A novel approach to calculate GPDs in Lattice QCD from non-symmetric frames

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Yong Zhao (ANL)

QNP 2022: The 9th International Conference on Quarks and Nuclear Physics

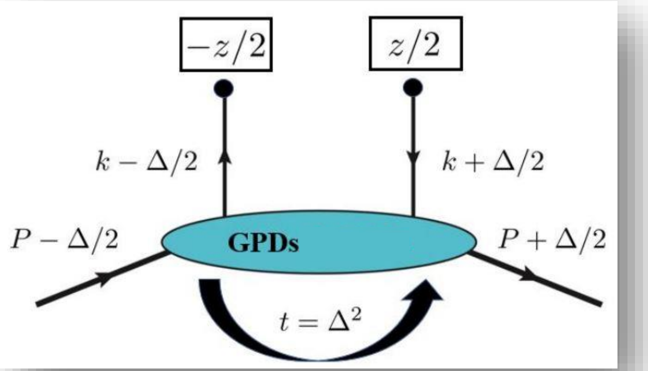
Florida State University

Background

What? Why? How?

Background

What? Why? How?



Generalized Parton Distributions (GPDs): (See Diehl, arXiv: 0307382)

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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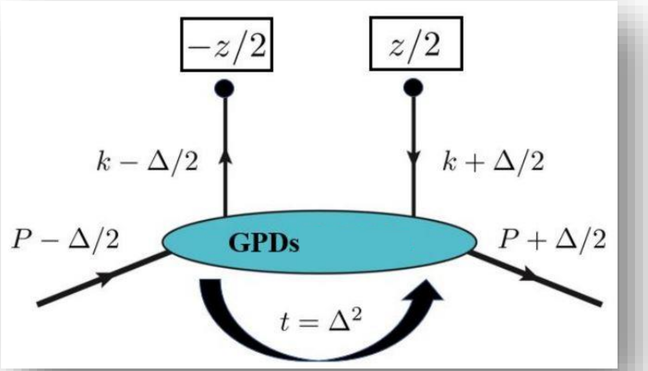

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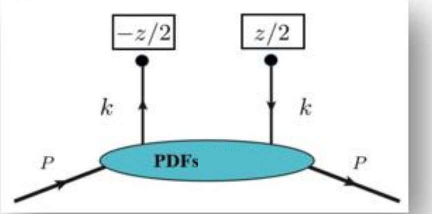
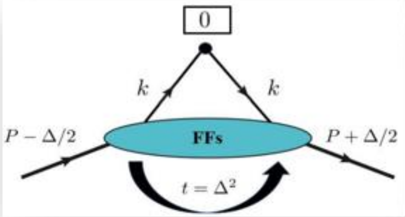
Relation with PDFs & FFs:

$$\textbf{GPDs}\ (x,\Delta) \xrightarrow{\Delta=0} \textbf{PDFs}\ (x), \qquad \textbf{GPDs}\ (x,\Delta) \xrightarrow{\int dx} \textbf{FFs}\ (\Delta)$$





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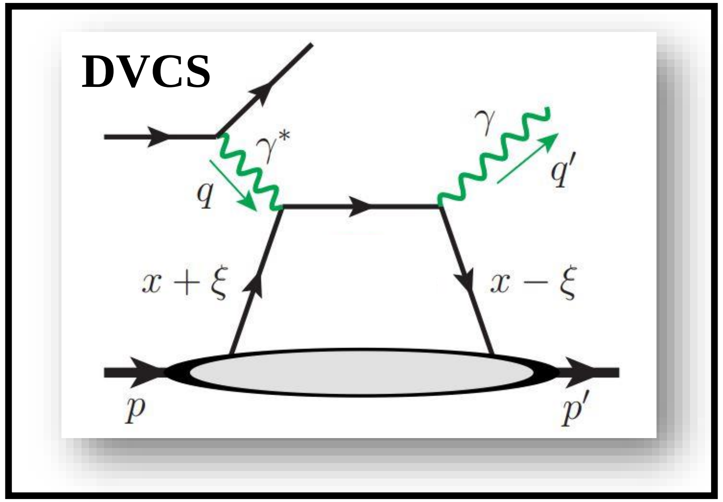
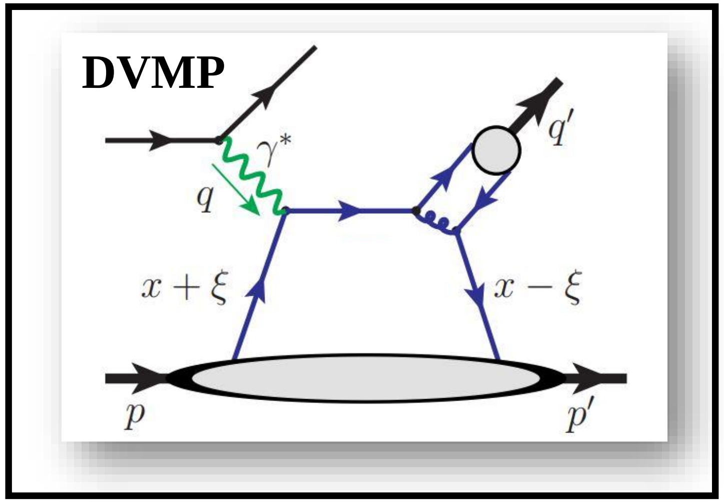
What? **Why?** How?

Physical processes giving access to GPDs:





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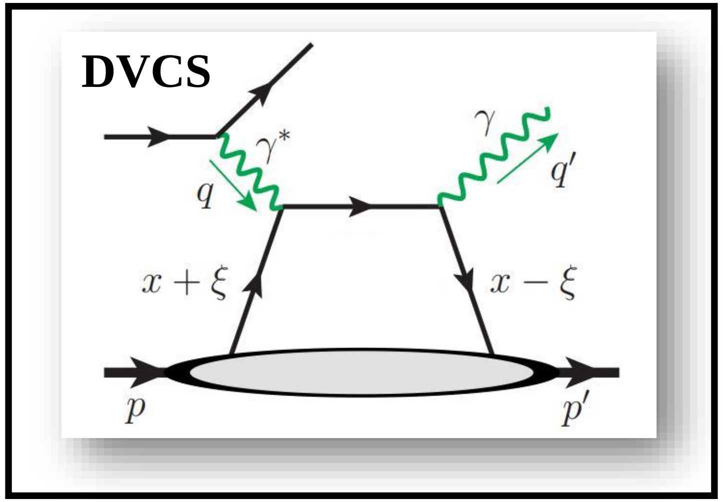

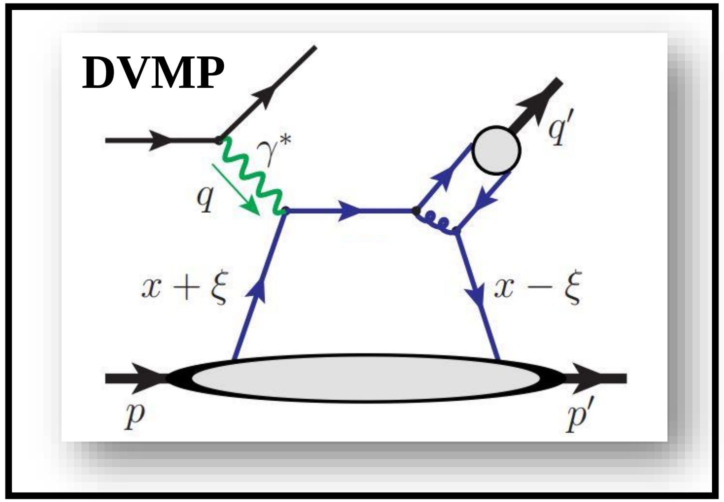

Amplitude:

$$\boxed{\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}}$$

x-dependence lost!

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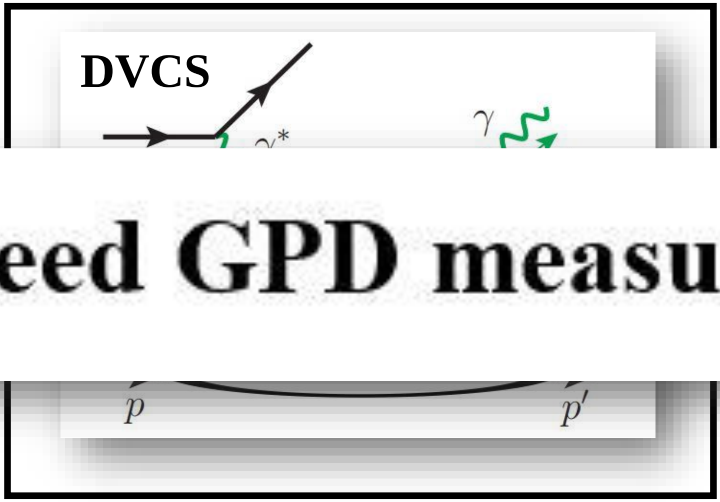
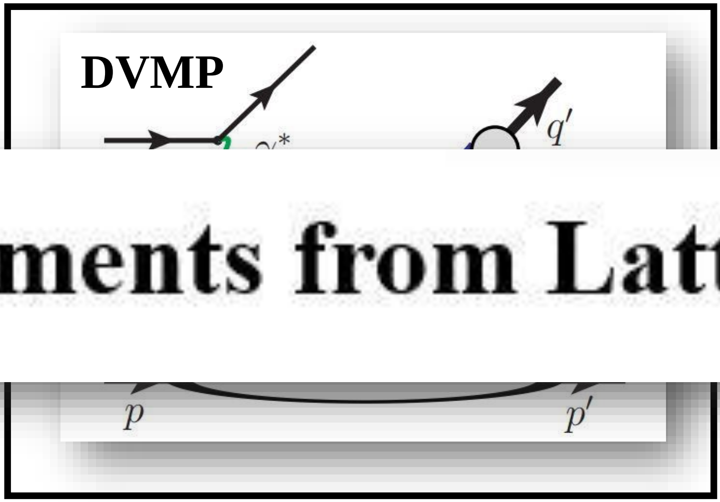
Physical processes giving access to GPDs:





We need GPD measurements from Lattice QCD

Amplitude:

$$\boxed{\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}}$$

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Can we extract these quantities from lattice QCD?

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Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x, \Delta; \lambda, \lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p'; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p; \lambda \rangle \Big|_{z^+ = \vec{z}_\perp = 0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **Cannot be computed on Euclidean lattice**

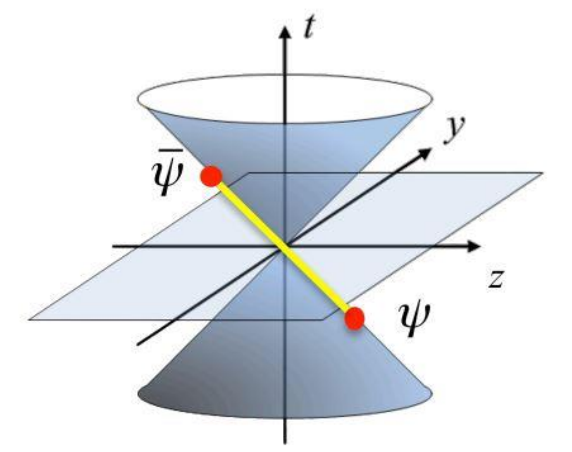

Figure courtesy: Yong Zhao

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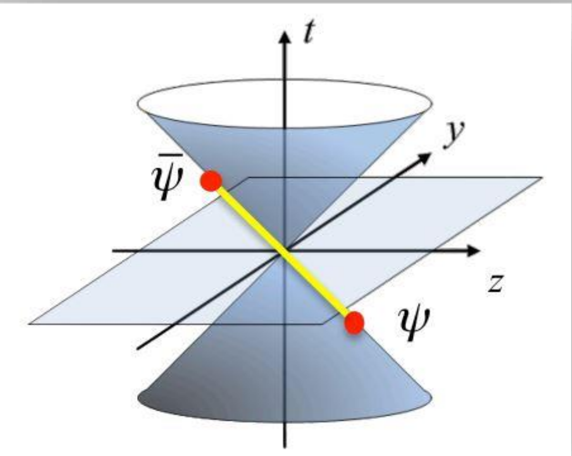
- **<u>Cannot</u> be computed on Euclidean lattice**

Correlator for quasi-GPDs (Ji, 2013) $-\infty \leq x \leq \infty$

$$F_{\rm Q}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\rm Q}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**



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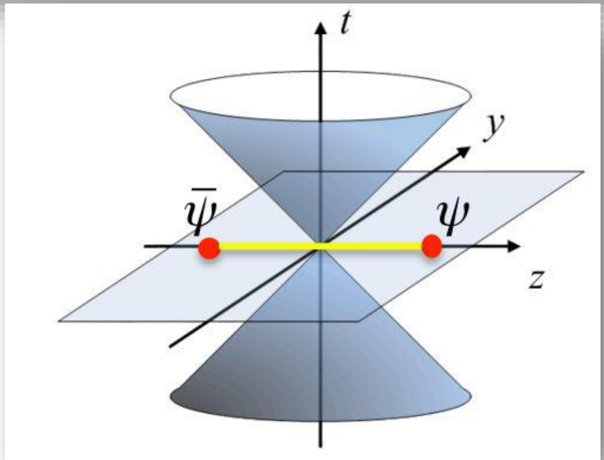


Figure courtesy: Yong Zhao

Can we extract these quantities from lattice QCD?

Light-cone (standard) correlator $-1 \le x \le 1$

$F^{[\Gamma]}(x, \Delta; \lambda, \lambda')$ … $\times \langle p'; \lambda' |$ …

- Time dependence : …
- Cannot be computed o…

Correlator for quasi-GPDs (Ji, 2013) $-\infty \le x \le \infty$

$F_{\mathrm{Q}}^{[\Gamma]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} e^{ik \cdot z}$ … $\psi(\tfrac{z}{2}) | p, \lambda \rangle \Big|_{z^0 = \vec{z}_\perp = 0}$

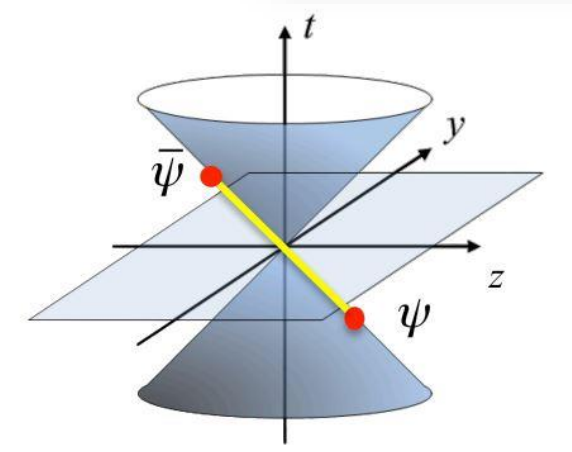
- … on z^3
- Can be computed on Euclidean lattice

Matching formula:

$$\boxed{H(x, \ldots) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} \underbrace{C\left(\xi, \frac{\mu^2}{P_3^2}, \ldots\right)}_{\text{Matching coefficient}} H_{\mathrm{Q}}\left(\frac{x}{\xi}, \ldots\right)}$$

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

(Schematic structure)



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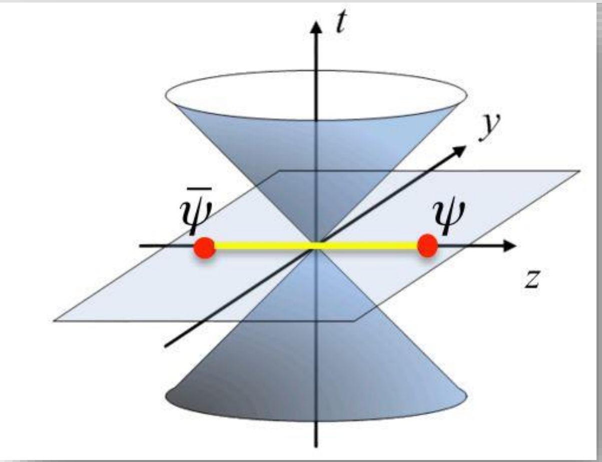

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Pioneering Lattice QCD calculations of GPDs:

<u>Quasi-distribution formalism</u>

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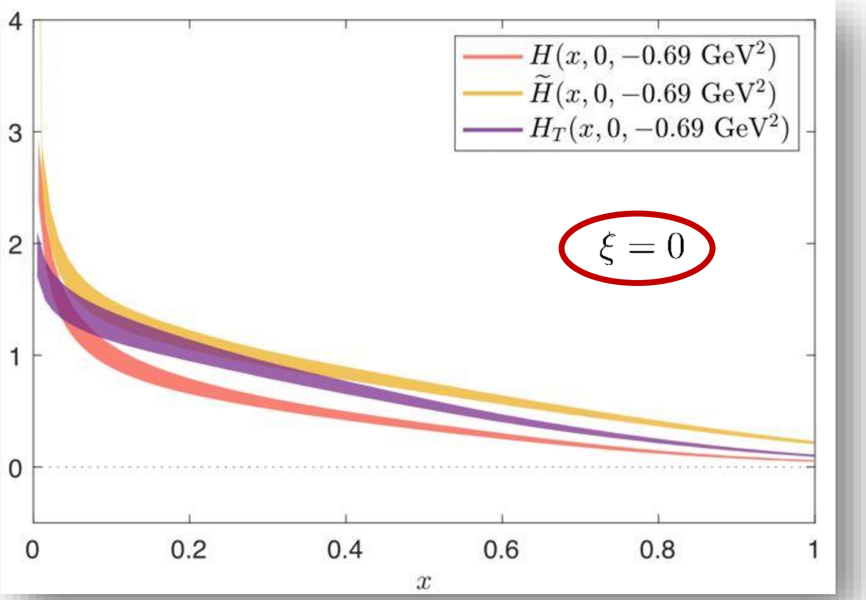

