



UNIVERSITÀ DI PAVIA



Istituto Nazionale di Fisica Nucleare

# MAPTMD22:

**A new fit of unpolarized TMDs**

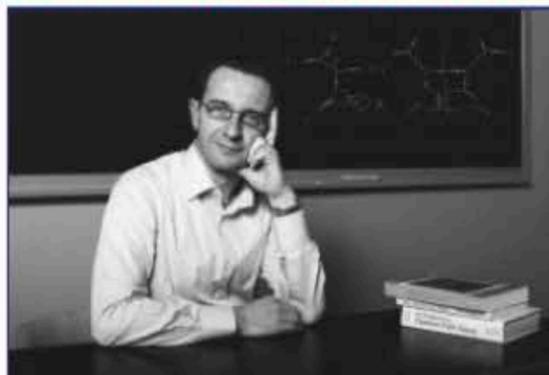
**arXiv: 2206.07598 [hep-ph]**

***MAP Collaboration***

***Matteo Cerutti***

# Results obtained with contribution from

**Alessandro Bacchetta**



**Marco Radici**



**Andrea Signori**



**Valerio Bertone**



**Chiara Bissoletti**



**Giuseppe Bozzi**



**Fulvio Piacenza**



# A new extraction of unpolarized quark TMDs

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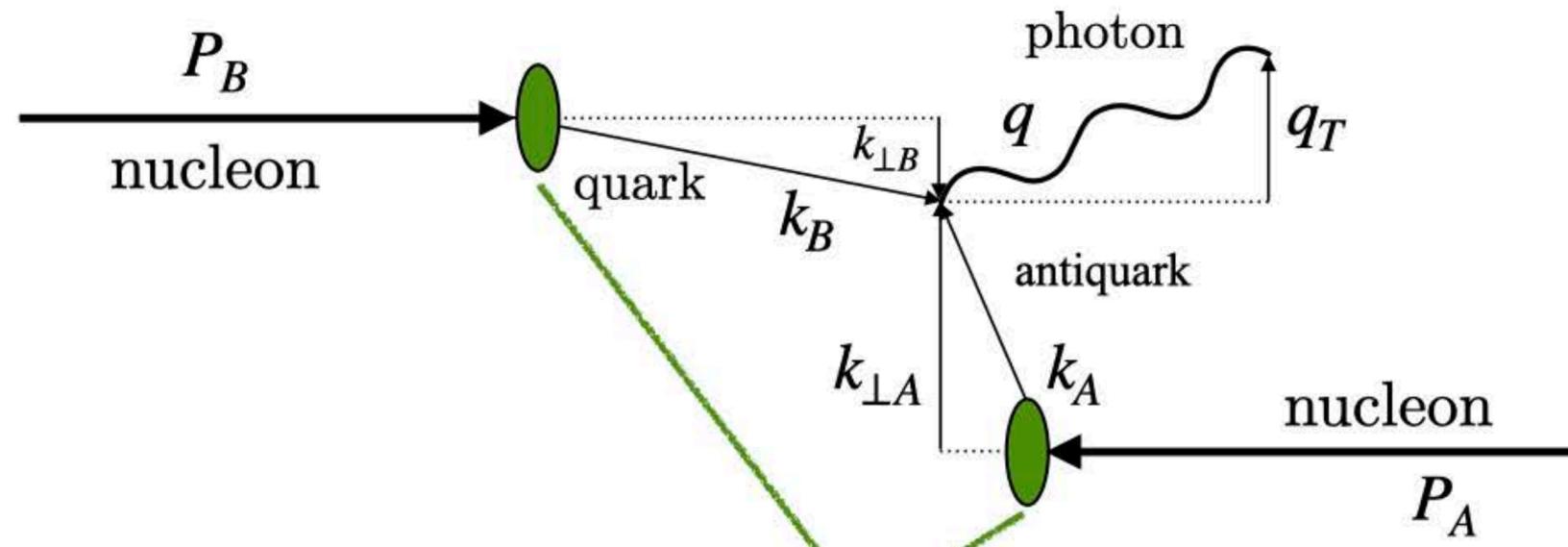
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- Number of fitted parameters: **21**

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy:  **$N^3LL^-$**
- ***Normalization*** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description:  **$\chi^2 / N_{data} = 1.06$**

# TMD factorization – Drell-Yan process



TMD PDF

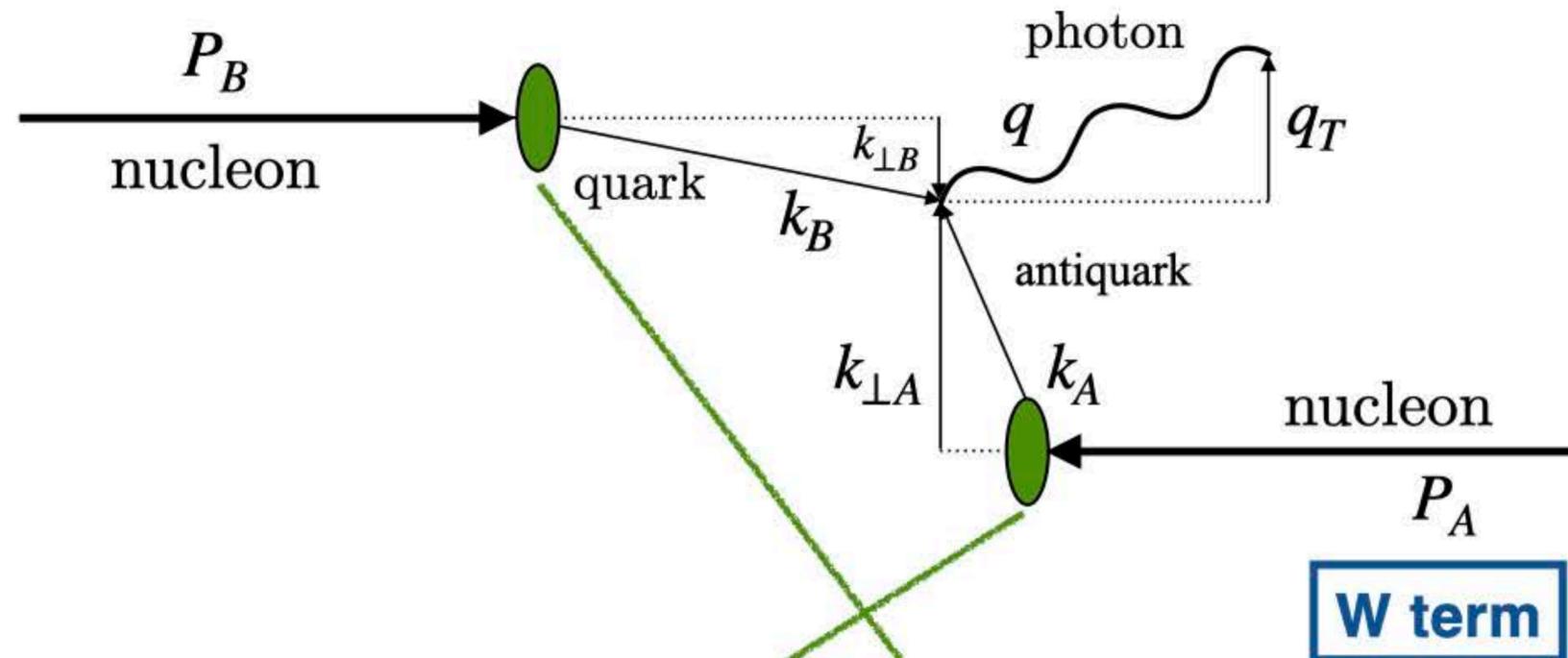
W term

$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) + Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Arnold, Metz and Schlegel, *Phys.Rev.D* 79 (2009)

# TMD factorization – Drell-Yan process



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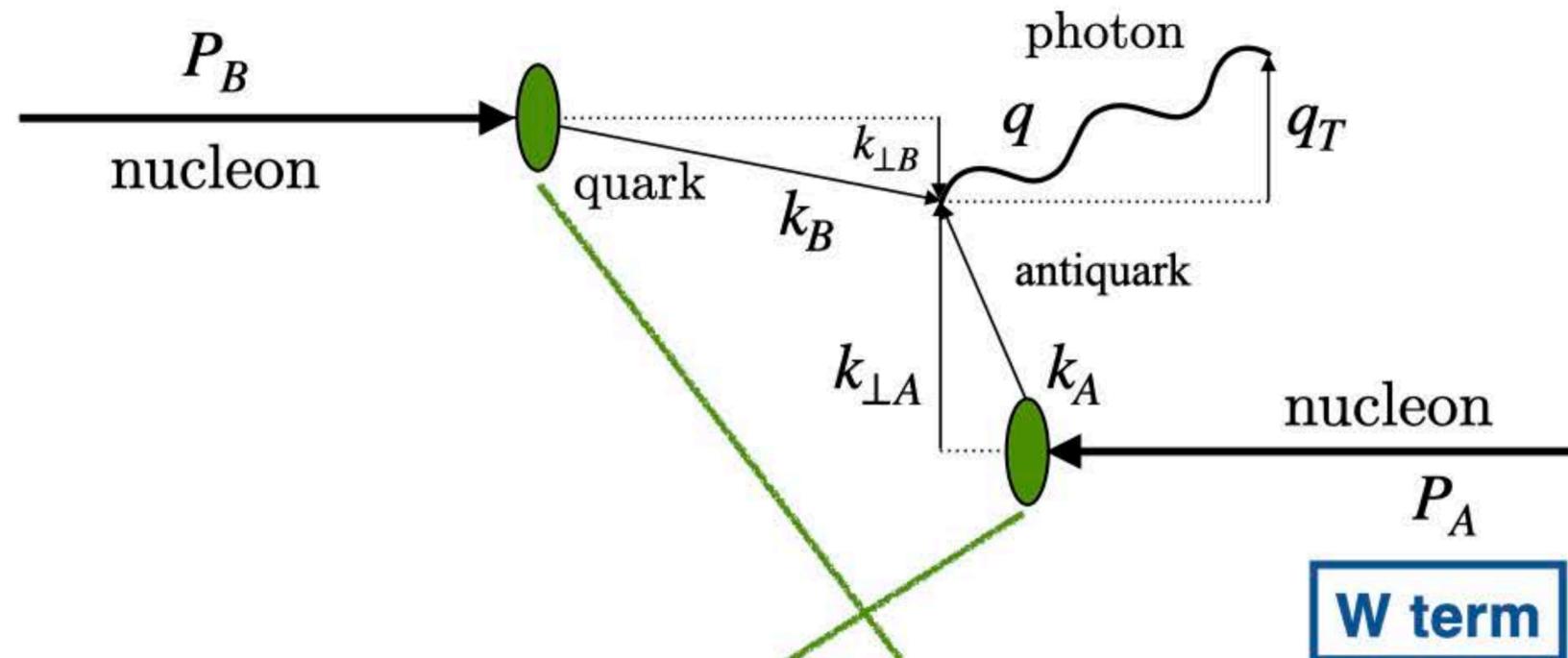
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- The **W term** dominates in the region where  $\mathbf{q}_T \ll Q$

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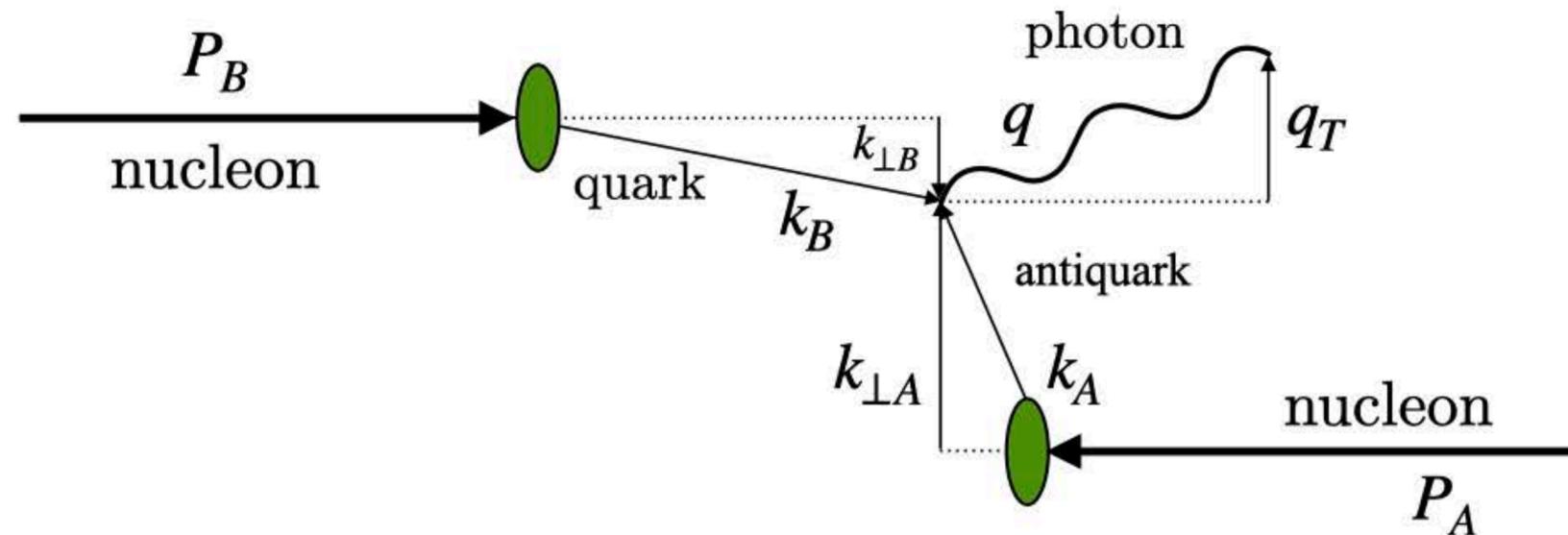
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- The **W term** dominates in the region where  $\mathbf{q}_T \ll Q$
- Y term has been excluded in the Pavia analyses

# TMD factorization — Drell-Yan process



**TMD PDF**

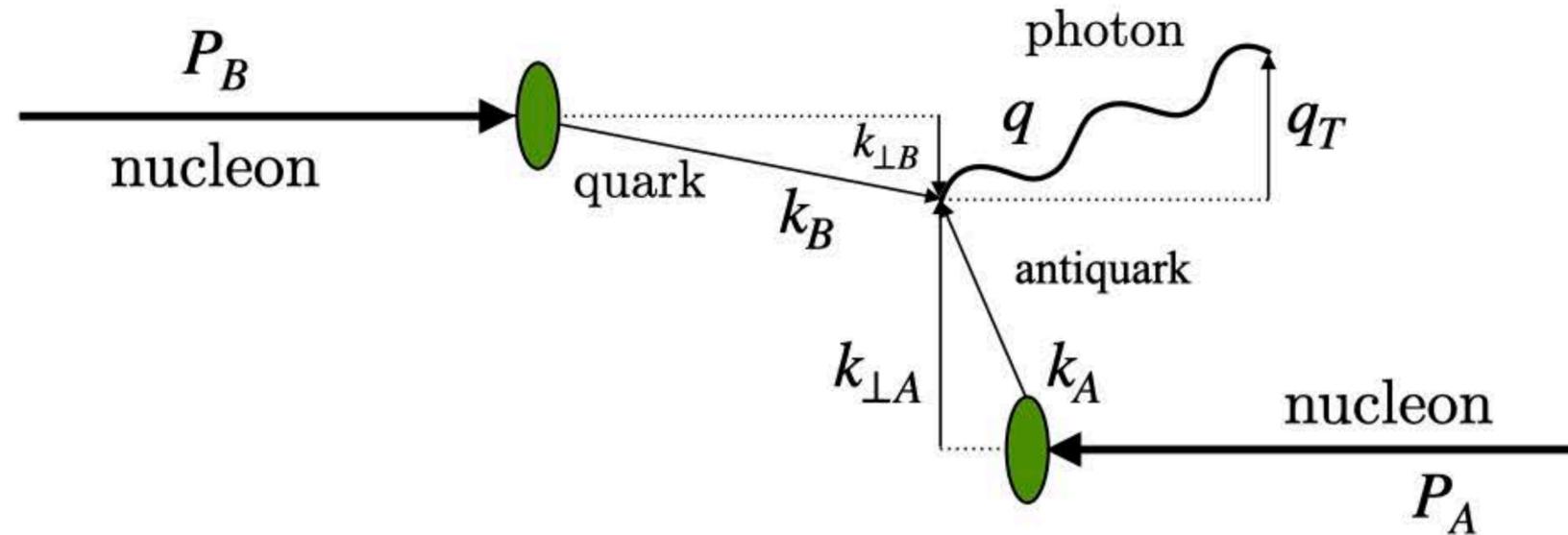
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

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**TMD PDF**

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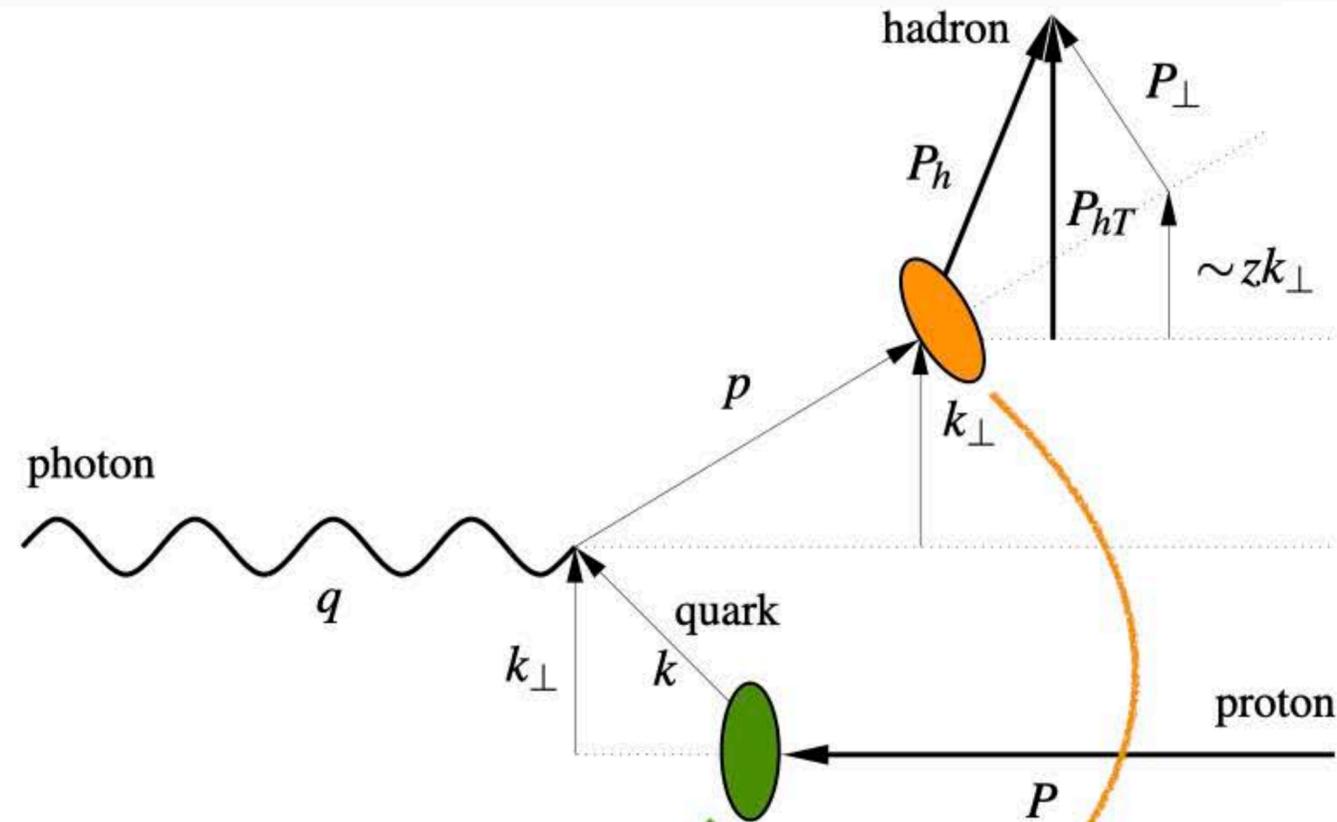
- Fourier-transformed space to avoid convolutions
- TMDs formally depend on two scales, but we set them equal

