Two-photon transitions of charmonia

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In collaboration with:
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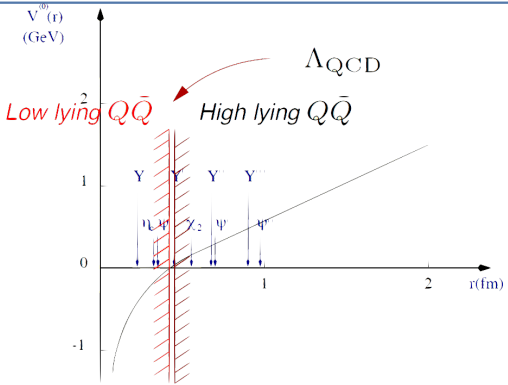
Outline

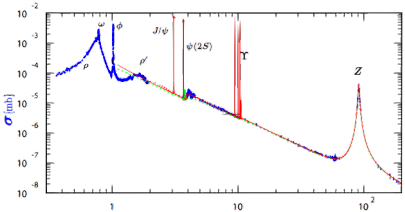
- Physics of two-photon transition of charmonium
- Light front dynamics and basis light-front quantization
- Numerical results: two-photon width and transition form factors
- Summary

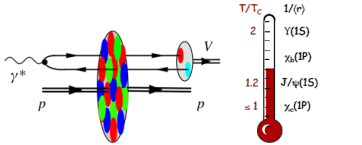
Based on: YL, M. Li (李枚键) and J.P. Vary, Phys. Rev. D letter **105** (2022); arXiv:2111.14178 [hep-ph]
Wave functions available on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2

Charmonium: ``a golden system to study strong interactions"

November Revolution







Tremendous applications in ee, ep, eA, pp, AA collisions

- Theoretically a hard problem: multiscale, multi-physics

$$\Lambda_{QCD} \lesssim \alpha_s^2 m_c < \alpha_s m_c < m_c$$

- Physically a simple system: nonrelativistic ($v_c \ll 1$), perturbative ($\alpha_s \ll 1$)

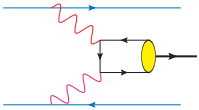
$$v_c^2 \sim 0.3,\ \alpha_s(m_c) \sim (0.3 - 0.6)\ (?)$$

- Potential model, pNRQCD, Lattice QCD

[Brambilla '14]

Two-photon transitions of charmonia

A clean & important probe to hadron structures [Reviews: Berger '87]

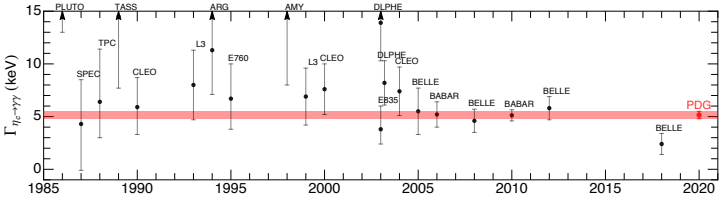


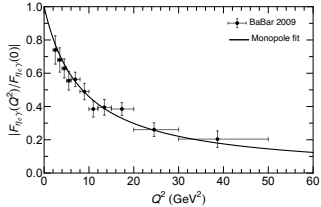
- Decay $H \to \gamma + \gamma$: golden channel for hadron identification
  - *Selection rules: P, C, angular momentum, gauge symmetry, ...*
- Exclusive photoproductions $\gamma + \gamma \to H$: channel of discovery
  - *pQCD factorization at large* Q^2, [Lepage '80 & Chernyak '84]

$$F_{\gamma\eta_c}(Q^2) = \int_0^1 \mathrm{d}x\, T_H(x, Q^2)\phi_{\eta_c}(x;\mu),$$

Experimental measurements [Review of particle physics 2020]

- Diphoton width: extensive measurements for $\eta_c, \eta_c', \chi_{c0}, \chi_{c2}, X(3872)$;
- Transition form factors: $F_{\eta_c\gamma}(Q^2)$ by BABAR 2010; $F_{\chi_{cJ}\gamma}(Q^2)$ by Belle 2017 with limited statistics





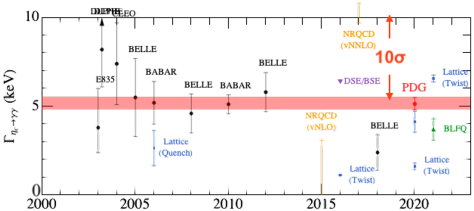
A crisis in theories for charmonium?