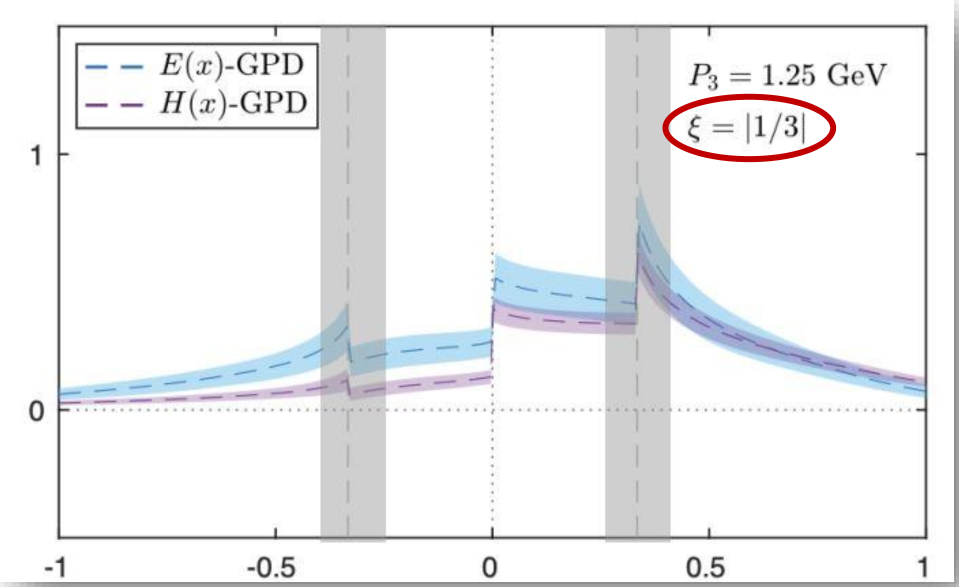

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

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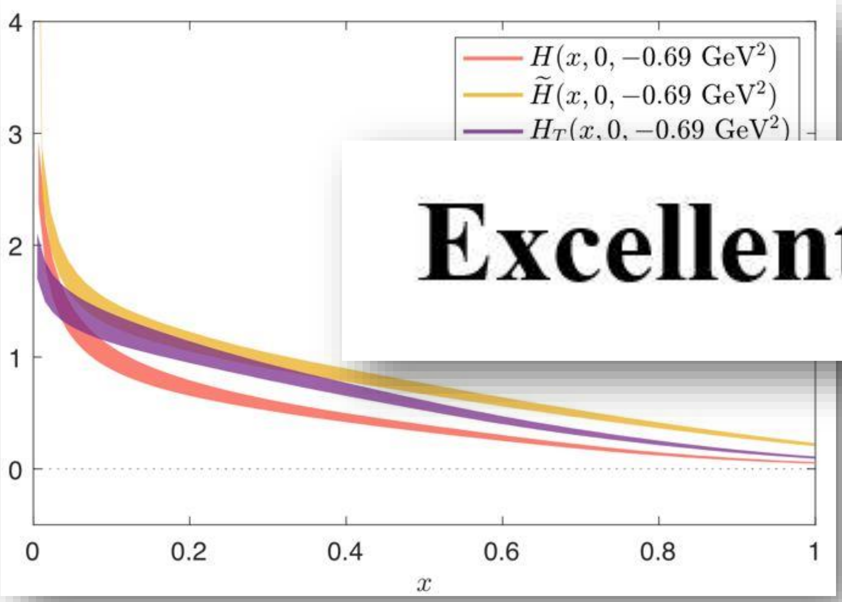
Background

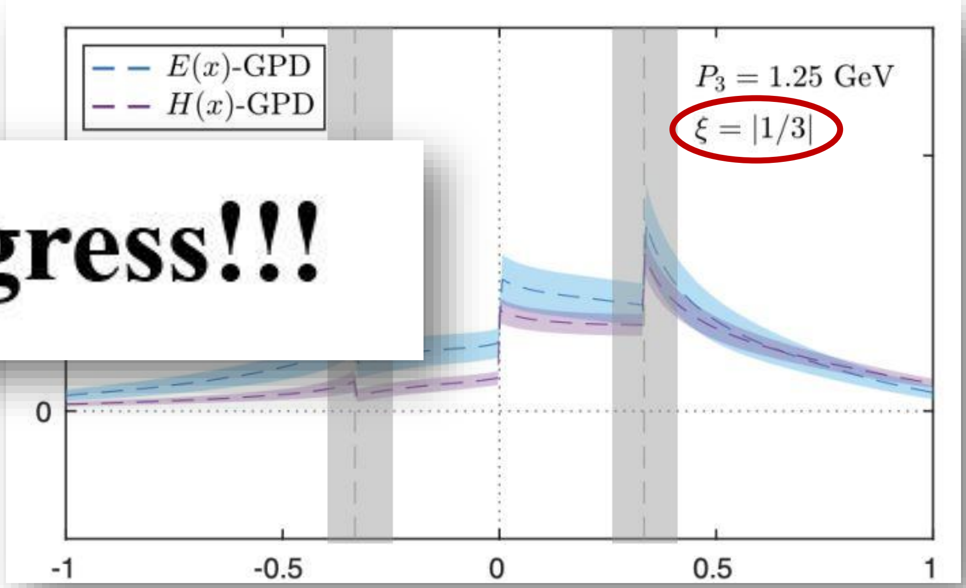
What? **Why?** How?

Pioneering Lattice QCD calculations of GPDs:

Quasi-distribution formalism



Excellent progress!!!



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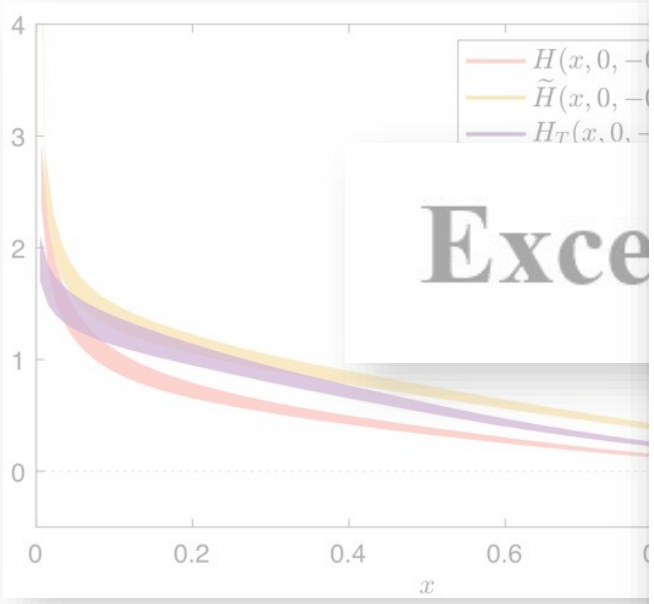
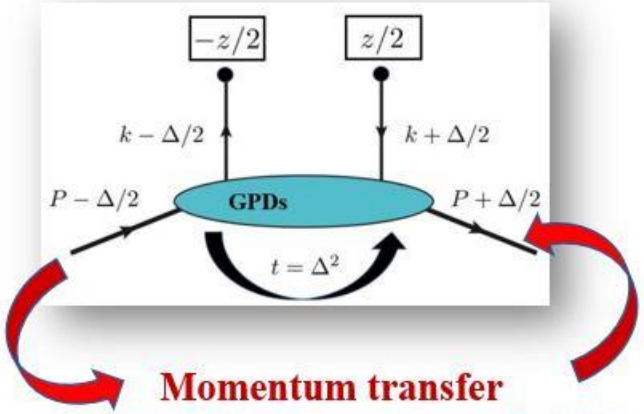


C. Alexandrou et. al. (PRL 125 (202

Practical drawback



Momentum transfer symmetric between source & sink

Lattice QCD calculations in symmetric frames are expensive

Background

What? Why? How?

Resolution:

Background

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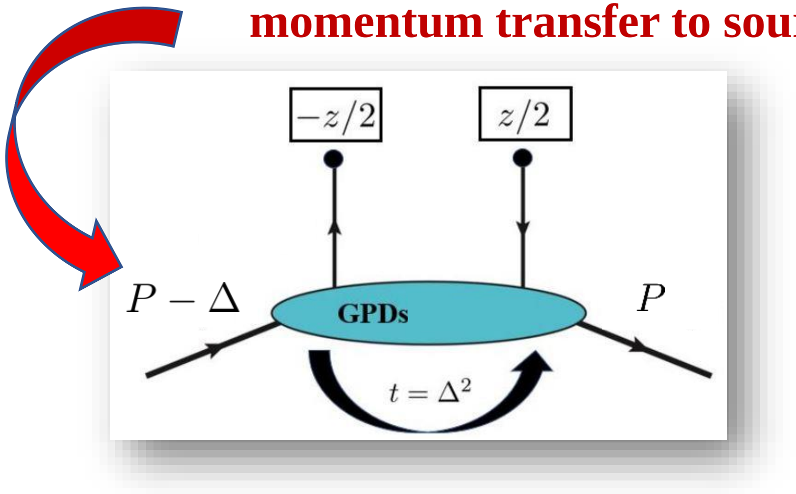
Resolution:

All
momentum transfer to source



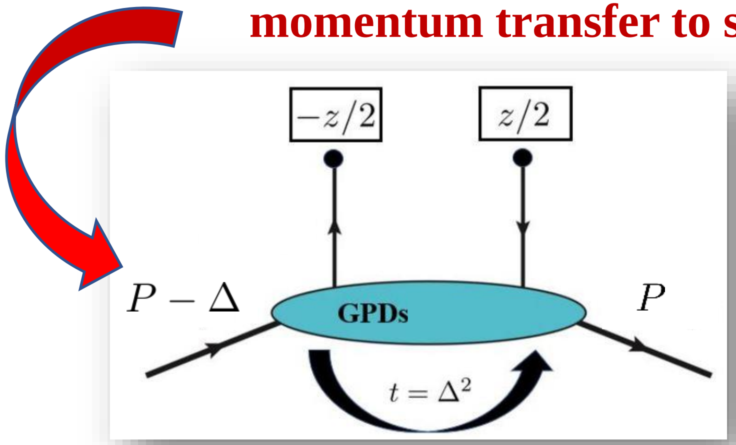
- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

Background

What? Why? How?

<u>Our contribution in a nutshell</u>:

All
momentum transfer to source



Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

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Our contribution in a nutshell:

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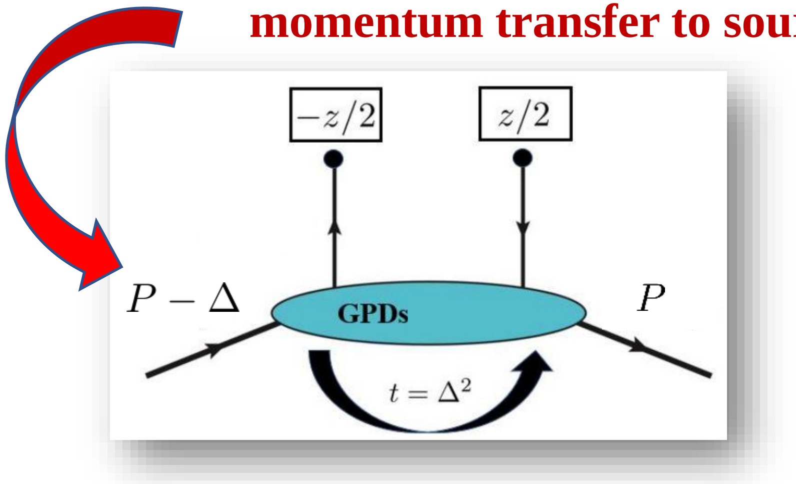
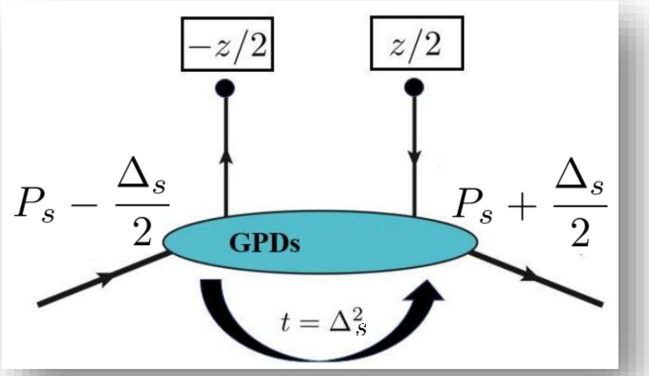


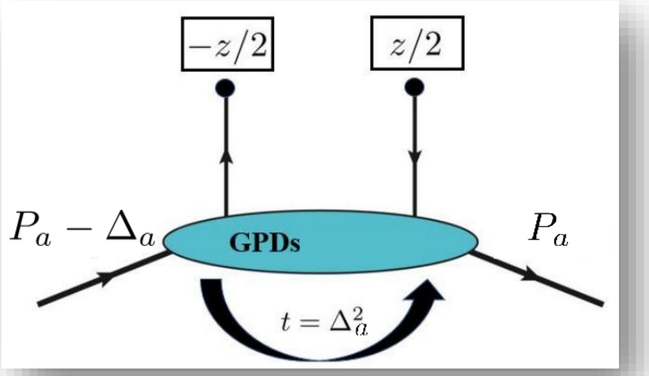
Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of (frame-dependent) power corrections allowing faster convergence to light-cone GPDs at LO**

Main results

Symmetric & asymmetric frames





Main results

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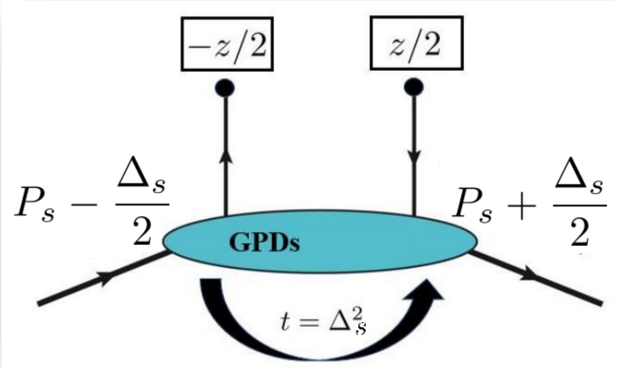
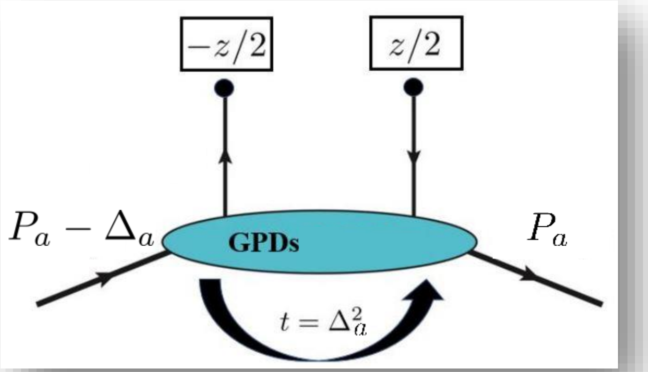
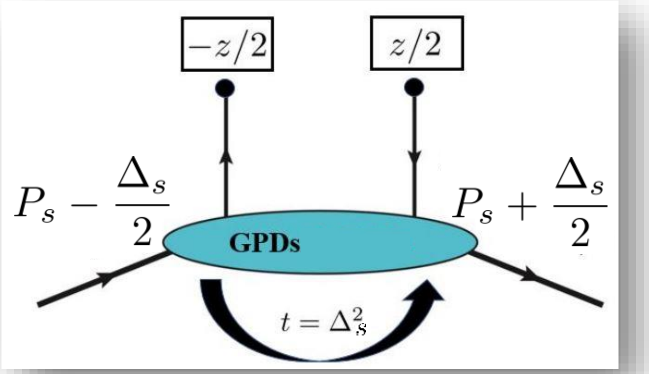




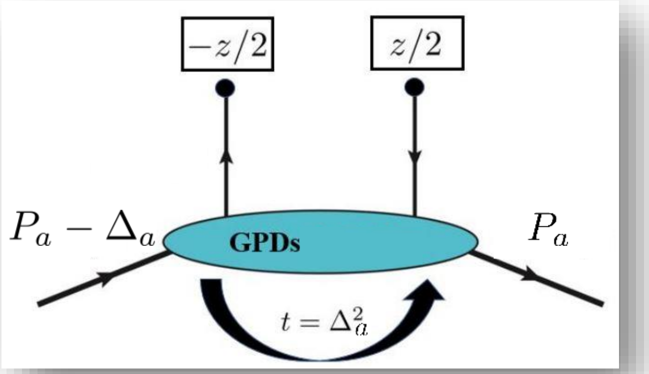
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Main results

Symmetric & asymmetric frames



Related via
Lorentz transformation?



