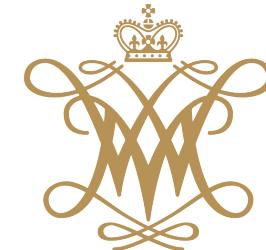


$\pi\pi$ scattering and the σ resonance

QNP 2022

Arkaitz Rodas

had spec



WILLIAM & MARY
CHARTERED 1693

Jefferson Lab
Thomas Jefferson National Accelerator Facility

$\pi\pi$

Pseudo-goldstone bosons

Spontaneous symmetry breaking

Populate final states

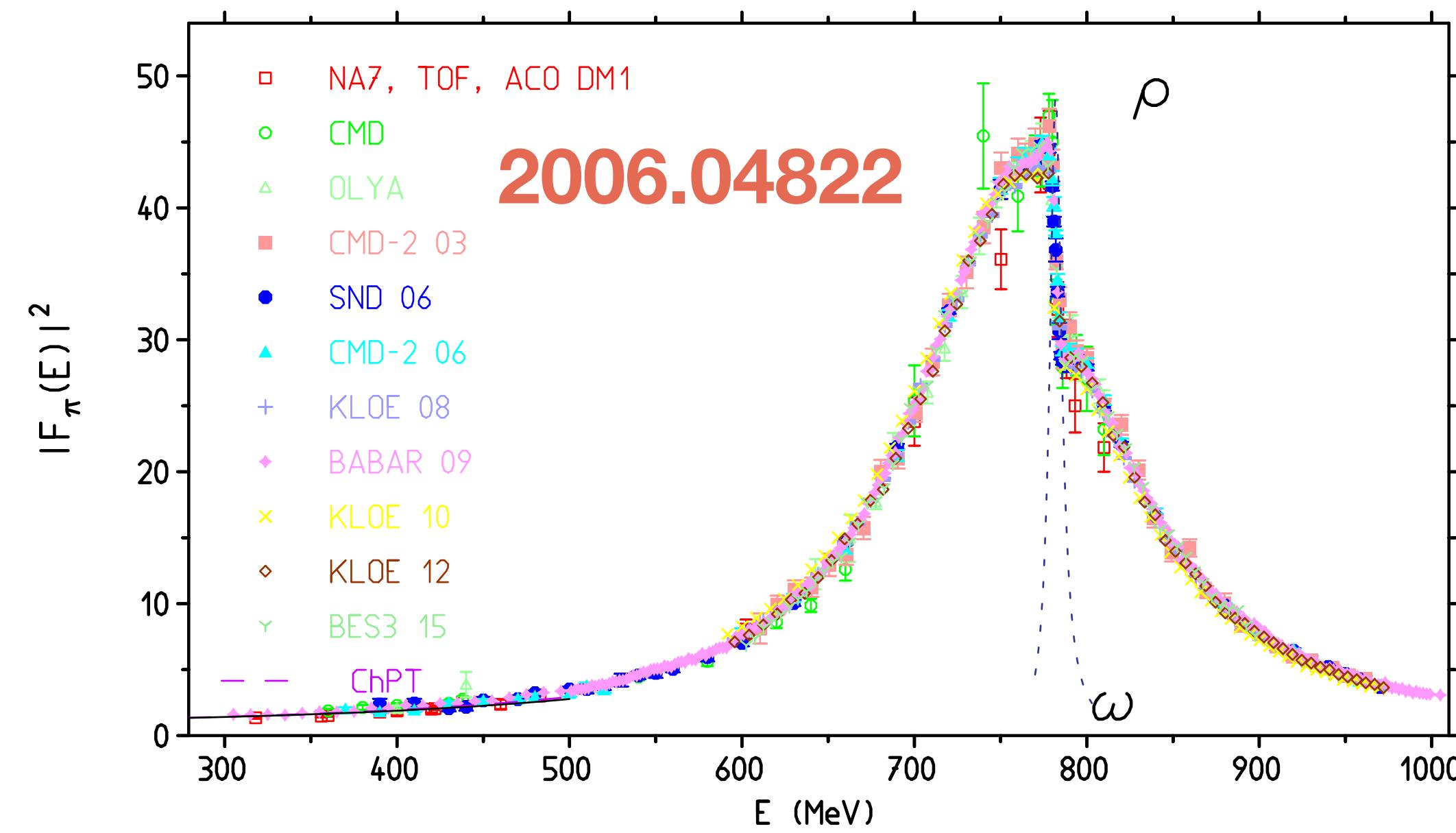
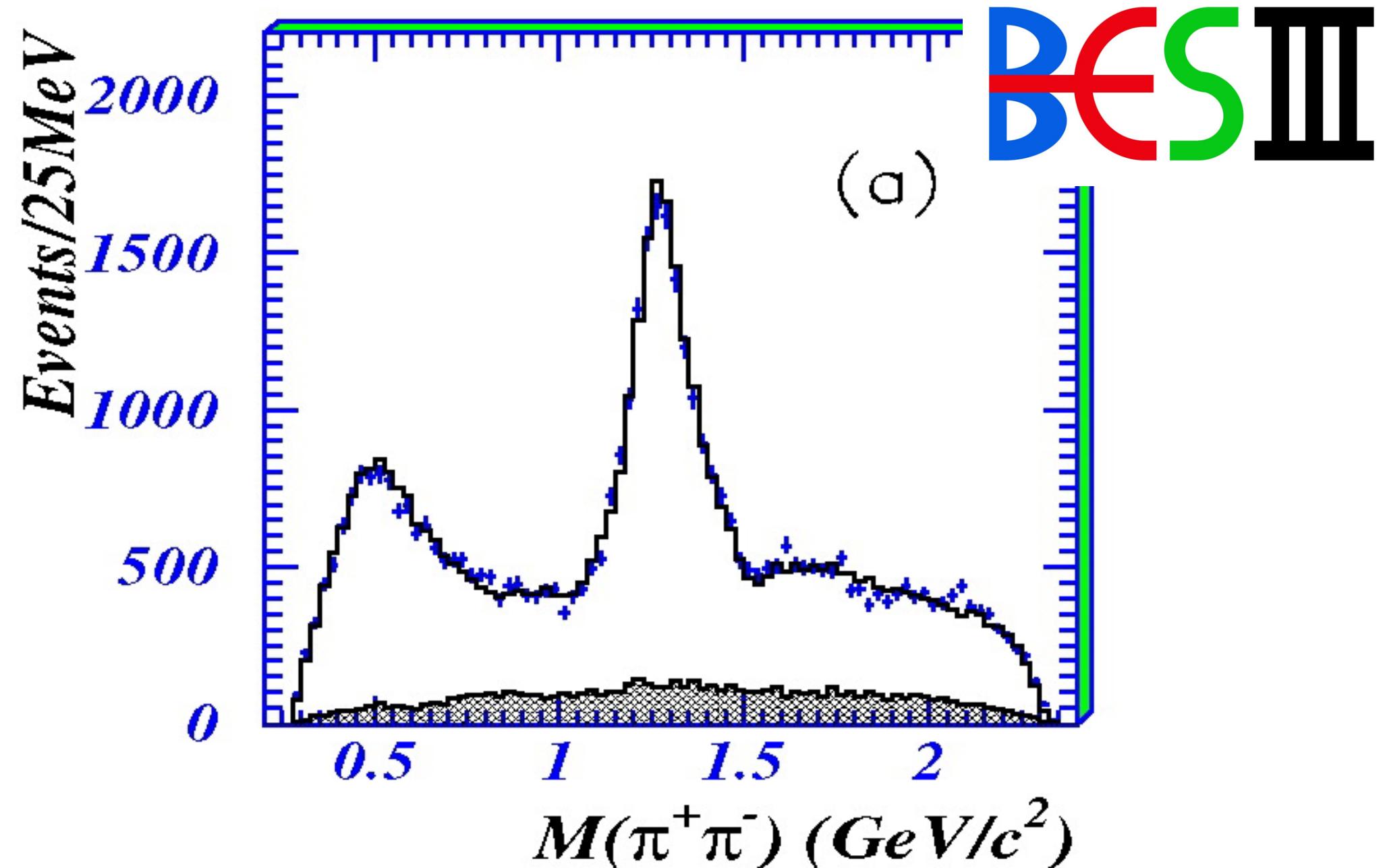
Crucial for hadron physics

HLbL

HVP

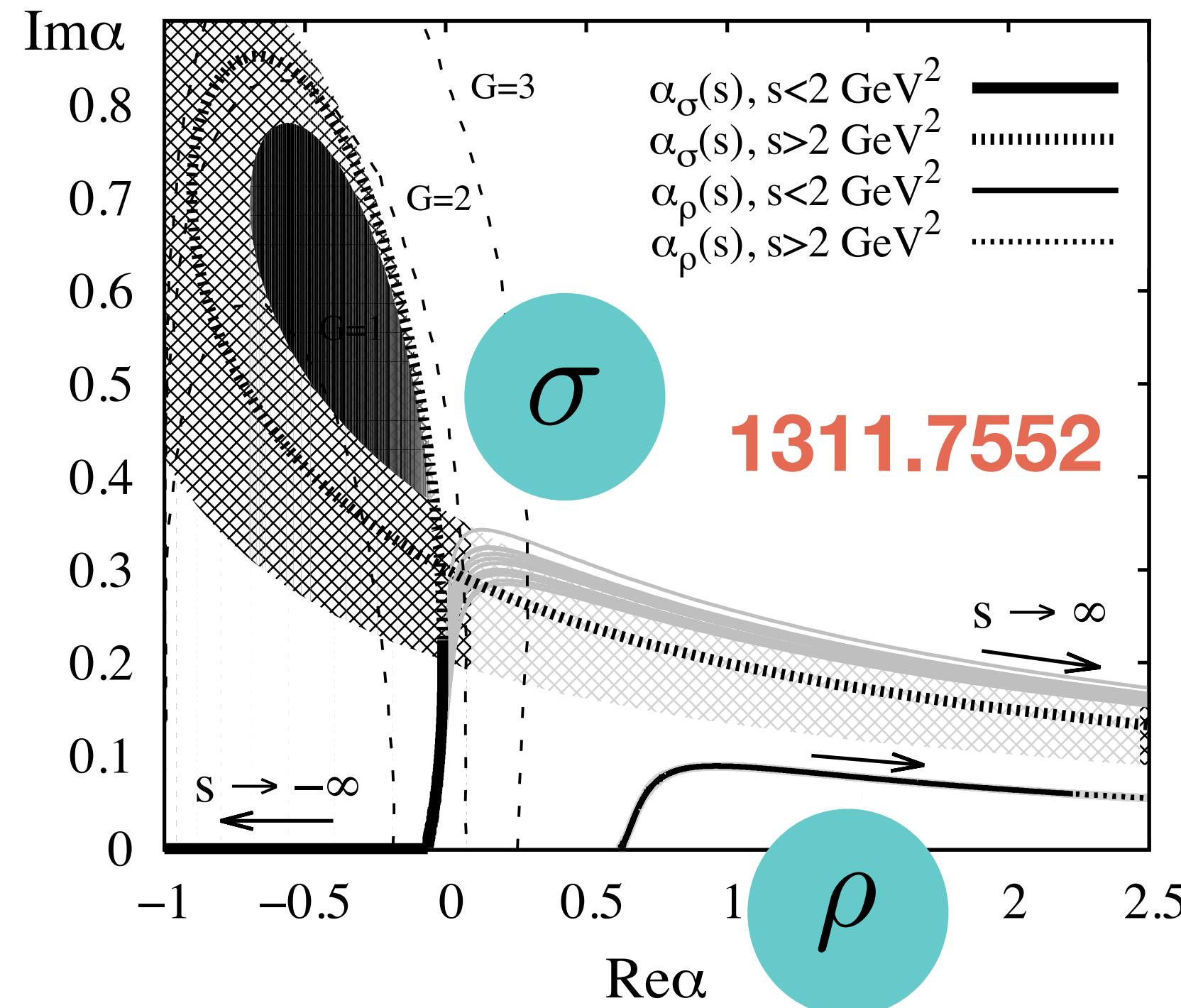
Resonances

Ordinary and non-ordinary

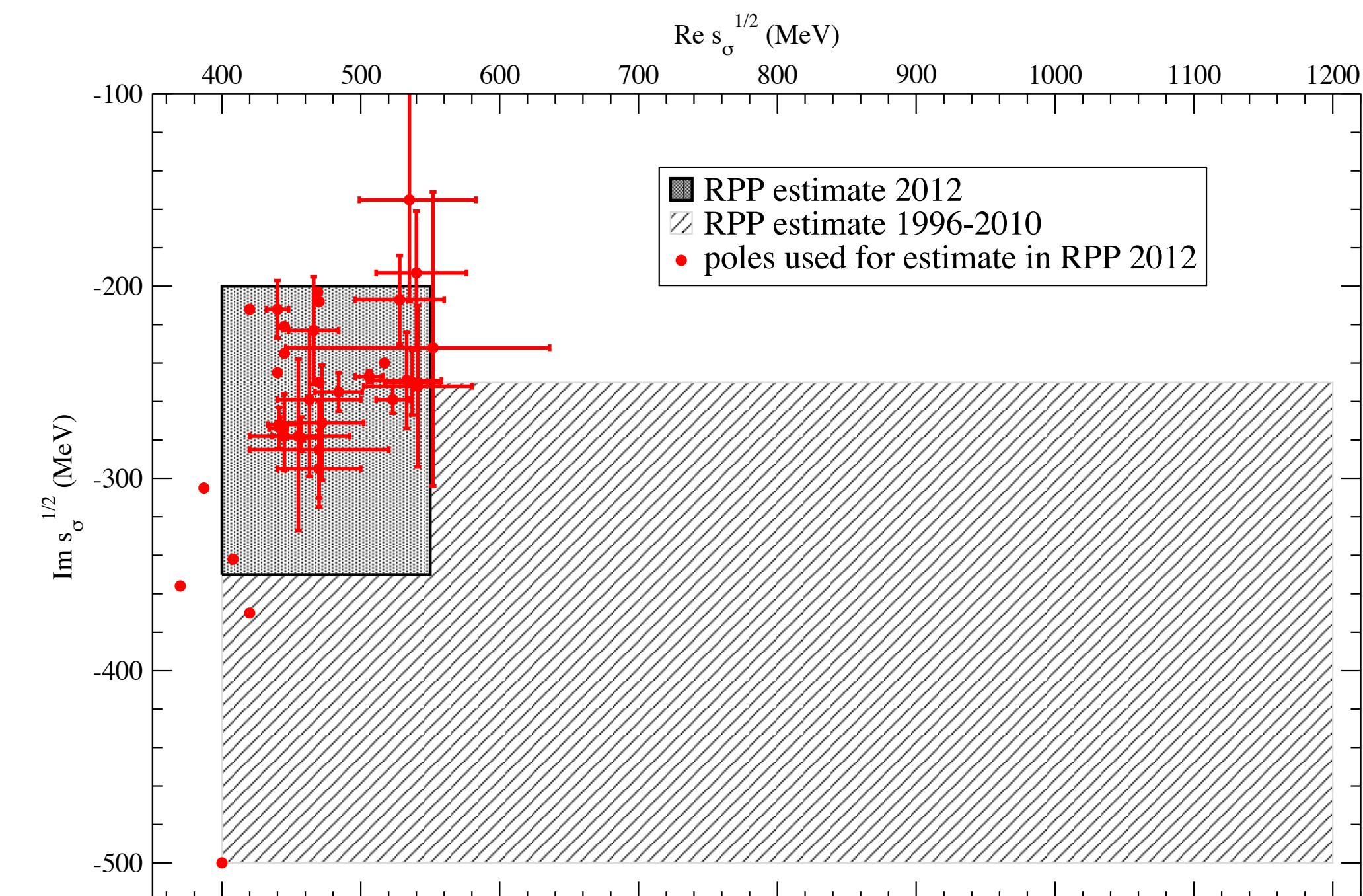


Light Scalars: the σ

Extremely broad object at phys. m_π



PDG



Non-ordinary resonance
Compositeness?

Debated for over half a century!!

The σ : Why lattice QCD?

Exp. “data” is not data

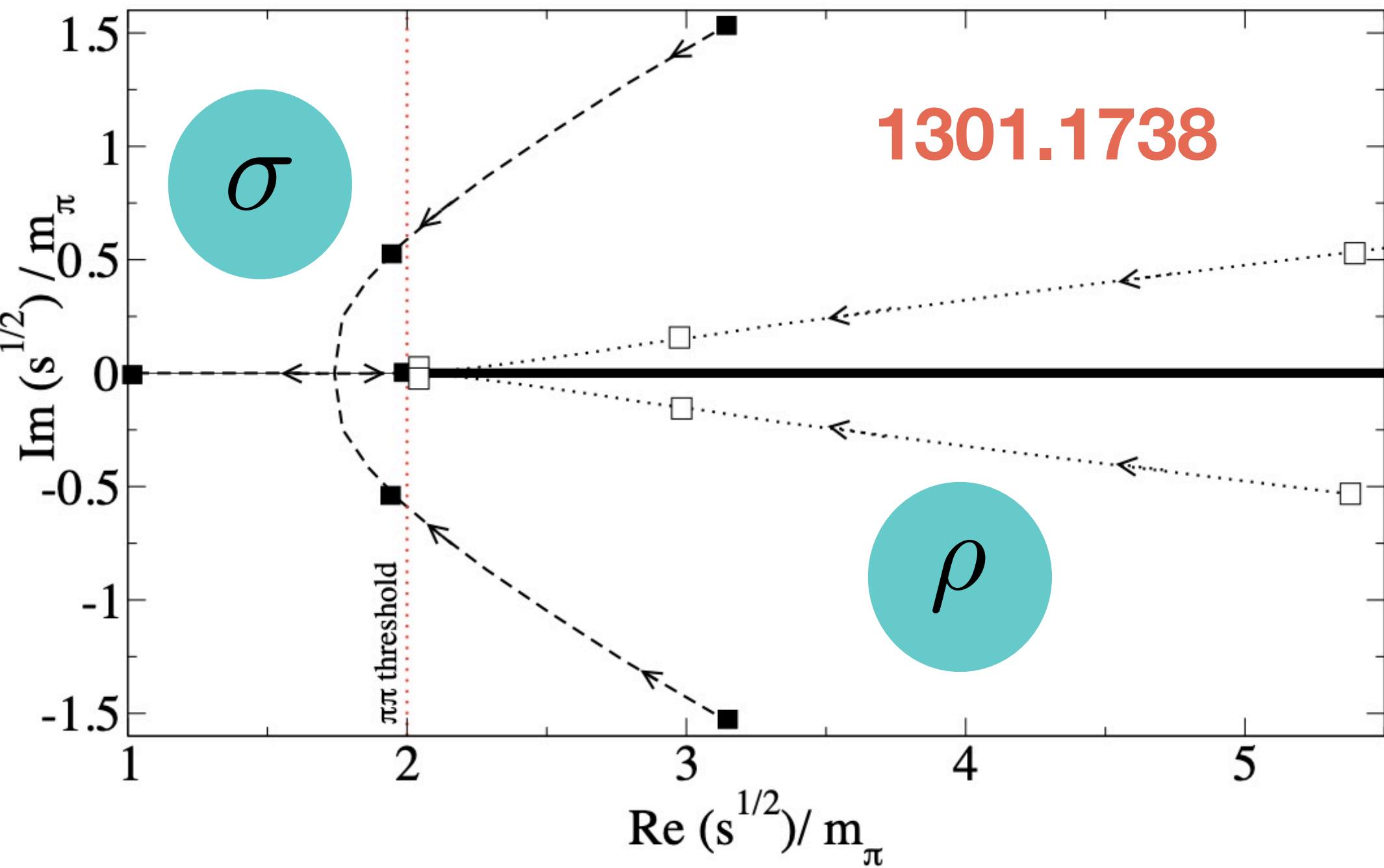
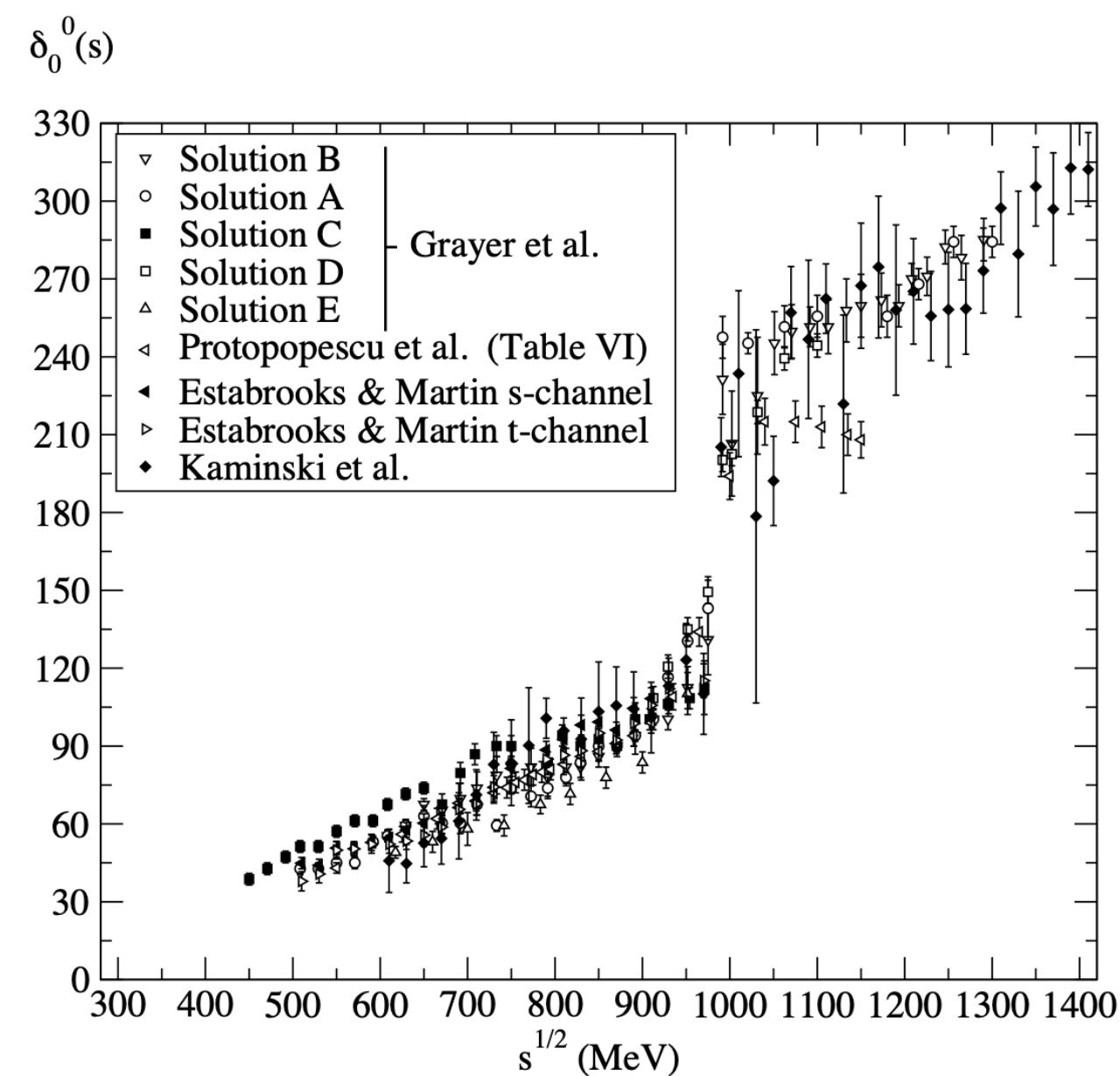
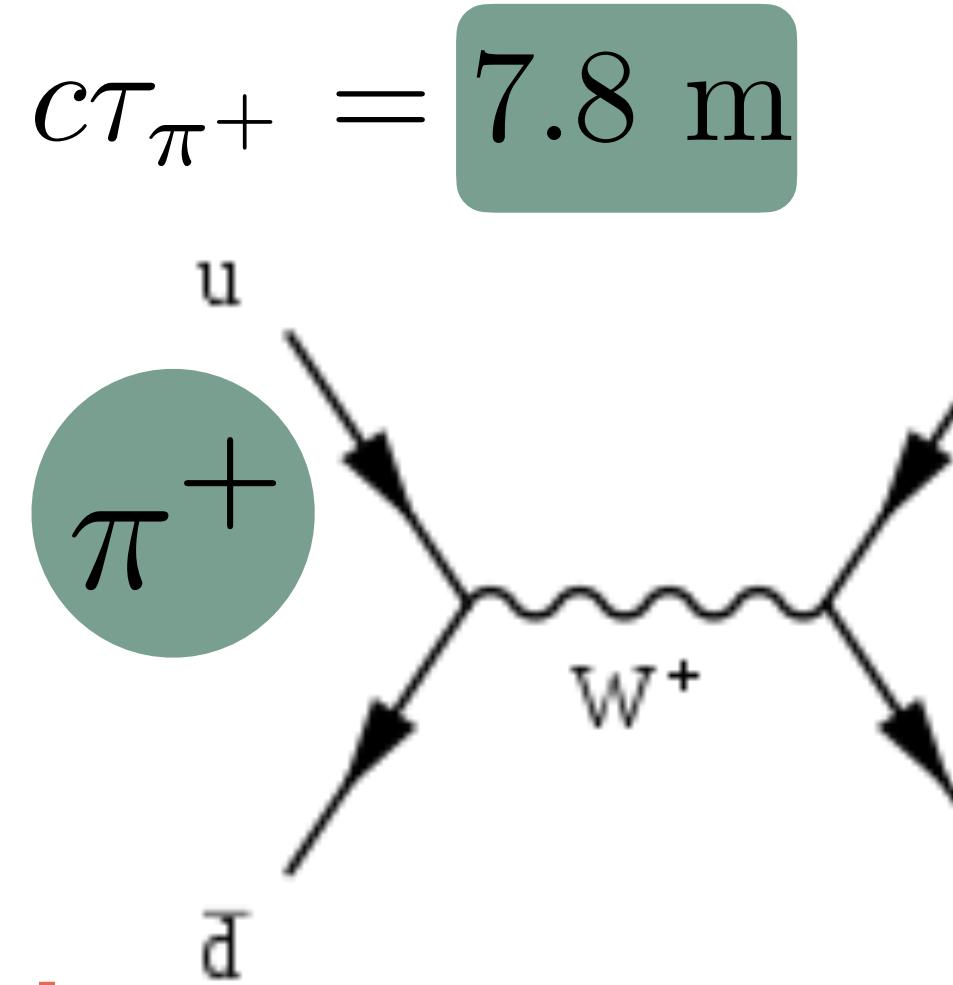
One beam (maybe)

For meson-meson “data” is always modeled

Interesting “modeled” m_q dependence

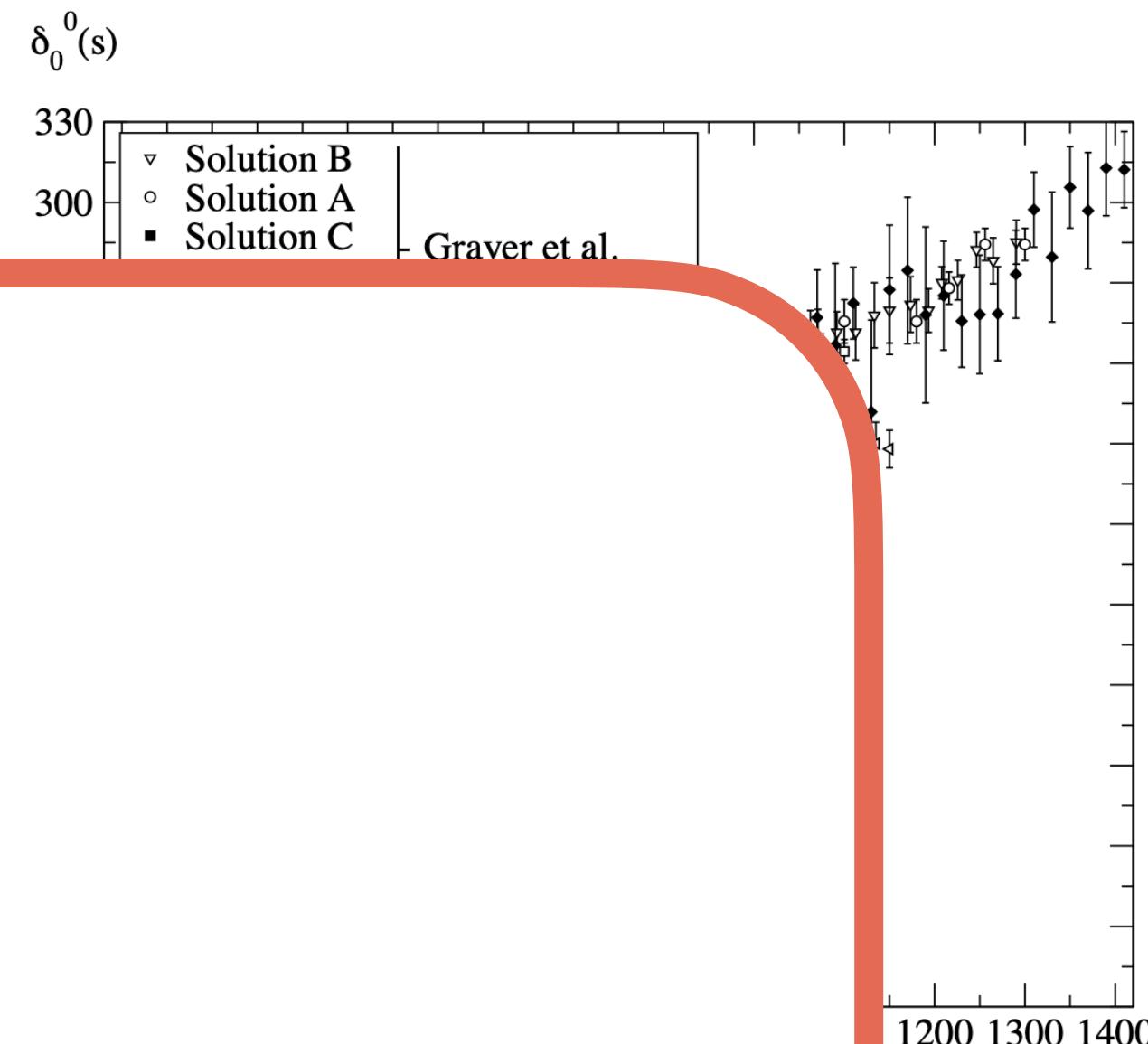
Can be related to its compositeness

Studied through its final products



The σ : Why lattice QCD?

