Effects of fireball sizes and shapes and critical fluctuations on light-nuclei production in heavy-ion collisions

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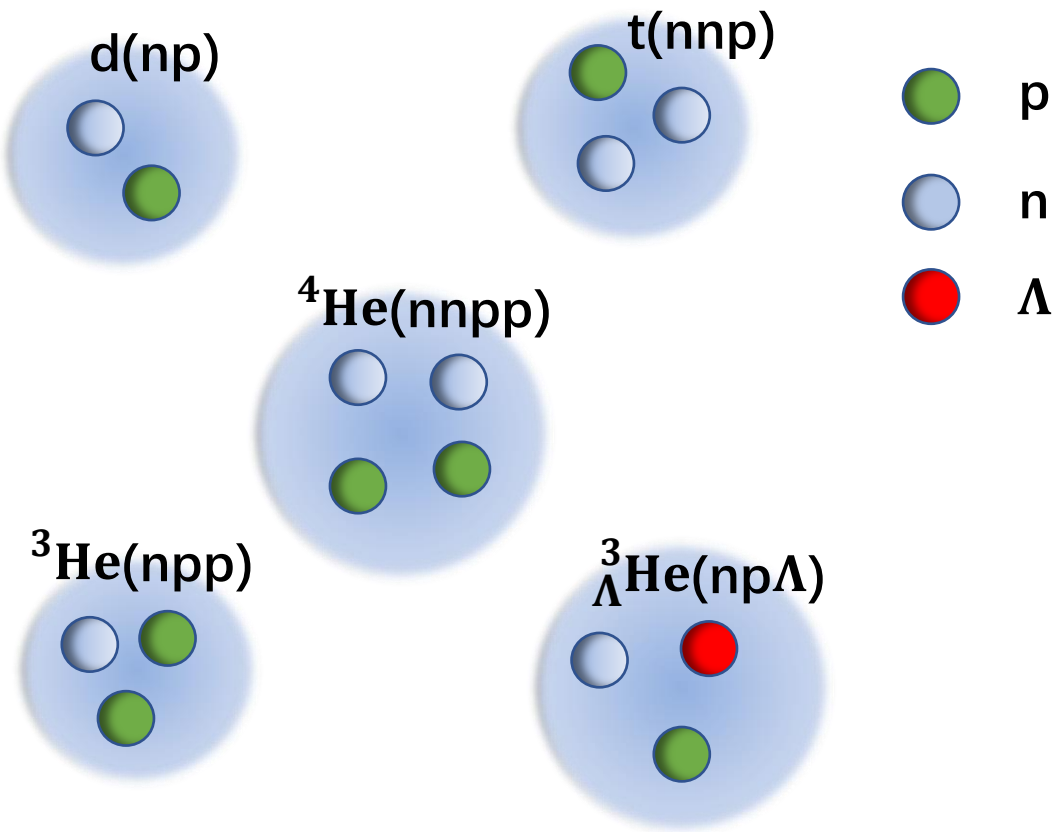
Light Nuclei Cluster

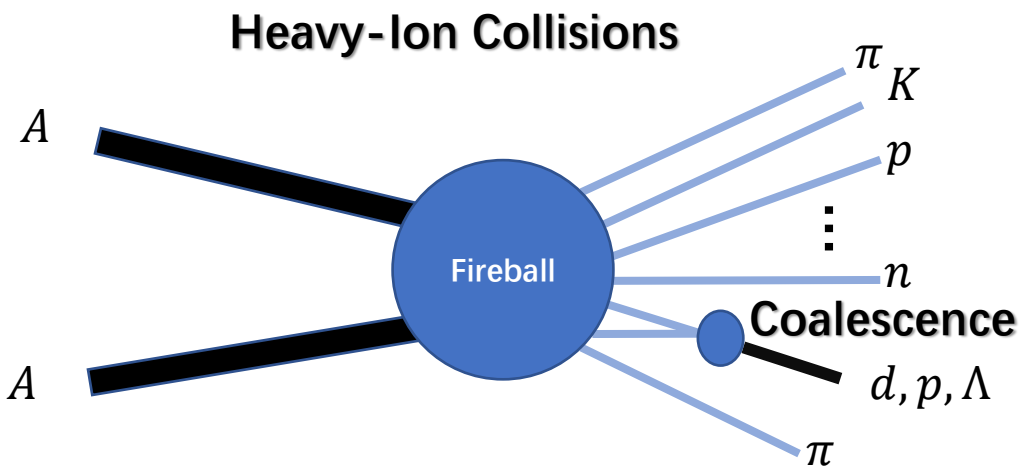


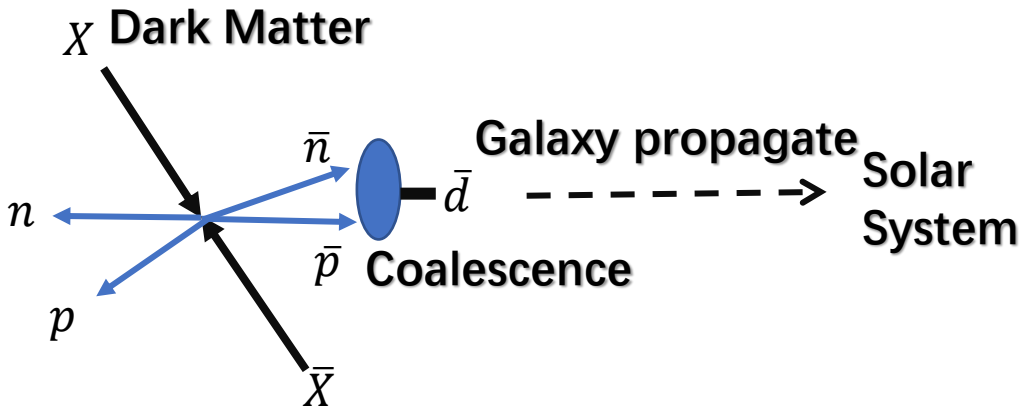
Loosely bounded objects (~MeV)