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta)$$

# TMD factorization – SIDIS process

TMD FF

TMD PDF



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

$$= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)$$

# MAPTMD22 – Datasets included

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## Drell-Yan

Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

$9 \lesssim Q \lesssim 11$  GeV excluded ( $\Upsilon$  resonance)

$$q_T|_{\max} = 0.2Q$$

**484 experimental points**

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**484 experimental points**

## SIDIS

HERMES data

COMPASS data

$$Q > 1.3 \text{ GeV}$$

$$0.2 < z < 0.7$$

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

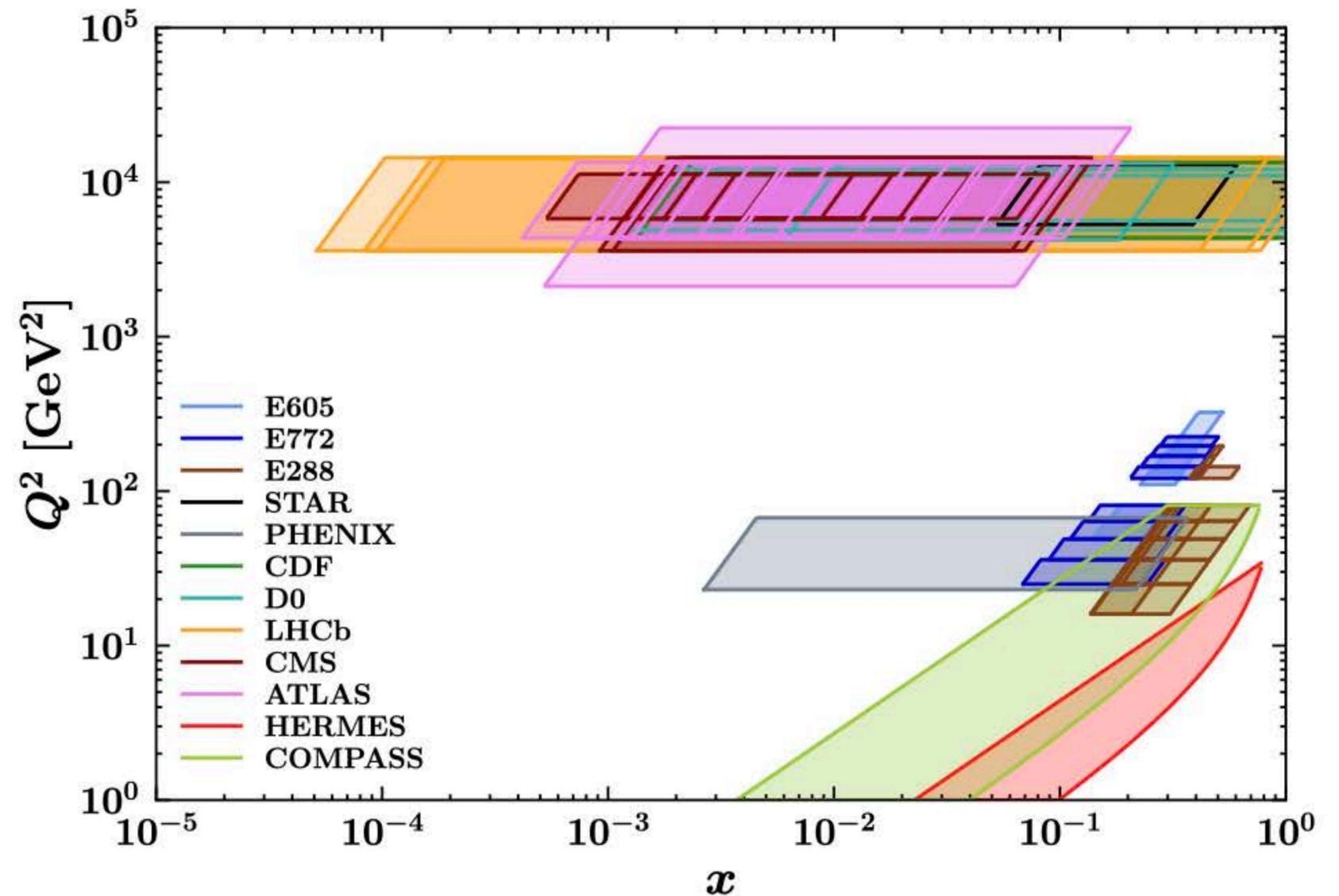
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# MAPTMD22 – Datasets included

484 experimental points

1547 experimental points

**Total: 2031 fitted experimental points**



# TMD factorization — expression of a TMD

Structure of a TMD in  $b_T$ -space

*Collins, "Foundations of Perturbative QCD"*

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu, \zeta)$$

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collinear PDF

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perturbative Sudakov  
form factor

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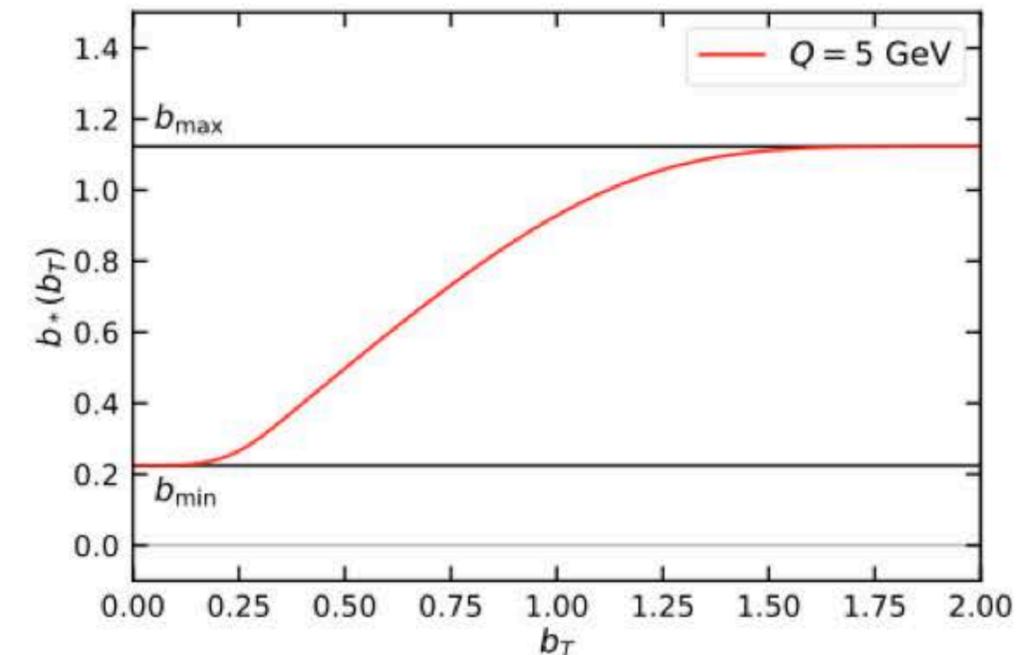
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$$b_*(b_T) = b_{\text{max}} \left( \frac{1 - \exp\left(-\frac{b_T^4}{b_{\text{max}}^4}\right)}{1 - \exp\left(-\frac{b_T^4}{b_{\text{min}}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = \frac{2e^{-\gamma_E}}{Q}$$



Bacchetta, Delcarro, Pisano, et al., JHEP 06 (2017)

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

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nonperturbative part  
of evolution

$$\hat{f}_1(x, b_T; \mu, \zeta) = \left[ \frac{\hat{f}_1(x, b_T; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T); \mu, \zeta) \equiv f_{\text{NP}}(x, b_T; \zeta) \hat{f}_1(x, b_*(b_T); \mu, \zeta)$$

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nonperturbative part  
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nonperturbative part  
of TMD

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# TMD factorization – Logarithmic counting

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Orders in powers of  $\alpha_S$

# TMD factorization – Logarithmic counting

Orders in powers of  $\alpha_S$

Accuracy	Hard factor and matching coefficient	Ingredients in perturbative Sudakov form factor		PDFs/FFs and $\alpha_S$ evol.
	$H$ and $C$	$K$ and $\gamma_F$	$\gamma_K$	
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N <sup>3</sup> LL <sup>-</sup>	2	3	4	NNLO (NLO FF)
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# TMD factorization – Logarithmic counting

Orders in powers of  $\alpha_S$

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NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
$N^3LL^-$	2	3	4	NNLO (NLO FF)
$N^3LL$	2	3	4	NNLO

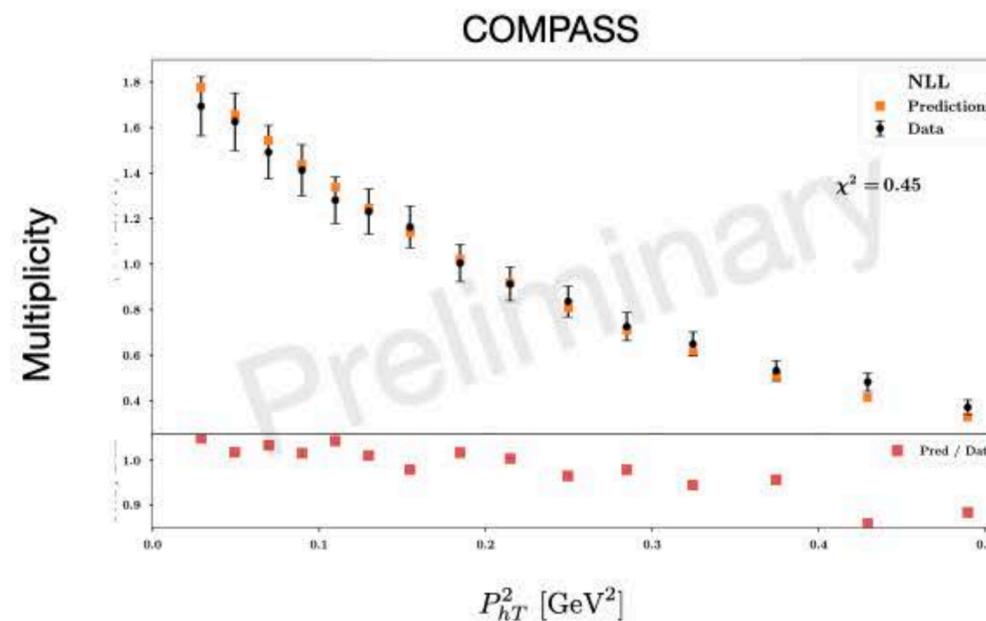
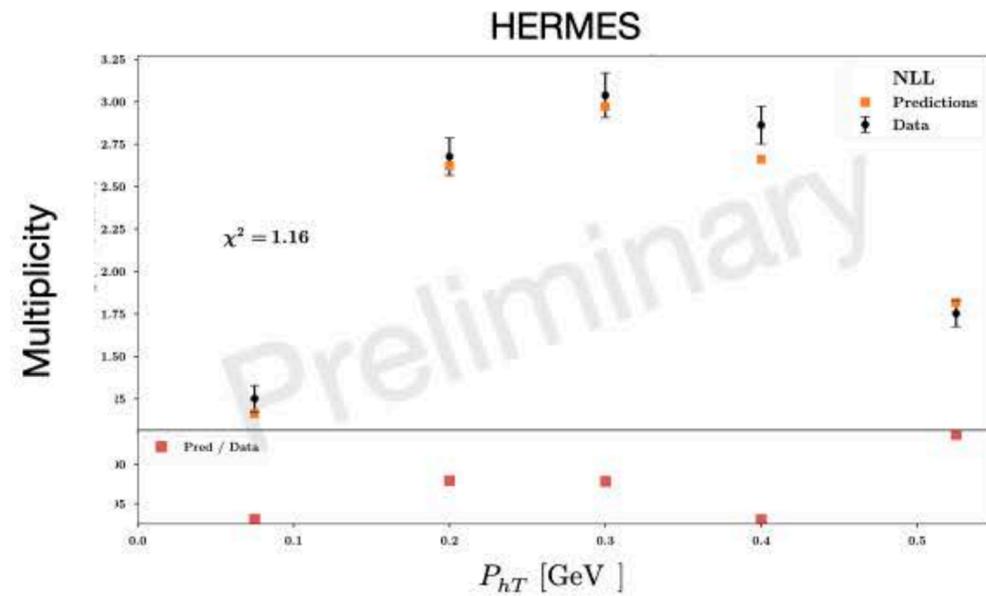
**$N^3LL^- = N^3LL$  but with NLO collinear FF**

# Available Global fits

	Accuracy	SIDIS	DY	Z production	N of points	$\chi^2/N_{\text{data}}$
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N <sup>3</sup> LL <sup>-</sup>	✓	✓	✓	1039	1.06
<b>MAPTMD22</b>	N <sup>3</sup> LL <sup>-</sup>	✓	✓	✓	<b>2031</b>	<b>1.06</b>

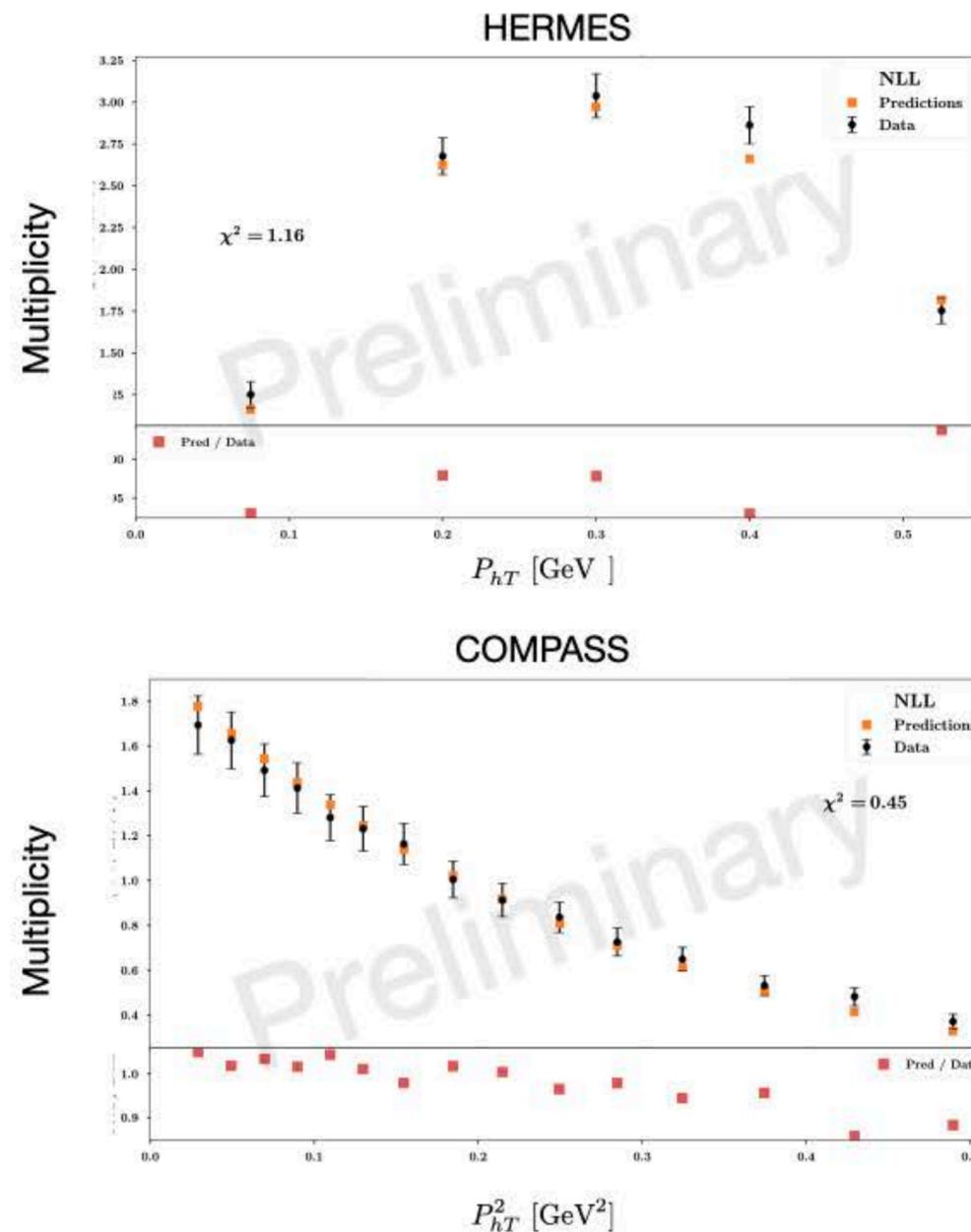
# MAPTMD22 – Normalization of SIDIS

## SIDIS multiplicities at NLL

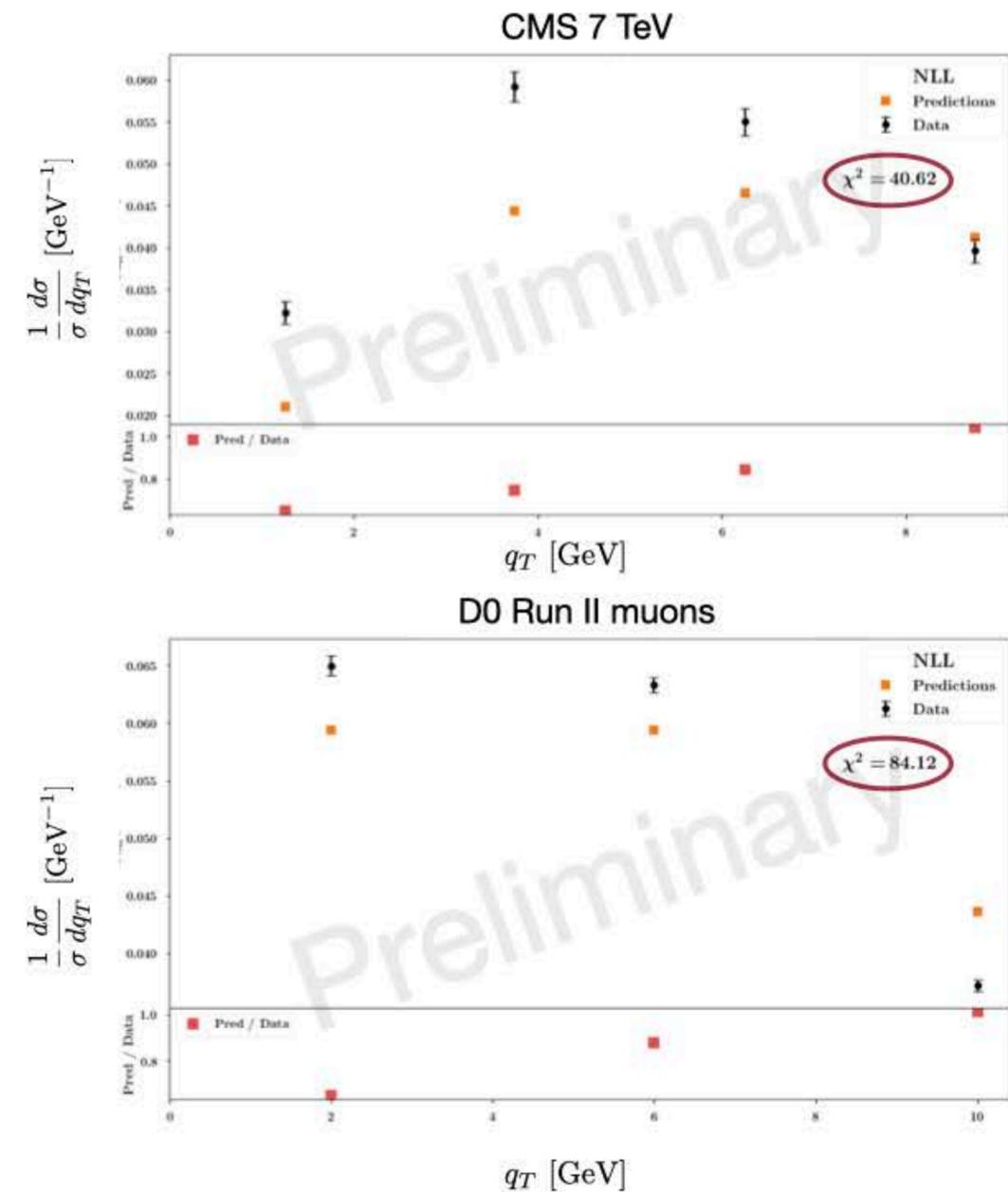


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## SIDIS multiplicities at NLL



## High-Energy Drell-Yan at NLL



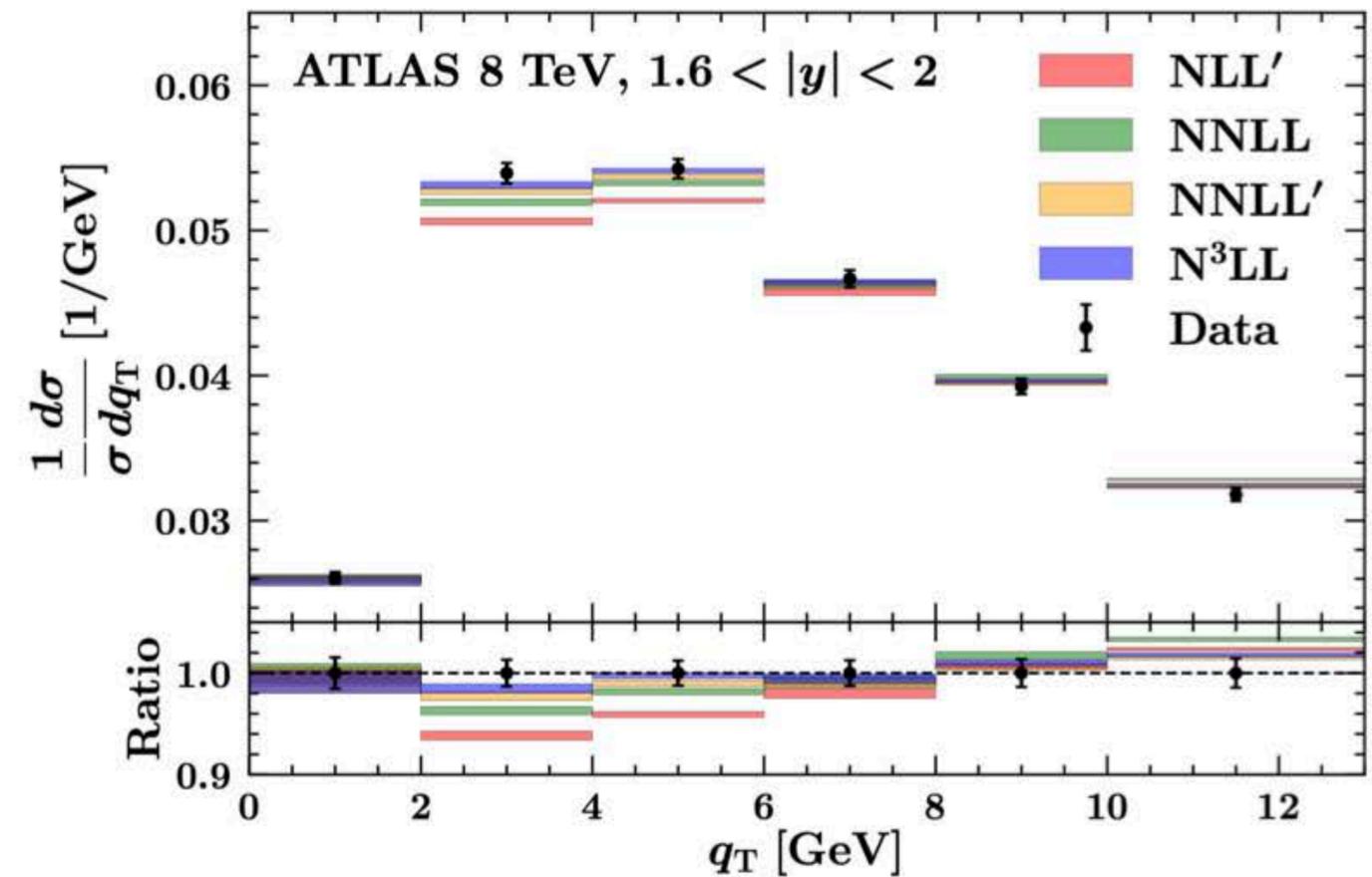
# MAPTMD22 – Normalization of SIDIS

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# MAPTMD22 – Normalization of SIDIS