$$c\tau_{\pi^+} = 7.8 \text{ m}$$



Or

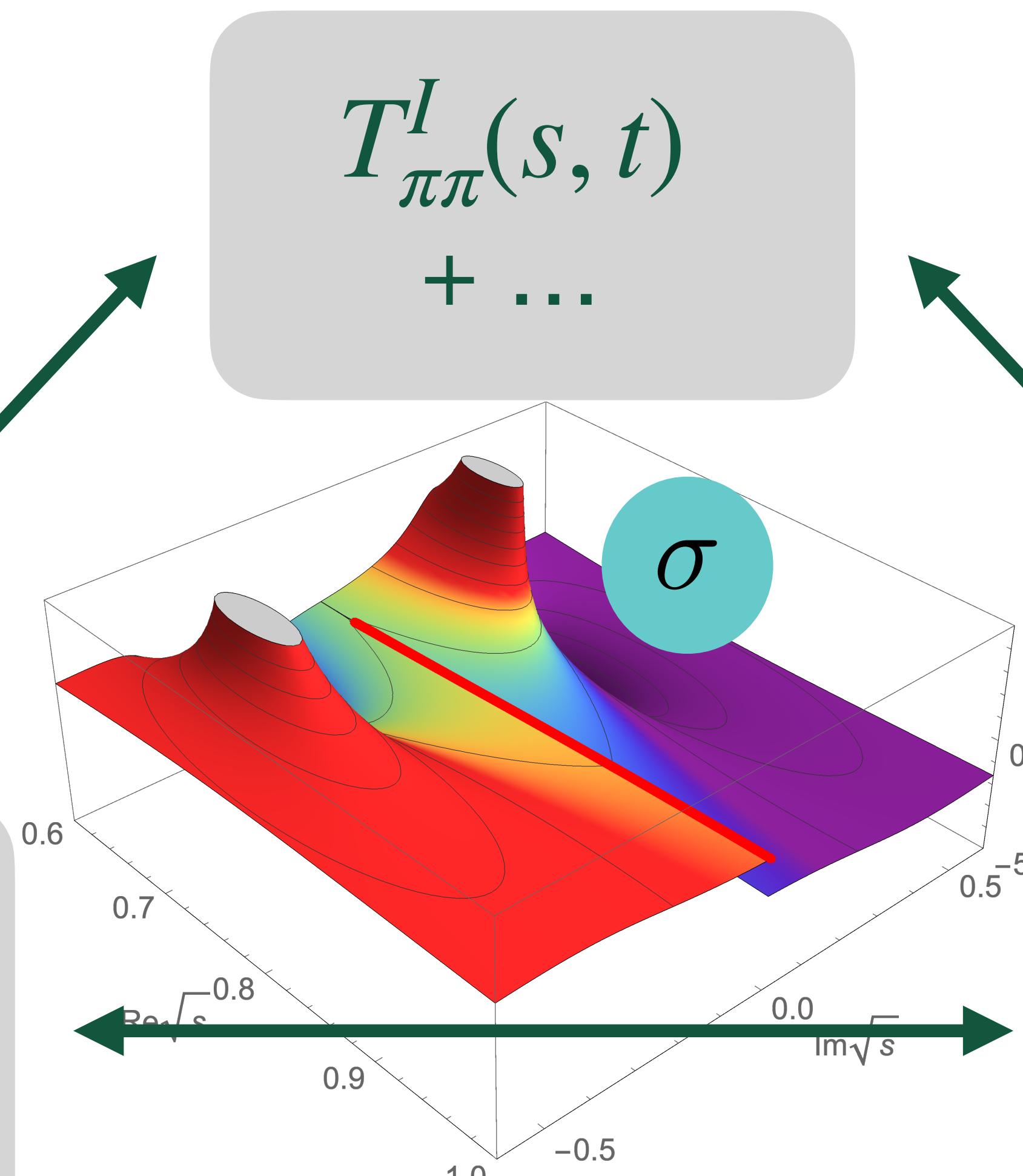
For

Inter

Cou

St

$s_p + |g|$
Form factors
Structure



QCD

$\text{Re}(s)/m_\pi$

The σ as intermediate state

$$N_f = 2 + 1$$

$m_\pi \sim 391 \text{ MeV} \rightarrow \text{BS}$

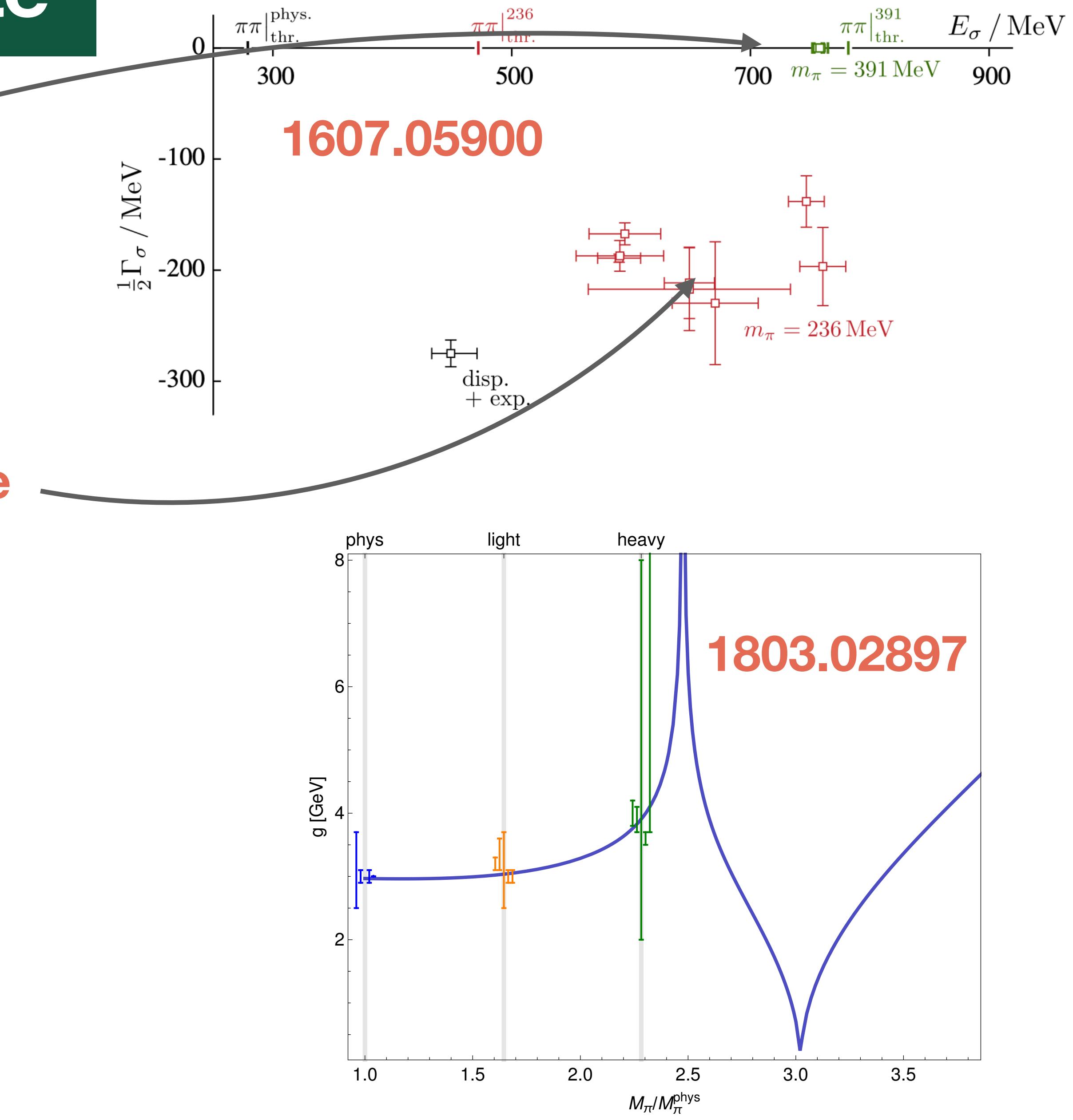
$m_\pi \sim ? \text{ MeV} \rightarrow \text{THIS TALK}$

$m_\pi \sim 236 \text{ MeV} \rightarrow \text{Broad resonance}$

$$N_f = 2$$

$m_\pi \sim 315 \text{ MeV} \rightarrow \text{VB vs resonance}$

$m_\pi \sim 227 \text{ MeV} \rightarrow \text{Broad resonance}$

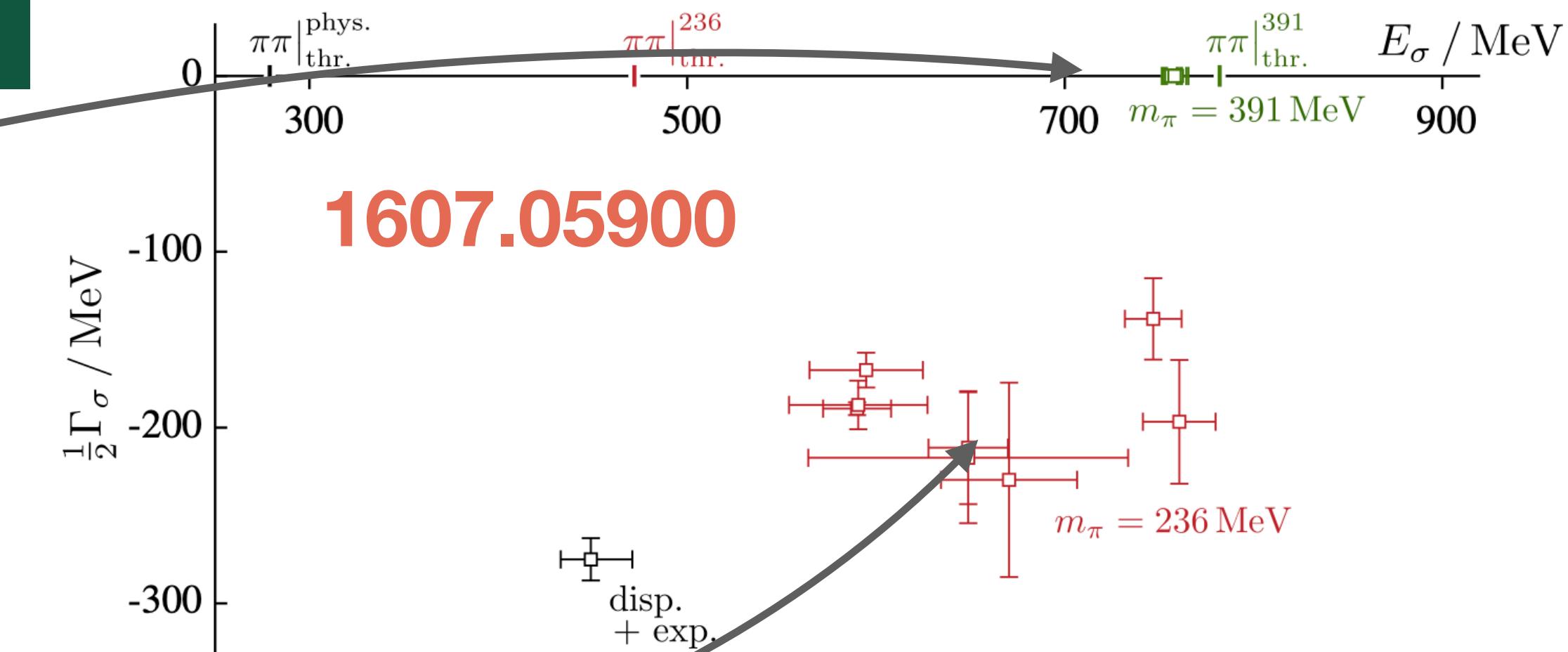


The σ as intermediate state

$$N_f = 2 + 1$$

$m_\pi \sim 391 \text{ MeV} \rightarrow \text{BS}$

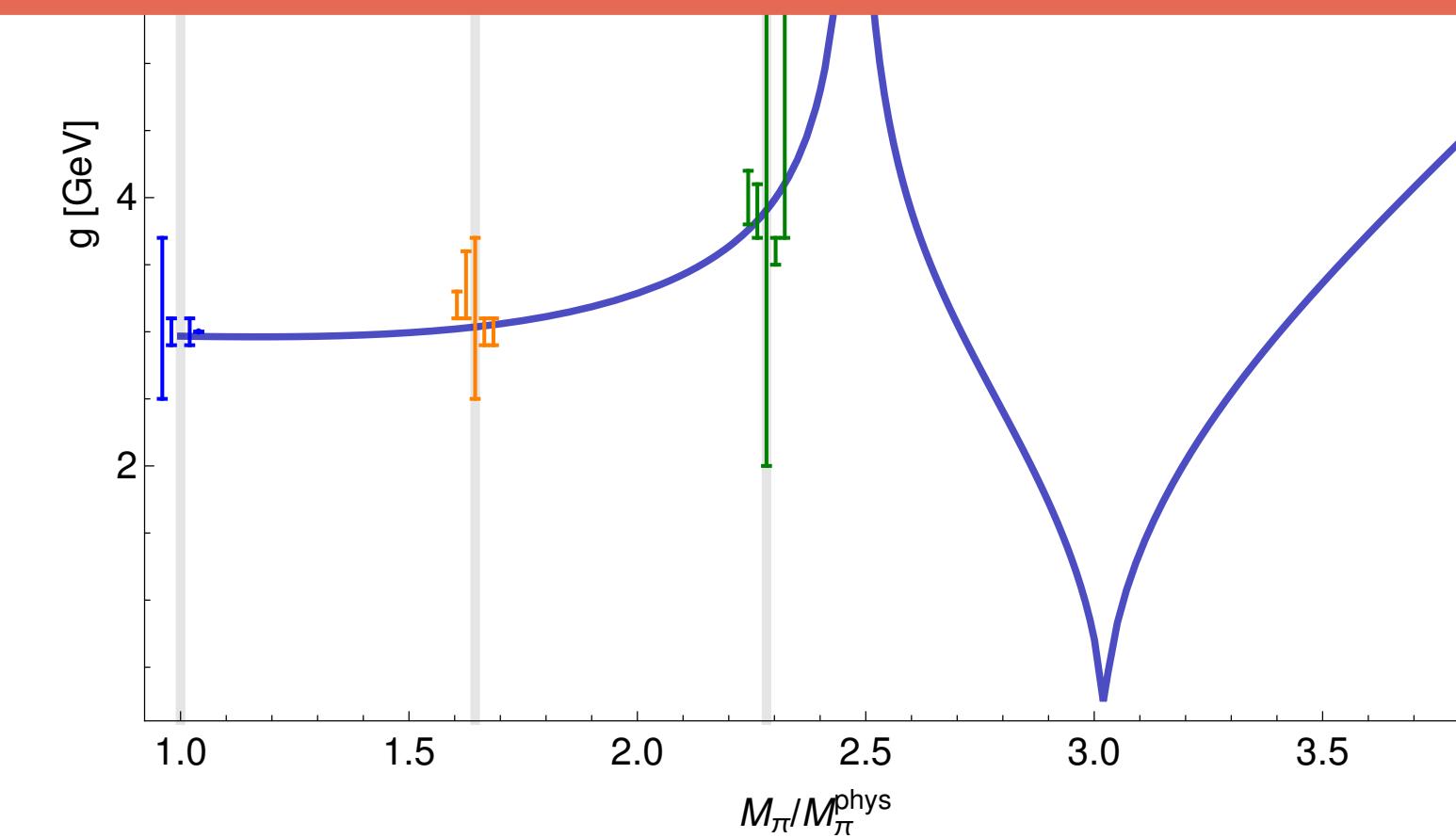
$m_\pi \sim ? \text{ MeV} \rightarrow \text{THIS TALK}$



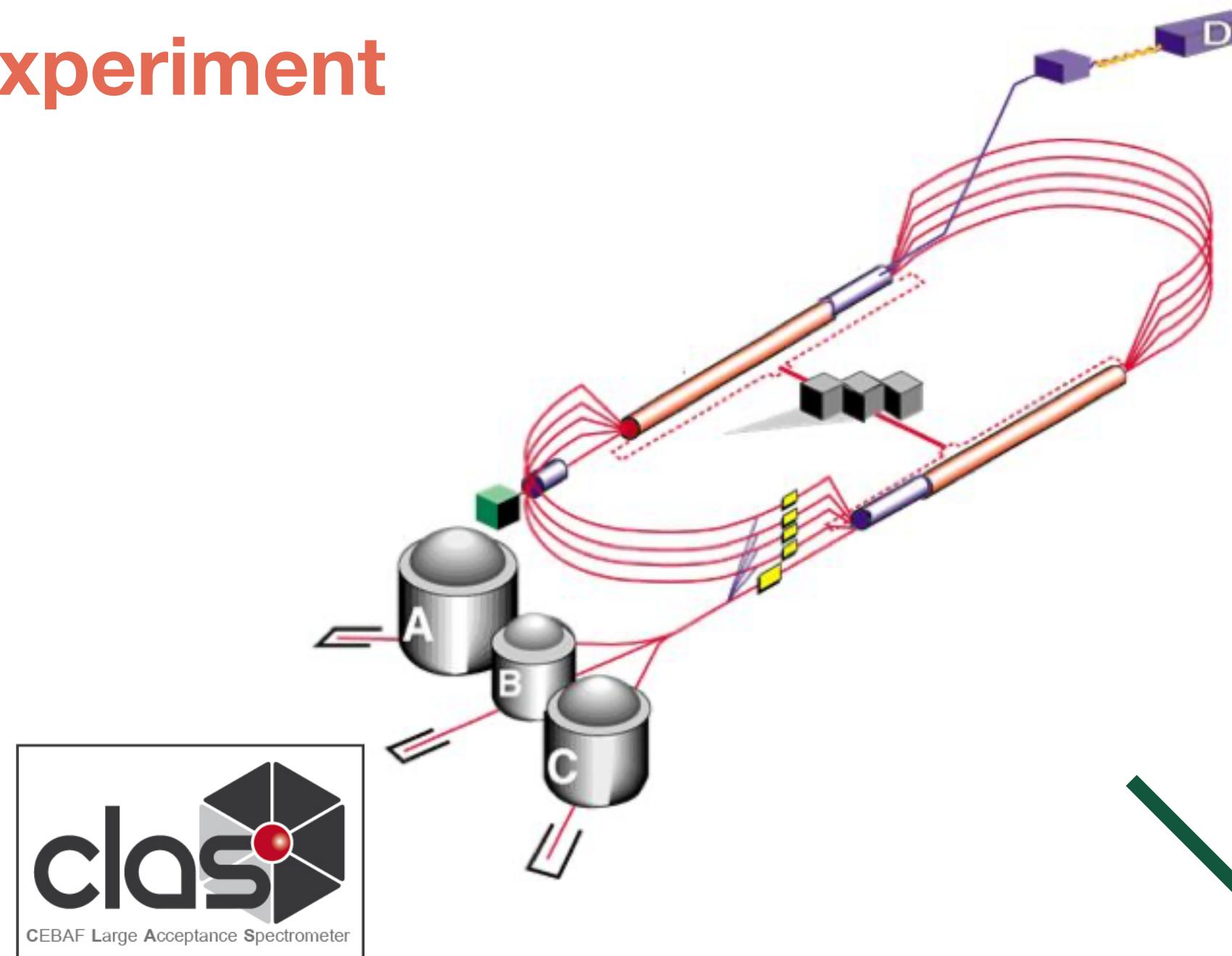
$-a_t m_\ell$	$(L/a_s)^3 \times T/a_t$	N_{cfgs}	N_{vecs}	$N_{t_{\text{src}}}$	$a_t m_\pi$	$a_t m_K$	$a_t m_\eta$	$a_t m_\Omega$	ξ	m_π/MeV
0.0850	$24^3 \times 256$	482	162	8–16	0.05634(14)	0.09027(15)	0.09790(100)	0.2857(8)	3.463(7)	330
0.0856	$24^3, 32^3 \times 256$	400, 484	162, 256	8	0.04724(13)	0.08659(14)	0.09602(70)	0.2793(8)	3.461(5)	283

$m_\pi \sim 315 \text{ MeV} \rightarrow \text{VB vs resonance}$

$m_\pi \sim 227 \text{ MeV} \rightarrow \text{Broad resonance}$

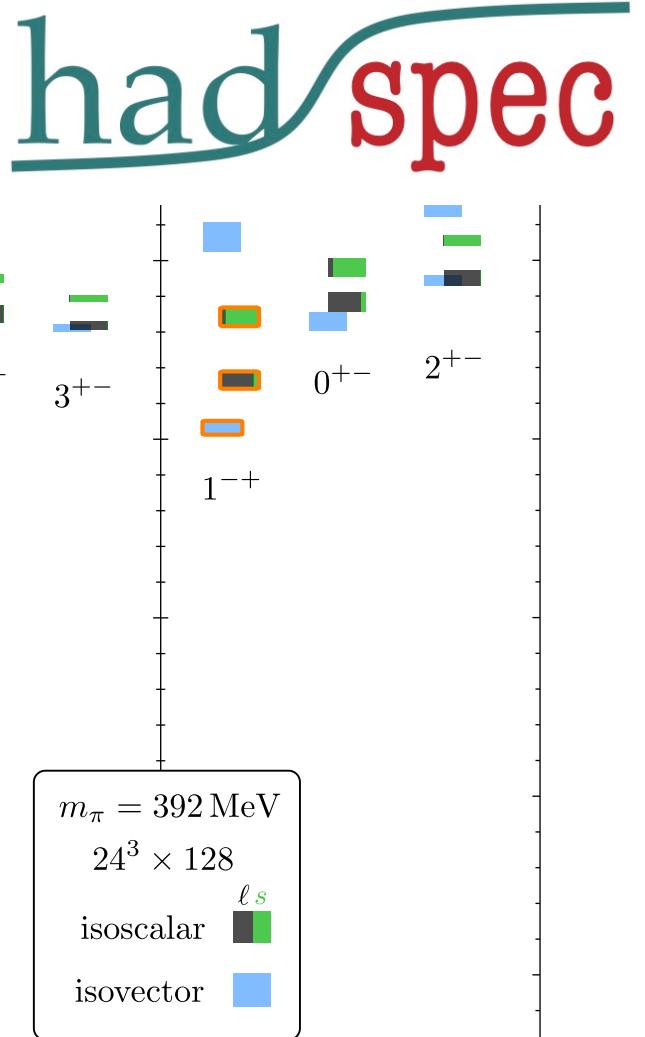
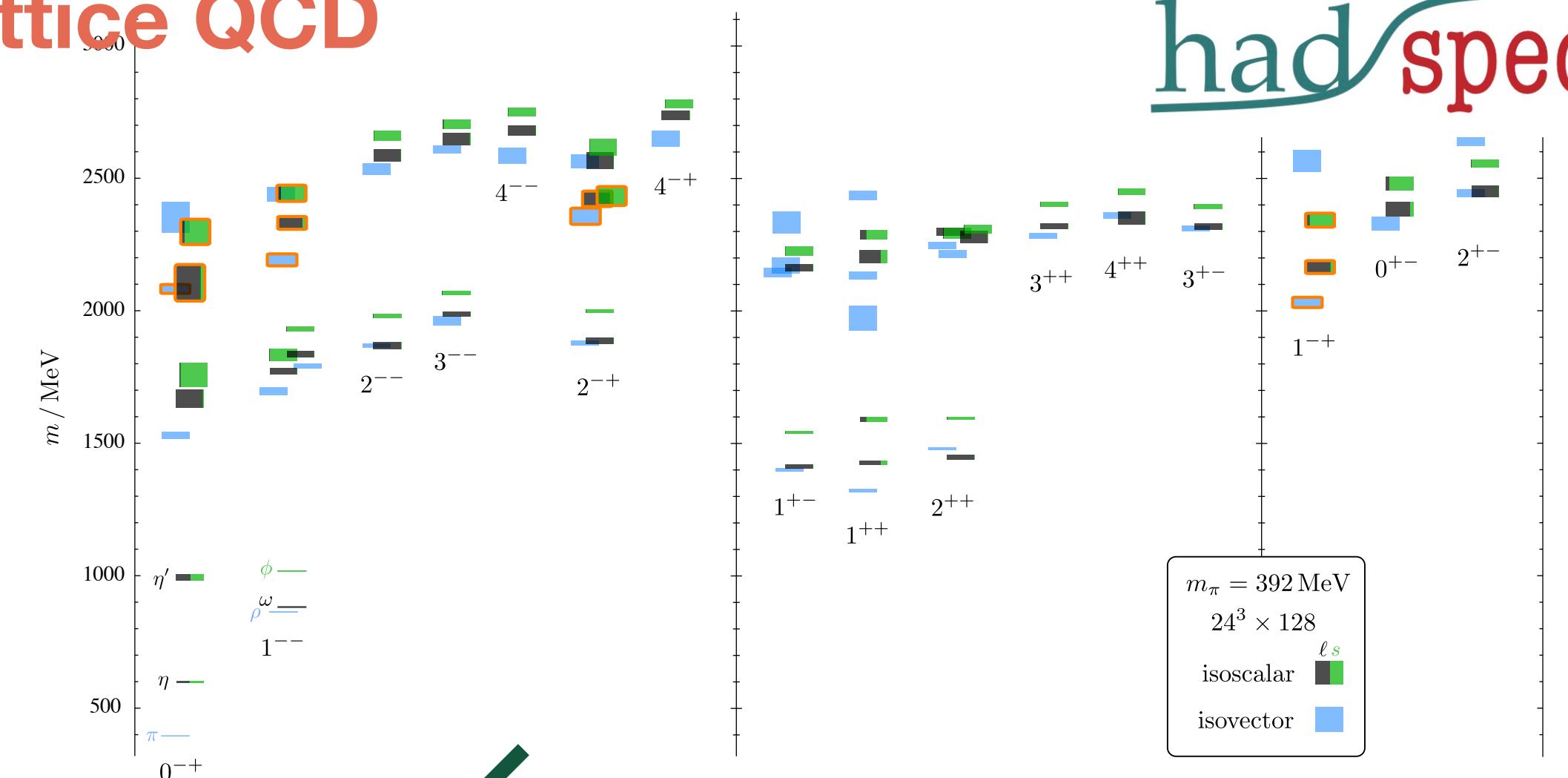


Experiment

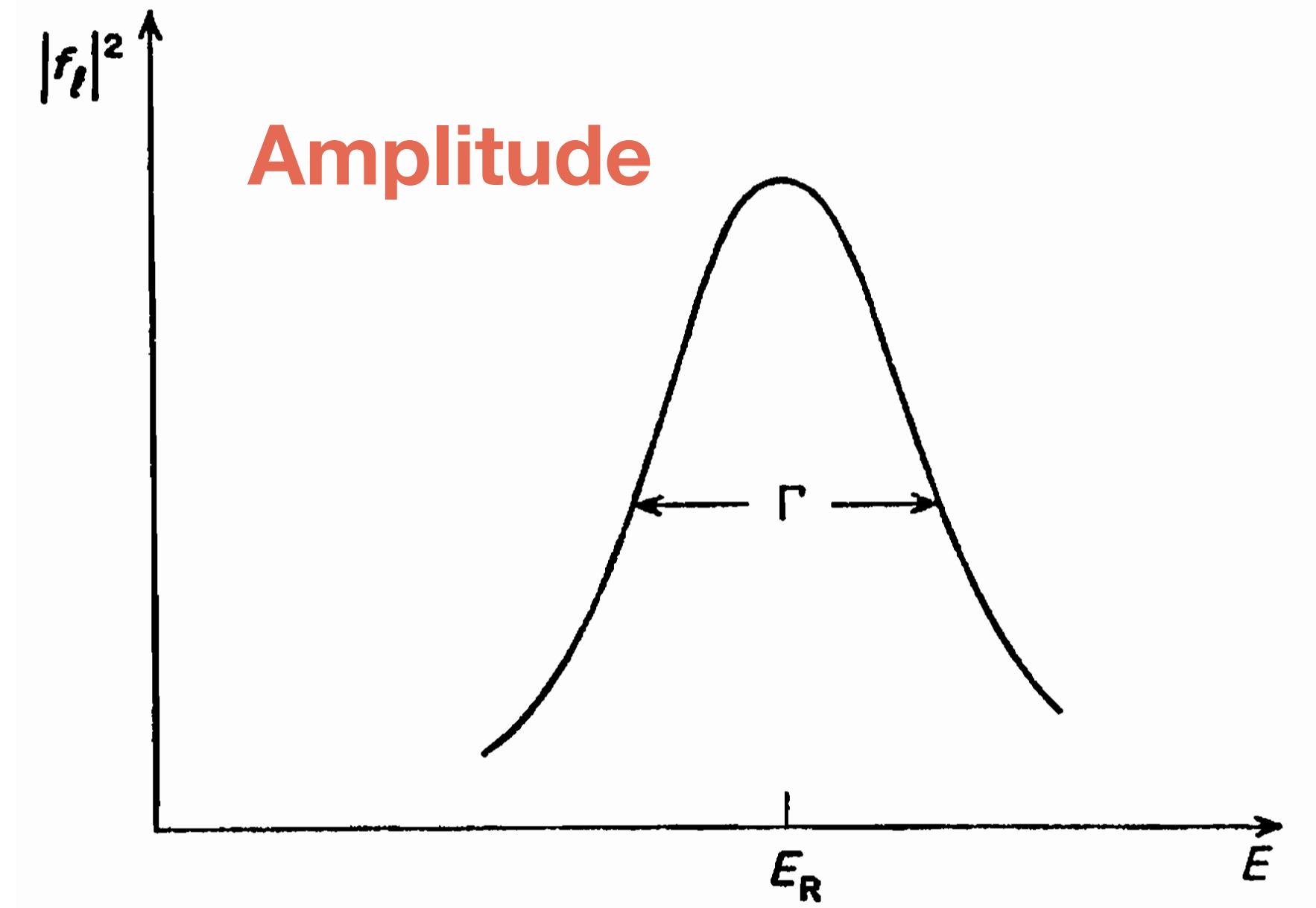


GLUEX

Lattice QCD



Resonances manifest
as amplitudes

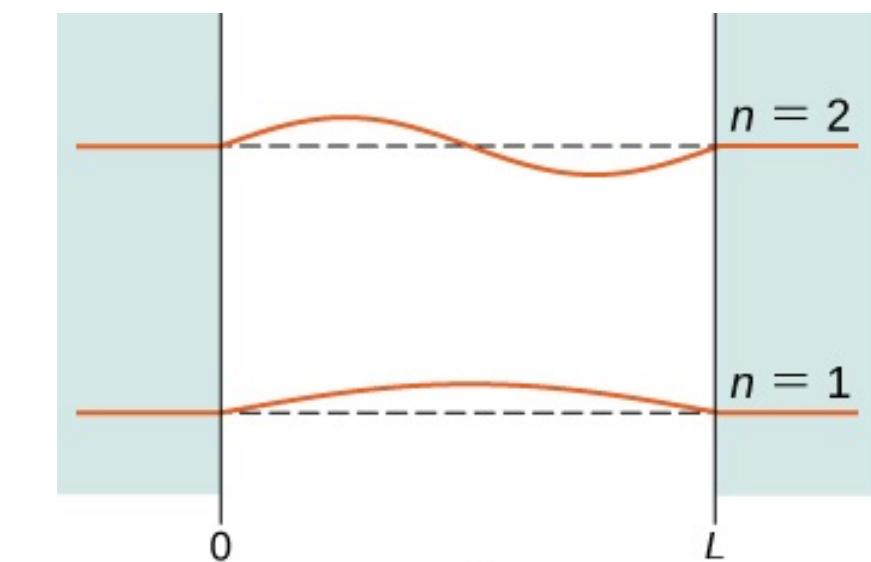


Lattice QCD

In

Out

Boundary



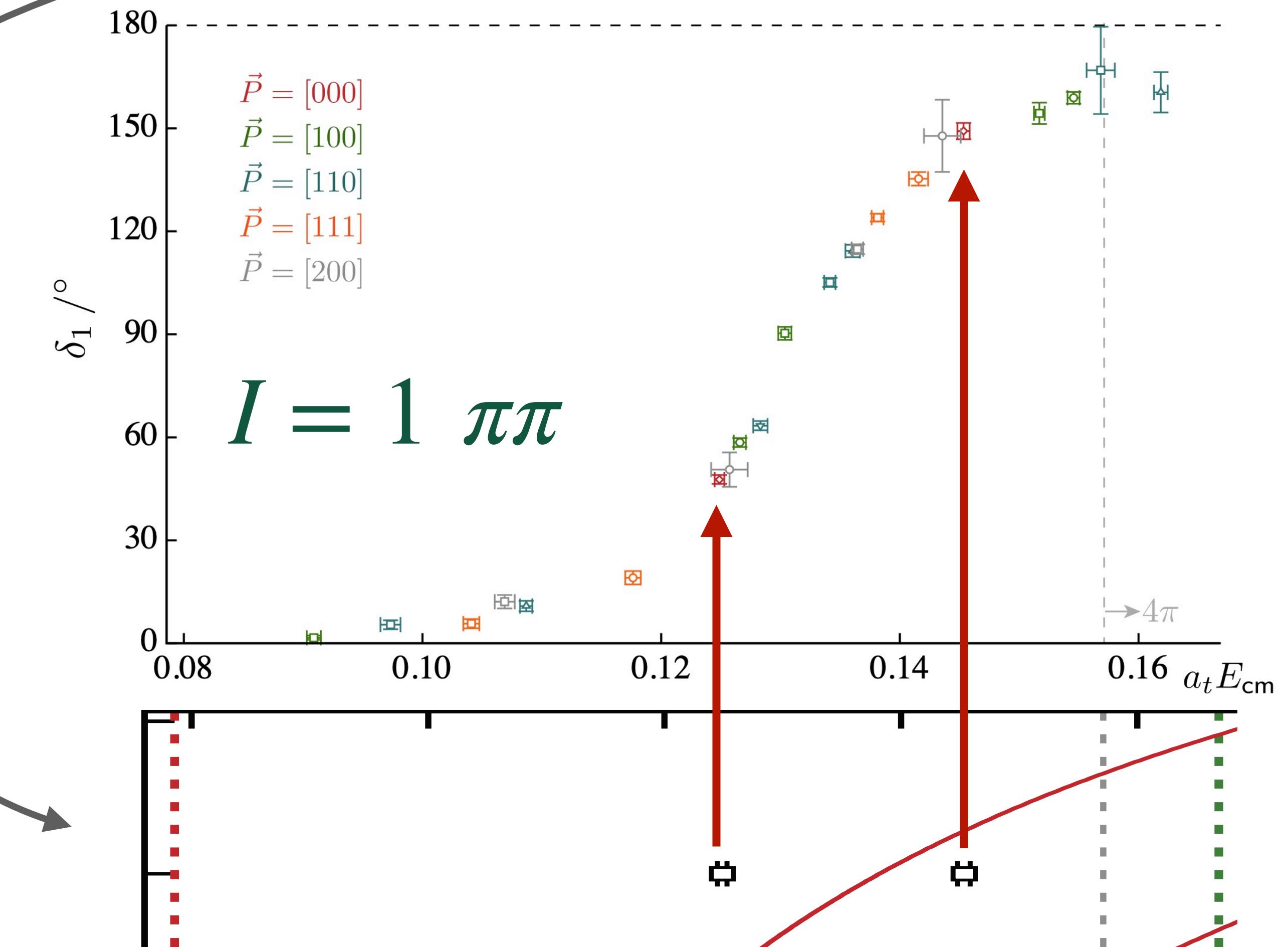
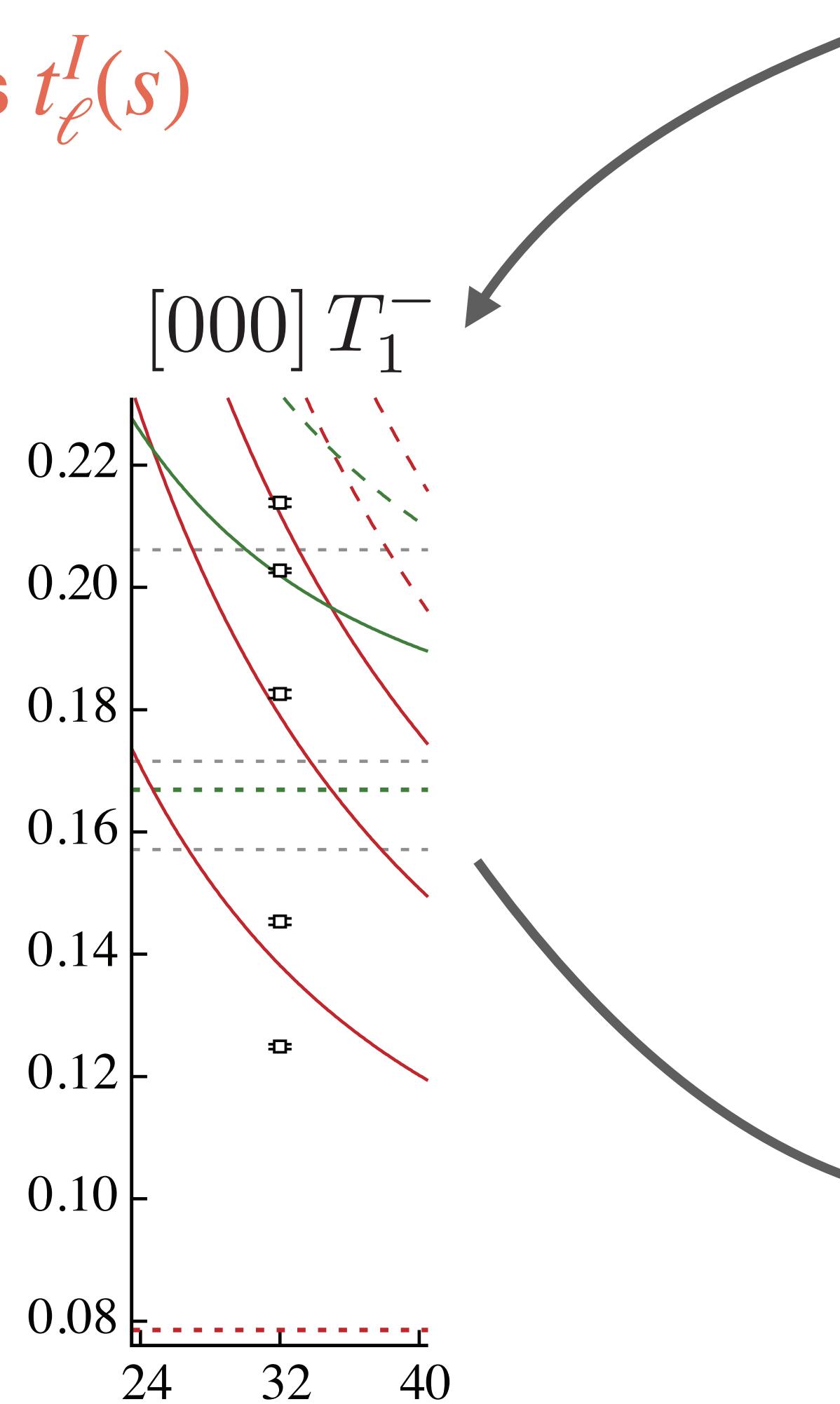
Lüscher