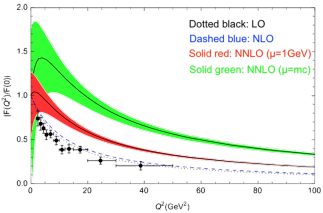
Status of theoretical predictions

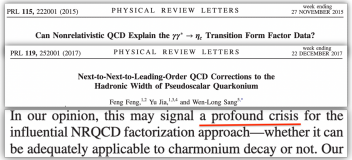
- Potential model: large relativistic corrections [e.g. Babiarz '19]
- NRQCD: 10σ discrepancy at NNLO -- a crisis for NRQCD? [Feng PRL '15&'17]
- Lattice QCD: challenge of representing γ^* on the lattice [e.g., Liu '20]

Why charmonium is so challenging?

- $\alpha_s \sim (0.3 - 0.6)$ is not that small -- non-perturbative effects
- $v_c^2 \sim 0.3$ is not that small -- relativistic effects
- $am_c \sim 0.5$ is not that small -- high order $O(a^2)$ effects






PRL 115, 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

PRL 119, 252001 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Next-to-Next-to-Leading-Order QCD Corrections to the Hadronic Width of Pseudoscalar Quarkonium

Feng Feng,[1,2] Yu Jia,[1,3,4] and Wen-Long Sang[5,*]

In our opinion, this may signal a profound crisis for the influential NRQCD factorization approach—whether it can be adequately applicable to charmonium decay or not. Our

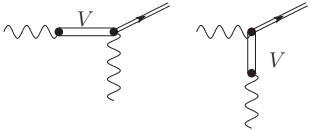
TγP: from vector meson dominance to light-cone dominance [Berger '87]

$Q^2 \to 0$

$Q^2 \to \infty$

- Low Q^2: vector meson dominance [Sakurai '63, Novikov '78]

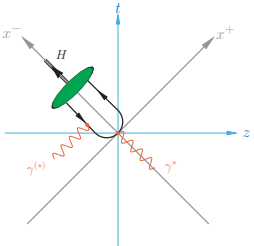
$$i\mathcal{M} \sim (Q^2 + M_V^2)^{-1} \underbrace{\psi(\vec{r}=0)}_{\text{wave function at origin}}$$



- Large Q^2: light-cone dominance [Lepage '80 & Chernyak '84]

$$i\mathcal{M} = \varepsilon_\mu \varepsilon_\nu^* \int \mathrm{d}^4x e^{iq\cdot z} \langle 0|J^\mu(z)J^\nu(0)|P\rangle \sim \int \mathrm{d}x\, T_H(x,Q^2) \underbrace{\phi_P(x;\mu)}_{\text{light-cone distribution amplitude}}$$

Large-Q^2 limit: $z^2 \sim 1/Q^2 \to 0$ *(the light cone)* [Gribov '83, Nandi '07 & Li '09]



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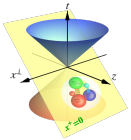
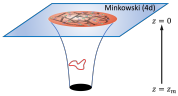
- Intermediate Q^2: get off the light cone ($z^2 = 0$) → light front ($z^+ = 0$)

$$i\mathcal{M} \sim \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} T_H(x, \vec{k}_\perp; Q^2)\, \underbrace{\psi_P(x, \vec{k}_\perp; \mu)}_{\text{light-front wave function}}$$