Main results

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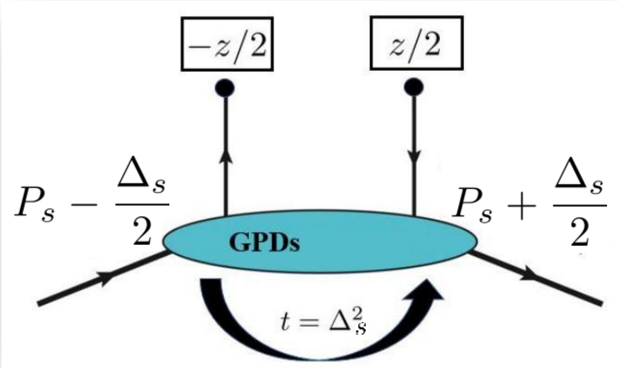

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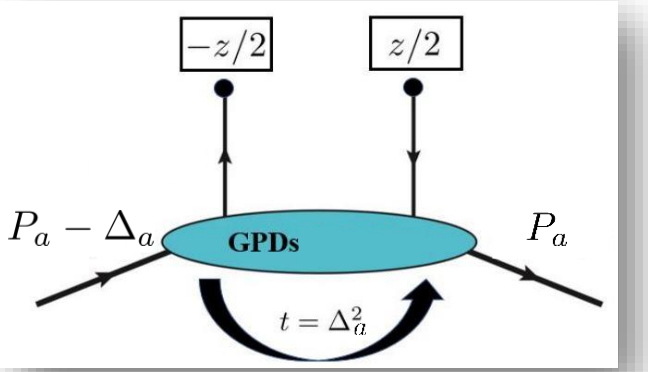
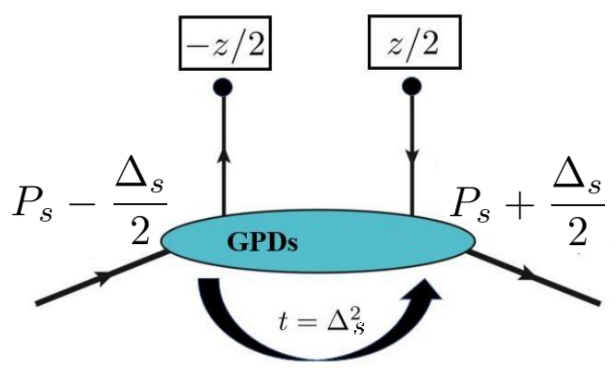


Yes, since symmetric & asymmetric frames are connected via Lorentz transformation

Main results

Symmetric & asymmetric frames



Related via
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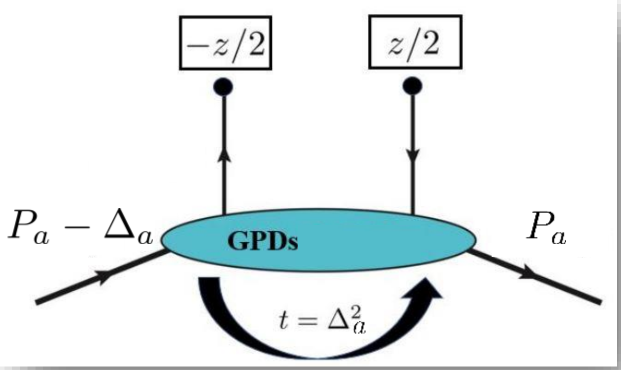
↔

What kind?



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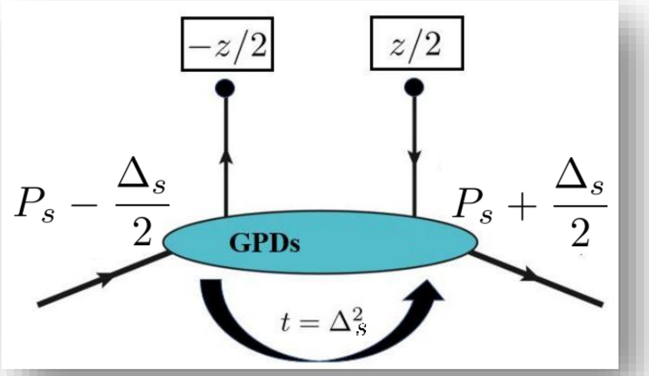

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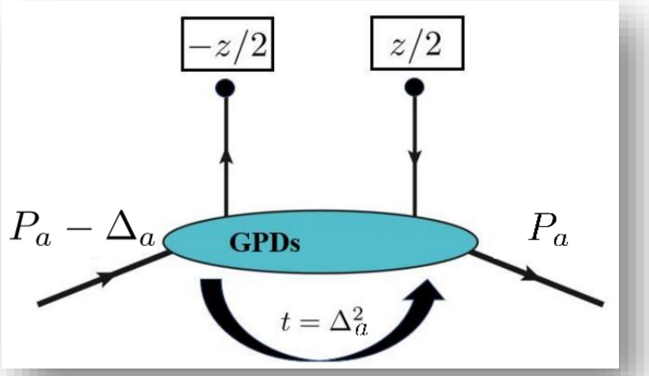
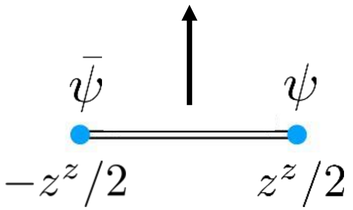


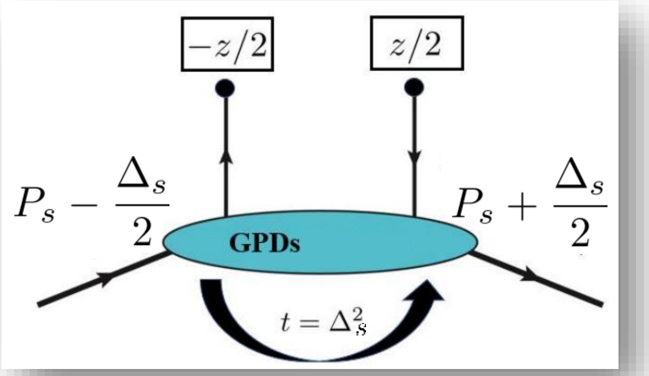
Case 1: Lorentz transformation in the z-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Main results

Symmetric & asymmetric frames



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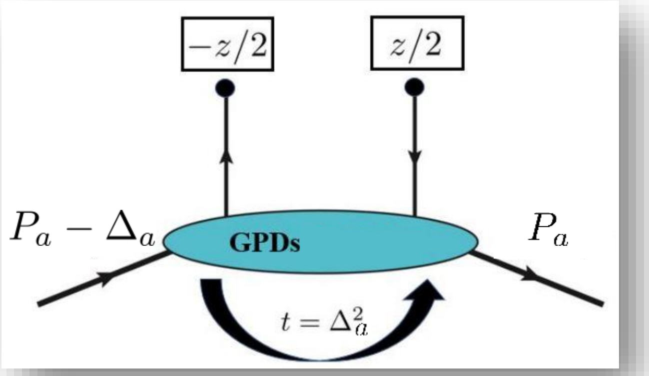

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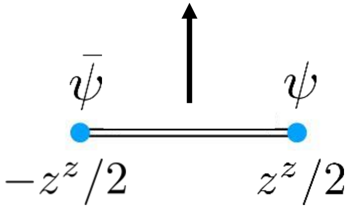


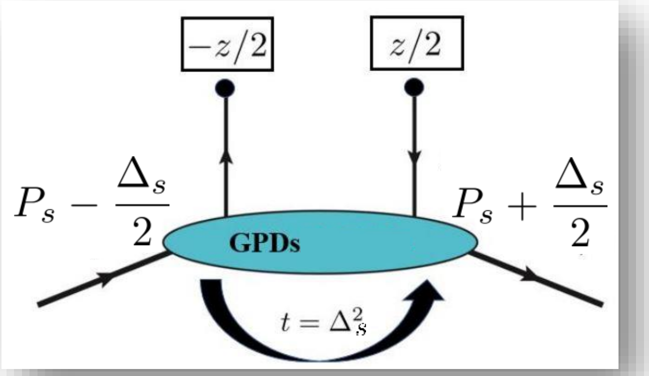
Results:

$$\boxed{\begin{aligned} z_s^0 &= -\gamma\beta z_a^z \\ z_s^z &= \gamma z_a^z \end{aligned}}$$

Operator distance develops a non-zero temporal component

Main results

Symmetric & asymmetric frames



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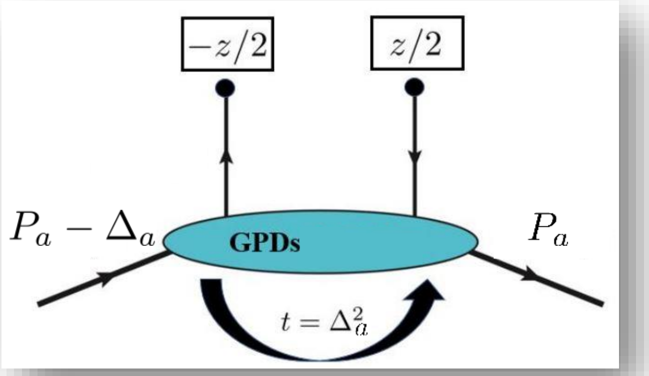
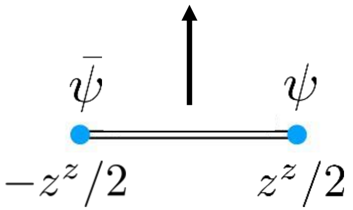


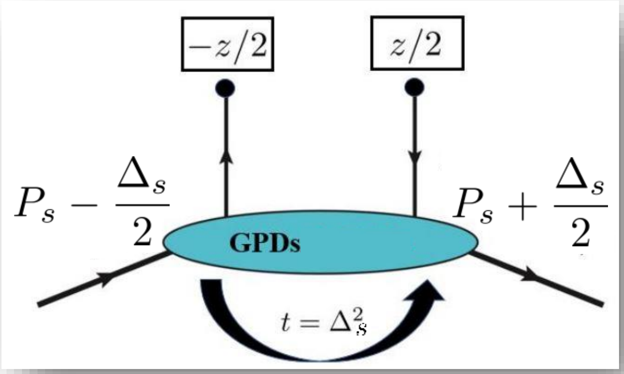
Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Main results

Symmetric & asymmetric frames



Related via
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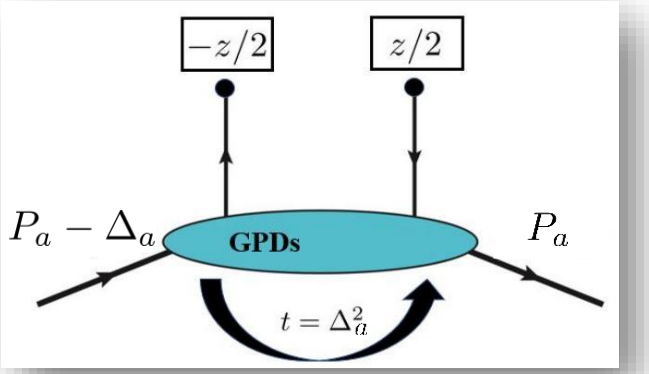

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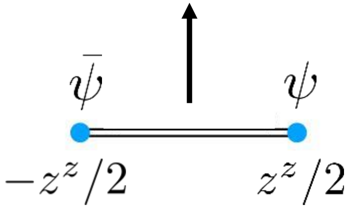


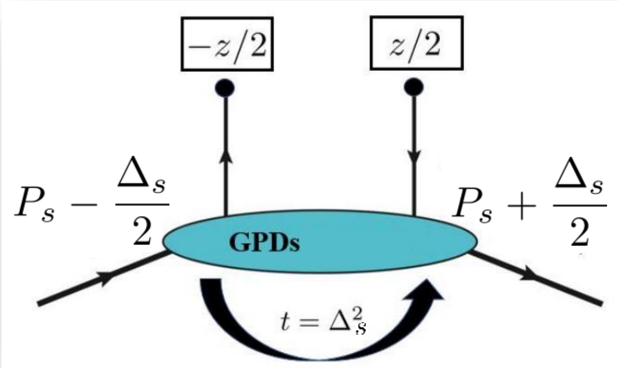
Results:

$$\boxed{\begin{aligned} z_s^0 &= 0 \\ z_s^z &= z_a^z \end{aligned}}$$

Operator distance remains spatial (& same)

Main results

Symmetric & asymmetric frames



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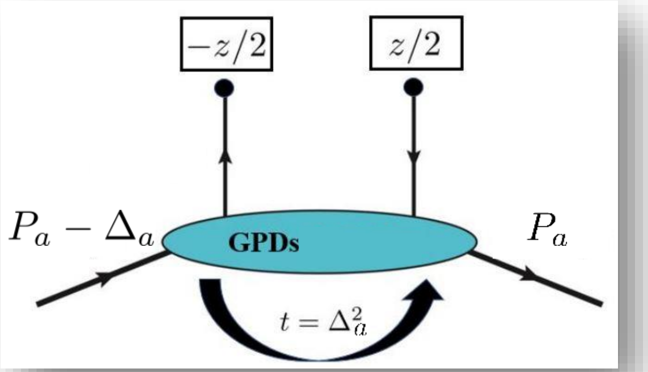


Case 2: Transver

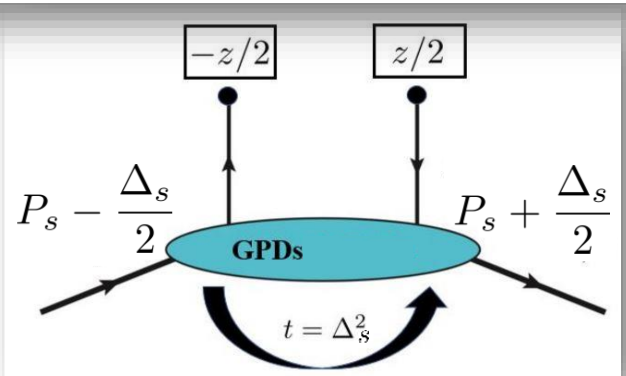
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

$-z^z/2$ $z^z/2$

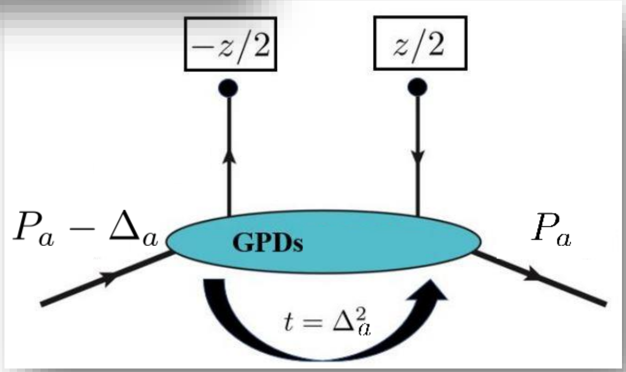
Main results

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?



Related via
Lorentz transformation?

What kind?



Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$

$\bar{\psi}$ ψ

$-z^z/2$ $z^z/2$

Results:

$$\boxed{\begin{aligned} z_s^0 &= 0 \\ z_s^z &= z_a^z \end{aligned}}$$

Operator distance remains spatial (& same)

Main results

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

P_s

GPDs on the light-cone:

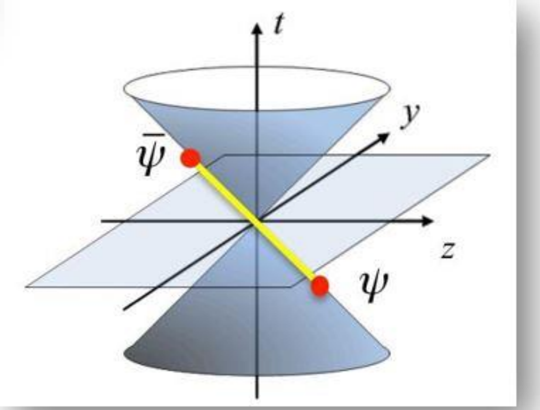


Figure courtesy: Yong Zhao

$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\gamma^+ q|p\rangle \qquad z = (0, z^-, 0_\perp)$$

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\not{z} q|p\rangle$$ Arbitrary light-like z

GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Trans

$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix}$ ... $0 \quad 0 \quad 1$ ... z_a^z

$\boxed{z_s^z = z_a^z}$

Operator distance remains spatial (& same)

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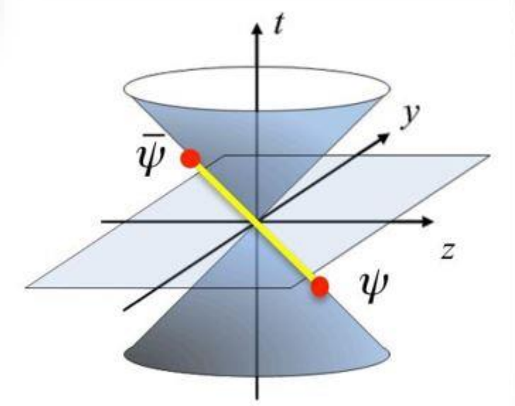

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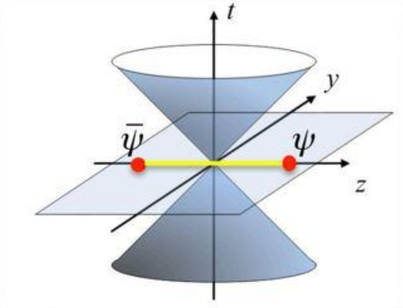


Figure courtesy: Yong Zhao

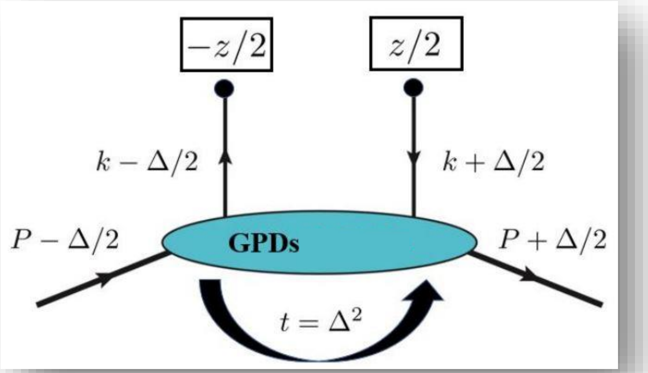
Are quasi-GPDs Lorentz-invariant?

