Semileptonic B_c decays to charmonia and new paths to identify X(3872)

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QNP 2022
9th International Conference on Quark and Nuclear Physics
September 5-9
Florida State University

based on works in collaboration with
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Outline

- Motivation to study semileptonic $b \to c$ transitions and B_c decays
- Spin symmetry + NRQCD to relate form factors required in the SM and BSM
- Relations among $B_c \to J/\psi$ and $B_c \to \eta_c$ form factors
- B_c to P-wave charmonia and the identification of X(3872)
- Summary

b → c transitions

b → c

- Possibility to precisely measure SM parameters, i.e. V_{cb}
  getting insights on the tension from inclusive/exclusive determinations

- Probe the presence of anomalies already shown up in modes induced by $b \to c\, \ell\, \nu_\ell$ transition

B_c modes

- Interest in B_c decays and properties

- Probing the structure of the charmonia produced in the decay

Semileptonic $B_c \to \chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$ decays

$(\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$: p-wave charmonia with $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$

X(3872)

- Discovered by Belle in 2003 and confirmed by CDF, D0 and BaBar
- In 2015 LHCb fixes $J^P=1^{++}$ → candidate for identification with $\chi_{c1}(2P)$
- Many other possible interpretations have been put forward
  - tetraquark
  - $D\,D^*$ molecule (proximity to the threshold)
  - $c\,\bar{c}\,g$ hybrid
  - ....
- Isospin violation disfavours the charmonium interpretation (but phase space suppression is at work)
- The preference of $\psi(2S)\,\gamma$ wrt $J/\psi\,\gamma$ favours the interpretation as $\chi_{c1}(2P)$

Looking for further information:

does X(3872) fulfill the expectations for the production of $\chi_{c1}(2P)$ in semileptonic B_c decays?

Semileptonic B_c decays

B_c

- discovered at Tevatron in 1998
- m_{Bc}=6.274 GeV
- τ_{Bc}=0.510 ps
- decays weakly
- possible modes: annihilation, b-decays, c-decays (dominant)
- environment to explore BSM effects

Control of theoretical uncertainties in phenomenological analyses requires reliable determination of the hadronic form factors

- Possibility to exploit NRQCD methods + HQ spin symmetry

Semileptonic B$_c$ decays

Generalized effective Hamiltonian

SM

$$
\begin{aligned}
H_{\rm eff}^{b\to c\ell\bar{\nu}} = \frac{G_F}{\sqrt{2}} V_{cb}\Big[& (1+\epsilon_V^\ell)\,(\bar{c}\gamma_\mu(1-\gamma_5)b)\,(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell) \\
& + \epsilon_R^\ell\,(\bar{c}\gamma_\mu(1+\gamma_5)b)\,(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell) \\
& + \epsilon_S^\ell\,(\bar{c}b)\,(\bar{\ell}(1-\gamma_5)\nu_\ell) \\
& + \epsilon_P^\ell\,(\bar{c}\gamma_5 b)\,(\bar{\ell}(1-\gamma_5)\nu_\ell) \\
& + \epsilon_T^\ell\,(\bar{c}\sigma_{\mu\nu}(1-\gamma_5)b)\,(\bar{\ell}\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell)\Big]
\end{aligned}
$$

complex
lepton flavour dependent couplings

Generalized effective Hamiltonian

SM

$$H_{\rm eff}^{b\to c\ell\bar\nu} = \frac{G_F}{\sqrt{2}} V_{cb}\Big[(1+\epsilon_V^\ell)\,(\bar c\gamma_\mu(1-\gamma_5)b)\,\left(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\right)$$
$$+\,\epsilon_R^\ell\,(\bar c\gamma_\mu(1+\gamma_5)b)\,\left(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\right)$$
$$+\,\epsilon_S^\ell\,(\bar c b)\,\left(\bar\ell(1-\gamma_5)\nu_\ell\right)$$
$$+\,\epsilon_P^\ell\,(\bar c\gamma_5 b)\,\left(\bar\ell(1-\gamma_5)\nu_\ell\right)$$
$$+\,\epsilon_T^\ell\,(\bar c\sigma_{\mu\nu}(1-\gamma_5)b)\,\left(\bar\ell\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell\right)\Big]$$

A larger set of Form Factors is required wrt the SM case

complex
lepton flavour dependent couplings

Semileptonic B_c decays

$$\frac{d\Gamma(B_c \to C\ell\bar{\nu})}{dw} = \tilde{\Gamma}\Big\{|1+\epsilon_V|^2 \frac{d\Gamma}{dw}^{SM} + |\epsilon_R|^2 \frac{d\Gamma}{dw}^{R} + |\epsilon_X|^2 \frac{d\Gamma}{dw}^{X} + |\epsilon_T|^2 \frac{d\Gamma}{dw}^{T}$$
$$+ 2\,\mathrm{Re}\left[\epsilon_R(1+\epsilon_V^*)\right] \frac{d\Gamma}{dw}^{SMR} + 2\,\mathrm{Re}\left[\epsilon_X(1+\epsilon_V^*)\right] \frac{d\Gamma}{dw}^{SMX}$$
$$+ 2\,\mathrm{Re}\left[\epsilon_T(1+\epsilon_V^*)\right] \frac{d\Gamma}{dw}^{SMT} + 2\,\mathrm{Re}\left[\epsilon_R\epsilon_T^*\right] \frac{d\Gamma}{dw}^{RT}$$
$$+ 2\,\mathrm{Re}\left[\epsilon_X\epsilon_R^*\right] \frac{d\Gamma}{dw}^{XR} + 2\,\mathrm{Re}\left[\epsilon_X\epsilon_T^*\right] \frac{d\Gamma}{dw}^{XT}\Big\},$$

$$w = v \cdot v'$$

$$q^2 = m_{B_c}^2 + m_C^2 - 2m_{B_c}m_C\, w$$

The various terms encode the dependence on the FF

Semileptonic B_c decays

SM

NP

$B_c \to \eta_c$:

$$\langle \eta_c(v')|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle = \sqrt{m_P\, m_{B_c}}\left[h_+(w)\,(v+v')_\mu + h_-(w)\,(v-v')_\mu\right]$$
$$\langle \eta_c(v')|\bar{Q}'Q|B_c(v)\rangle = \sqrt{m_P\, m_{B_c}}\, h_S(w)(1+w)$$
$$\langle \eta_c(v')|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle = -i\,\sqrt{m_P\, m_{B_c}}\, h_T(w)\,(v_\mu\, v'_\nu - v_\nu\, v'_\mu)$$