## High-Energy Drell-Yan beyond NLL

$Q \sim 100$  GeV



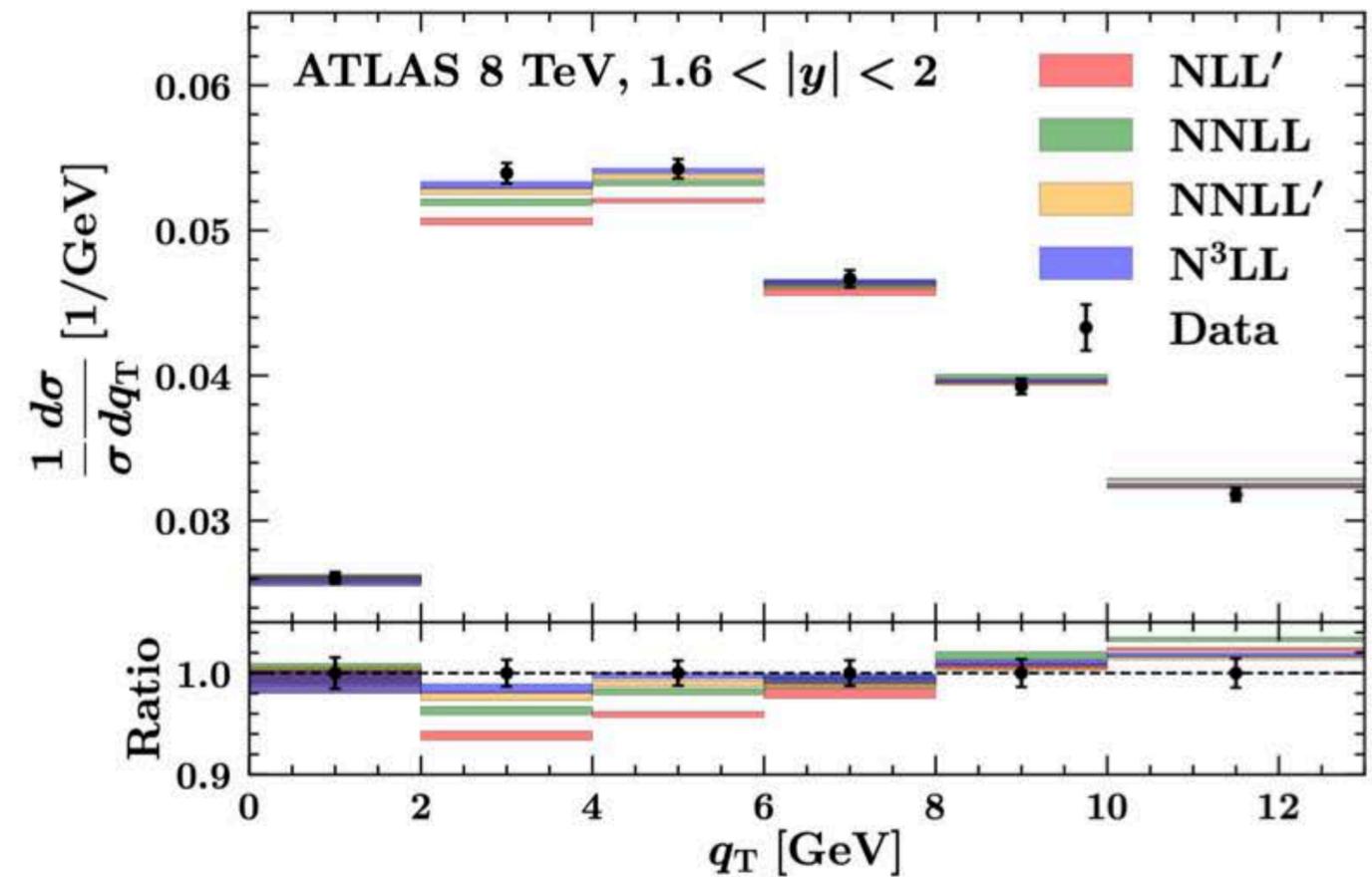
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, [arXiv:1912.07550](https://arxiv.org/abs/1912.07550)

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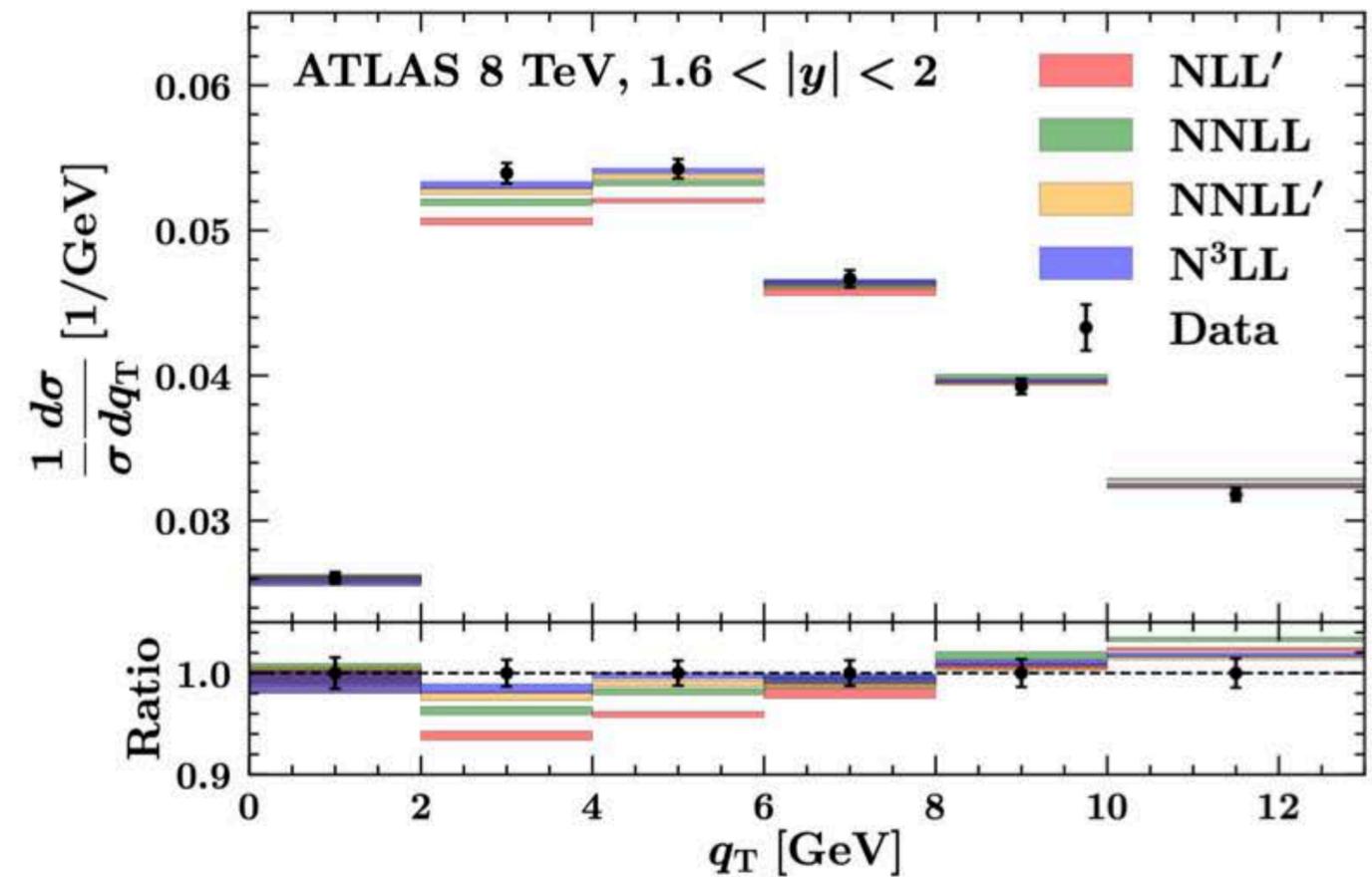
# MAPTMD22 – Normalization of SIDIS

## SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$

## High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$

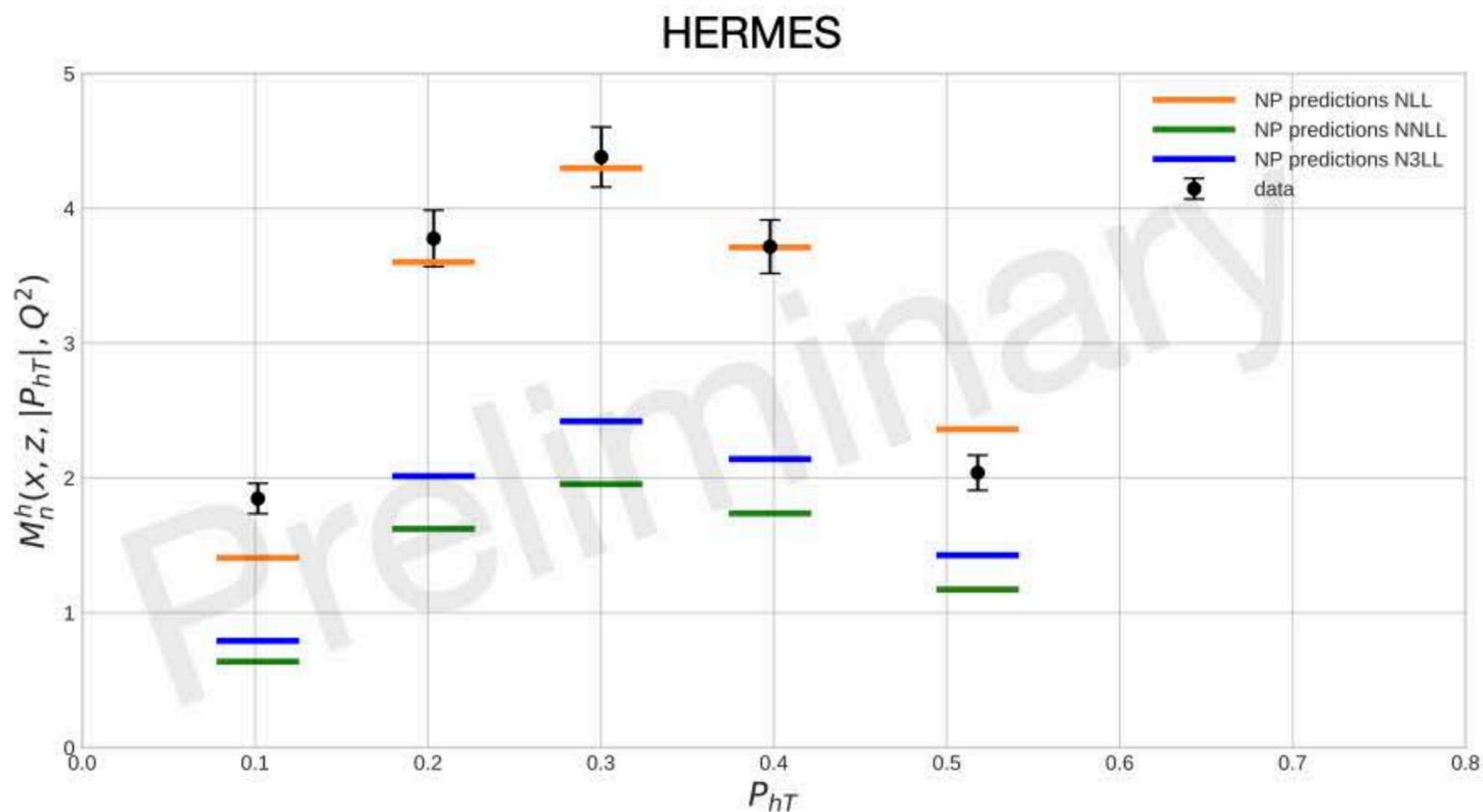


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, [arXiv:1912.07550](https://arxiv.org/abs/1912.07550)

# MAPTMD22 – Normalization of SIDIS

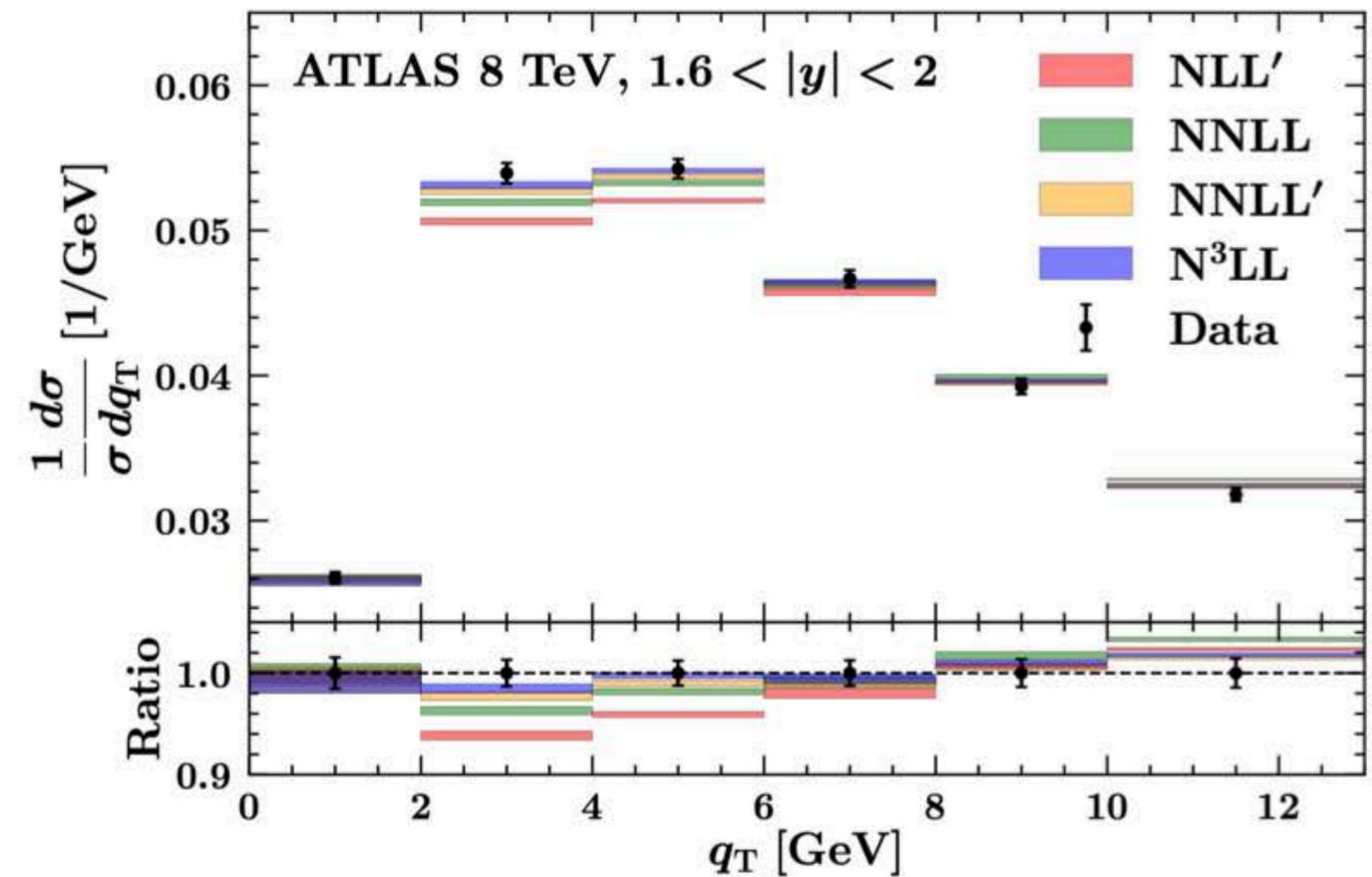
## SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$