Main results

Definitions of quasi-GPDs

Main results

Definitions of quasi-GPDs

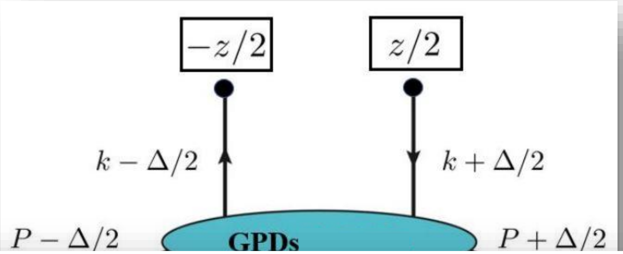


Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

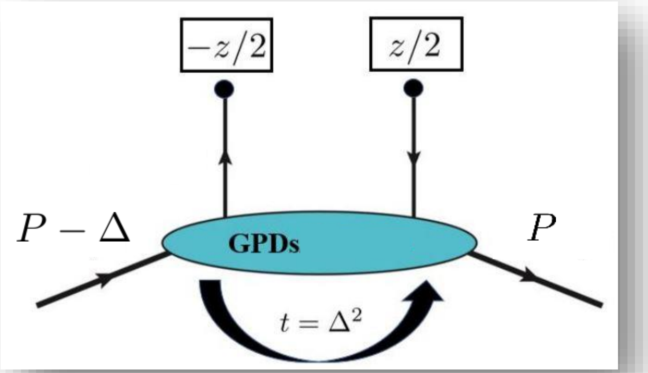
Main results

Definitions of quasi-GPDs



If this definition is Lorentz covariant:

Use γ^0



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Definition of quasi-GPDs in asymmetric frames:

$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar q(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec z_\perp=\vec 0_\perp}$$
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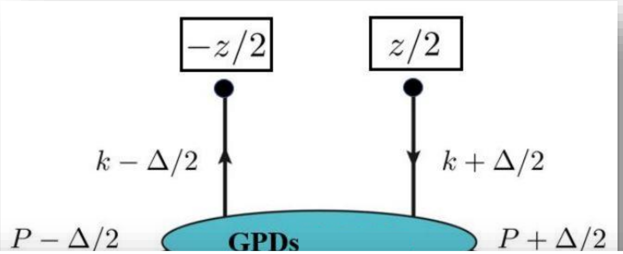
Main results

Definitions of quasi-GPDs



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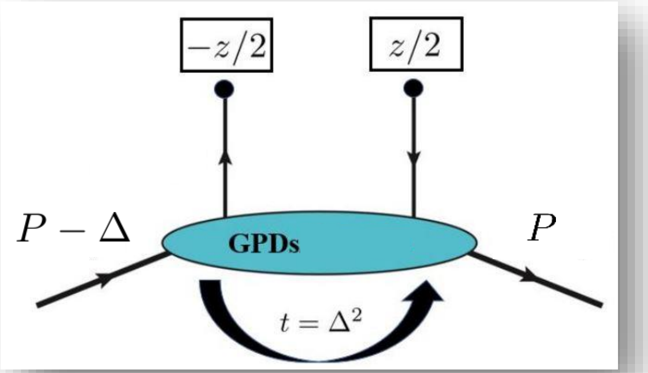
If this definition is Lorentz covariant:

Use γ^0

Use kinematics of asymmetric frame



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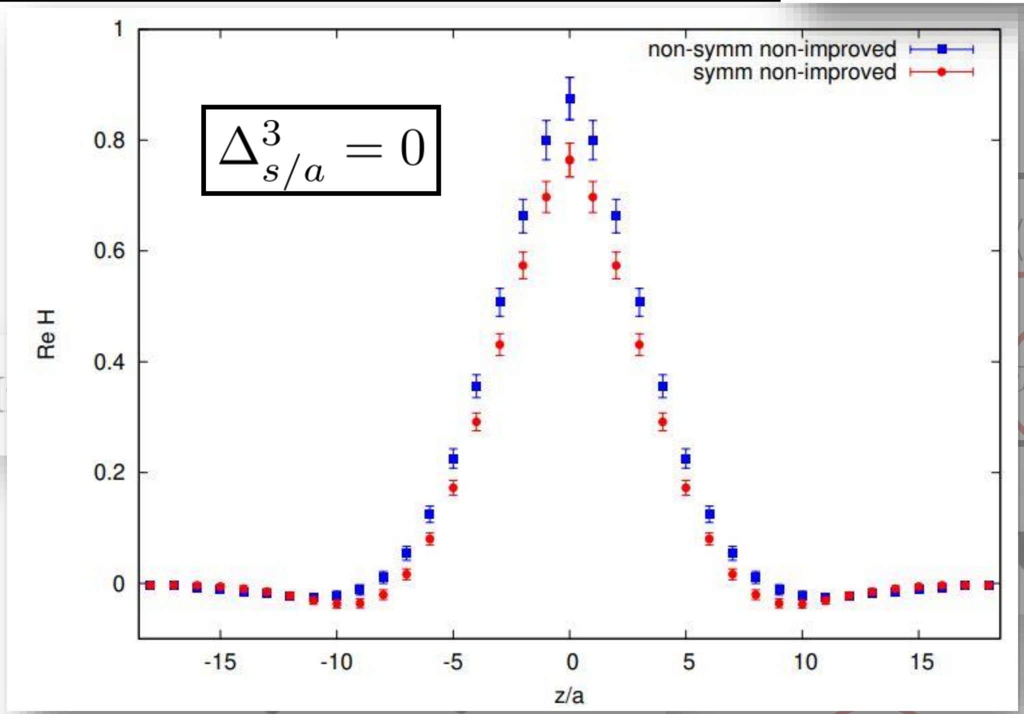
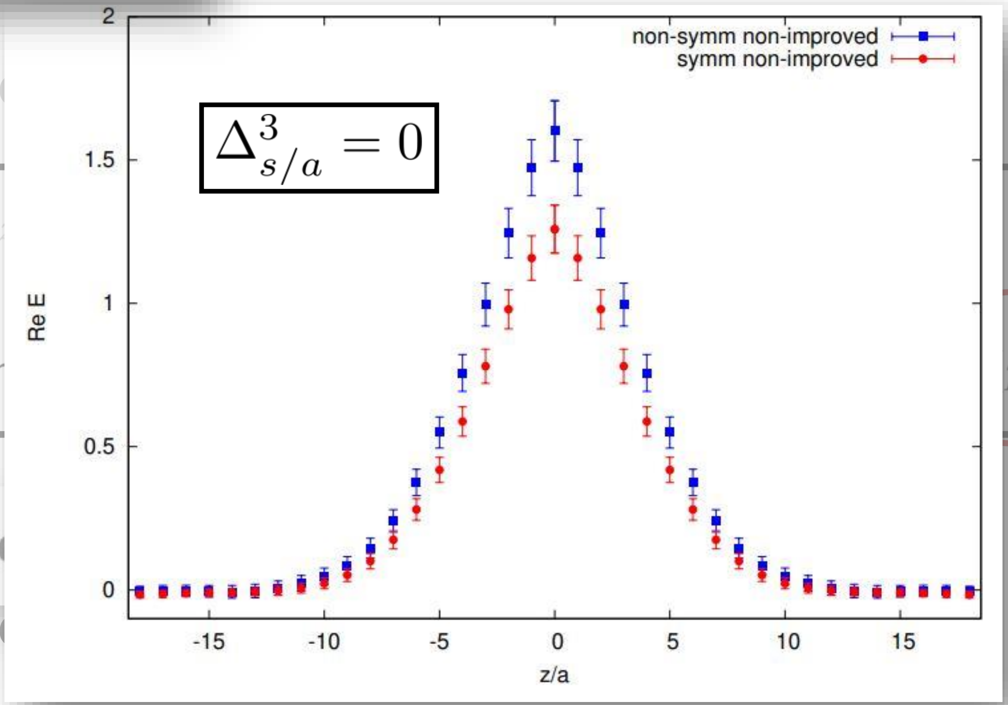
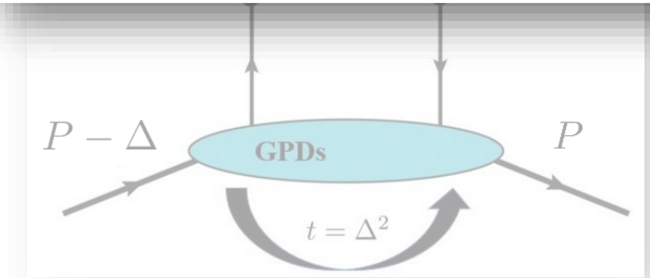
Main results

Lattice QCD results







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Main results

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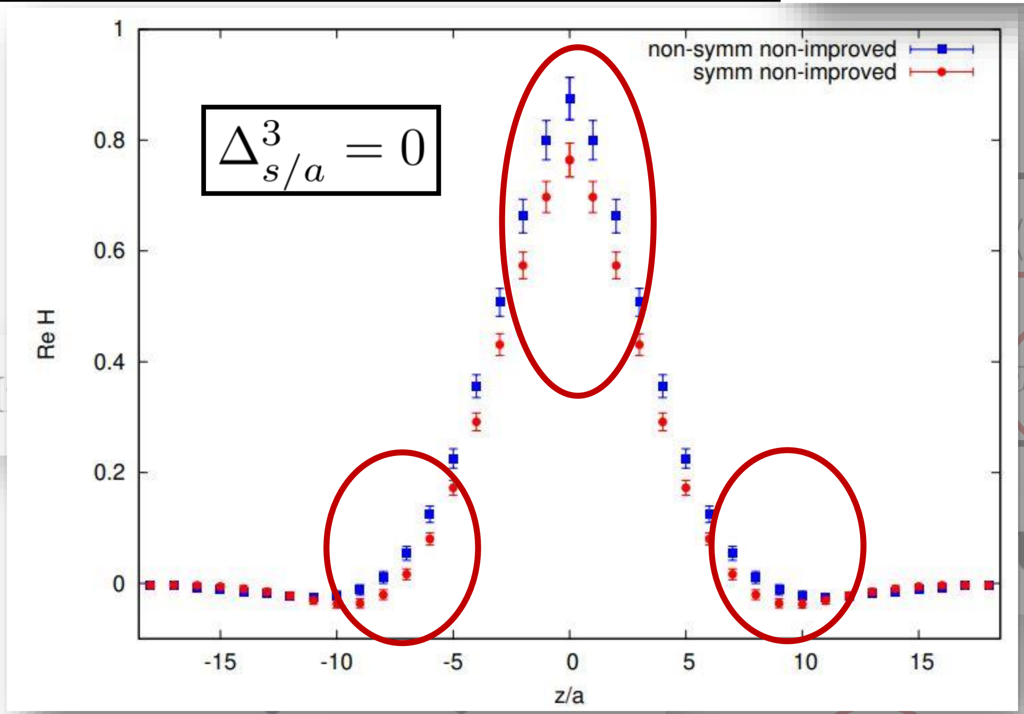
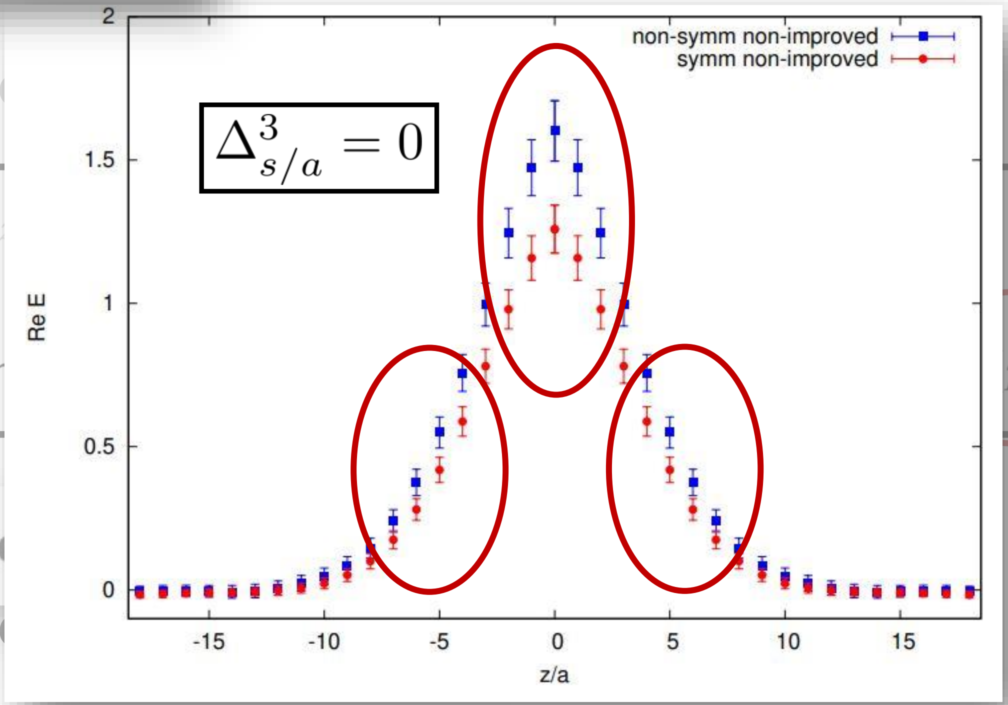


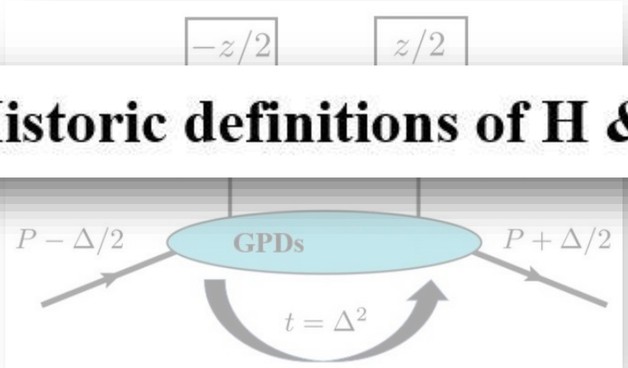


Qualitatively similar results for imaginary parts

Frame-dependence of quasi-GPDs

Main results

Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

$$\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

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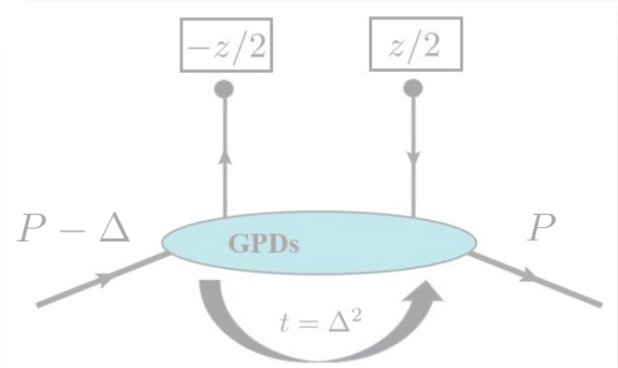

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Main results

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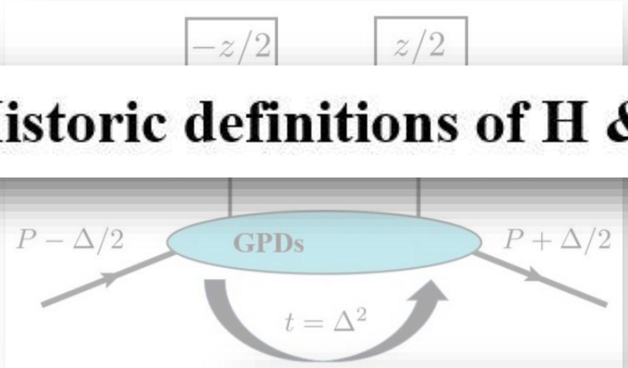

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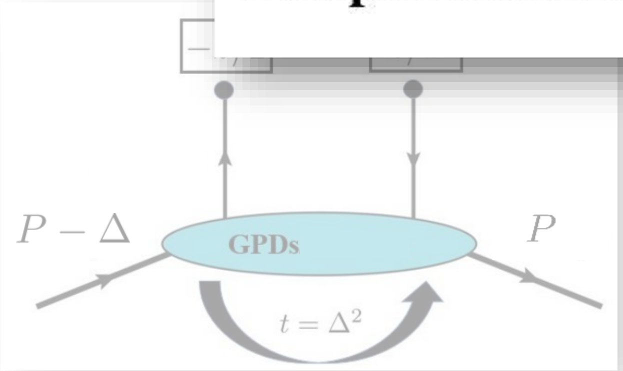
This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a complete basis for a <u>spatially-separated</u> bi-local operator at <u>finite momentum</u>



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We do not dismiss these definitions
since they do work in the large-momentum limit (I will show this formally later)

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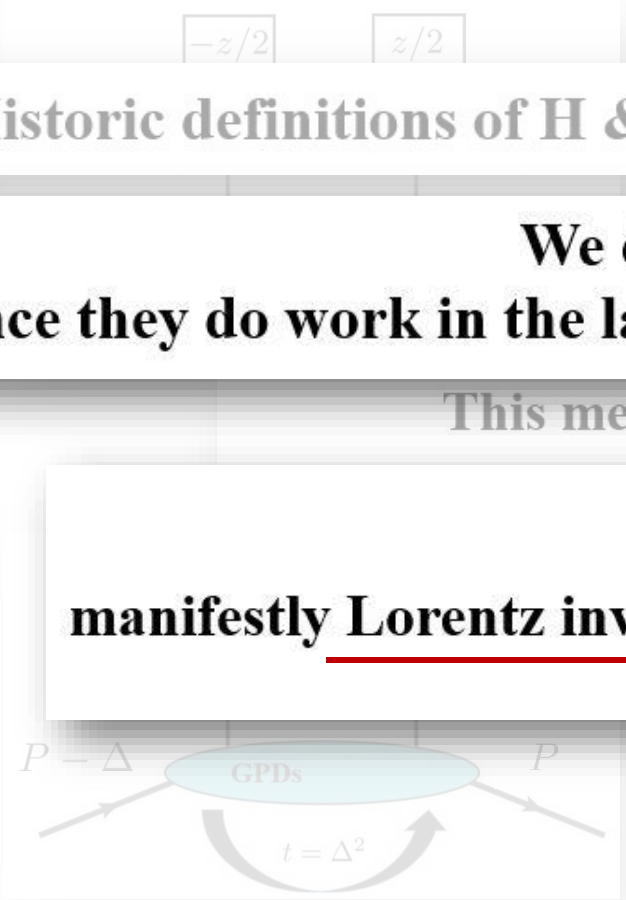

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Can we come up with a manifestly Lorentz invariant definition of quasi-GPDs for finite values of momentum?

