On the $Z_{cs}(3985)$ and $X(3960)$ states

Towards HQSS and $SU(3)$ multiplet descriptions

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

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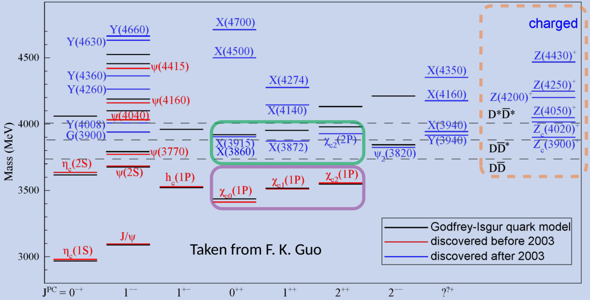
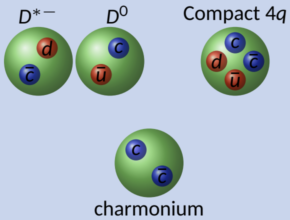
Outline

1. Introduction
2. Interactions
3. $X(3960)$
4. HQSS & $SU(3)$ multiplets
5. $Z_{cs}(3985)$ and $Z_c(3900)$

Quark model in the charmonium sector

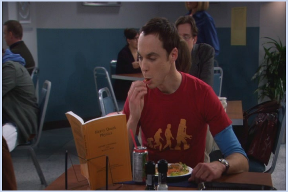




- $\chi_{cJ}(1P)$ well established, "very CQM model" state.
- $X(3872)$ discovered by Belle [PRL,91,262001('03)] (also 2003!)
  $J^{PC} = 1^{++}$ and $\Gamma \simeq 1$ MeV established by LHCb (*e.g.* [JHEP,08(2020),123]).
- $\chi_{cJ}(2P)$ Not established. Influence of open thresholds? Is $X(3872)$ a molecular states?
- Z_c states have $I = 1$, clearly "tetraquarks" ($c\bar{c}u\bar{d}$, ...)
- Many theoretical and lattice and experimental works: can't cite them properly here! (many references in [2207.08653])

HQSS and flavour $SU(3)$ LO lagrangian

[Grinstein *et al.*, NP,B380('92); Alfiky *et al.*, PL,B640('06), …]



- $SU(3)$ light flavour symmetry: $H_a^{(Q)} \sim (Q\bar{u}, Q\bar{d}, Q\bar{s}) \sim (D^0, D^+, D_s^+)$
- HQSS: $H_a^{(Q)} = \frac{1+\not{v}}{2}\left(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right)$ (with $v \cdot P_a^{*(Q)} = 0$)
- LO lagrangian (S-wave contact interactions):

$$\mathcal{L}_{4H} = \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a} H_b^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\right]\left(F_A\,\delta_a^b\delta_c^d + F_A^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)$$
$$+ \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a} H_b^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5\right]\left(F_B\,\delta_a^b\delta_c^d + F_B^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right),$$

- **Only 4 constants**, many ways to write the constants.

$$\mathcal{C}_{0a} = F_A + \frac{10F_A^\lambda}{3}, \qquad \mathcal{C}_{1a} = F_A - \frac{2}{3}F_A^\lambda,$$
$$\mathcal{C}_{0b} = F_B + \frac{10F_B^\lambda}{3}, \qquad \mathcal{C}_{1b} = F_B - \frac{2}{3}F_B^\lambda.$$

$D_s^+ D_s^-$ interaction and $B^+ \to D_s^+ D_s^- K^+$ decay [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

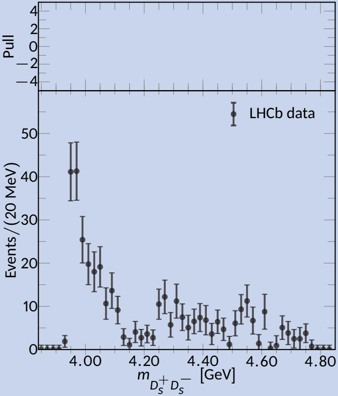
- Scattering amplitude: $T(E) = \dfrac{V}{1 - V\,G(E)}$
  - $V = 4m_{D_s}^2 \dfrac{C_{0a} + C_{1a}}{2}$
  - G loop functions, once-subtracted, $G(E_{\rm th}) = G_\Lambda(E_{\rm th})$
- Simple production model:

$$T_B(E) = P + P\,G(E)T(E) = P\frac{1}{1 - V\,G(E)}$$

- $$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3}\frac{k\,p}{4m_B^2}\left|T_B(E)\right|^2$$

[LHCb-PAPER-2022-018,019]

[LHC Seminar (July 5th, 2022)]