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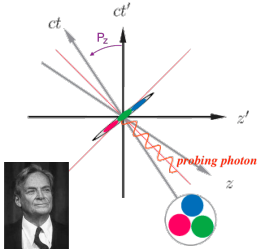
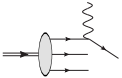
Physics on the light front

[Reviews: Brodsky '98, Burkardt '02, Bakker '14, Ji '21]

infinite momentum frame

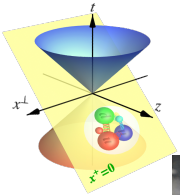
$P_z \to \infty$ (Fubini, '65)





light front quantization

$x^+ = 0$ (Dirac, '49)

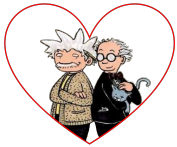


$$x^\pm = x^0 \pm x^3$$
$$P^\pm = P^0 \pm P^3$$
$$\underline{\mathcal{M}}^2 = P^+ \underline{P}^- - \vec{P}_\perp^2$$

$$i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle = \frac{1}{2}\underline{P}^-|\psi(x^+)\rangle$$

$$\Downarrow$$

$$\underline{\mathcal{M}}^2|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle$$



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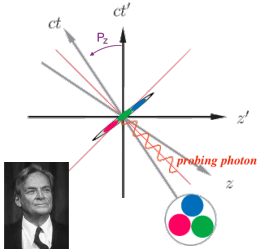
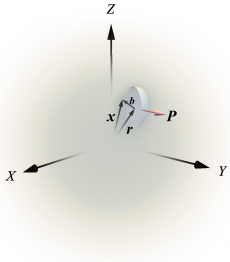


localized wavepacket

$R_z \to 0$ (2206.12903)



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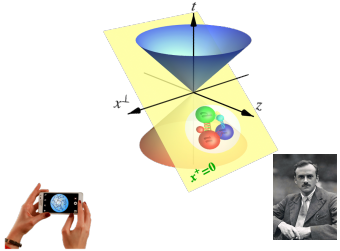

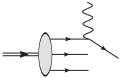

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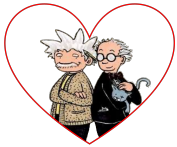
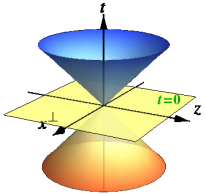
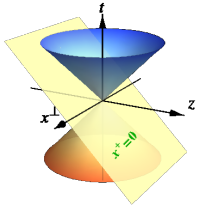
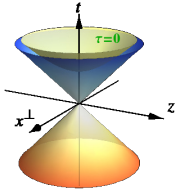


Dirac's forms of relativistic dynamics

[Dirac, Rev. Mov. Phys. '49]

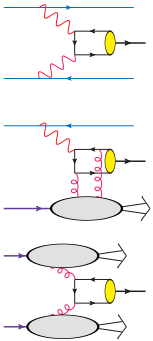
	instant form	***front form***	***point form***
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J_z$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$

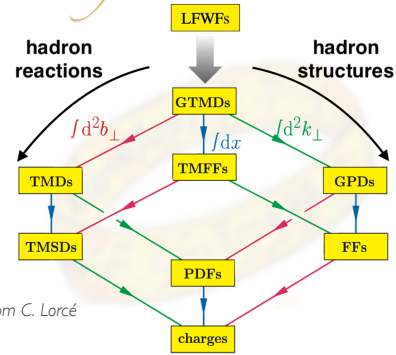
$P^\pm \triangleq P^0 \pm P^3$, $\vec{P}^\perp \triangleq (P^1, P^2)$, $x^\pm \triangleq x^0 \pm x^3$, $\vec{x}^\perp \triangleq (x^1, x^2)$, $E^i = M^{+i}$, $E^+ = M^{+-}$, $F^i = M^{-i}$, $K^i = M^{0i}$, $J^i = \frac{1}{2}\epsilon^{ijk}M^{jk}$.

Light-front wave functions (LFWFs) [Reviews: Brodsky '98, Diehl '03, Lorcé '11]

LFWFs are frame independent and can directly access the partonic information of hadrons

One wavefunction to rule them all!