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Main results

Lorentz covariant formalism

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Vector operator)

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Features:

- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures**
- **8 Lorentz-invariant amplitudes (or Form Factors)** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Validating the frame-independence of A's from Lattice QCD

Lorentz covariant formalism

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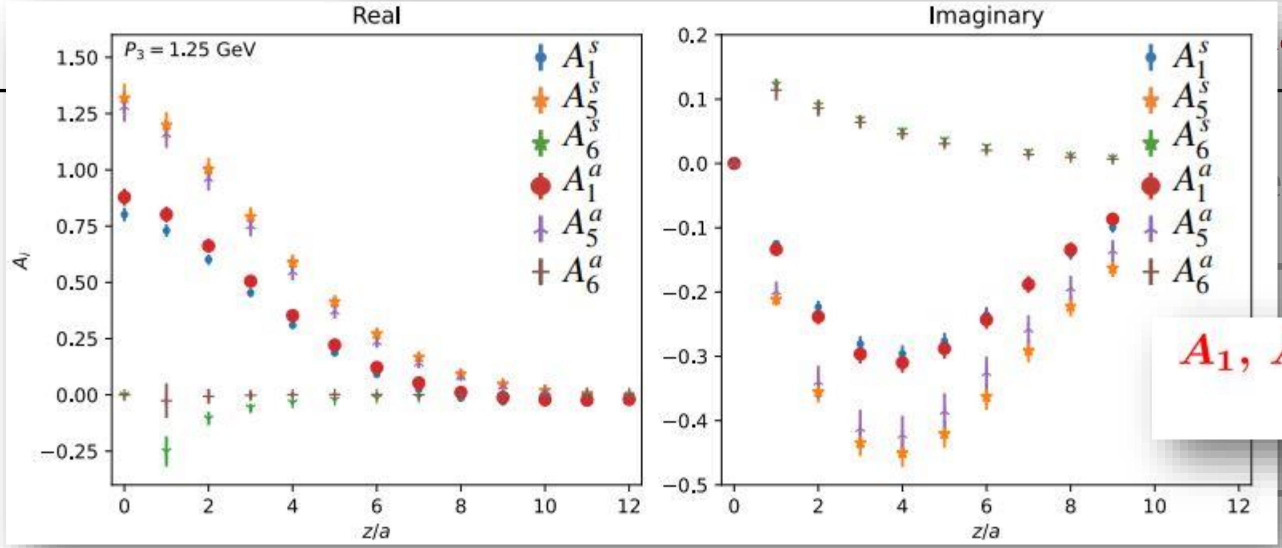

A_1, A_5 : Dominant contributions;
Full agreement in two frames for both Re & Im parts

A_6 : Small but non-negligible;
Tension between frames is a statistical effect

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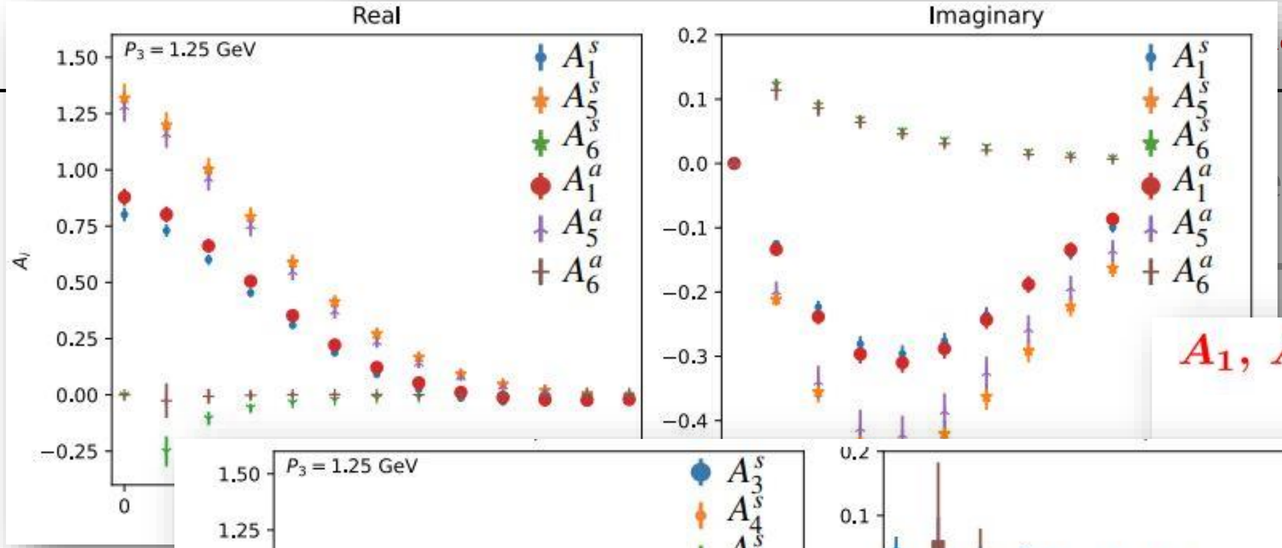

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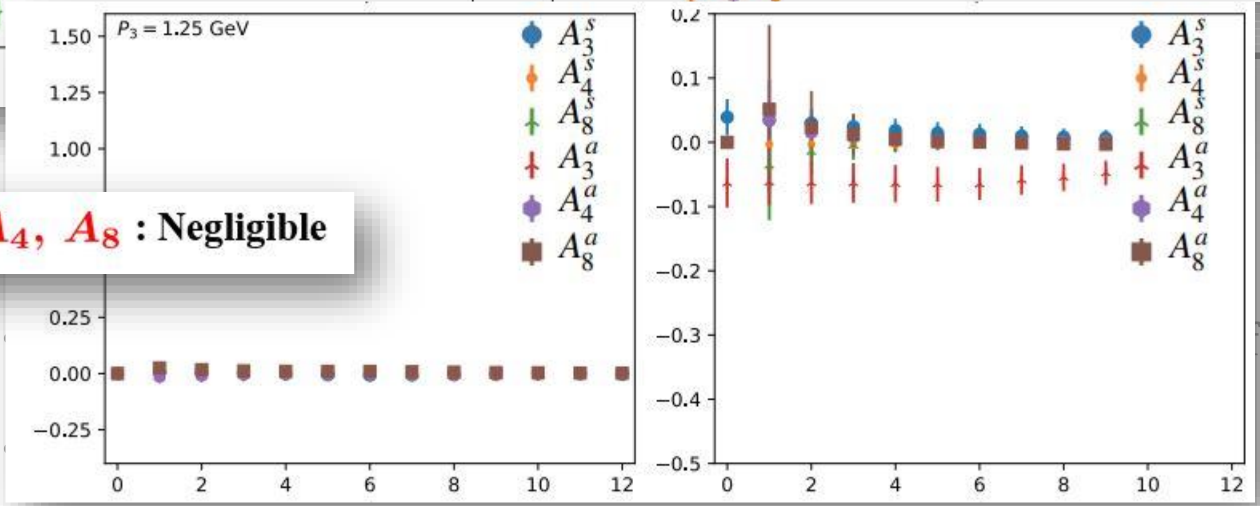

$A_3, \; A_4, \; A_8$: Negligible

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Validating the frame-independence of A’s from Lattice QCD

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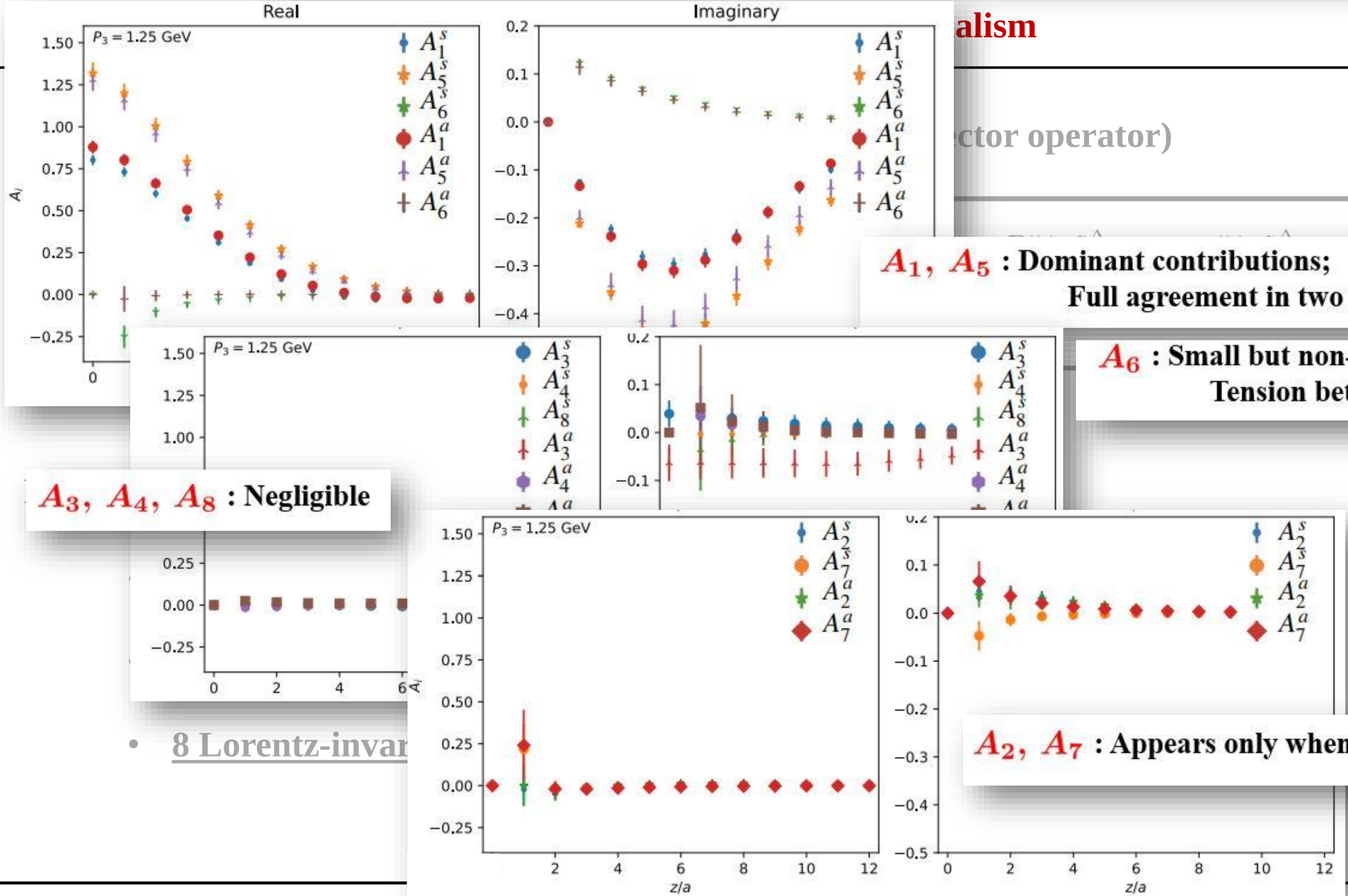

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A_3, A_4, A_8 : Negligible

A_2, A_7 : Appears only when you work with γ^3 ; Negligible

- 8 Lorentz-invar

Main results

Exploring historical definitions of quasi-GPDs

Mapping Form Factors to the historical definitions of quasi-GPDs:

Main results

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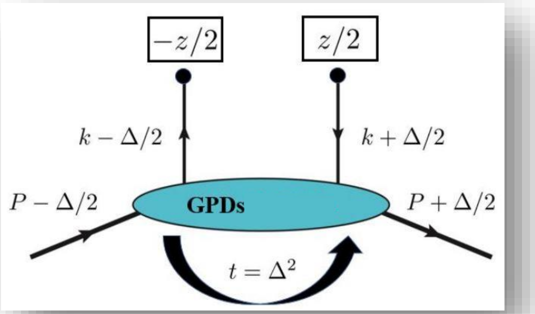

Symmetric frame:

$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6}$$
$$+ \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}$$

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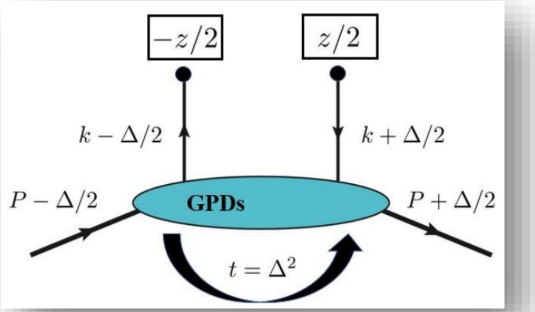

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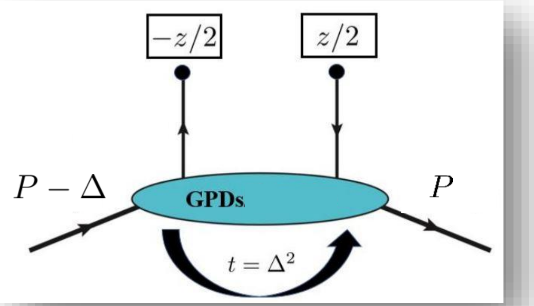


Asymmetric frame:

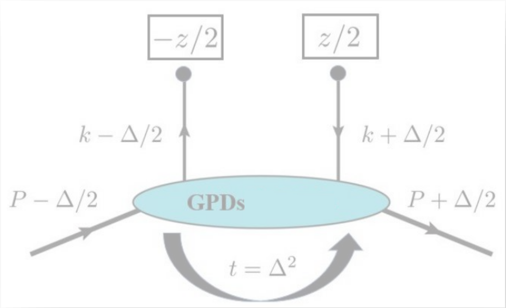
$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4}$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2 P^3_{avg,a}}\right)\boldsymbol{A_6}$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P^0_{avg,a}P^3_{avg,a}}\right)\boldsymbol{A_8}$$

Main results

Exploring historical definitions of quasi-GPDs

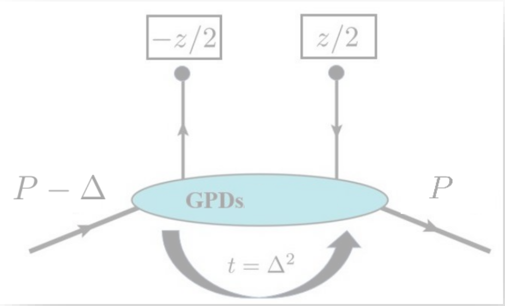
Frame-dependent expressions: Explicit non-invariance from kinematics factors

Symmetric frame:



$$
\begin{aligned}
H_{\mathrm{Q}(0)}(z, P_s, \Delta_s)\big|_s = {} & \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6} \\
& + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}
\end{aligned}
$$

Asymmetric frame:



$$
\begin{aligned}
H_{\mathrm{Q}(0)}\big|_a(z, P_a, \Delta_a) = {} & \boldsymbol{A_1} + \frac{\Delta_a^0}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a} P^3_{avg,a}} - \frac{1}{\left(1 + \frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{\Delta_a^0 \Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4} \\
& + \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{\left(1 + \frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P^3_{avg,a})^2} - \frac{1}{\left(1 + \frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{P^0_{avg,a}\Delta_a^0 \Delta_a^3 z^3}{2M^2 (P^3_{avg,a})^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P^3_{avg,a}}\right)\boldsymbol{A_6} \\
& + \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P^0_{avg,a} P^3_{avg,a}} - \frac{1}{\left(1 + \frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{\left(1 + \frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P^3_{avg,a})^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P^0_{avg,a} P^3_{avg,a}}\right)\boldsymbol{A_8}
\end{aligned}
$$

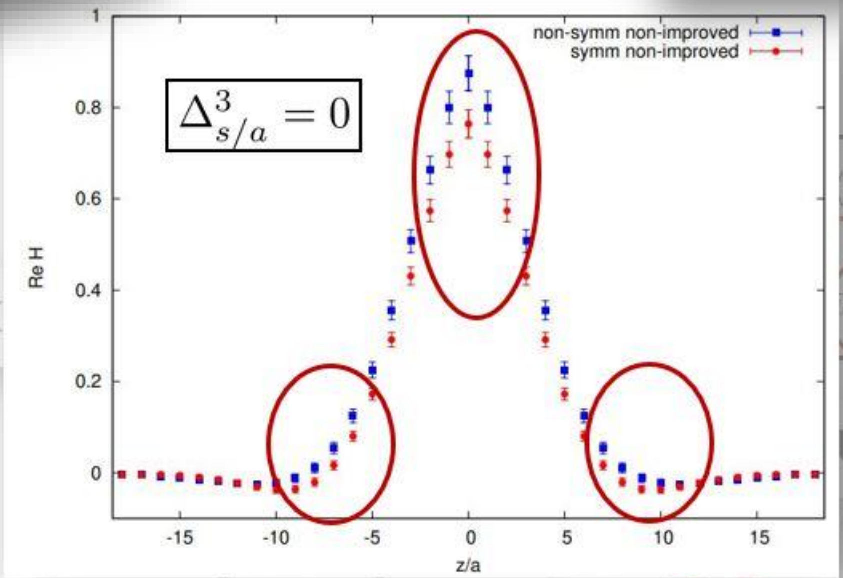
Main results

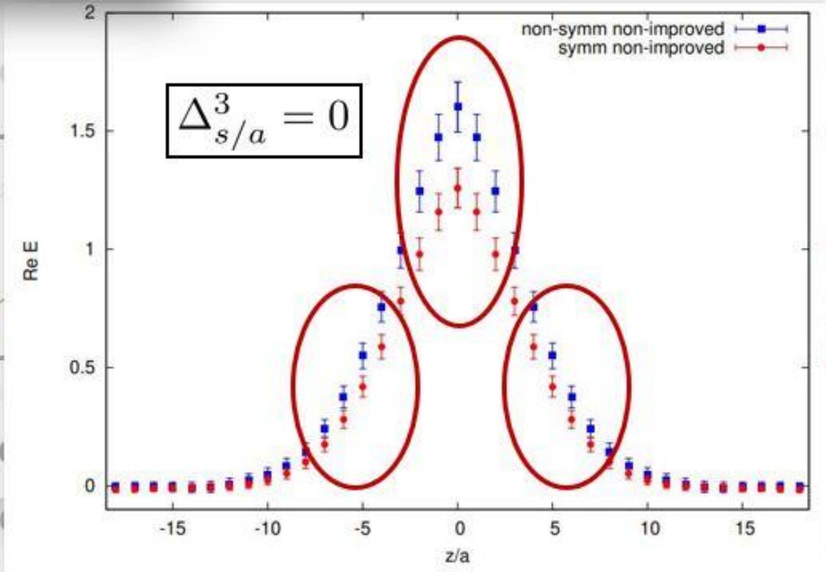
Exploring historical definitions of quasi-GPDs