$B_c \to J/\psi$:

$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle = i\,\sqrt{m_V\, m_{B_c}}\, h_V(w)\, \epsilon_{\mu\nu\alpha\beta}\,\epsilon^{*\nu}\, v'^\alpha\, v^\beta$$
$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_\mu\gamma_5\, Q|B_c(v)\rangle = \sqrt{m_V\, m_{B_c}}\left[h_{A_1}(w)\,(1+w)\,\epsilon^*_\mu - h_{A_2}(w)\,(\epsilon^*\cdot v)\,v_\mu - h_{A_3}(w)\,(\epsilon^*\cdot v)\,v'_\mu\right]$$
$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_5 Q|B_c(v)\rangle = -\sqrt{m_V\, m_{B_c}}\, h_P(w)\,(\epsilon^*\cdot v)$$
$$\begin{aligned}\langle J/\psi(v',\epsilon)|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle = -\sqrt{m_V\, m_{B_c}}\,\epsilon^{\mu\nu\alpha\beta}&\left[h_{T_1}(w)\epsilon^*_\alpha(v+v')_\beta + h_{T_2}(w)\epsilon^*_\alpha(v-v')_\beta\right.\\ &\left. + h_{T_3}(w)(\epsilon^*\cdot v)v_\alpha v'_\beta\right]\end{aligned}$$

Semileptonic B_c decays

$B_c \to \chi_{c0}$:

$$\langle \chi_{c0}(v')|\bar c \gamma_\mu \gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\,[g_+(w)(v+v')_\mu + g_-(w)(v-v')_\mu]$$
$$\langle \chi_{c0}(v')|\bar c \gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\, g_P(w)$$
$$\langle \chi_{c0}(v')|\bar c \sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\, g_T(w)\, \epsilon_{\mu\nu\alpha\beta} v^\alpha v'^\beta$$

$B_c \to h_c$:

$$\langle h_c(v',\epsilon)|\bar c \gamma_\mu b|B_c(v)\rangle = \sqrt{m_{h_c} m_{B_c}}\Big[f_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)\big(f_{V_2}(w)(v+v')_\mu + f_{V_3}(w)(v-v')_\mu\big)\Big]$$
$$\langle h_c(v',\epsilon)|\bar c \gamma_\mu \gamma_5 b|B_c(v)\rangle = i\sqrt{m_{h_c} m_{B_c}}\, f_A(w)\, \epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha} v^\beta v'^\sigma$$
$$\langle h_c(v',\epsilon)|\bar c b|B_c(v)\rangle = \sqrt{m_{h_c} m_{B_c}}\,(\epsilon^*\cdot v) f_S(w)$$
$$\langle h_c(v',\epsilon)|\bar c \sigma_{\mu\nu} b|B_c(v)\rangle = i\sqrt{m_{h_c} m_{B_c}}\Big[f_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + f_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + f_{T3}(w)\,(\epsilon^*\cdot v)\,(v_\mu v'_\nu - v_\nu v'_\mu)\Big].$$

$B_c \to \chi_{c1}$:

$$\langle \chi_{c1}(v',\epsilon)|\bar c \gamma_\mu b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}} m_{B_c}}\Big[g_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)[g_{V_2}(w)(v+v')_\mu + g_{V_3}(w)(v-v')_\mu]\Big]$$
$$\langle \chi_{c1}(v',\epsilon)|\bar c \gamma_\mu \gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}} m_{B_c}}\, g_A(w)\, \epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha} v^\beta v'^\sigma$$
$$\langle \chi_{c1}(v',\epsilon)|\bar c b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}} m_{B_c}}\, g_S(w)\,(\epsilon^*\cdot v)$$
$$\langle \chi_{c1}(v',\epsilon)|\bar c \sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}} m_{B_c}}\Big[g_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + g_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + g_{T_3}(w)(\epsilon^*\cdot v)(v_\mu v'_\nu - v_\nu v'_\mu)\Big]$$

$B_c \to \chi_{c2}$:

$$\langle \chi_{c2}(v',\eta)|\bar c \gamma_\mu b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\, i\, k_V(w)\, \epsilon_{\mu\alpha\beta\sigma}\eta^{*\alpha\tau} v_\tau v^\beta v'^\sigma$$
$$\langle \chi_{c2}(v',\eta)|\bar c \gamma_\mu \gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\,\big[k_{A_1}(w)\,\eta^*_{\mu\alpha} v^\alpha + \eta^*_{\alpha\beta} v^\alpha v^\beta\,\big(k_{A_2}(w) v_\mu + k_{A_3}(w) v'_\mu\big)\big]$$
$$\langle \chi_{c2}(v',\eta)|\bar c \gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\, k_P(w)\,\eta^*_{\alpha\beta} v^\alpha v^\beta$$
$$\langle \chi_{c2}(v',\eta)|\bar c \sigma_{\mu\nu}\gamma_5 b|B_c(v)\rangle = i\sqrt{m_{\chi_{c2}} m_{B_c}}\,\big[k_{T_1}(w)(\eta^{*\alpha}_{\mu} v_\alpha v_\nu - \eta^{*\alpha}_{\nu} v_\alpha v_\mu) + {} + k_{T_2}(w)(\eta^{*\alpha}_{\mu} v_\alpha v'_\nu - \eta^{*\alpha}_{\nu} v_\alpha v'_\mu) + k_{T_3}(w)\eta^*_{\alpha\beta} v^\alpha v^\beta (v_\mu v'_\nu - v_\nu v'_\mu)\big]$$