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$Q \sim 100 \text{ GeV}$

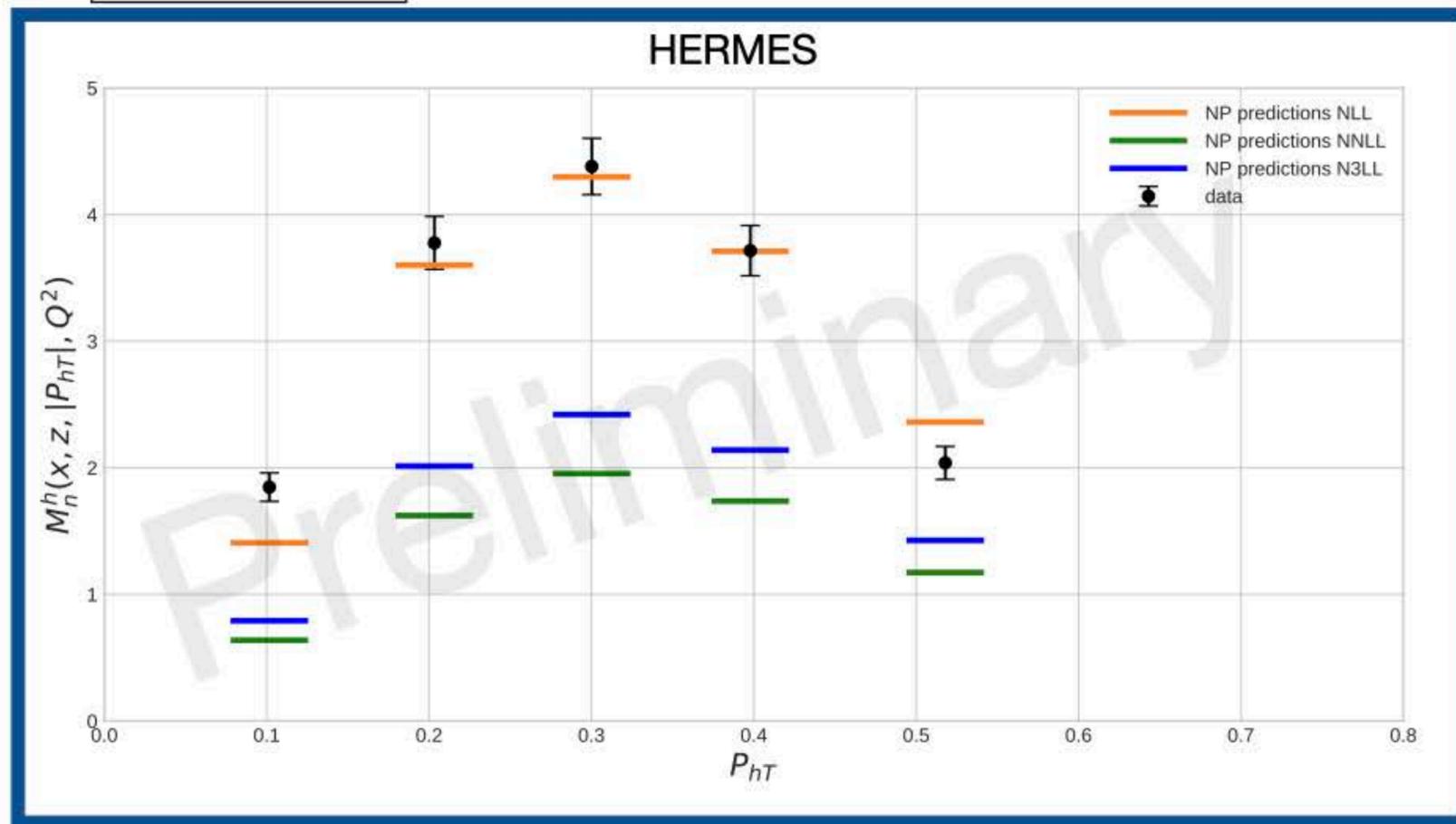


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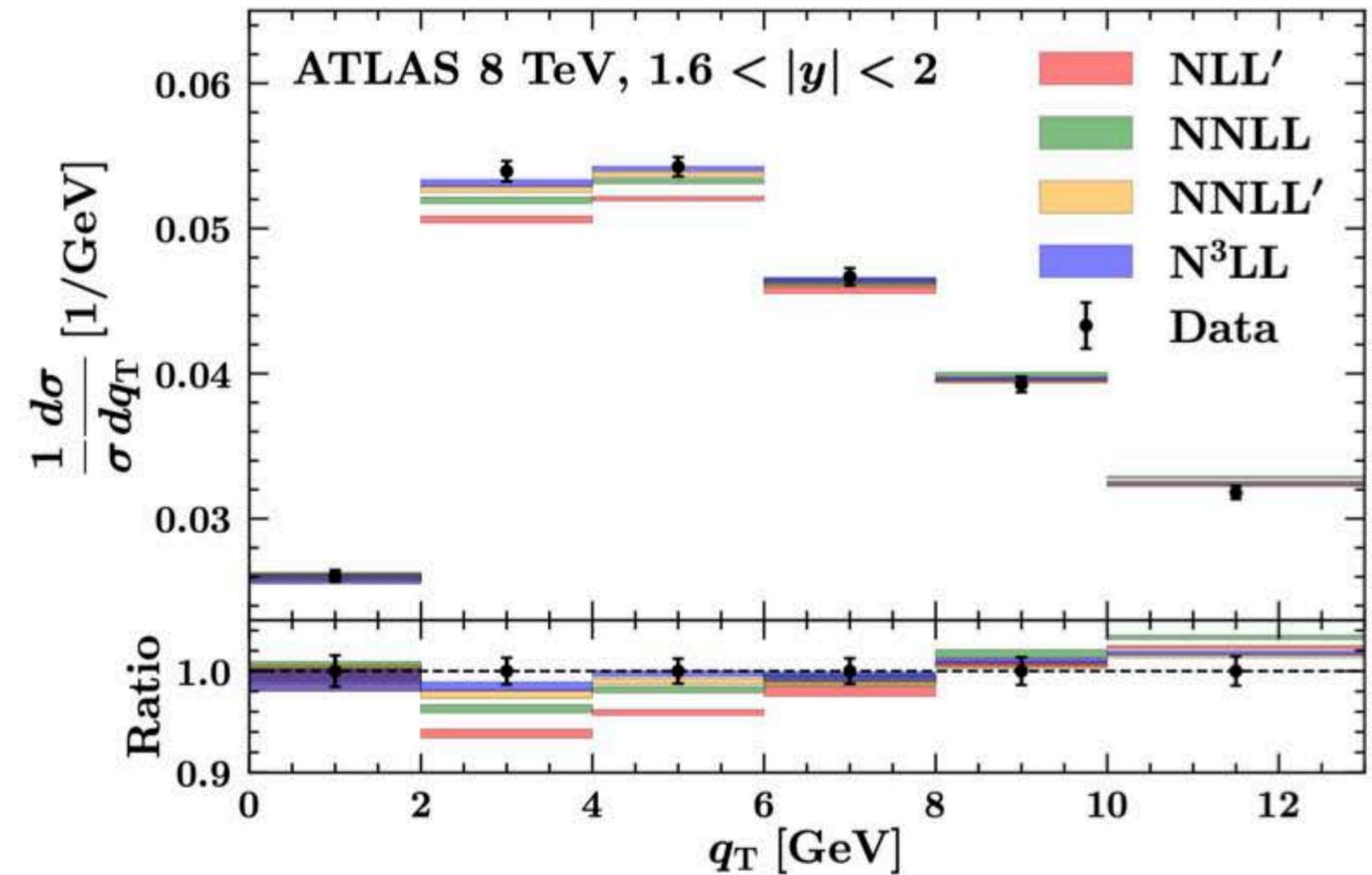
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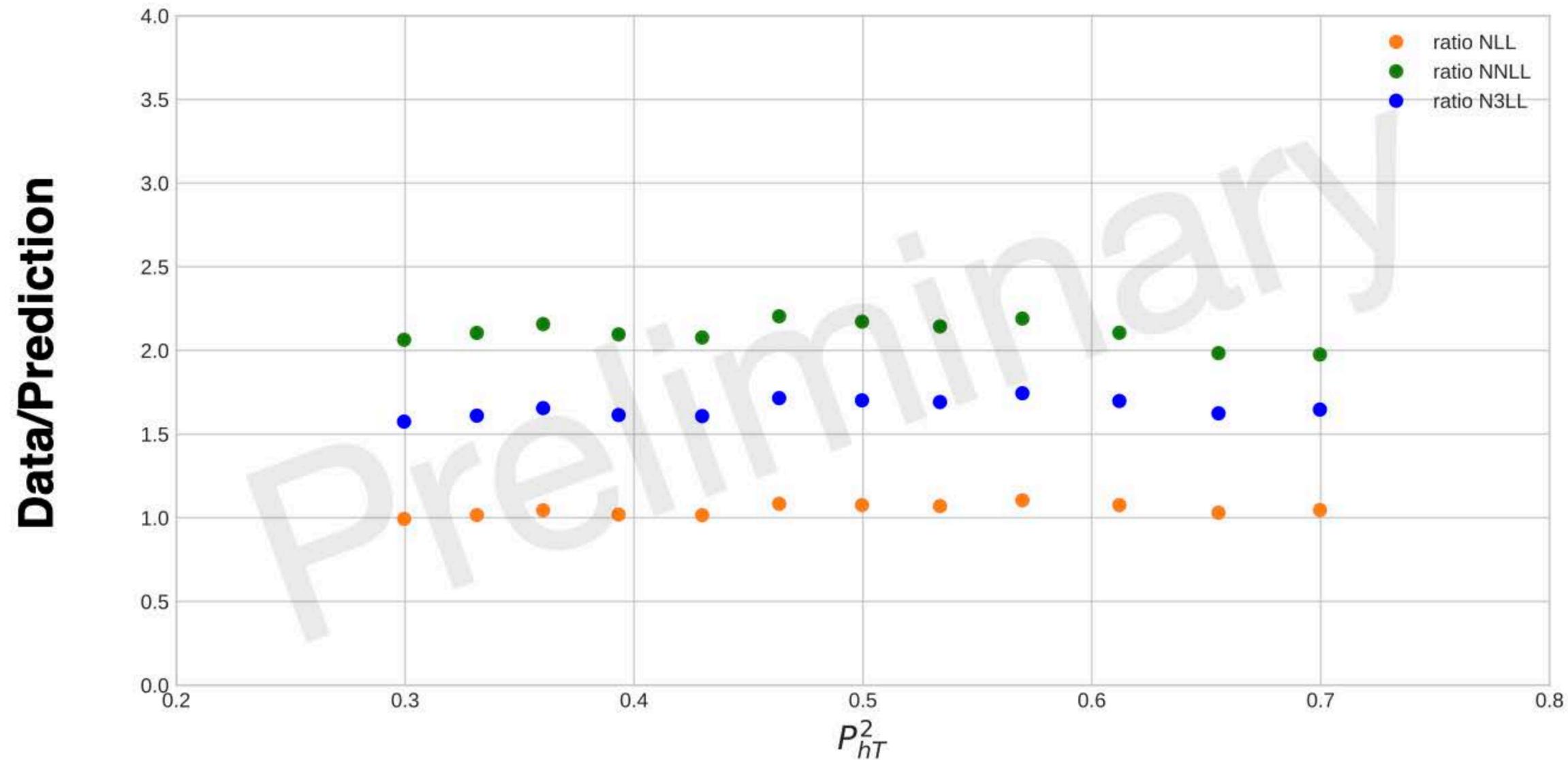
The description considerably worsens at higher orders!!

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, [arXiv:1912.07550](https://arxiv.org/abs/1912.07550)

# MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

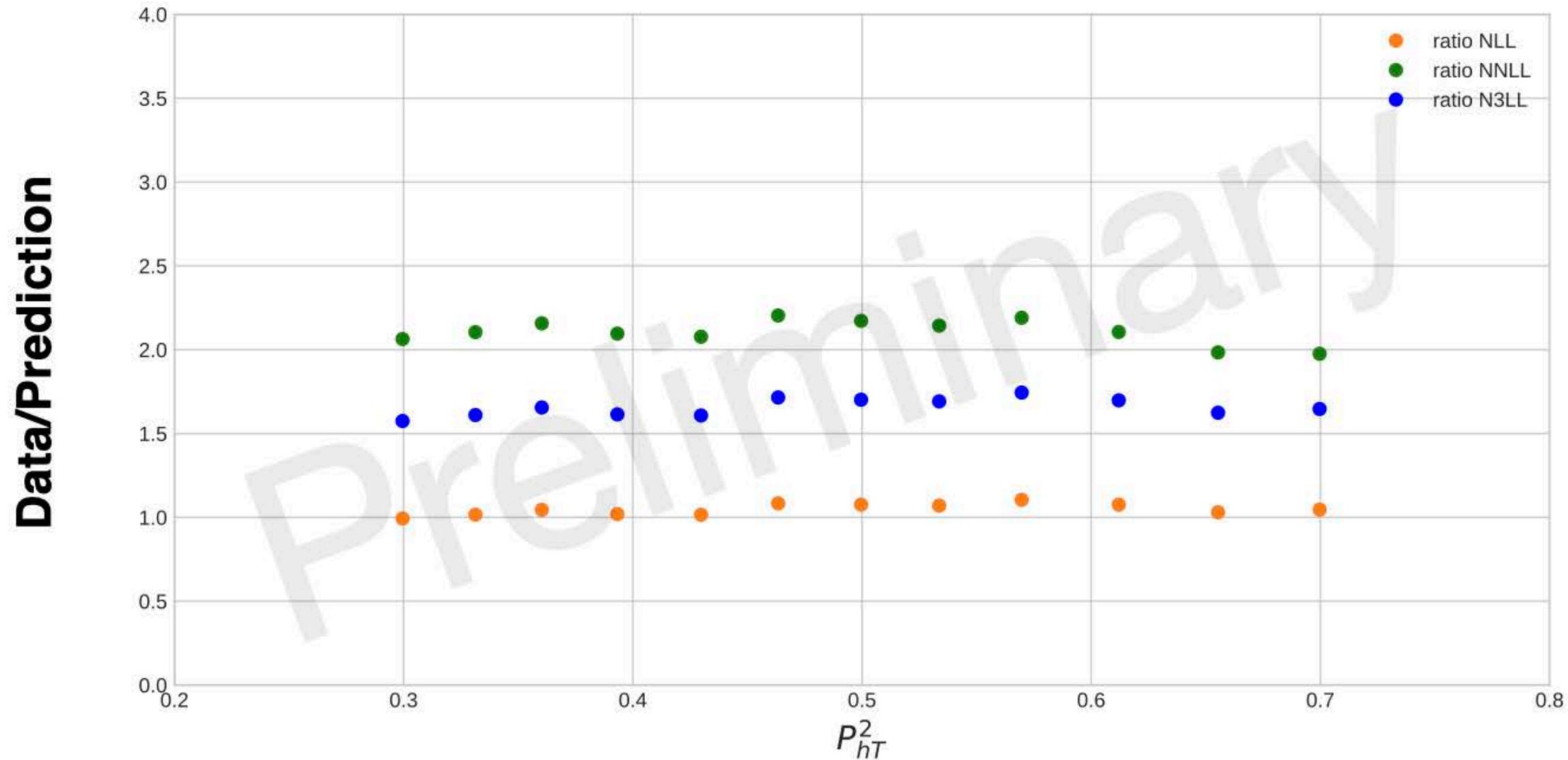
*J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176*



# MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

*J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176*



***The discrepancy amounts to an almost constant factor!!***

# MAPTMD22 – Normalization of SIDIS

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# MAPTMD22 – Normalization of SIDIS

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SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$$

# MAPTMD22 – Normalization of SIDIS

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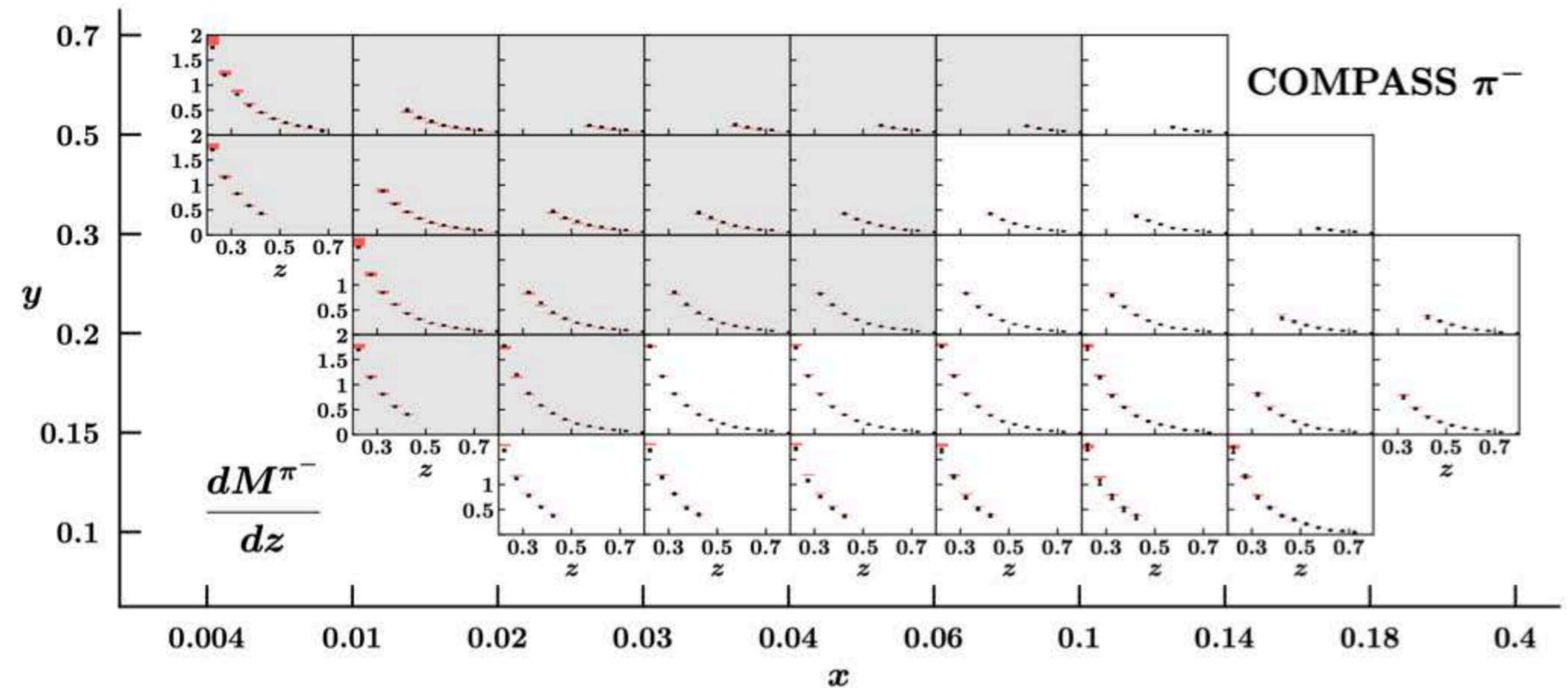
SIDIS multiplicity  $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} / \frac{d\sigma}{dx dQ}$

Collinear SIDIS cross section  $\frac{d\sigma}{dx dQ dz}$

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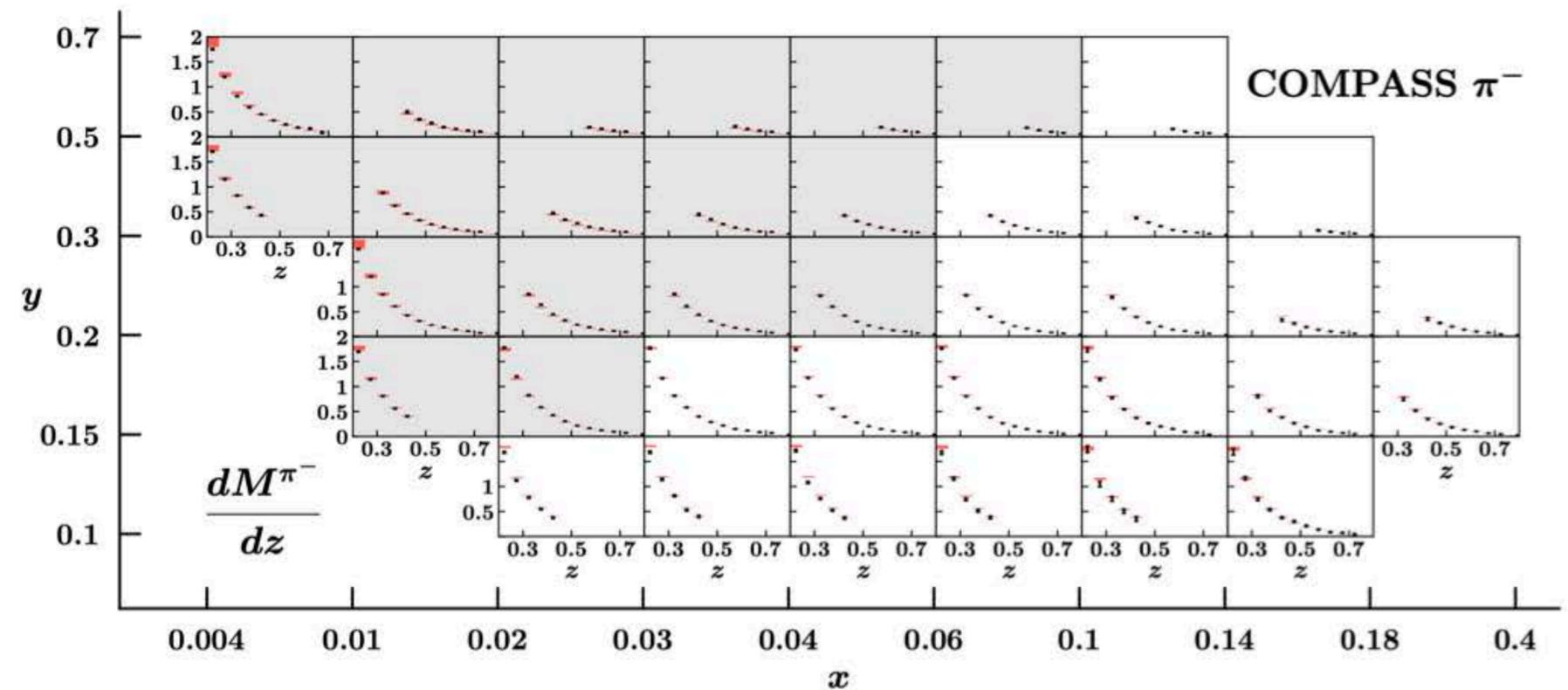
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Normalization of prediction such that