Frame-dependent expressions: Explicit non-invariance from kinematics factors

Reminder:

Lattice QCD results





Qualitatively similar results for imaginary parts

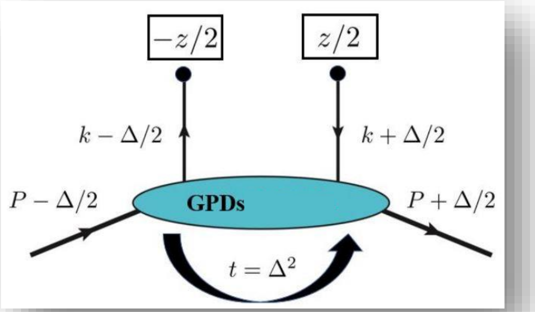
Frame-dependence of quasi-GPDs

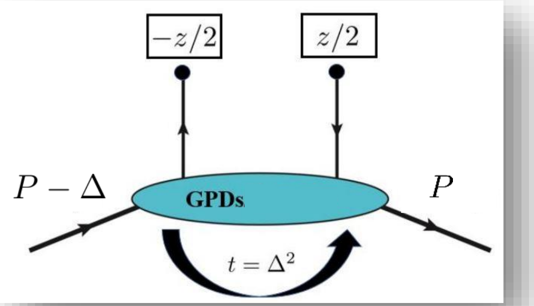
A_8

Main results

Light-cone GPDs

Mapping Form Factors to the light-cone GPDs: (Sample results)

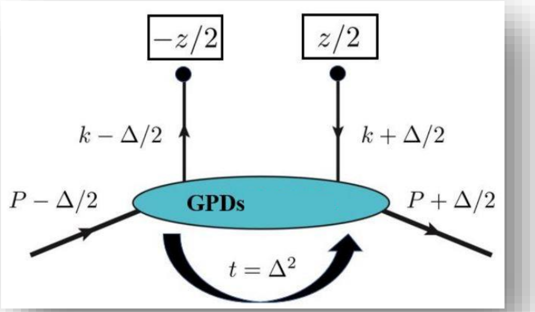




Main results

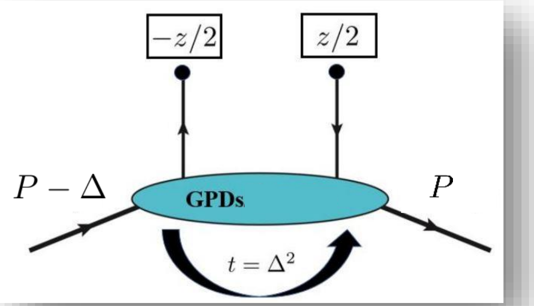
Light-cone GPDs

Mapping Form Factors to the light-cone GPDs: (Sample results)



Lorentz-invariant definition:

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\not{z}\,q|p\rangle$$



Relation between light-cone GPD H & Form Factors:

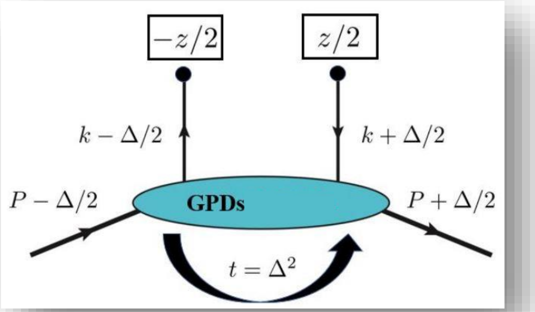
$$\boxed{H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}}$$

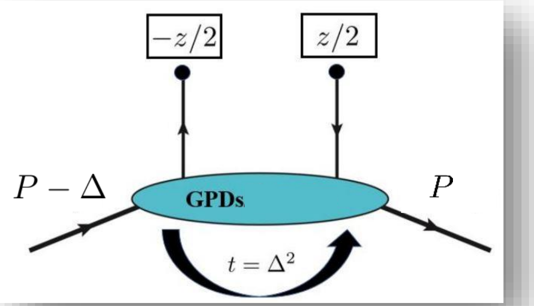
Lorentz-invariant expression

Main results

Sketch of the essence of a
Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism





Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

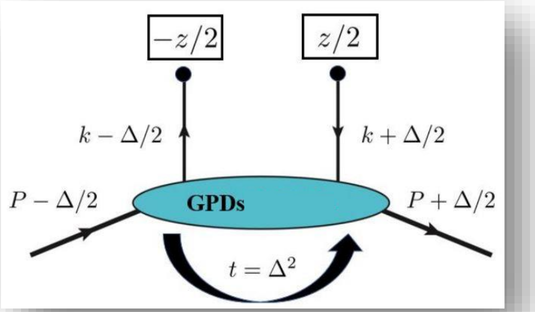
Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

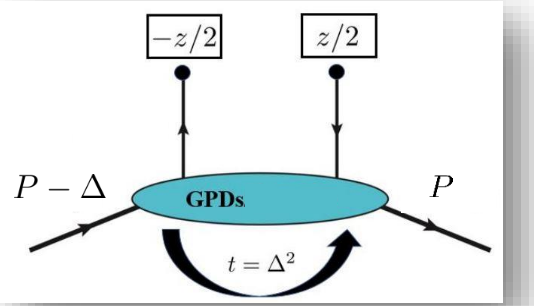
si-GPDs & Form Factors: (Sample results)

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6$$
$$+\left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0z^3}{2P_{avg,a}^0P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_{avg,a}^3}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3z^3}{2M^2P_{avg,a}^0P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3z^3}{4M^2P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P_{avg,a}^0P_{avg,a}^3}\right)A_8$$

Main results

Sketch of the essence of a
Lorentz-invariant definition of quasi-GPDs

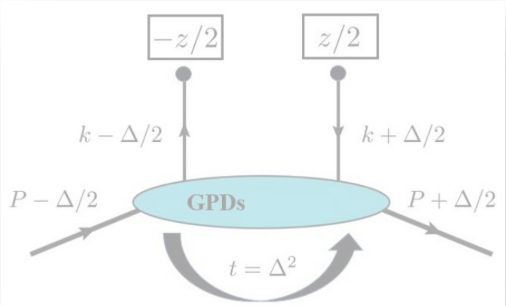
Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

si-GPDs & Form Factors: (Sample results)

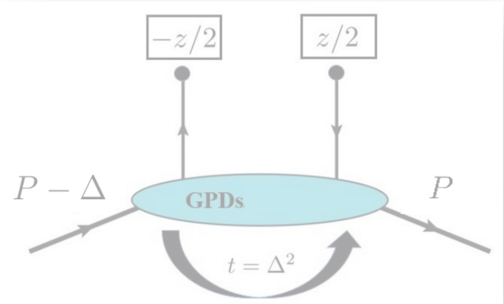
Symmetric frame:



$$H_{\mathrm{Q}(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Contamination from frame-dependent power corrections

Asymmetric frame:



$$H_{\mathrm{Q}(0)}\big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)A_8$$

Main results

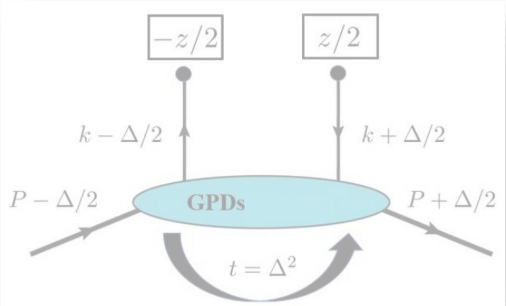
Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

si-GPDs & Form Factors: (Sample results)

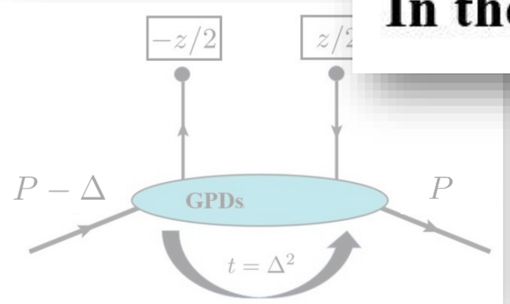


Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\Big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Contamination from frame-dependent power corrections

Asymmetric frame:



In the large-momentum limit, these expressions reduce to light-cone results

$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)A_6$$

$$+\left(\frac{(\Delta_a^0)^3z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

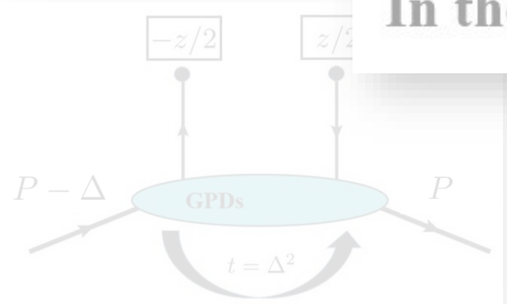
Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Contamination from frame-dependent power corrections

Asymmetric frame:

In the large-momentum limit, these expressions reduce to light-cone results



$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6$$

$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)A_8$$

Main results

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Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \tag{12}$$

type, where $\hat E(0,z;A)$ is the standard $0 \to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)\,. \tag{13}$$

2 Amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

Contamination from frame-dependent power corrections

... limit, these expressions reduce to light-cone results

Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

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$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp), -z^2) + z^\alpha \mathcal{M}_z(-(zp), -z^2)\,. \qquad (13)$$

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The $\mathcal{M}_p(-(zp), -z^2)$ part gives the twist-2 distribution when $z^2\to 0$, while $\mathcal{M}_z((zp), -z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

Contamination from frame-dependent power corrections

If one takes $z=(z_-, z_\perp)$ in the $\alpha=+$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

... limit, these expressions reduce to light-cone results

A_6

A_8

Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin
Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

$$\left.\frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right) A_6$$

Contamination from frame-dependent power corrections

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

type, where $\hat E(0,z;A)$ is the standard $0\to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)\,. \qquad (13)$$

2 Amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2\to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

If one takes $z=(z_-,z_\perp)$ in the $\alpha=+$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

... limit, these expressions reduce to light-cone results

formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0\int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \qquad (14)$$

Therefore, γ^0 is better behaved than γ^3 with respect to power corrections

Main results

Relation between light-cone GPD H & Form Factors:

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Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

Old Domini
Thomas Jefferson Natio

Statement needs a qualifier: Situation more complicated for quasi-GPDs

(See next slide)

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

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Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

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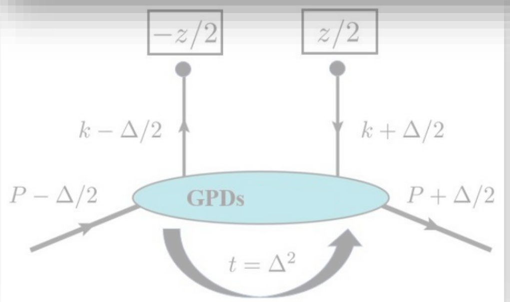
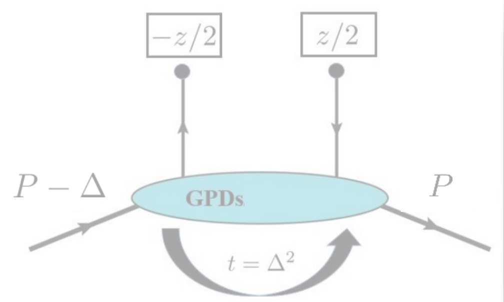
Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is plagued with (frame-dependent) power corrections



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0z^3}{2P_{avg,a}^0P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_{avg,a}^3}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3z^3}{2M^2P_{avg,a}^0P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3z^3}{4M^2P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P_{avg,a}^0P_{avg,a}^3}\right)A_8$$

Main results

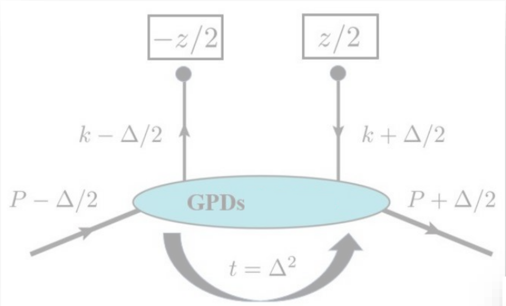
Sketch of the essence of a
Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

si-GPDs & Form Factors: (Sample results)



Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6$$

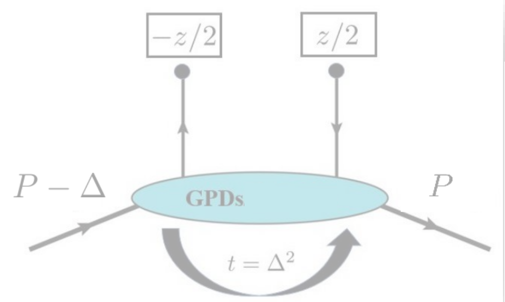
$$+\left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)}{2M^2}\cdots\right.$$

In spirit of what's done for PDFs:

You can think of eliminating power corrections by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 + \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$

$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2 P^3_{avg,a}}\right)A_6$$

$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

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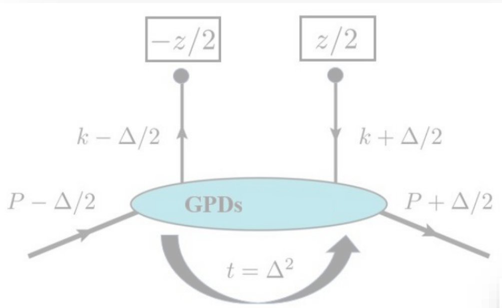
si-GPDs & Form Factors: (Sample results)



Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6$$

$$+\left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)}{2M^2}\right.$$

You can think of eliminating power corrections by the addition of other operators:

$$(\gamma^1,\gamma^2)$$

In spirit of what's done for PDFs:

$$\frac{\Delta_a^0\Delta_a^3z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\Big)A_4$$

$$-\frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\Big)A_6$$

$$\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\Big)A_8$$

Main finding:

Lorentz-invariant definition of quasi-GPDs:

Schematic structure: $H_Q \to c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel frame-dependent power corrections to project quasi-GPD to the light-cone result

Main results

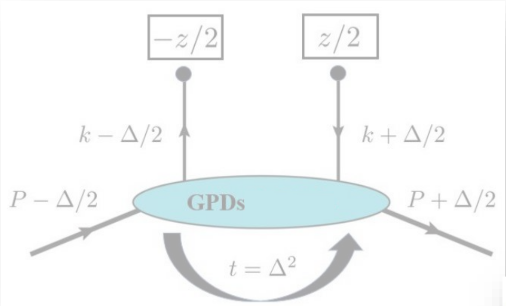
Sketch of the essence of a
Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

si-GPDs & Form Factors: (Sample results)



Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 + \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6$$

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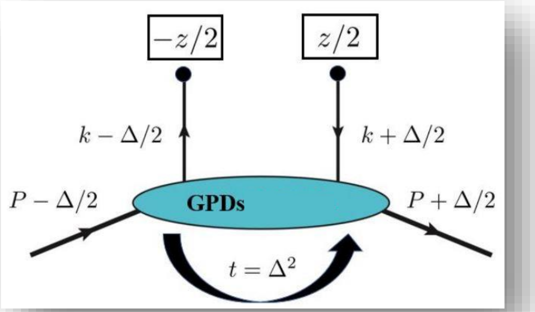
Note: Here c's are frame-dependent kinematic factors that cancel frame-dependent power corrections to project quasi-GPD to the light-cone result

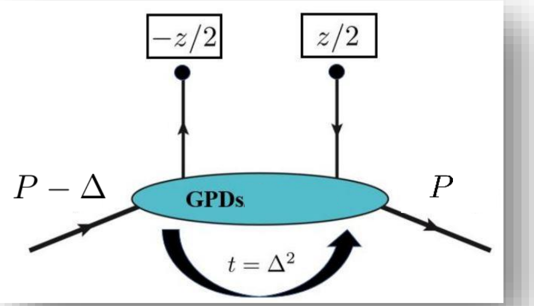
Agreement of results for H & E between frames confirmed by Lattice results

Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

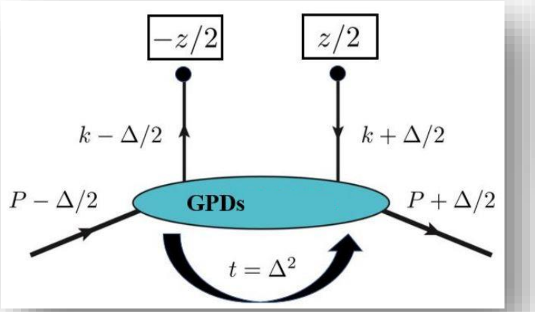




Main results

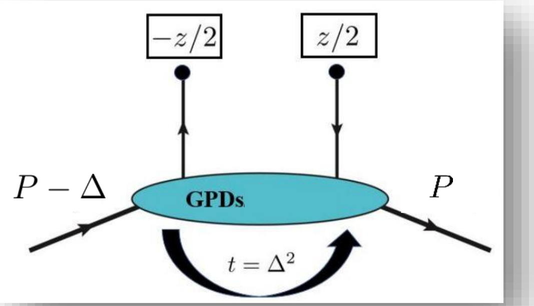
Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



Matching equation:

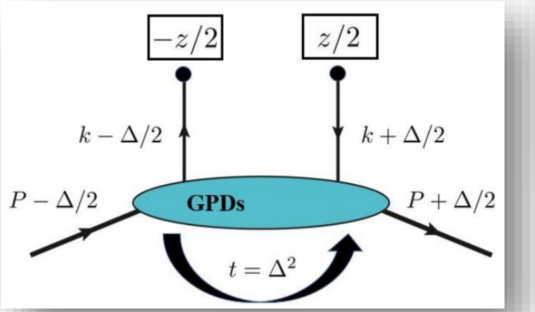
$$H_{\rm Q}(z\cdot P, z\cdot\Delta, t, z^2, \mu) = \int_{-1}^{1} du\, \bar{C}\,(u, z\cdot P, z\cdot\Delta, z^2, \mu^2)\, H(u, z\cdot P, z\cdot\Delta, t, \mu)$$



Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



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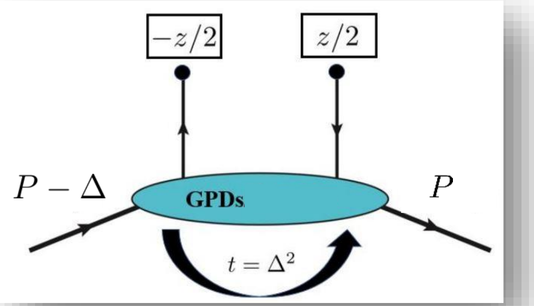
$$H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, t, z^2, \mu) = \int_{-1}^{1} du\, \bar{C}\,(u, z\cdot P, z\cdot\Delta, z^2, \mu^2)\, H(u, z\cdot P, z\cdot\Delta, t, \mu)$$

Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

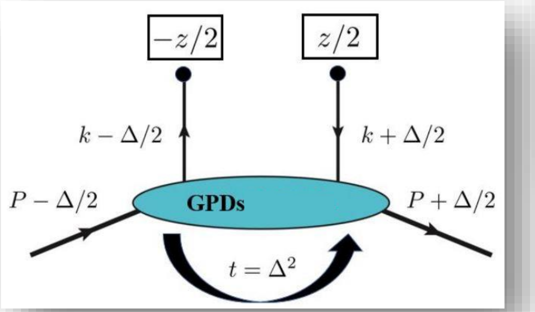
$$\lim_{z^2\to 0} H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = H(z\cdot P, z\cdot\Delta, \Delta^2)$$



Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



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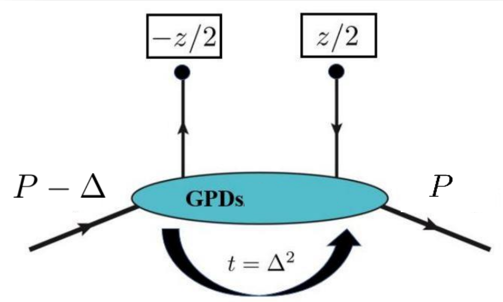


Relation between light-cone GPD H & Form Factors:

$$\boxed{H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}}$$

Natural candidate:

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$

$$\boxed{H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}}$$

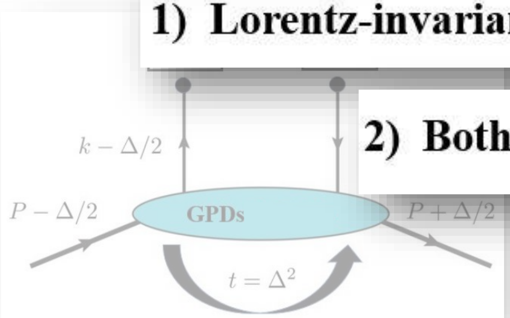
Main results

Lorentz covariant formalism

Sketch of the essence of a
L t definition of quasi-GPDs

Key points:

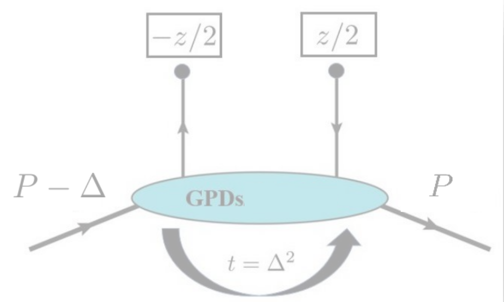
1) Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ should converge faster at LO



2) Both sides Lorentz invariant → NLO differences suppressed by frame-independent power corrections

Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

$$\lim_{z^2 \to 0} H_Q(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = H(z \cdot P, z \cdot \Delta, \Delta^2)$$



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Main results

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Lorentz covariant formalism

Sample results for E

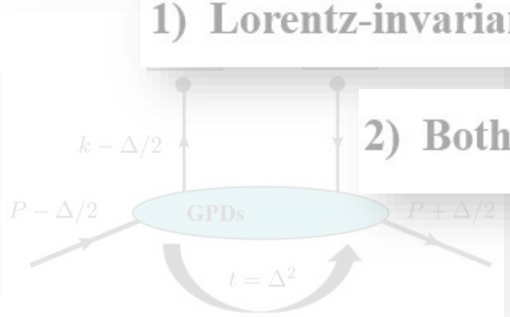
of the essence of a

t definition of quasi-GPDs

Key points:

1) Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ should converge faster at LO

2) Both sides Lorentz invariant → NLO differences suppressed by frame-independent power corrections



Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

$$\lim_{z^2 \to 0} H_Q(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = H(z \cdot P, z \cdot \Delta, \Delta^2)$$

Relation between light-cone GPD H & Form Factors:

$$\boxed{H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3}$$

Natural candidate:

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$

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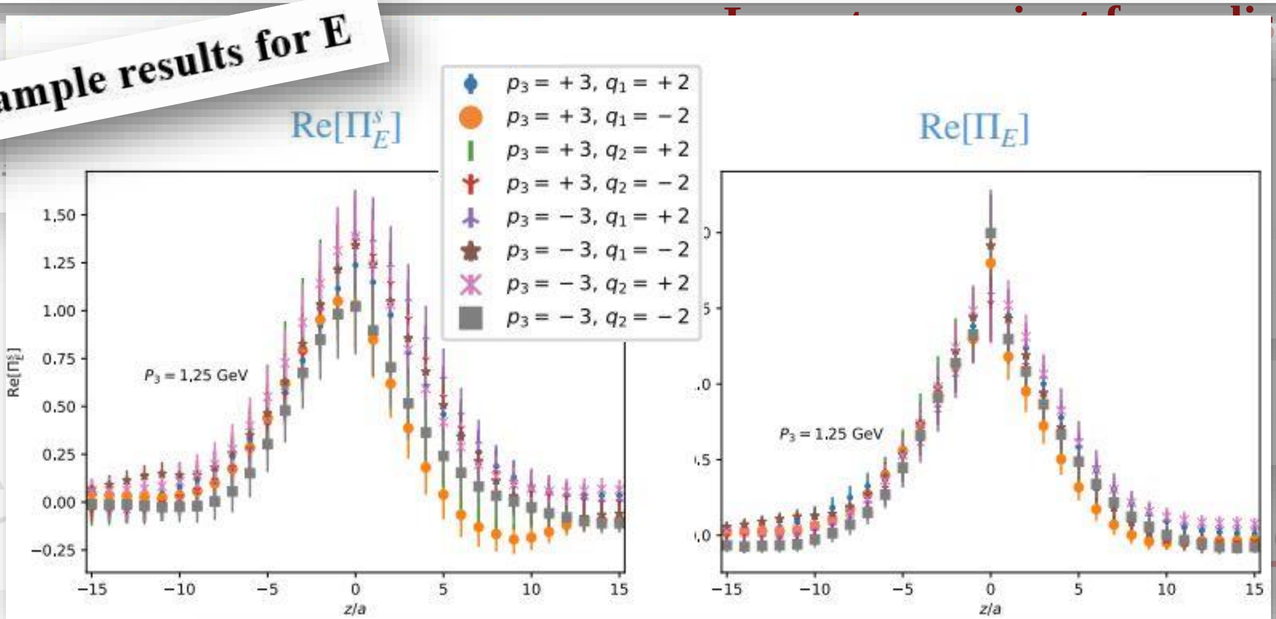
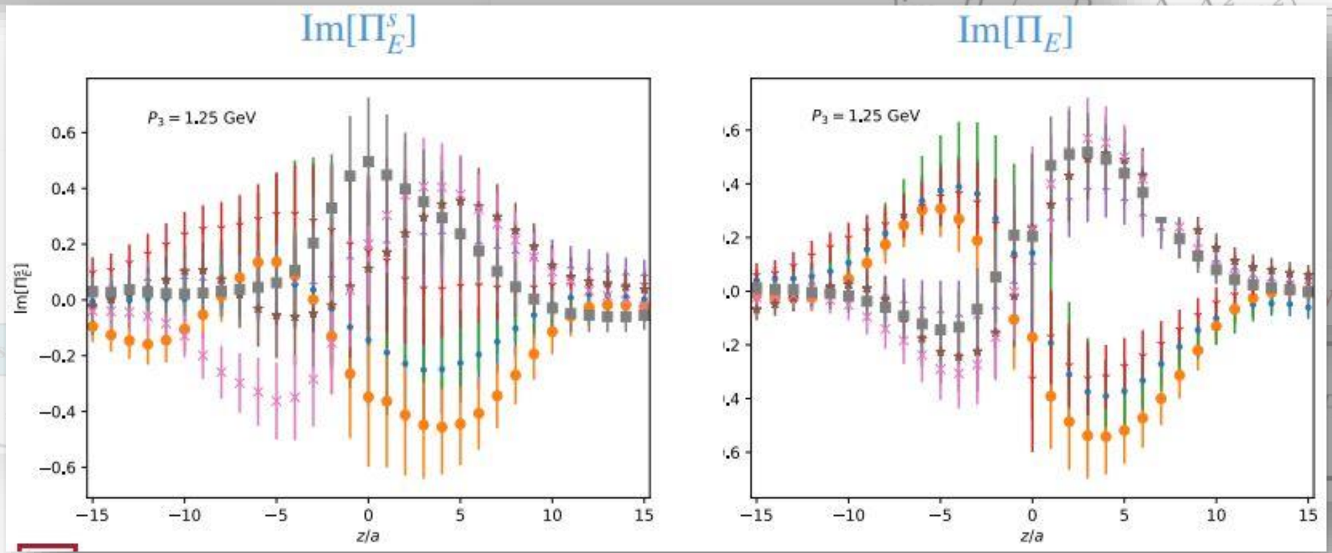
Main results

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E





should converge faster at LO

suppressed by frame-independent power corrections

of light-cone & quasi-GPDs at LO

$= H(z \cdot P, z \cdot \Delta, \Delta^2)$

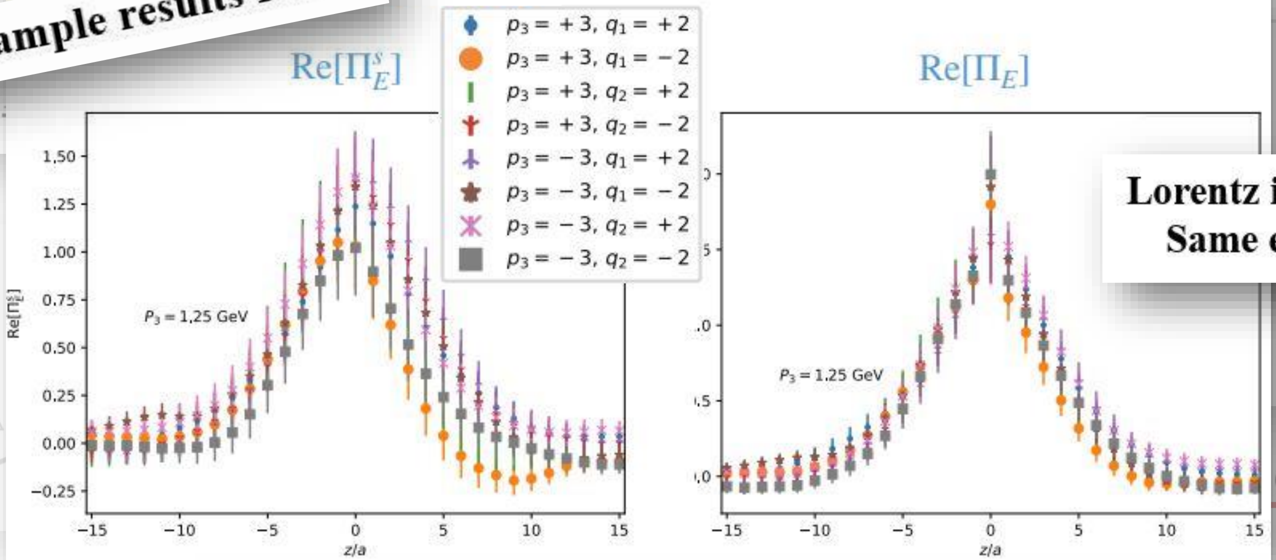
Natural candidate:

variant generalization of LC definition to $z^2 \neq 0$

$$P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3$$

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E

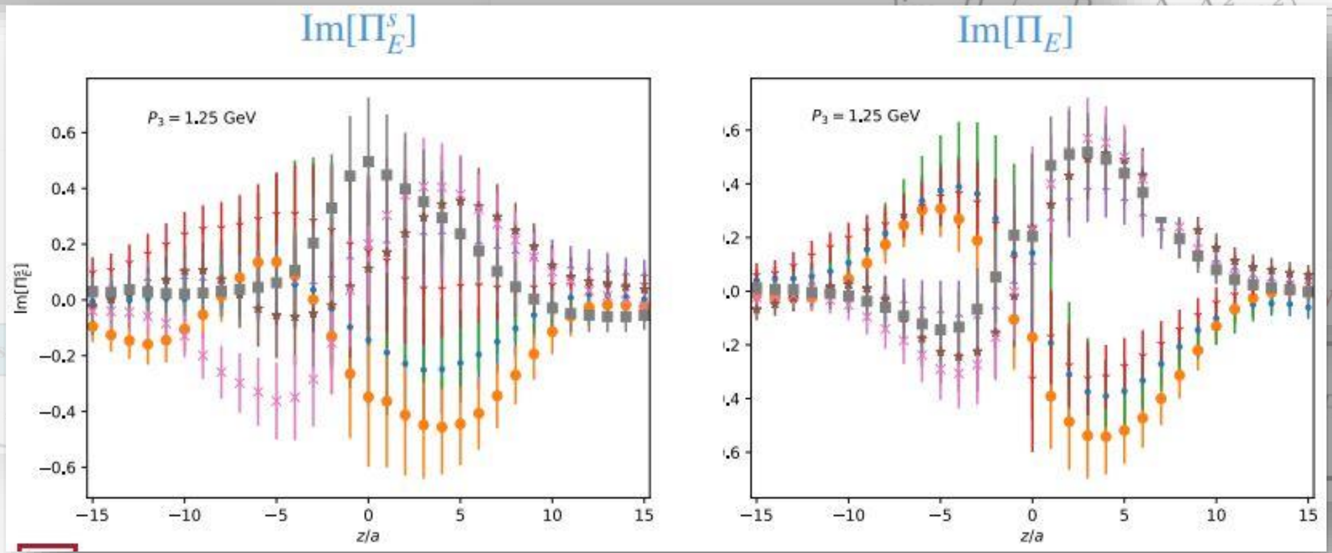


Lorentz invariant definition leads to more precise results for E;
Same effect of improvement for asymmetric frame

suppressed by frame-independent power corrections

of light-cone & quasi-GPDs at LO

$= H(z \cdot P, z \cdot \Delta, \Delta^2)$



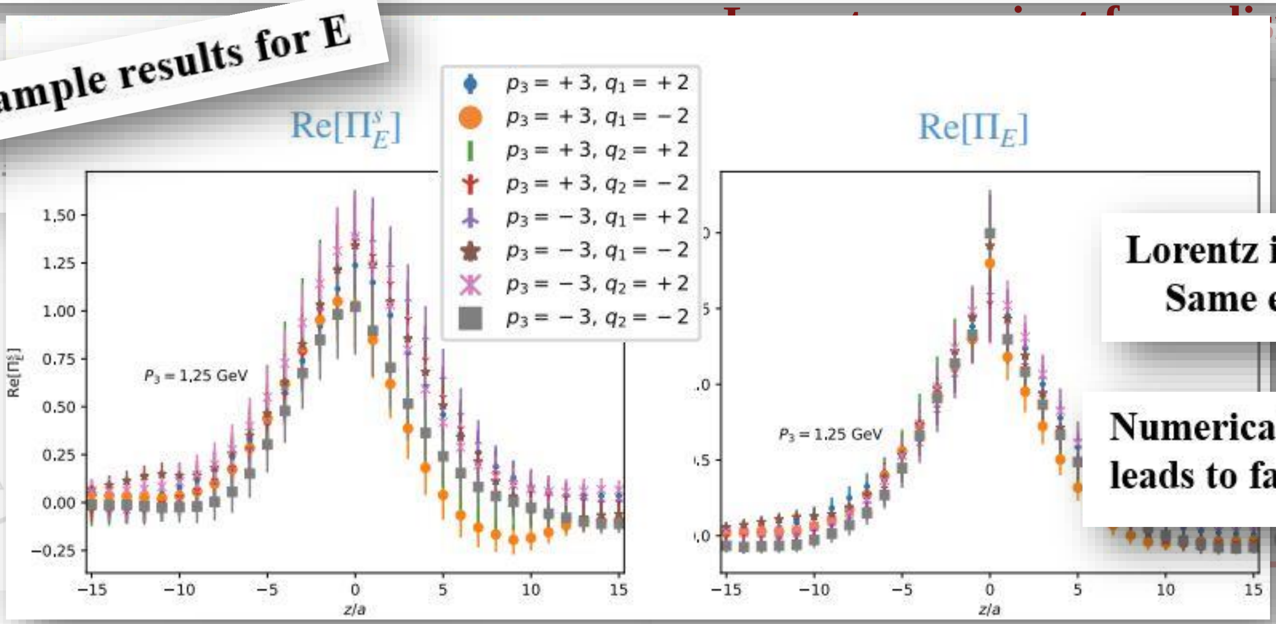
Natural candidate:

variant generalization of LC definition to $z^2 \neq 0$

$$P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3$$

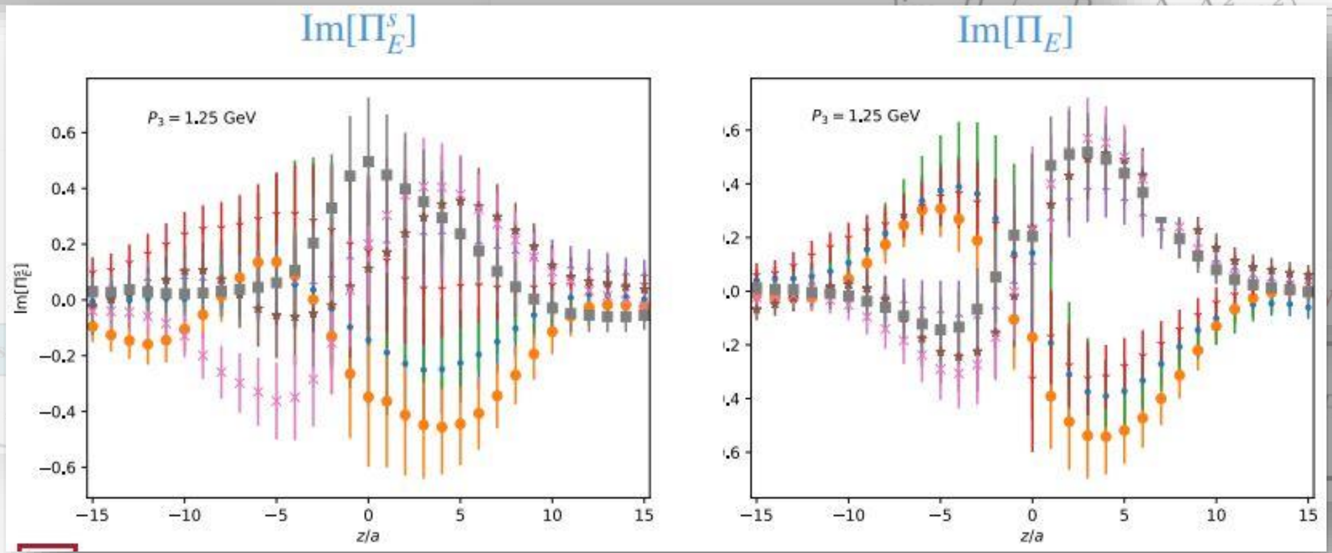
Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E