SM

NP

HQ spin symmetry in B_c decays

HQ limit:

- Heavy-light mesons → HQ spin & flavour symmetry
- Heavy-heavy mesons → HQ spin symmetry (HQSS)

Relations among the FF describing weak matrix elements

Well known example:
FF of weak matrix elements between heavy-light ground state mesons are all described by the
Isgur-Wise function

Less known case:
Heavy-heavy mesons decays
Infrared divergences present in the heavy quarkonium regulated in the HQ limit by the kinetic energy operator that breaks flavour symmetry ⇢ only spin symmetry can be exploited

Thacker and Lepage, PRD43 (1991) 196

Form Factors in the effective theory

Expansion parameters for a system with 2 Heavy Quarks:

- Relative HQ 3-velocity in the hadron rest-frame (NRQCD)
- Inverse HQ mass $1/m_Q$ (HQET)

- HQ field: $$Q(x) = e^{-im_Q v\cdot x}\psi(x) = e^{-im_Q v\cdot x}\Big(\psi_+(x) + \psi_-(x)\Big) \qquad \psi_\pm(x) = P_\pm \psi(x) = \frac{1 \pm \slashed{v}}{2}\psi(x)$$

$$\Rightarrow \quad Q(x) = e^{-im_Q v\cdot x}\left(1 + \frac{i\slashed{D}_\perp}{2m_Q} + \frac{(-iv\cdot D)}{2m_Q}\frac{i\slashed{D}_\perp}{2m_Q} + \ldots\right)\psi_+(x) \qquad D_{\perp\mu} = D_\mu - (v\cdot D)v_\mu$$

- Expand the HQ QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\slashed{D}_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\slashed{D}_\perp) + \ldots\right)\psi_+(x)$$

Form Factors in the effective theory

Power counting in NRQCD

$$\psi_+ \sim \tilde{v}^{3/2}$$

$$D_\perp \sim \tilde{v} \qquad D_t \sim \tilde{v}^2$$

$$E_i = G_{0i} \sim \tilde{v}^3 \qquad B_i = \frac{1}{2}\epsilon_{ijk}G^{jk} \sim \tilde{v}^4$$

Form Factors in the effective theory

$$\mathcal{L}_{QCD} = \bar{\psi}_+(x)\left(\underbrace{iv\cdot D + \frac{(iD_\perp)^2}{2m_Q}}_{\mathcal{O}(\tilde{v}^2)} + \underbrace{\frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\not{D}_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\not{D}_\perp)}_{\mathcal{O}(\tilde{v}^4)} + \ldots\right)\psi_+(x)$$

$\mathcal{O}(\tilde{v}^2)$ Leading order term

$\mathcal{O}(\tilde{v}^4)$ NLOterm

$$\boxed{\mathcal{L}_0 = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q}\right)\psi_+(x)}$$

$$\boxed{\mathcal{L}_1 = \mathcal{L}_{1,1} + \mathcal{L}_{1,2}}$$

Form Factors in the effective theory

Goal:

$$\boxed{\langle C|\bar{Q}'\Gamma Q|B_c\rangle}$$

$$C = \eta_c,\ J/\psi \qquad C = \chi_{c0},\ \chi_{c1},\ \chi_{c2},\ h_c$$

I. expand the current:

$$\boxed{\bar{Q}'(x)\Gamma Q(x) = J_0 + \left(\frac{J_{1,0}}{2m_Q} + \frac{J_{0,1}}{2m_{Q'}}\right) + \left(-\frac{J_{2,0}}{4m_Q^2} - \frac{J_{0,2}}{4m_{Q'}^2} + \frac{J_{1,1}}{4m_Q m_{Q'}}\right)}$$

$$J_0 = \bar{\psi}'_+\Gamma\psi_+$$

$$J_{1,0} = \bar{\psi}'_+\Gamma i\overrightarrow{\not{D}}_\perp\psi_+ \qquad J_{0,1} = \bar{\psi}'_+\left(-i\overleftarrow{\not{D}}'_\perp\right)\Gamma\psi_+$$

$$J_{2,0} = \bar{\psi}'_+\Gamma\left(iv\cdot\overrightarrow{D}\right)i\overrightarrow{\not{D}}_\perp\psi_+ \qquad J_{0,2} = \bar{\psi}'_+ i\overleftarrow{\not{D}}'_\perp\left(iv'\cdot\overleftarrow{D}\right)\Gamma\psi_+ \qquad J_{1,1} = \bar{\psi}'_+\left(-i\overleftarrow{\not{D}}'_\perp\right)\Gamma\left(i\overrightarrow{\not{D}}_\perp\right)\psi_+$$

Form Factors in the effective theory

II: exploit spin symmetry for large HQ mass:

doublet of negative parity states: $(B_c, B_c^*) \longrightarrow$ $\boxed{\mathcal{M}(v) = P_+(v)\ [B_c^{*\mu}\,\gamma_\mu - B_c\,\gamma_5]\ P_-(v)}$

$(\eta_c,\ J/\psi) \longrightarrow$ $\boxed{\mathcal{M}'(v') = P_+(v')\Big[\Psi^{*\mu}\gamma_\mu - \eta_c\,\gamma_5\Big]P_-(v')}$

4-plet of positive parity states $(\chi_{c0,1,2}, h_c)$

$$\boxed{\mathcal{M}'^{\mu}(v') = P_+(v')\left[\chi_{c2}^{\mu\nu}\,\gamma_\nu + \frac{1}{\sqrt{2}}\,\chi_{c1,\gamma}\,\epsilon^{\mu\alpha\beta\gamma}\,v'_\alpha\,\gamma_\beta + \frac{1}{\sqrt{3}}\,\chi_{c0}\,(\gamma^\mu - v'^\mu) + h_c^\mu\,\gamma_5\right] P_-(v')} \qquad v'_\mu\,\mathcal{M}'^{\mu} = 0$$

analogous for 2P charmonia

Form Factors in the effective theory

III. trace formalism:

$$\langle C|\bar{Q}'\Gamma D_{\mu_1} D_{\mu_2} \dots Q|B_c\rangle = -\mathrm{Tr}\left[\mathcal{F}_{\mu\,\mu_1\mu_2\dots}\bar{\mathcal{M}}'^{\mu}\Gamma\mathcal{M}\right]$$

$$\Downarrow$$

Universal functions: i.e. the same for all the members of the multiplet of final states