Nucleons close each other in phase-space (homogeneous):

- **Phase-space**
- nucleons interaction

Coalescence is widely used model



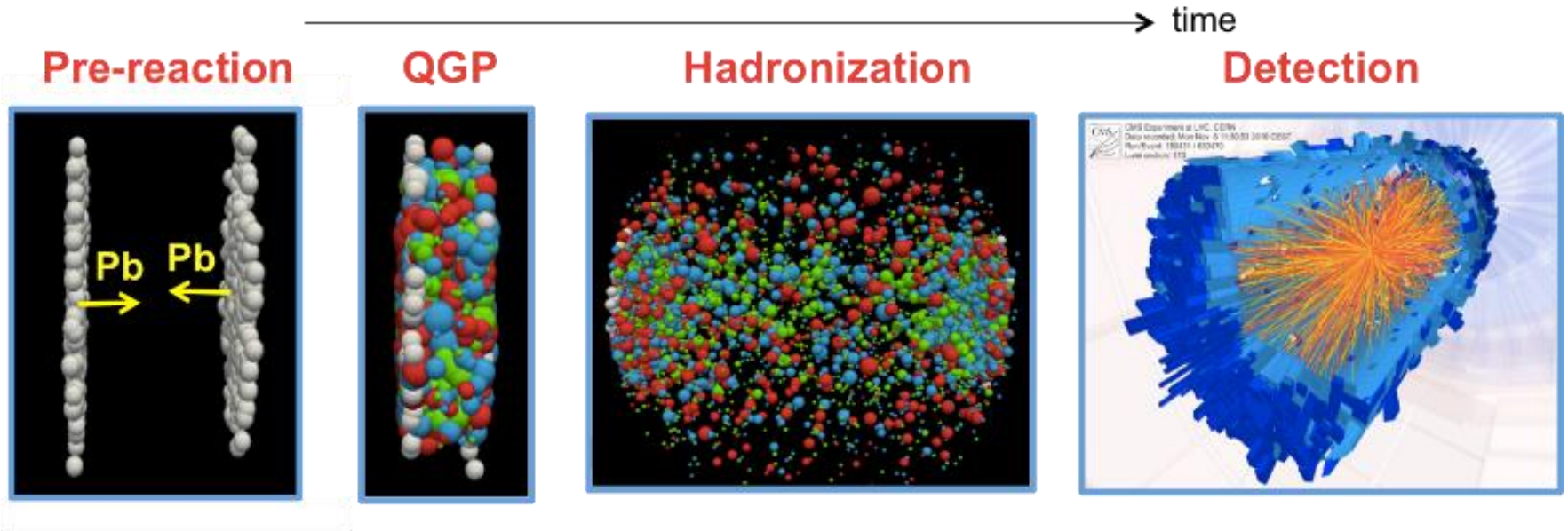


Examples in Heavy-Ion Collisions

- **quark + quark -> hardon**
- **proton + neutron -> light nuclei**

Anti Light nuclei as Indirect detection of Dark Matter

Heavy-Ion collisions



Heavy-Ion Collisions

- **Quark-Gluon Plasma formed**
- **Lower collision energy, higher baryon chemical potential**

QCD phase diagram

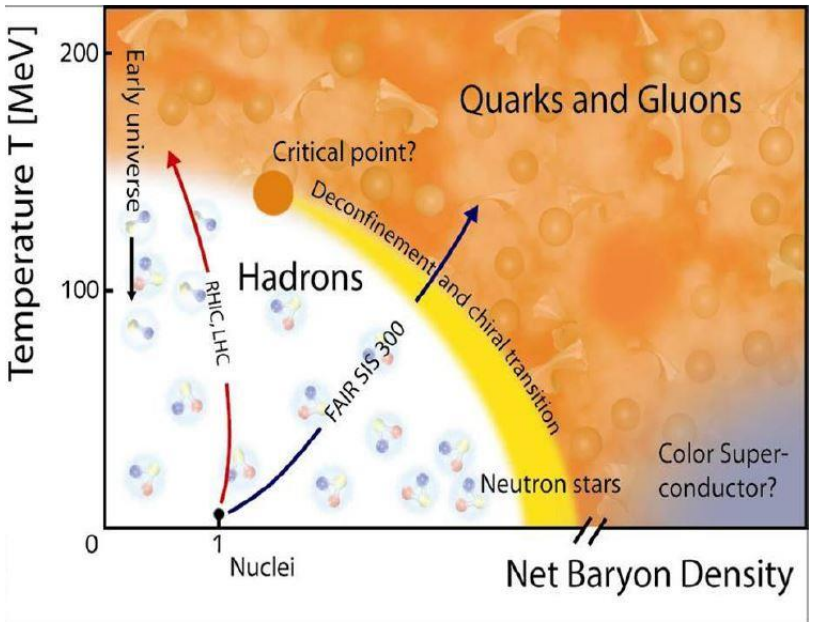
- Lattice QCD (small μ_B finite T):
  - Crossover
- Effective models(large μ_B)
  - 1st order phase trans.