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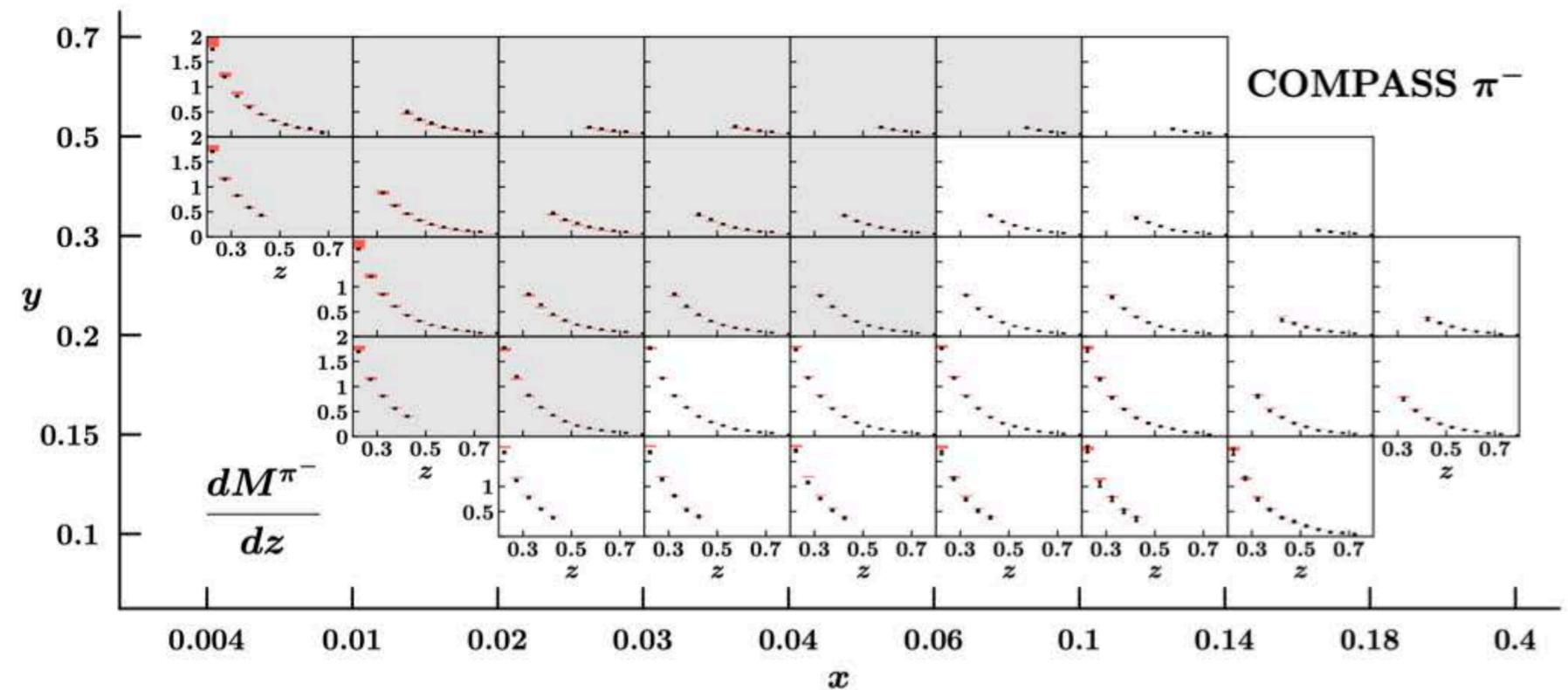
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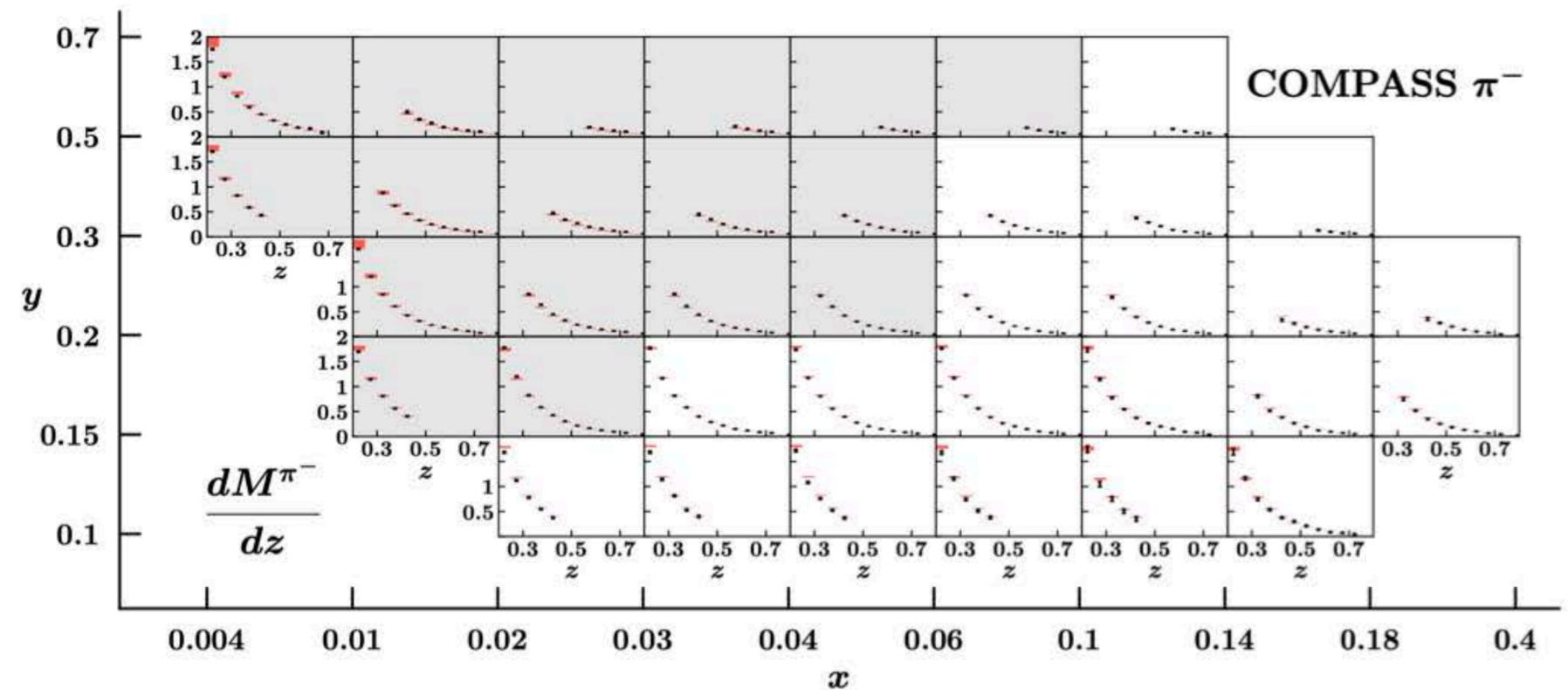
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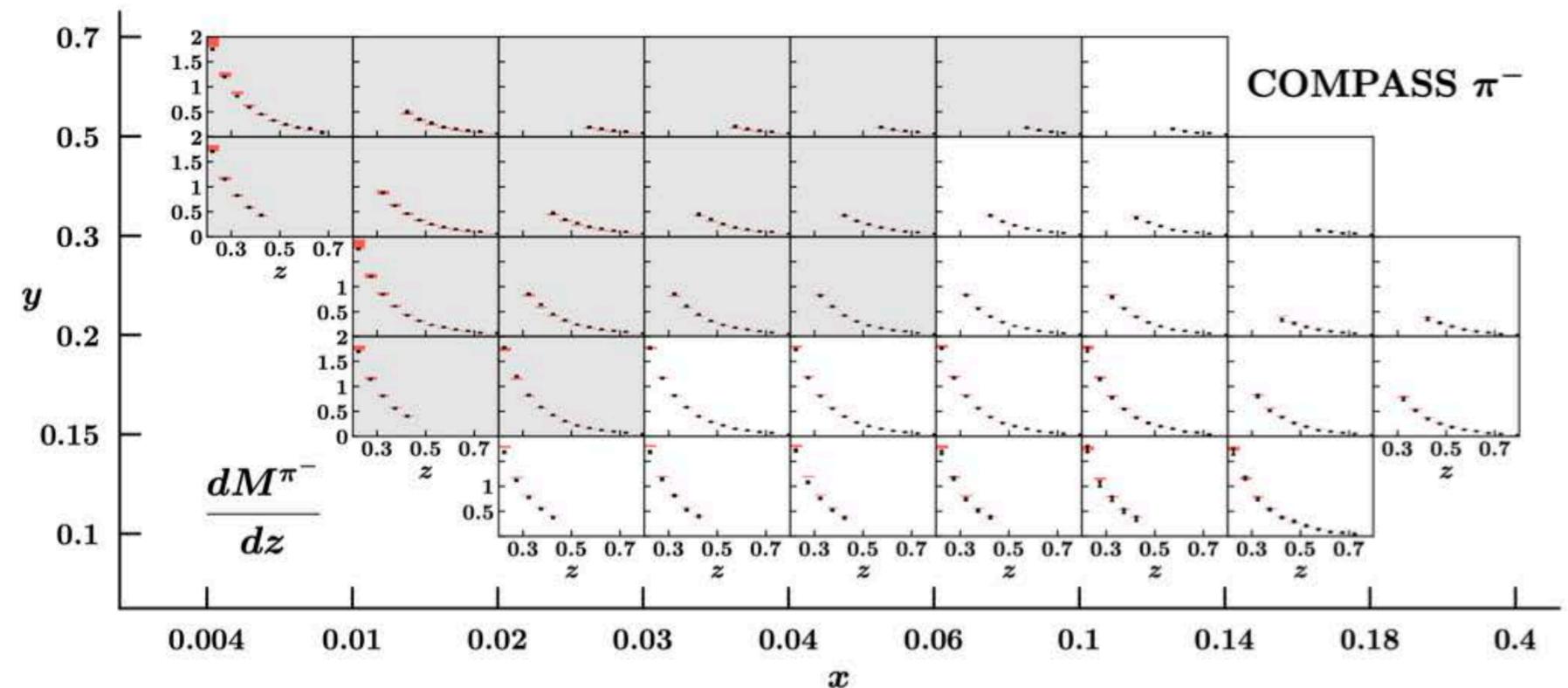
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**Independent of the fitting parameters!!**



# MAPTMD22 — Parameterization of TMDs

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$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left( e^{-\frac{k_{\perp}^2}{g_{1A}}} + \lambda_B k_{\perp}^2 e^{-\frac{k_{\perp}^2}{g_{1B}}} + \lambda_C e^{-\frac{k_{\perp}^2}{g_{1C}}} \right)$$

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$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

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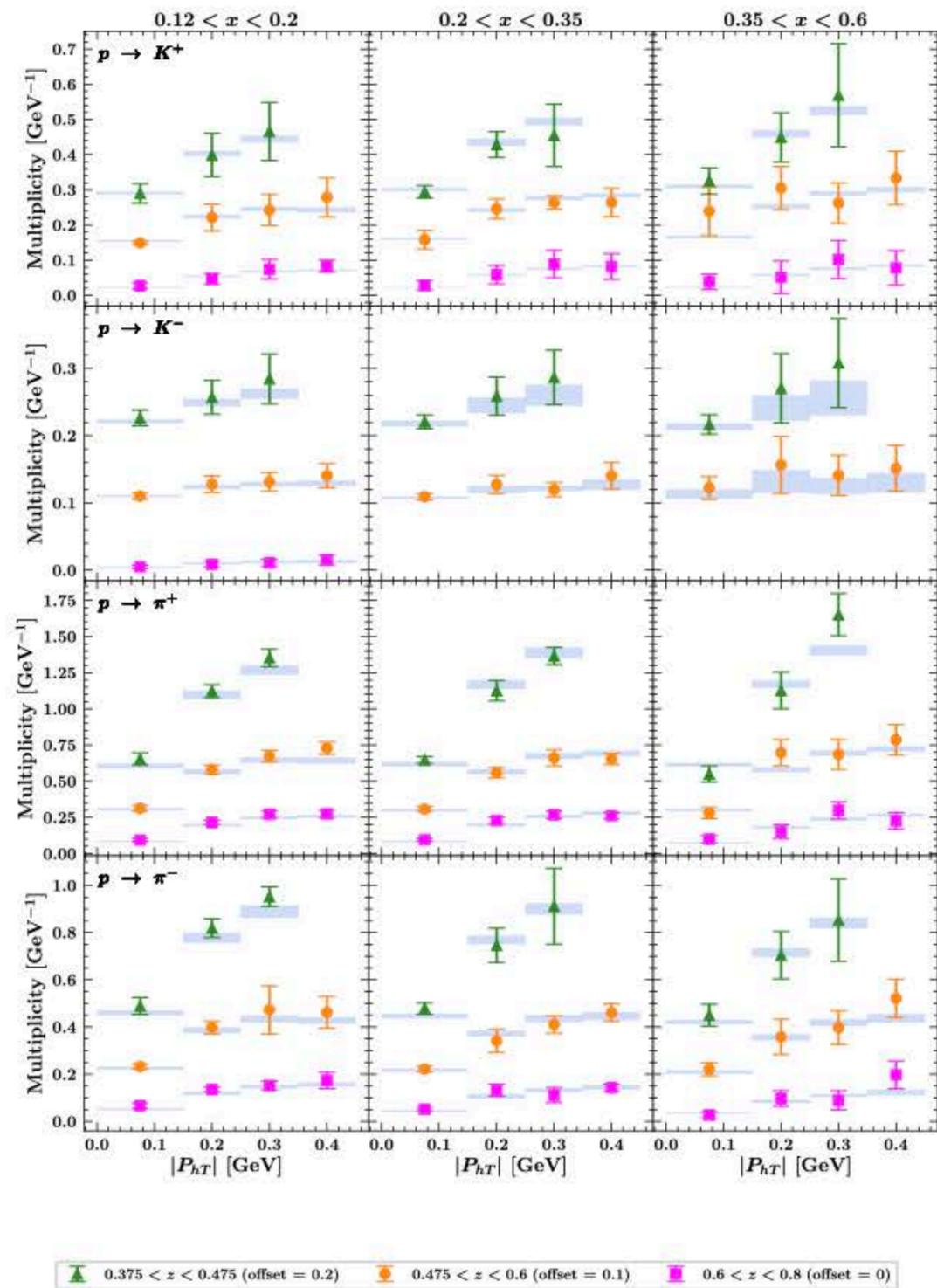
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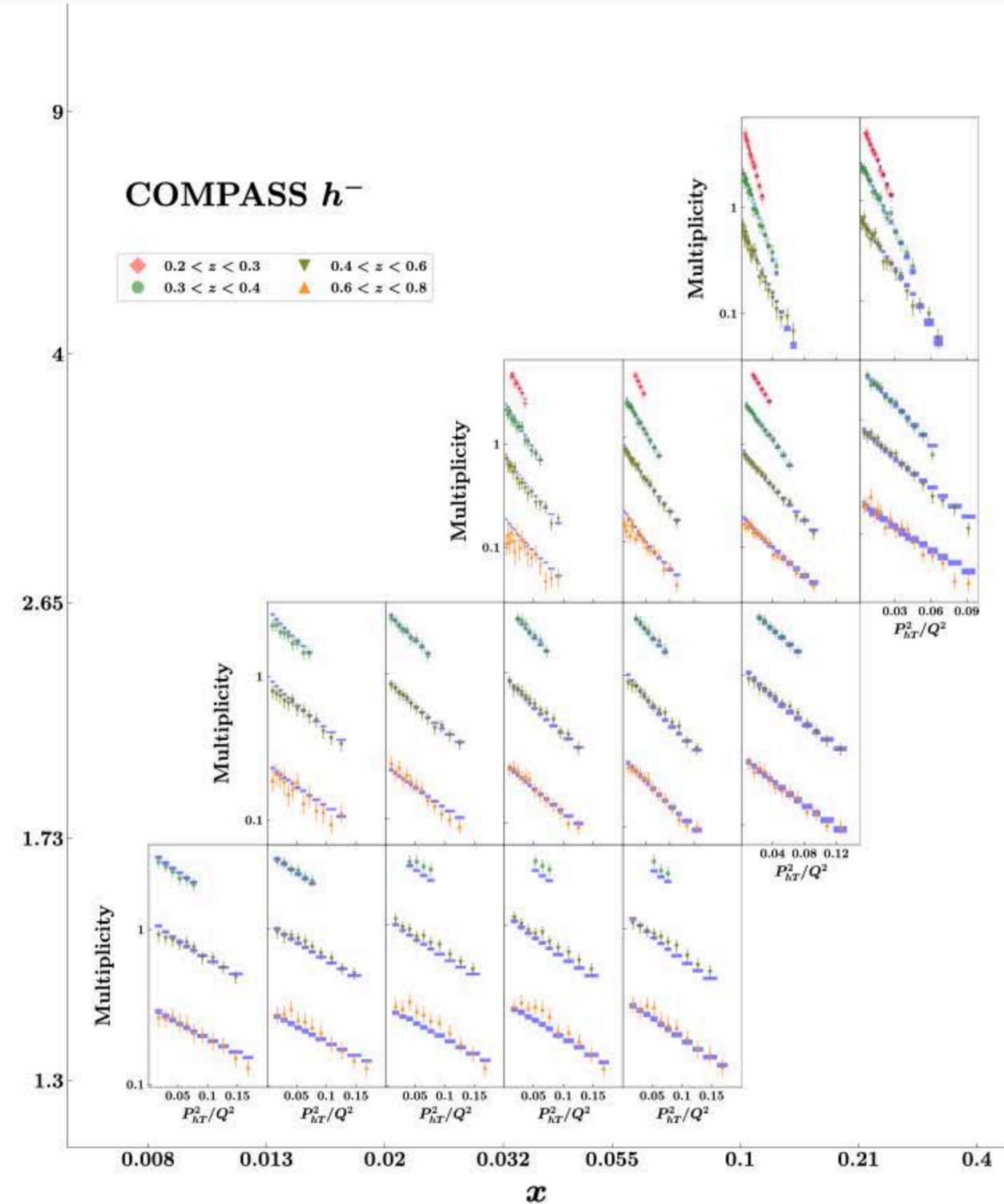
**11 parameters for TMD PDF  
+ 1 for NP evolution + 9 for TMD FF  
= 21 free parameters**

# MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$

HERMES

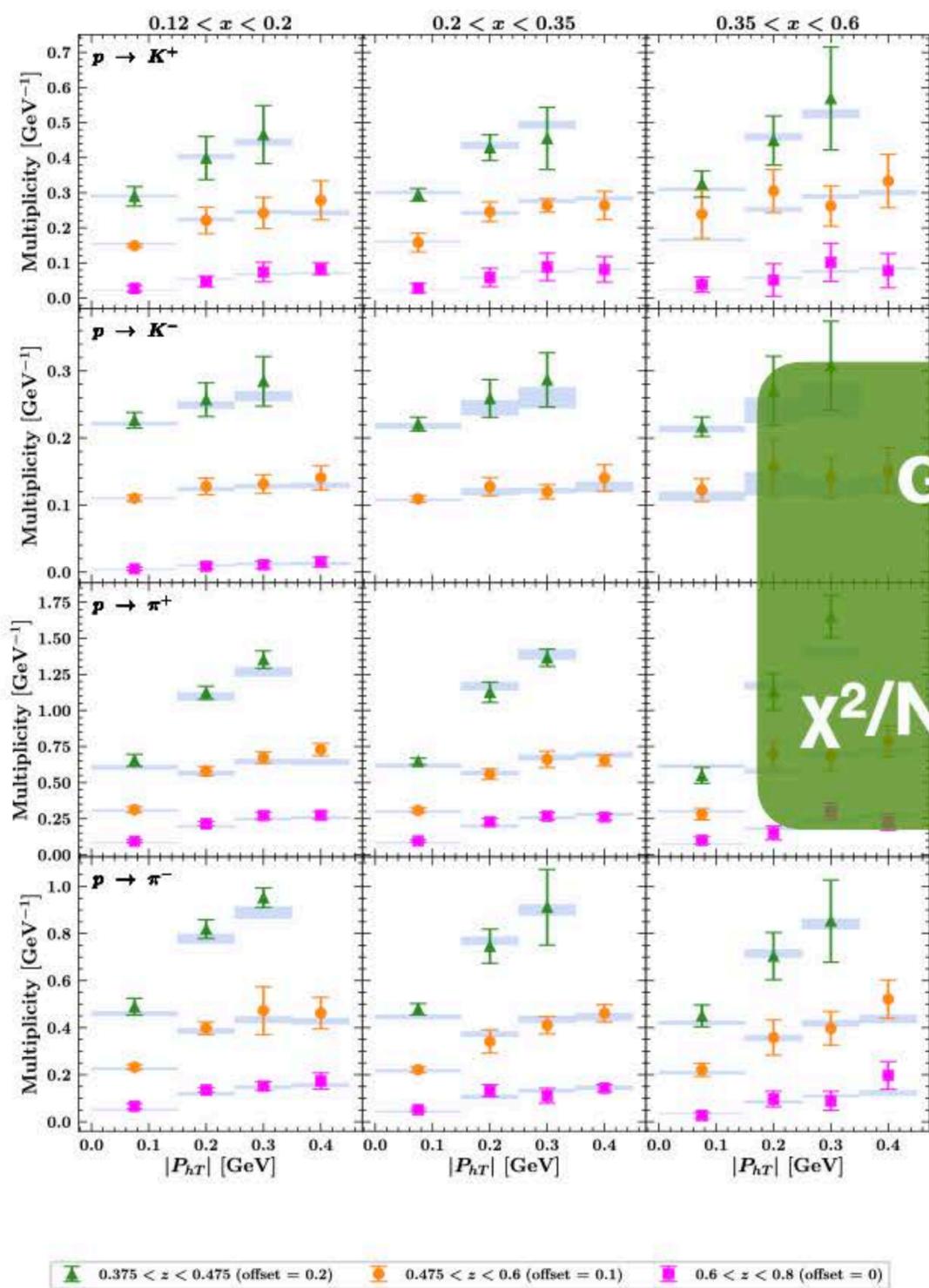


$Q$  [GeV]