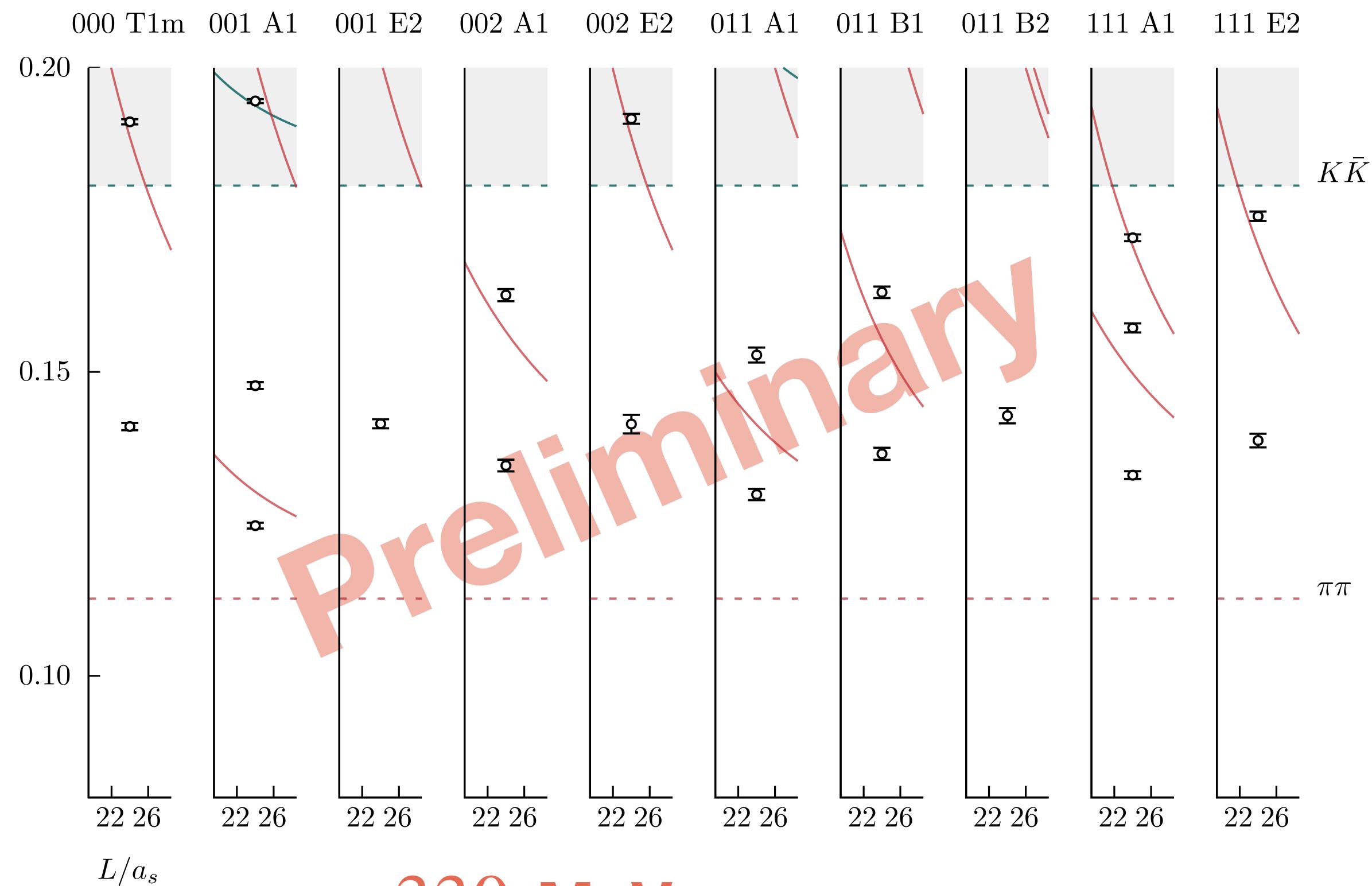
$$e^{ikL + 2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

Pathway towards $t_\ell^I(s)$

Spectrum (GEVP)

Our E_n become the x axis

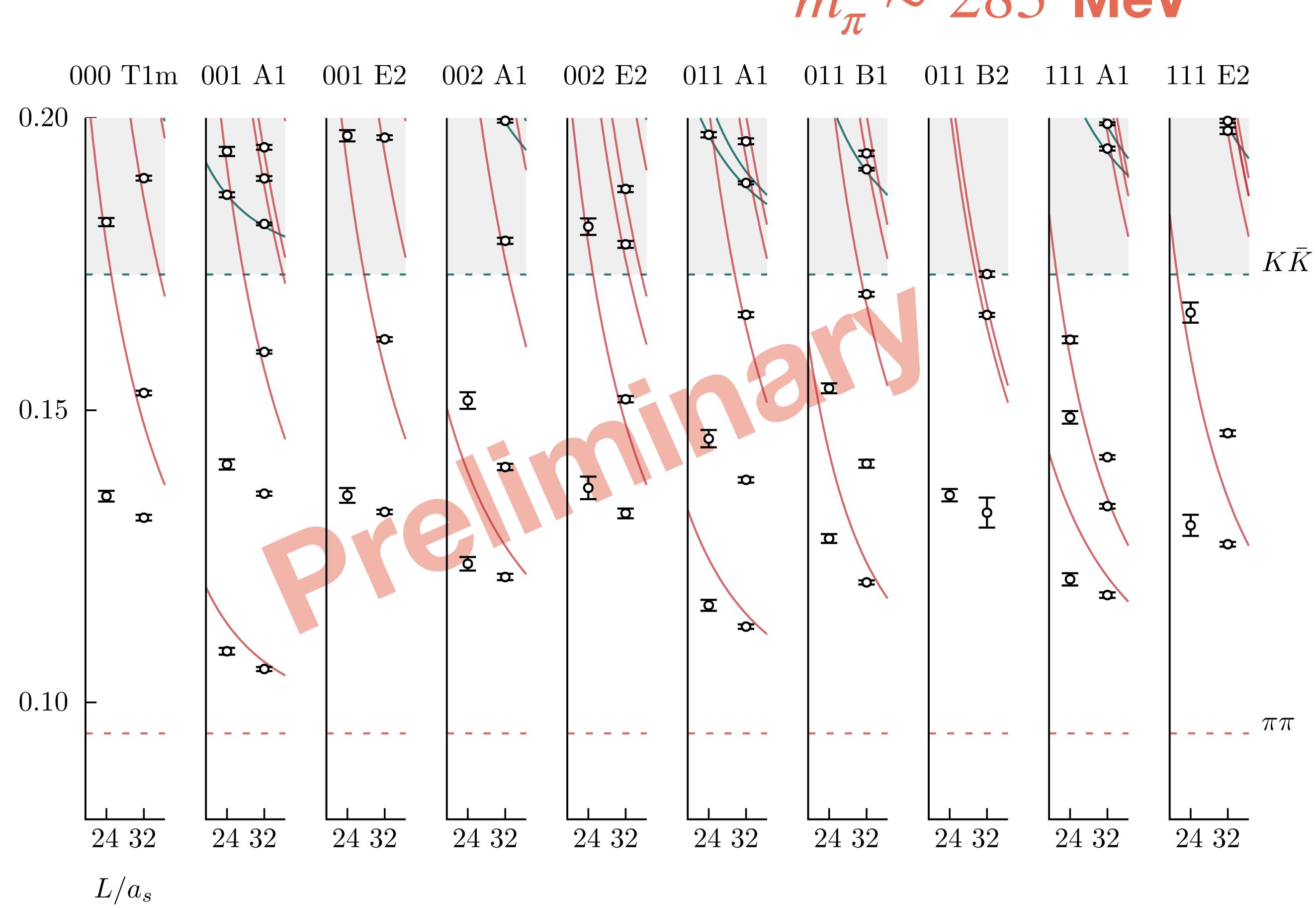




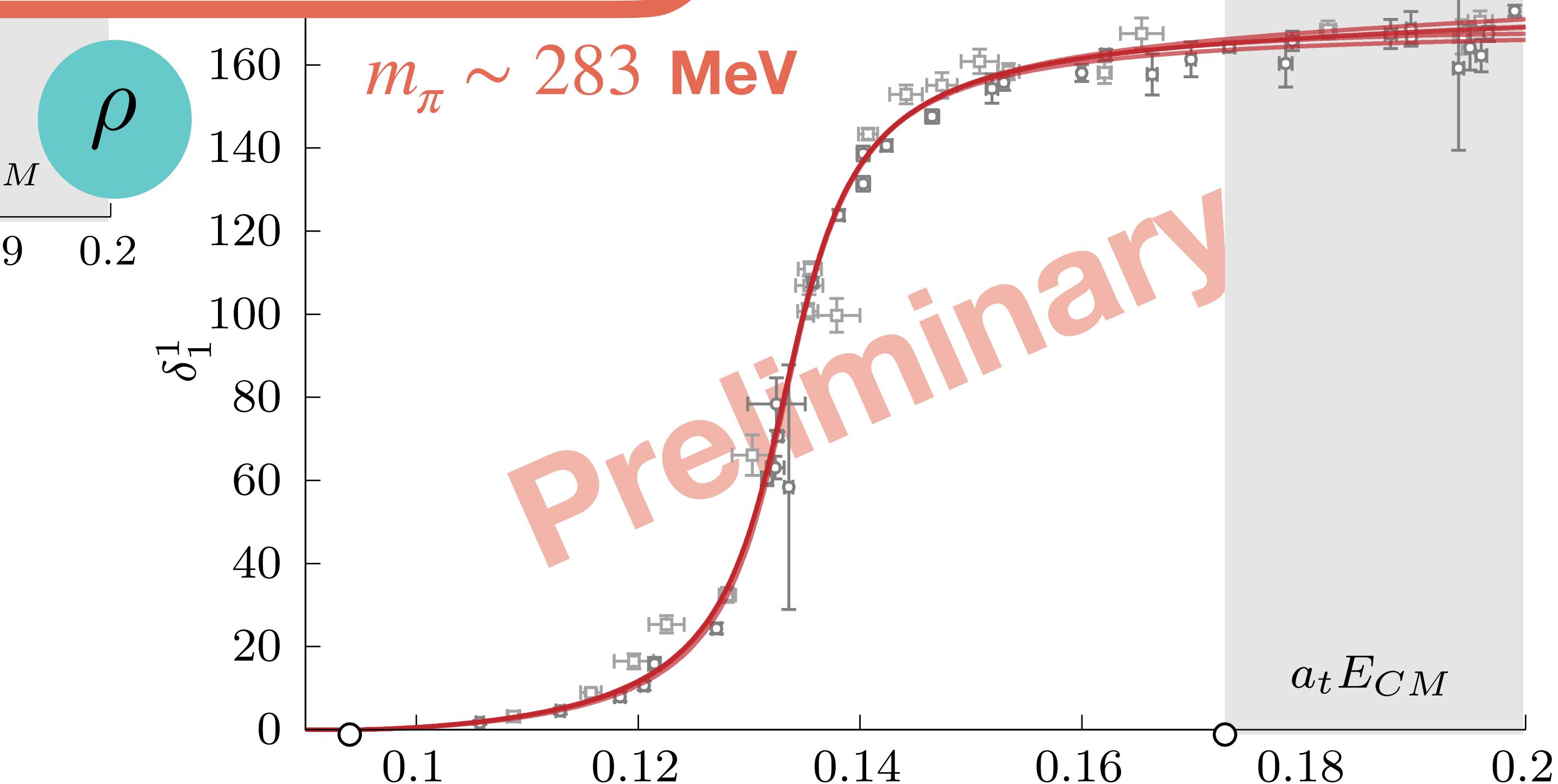
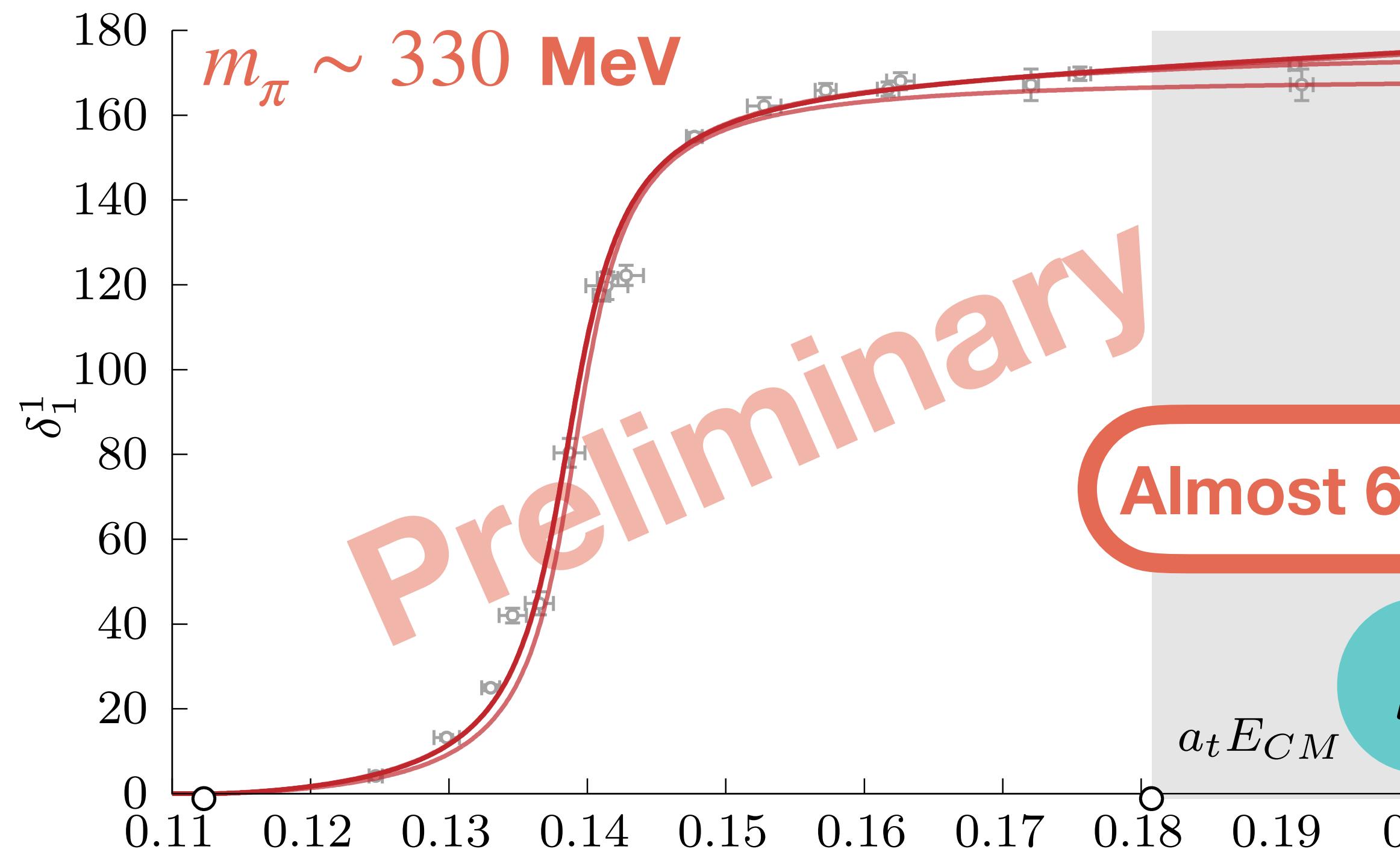
$m_\pi \sim 330$ MeV

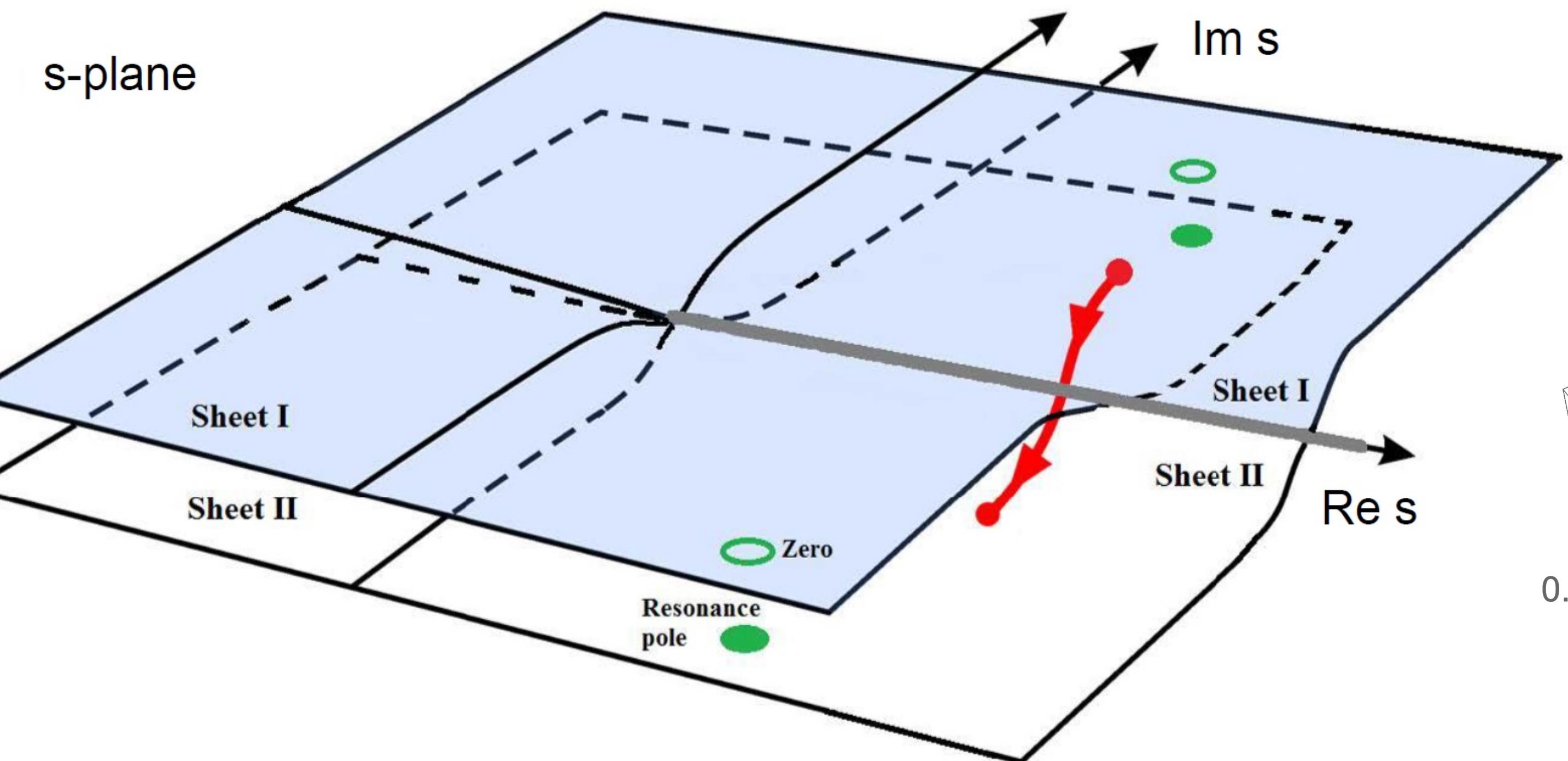
Allows us to study the
 ρ resonance m_q dependence