Relations among the various modes can be derived

Form Factors in the effective theory

III. trace formalism: At LO in the HQ expansion all the matrix elements involve a single universal function

$$\boxed{\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\, v_\mu \, \mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]}$$

Form Factors in the effective theory

III. trace formalism: At LO in the HQ expansion all the matrix elements involve a single universal function

$$\boxed{\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\, v_\mu\, \mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]}$$

At $O(1/m_Q)$

$$\langle M'(v')|\bar{\psi}'_+\, \Gamma\, i\, \overrightarrow{D}_\alpha\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(b)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\langle M'(v')|\bar{\psi}'_+\, (-i\, \overleftarrow{D}_\alpha)\, \Gamma\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(c)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\Sigma^{(Q)}_{\mu\alpha} = \Sigma^{(Q)}_1\, g_{\mu\alpha} + \Sigma^{(Q)}_2\, v_\mu\, v_\alpha + \Sigma^{(Q)}_3\, v_\mu\, v'_\alpha + \Sigma^{(Q)}_4\, v_\mu\, \gamma_\alpha + \Sigma^{(Q)}_5\, \gamma_\mu\, v_\alpha + \Sigma^{(Q)}_6\, \gamma_\mu\, v'_\alpha + \Sigma^{(Q)}_7\, i\, \sigma_{\mu\alpha}$$

Constraints:

$$\Sigma^{(b)}_i(w) - \Sigma^{(c)}_i(w) = 0 \qquad\qquad i = 1, 4, 5, 6, 7$$

$$\Sigma^{(b)}_2(w) - \Sigma^{(c)}_2(w) = \tilde{\Lambda}\, \Xi\,,$$

$$\Sigma^{(b)}_3(w) - \Sigma^{(c)}_3(w) = -\tilde{\Lambda}'\, \Xi(w)\,.$$

Form Factors in the effective theory

III. trace formalism: At LO in the HQ expansion all the matrix elements involve a single universal function

$$\boxed{\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\, v_\mu\, \mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]}$$

At $O(1/m_Q)$

$$\langle M'(v')|\bar{\psi}'_+\, \Gamma\, i\, \overrightarrow{D}_\alpha\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(b)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\langle M'(v')|\bar{\psi}'_+\, (-i\, \overleftarrow{D}_\alpha)\, \Gamma\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(c)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

At $O(1/m_Q)^2$

$$\langle M'(v')|\bar{\psi}'_+\, \Gamma\, i\, \overrightarrow{D}_\alpha\, i\, \overrightarrow{D}_\beta\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Omega^{(b)}_{\mu\alpha\beta}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\langle M'(v')|\bar{\psi}'_+\, i\, \overleftarrow{D}_\alpha\, i\, \overleftarrow{D}_\beta\, \Gamma\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Omega^{(c)}_{\mu\alpha\beta}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

Constraints:

$$\Omega^{(b)}_{\mu\alpha\beta} - \Omega^{(c)}_{\mu\alpha\beta} = \left(\tilde{\Lambda}\, v_\alpha - \tilde{\Lambda}'\, v'_\alpha\right) \Sigma^{(b)}_{\mu\beta} + \left(\tilde{\Lambda}\, v_\beta - \tilde{\Lambda}'\, v'_\beta\right) \Sigma^{(c)}_{\mu\alpha}$$

Other corrections stem from the expansion of the states
(non-local corrections)

$$\langle M'(v')|i\int \mathrm{d}^4x\, \mathrm{T}\,[J_0(0),\mathcal{L}_1(x)]\,|M(v)\rangle =$$

$$-\frac{1}{4\,m_b}\underbrace{\left(-\frac{i}{2}\right)\mathrm{Tr}\big[\Upsilon^{(b)}_{2\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,P_+\,\sigma^{\alpha\beta}\,\mathcal{M}\big]}_{G^{(b)}}-\frac{1}{2\,m_b^2}\underbrace{\mathrm{Tr}\big[\Upsilon^{(b)}_{1\mu}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\big]}_{K^{(b)}},$$

$$\langle M'(v')|i\int \mathrm{d}^4x\, \mathrm{T}\,[J_0(0),\mathcal{L}'_1(x)]\,|M(v)\rangle =$$

$$-\frac{1}{4\,m_c}\underbrace{\left(-\frac{i}{2}\right)\mathrm{Tr}\big[\Upsilon^{(c)}_{2\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\sigma^{\alpha\beta}\,P'_+\,\Gamma\,\mathcal{M}\big]}_{G^{(c)}}-\frac{1}{2\,m_c^2}\underbrace{\mathrm{Tr}\big[\Upsilon^{(c)}_{1\mu}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\big]}_{K^{(c)}},$$

→ Other universal functions are involved

$B_c \to J/\psi$, η_c Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of the same decay mode

$B_c \to J/\psi$

$$h_{T_1}(w) = \frac{1}{2}\Big((1+w)h_{A_1}(w) - (w-1)h_V(w)\Big)$$

$$h_{T_2}(w) = \frac{1+w}{2(m_b+3m_c)}\Big((m_b-3m_c)h_{A_1}(w) + 2m_c(h_{A_2}(w)+h_{A_3}(w)) - (m_b-m_c)h_V(w)\Big)$$

$$h_{T_3}(w) = h_{A_3}(w) - h_V(w)$$

$$h_P(w) = \frac{1}{m_b+3m_c}\Big((1+w)\,(m_b h_{A_1}(w) + 2m_c h_V(w)) + (-m_b + (w-2)m_c)\,h_{A_2}(w) - (w\,m_b + (2w-1)m_c)\,h_{A_3}(w)\Big)$$