→ Critical point

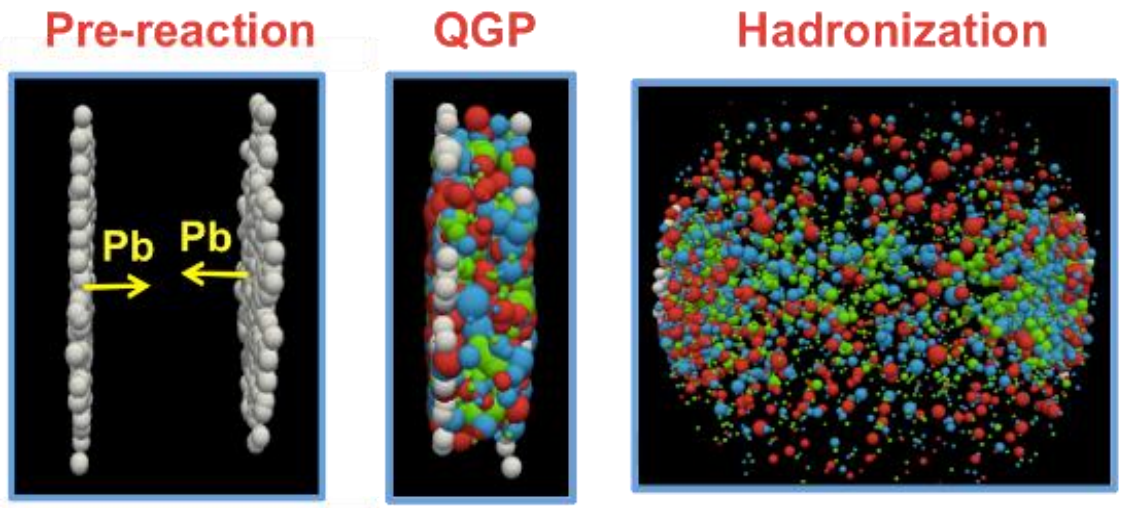
- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

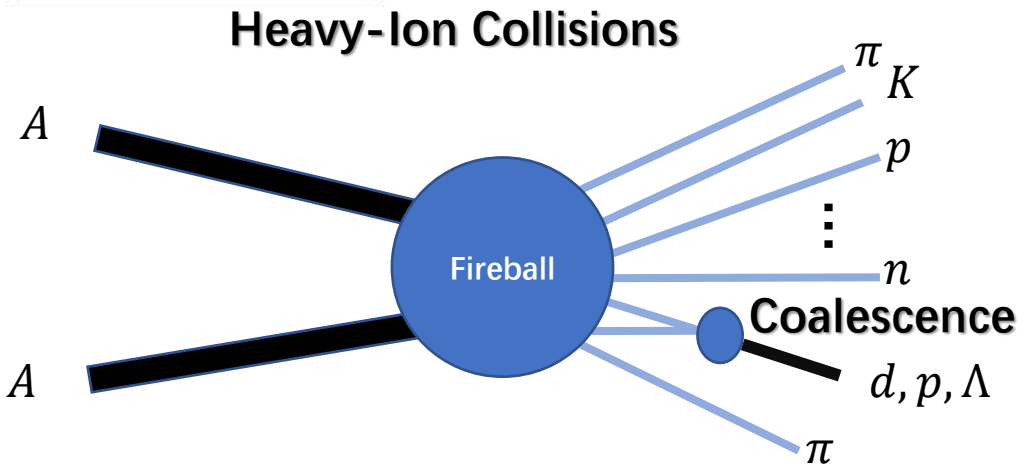
→ Heavy-ion collisions :

  - Changing collision energy, mapping $T-\mu$: RHIC(BES),NICA,FAIR,J_PARC⋯.