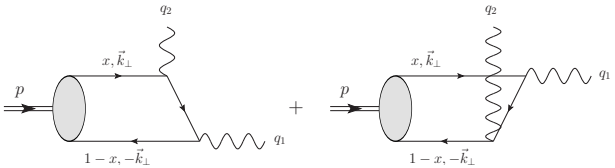
Adapted from C. Lorcé

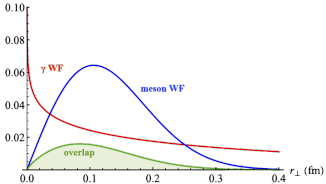
Hadron Physics without LFWFs is like Biology without DNA!

--- Stanley J. Brodsky

must have the precious!

LFWF representation of two-photon transitions





- The amplitude can be accessed in light-cone perturbation theory and also through hadronic matrix elements, [Lepage '80, Feldmann '97, Kroll '10, Babiarz '19]

$$\varepsilon^*_\mu(q_1,\lambda_1)\varepsilon^*_\nu(q_2,\lambda_2)e_\alpha(p,\lambda)\mathcal{M}^{\mu\nu\alpha} = \varepsilon^*_\nu(q_2,\lambda_2)\langle\gamma^*(q_1,\lambda_1)|J^\nu(0)|H(p,\lambda)\rangle.$$

It is convenient to adopt a frame in which $q_1^- = q_2^+ = 0$, i.e. with manifest light-cone dominance.

- Example: LFWF representation of a pseudoscalar meson (0^{-+}),

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}+\cdots$$

Intuitively, this is the overlap of the photon wave function with the meson wave function. [Beuf '16, Lappi '20]

Light-front dynamics $\neq$ NR dynamics w. relativistic corrections

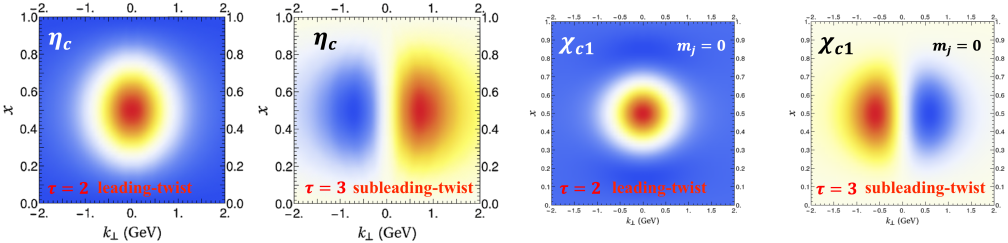
N.B. wave functions from non-relativistic dynamics with relativistic corrections in general are different from wave functions from relativistic dynamics, e.g. LFD.

- Parities in NRQM: $\mathsf{P} = (-1)^{L+1}, \mathsf{C} = (-1)^{L+S}$ are approximations since L is not a good quantum number

  NRQM + relativistic correction: spin-orbital coupling $\rightarrow$ partial wave mixing subject to parities

- Parities in LFD: $m_{\mathsf{P}} = (-1)^{L_z+S+1}, \mathsf{C} = (-1)^{L_z+S+\ell}$ are exact

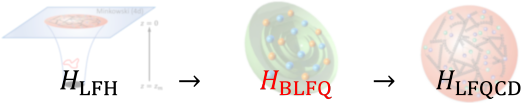
  There exist leading-twist wavefunctions that are absent in NRQM (including relativistic corrections) due to parities



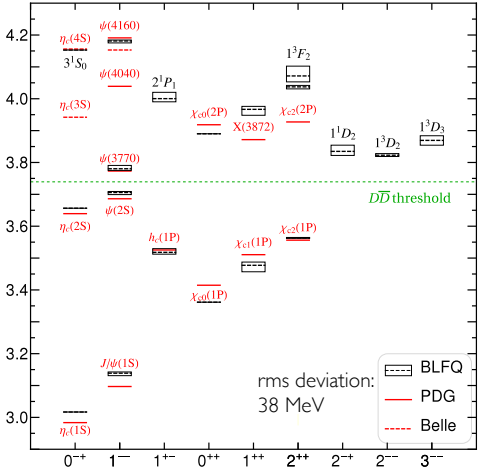
(a) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (b) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$ (c) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (d) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$