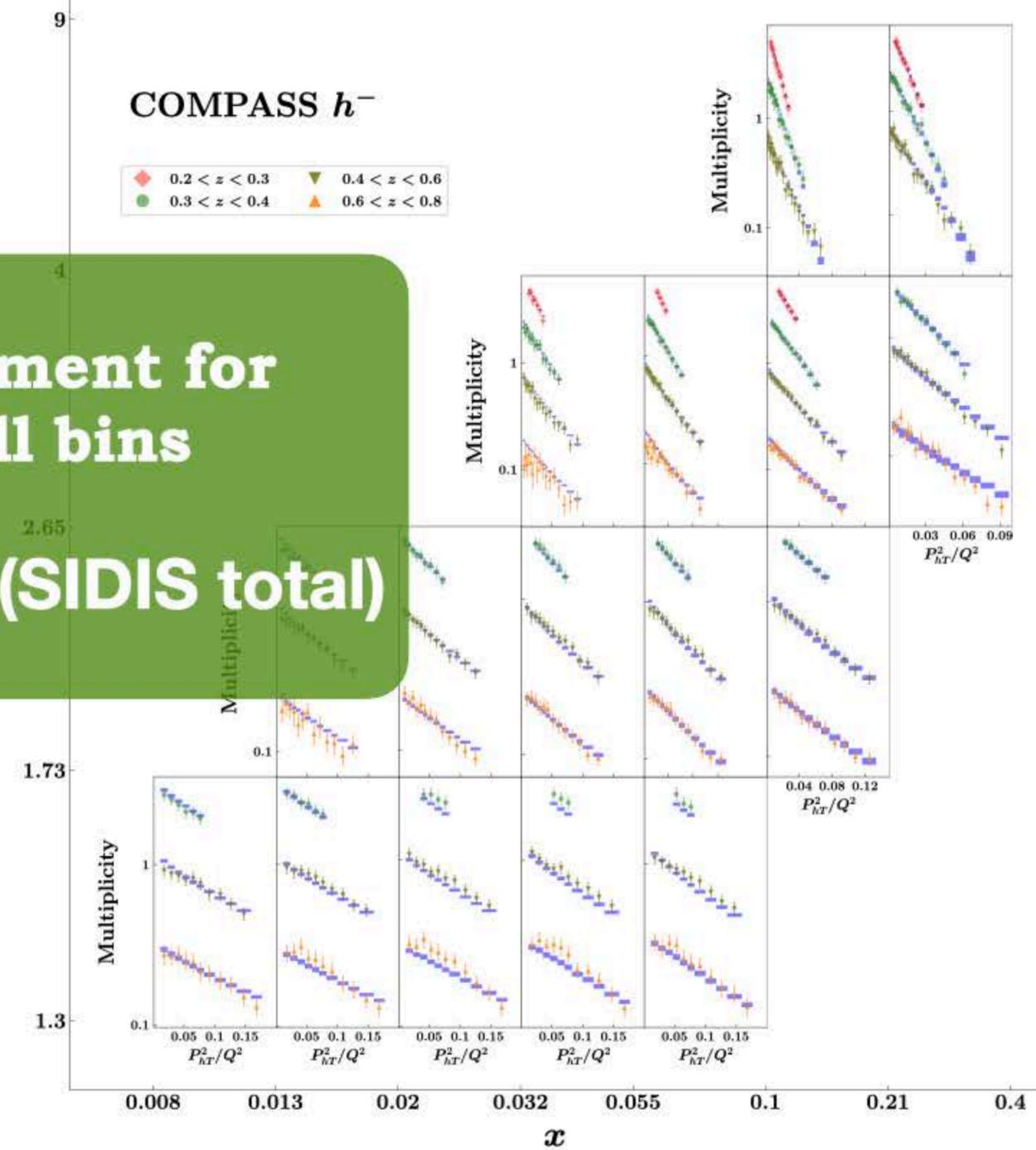
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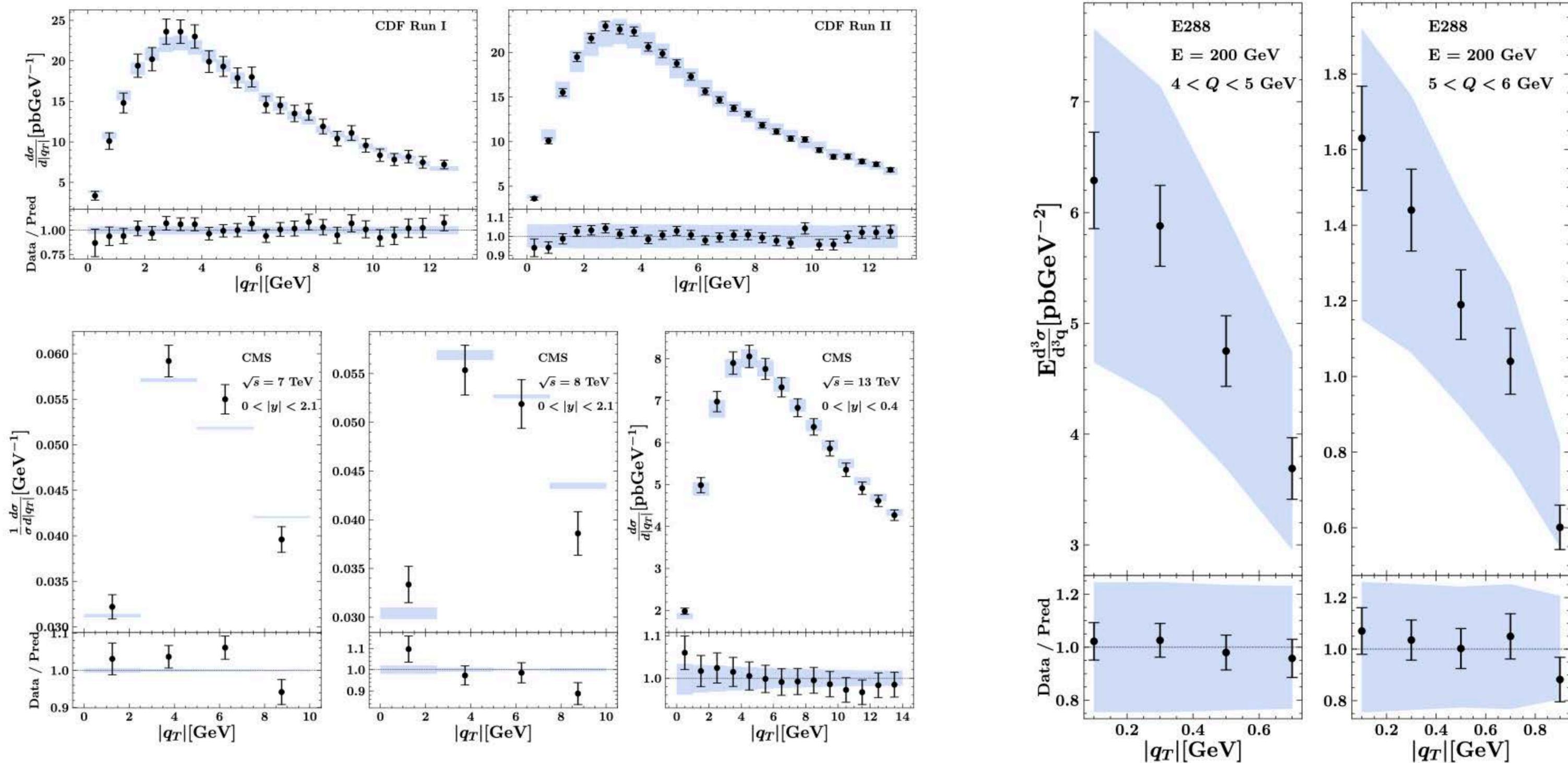


Good agreement for almost all bins

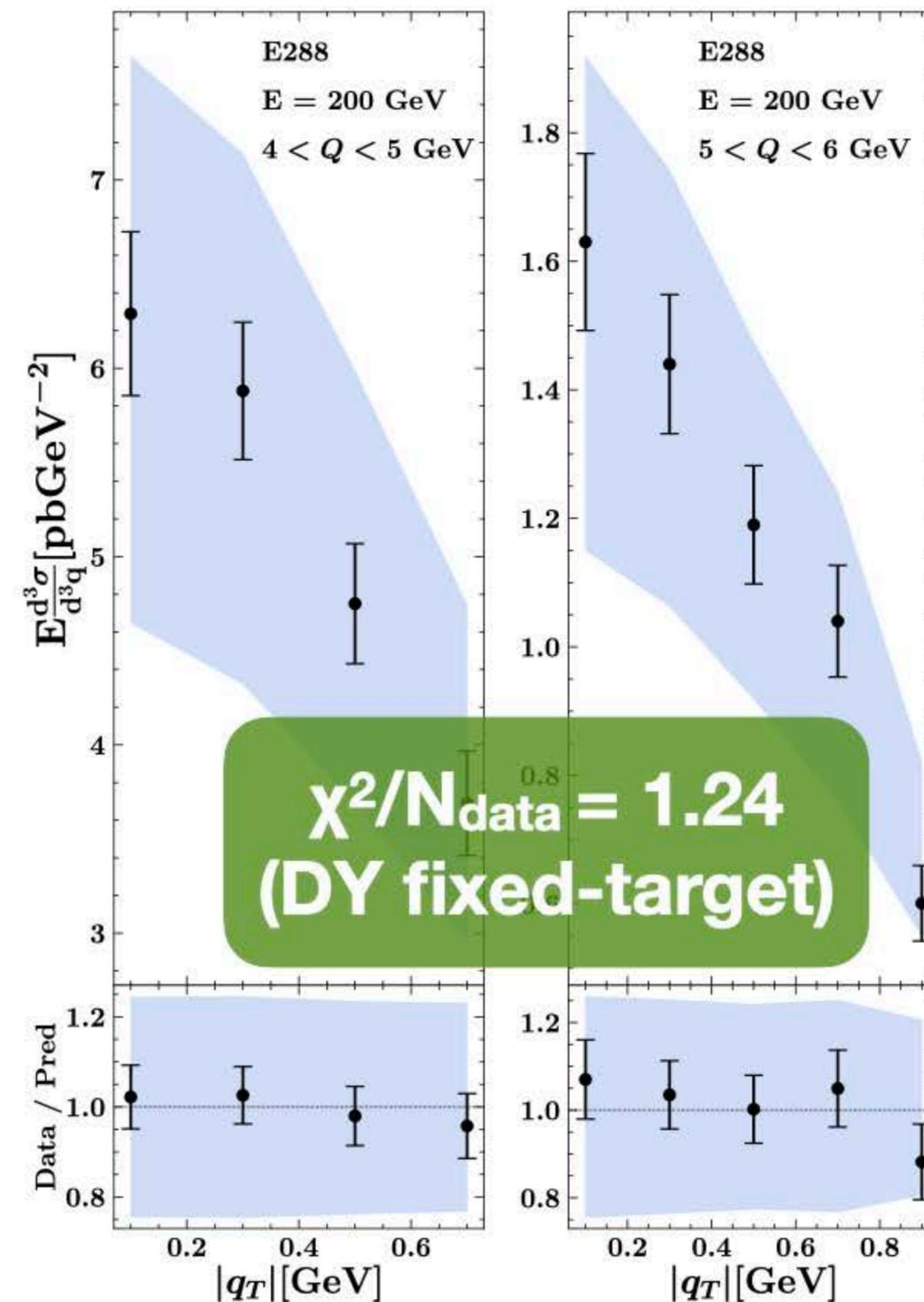
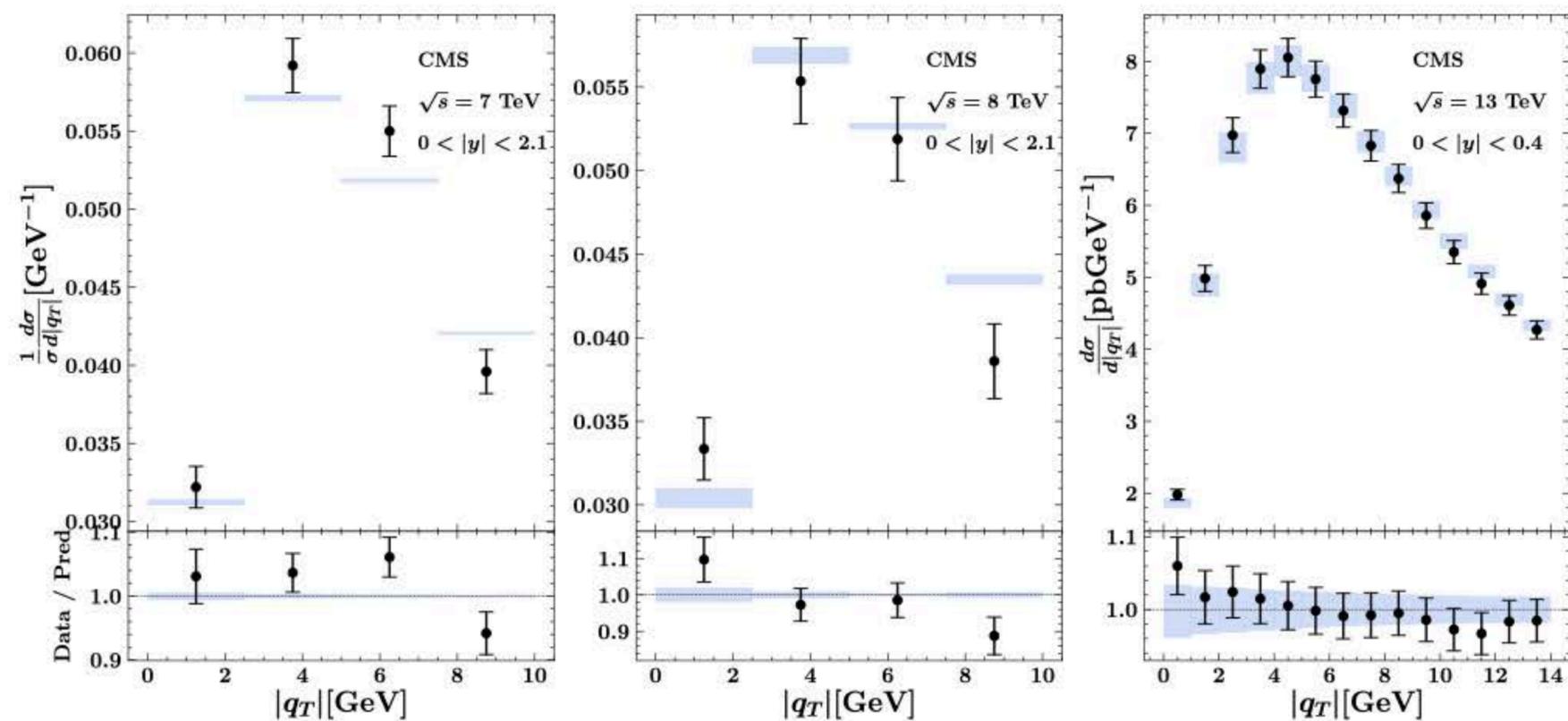
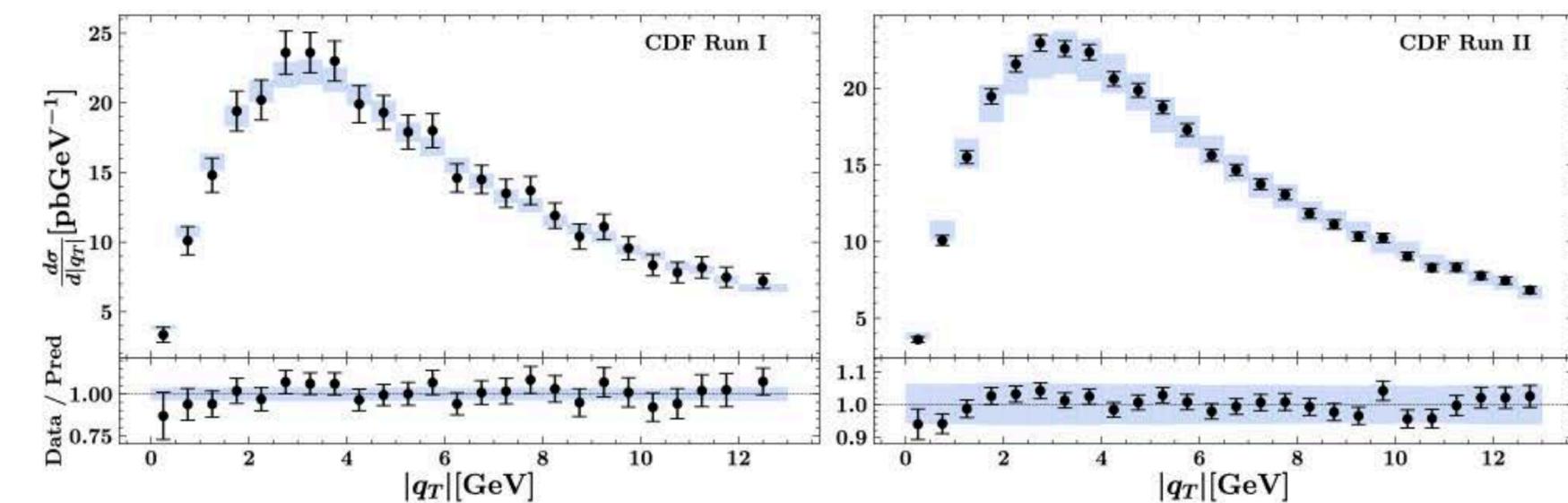
$\chi^2/N_{data} = 0.87$  (SIDIS total)