Similar spectrum to
 previous masses

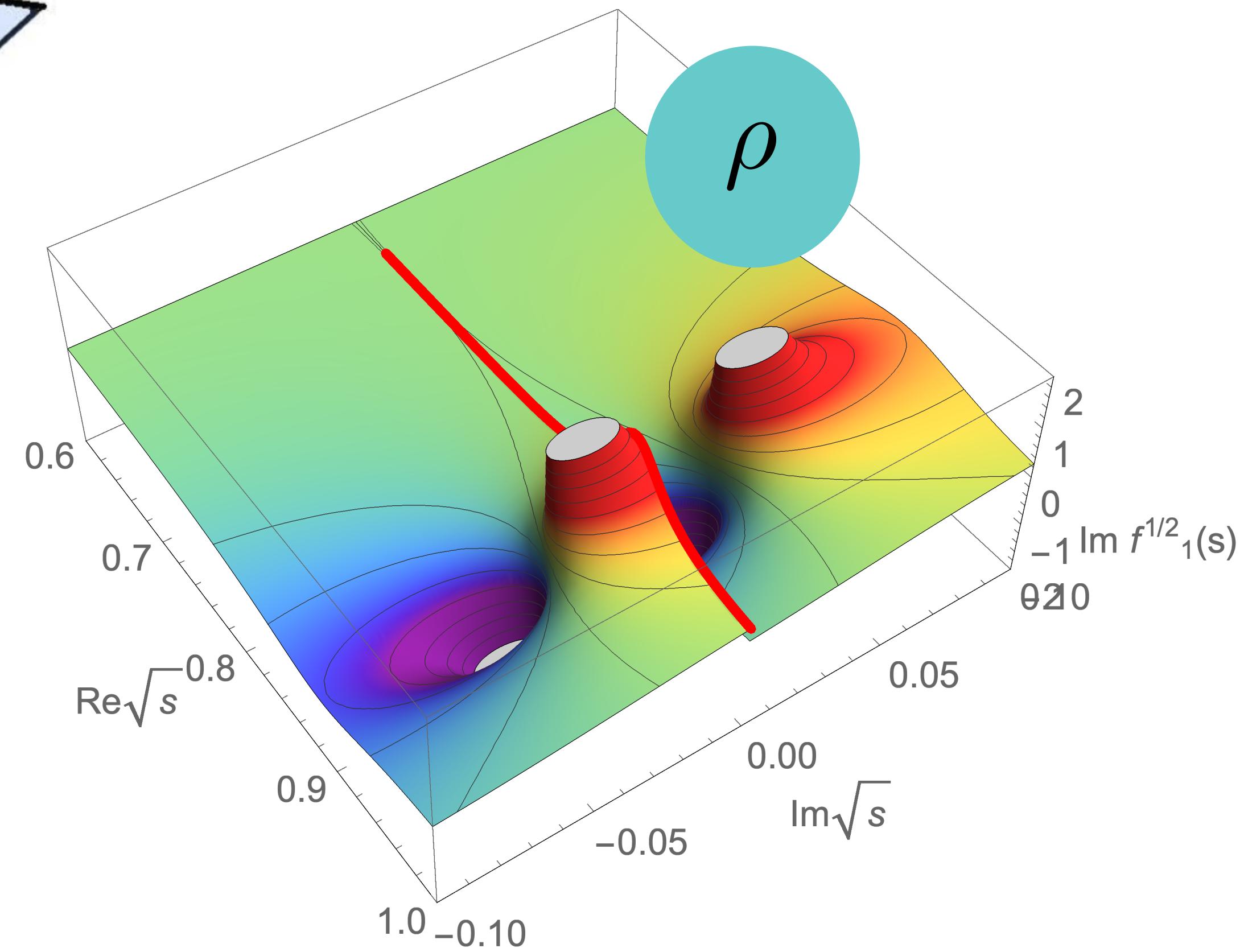


$m_\pi \sim 283$ MeV



Unitarity**Exercise**

$$S_\ell^{II}(s) = \frac{1}{S_\ell^I(s)}$$



$I = 1 \pi\pi$

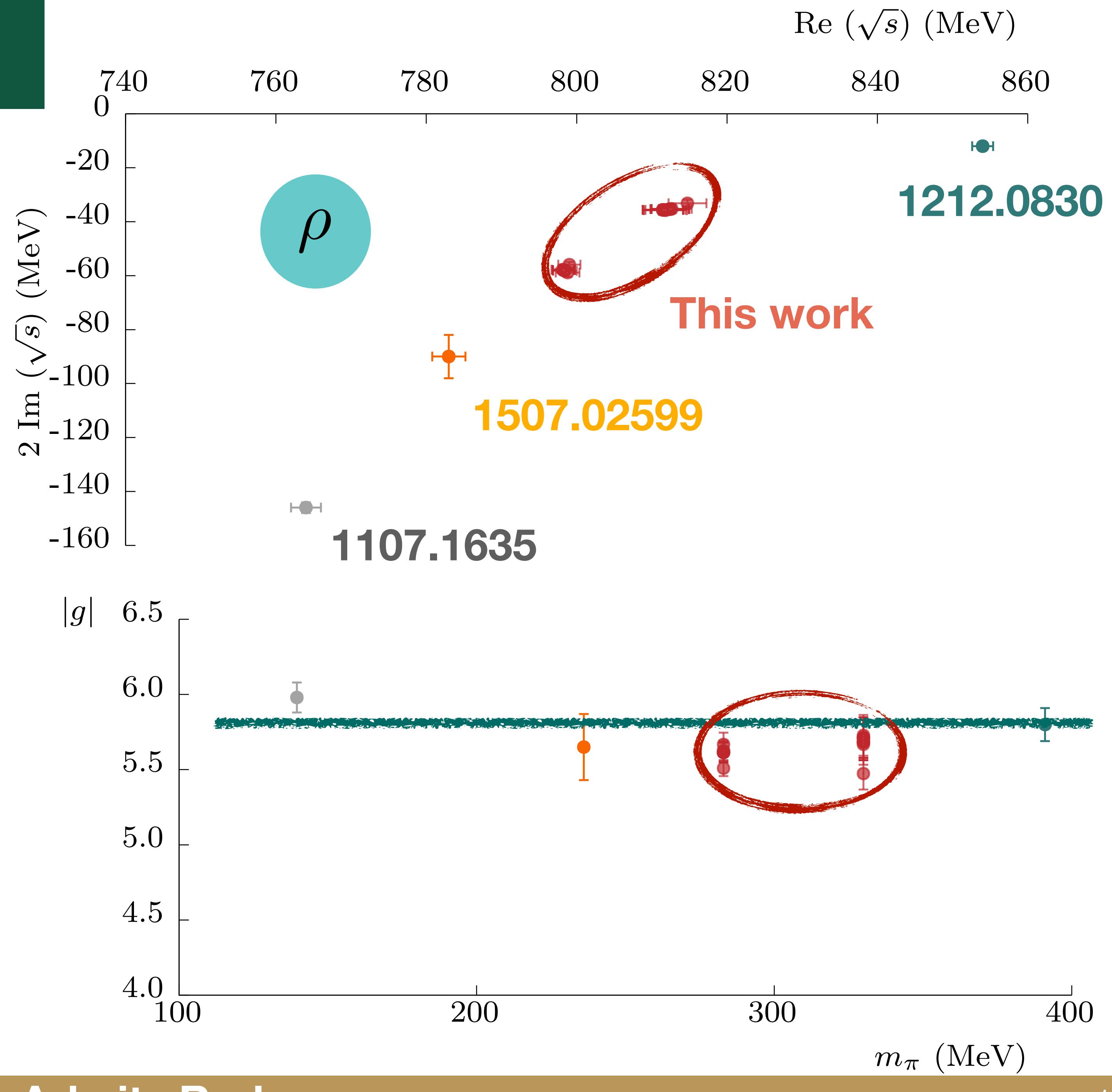
had spec

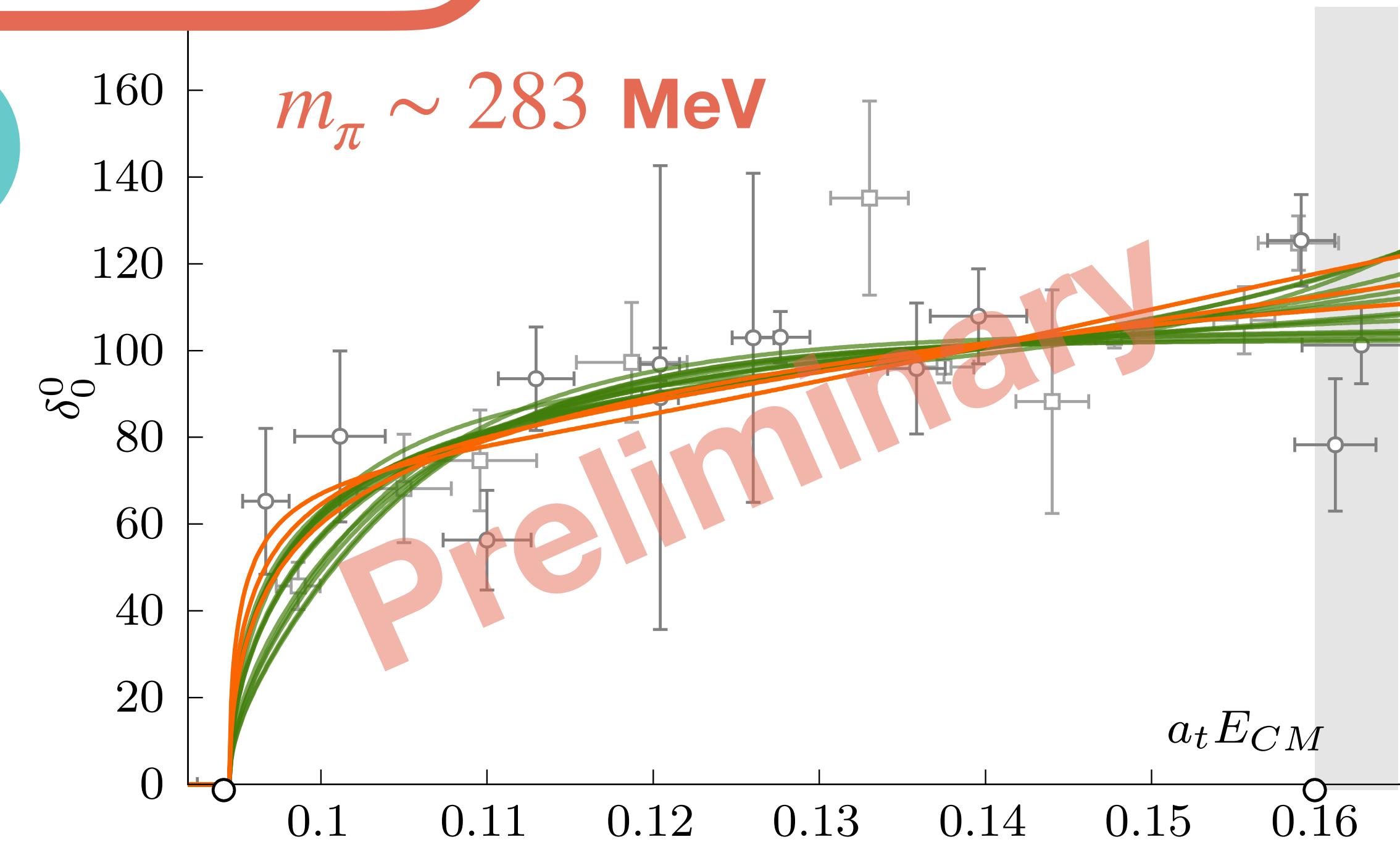
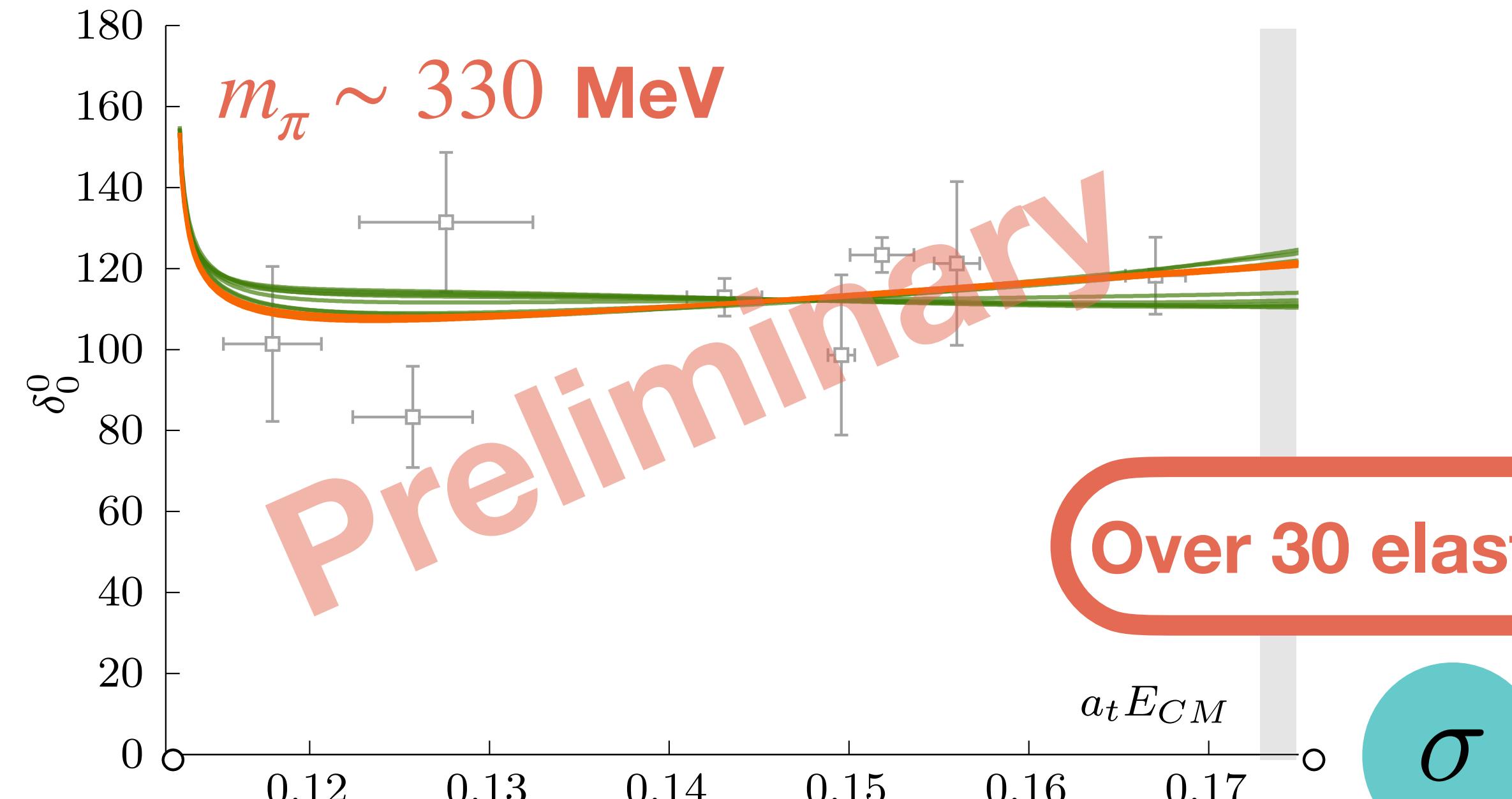
Ordinary m_q dependence

$$M \sim a + b m_q$$

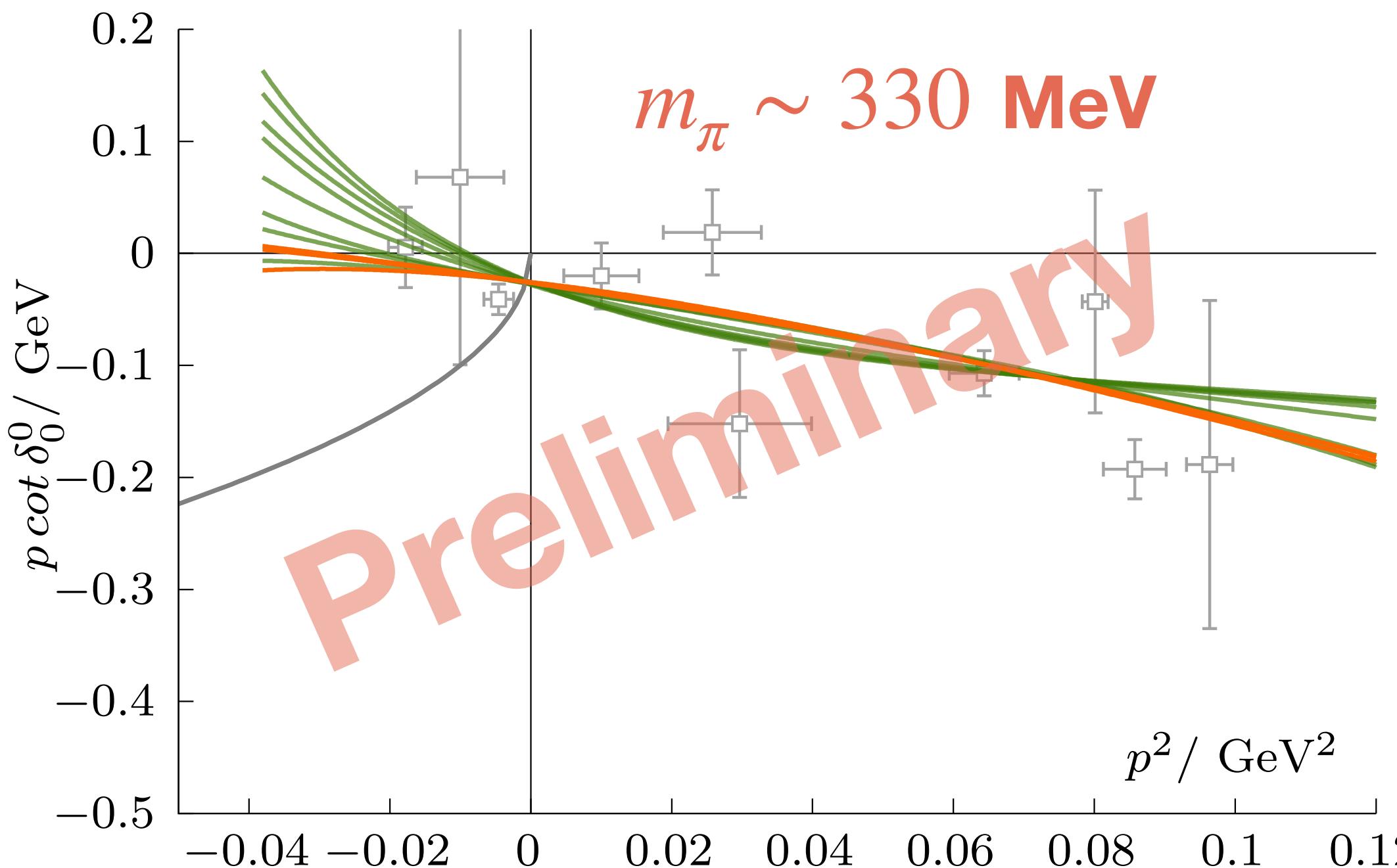
$$\Gamma \sim q^2$$

g constant





$I = 0 \pi\pi$



Large, positive a

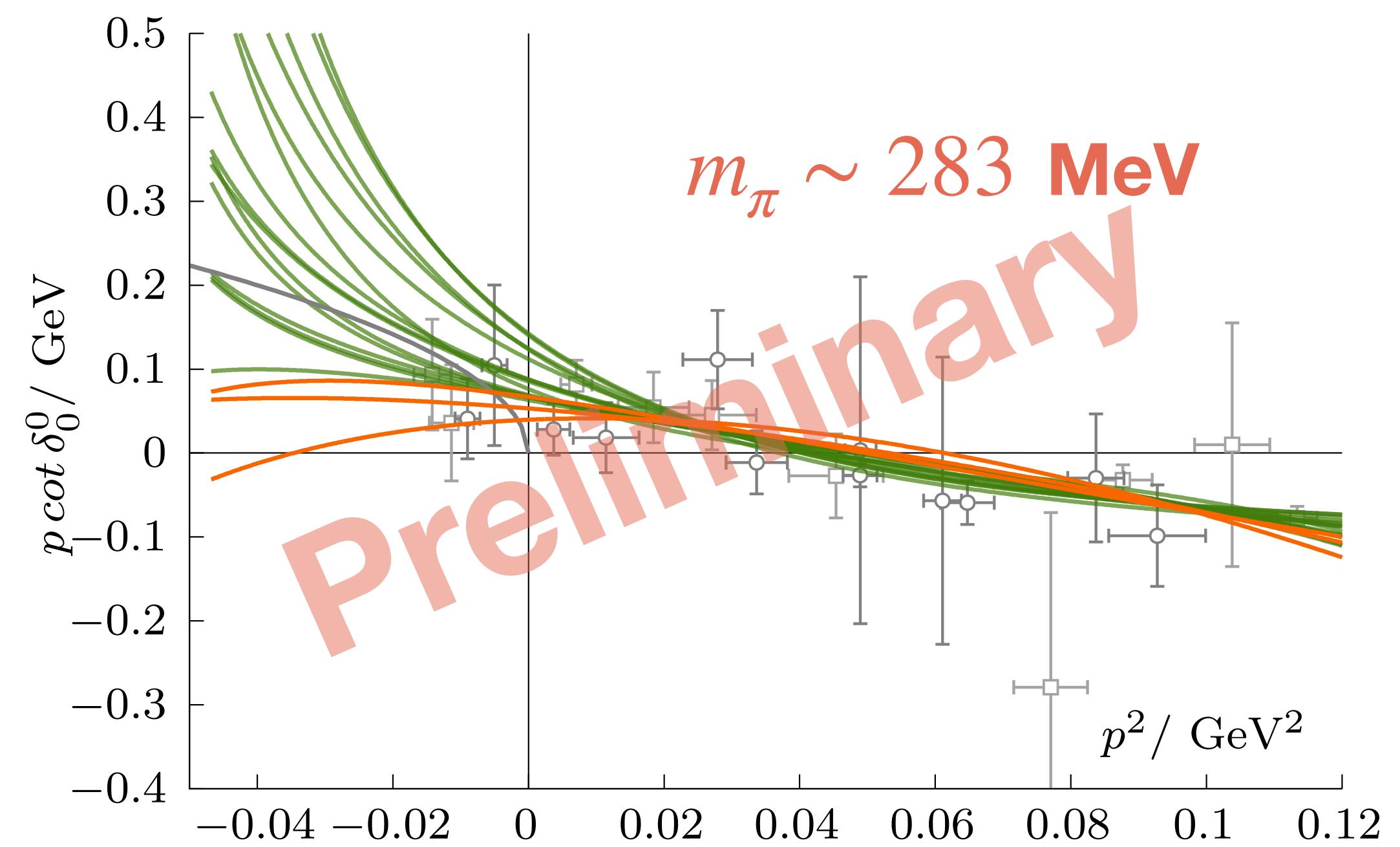
Large spread at threshold

$$p \cot \delta(s) \sim \frac{1}{a}$$

Large, negative a

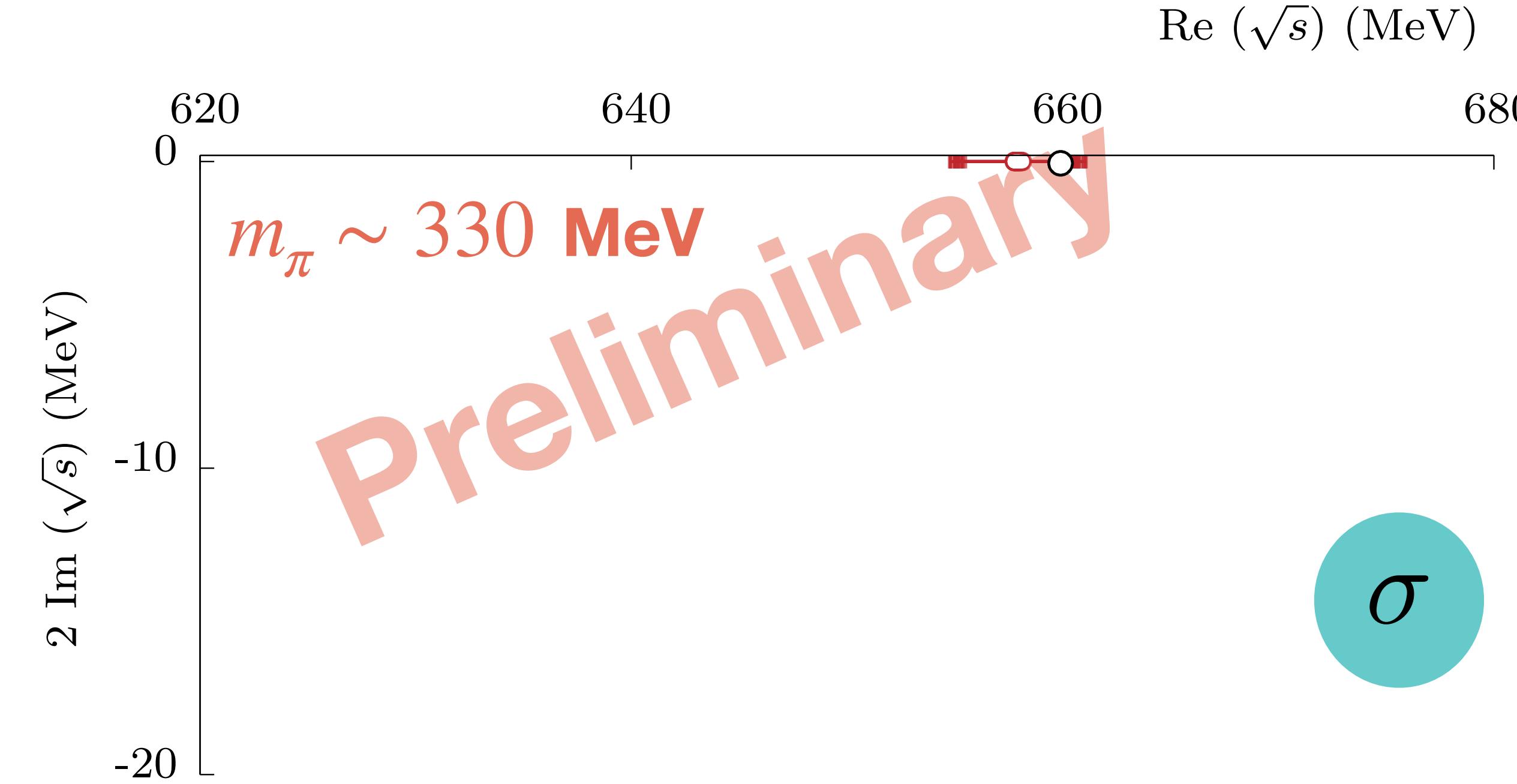
Extremely precise at threshold

σ



Light Scalars: the σ

had spec



2nd Riemann sheet

VB vs resonance

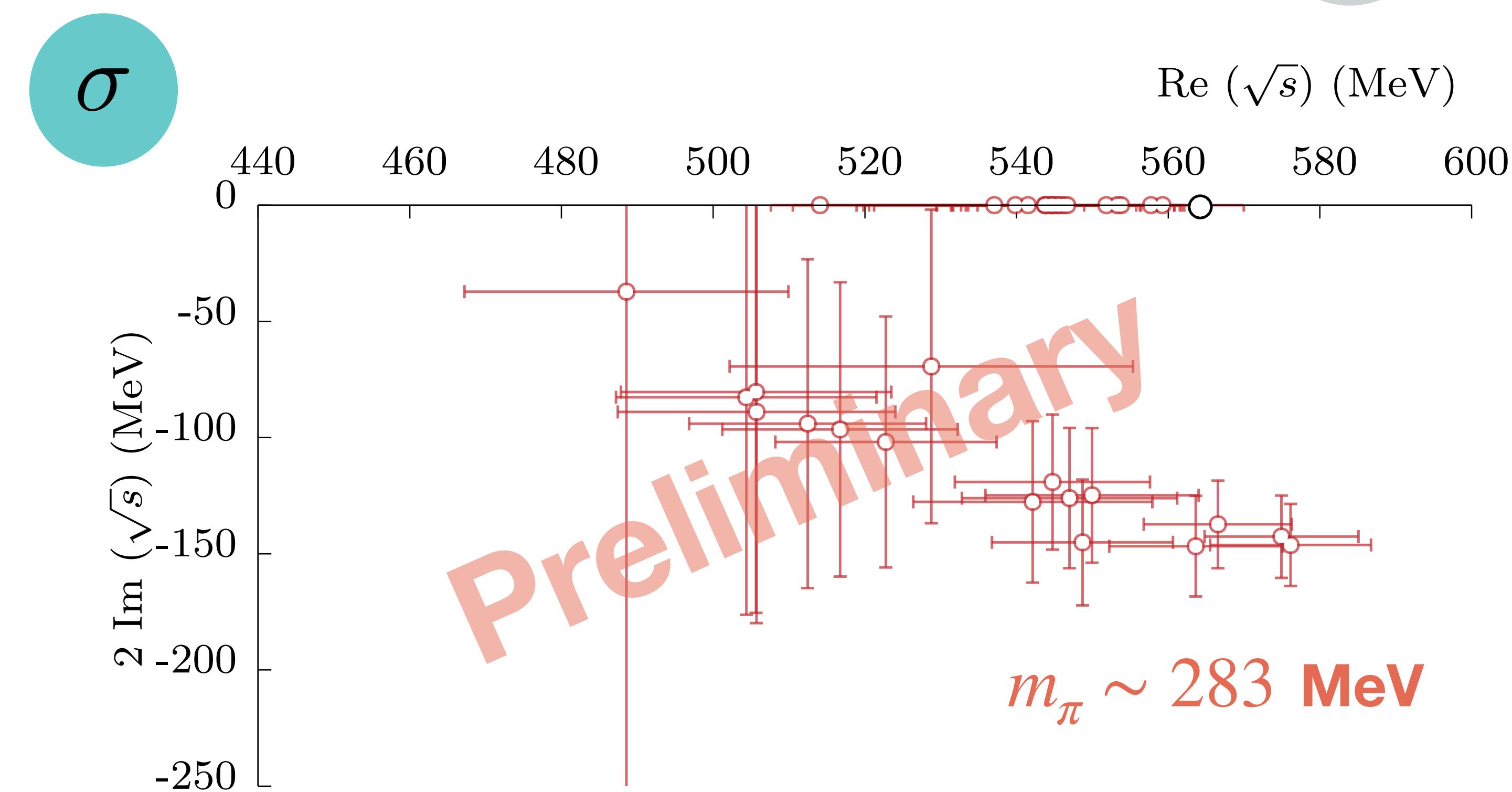
Lower $a \rightarrow$ broader σ

Extremely “noisy”