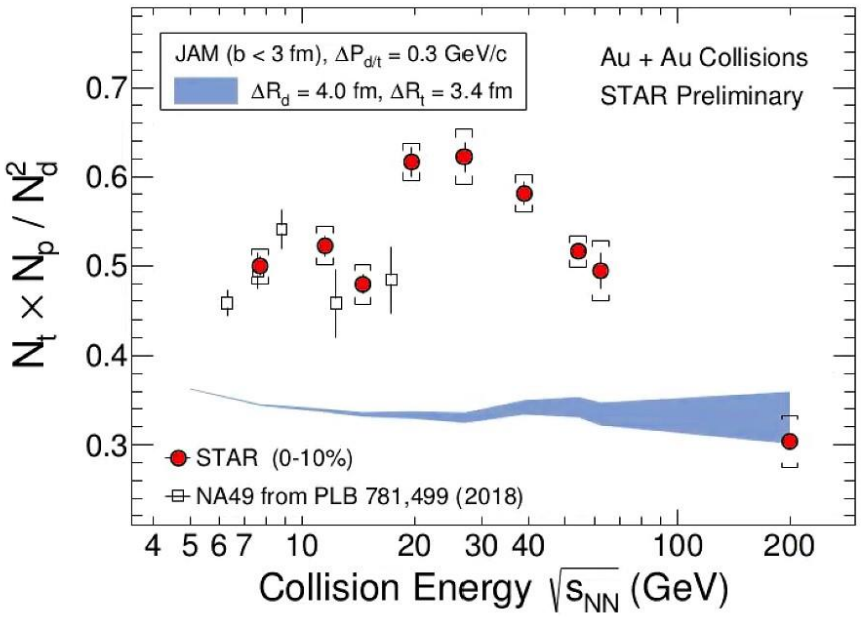


Light nuclei in heavy ion collisions



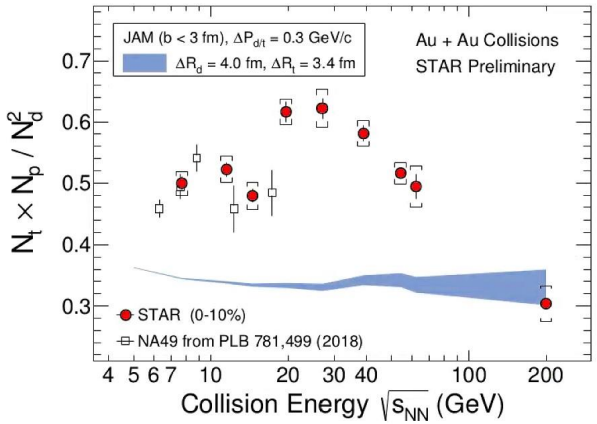


H. Liu et al., Phys. Lett. B805, 135452 (2020)

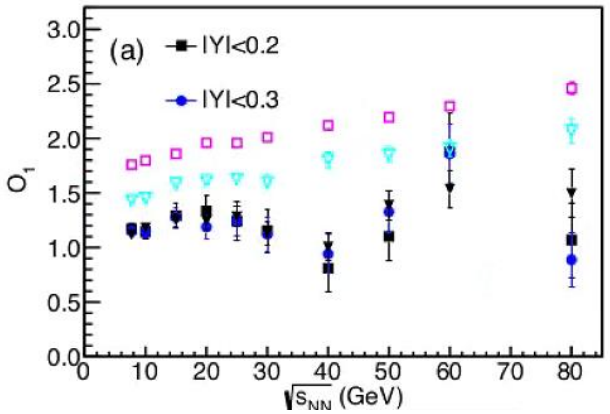


- Light nuclei formed at late stage
- Light nuclei yield ratio shows non-monotonic behavior

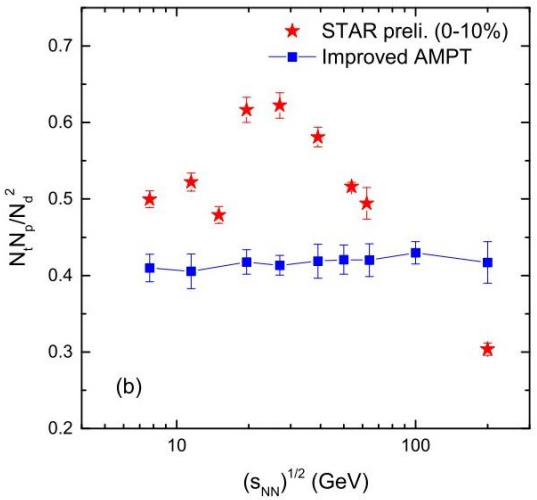
Current models Can't describe the data



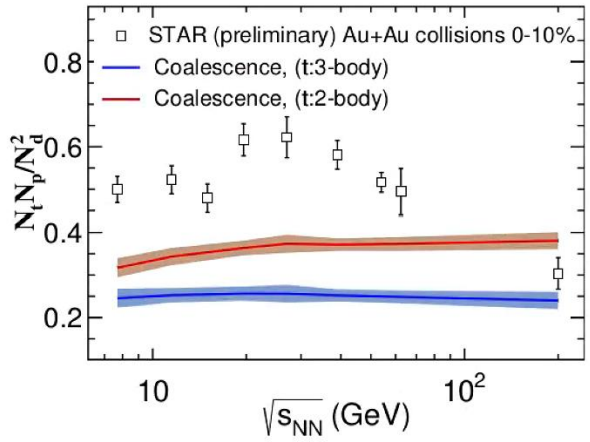
Hui Liu et al., PLB (2020)



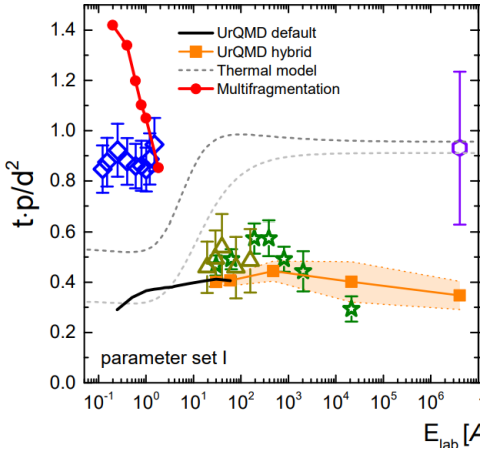
X.Deng et al., PLB (2020)



K.Sun et al., PRC (2021)



W. Zhao et al., PRC (2018)



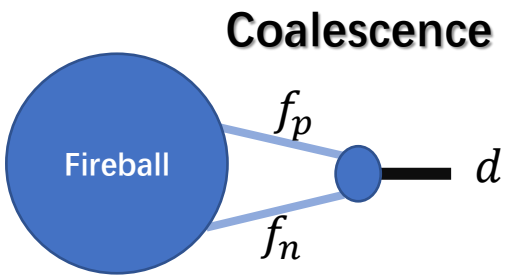
P.Hillmann et al., 2109.05972

And others....

Phase-space produced in HIC

No clear non-monotonic on the model so far

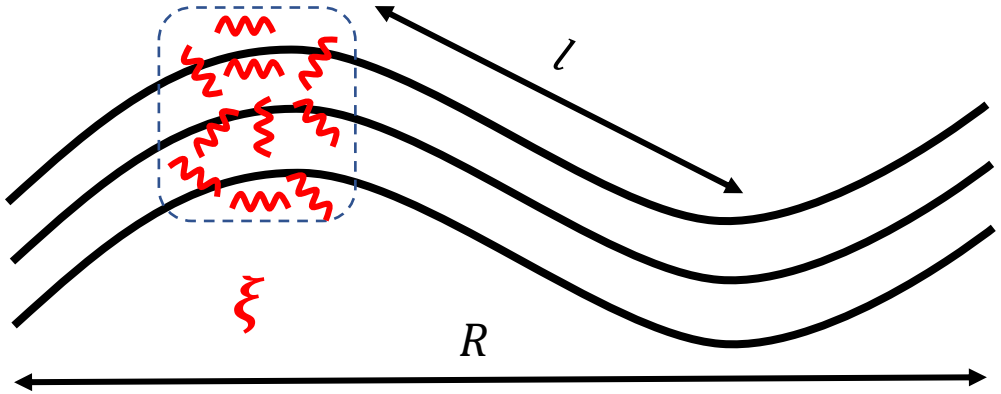
Can light nuclei detect critical point effects?