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- Fit: two solutions (virtual or bound), in both:
  $M_{X(3960)} = 3928(3)$ MeV
  $2M_{D_s} - M_{X(3960)} = 8(3)$ MeV
- LHCb: $M = 3956(5)(11)$ MeV, $\Gamma = 43(13)(7)$ MeV
- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8}$ MeV
  (*cf.* also [Bayar, Feijoo, Oset, 2207.08490])

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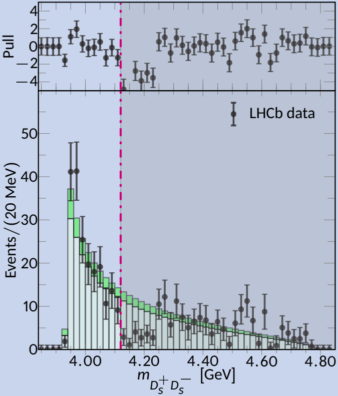

	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV
$C_{D_s\bar{D}_s}$ (fm^2)	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

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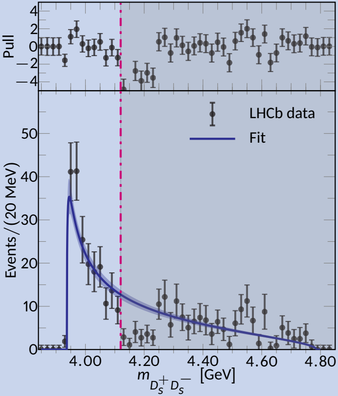

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Fixing all constants [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

- $X(3960)$: fixes $C_{D_s\bar{D}_s} = (C_{0a} + C_{1a})/2$ (as previously seen)
- $Z_c(3900)$: fixes $V = C_{1a} - C_{1b}$
  - Assume virtual state $M = 3813^{+28}_{-21}$ MeV ([2201.08253; 1512.03638] from a fit to BESIII data)
- $X(3872)$: fixes $\begin{cases} C_{0X} = (C_{0a} + C_{0b})/2\,, \\ C_{1X} = (C_{1a} + C_{1b})/2 \end{cases}$
  - Experimental information:

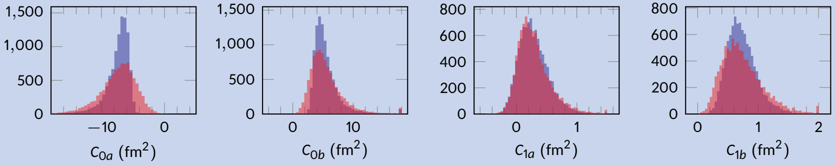
$$\begin{cases} \text{[LHCb, 2204.12597]} & R^{\rm exp}_{X(3872)} = 0.29(4) \\ \text{[LHCb, PR,D 102,092005('20)]} & B^{\rm exp}_{X(3872)} = [-150, 0]\ \text{keV} \longleftarrow M^{\rm exp}_{X(3872)} = 3871.69^{+0.00\,+0.05}_{-0.04\,-0.13}\ \text{MeV} \end{cases}$$

  - Theoretically: [0911.4407; 1210.5431; 1504.00861]

$$V = \frac{1}{2}\begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}, \qquad T = (\mathbb{I} - V\,G)^{-1}\,V\,.$$

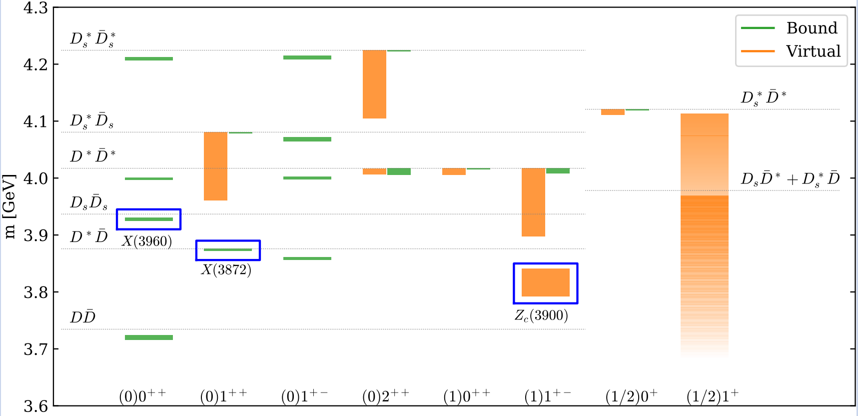
$$R_{X(3872)} = \frac{\hat{\Psi}_n - \hat{\Psi}_c}{\hat{\Psi}_n + \hat{\Psi}_c}, \quad \frac{\hat{\Psi}_n}{\hat{\Psi}_c} = \frac{1 - (2m_{D^+}m_{D^{*-}})\,G_2\,(C_{0X} + C_{1X})}{(2m_{D^+}m_{D^{*-}})\,G_2\,(C_{0X} - C_{1X})} = \frac{(2m_{D^0}m_{D^{*0}})\,G_1\,(C_{0X} - C_{1X})}{1 - (2m_{D^0}m_{D^{*0}})\,G_1\,(C_{0X} + C_{1X})},$$