Extremely precise, shallow bound state

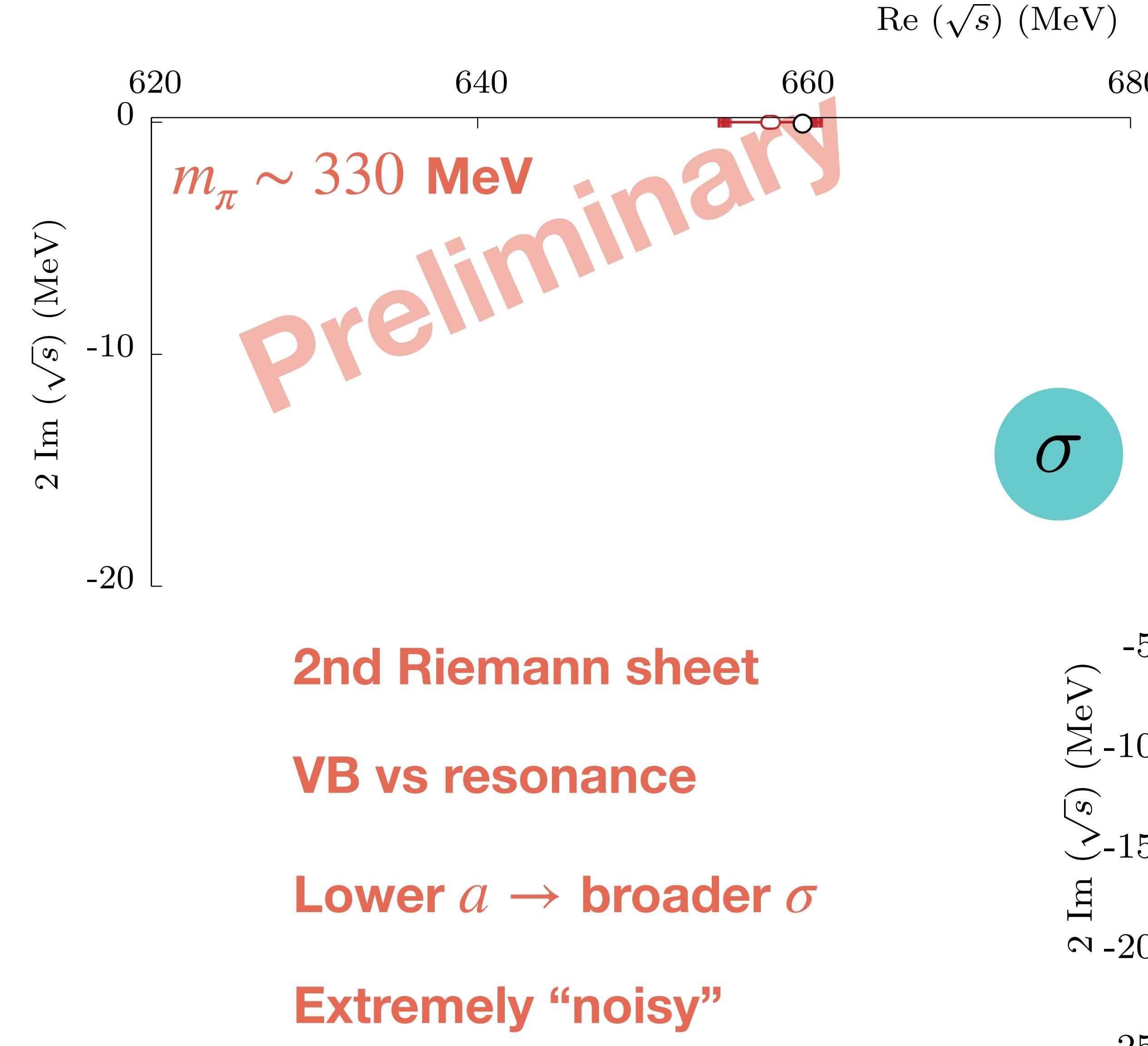
1st Riemann sheet

Weinberg's criterion $\rightarrow Z \sim 0$



Light Scalars: the σ

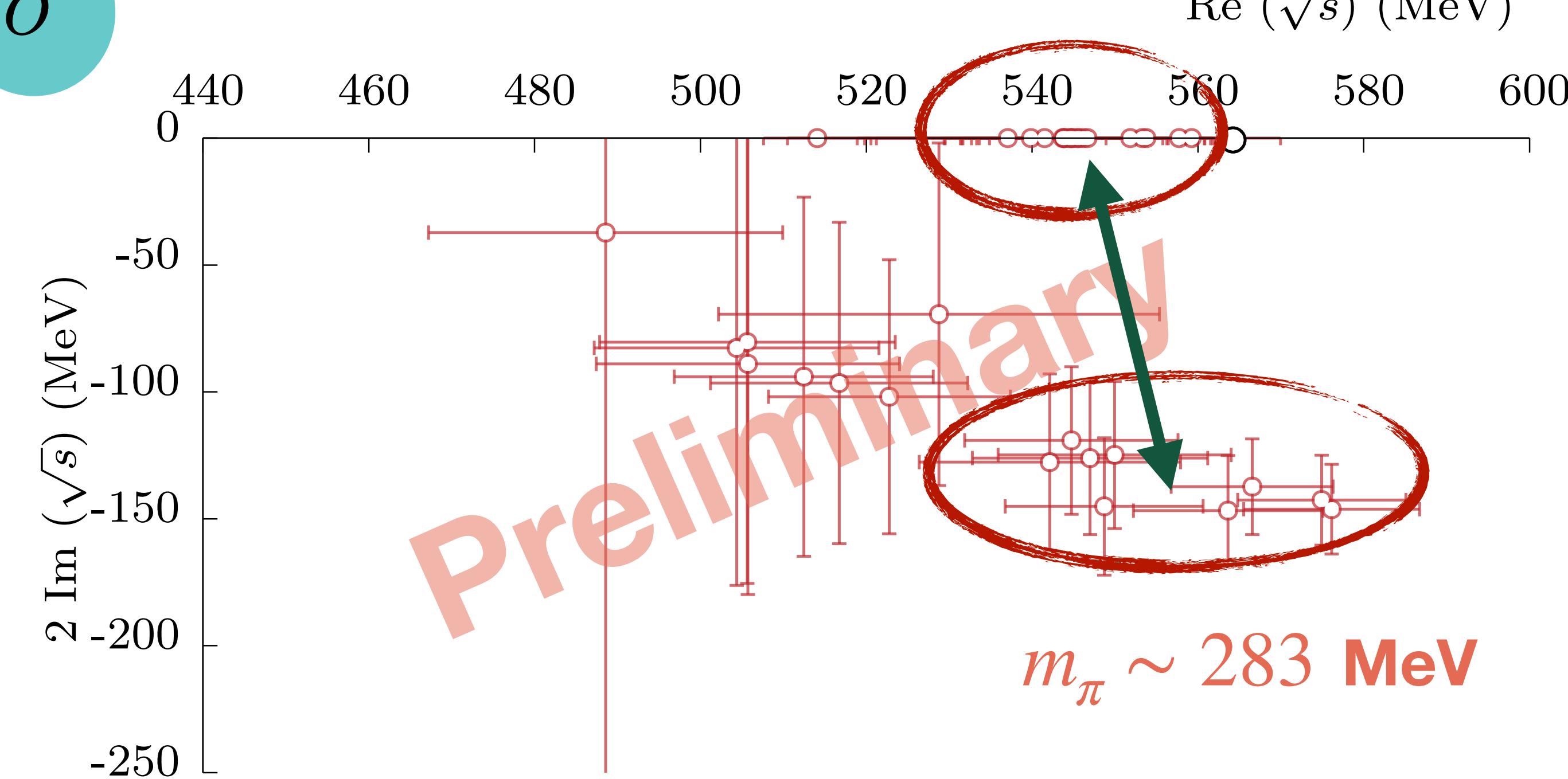
had spec



Extremely precise, shallow bound state

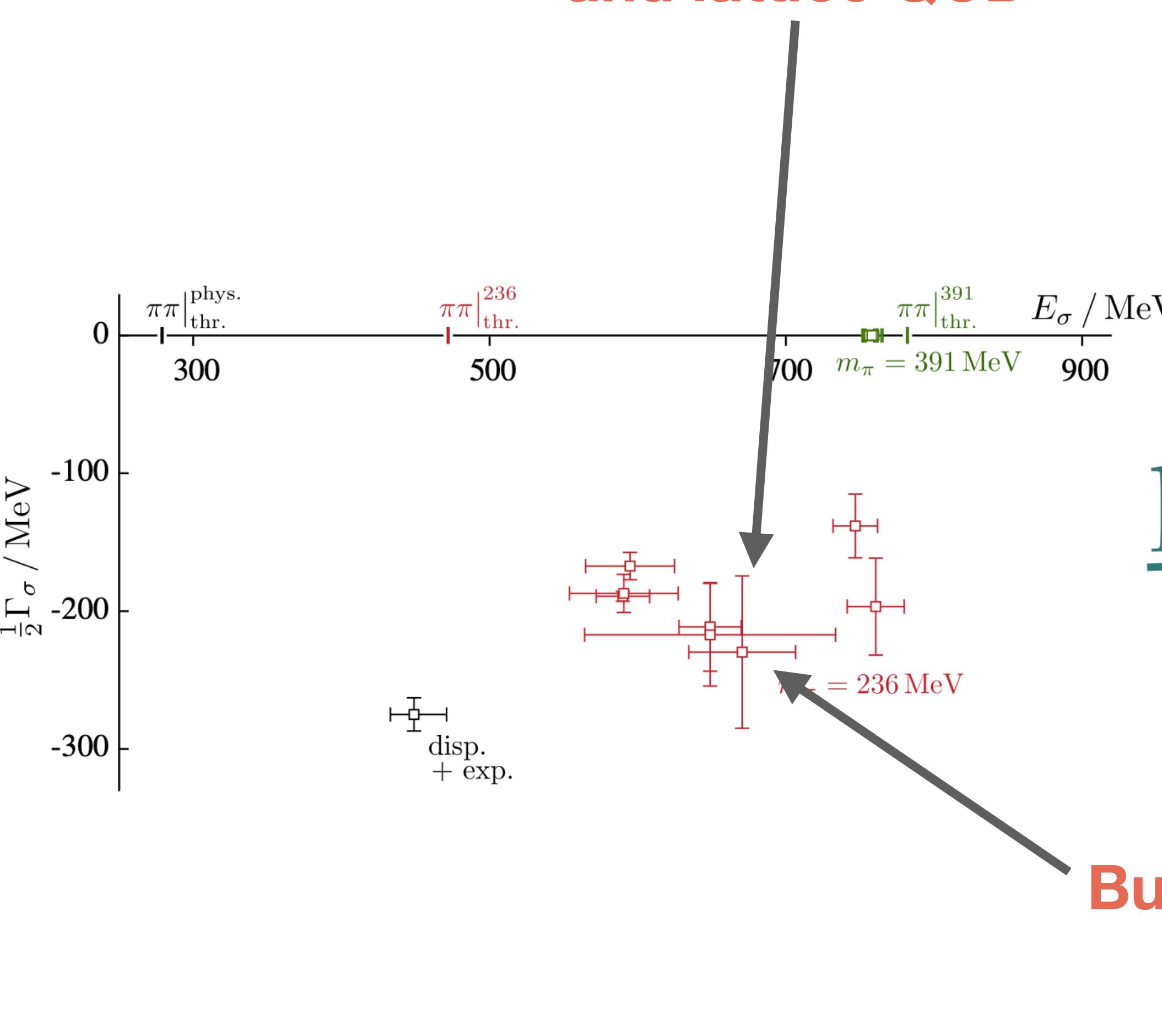
1st Riemann sheet

Weinberg's criterion $\rightarrow Z \sim 0$



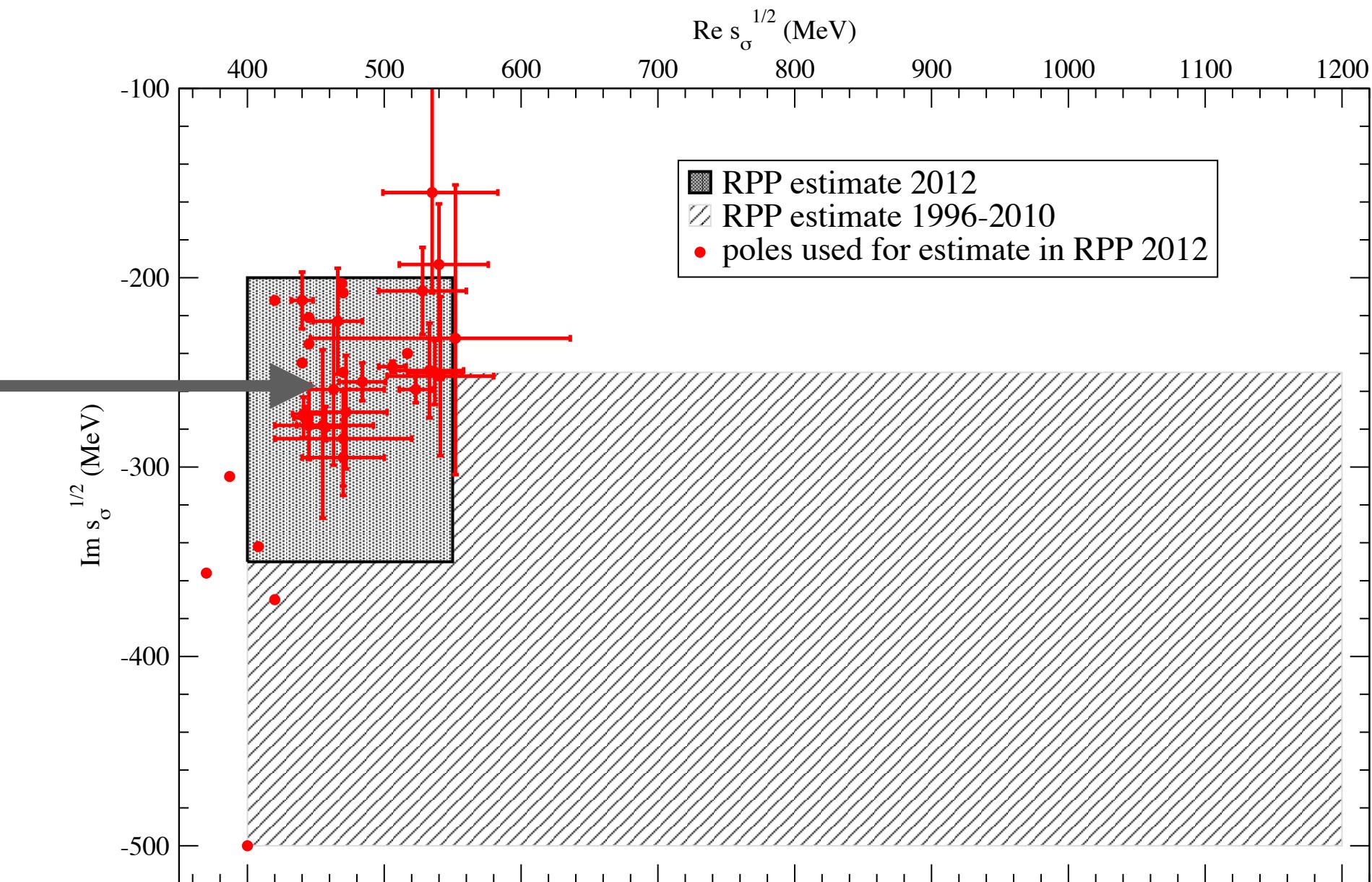
Light Scalars: the σ

It affects both experiment and lattice QCD

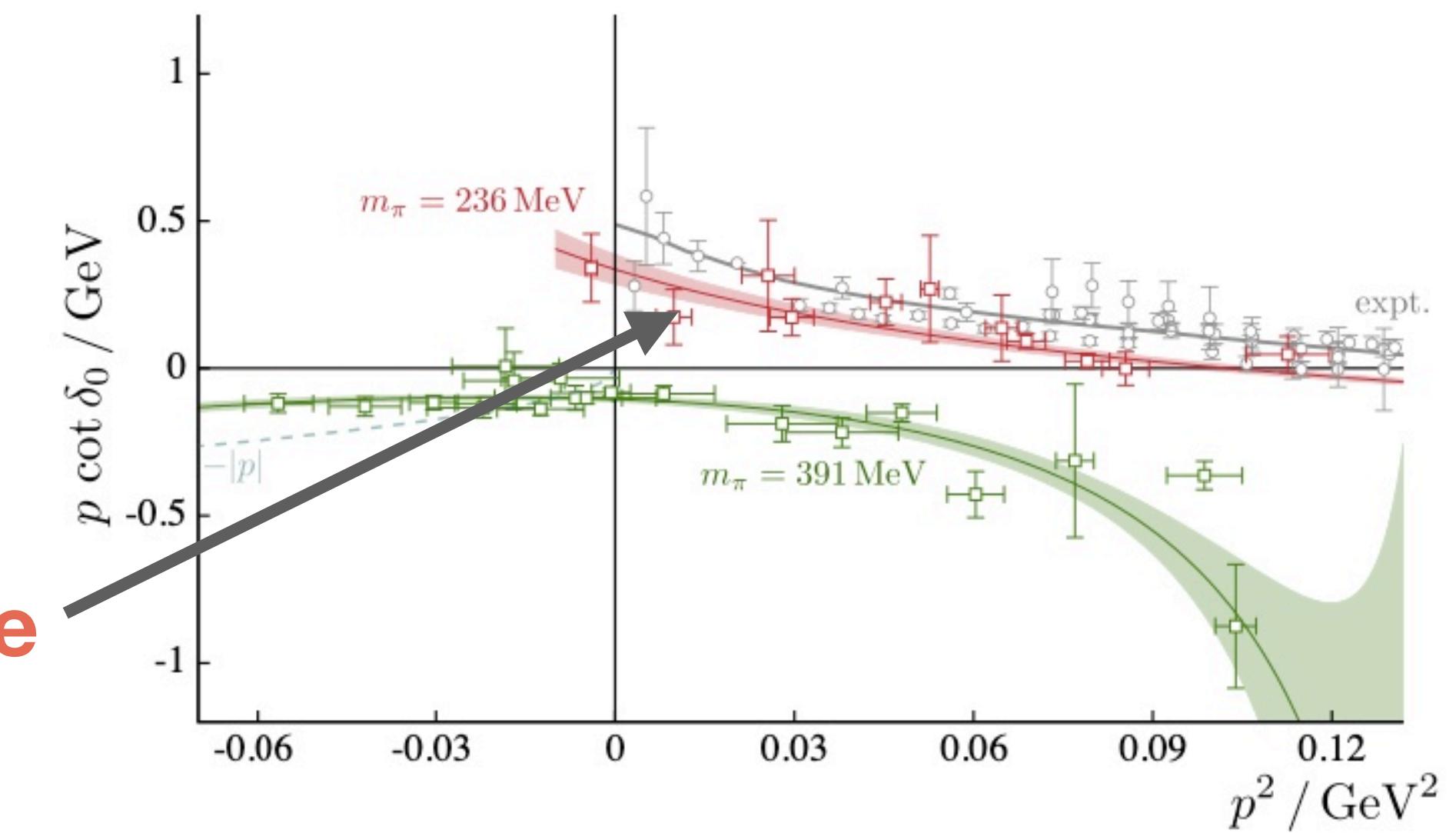


had spec
1607.05900

But data is precise



PDG



Summary and outlook

New, precise data for $m_\pi = 330, 283$ MeV

Robust systematic analysis

Small statistic uncertainties

ρ trajectory at various m_q points

Ordinary behavior

Sub-percent errors

σ found at both masses

$m_\pi = 330$ MeV \rightarrow Very shallow bound state

$m_\pi = 283$ MeV \rightarrow Either VS or resonance

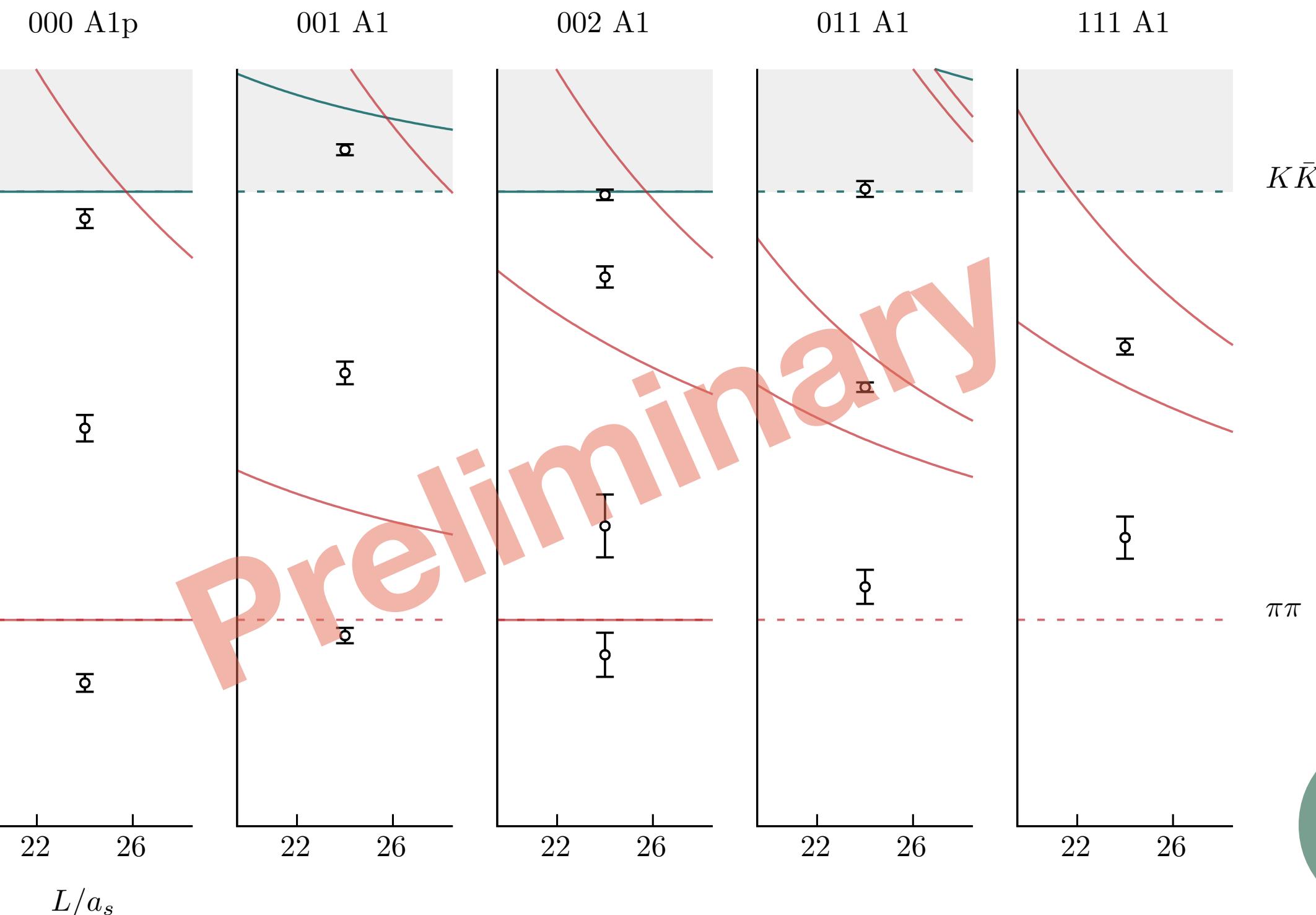
Systematics \rightarrow param. model dependence

Lighter $m_\pi \rightarrow$ problem worsens

A “model-free” σ resonance??

Extra slides

$I = 0 \pi\pi$

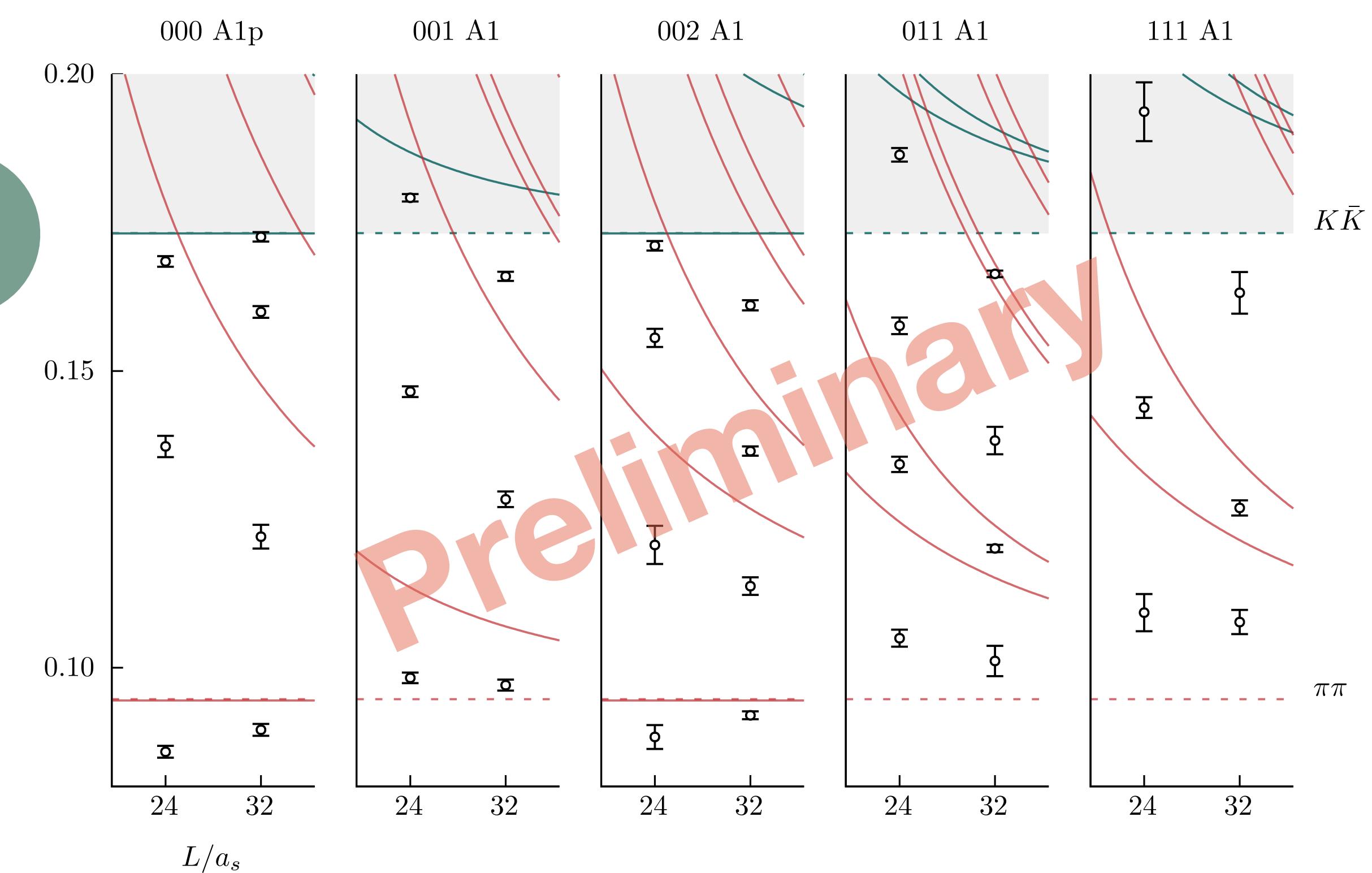


$m_\pi \sim 330 \text{ MeV}$

Around 60 “elastic” levels for $I=0$

Similar spectrum to previous masses

σ



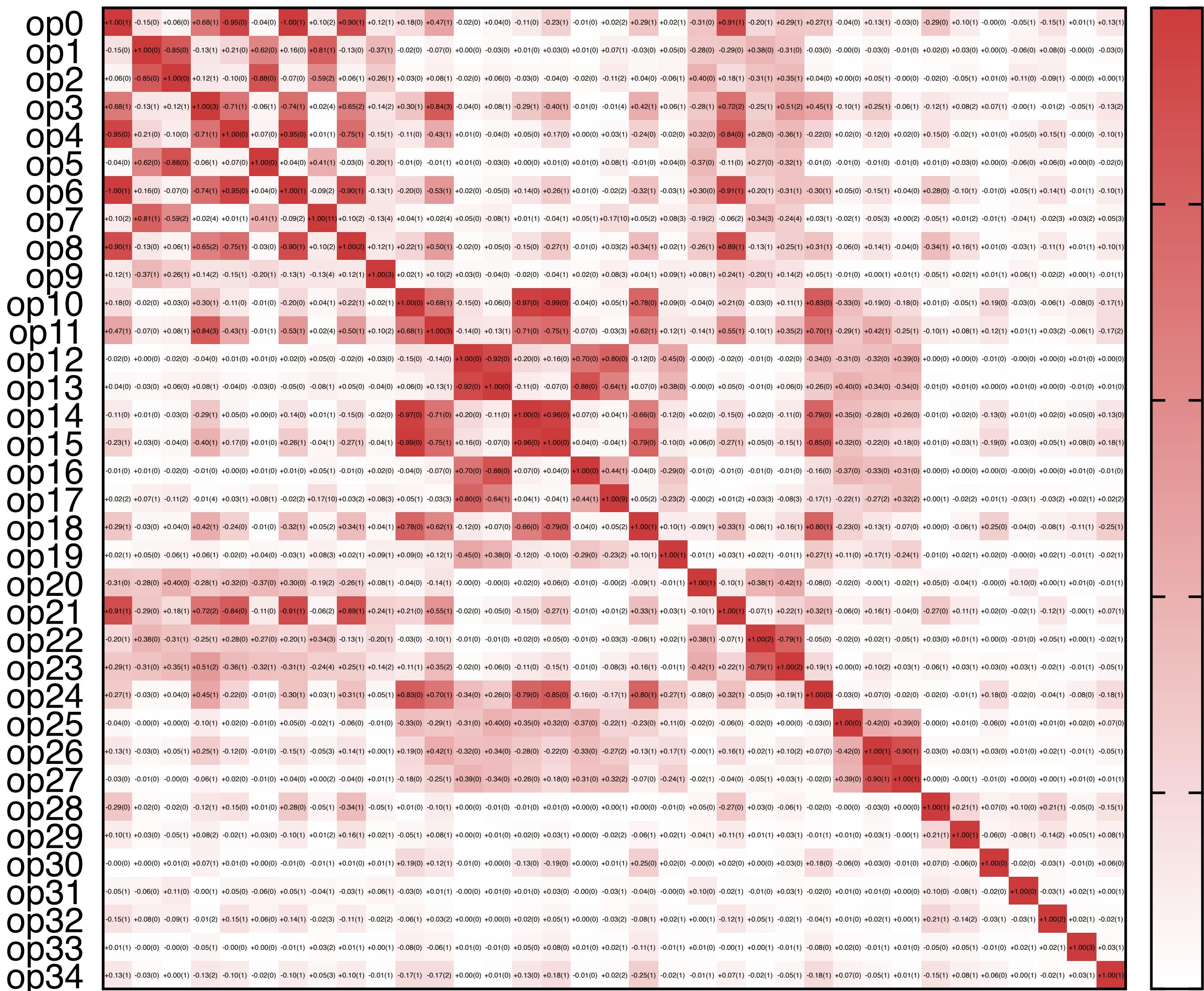
Many ops. for a good GEVP

Distillation → 0905.2160

Time Avg. correlations

Some highly correlated

More than a few relevant ops.



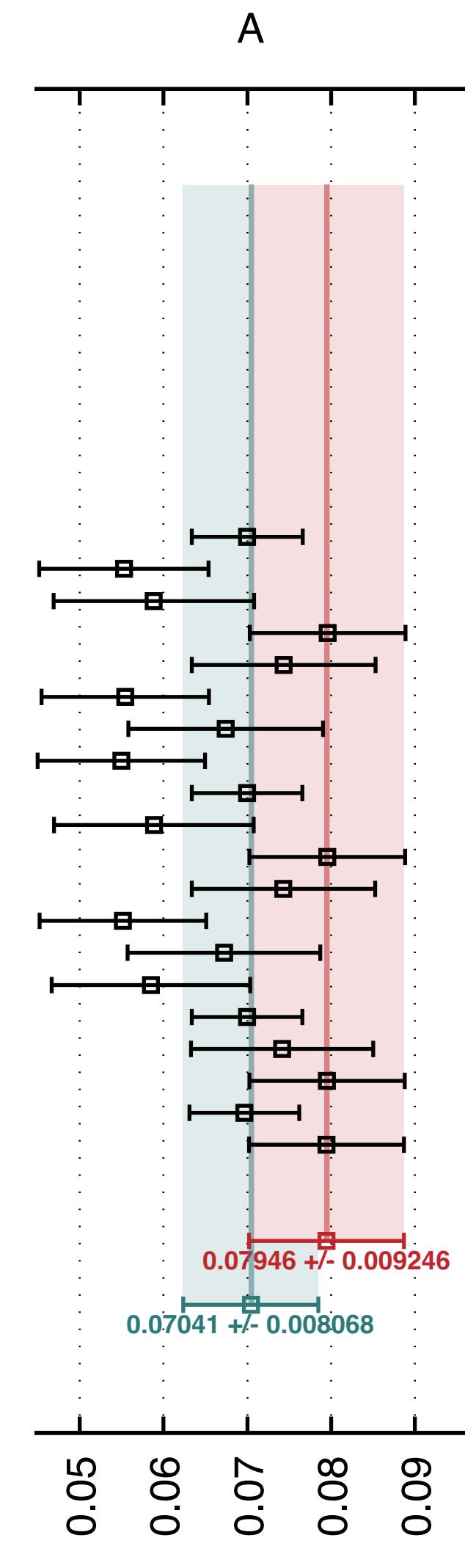
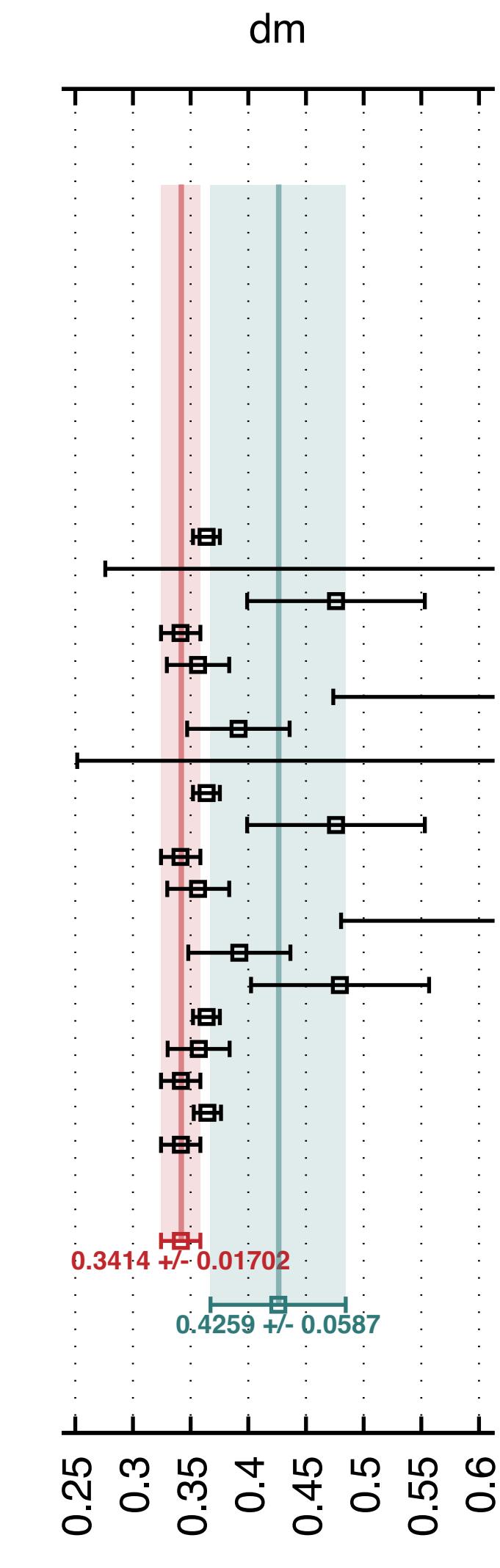
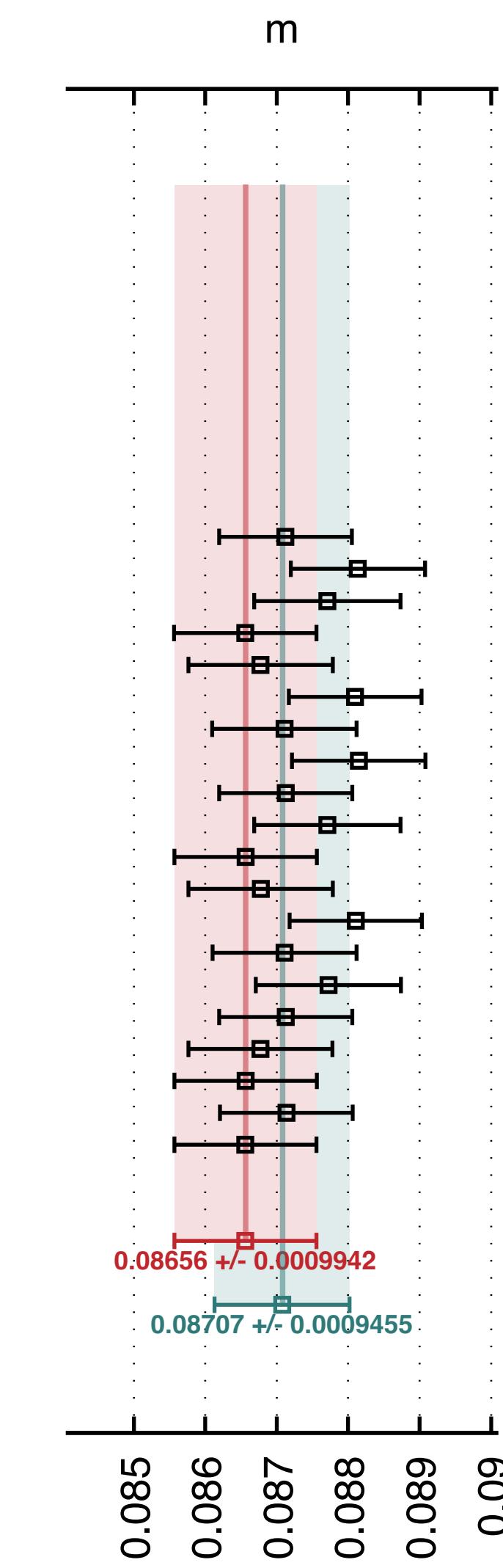
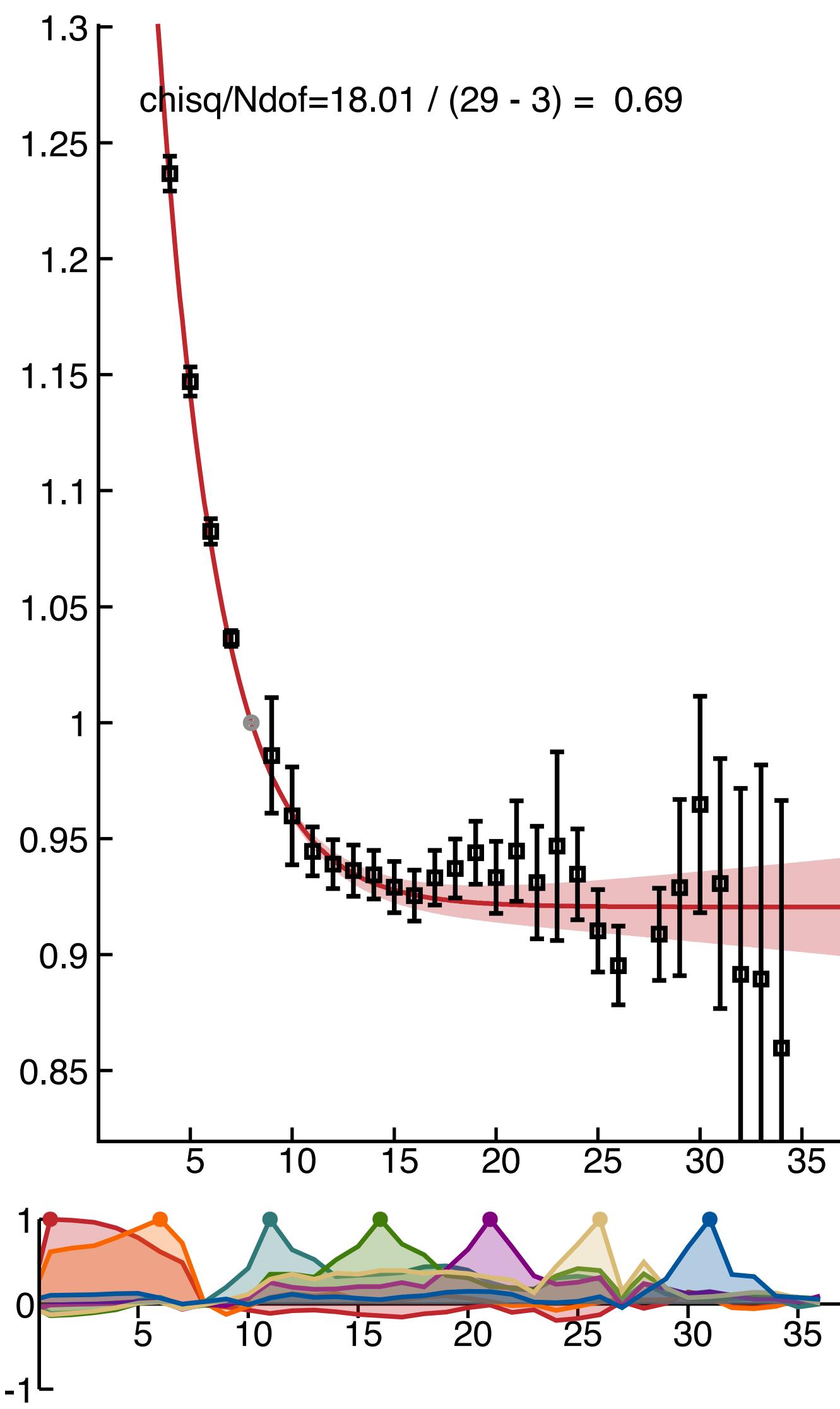
Many fits for a good E_n

Many fits for diff.