Basis light-front quantization

[Vary et al. PRC '09; YL, Maris, Zhao, Vary, PLB '16]



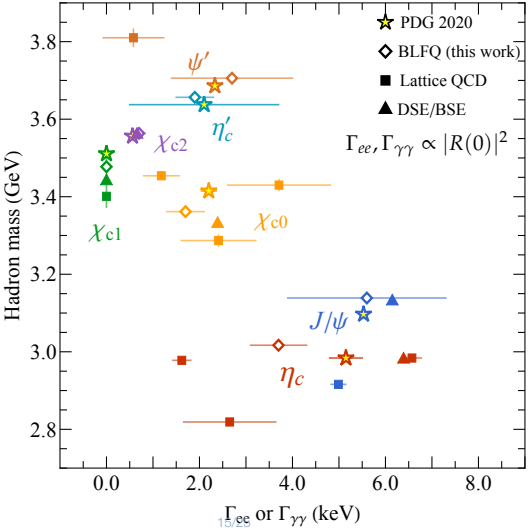
- Holographic light-front QCD confinement plus one-gluon-exchange interaction [Review: Brodsky '14]
- Solved in basis function approach, $\Lambda_{\rm UV} \approx b\sqrt{N_{\rm max}}$.
- Two free parameters m_c, κ fitted to the mass spectrum. Posterior rms deviation: $\lesssim$ 40 MeV
- Application to a variety of systems:
  - $c\bar{c}, b\bar{b}$: YL, PLB '16 & PRD '17
  - $b\bar{c}, b\bar{q}, c\bar{q}$: Tang, PRD '18 & EPJC '20
  - $q\bar{q}$: Jia, PRC '19; Qian, PRC '20; YL, '21
  - Baryons: Mondal, PRD '20; Xu, '21
- Access to a variety of observables
  - Form factors: YL, PRD '18; Mondal, PRD '20
  - (Semi-)leptonic decay: Li, PRD '18 & '19; Tang, PRD '21
  - PDFs/GPDs: Lan, PRL '19 & PRD '20; Adhikari, PRC '18 & '21
  - Diffractive production: Chen, PLB '17 & PRC '18



Numerical results: overview

Parameter-free predictions:

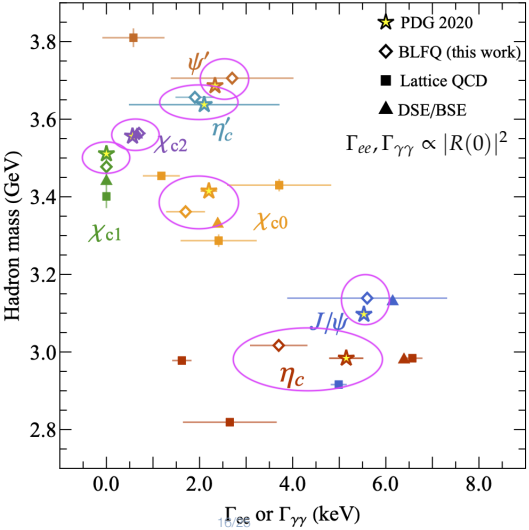
[Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21; DSE: Chen '16]



Numerical results: overview

Parameter-free predictions:

[Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21; DSE: Chen '16]