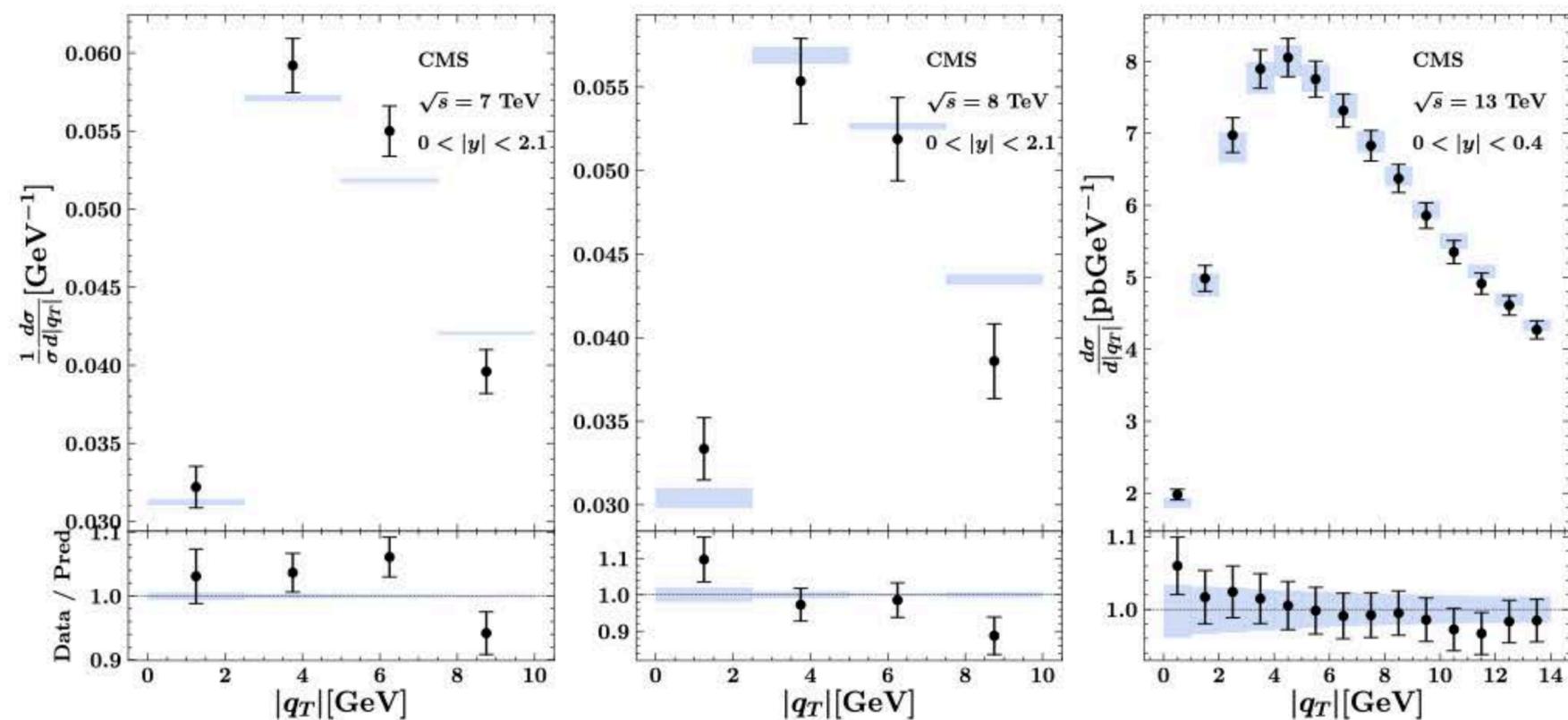
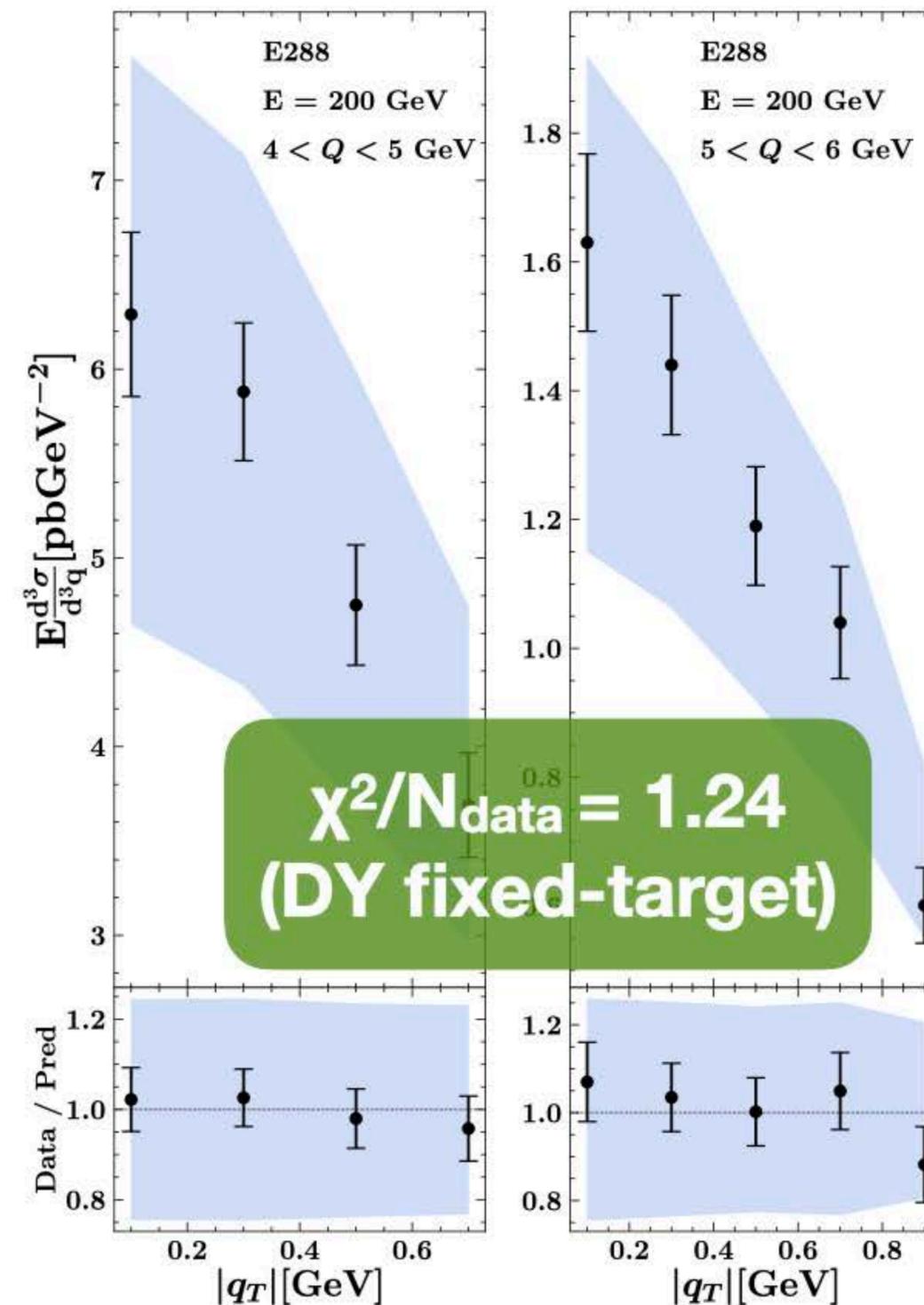
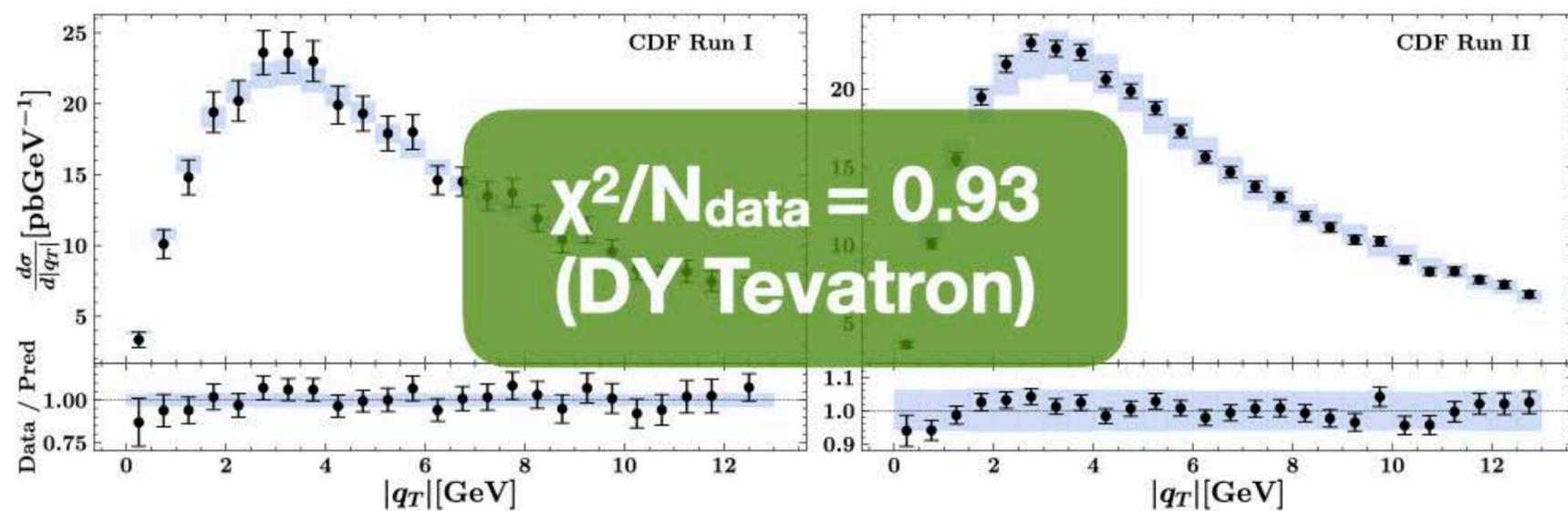
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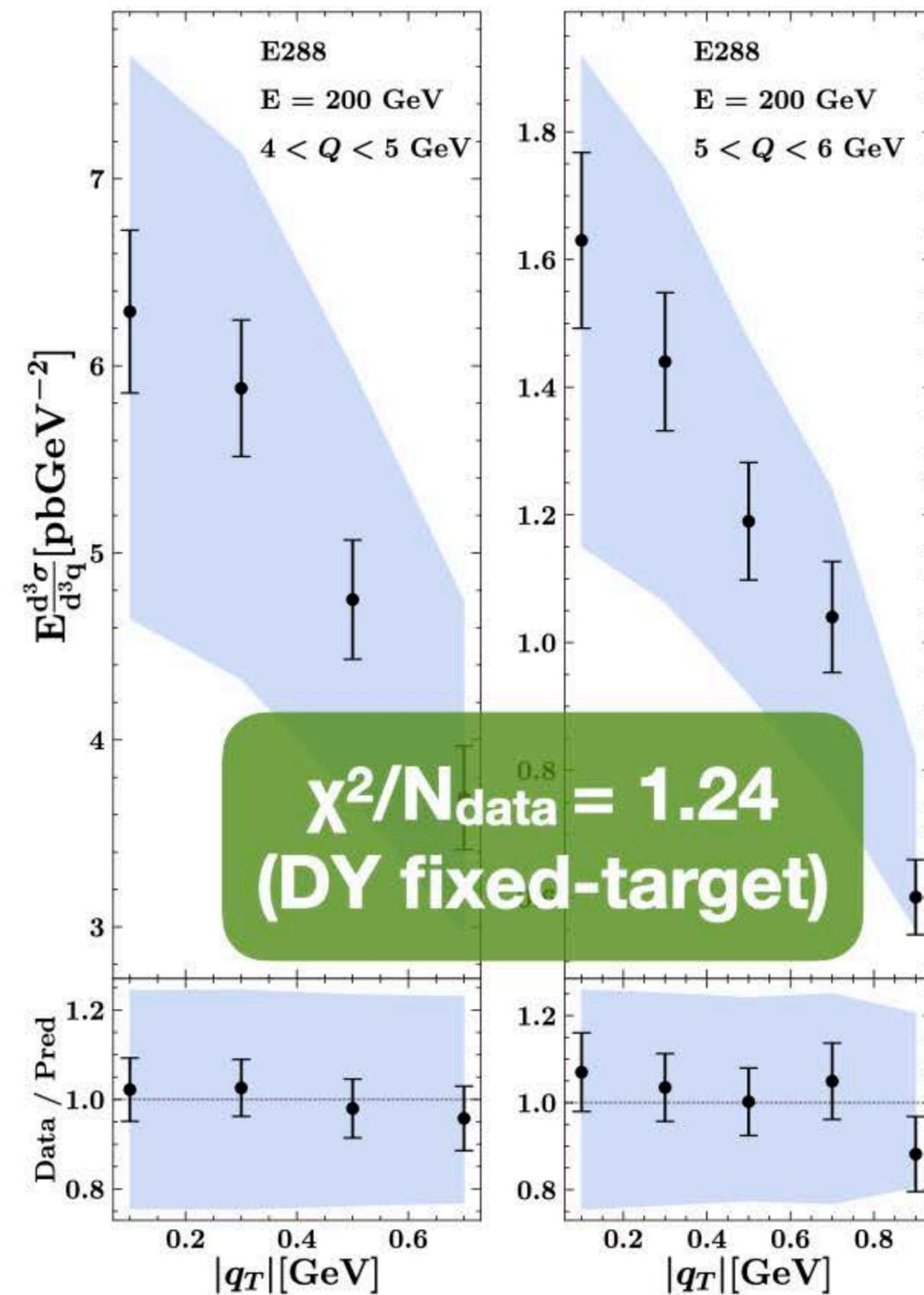
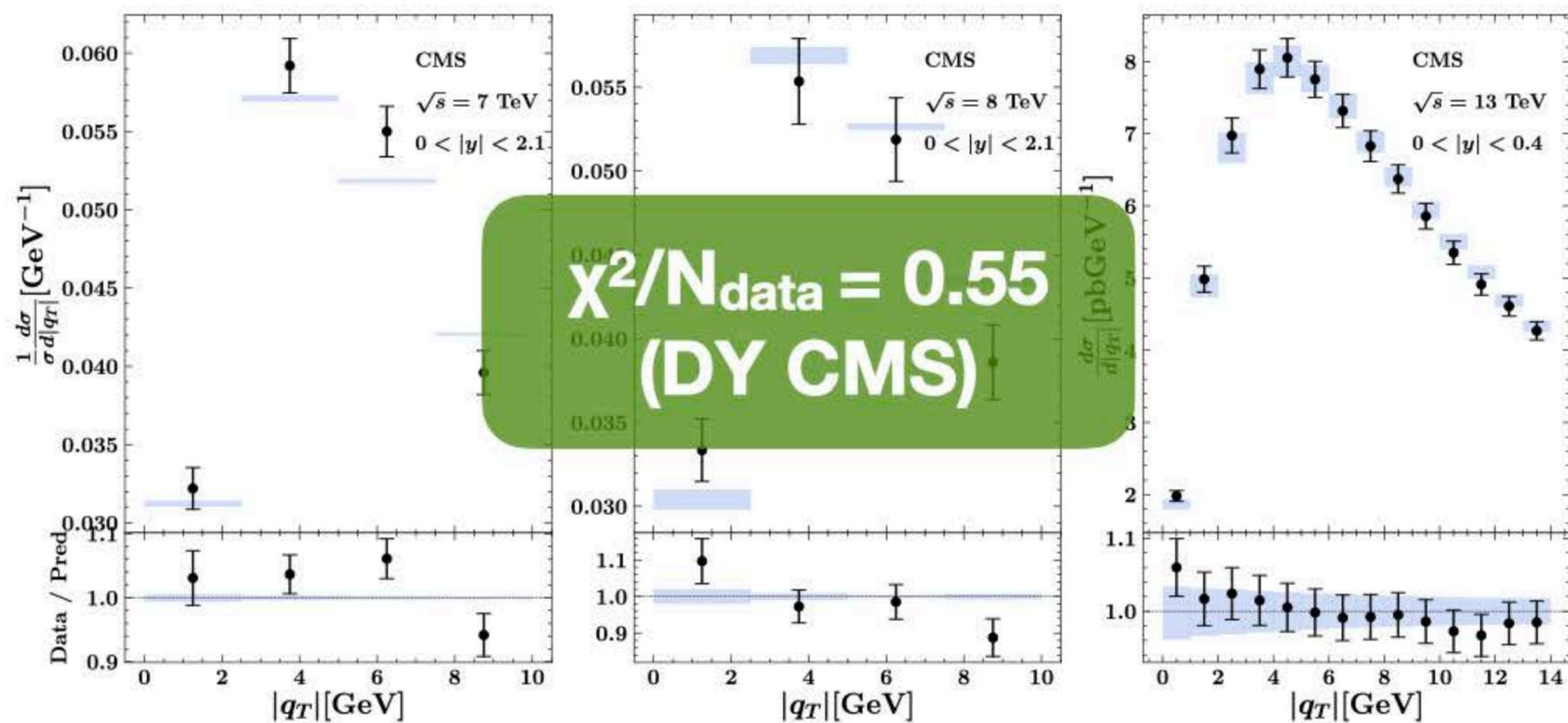
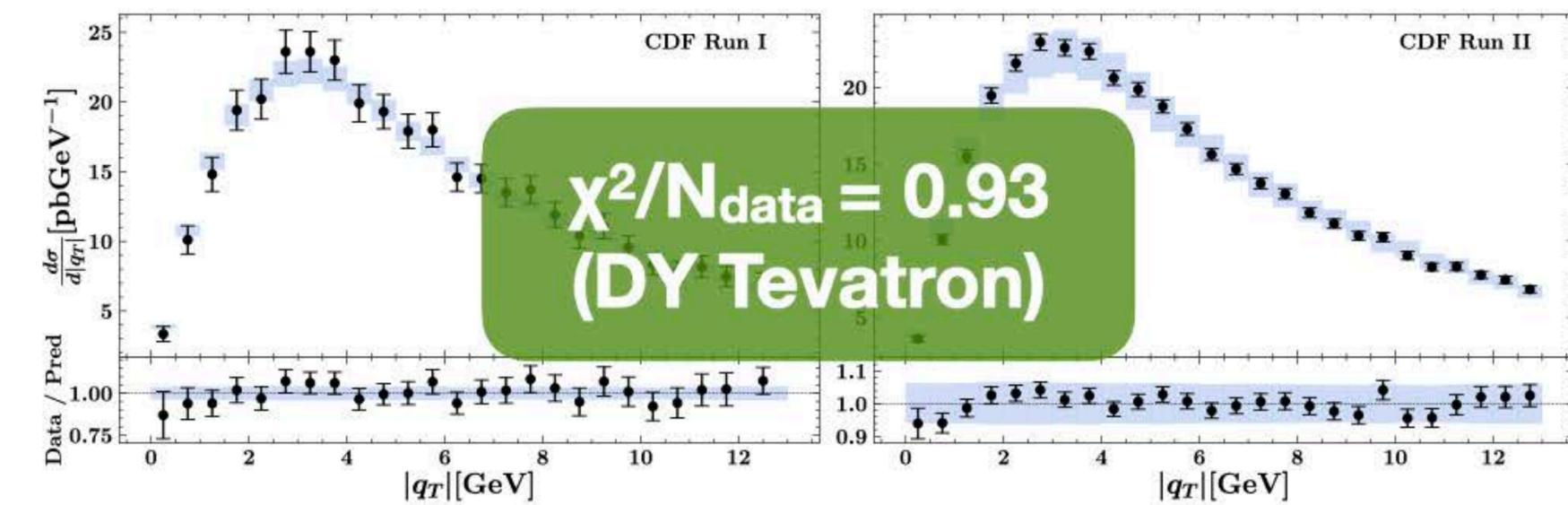
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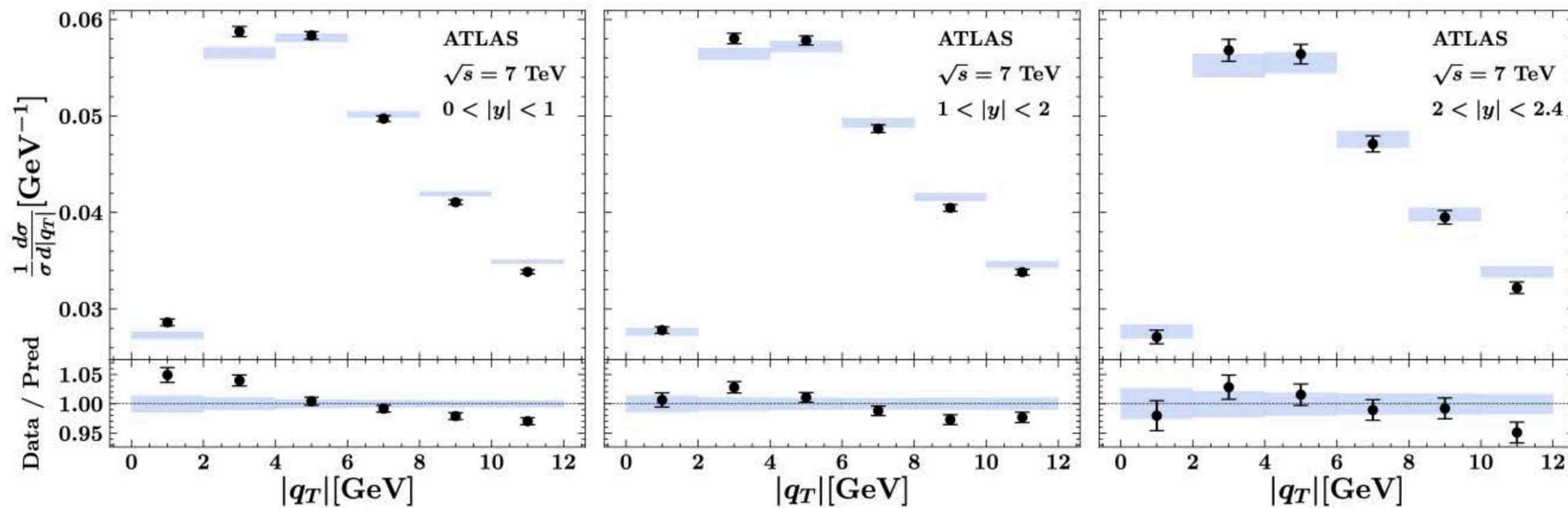
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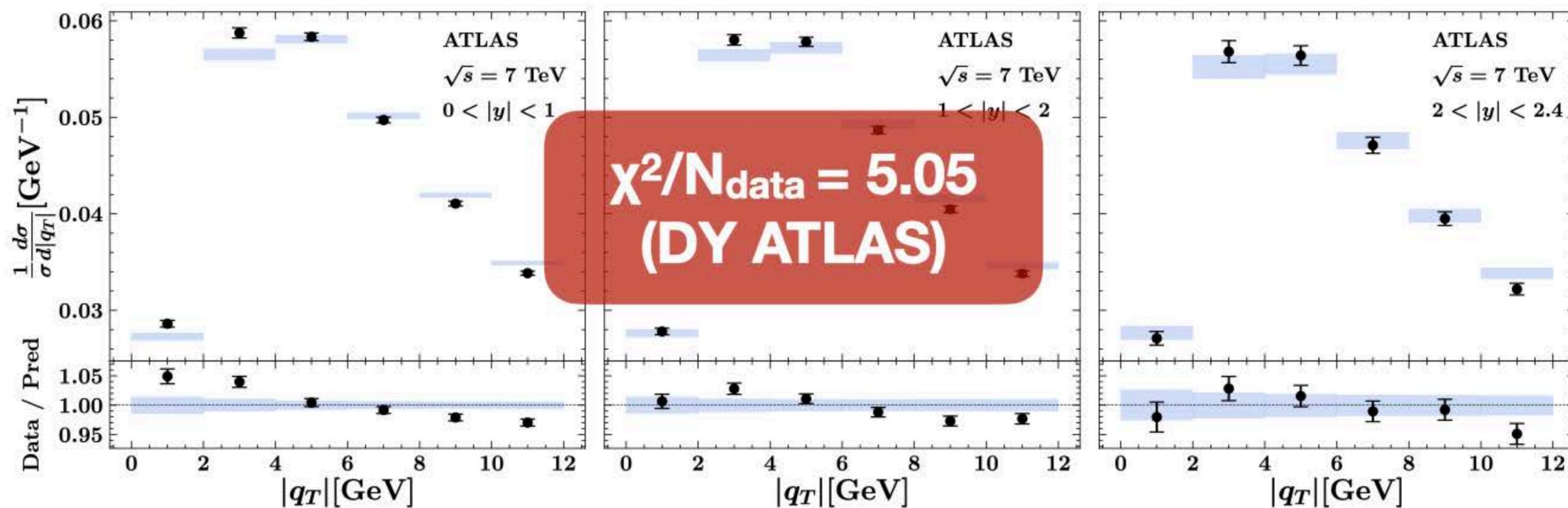
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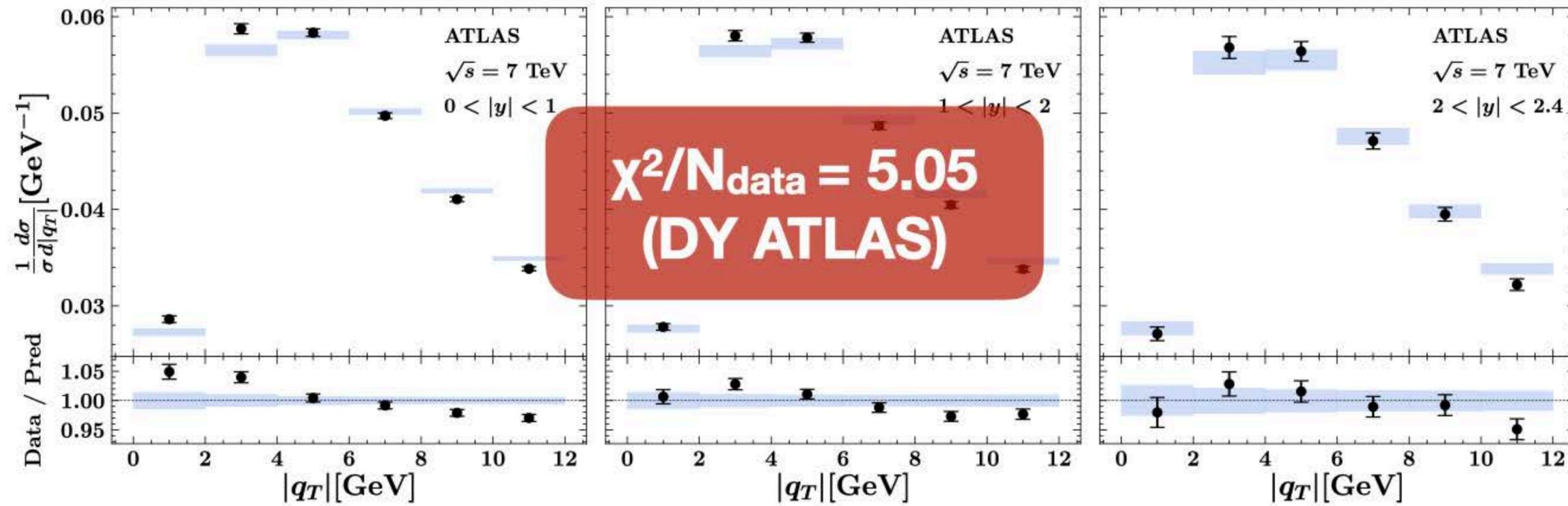
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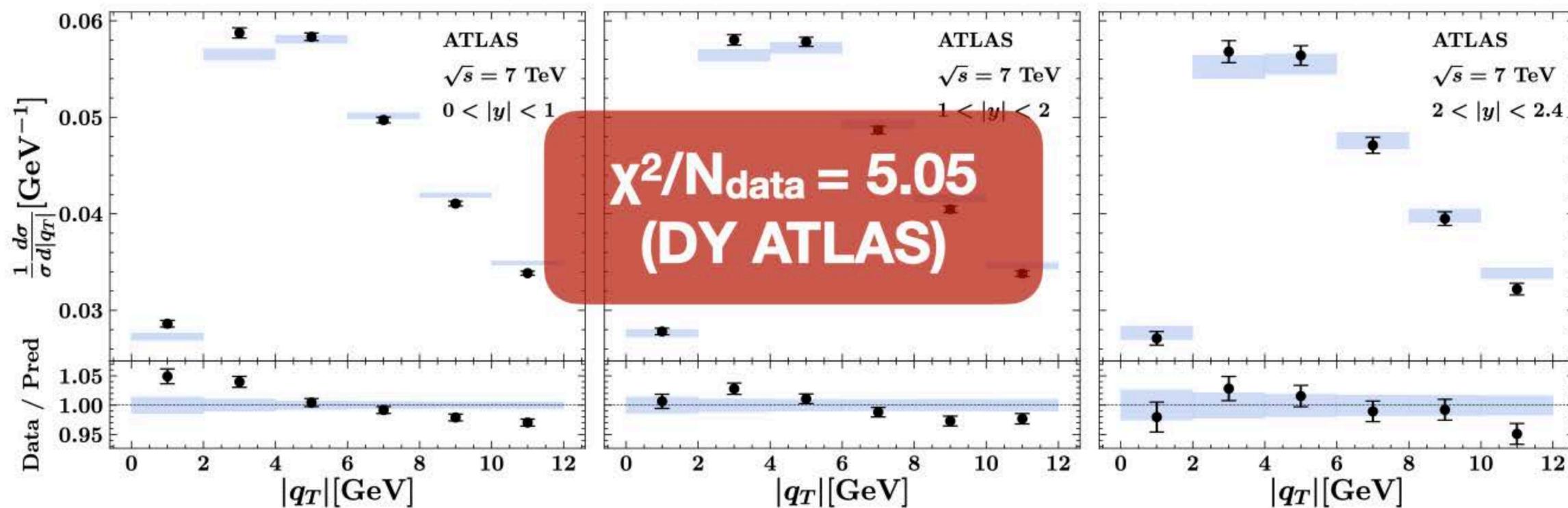