Lorentz invariant definition leads to more precise results for E;
Same effect of improvement for asymmetric frame

Numerical indications that using Lorentz invariant definition for E leads to faster convergence to LC GPD with respect to P^3

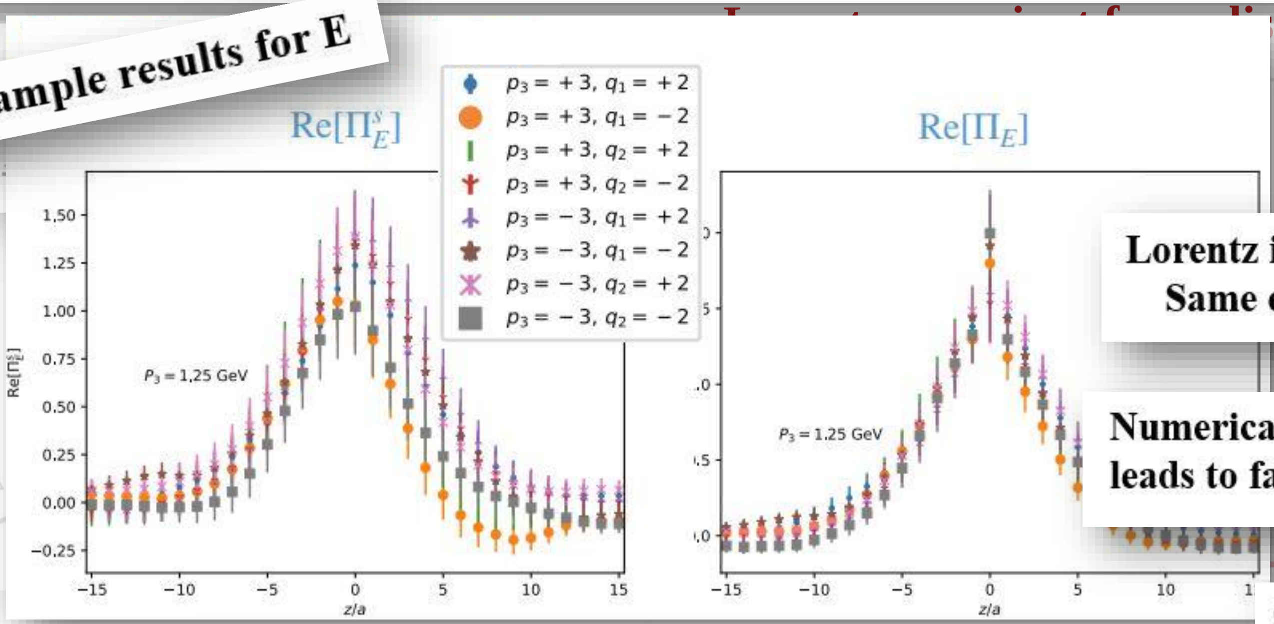


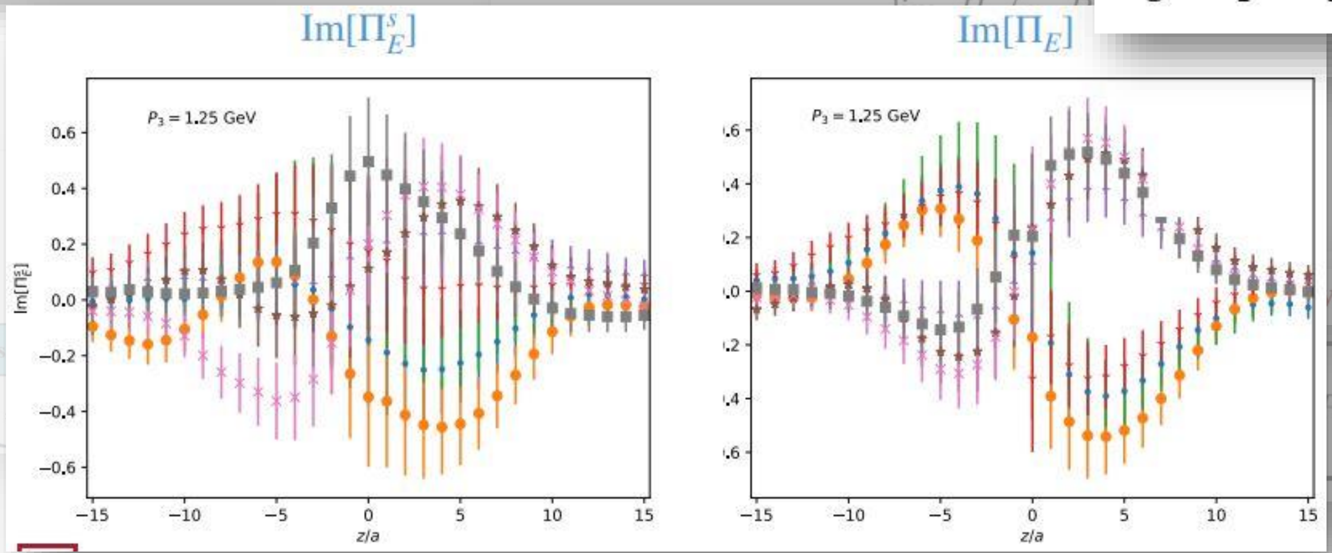
Natural candidate:

Main results

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E





Lorentz invariant definition leads to more precise results for E;
Same effect of improvement for asymmetric frame

Numerical indications that using Lorentz invariant definition for E leads to faster convergence to LC GPD with respect to P^3

Signal quality for H same for all cases (not shown)

of light-cone & quasi-GPDs at LO

Natural candidate:

variant generalization of LC definition to $z^2 \neq 0$

$$P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Summary

Connecting dots: Ending with what I started with

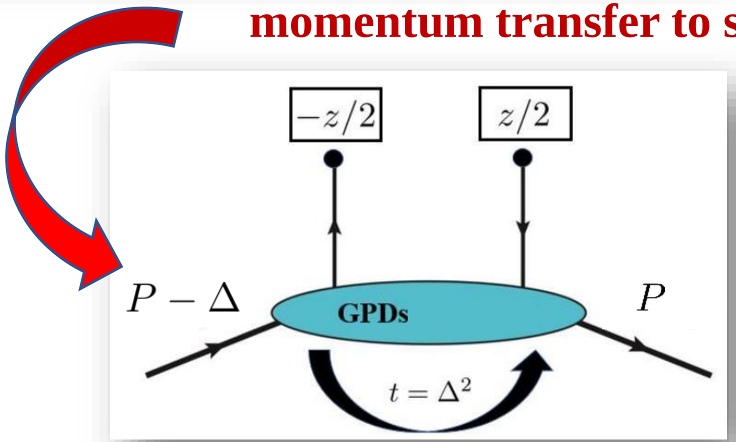
Summary

Goal:

Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

All momentum transfer to source



Summary

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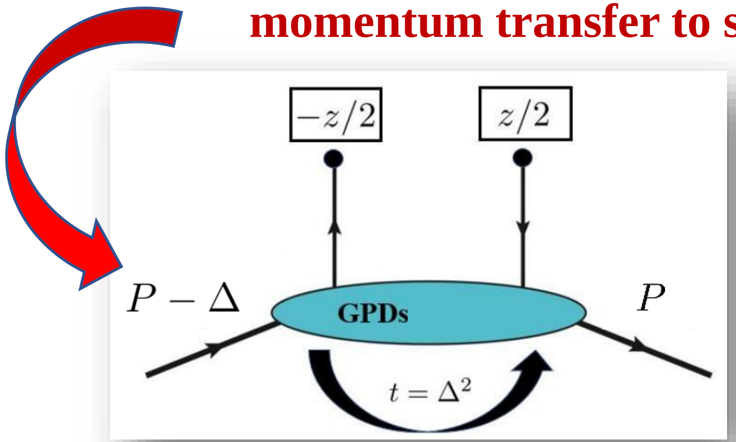


Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

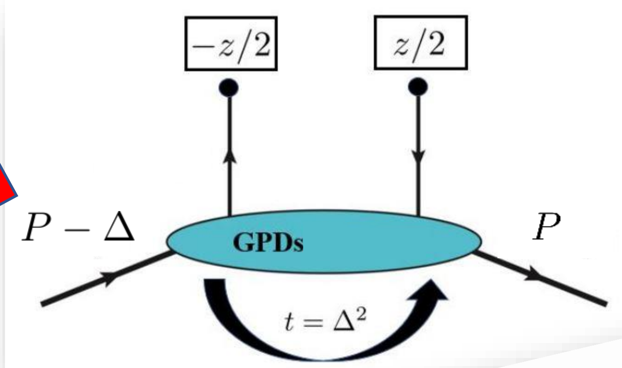
Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



Key findings: QCD calc... asymmetric fram...

1) **Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant**

$$H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle$$

Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

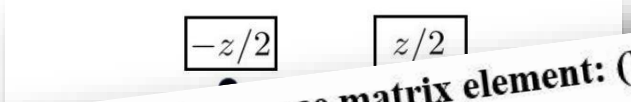
Contamination from frame-dependent power corrections

Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

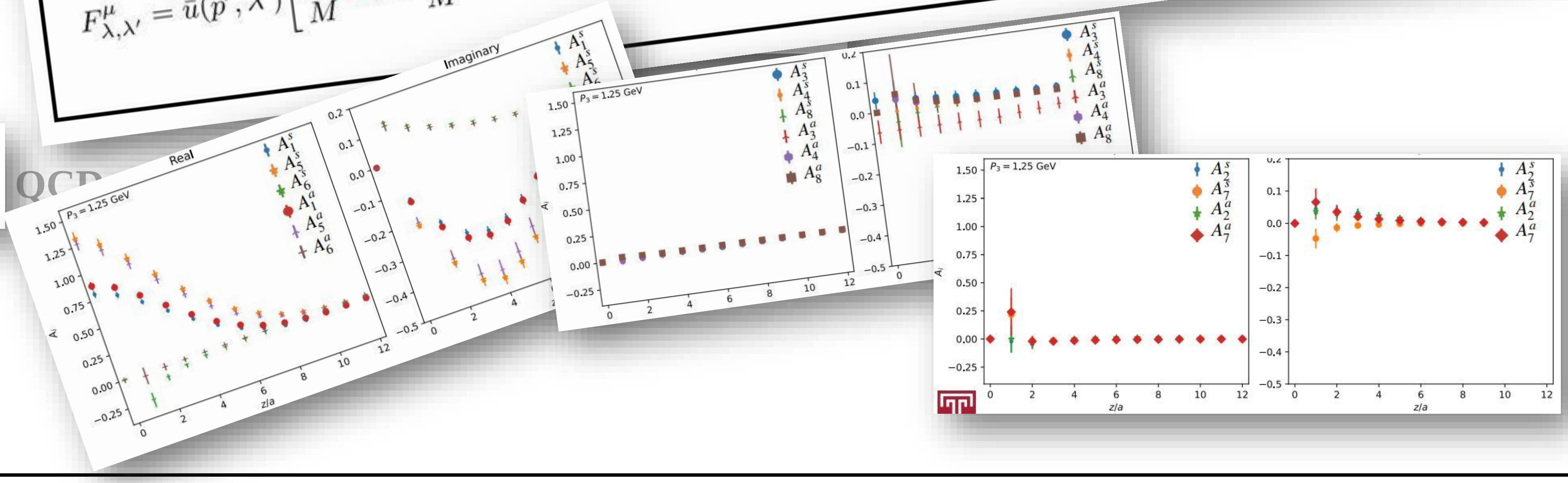
All momentum transfer to source



2) **Novel parameterization of position-space matrix element: (Vector operator)**

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\mathbf{A_1} + \frac{z^{\mu}}{M}\mathbf{A_2} + \frac{\Delta^{\mu}}{M}\mathbf{A_3} + \frac{i\sigma^{\mu z}}{M}\mathbf{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\mathbf{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\mathbf{A_8}\right]u(p,\lambda)$$

Key findings:

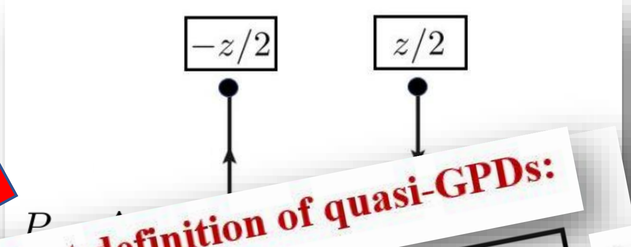


Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



3)

Lorentz-invariant definition of quasi-GPDs:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$H_Q \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of (frame-dependent) power corrections allowing faster convergence to light-cone GPDs at LO**

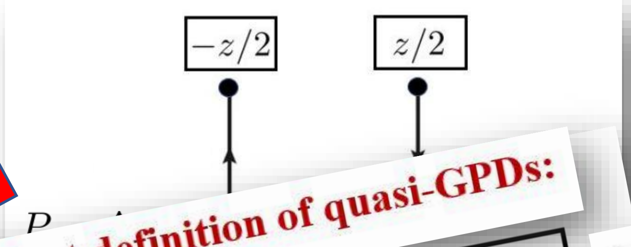
Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All

momentum transfer to source



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Lorentz-invariant definition of quasi-GPDs:

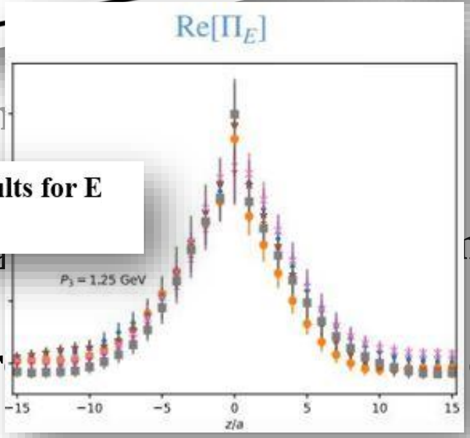
$$H_Q(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = \mathbf{A_1} + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} \mathbf{A_3}$$

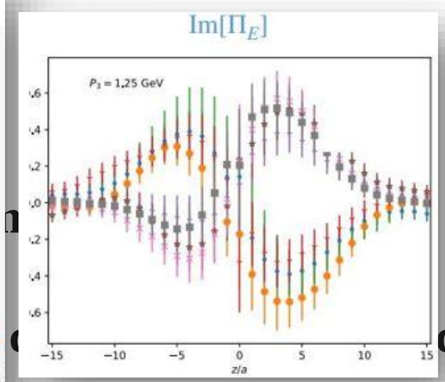
$$H_Q \to c_0 \langle \bar{\psi} \gamma^0 \psi \rangle + c_1 \langle \bar{\psi} \gamma^1 \psi \rangle + c_2 \langle \bar{\psi} \gamma^2 \psi \rangle$$

Key findings:

QCD ... lations of GPDs in

Lorentz invariant definition leads to more precise results for E





- Lorentz cova...
- Elimination of (frame-dependent) power corr... cone GPDs at LO

Backup slides

Main results

Renormalization: Sketch

Few words on operators:

- **Schematic structure of Lorentz non-invariant quasi-GPD:** $$\boxed{H_{\mathrm{Q}} \to c\langle\bar{\psi}\gamma^0\psi\rangle}$$

- **Schematic structure of Lorentz invariant quasi-GPD:** $$\boxed{H_{\mathrm{Q}} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$$

How to renormalize?

Main results

Renormalization: Sketch

Few words on operators:

- **Schematic structure of Lorentz non-invariant quasi-GPD:** $\boxed{H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle}$

- **Schematic structure of Lorentz invariant quasi-GPD:** $\boxed{H_Q \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$

Few words on renormalization:

RI-MOM

- **Renormalization factors are different for** $\langle\bar{\psi}\gamma^0\psi\rangle$, $\langle\bar{\psi}\gamma^1\psi\rangle$, $\langle\bar{\psi}\gamma^2\psi\rangle$
  --- UV-divergent terms same
  --- Finite terms different

- **Matching:**
  --- Frame-independent
  --- Available for only γ^0
  --- Takes care of finite terms for γ^0

- **Strategy to renormalize: Use Renormalization factor for operator whose matching is known**