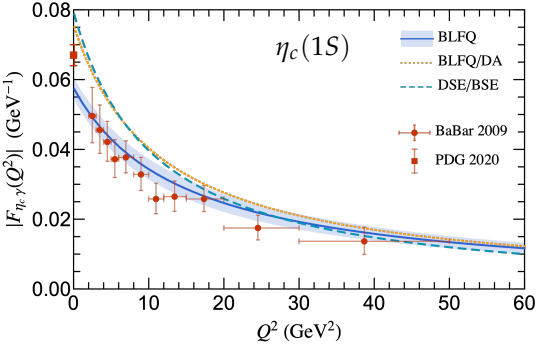
Transition form factor: η_c

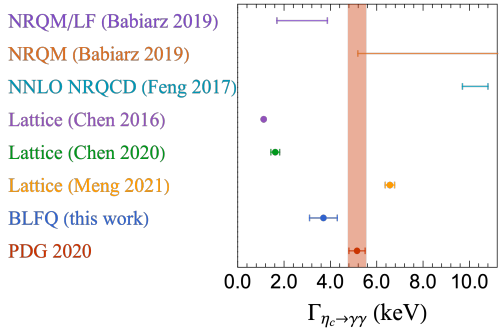
$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2,q_2^2), \quad F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2,0) = F_{P\gamma\gamma}(0,Q^2)$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_P^3 \left|F_{P\gamma\gamma}(0,0)\right|^2.$

[Lepage '81, Babiarz '19, Hoferichter '20]

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{{\rm d}x}{2\sqrt{x(1-x)}}\int\frac{{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}$$





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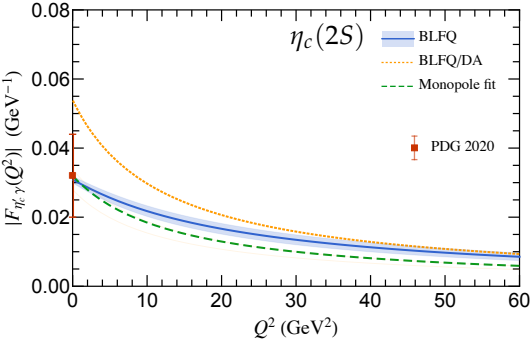


- No experimental measurement yet.
- A monopole fit using $\Lambda^2 = M_{\psi'}^2$ is included for comparison.
- Note that a VMD prediction requires the off-shell coupling $g_V(Q^2) = V_{PV\gamma}(Q^2)$: [Lakhina '06]

$$F_{P\gamma}^{\rm (VMD)}(Q^2) = \sum_V \frac{e_f^2 f_V}{1+\frac{M_P}{M_V}}\left[\frac{g_V(0)}{M_V^2+Q^2}+\frac{g_V(Q^2)}{M_V^2}\right]$$

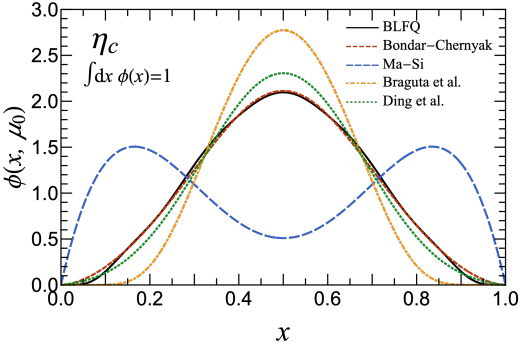
Light cone distribution amplitude (LCDA)

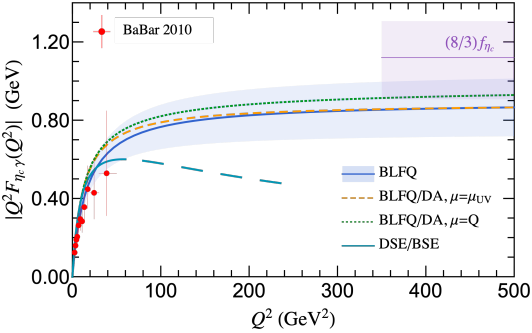
At large-Q^2, viz. $Q^2 + \langle m_f^2/x(1-x)\rangle \gg \langle k_\perp^2/x(1-x)\rangle$,

$$F_{P\gamma}(Q^2) \approx e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2 + m_f^2} \xrightarrow{Q\to\infty} \frac{6e_f^2 f_P}{Q^2}.$$

- LCDA plays a pivotal role in hard exclusive charmonium production. [See, e.g., Braguta '12]
- Our LCDA agrees with the Bondar-Chernyak model. Both fit the BABAR normalized TFF well.