Nucleons close to each other in r space have similar momentum p
=>Homogeneity length $l \sim 1/\partial_\mu u^\mu$

$R, l \gg \xi$, when not so close to critical regime.

Background is large for N_A



$\boldsymbol{R}$: Fireball size
$\boldsymbol{l}$:homogeneity length
ξ: correlation length

Light Nuclei Yield Ratio (Background~~+Critical~~):

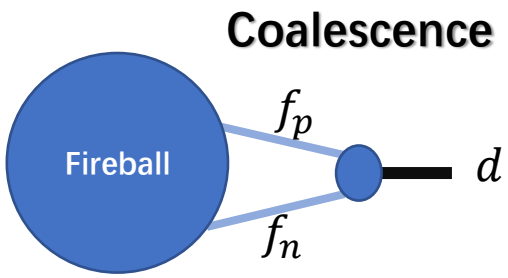
Canceling the background

SW, K.Murase, S.Tang, H.Song, 2205.14302

Coalescence model (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$



Similar Coalescence Model for Dark Matter search

$$F_{\bar{d}}(\sqrt{s}, \vec{k}_{\bar{d}}) = \int F_{(\bar{p}\bar{n})}(\sqrt{s}, \vec{k}_{\bar{p}}, \vec{k}_{\bar{n}})\, \mathcal{C}(\sqrt{s}, \vec{k}_{\bar{p}}, \vec{k}_{\bar{n}} | \vec{k}_{\bar{d}})\, d^3\vec{k}_{\bar{n}}\, d^3\vec{k}_{\bar{n}}$$

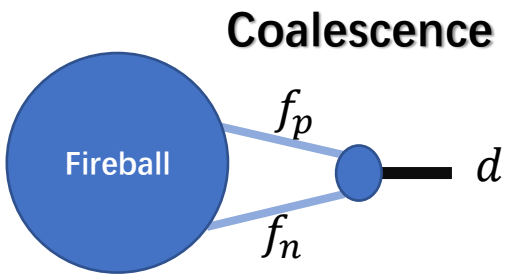
Coalescence model (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

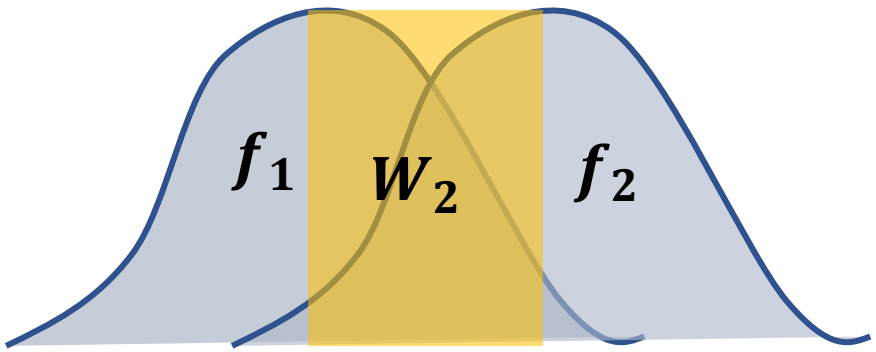
$$N_A = g_A \int$$

Phase space density

Wigner function



- Wigner function(probability to produce the light nuclei): depends on the relative distance of nucleons in phase space



Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

$$N_A = g_A \int$$

Phase space density

Wigner function

Gauss → Constant

Non-Gauss → Non-trivial

$$\varphi_X(t) = E[e^{itX}] = 1 + \frac{it\,E[X]}{1} - \frac{t^2 E[X^2]}{2!} + \cdots + \frac{(it)^n E[X^n]}{n!})$$

- Wigner function: only relative distance of nucleons
- => N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

$$N_A = g_A \int \boxed{\text{Phase space density}} \boxed{\text{Wigner function}}$$

Gauss

Non-Gauss

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}} \right]^{A-1} \cdot [1 + \mathcal{O}(\{\mathcal{C}_\alpha\}_{|\alpha|\geq 3})]$$

Fireball size Homog. Length

$$C_2 = 2 \begin{pmatrix} \frac{\langle \boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} \rangle}{\sigma_A^2} & \langle \boldsymbol{r}\boldsymbol{p}^{\mathrm{T}} \rangle \\ \langle \boldsymbol{p}\boldsymbol{r}^{\mathrm{T}} \rangle & \sigma_A^2 \langle \boldsymbol{p}\boldsymbol{p}^{\mathrm{T}} \rangle \end{pmatrix}$$

$$\langle r\, r^T \rangle \sim \int dr dp\, f(r,p) r r^T \sim R^2$$

$$\langle r\, p^T \rangle \text{ relates to } \frac{1}{\partial_\mu u^\mu} \sim l$$

Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

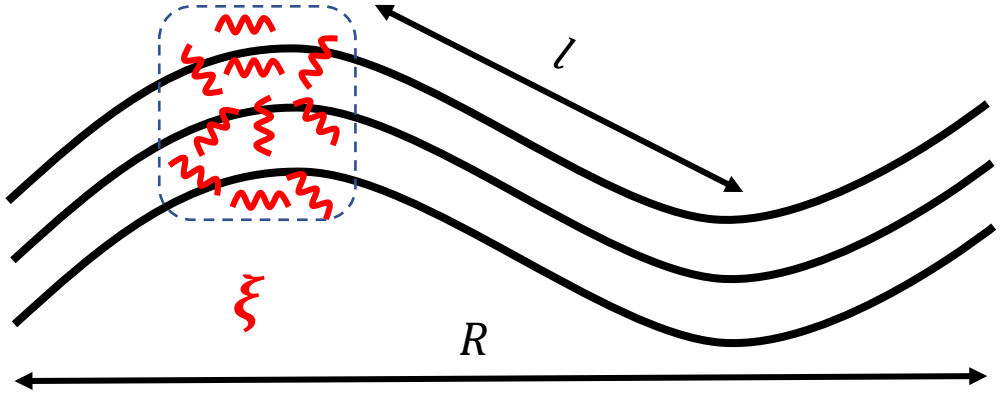
$$N_A = g_A \int \boxed{\text{Phase space density}} \boxed{\text{Wigner function}}$$

Gauss | Non-Gauss

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}} \right]^{A-1} \cdot [1 + \mathcal{O}(\{\mathcal{C}_\alpha\}_{|\alpha|\geq 3})]$$

Fireball size Homog. Length

$$C_2 = 2 \begin{pmatrix} \frac{\langle \boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} \rangle}{\sigma_A^2} & \langle \boldsymbol{r}\boldsymbol{p}^{\mathrm{T}} \rangle \\ \langle \boldsymbol{p}\boldsymbol{r}^{\mathrm{T}} \rangle & \sigma_A^2 \langle \boldsymbol{p}\boldsymbol{p}^{\mathrm{T}} \rangle \end{pmatrix}$$



R : Fireball size
l:homogeneity length
ξ: correlation length

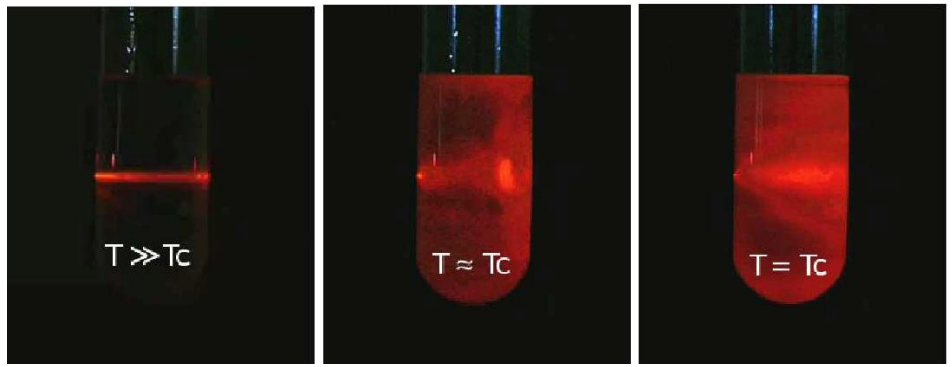
Light Nuclei Ratio Near QCD Critical Point:

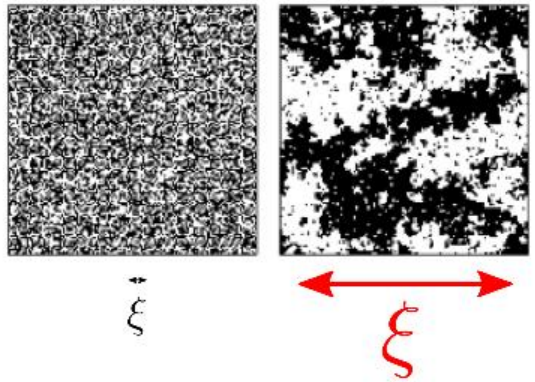
(Background+Critical)

SW, K.Murase, S.Zhao, H.Song, to appear

Properties of critical point

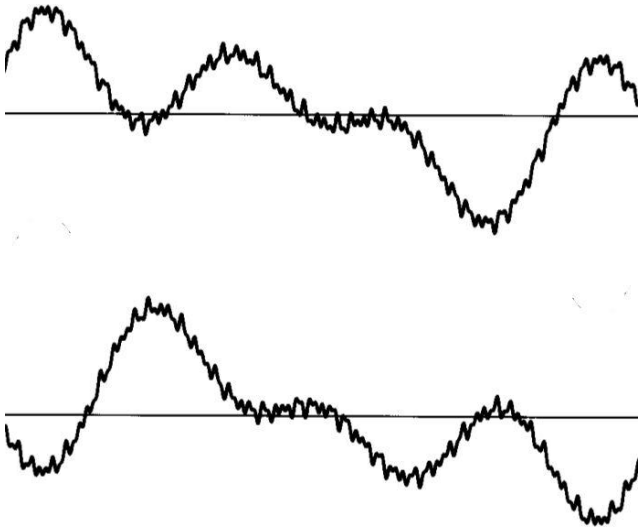
- Long range correl. (e.g., critical opalescence)
- Singularity
- Universal scaling
- Critical slowing down





- Large fluctuations

$$f = f_0 + \delta f$$



Critical contribution δf in phase-space

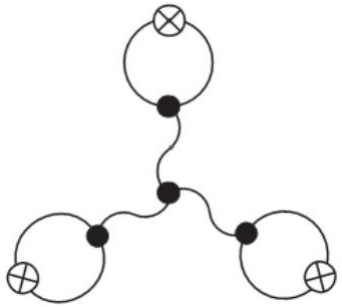
SW, K.Murase, S.Zhao, H.Song, to appear

$$N_A \sim \langle (f_0 + \delta f)^A \rangle_\sigma \sim f_0^A + \langle (\delta f)^2 \rangle_\sigma^{\beta_2} + \langle (\delta f)^3 \rangle_\sigma^{\beta_3} + \cdots + \langle (\delta f)^A \rangle_\sigma^{\beta_4}$$

