
  - $V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2}$
  - $G$ loop functions, once-subtracted, $G(E_{th}) = G_\Lambda(E_{th})$

- Simple production model:
  - $T_B(E) = P + PG(E)T(E) = P \frac{1}{1 - V G(E)}$
  - $d\Gamma \over dE = \frac{1}{(2\pi)^3 \frac{k p}{4m_B^2}} |T_B(E)|^2$

- Fit: two solutions (virtual or bound), in both:
  - $M_{X(3960)} = 3928(3)$ MeV
  - $2M_{D_s} - M_{X(3960)} = 8(3)$ MeV
  - LHCb: $M = 3956(5)(11)$ MeV, $\Gamma = 43(13)(7)$ MeV

[Prelovsek et al., JHEP 06,035('20)]: Bound state $B = 6.2_{-3.8}^{+2.0}$ MeV
(cf. also [Bayar, Feijoo, Oset, 2207.08490])

### Table: $C_{D_s\bar{D}_s} (fm^2)$

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</table>
Fixing all constants [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- **X(3960):** fixes $C_{D_3 \bar{D}_s} = (C_{0a} + C_{1a}) / 2$ (as previously seen)

- **Z_c(3900):** fixes $V = C_{1a} - C_{1b}$
  - Assume virtual state $M = 3813^{+28}_{-21}$ MeV ([2201.08253; 1512.03638] from a fit to BESIII data)

- **X(3872):** fixes
  \[
  \begin{align*}
  C_{0X} = (C_{0a} + C_{0b}) / 2, \\
  C_{1X} = (C_{1a} + C_{1b}) / 2
  \end{align*}
  \]

  - Experimental information:
    \[
    \begin{align*}
    R_{X(3872)}^{\exp} & = 0.29(4) \\
    B_{X(3872)}^{\exp} & = [-150, 0] \text{ keV} \quad \leftarrow \quad M_{X(3872)}^{\exp} = 3871.69^{+0.00 +0.05}_{-0.04 -0.13} \text{ MeV}
    \end{align*}
    \]

  - Theoretically: [0911.4407; 1210.5431; 1504.00861]
    \[
    V = \frac{1}{2} \begin{pmatrix}
    C_{0X} + C_{1X} & C_{0X} - C_{1X} \\
    C_{0X} - C_{1X} & C_{0X} + C_{1X}
    \end{pmatrix} , \quad T = (I - VG)^{-1} V
    \]

    \[
    R_{X(3872)} = \frac{\hat{\Psi}_n - \hat{\Psi}_c}{\hat{\Psi}_n + \hat{\Psi}_c} , \quad \frac{\hat{\Psi}_n}{\hat{\Psi}_c} = \frac{1 - (2m_D + m_{D^* -}) G_2 (C_{0X} + C_{1X})}{(2m_D + m_{D^* -}) G_2 (C_{0X} - C_{1X})} = \frac{(2m_D^0 m_{D^* 0} G_1 (C_{0X} - C_{1X})}{1 - (2m_D^0 m_{D^* 0} G_1 (C_{0X} + C_{1X})},
    \]

- **SU(3) and HQSS breaking corrections** (30%) are taken into account for the LECs
Predictions (complete multiplet): \(X(3960)\) as bound

With the **four LECs** \((C_{0a}, C_{0b}, C_{1a}, C_{1b})\) fixed, poles can be looked for in every sector.
Predictions (complete multiplet): $X(3960)$ as virtual [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

With the four LECs ($C_{0a}, C_{0b}, C_{1a}, C_{1b}$) fixed, poles can be looked for in every sector.
A closer look to $Z_c(3900)$ and $Z_{cs}(3985)$ [Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\langle D\bar{D}^*; 1(1^+) | \hat{T} | D\bar{D}^*; 1(1^+) \rangle = \langle D_s\bar{D}^*; 1/2 (1^-) | \hat{T} | D_s\bar{D}^*; 1/2 (1^-) \rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant $V$ and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach by:
  1. Coupled channels:
     \[
     I \quad (l = 1) \quad \left( \frac{1}{\sqrt{2}} [D\bar{D}^* - D^*\bar{D}], J/\psi \pi \right)
     \]
     (also necessary for $e^+e^- \to J/\psi \pi\pi$ data)
  2. Energy dependence: $C_{1Z} \to C_{1Z} + b \frac{s - E_{th}^2}{2E_{th}}$
- Production mechanism for $Y \to D^0 \bar{D}^* - \pi^+$

\[
|A_2(s,t)|^2 = \left| \frac{1}{t-m_D^2} + l(s)T_{22}(s) \right|^2 q_\pi^4(s) + \frac{1}{2} E_\pi(s) \frac{h_S}{h_D} \left[ \frac{1}{t-m_D^2} + l(s)T_{22}(s) \right] + \beta \left[ 1 + G_2(s)T_{22}(s) \right]
\]
A closer look to $Z_c(3900)$ and $Z_{cs}(3985)$ [Du, MA, Guo, Nieves, PR,D105,074018(’22)]