[LCDAs: Ma '04, Bondar '05, Braguta '07, Ding '16]





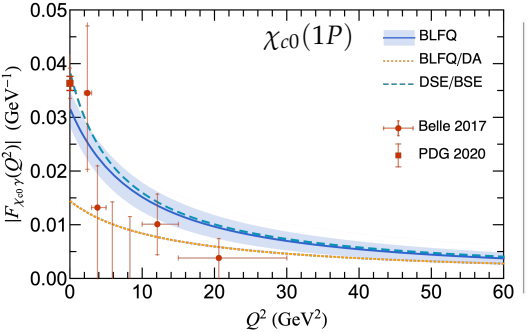
Transition form factor: χ_{c0}

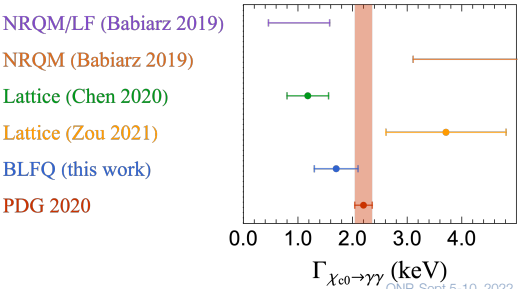
[Babiarz '20, Hoferichter '20; DSE/BSE: Chen '17]

$$\mathcal{M}^{\mu\nu}_{S\to\gamma\gamma} = 4\pi\alpha_{\rm em}\Big\{\big[(q_1\cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu\big]F_1^S(q_1^2,q_2^2) + \frac{1}{M_S^2}\big[q_1^2q_2^2g^{\mu\nu} + (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2q_2^\mu q_2^\nu - q_2^2q_1^\mu q_1^\nu\big]F_2^S(q_1^2,q_2^2)\Big\}$$

Single-tag TFF: $F_{S\gamma}(q^2) = F_1^S(q^2,0) = F_1^S(0,q^2)$. Width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha_{\rm em}^2}{4}M_S^3|F_{S\gamma}(0)|^2$. Belle provides the first measurement of the TFF, albeit with limited statistics.

[Belle, PRD 2017]



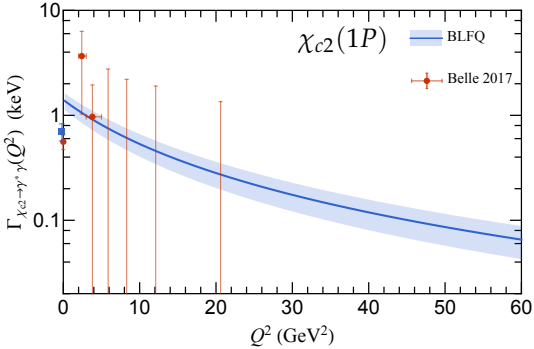


Transition form factor: χ_{c2}

[Babiarz '20, Hoferichter '20; DSE/BSE: Chen '17]

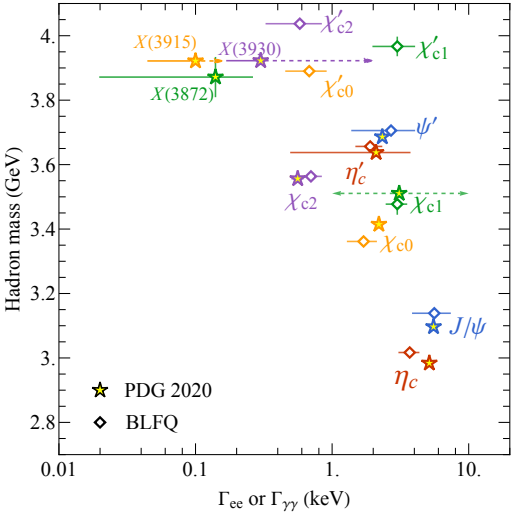
$$\Gamma_{T\to\gamma\gamma} = \frac{\pi\alpha_{\rm em}^2}{5M_T}\left(|\mathcal{M}_{++;0}|^2 + |\mathcal{M}_{+-;2}|^2\right).$$