$B_c \to \eta_c$

$$h_-(w) = \frac{m_b-m_c}{2(m_b+3m_c)}(1+w)\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big)$$

$$h_T(w) - h_+(w) = -\frac{m_b+m_c}{2(m_b+3m_c)}(1+w)\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big)$$

$$h_T(w) - h_S(w) = -\frac{m_b+m_c}{(m_b+3m_c)}\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big).$$

can all be related to

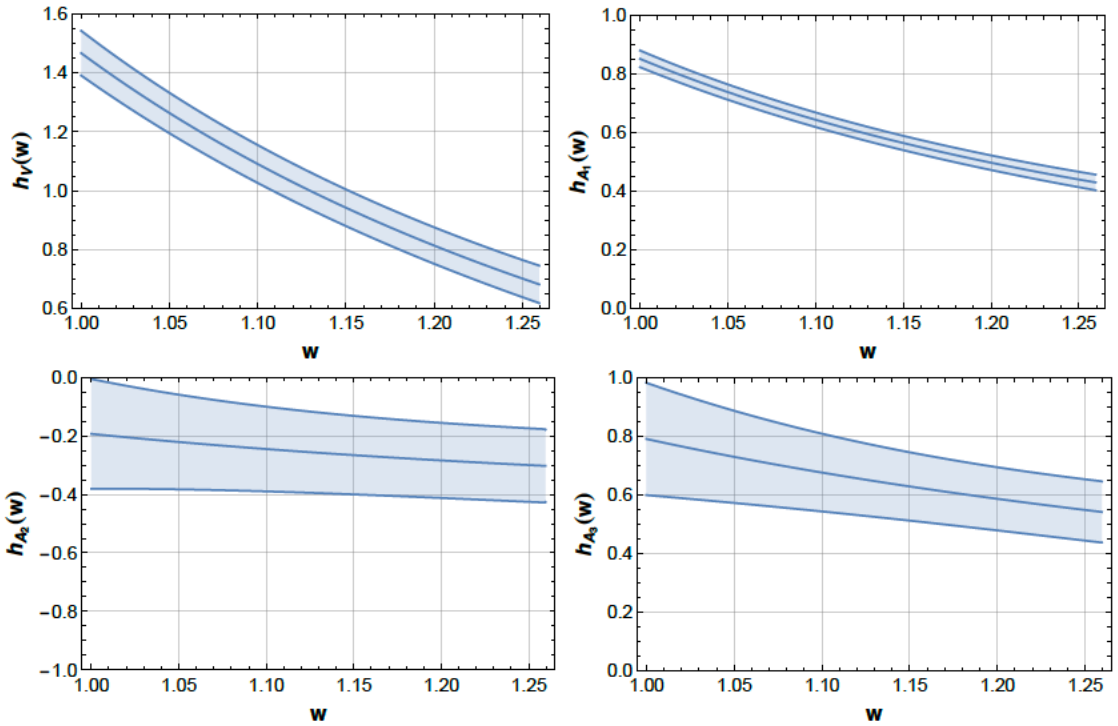
$h_V(w)$
$h_{A1}(w)$ $h_{A2}(w)$ $h_{A3}(w)$

P.Colangelo, F. Loparco, N. Losacco, M. Novoa Brunet, FDF
arXiv:2205.08933,
JHEP09 (2022) 028

Bc →J/ψ, η_c Form Factors in the effective theory: relations at $O(1/m_Q)$

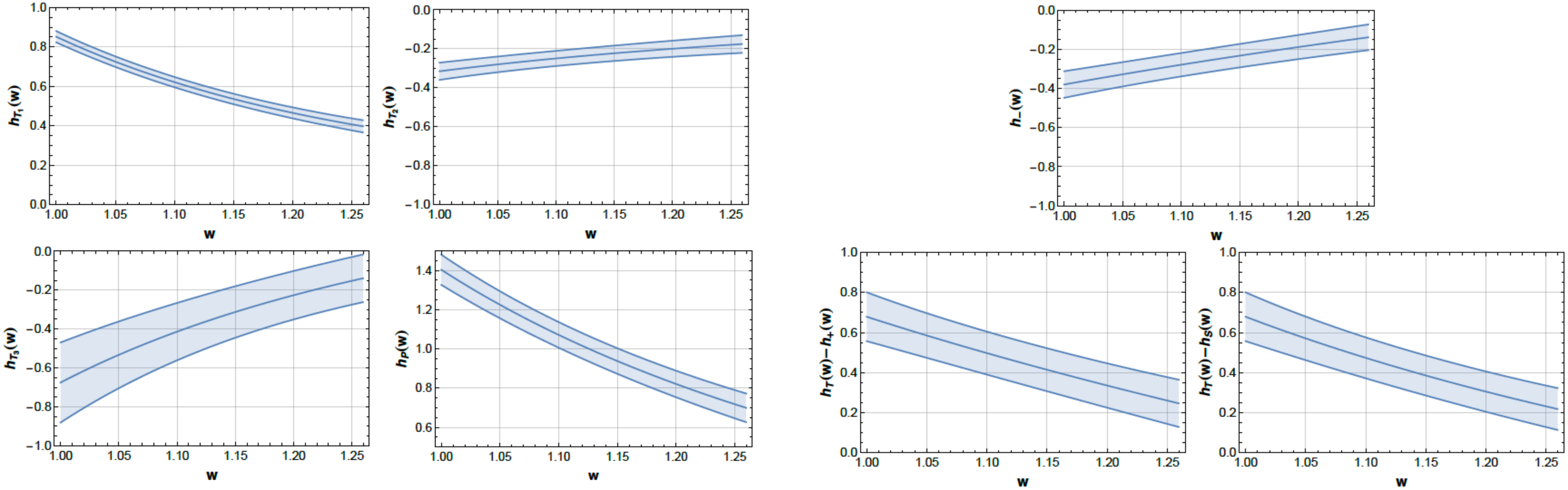
available lattice results

HPQCD Collab.
arXiv:2007.06957



$B_c \to J/\psi$, η_c Form Factors in the effective theory: relations at $O(1/m_Q)$

results



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$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of the same decay mode