- t_0
- t_{min}, t_{max}
- N_{exp}

Model averaging technique

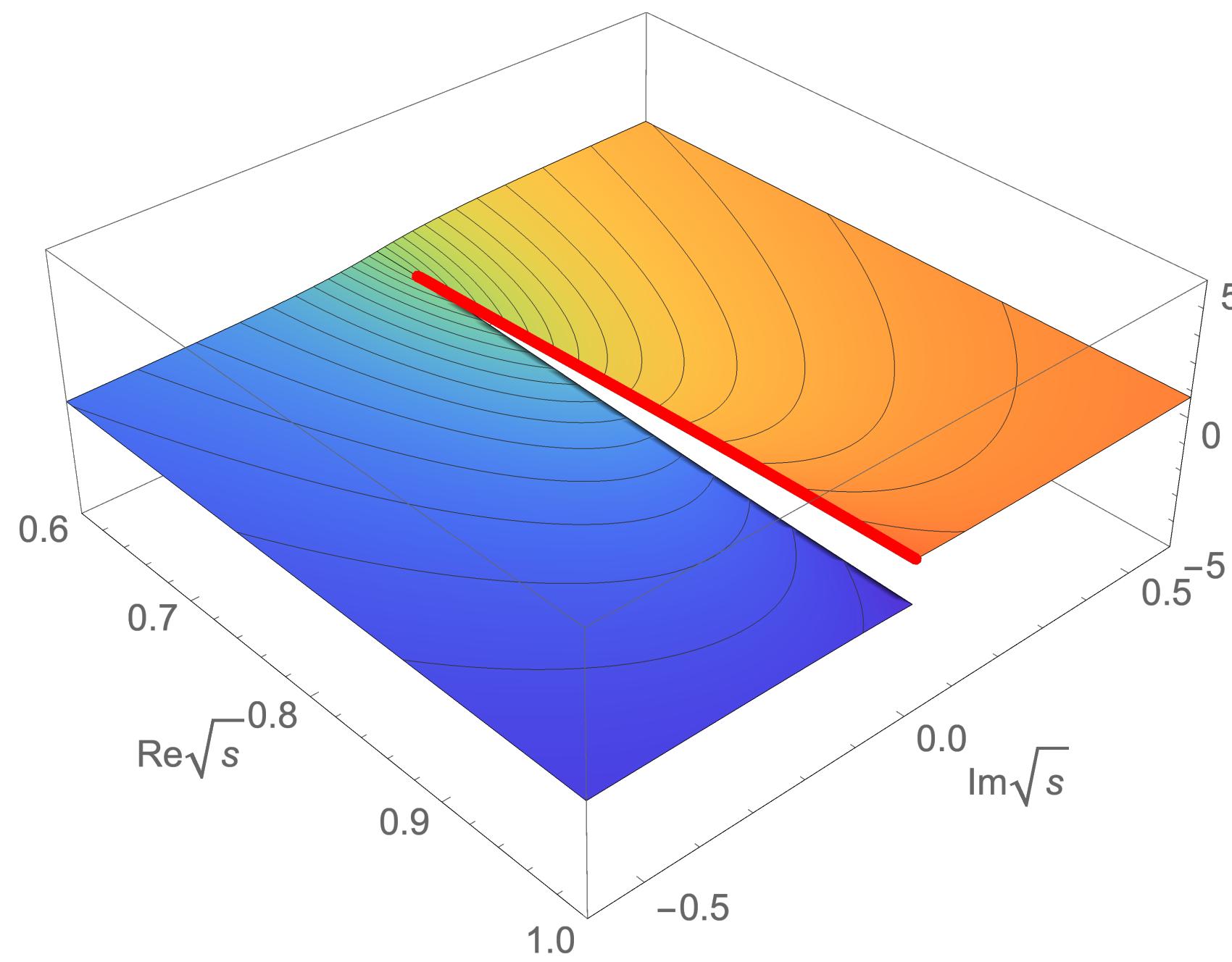
2008.01069



Scattering

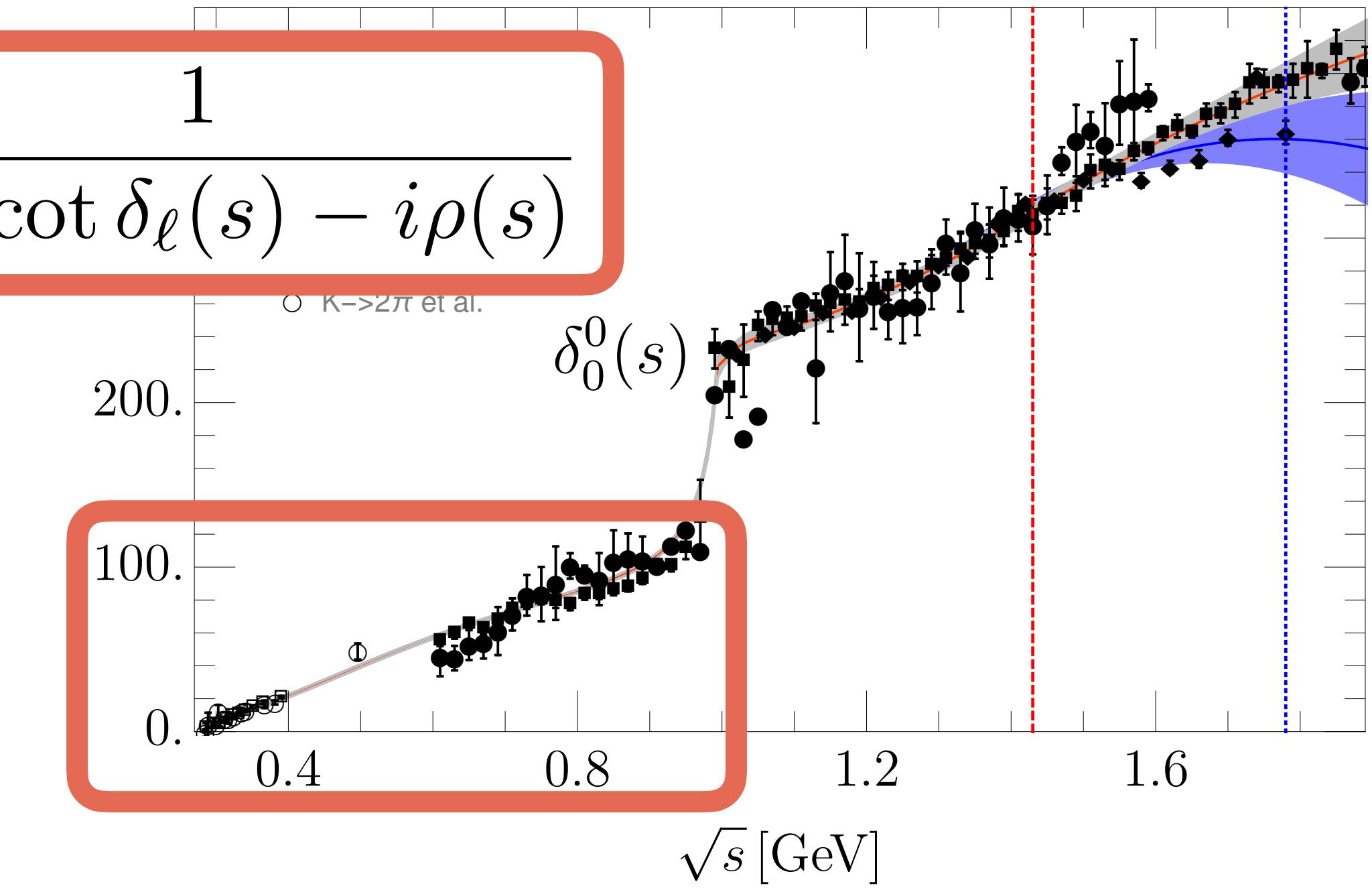
My fits

I sheet clear of poles



Analiticity

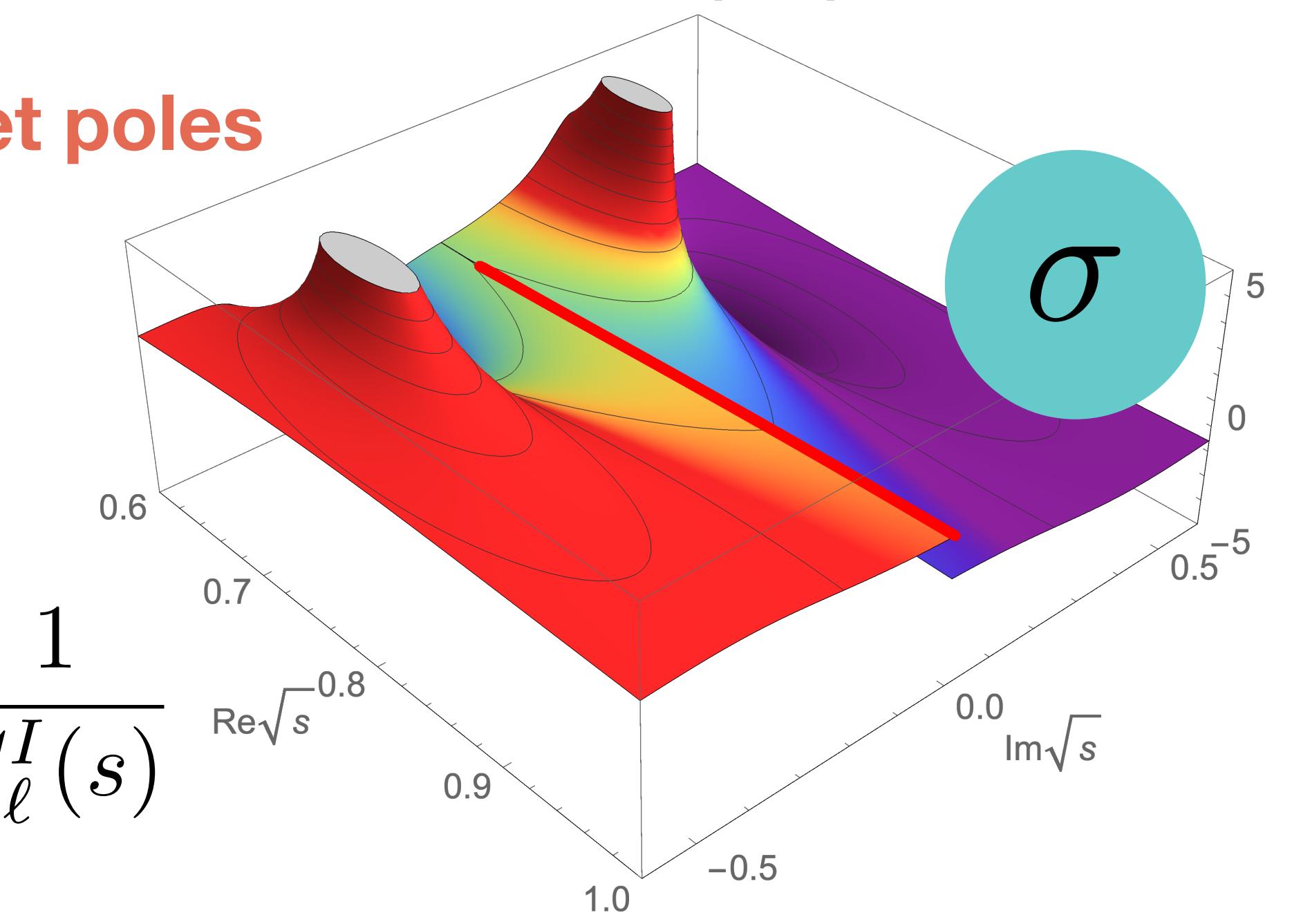
$$t_\ell(s) = \frac{1}{\rho(s) \cot \delta_\ell(s) - i\rho(s)}$$



II sheet poles

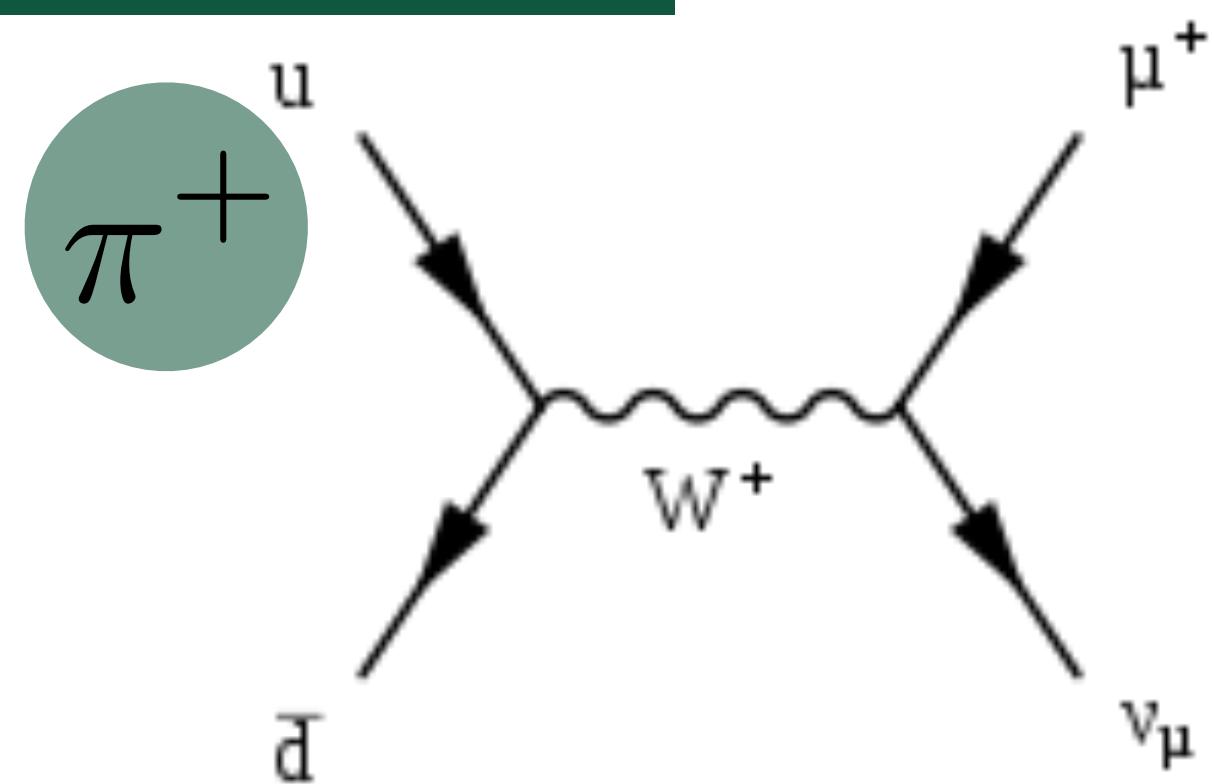
Unitarity

$$S_\ell^{II}(s) = \frac{1}{S_\ell^I(s)}$$



Exp. Problem

$$c\tau_{\pi^+} = 7.8 \text{ m}$$

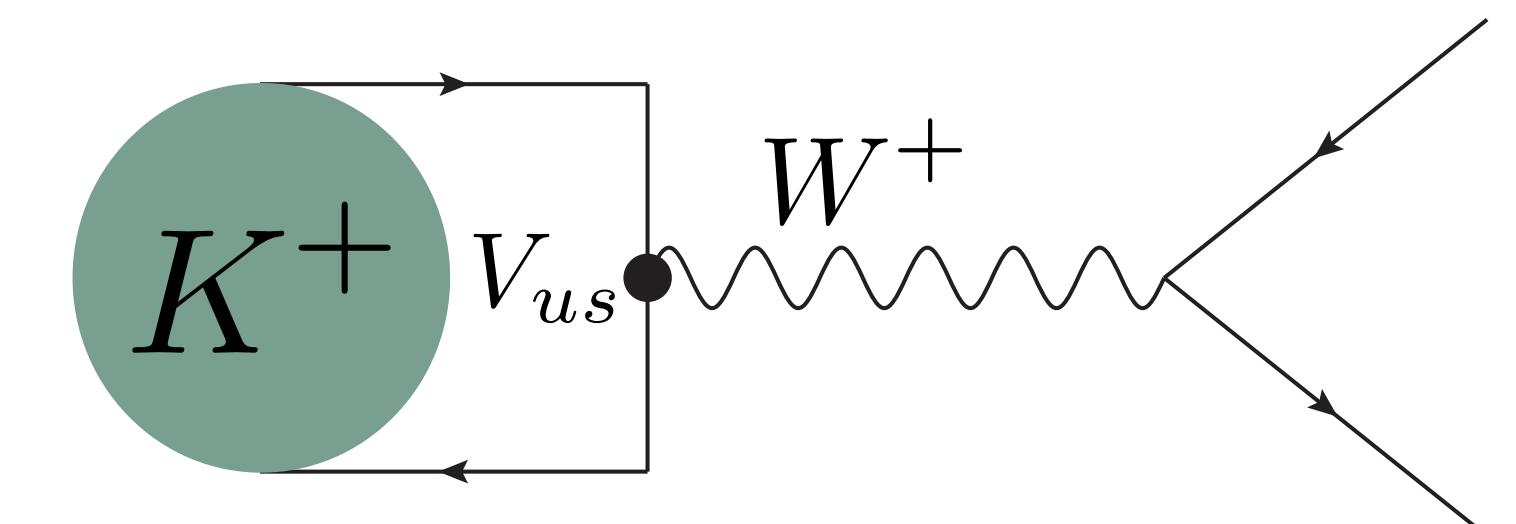


Mesons decay

One beam (maybe)

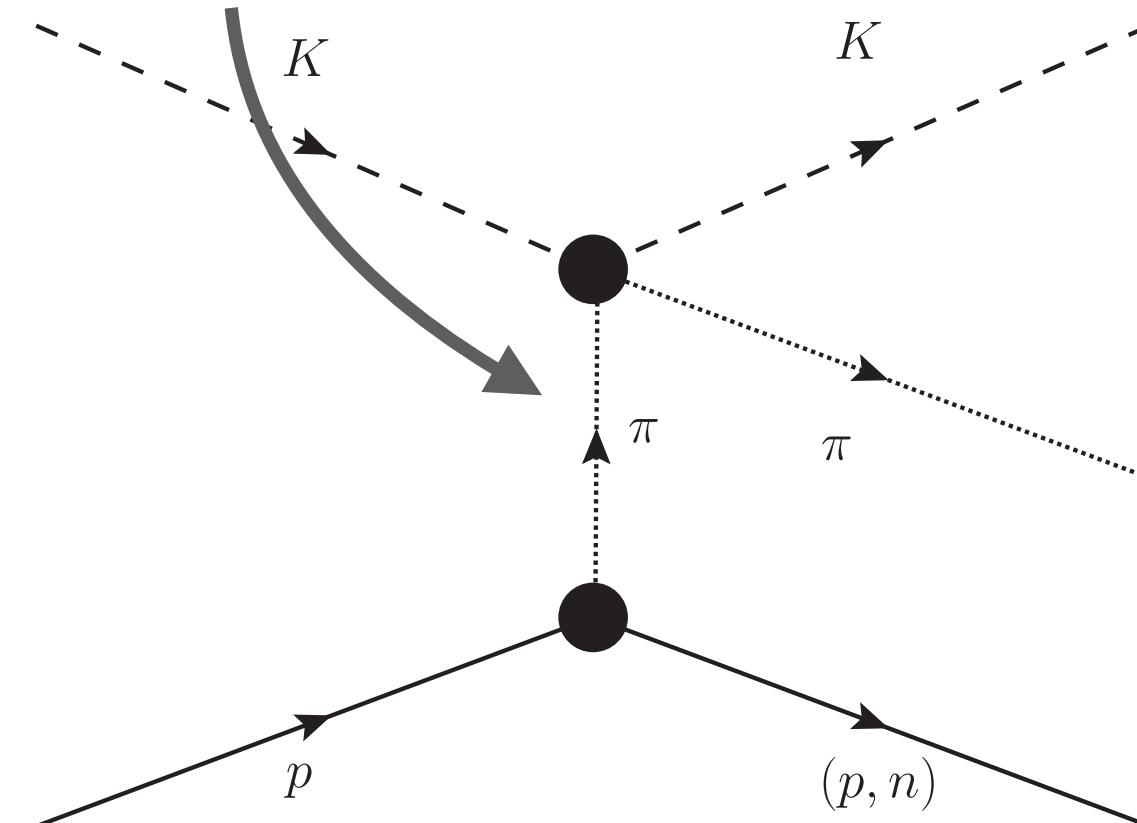
Not two beams

$$c\tau_K = 3.7 \text{ m}$$



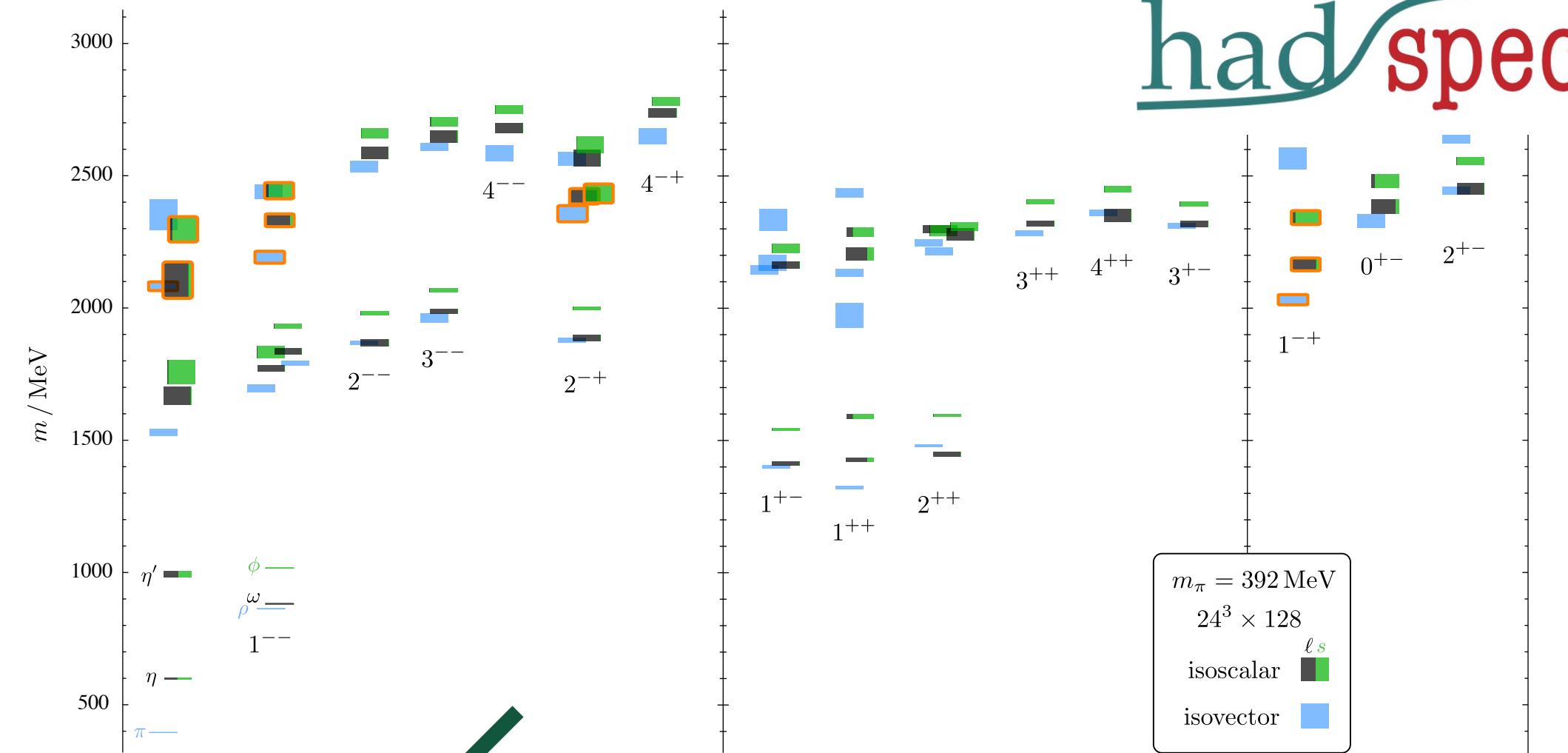
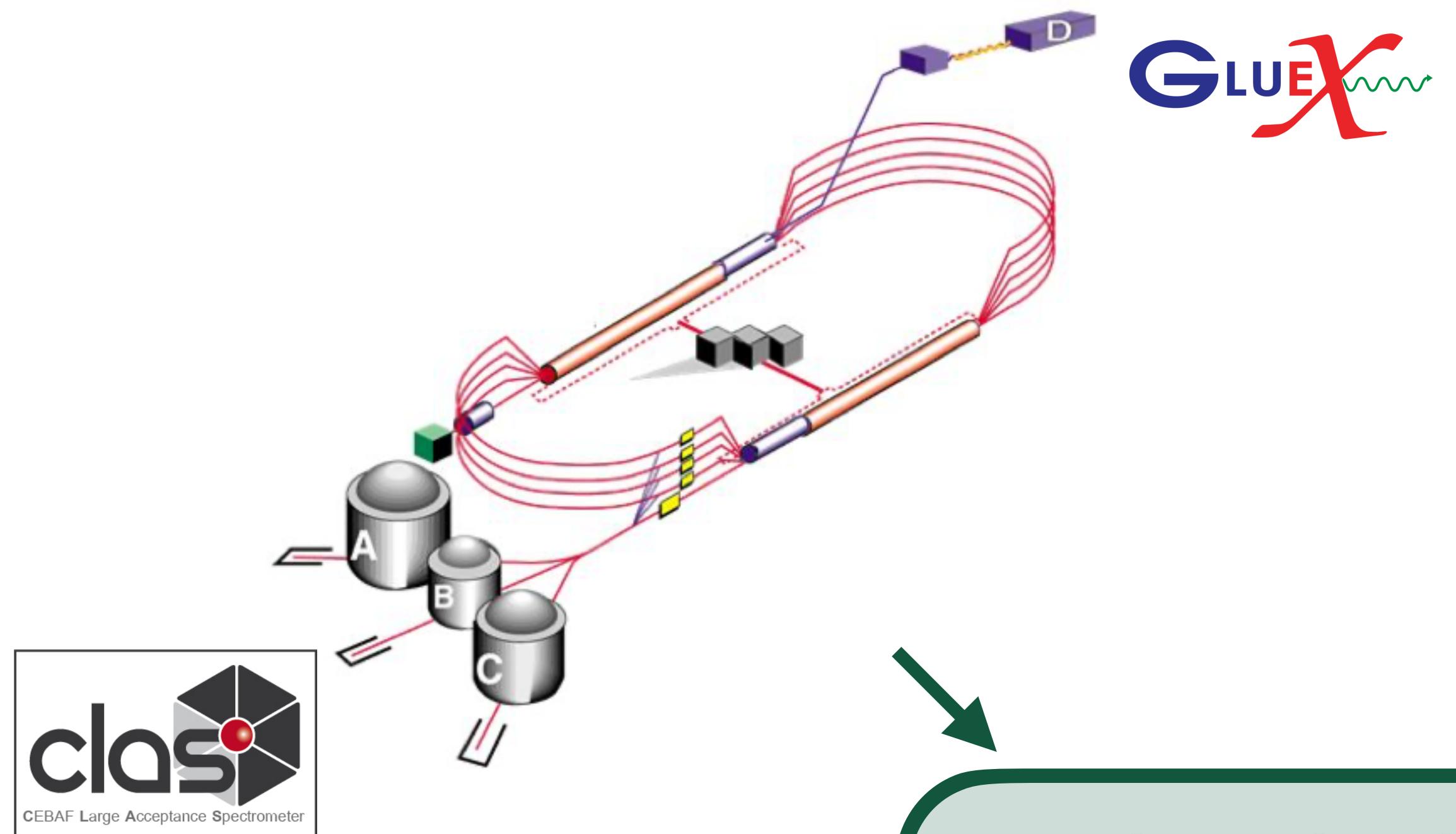
“Data” is not data

Virtual pion



For meson-meson data must be always modeled

Some systematic uncertainties unknown



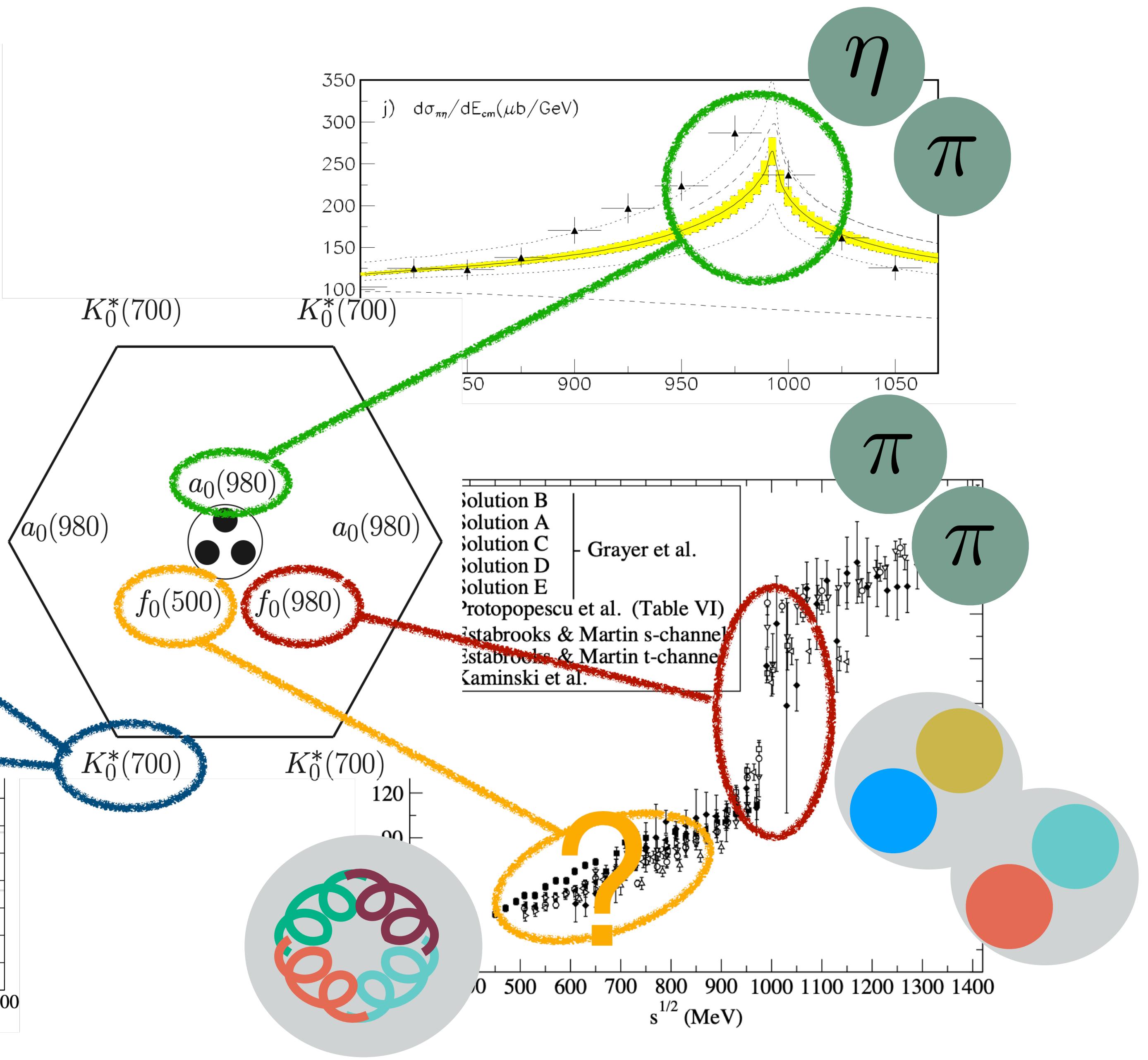
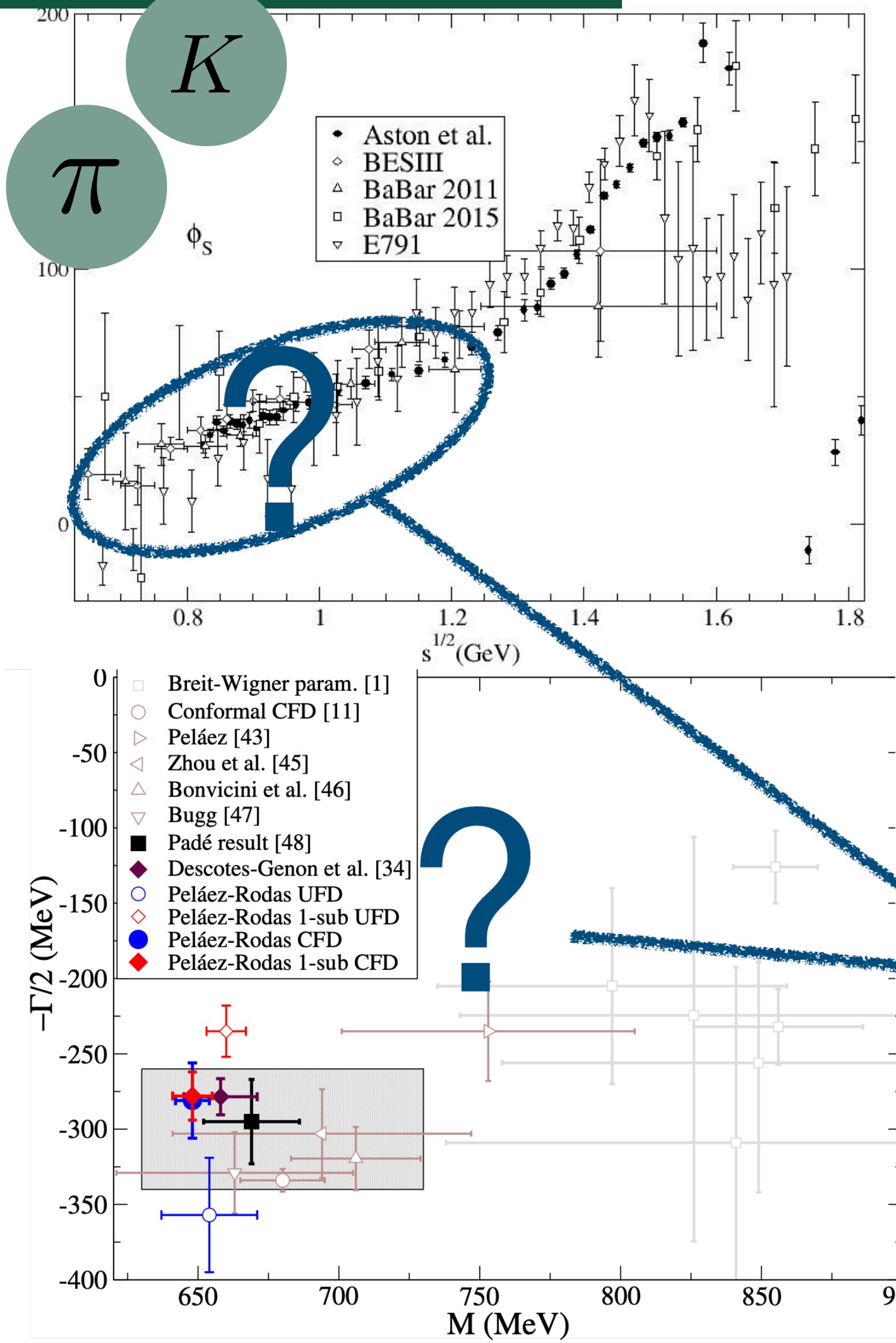
Amplitude analyses