- $SU(3)$ and HQSS **breaking corrections** (30%) are taken into account for the LECs



Predictions (complete multiplet): $X(3960)$ as bound [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

With the **four LECs** (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



Predictions (complete multiplet): $X(3960)$ as virtual [Ji, Dong, MA, Du, Guo, Nieves, 2207.08563]

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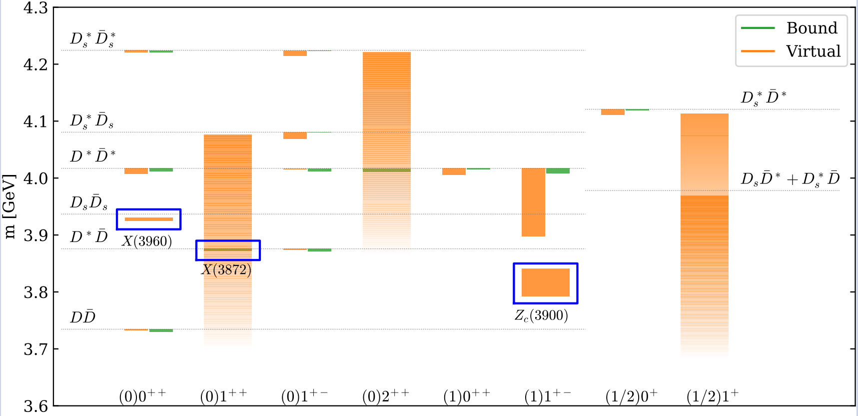
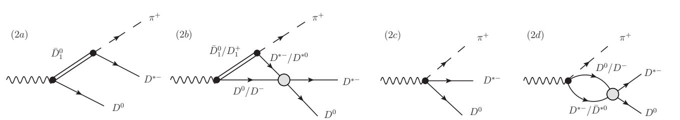


A closer look to $Z_c(3900)$ and $Z_{cs}(3985)$ [Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\left\langle D\bar{D}^*;\, 1(1^{+-})\right| \hat{T} \left|D\bar{D}^*;\, 1(1^{+-})\right\rangle = \left\langle D_s\bar{D}^*;\, \frac{1}{2}(1^{+})\right| \hat{T} \left|D_s\bar{D}^*;\, \frac{1}{2}(1^{+})\right\rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach by:
  1. Coupled channels: $\begin{cases} I=1 & \left(\frac{1}{\sqrt{2}}\left[D\bar{D}^* - D^*\bar{D}\right],\, J/\psi\,\pi\right) \\ I=\frac{1}{2} & \left(\frac{1}{\sqrt{2}}\left[D_s\bar{D}^* - D_s^*\bar{D}\right],\, J/\psi\,K\right) \end{cases}$ (also necessary for $e^+e^- \to J/\psi\pi\pi$ data)
  2. Energy dependence: $C_{1Z} \to C_{1Z} + b\,\dfrac{s - E_{\rm th}^2}{2E_{\rm th}}$
- Production mechanism for $Y \to D^0\,\bar{D}^{*-}\,\pi^+$



$$\overline{|\mathcal{A}_2(s,t)|^2} = \left|\frac{1}{t-m_{D_1}^2} + I(s)T_{22}(s)\right|^2 q_\pi^4(s) + \frac{1}{2}\left|E_\pi(s)\frac{h_S}{h_D}\left[\frac{1}{t-m_{D_1}^2} + I(s)T_{22}(s)\right] + \beta\left[1 + G_2(s)T_{22}(s)\right]\right|^2$$