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M. Novoa Brunet, FDF
arXiv:2208.13398

- $B_c \to \chi_{c0}$

$$g_T(w) = -\frac{1}{w+1}\left[2g_-(w) + g_P(w)\right]$$

- $B_c \to \chi_{c1}$

$$g_{T_2}(w) = -\frac{1}{2}\left[g_{V_1}(w) - (1+w)g_A(w)\right]$$

$$g_{T_3}(w) = -\frac{1}{2(w-1)}\left[g_{V_1}(w) + 4g_{V_2}(w)\right] + \frac{1}{2}g_A(w) + \frac{1}{w-1}\left[g_S(w) + g_{T_1}(w)\right]$$

- $B_c \to \chi_{c2}$

$$k_{T_1}(w) = -wk_V(w) + k_{A_2}(w) + wk_{A_3}(w) + k_P(w)$$

$$k_{T_2}(w) = k_V(w) - k_{A_1}(w) - k_{A_2}(w) - wk_{A_3}(w) - k_P(w)$$

$$k_{T_3}(w) = -k_V(w) + k_{A_3}(w)$$

- $B_c \to h_c$

$$f_{T_2}(w) = \frac{1}{2}\left[f_{V_1}(w) + (1+w)f_A(w)\right]$$

$$f_{T_3}(w) = \frac{1}{2(w-1)}\left[f_{V_1}(w) + 4f_{V_2}(w)\right] + \frac{1}{2}f_A(w) - \frac{1}{w-1}\left[f_S(w) - f_{T_1}(w)\right]$$

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of pairs of decay modes

- $B_c \to \chi_{c0}$ and $B_c \to \chi_{c1}$

$$
\begin{aligned}
&(w+1)g_+(w) - (w-1)g_-(w) + g_P(w) = \\
&\quad \frac{w+1}{\sqrt{6}}\left\{2g_{V_1}(w) + (w+1)g_{V_2}(w) - (w-1)\left[g_{V_3}(w) + g_A(w)\right] - g_S(w) + 2g_{T_1}(w)\right\}
\end{aligned}
$$

- $B_c \to h_c$ and $B_c \to \chi_{c1}$

$$
\begin{aligned}
&f_{V_1}(w) + (w-1)f_A(w) - 2f_{T_1}(w) = \\
&\quad \sqrt{2}\left\{g_{V_1}(w) + (w+1)g_{V_2}(w) - (w-1)g_{V_3}(w) - g_S(w)\right\} \\
&3f_{V_1}(w) + 2(w+1)f_{V_2}(w) - (w-1)\left[2f_{V_3}(w) - f_A(w)\right] - 2\left[f_S(w) + f_{T_1}(w)\right] = \\
&\quad \sqrt{2}\left\{g_{V_1}(w) - (w-1)g_A(w) + 2g_{T_1}(w)\right\}
\end{aligned}
$$

Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) Form Factors in the effective theory: relations at LO

$$g_+(w) = 0$$

$$g_S(w) = g_{T_1}(w) = 0$$

$$k_{A_2}(w) = k_{T_3}(w) = 0$$

$$f_{V_1}(w) = f_{V_3}(w) = f_A(w) = f_{T_1}(w) = f_{T_2}(w) = 0$$

χ_{c0} (red)

χ_{c1} (blue)

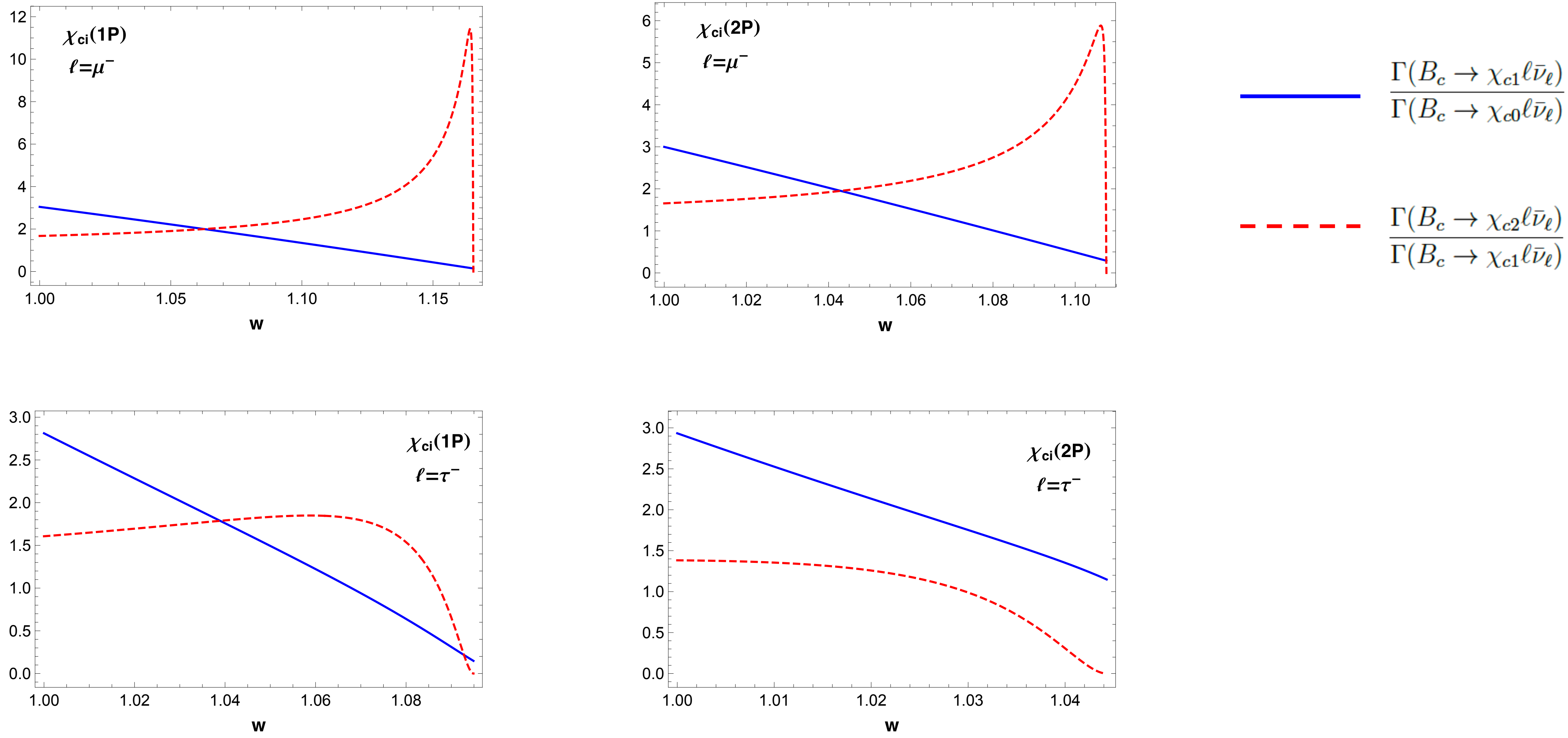
χ_{c2} (orange)