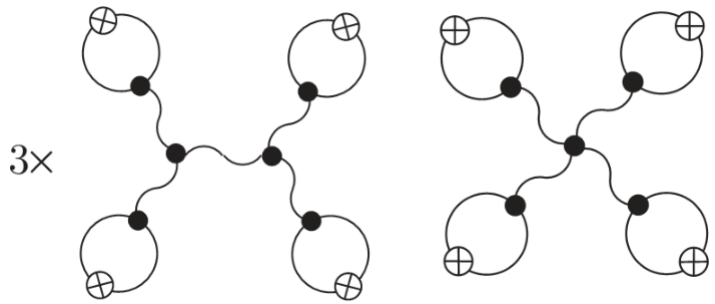
Critical δf: A constituent nucleons relates to 2,3, … A-point critical correlator

$\langle \delta f_1 \delta f_2 \rangle_\sigma \sim \Xi(A, 2)$

$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma \sim \Xi(A, 3)$



$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma \sim \Xi(A, 4)$



Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, to appear

$$N_A = g_A \int$$

Phase space density: Background+Critical

Wigner func

Characteristic func.

Fourier transf.

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}}\right]^{A-1} \cdot [1 + \sum_{i=2}^{A} \Xi(A,i)]$$

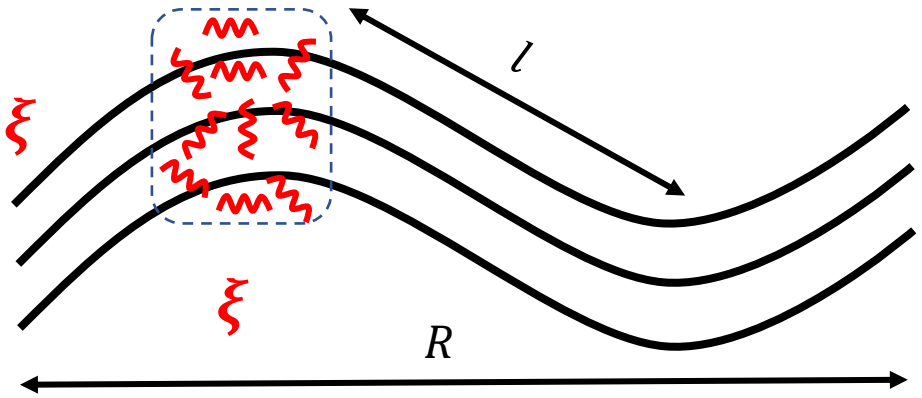
Scale of ξ

Scales of R, l

Bkg

Cri

N_A share a common structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$
=> The ratios of N_A cancel Bkg and highlight Cri



R : Fireball size
l:homogeneity length
ξ: correlation length

Light nuclei yield: Background+Critical

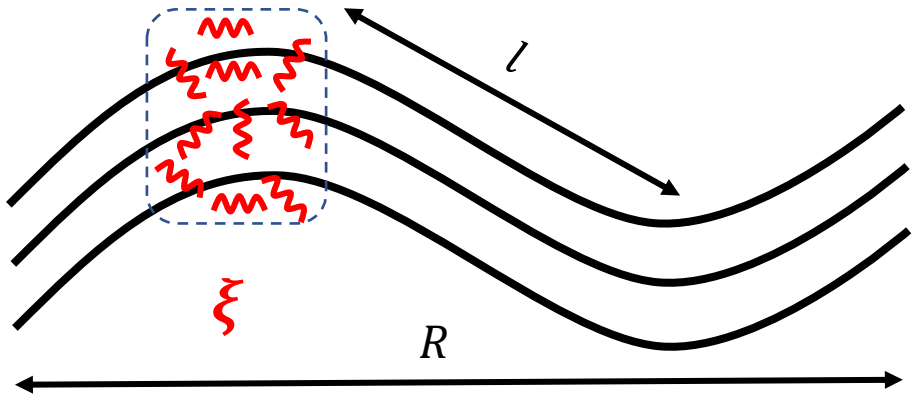
SW, K.Murase, S.Zhao, H.Song, to appear

$$R_{A,B}^{1-B,A-1} = \frac{N_p^{B-A} N_B^{A-1}}{N_A^{B-1}}$$

$$\tilde{R}(A,B) \equiv R_{A,B}^{1-B,A-1} - g_B^{A-1} g_A^{-(B-1)} \sim \mathcal{O}(\xi)$$

$$\tilde{R}(A,B,D) \equiv R_{A,B}^{1-B,A-1} - g_B^{A-1} g_D^{-(A-1)(B-1)/(D-1)} [R_{A,D}^{1-D,A-1}]^{(B-1)/(D-1)} \sim \mathcal{O}(\xi)$$

N_A share a common structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$
=> The ratios of N_A cancel Bkg and highlight Cri