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- $\left\langle D\bar{D}^*;\, 1(1^{+-})\right| \hat{T} \left|D\bar{D}^*;\, 1(1^{+-})\right\rangle = \left\langle D_s\bar{D}^*;\, \frac{1}{2}(1^{+})\right| \hat{T} \left|D_s\bar{D}^*;\, \frac{1}{2}(1^{+})\right\rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
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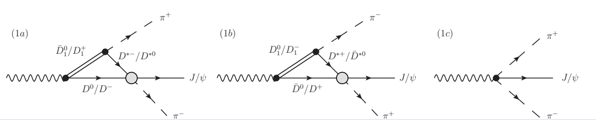


$$\overline{|\mathcal{A}_1(s,t)|^2} = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3\cos^2\theta - 1}{2}\left[\tau(s)\tau(t)^* + \tau(s)^*\tau(t)\right] q_\pi^2(s) q_\pi^2(t)$$
$$+ \frac{1}{2}\left\{|\tau'(s)|^2 E_\pi^2(s) + |\tau'(t)|^2 E_\pi^2(t) + \left[\tau'(s)^*\tau'(t) + \tau'(s)\tau'(t)^*\right] E_\pi(s) E_\pi(t)\right\}$$

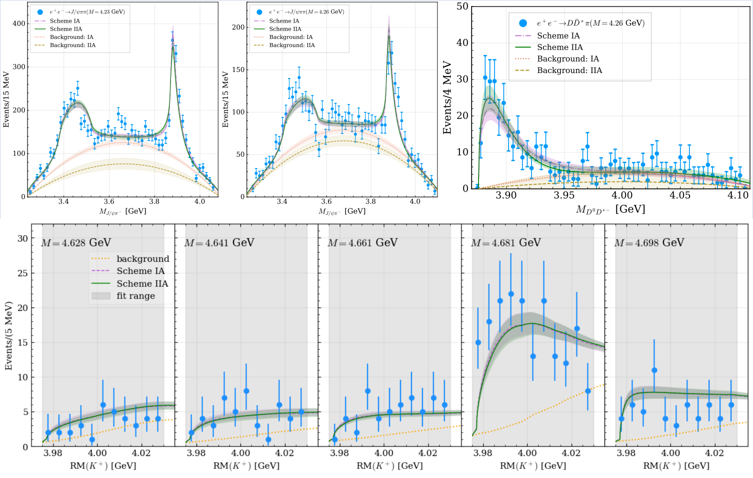
$$\tau(s) = \sqrt{2} I(s) T_{12}(s) + \alpha \qquad \tau'(s) = \frac{h_S}{h_D}\sqrt{2} I(s) T_{12}(s)$$

Fit to data [Du, MA, Guo, Nieves, PR,D105,074018('22)]

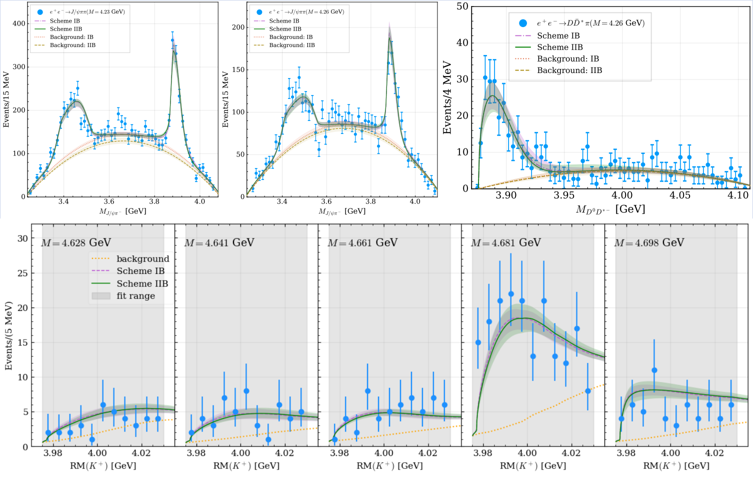
- Fitted data:
  - $J/\psi\pi^-$ distribution in $e^+e^- \to J/\psi\pi^+\pi^-$ [BESIII, PRL,119('17)]
  - D^0D^{*-} distribution in $e^+e^- \to D^0D^{*-}\pi^+$ [BESIII,PR, D92('15)]
  - $e^+e^- \to (D^{*0}D_s^- + D^0D_s^{*-})K^+$ [BESIII, PRL,126('21)]
- Some production/background/normalization constants are also fitted (not shown here)
- Four schemes:
  - A or B: $b = 0$ or free
  - I or II: $h_s = 0$ or not (D- or S- and D-waves)

Scheme	$D_1D^*\pi$	$a_2(\mu)$	χ^2/dof	C_{12} [fm^2]	C_Z [fm^2]	b [fm^3]
IA	D	-2.5	1.62	0.005(1)	$-0.226(10)$	0^*
		-3.0	1.62	0.005(1)	$-0.177(6)$	0^*
IIA	S+D	-2.5	1.83	0.006(1)	$-0.217(10)$	0^*
		-3.0	1.83	0.006(1)	$-0.171(6)$	0^*
IB	D	-2.5	1.24	0.007(4)	$-0.222(6)$	$-0.447(44)$
		-3.0	1.21	0.008(1)	$-0.177(4)$	$-0.255(30)$
IIB	S+D	-2.5	1.37	0.005(1)	$-0.203(7)$	$-0.473(45)$
		-3.0	1.27	0.005(1)	$-0.171(5)$	$-0.270(30)$