- $\langle D\bar{D}^*; 1(1^-) | \hat{T} | D\bar{D}^*; 1(1^-) \rangle = \langle D_s\bar{D}^*; \frac{1}{2}(1^-) | \hat{T} | D_s\bar{D}^*; \frac{1}{2}(1^-) \rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant $V$ and single channel: only virtual or bound states ($V^{-1} = G$)
- Extension of the previous approach by:
  1. Coupled channels: $I = 1$ \( \left( \frac{1}{\sqrt{2}} [D\bar{D}^* - D^*\bar{D}] , J/\psi \pi \right) \) (also necessary for $e^+e^- \rightarrow J/\psi\pi\pi$ data)
  2. Energy dependence: $C_{1Z} \rightarrow C_{1Z} + b \frac{s - E_{th}^2}{2E_{th}}$
- Production mechanism for $Y \rightarrow J/\psi \pi^+ \pi^-$

$$| A_1(s, t) |^2 = |\tau(s)|^2 q_\pi^2(s) + |\tau(t)|^2 q_\pi^2(t) + \frac{3\cos^2 \theta - 1}{2} \left[ \tau(s)\tau(t)^* + \tau(s)^* \tau(t) \right] q_\pi^2(s)q_\pi^2(t)$$
$$+ \frac{1}{2} \left\{ |\tau'(s)|^2 E_{\pi}^2(s) + |\tau'(t)|^2 E_{\pi}^2(t) + \left[ \tau'(s)^* \tau'(t) + \tau'(s)\tau'(t)^* \right] E_{\pi}(s)E_{\pi}(t) \right\}$$

$\tau(s) = \sqrt{2}l(s)T_{12}(s) + \alpha \quad \tau'(s) = \frac{h_s}{h_D} \sqrt{2}l(s)T_{12}(s)$
Fit to data [Du, MA, Guo, Nieves, PR,D105,074018(’22)]

- Fitted data:
  - $J/\psi\pi^-$ distribution in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ [BESIII, PRL,119(’17)]
  - $D^0D^{*-}$ distribution in $e^+e^- \rightarrow D^0D^{*-}\pi^+$ [BESIII,PR, D92(’15)]
  - $e^+e^- \rightarrow (D^{*0}D_s^- + D^0D_s^{*-})K^+$ [BESIII, PRL,126(’21)]

- Some production/background/normalization constants are also fitted (not shown here)

- Four schemes:
  - A or B: $b = 0$ or free
  - I or II: $h_s = 0$ or not ($D$- or $S$- and $D$-waves)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$D_1D^{*-}$</th>
<th>$a_2(\mu)$</th>
<th>$\chi^2$/dof</th>
<th>$C_{12}$ [fm$^2$]</th>
<th>$C_Z$ [fm$^2$]</th>
<th>$b$ [fm$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>$D$</td>
<td>$-2.5$</td>
<td>1.62</td>
<td>0.005(1)</td>
<td>$-0.226(10)$</td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3.0$</td>
<td>1.62</td>
<td>0.005(1)</td>
<td>$-0.177(6)$</td>
<td>0*</td>
</tr>
<tr>
<td>IIA</td>
<td>$S+D$</td>
<td>$-2.5$</td>
<td>1.83</td>
<td>0.006(1)</td>
<td>$-0.217(10)$</td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3.0$</td>
<td>1.83</td>
<td>0.006(1)</td>
<td>$-0.171(6)$</td>
<td>0*</td>
</tr>
<tr>
<td>IB</td>
<td>$D$</td>
<td>$-2.5$</td>
<td>1.24</td>
<td>0.007(4)</td>
<td>$-0.222(6)$</td>
<td>$-0.447(44)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3.0$</td>
<td>1.21</td>
<td>0.008(1)</td>
<td>$-0.177(4)$</td>
<td>$-0.255(30)$</td>
</tr>
<tr>
<td>IIB</td>
<td>$S+D$</td>
<td>$-2.5$</td>
<td>1.37</td>
<td>0.005(1)</td>
<td>$-0.203(7)$</td>
<td>$-0.473(45)$</td>
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<tr>
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<td>$-3.0$</td>
<td>1.27</td>
<td>0.005(1)</td>
<td>$-0.171(5)$</td>
<td>$-0.270(30)$</td>
</tr>
</tbody>
</table>
Fit to data (scheme A) [Du, MA, Guo, Nieves, PR,D105,074018('22)]
Fit to data (scheme B) [Du, MA, Guo, Nieves, PR,D105,074018(’22)]
\(Z_{CS}^{(*)}\) and \(Z_{C}^{(*)}\) poles [Du, MA, Guo, Nieves, PR,D105,074018('22)]