h_c (pink)

$$\begin{aligned}
\Xi(w) &= \frac{\sqrt{3}}{(w+1)} g_-(w) = -\frac{\sqrt{3}}{(w+1)} g_T(w) = \frac{\sqrt{3}}{(w^2-1)} g_P(w) \\
&= \frac{\sqrt{2}}{(w^2-1)} g_{V_1}(w) = -\frac{2\sqrt{2}}{(w-1)} g_{V_2}(w) = \frac{2\sqrt{2}}{(w+1)} g_{V_3}(w) = \frac{\sqrt{2}}{(w+1)} g_A(w) = \frac{\sqrt{2}}{(w+1)} g_{T_2}(w) \\
&= -k_V(w) = \frac{1}{w+1} k_{A_1}(w) = -k_{A_3}(w) = -k_P(w) = -k_{T_1}(w) = -k_{T_2}(w) \\
&= -f_{V_1}(w) = -f_{V_2}(w) = -\frac{1}{w+1} f_S(w) = f_{T_3}(w)
\end{aligned}$$

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

$$\frac{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c0}\ell\bar{\nu})/dw} \qquad \frac{d\Gamma(B_c\to\chi_{c2}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}$$



Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) exploiting FF relations at LO

- Constraint that holds at LO both in SM and for generic NP

$$\boxed{2\frac{d\Gamma}{dw}(B_c \to \chi_{c0}\ell\bar{\nu}_\ell) + \frac{d\Gamma}{dw}(B_c \to \chi_{c1}\ell\bar{\nu}_\ell) - \frac{d\Gamma}{dw}(B_c \to \chi_{c2}\ell\bar{\nu}_\ell) = 0.}$$

has to be satisfied by the three members of the 4-plet

Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) exploiting FF relations at LO

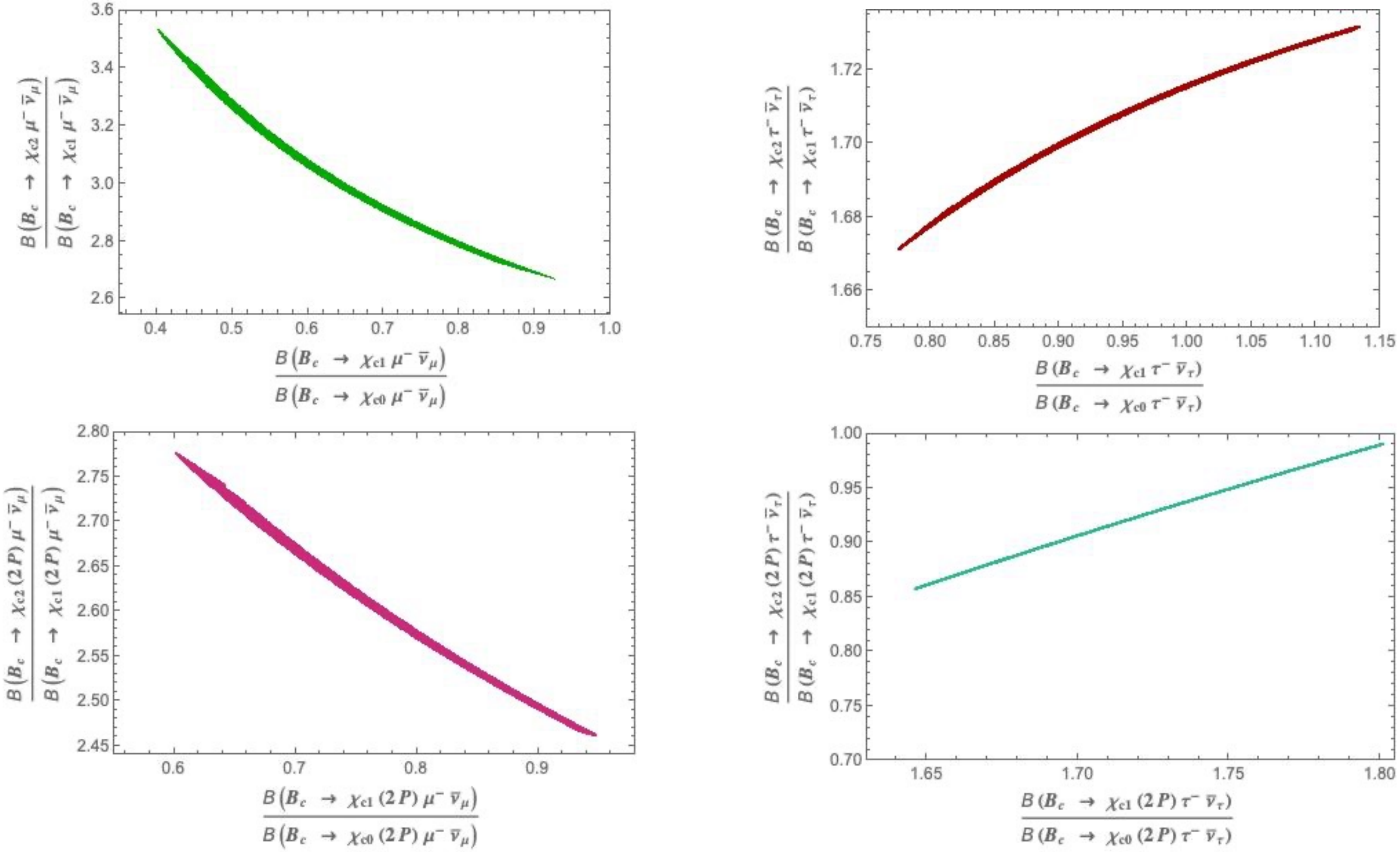
Parametrization: $\boxed{\Xi(w) = \Xi_0 + \Xi_1(w-1) + \Xi_2(w-1)^2}$