- Unitarity
- Analyticity
- Crossing

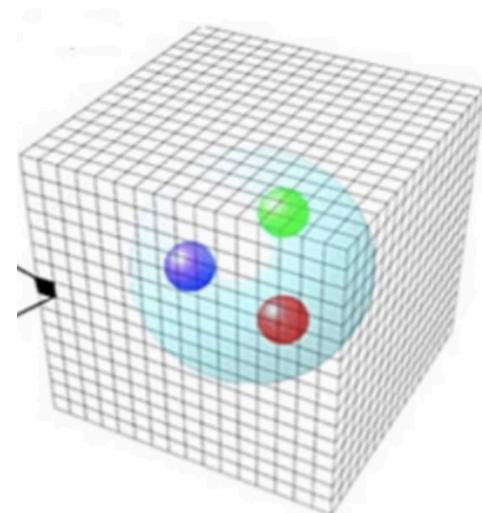
QCD

Observables

Light Scalars

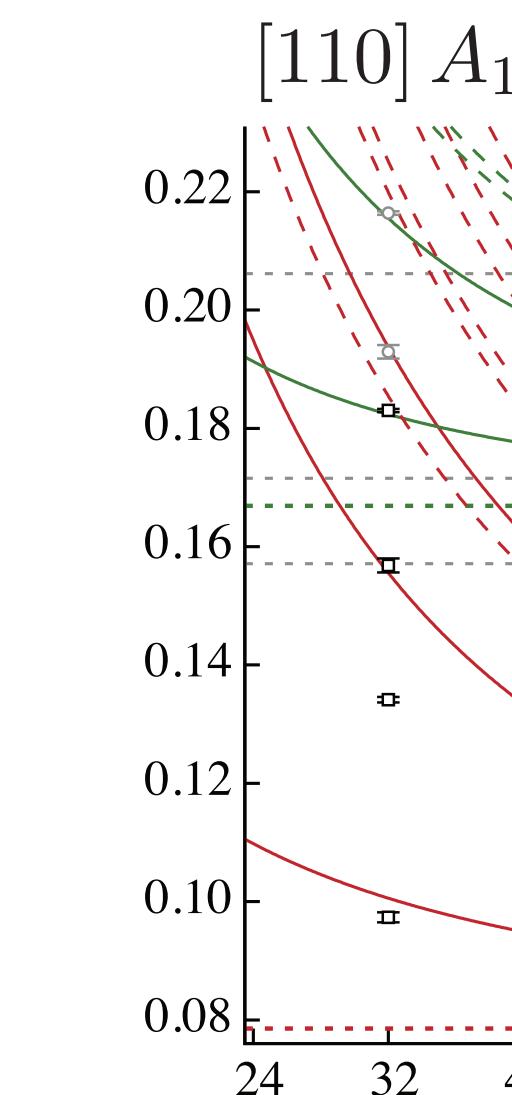
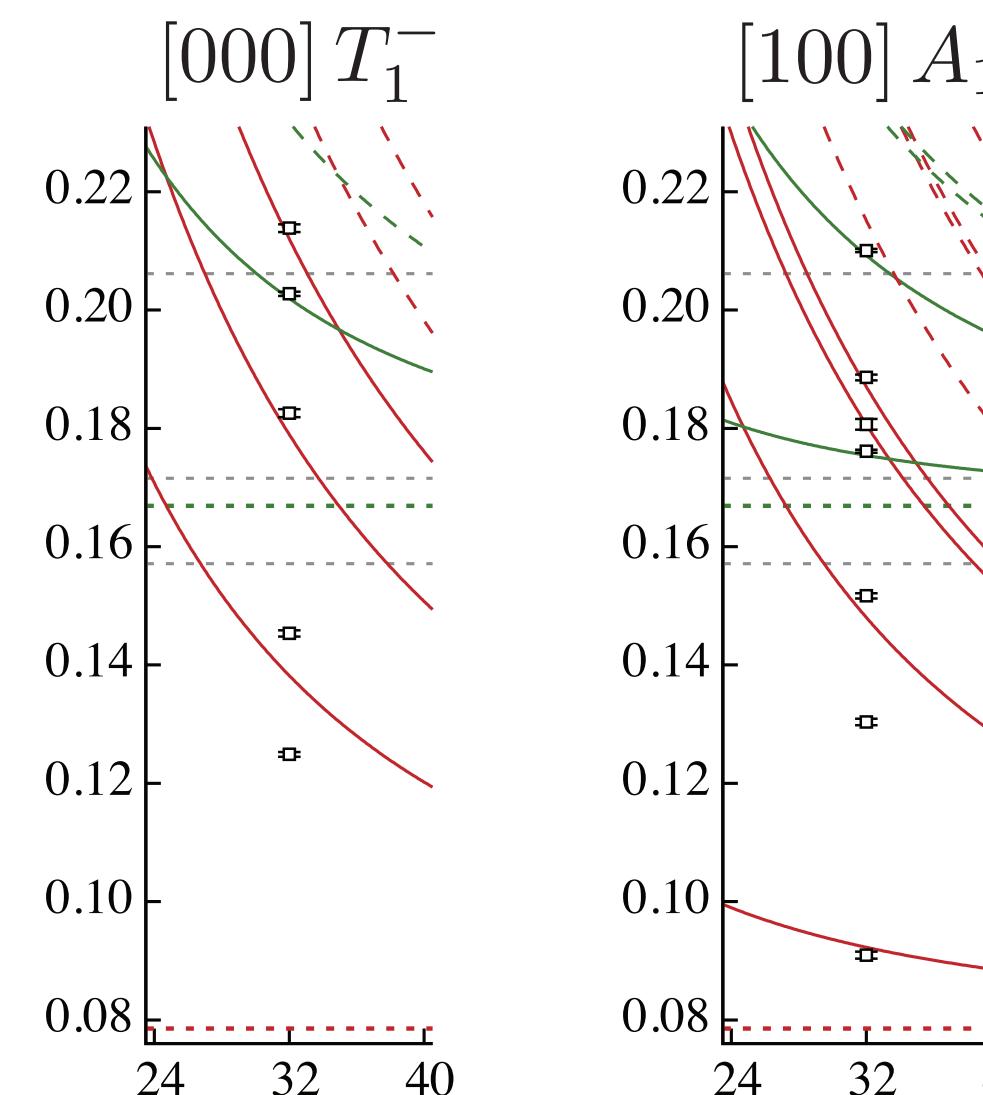


$\pi\pi$ scattering on the lattice



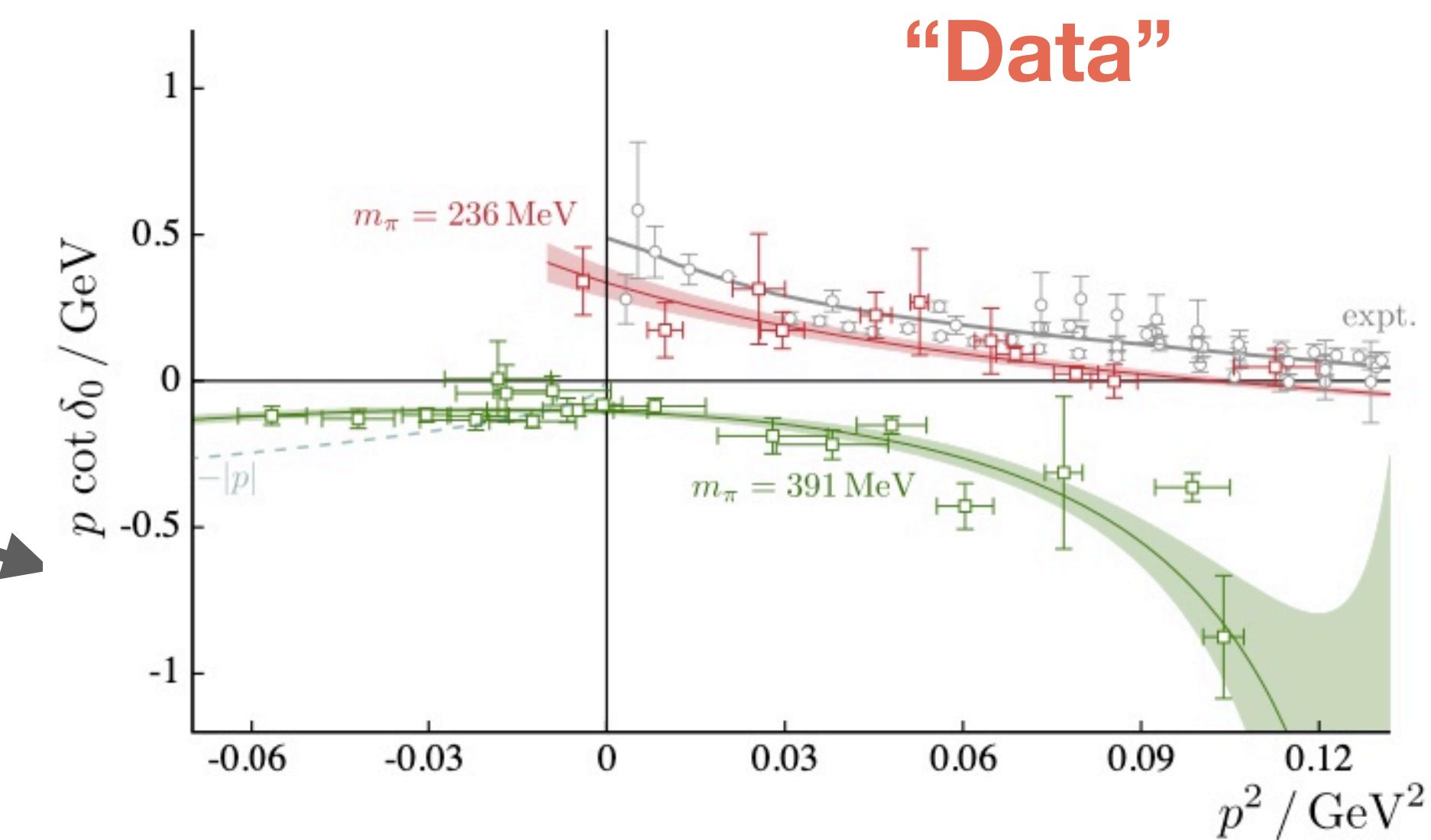
$$t \rightarrow -it$$

$$-iS = -i \int d^3x dt \mathcal{L} \rightarrow - \int d^3x dt \mathcal{L}_E = -S_E$$



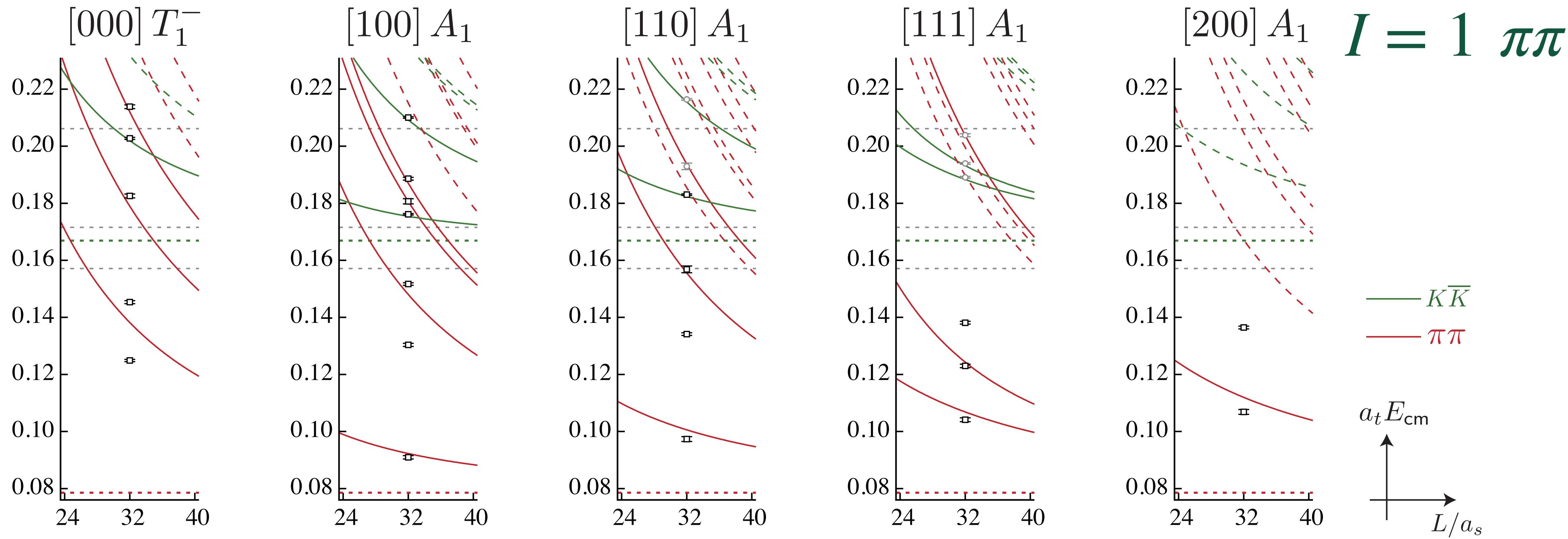
Lüscher

Quantized levels in a box



Hamiltonian evolution

$$e^{-iHt} \xrightarrow{t \rightarrow -it} \sum_n e^{-E_n t} \langle 0 | O_f(0) | n \rangle \langle n | O_i^\dagger(0) | 0 \rangle$$



Lattice QCD

In a 1D box

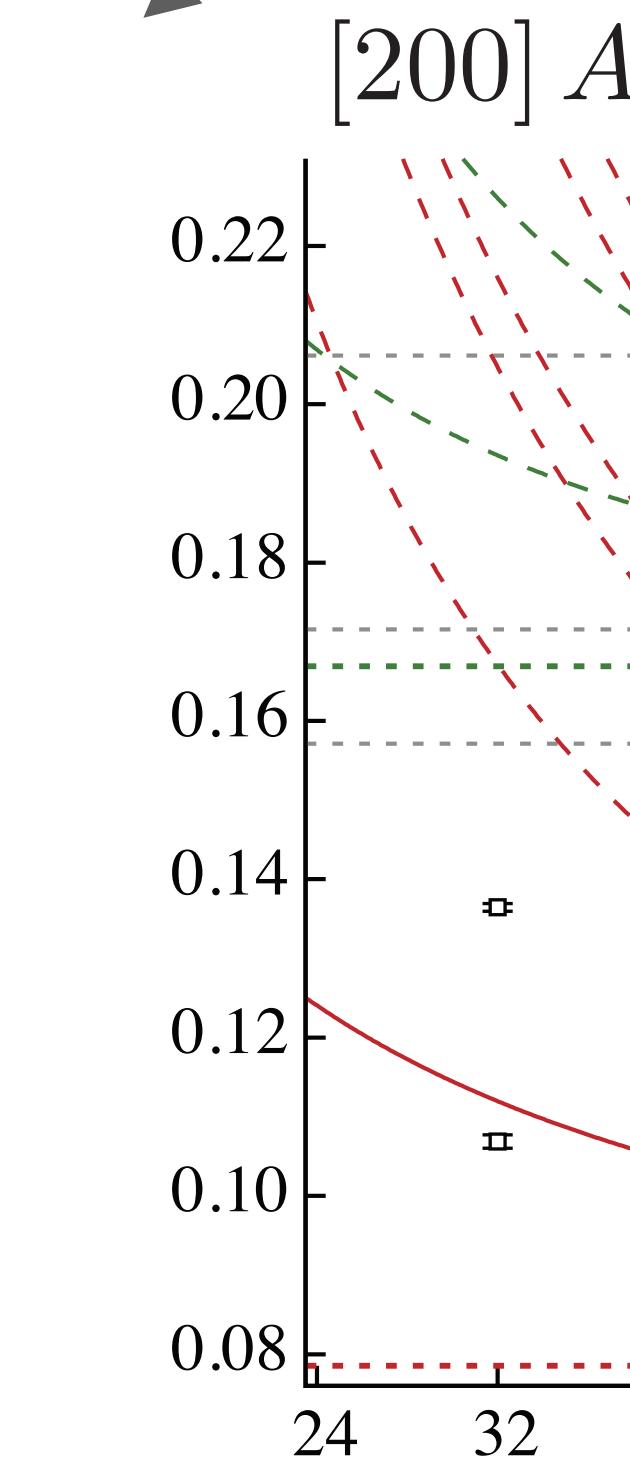
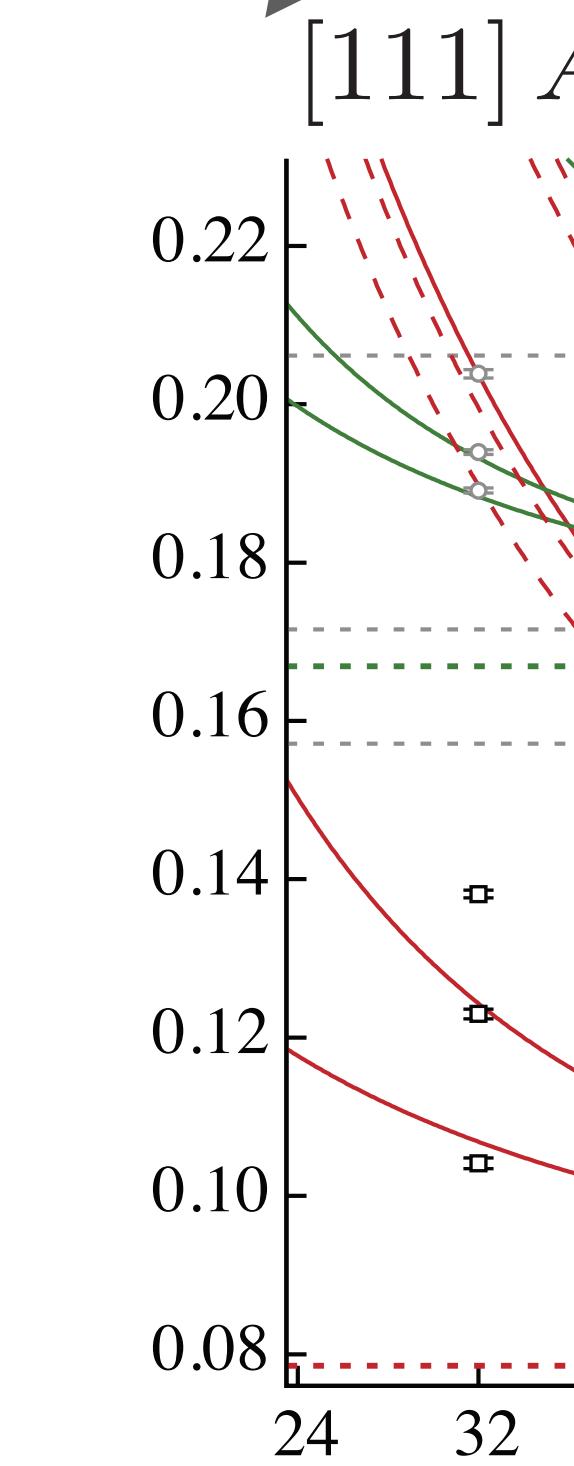
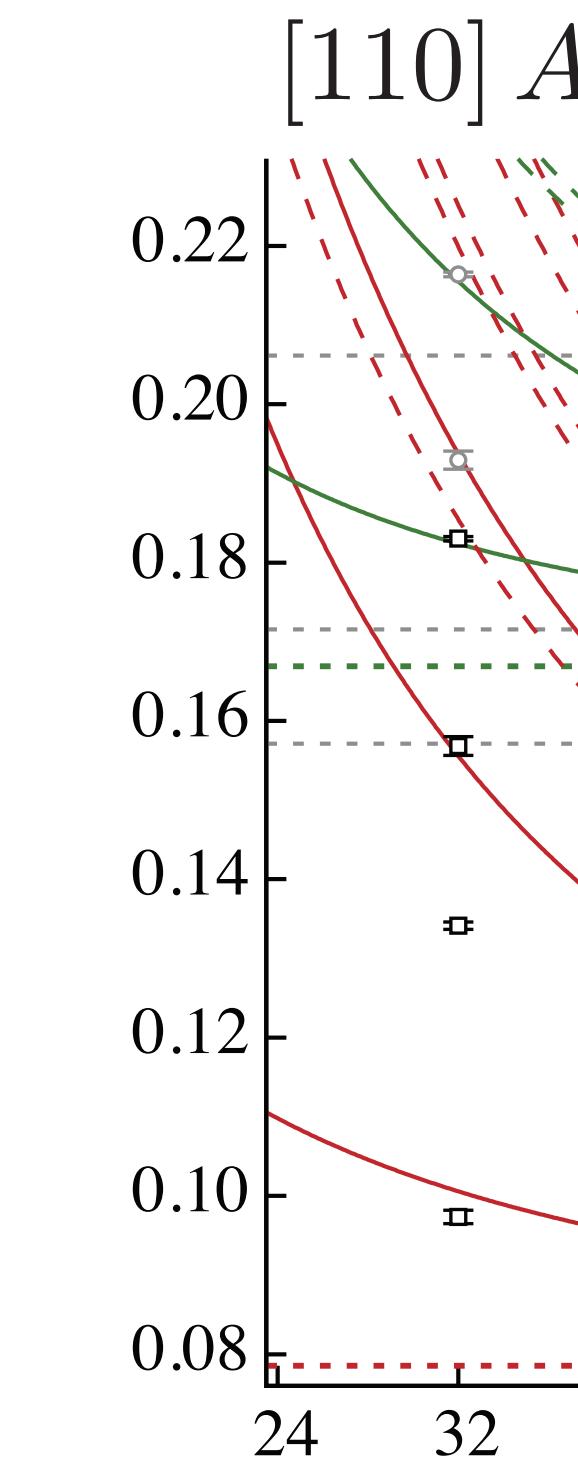
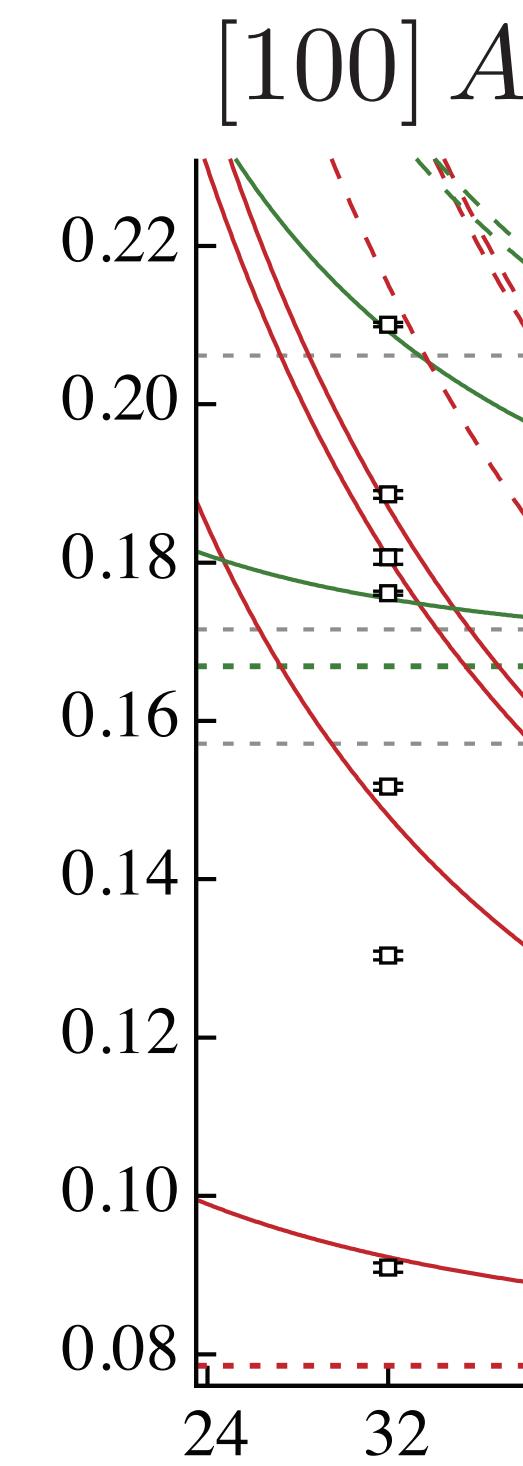
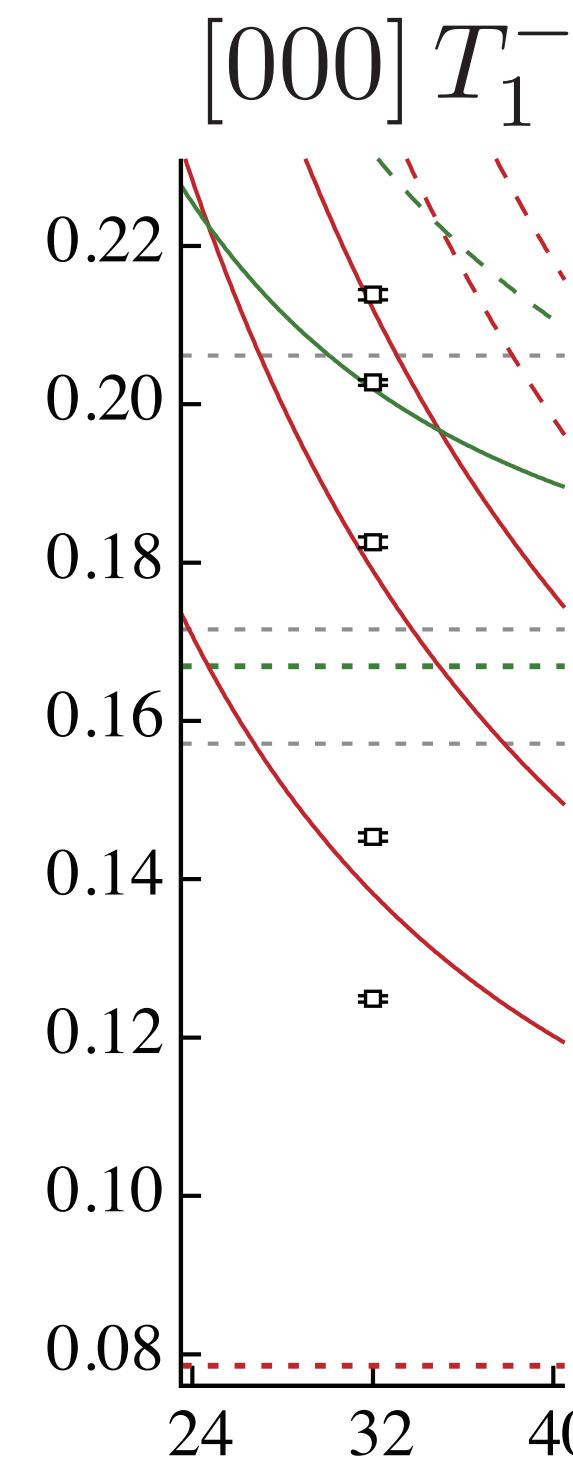
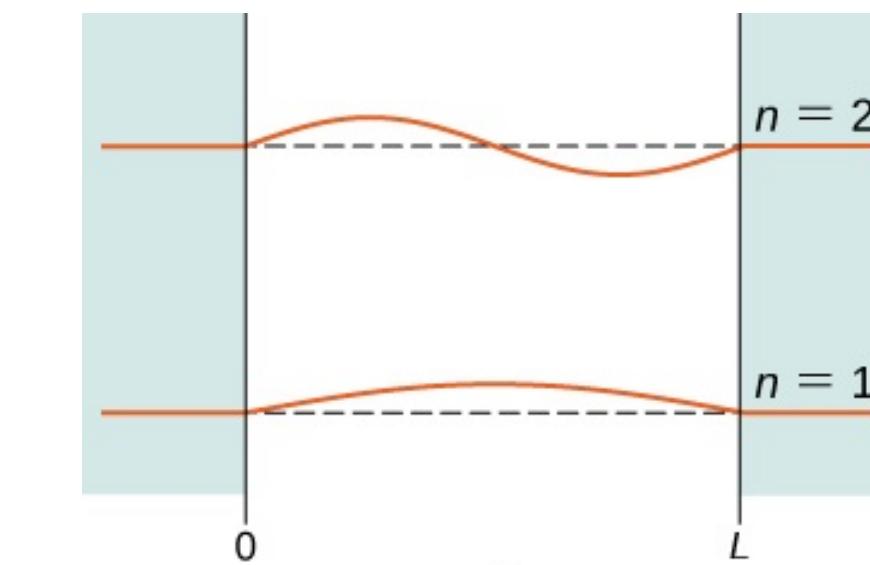
$$R \ll L$$