<table>
<thead>
<tr>
<th>(D_1D^*)</th>
<th>(a_2(\mu))</th>
<th>(Z_c) [MeV] (\Gamma/2)</th>
<th>(Z_{CS}) [MeV] (\Gamma/2)</th>
<th>(Z_c^*) [MeV] (\Gamma/2)</th>
<th>(Z_{CS}^*) [MeV] (\Gamma/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>D</td>
<td>$-2.5$</td>
<td>$3813^{+21}_{-28}$ vir.</td>
<td>$3920^{+18}_{-26}$ vir.</td>
<td>$3962^{+19}_{-25}$ vir.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3.0$</td>
<td>$3812^{+22}_{-26}$ vir.</td>
<td>$3924^{+19}_{-23}$ vir.</td>
<td>$3967^{+19}_{-22}$ vir.</td>
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<td>IIA</td>
<td>S+D</td>
<td>$-2.5$</td>
<td>$3799^{+22}_{-33}$ vir.</td>
<td>$3907^{+22}_{-31}$ vir.</td>
<td>$3949^{+22}_{-30}$ vir.</td>
</tr>
<tr>
<td></td>
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<td>$-3.0$</td>
<td>$3798^{+25}_{-31}$ vir.</td>
<td>$3911^{+17}_{-27}$ vir.</td>
<td>$3955^{+22}_{-27}$ vir.</td>
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<tr>
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<td>D</td>
<td>$-2.5$</td>
<td>$3897^{+4}_{-4}$</td>
<td>$37^{+8}_{-6}$</td>
<td>$4035^{+4}_{-4}$</td>
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<td></td>
<td></td>
<td>$-3.0$</td>
<td>$3898^{+5}_{-5}$</td>
<td>$38^{+10}_{-7}$</td>
<td>$4035^{+5}_{-5}$</td>
</tr>
<tr>
<td>IIB</td>
<td>S+D</td>
<td>$-2.5$</td>
<td>$3902^{+6}_{-6}$</td>
<td>$38^{+9}_{-6}$</td>
<td>$4042^{+5}_{-5}$</td>
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<tr>
<td></td>
<td></td>
<td>$-3.0$</td>
<td>$3902^{+5}_{-5}$</td>
<td>$37^{+9}_{-6}$</td>
<td>$4039^{+5}_{-6}$</td>
</tr>
</tbody>
</table>

- \(Z_c\) exp.: 
  \[M = 3881.2(4.2)(52.7)\text{ MeV}\]
  \[\Gamma/2 = 25.9(2.3)(18.0)\text{ MeV}\]

- \(Z_{CS}\) exp.: 
  \[M = 3982.5(2.6)(2.1)\text{ MeV}\]
  \[\Gamma/2 = 6.4(2.6)(1.5)\text{ MeV}\]

- Both schemes (A and B)
- Including \(SU(3)\) breaking effects
  - [Yang et al., PR,D103('21): \(Z_{CS}\) and distribution]
  - [Ikeno, Molina, Oset, PL,B814('21)] \(Z_{CS}\) threshold effect)
  - [JPAC, PL,B772('17)] Several possibilities for \(Z_c\)
## Summary and conclusions

- We have considered $D^{(*)}(s)\bar{D}^{(*)}(s)$ interactions with HQSS and SU(3) light-flavour symmetry.
- The $X(3960)$ structure in LHCb data on $B^+ \rightarrow D_s^+ D_s^- K^+$ can be explained with a bound or virtual state.
- The experimental information coming from $X(3960)$, $X(3872)$, and (a virtual) $Z_c(3900)$ allows to fix the four constants appearing in the LO lagrangian.
- Predictions are made based on these constants for multiplet partners of these states in other sectors.
- Considering a generalization of the interactions, the BESIII data for the $Z_c$ and $Z_{cS}$ states can be well reproduced, being $Z_c$ and $Z_{cS}$ flavour partners.