Belle provides the first measurement of the Q^2 dependent width, albeit with limited statistics. [Belle, PRD 2017]



The $c\bar{c}$ contents of the X's

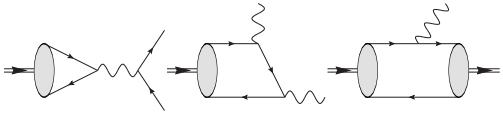
Belle recently measured the reduced diphoton width of X(3872). The value is much smaller than our prediction of a pure $c\bar{c}$ $\chi_{c1}(2P)$, indicating large portion of non-$c\bar{c}$ content (model dependence?) [Belle, PRL 2021]



Radiative transitions

Leptonic and radiative transitions probe the fundamental structure of the hadrons:

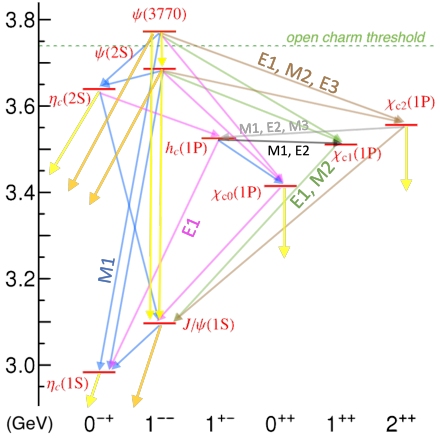
[Review: Barnes & Yuan, Int. J. Mod. Phys. A 2009]



decay width (keV)		Γ_{ee}	$\Gamma_{\gamma\gamma}$	$\Gamma_{\eta_c\gamma}$	$\Gamma_{J/\psi\gamma}$
η_c	PDG	-	5.15(35)		
	BLFQ	-	3.7(6)		
J/ψ	PDG	5.53(10)	-	1.6(4)	
	BLFQ	5.7(1.9)	-	2.6(1)	
χ_{c0}	PDG	-	2.1(1.6)	-	$15(1)\times 10^3$
	BLFQ	-	1.9(4)	-	in progress
χ_{c1}	PDG	-	-	-	288(16)
	BLFQ	-	-	-	in progress
⋮					

[YL, PRD '17; Li, PRD '18; Chen, in progress]

[PDG, PTEP '20 + '21(update)]

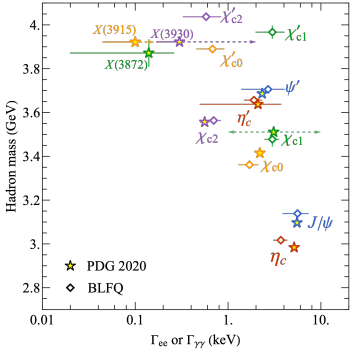


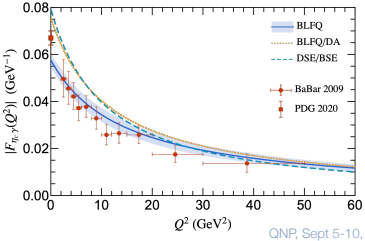
Summary

- Light-front Hamiltonian formalism provides unique tools to access the hadronic observables
  - Light-cone dominance
  - Collinear factorization and k_T factorization
- We computed the two-photon width and transition form factors of charmonia within the basis light-front quantization approach.
  - Excellent agreements with the available experimental measurements.
  - No parameters are dialed to obtain these results.
  - Reveal relativistic nature of charmonium system
- The obtained wave functions await further experimental measurements and further applications.

Based on: YL, M. Li (李枚键) and J.P. Vary, Phys. Rev. D **L105** (2022); arXiv:2111.14178 [hep-ph]

LFWFs available on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2





fin

Thank you for your attention.