In

Out

Boundary

$$e^{ikL + 2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

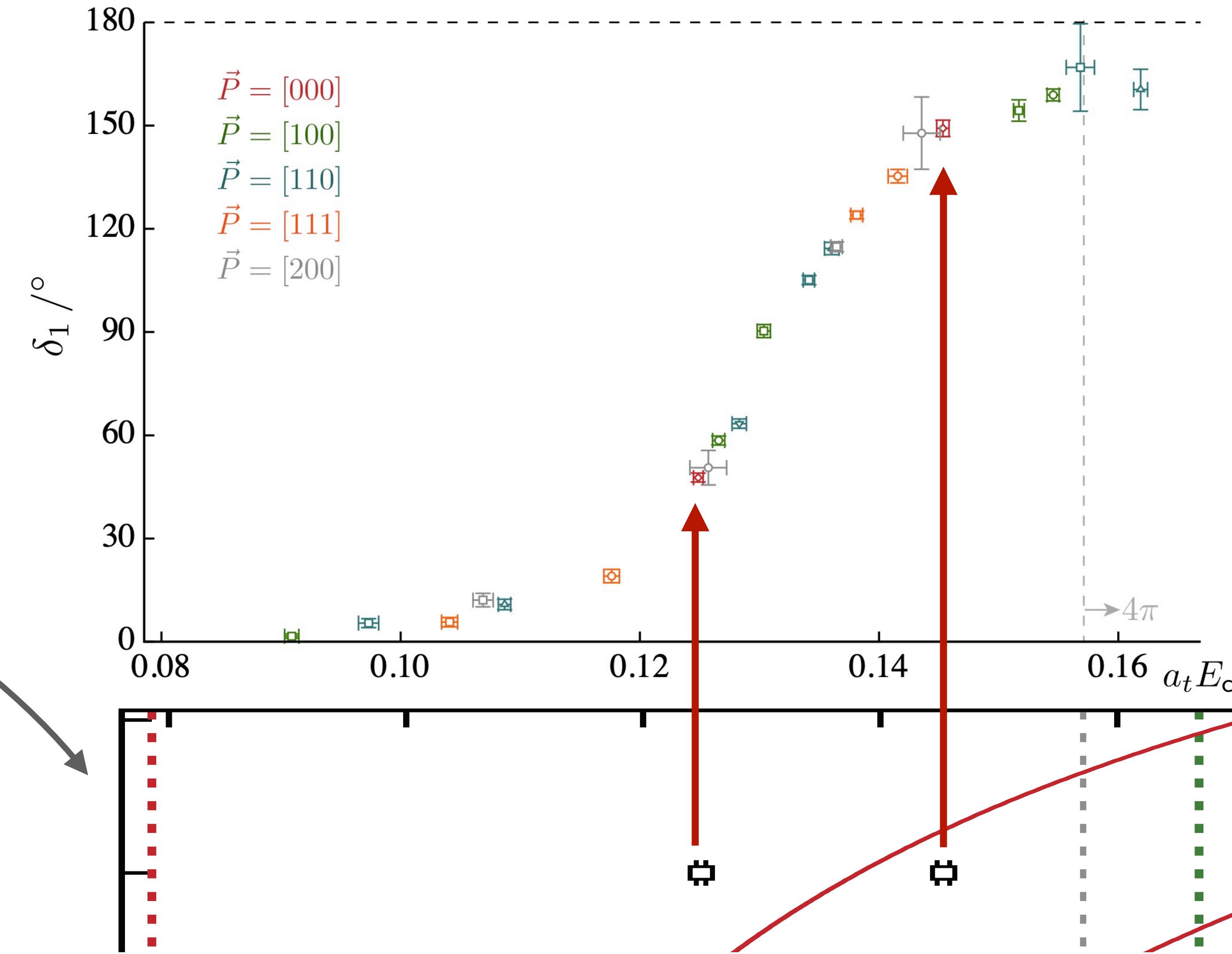
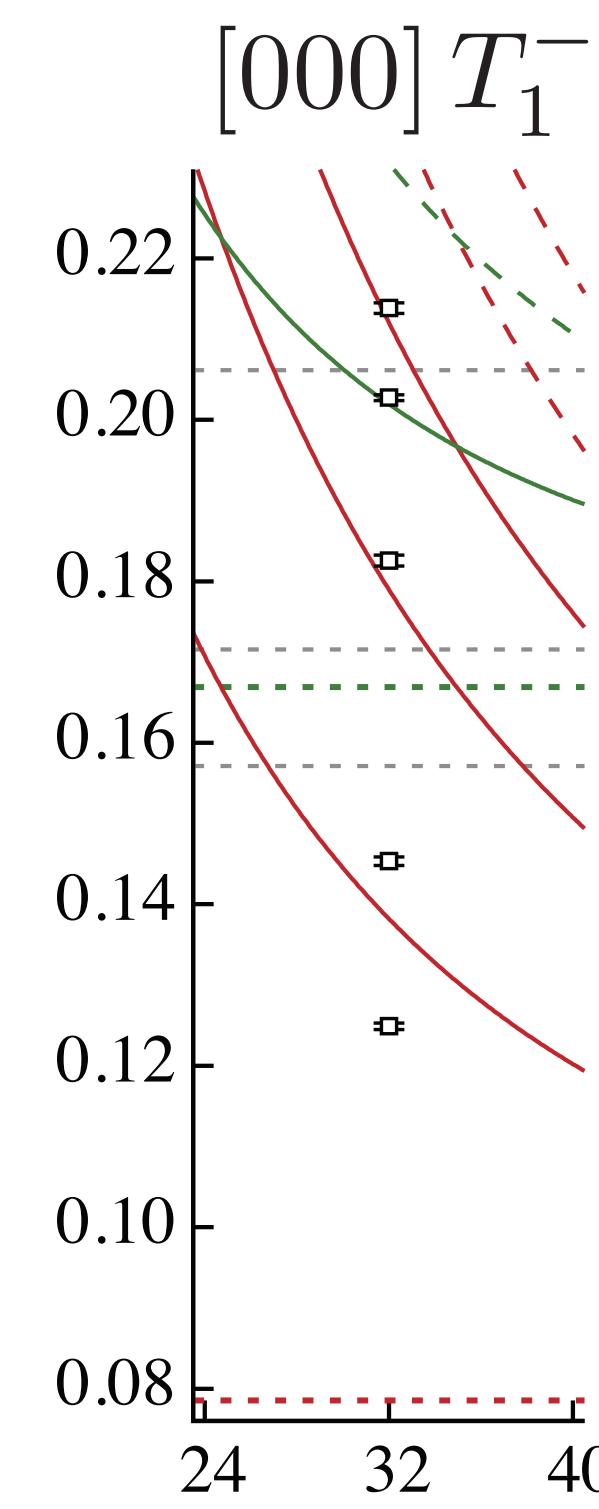


$\text{--- } K\bar{K}$
 $\text{— } \pi\pi$
 $a_t E_{\text{cm}}$
 L/a_s

$$e^{ikL+2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

We recover $\delta(s)$

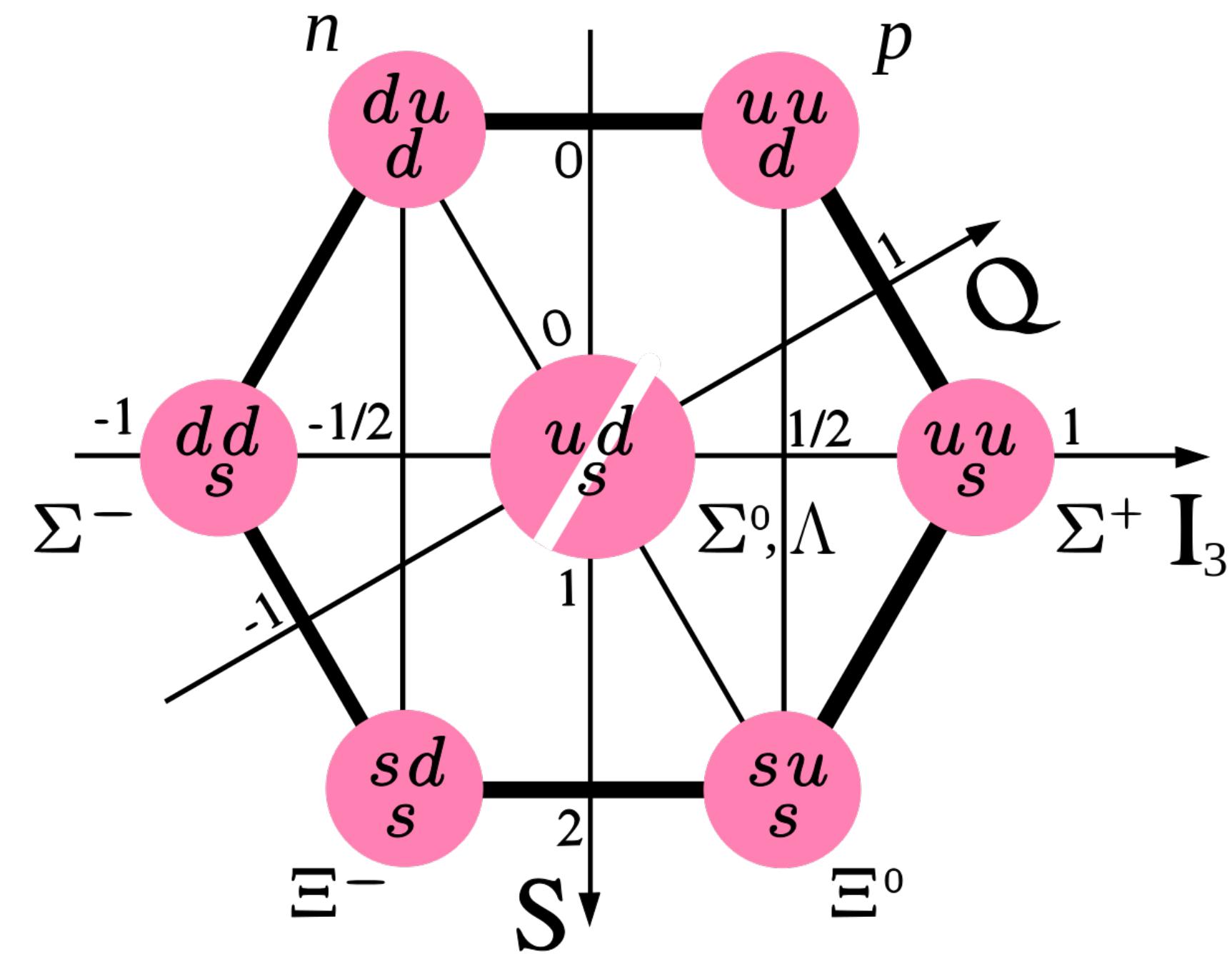
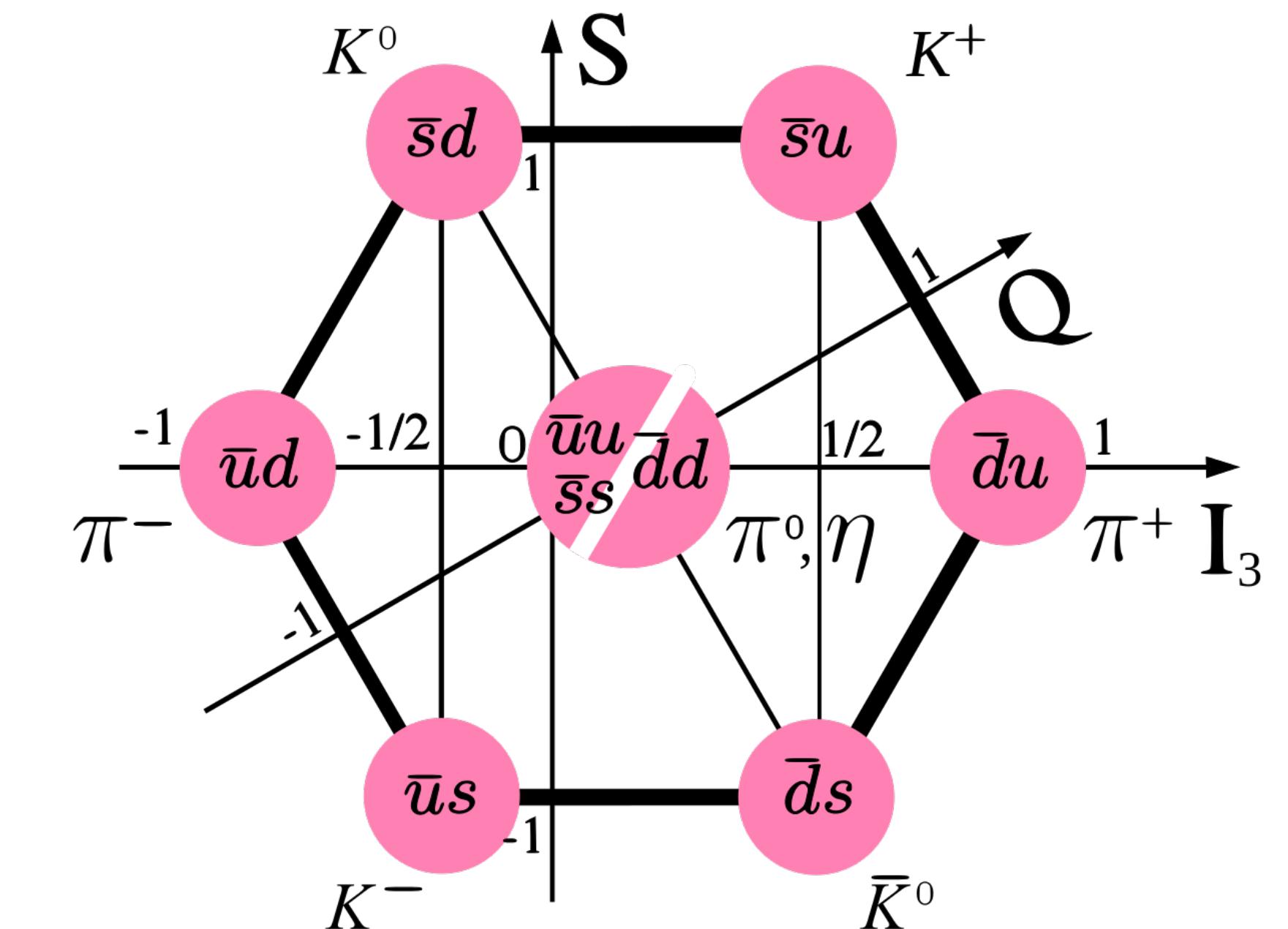
Our E_n become the x axis



Multiplets

Classify particles using symmetries

$$m_u \sim m_d \ll m_s \ll m_p$$



They are strongly related

Masses
Compositeness
Sizes

Pseudo-scalars // Pseudo-goldstone

Chiral symmetry

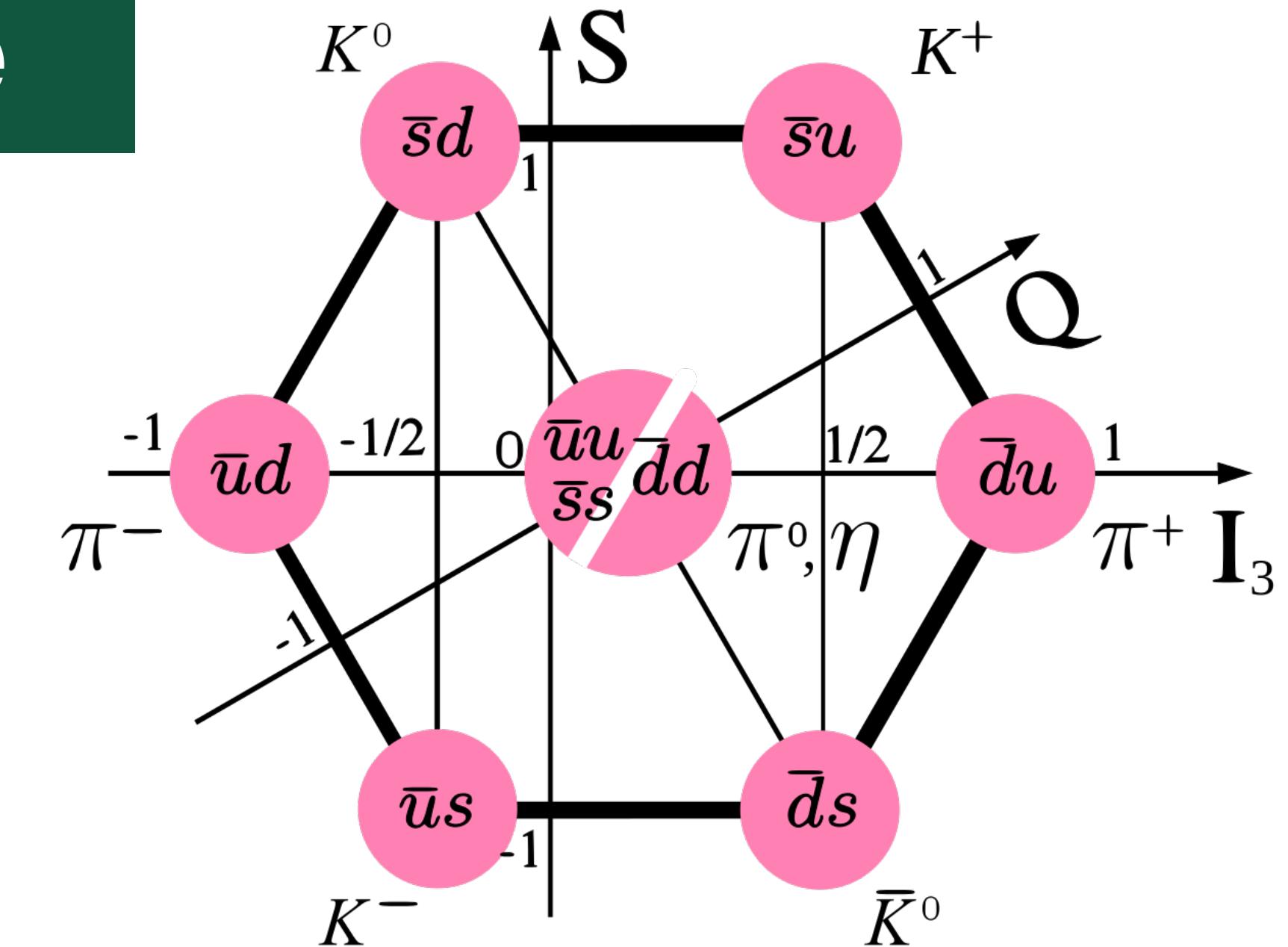
$$m_u \simeq m_d \simeq 0$$

Invariant

\mathcal{L}

$$q_{L,R} \rightarrow \exp\left(-i\theta_a^{L,R} \frac{T_a}{2}\right) q_{L,R}$$

$$q_{L,R} = \left(\frac{1 \mp \gamma_5}{2} \right) q$$



Noether's theorem \Rightarrow Conserved currents

$$V_a^\mu = \bar{q} \gamma^\mu T_a q$$

$$A_a^\mu = \bar{q} \gamma^\mu \gamma_5 T_a q$$

Pseudo-scalars // Pseudo-goldstone

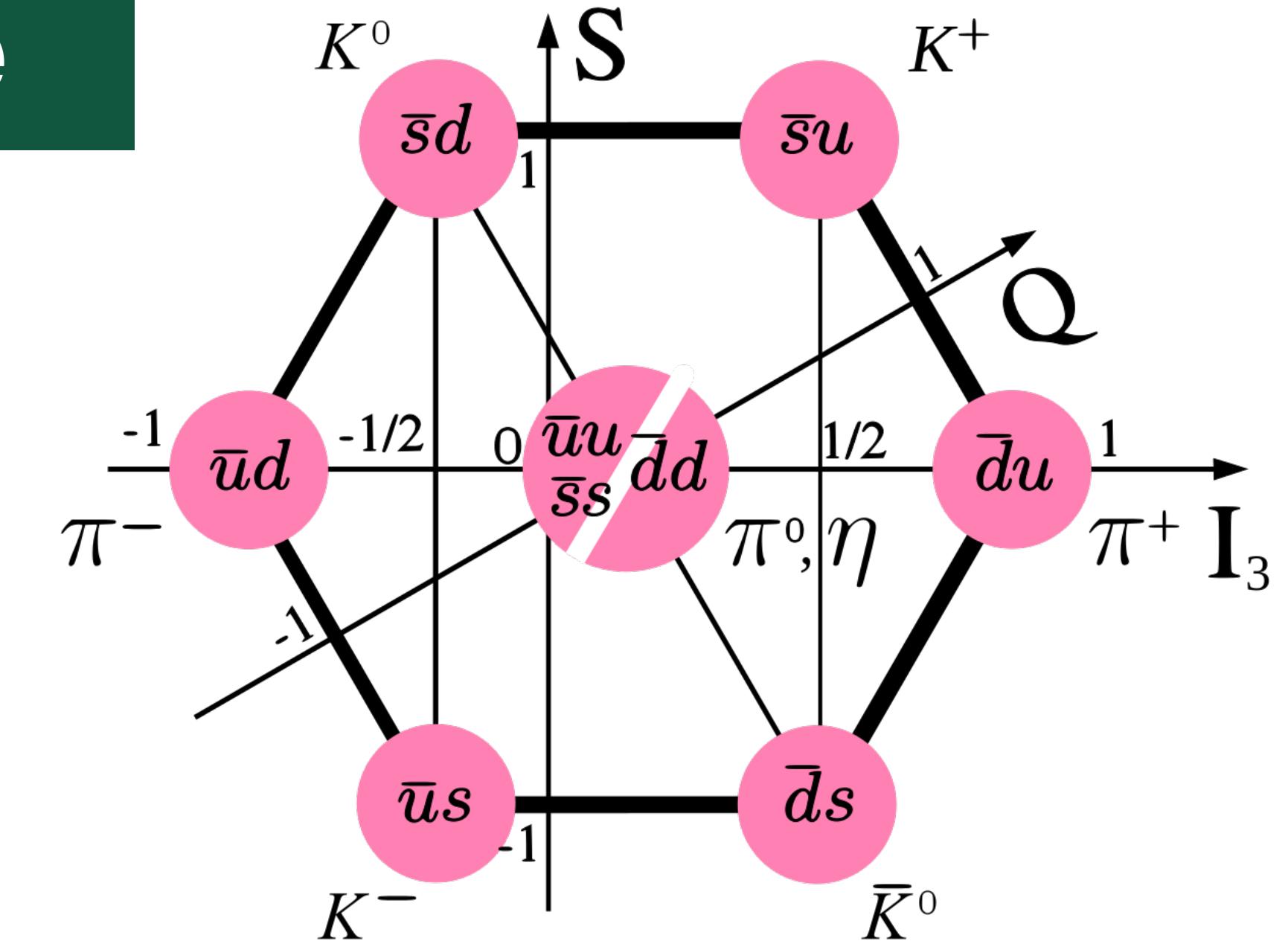
Chiral symmetry

m_u

Invariant

$$q = \begin{pmatrix} 1 & \mp \gamma_5 \\ 2 & 2 \end{pmatrix} q_{L,R}$$

Spontaneously broken

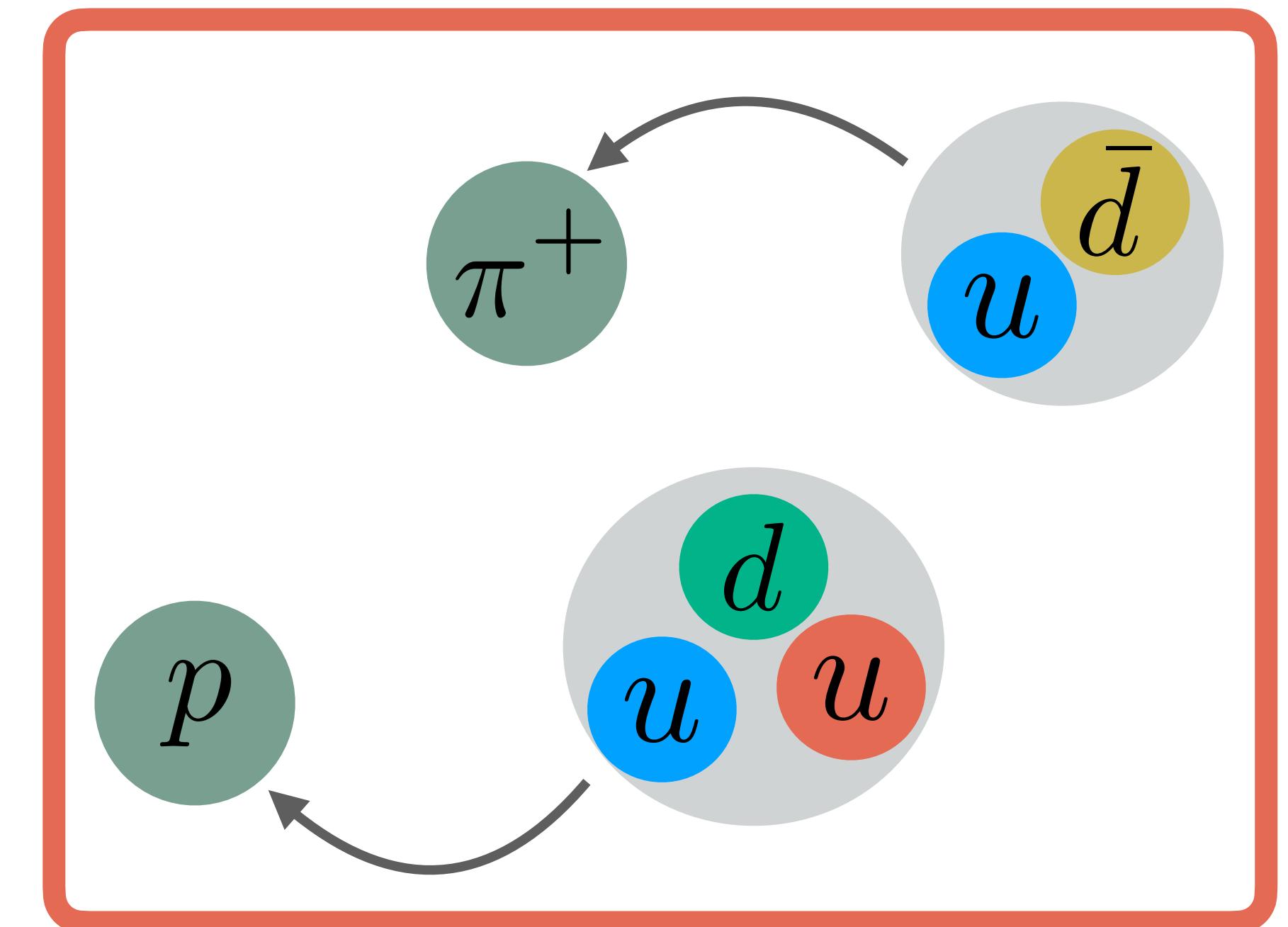


Noether's theorem \Rightarrow Conserved currents

$$V_a^\mu = \bar{q} \gamma^\mu T_a q$$

$$A_a^\mu = \bar{q} \gamma^\mu \gamma_5 T_a q$$

$$m_\pi \ll m_p$$



Vectors

Ordinary mass hierarchy

Only light quarks

$$m_\rho \simeq 770 \text{ MeV}$$

$$m_\omega \simeq 782 \text{ MeV}$$

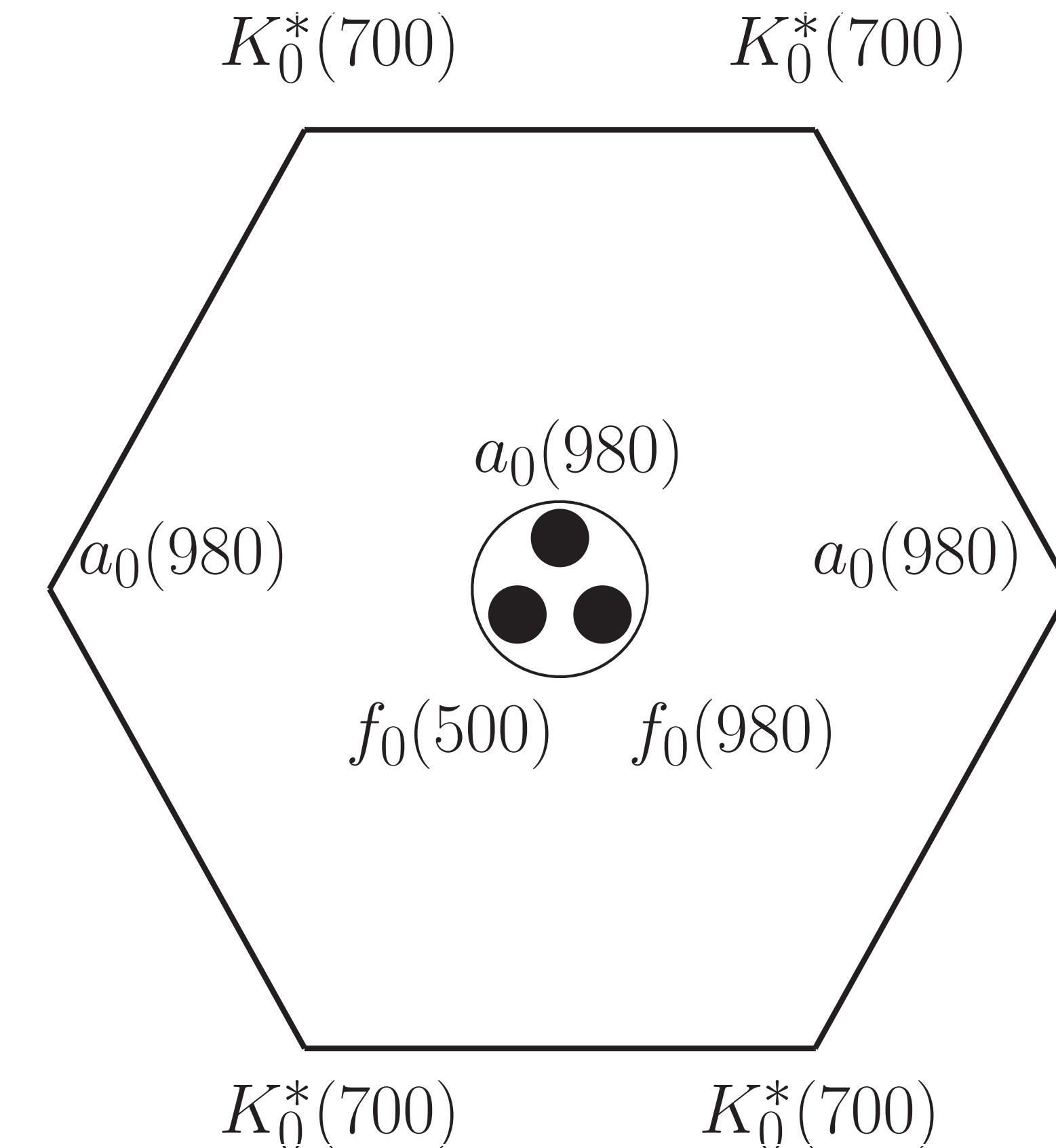
Light+strange

$$m_K^* \simeq 892 \text{ MeV}$$

Only strange

$$m_\phi \simeq 1020 \text{ MeV}$$

Multiplets work...



Scalars

Non-ordinary mass hierarchy

Only light quarks

$$m_\sigma \sim 450 \text{ MeV}$$

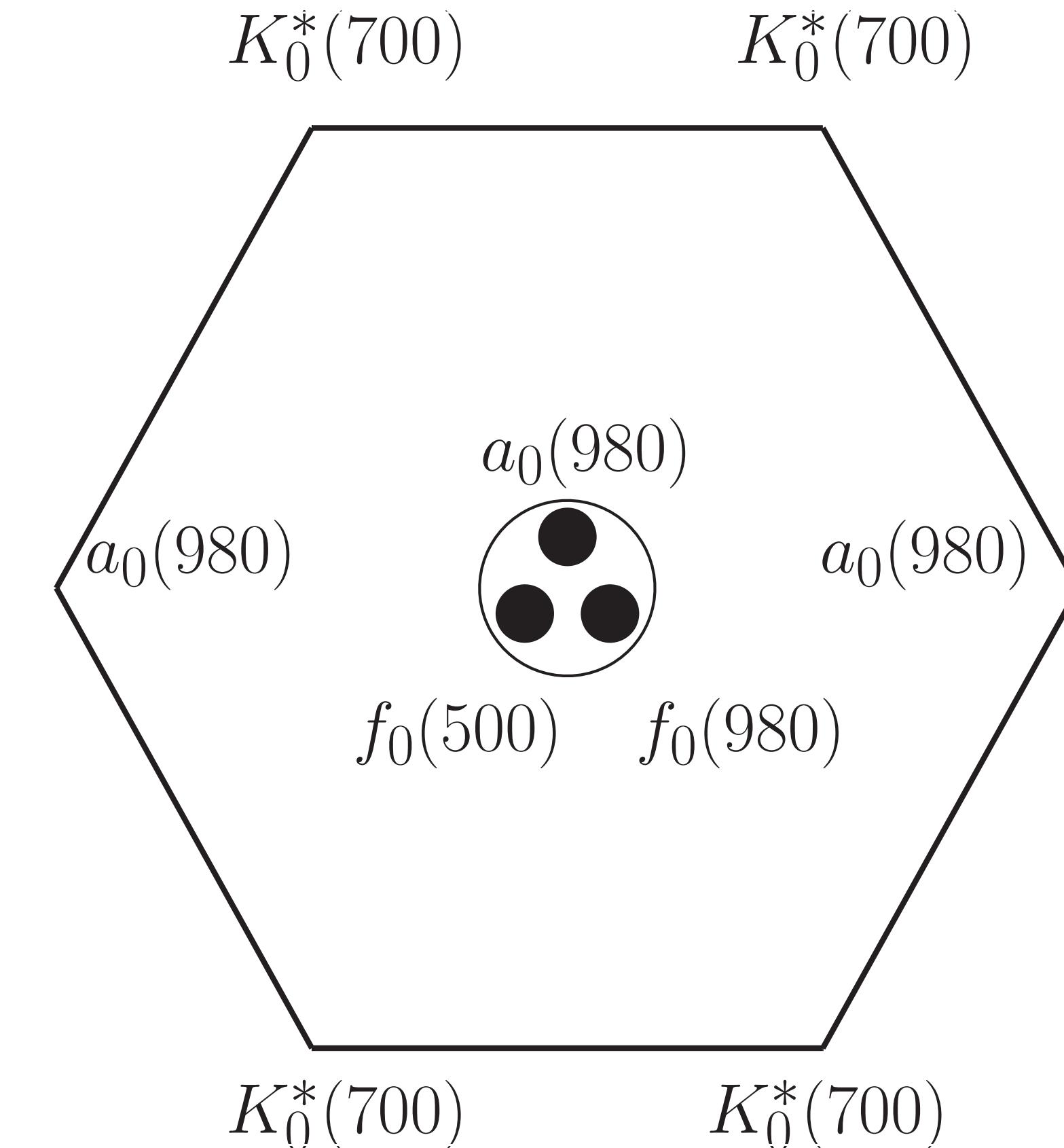
$$m_{f_0} \sim 980 \text{ MeV}$$

Light+strange

$$m_\kappa \sim 650 \text{ MeV}$$

Only strange

$$m_{a_0} \sim 980 \text{ MeV}$$



Multiplets work...

Sometimes