Fit to data (scheme A) [Du, MA, Guo, Nieves, PR,D105,074018('22)]



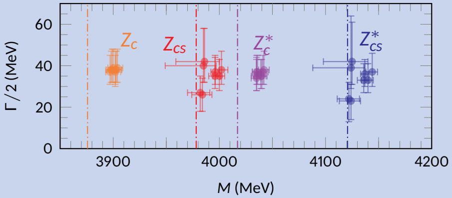
Fit to data (scheme B) [Du, MA, Guo, Nieves, PR,D105,074018('22)]



$Z_{cs}^{(*)}$ and $Z_c^{(*)}$ poles [Du, MA, Guo, Nieves, PR,D105,074018('22)]

	$D_1D^*\pi$	$a_2(\mu)$	Z_c [MeV] Mass	Z_c $\Gamma/2$	Z_{cs} [MeV] Mass	Z_{cs} $\Gamma/2$	Z_c^* [MeV] Mass	Z_c^* $\Gamma/2$	Z_{cs}^* [MeV] Mass	Z_{cs}^* $\Gamma/2$
IA	D	-2.5	3813^{+21}_{-28}	vir.	3920^{+18}_{-26}	vir.	3962^{+19}_{-25}	vir.	4069^{+12}_{-16}	vir.
		-3.0	3812^{+22}_{-26}	vir.	3924^{+19}_{-23}	vir.	3967^{+19}_{-22}	vir.	4078^{+17}_{-13}	vir.
IIA	S+D	-2.5	3799^{+24}_{-33}	vir.	3907^{+22}_{-31}	vir.	3949^{+22}_{-30}	vir.	4057^{+20}_{-28}	vir.
		-3.0	3798^{+25}_{-31}	vir.	3911^{+17}_{-27}	vir.	3955^{+22}_{-27}	vir.	4067^{+19}_{-25}	vir.
IB	D	-2.5	3897^{+4}_{-4}	37^{+8}_{-6}	3996^{+4}_{-4}	37^{+8}_{-6}	4035^{+4}_{-4}	37^{+8}_{-6}	4137^{+4}_{-4}	36^{+7}_{-6}
		-3.0	3898^{+5}_{-5}	38^{+10}_{-7}	3996^{+5}_{-6}	35^{+9}_{-6}	4035^{+4}_{-5}	34^{+9}_{-6}	4136^{+5}_{-6}	33^{+8}_{-6}
IIB	S+D	-2.5	3902^{+6}_{-6}	38^{+9}_{-6}	4002^{+6}_{-6}	38^{+9}_{-7}	4042^{+5}_{-5}	38^{+9}_{-7}	4144^{+5}_{-6}	37^{+9}_{-7}
		-3.0	3902^{+5}_{-5}	37^{+9}_{-6}	4000^{+5}_{-6}	35^{+8}_{-7}	4039^{+5}_{-6}	35^{+8}_{-6}	4140^{+5}_{-6}	33^{+8}_{-6}

- Z_c exp.:
  $M = 3881.2(4.2)(52.7)$ MeV
  $\Gamma/2 = 25.9(2.3)(18.0)$ MeV
- Z_{cs} exp.:
  $M = 3982.5(2.6)(2.1)$ MeV
  $\Gamma/2 = 6.4(2.6)(1.5)$ MeV
- Both schemes (A and B)
- Including $SU(3)$ breaking effects
- [Yang *et al.*, PR,D103('21): Z_{cs} and distribution]
- [Ikeno, Molina, Oset, PL,B814('21)] (Z_{cs} threshold effect)
- [JPAC, PL,B772('17)] Several possibilities for Z_c



Summary and conclusions

- We have considered $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ interactions with **HQSS** and **$SU(3)$ light-flavour symmetry**.
- The $X(3960)$ structure in LHCb data on $B^+ \to D_s^+ D_s^- K^+$ can be explained with a **bound or virtual** state.
- The experimental information coming from $X(3960)$, $X(3872)$, and (a virtual) $Z_c(3900)$ allows to fix the **four constants** appearing in the LO lagrangian.
- Predictions are made based on these constants for **multiplet partners** of these states in other sectors
- Considering a generalization of the interactions, the BESIII data for the Z_c and Z_{cs} states can be well reproduced, being **Z_c and Z_{cs} flavour partners**.