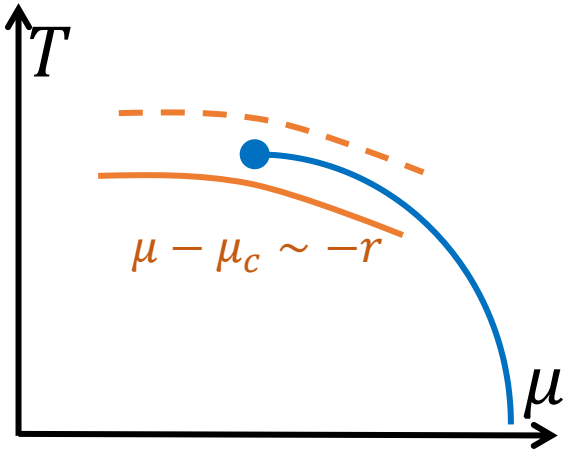


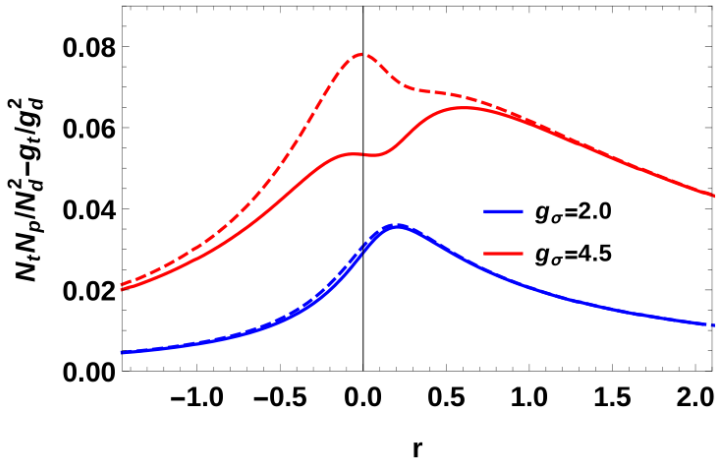
R : Fireball size
l:homogeneity length
ξ: correlation length

The ratios of N_A proportional to Cri
=> In the ratios of N_A, large scales R, L are unimportant but ξ matters

Example: in the Ising critical regime

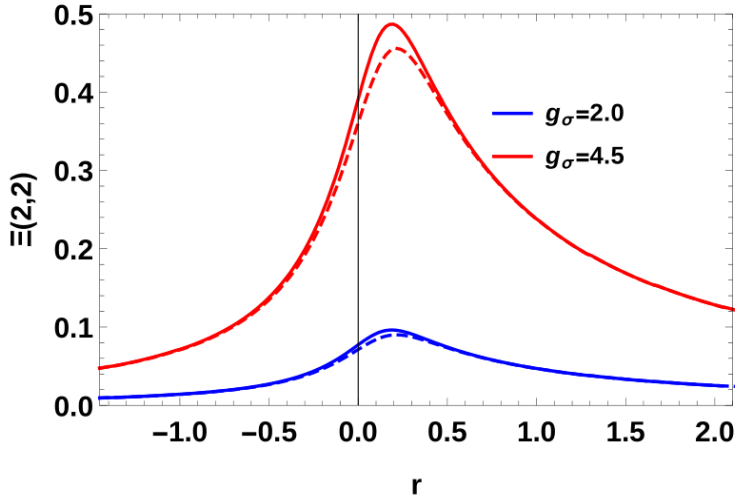
SW, K.Murase, S.Zhao, H.Song, to appear

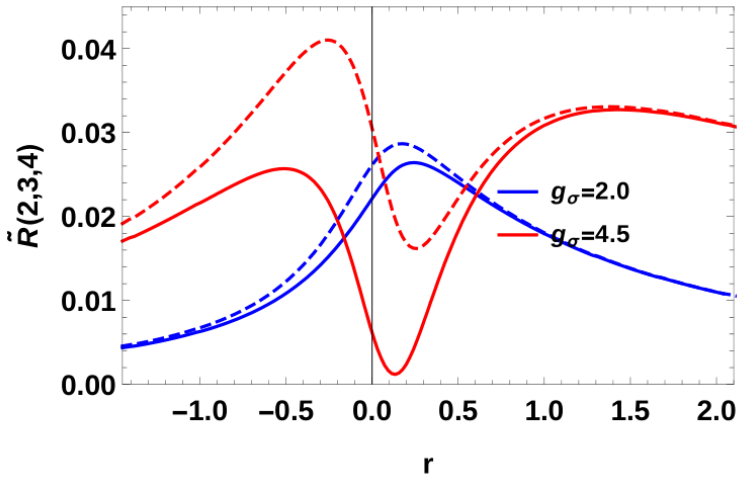




$$\frac{g_t}{g_d^2}\frac{3\Xi(3,2)-\Xi(3,3)-2\Xi(2,2)-\Xi(2,2)^2}{[1+\Xi(2,2)]^2}.$$

~2pt - 3pt - (2pt)^2





$$\tilde{R}(2,3,4) = \frac{\langle N_t\rangle_\sigma N_p}{\langle N_d\rangle_\sigma^2} - \frac{g_t}{g_{4He}^{2/3}}\left[\frac{\langle N_{4He}\rangle_\sigma N_p^2}{\langle N_d\rangle_\sigma^2}\right]^{2/3}$$

~2pt -4 (2pt)^2

Conclusion and Outlook

- Phase space distribution in Coalescence Model:
  - Lower order phase-space cumulants ($C_\alpha, |\alpha| < 3$) play similar role for different light-nuclei production N_A
    - => Fireball size R, Homogeneity length l play similar role.
    - => Higher order phase-space cumulants ($C_\alpha, |\alpha| \geq 3$) are important to light-nuclei yield ratios.
- Proper ratios of light nuclei largely cancel the effects from the scales of fireball size, homogeneity length, etc. But critical correlation length can not be canceled.
- $2 \sim A$ point correlators contribute to N_A, square and higher order terms of 2-point correlator result in dip inside the peak near the critical point.
- This property arises from the fact the coalescence process only depends on the relative distance in phase space and is general can be applied in other context.