$$\Xi_0 \in [0.1, 1],\ \Xi_1 \in [-1, 0] \text{ and } \Xi_2 \in [-1, 1]$$

Varied to reproduce

$$\mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) = (2.4\pm^{0.9}_{0.8}) \times 10^{-5}$$

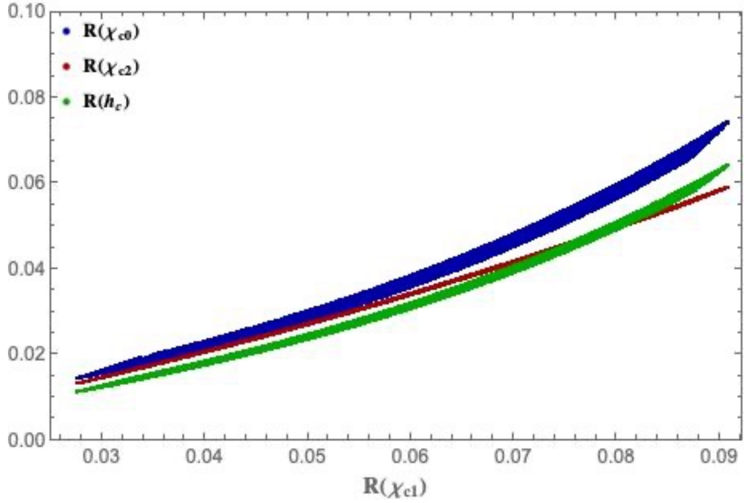
Correlations predicted:

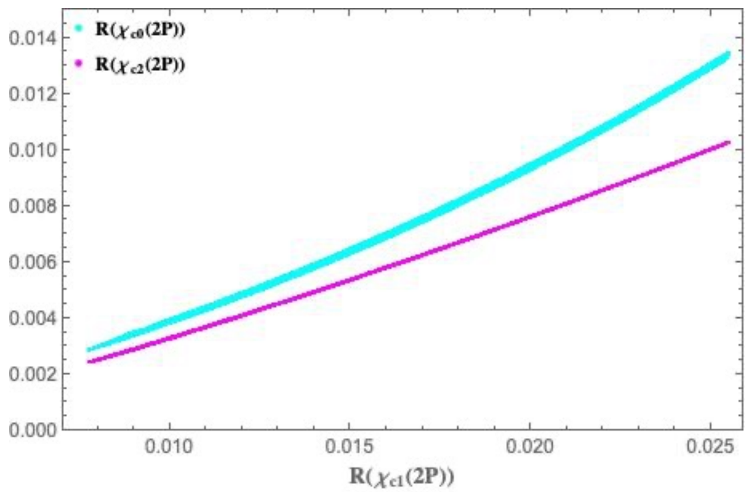


$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

Tests of LFU:

$$\boxed{R(C) = \Gamma(B_c \to C\tau\bar{\nu}_\tau)/\Gamma(B_c \to C\ell\bar{\nu}_\ell)}$$





$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at NLO

At NLO the number of universal functions increase. However:

- They enter in different modes, so model independent predictions are obtained
- They can be used also in other processes, i.e. non leptonic transitions
- Being model independent they serve as test of other approaches:
  the outcome should satisfy the effective theory predictions
- Once reliable determinations for a few form factors is available (i.e. by lattice) the others can be predicted
- a reduced number of structures contributes close to w=1:

$$\lim_{w\to 1} \frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c \to \chi_{c0}\ell\bar\nu_\ell) = 18\,\hat m_\ell^2(\epsilon_b+\epsilon_c)^2\left[\Sigma^{(b)}_{\chi_{c1},1}(1)\right]^2$$

$$\lim_{w\to 1} \frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c \to \chi_{c1}\ell\bar\nu_\ell) = 12\left[2(1-r_1)^2+\hat m_\ell^2\right]\left[\epsilon_b\Sigma^{(b)}_{\chi_{c1},1}(1)-\epsilon_c\Sigma^{(c)}_{\chi_{c1},1}(1)\right]^2$$

$$\lim_{w\to 1} \frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c \to h_c\ell\bar\nu_\ell) = 6\left[2(1-r_h)^2+\hat m_\ell^2\right]\left[(\epsilon_b-\epsilon_c)\Sigma^{(b)}_{\chi_{c1},1}(1)+2\epsilon_c\Sigma^{(c)}_{\chi_{c1},1}(1)\right]^2$$

$$\hat m_\ell^2 = \frac{m_\ell^2}{m_{B_c}^2}$$

$$r = m_C/m_{B_c} \qquad C = m_{\chi_{c0}},\, \chi_{c1},\, \chi_{c2},\, h_c$$

$$\epsilon_b = \frac{1}{2m_b} \qquad \epsilon_c = \frac{1}{2m_c}$$

If X(3872) is $\chi_{c1}(2P)$ it should fulfill these relations (though hard to test…)

Conclusions

Bc decays represent a further testing ground for:

- the determination of V_{cb}
- flavour anomalies
- probing the structure of the hadrons in the final state

Predictions based on NRQCD + HQE allow to

- Relate FFs among themselves
- Work out relations to be fulfilled by hadrons in the final state related by HQSS

  - reduction of the theoretical uncertainty
  - model independent results: possibility to test the outcome of other approaches