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Possible justifications:

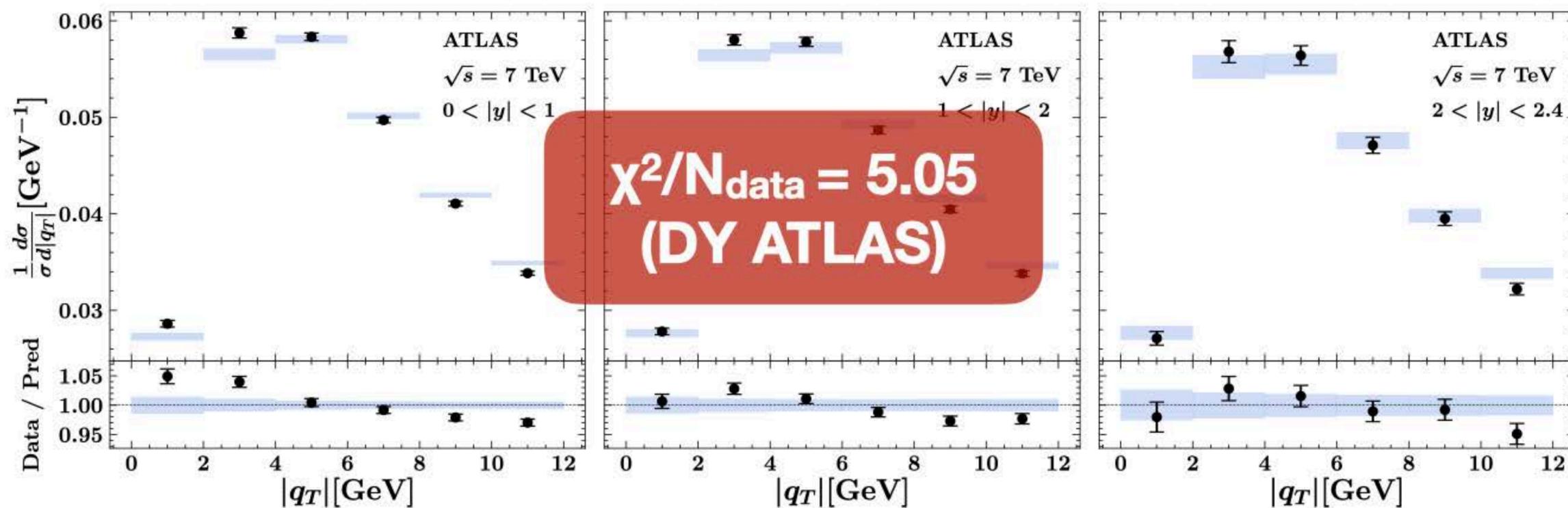
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## Possible justifications:

- ⊖ Small experimental uncertainties

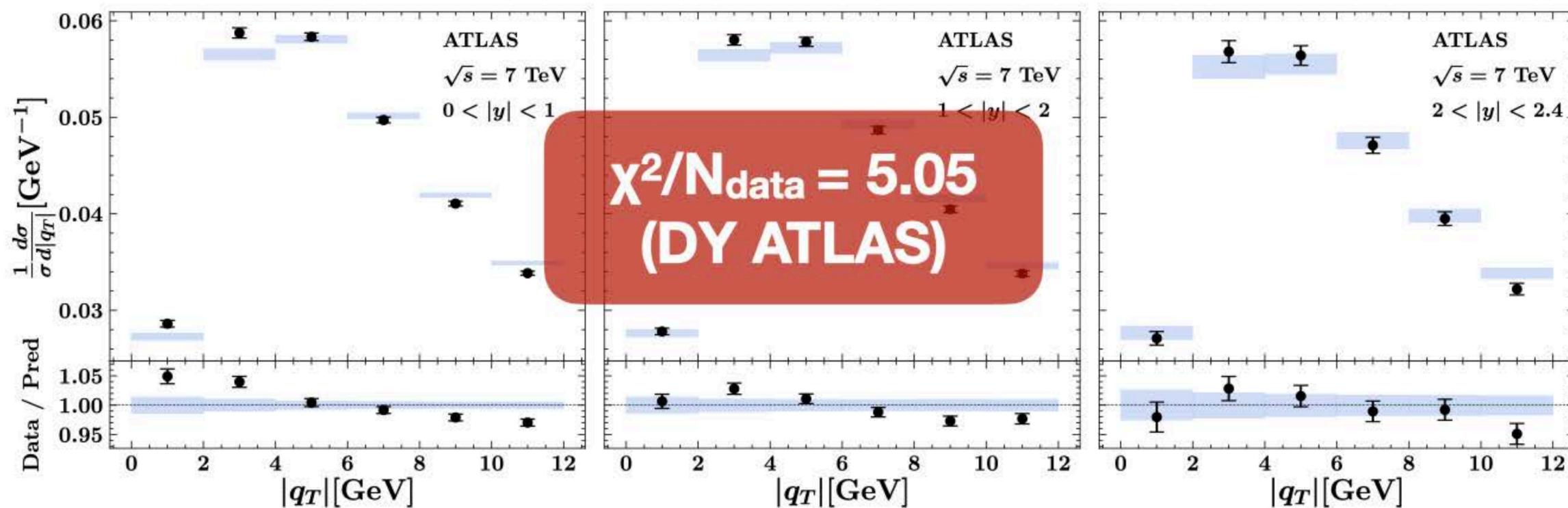
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## Possible justifications:

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- ❌ Implementation of lepton cuts

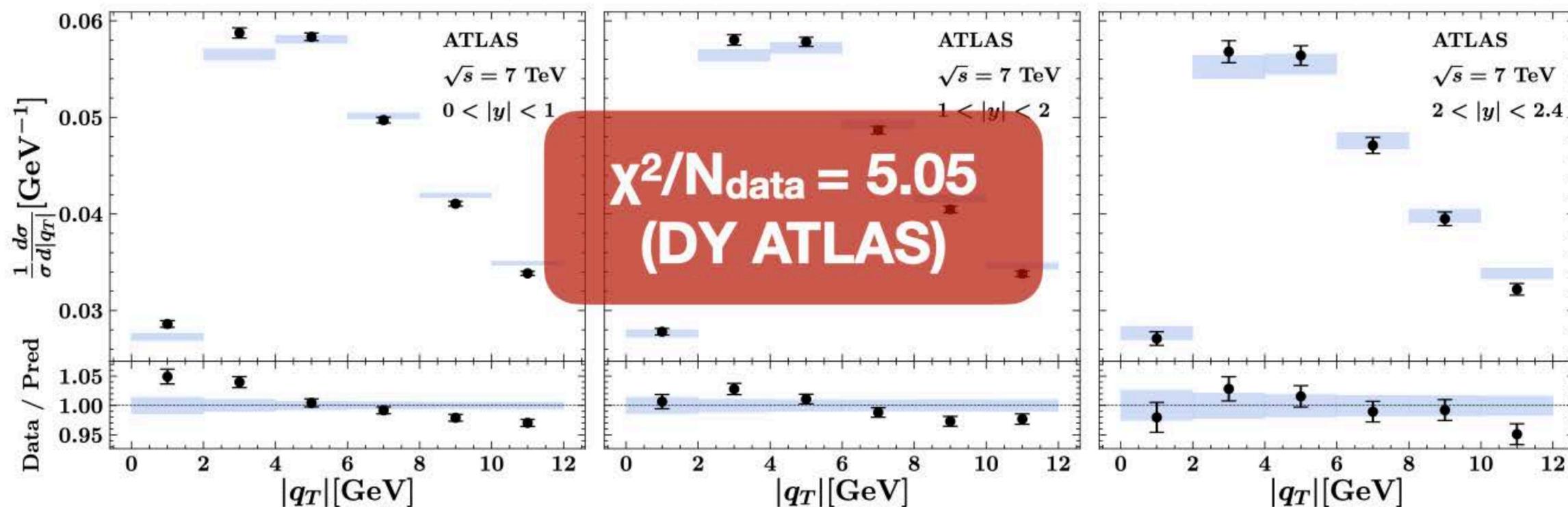
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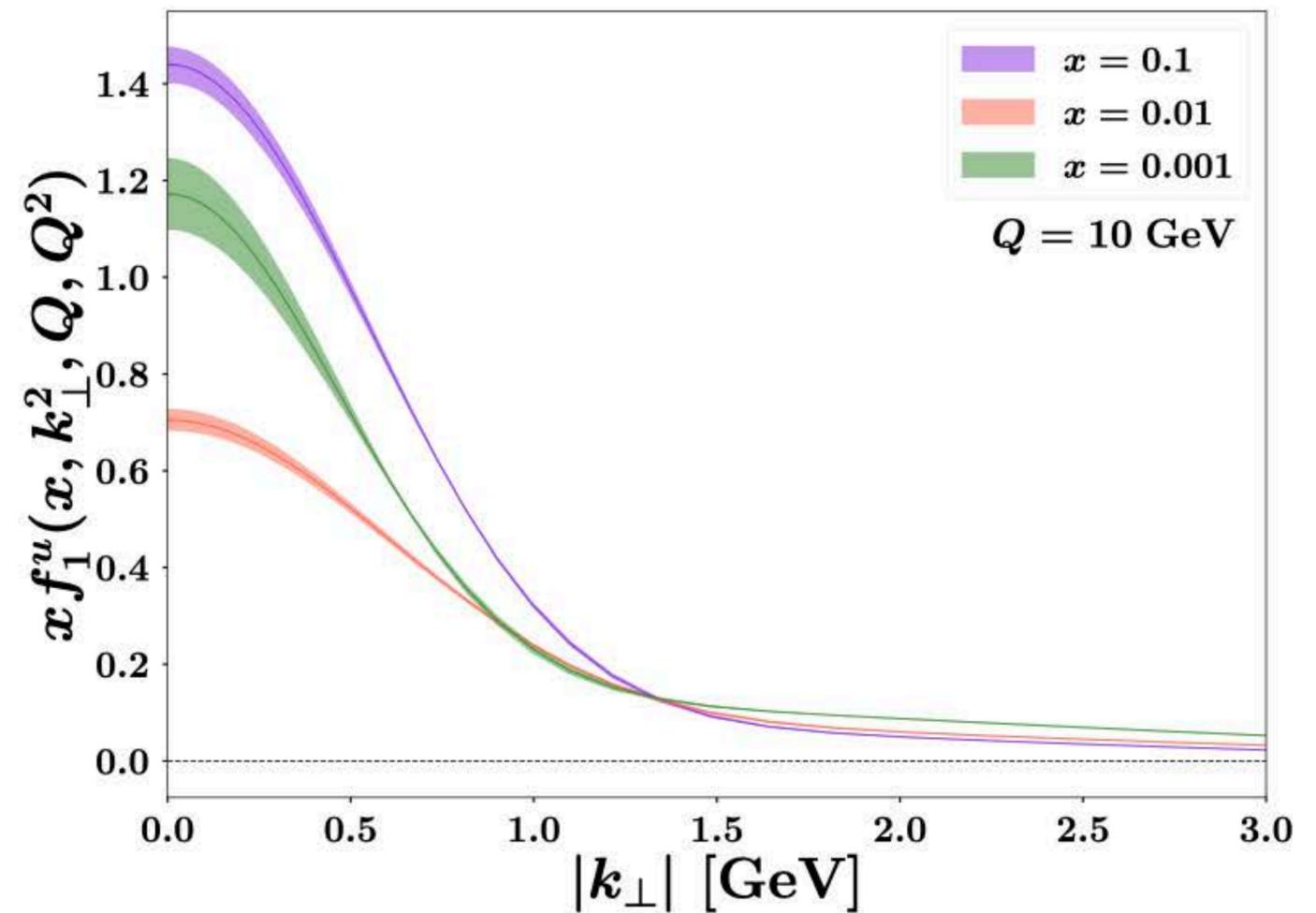
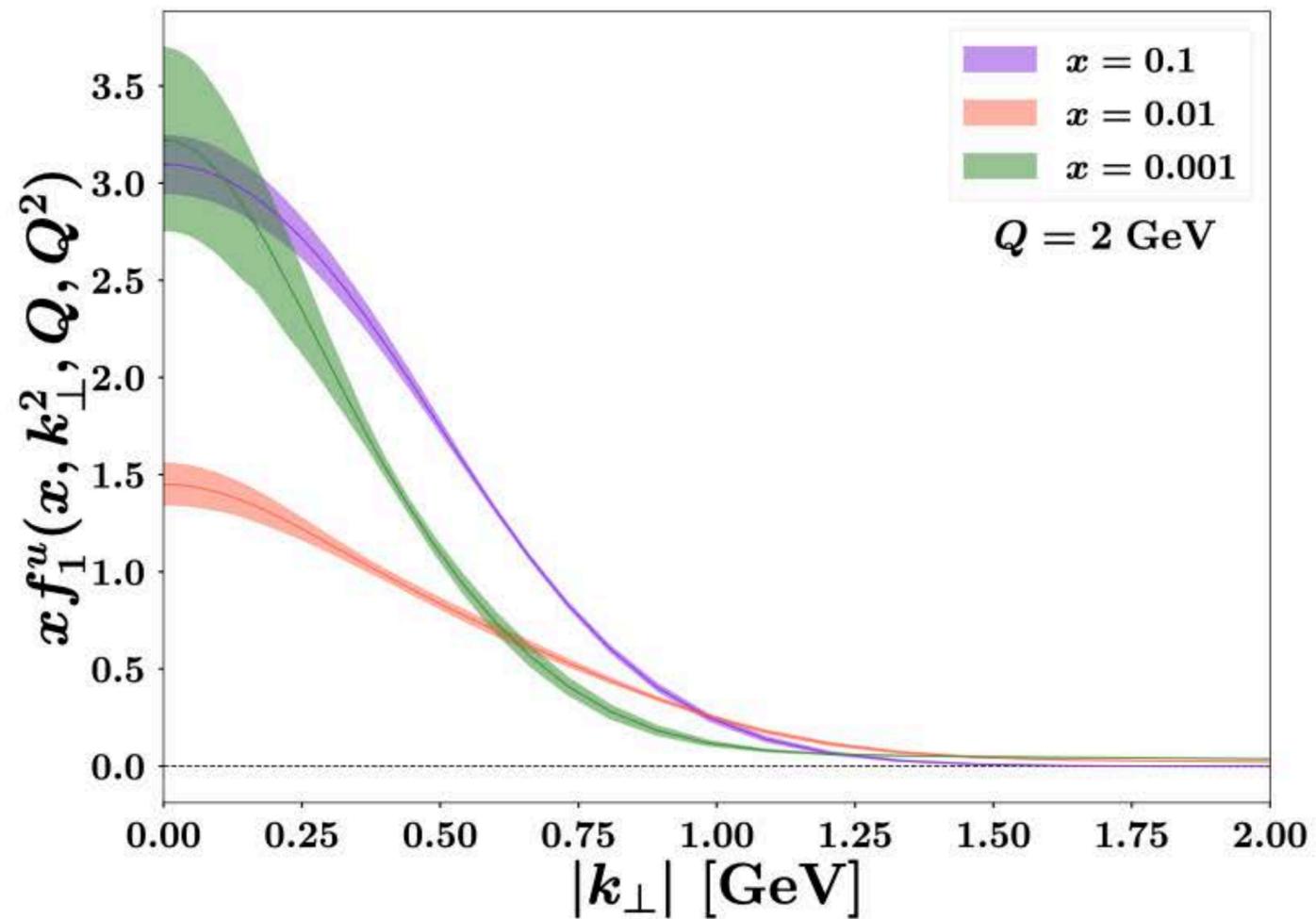


## Possible justifications:

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- Effects of power corrections
- Implementation of lepton cuts
- Effects of the matching between perturbative and non-perturbative physics

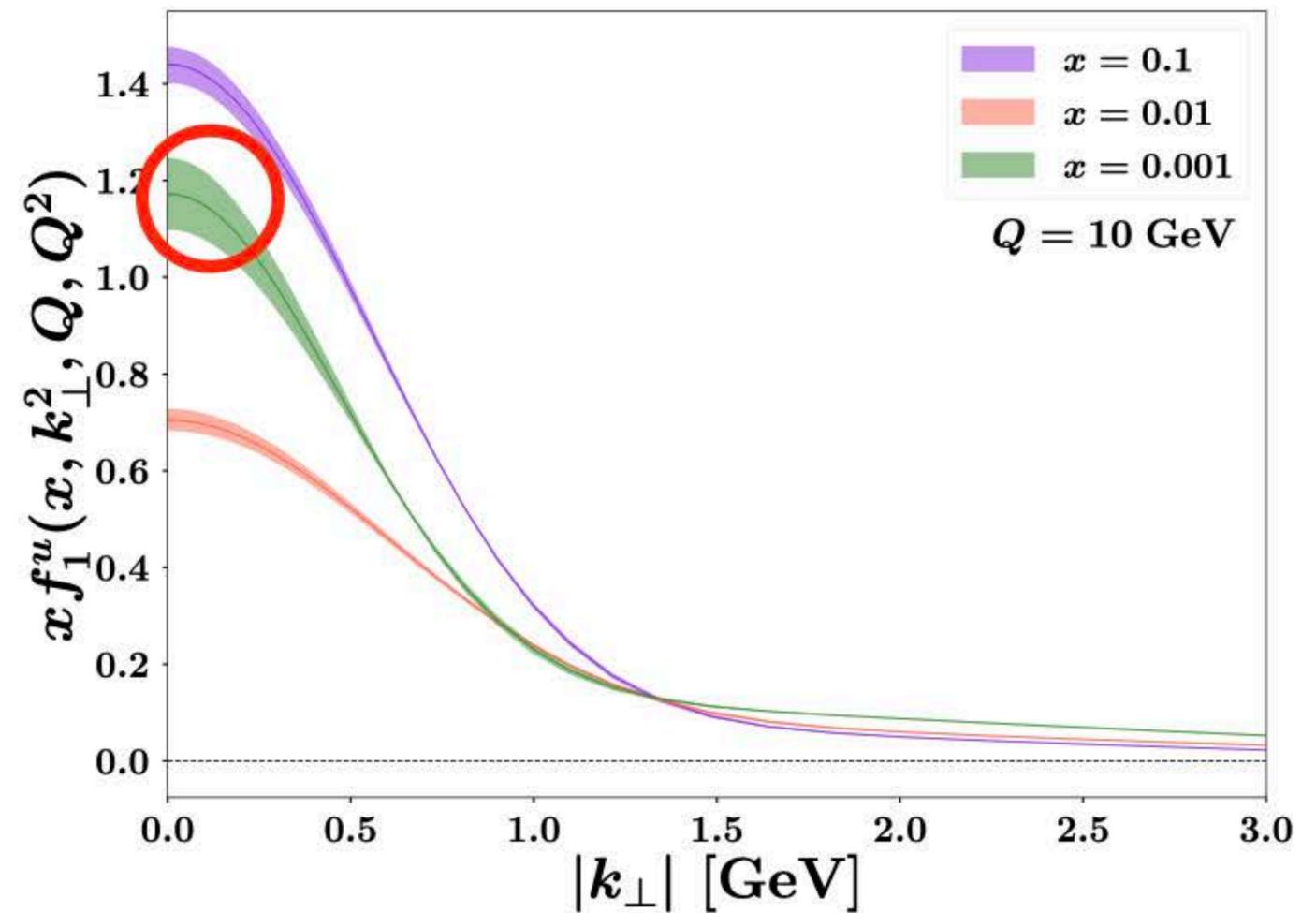
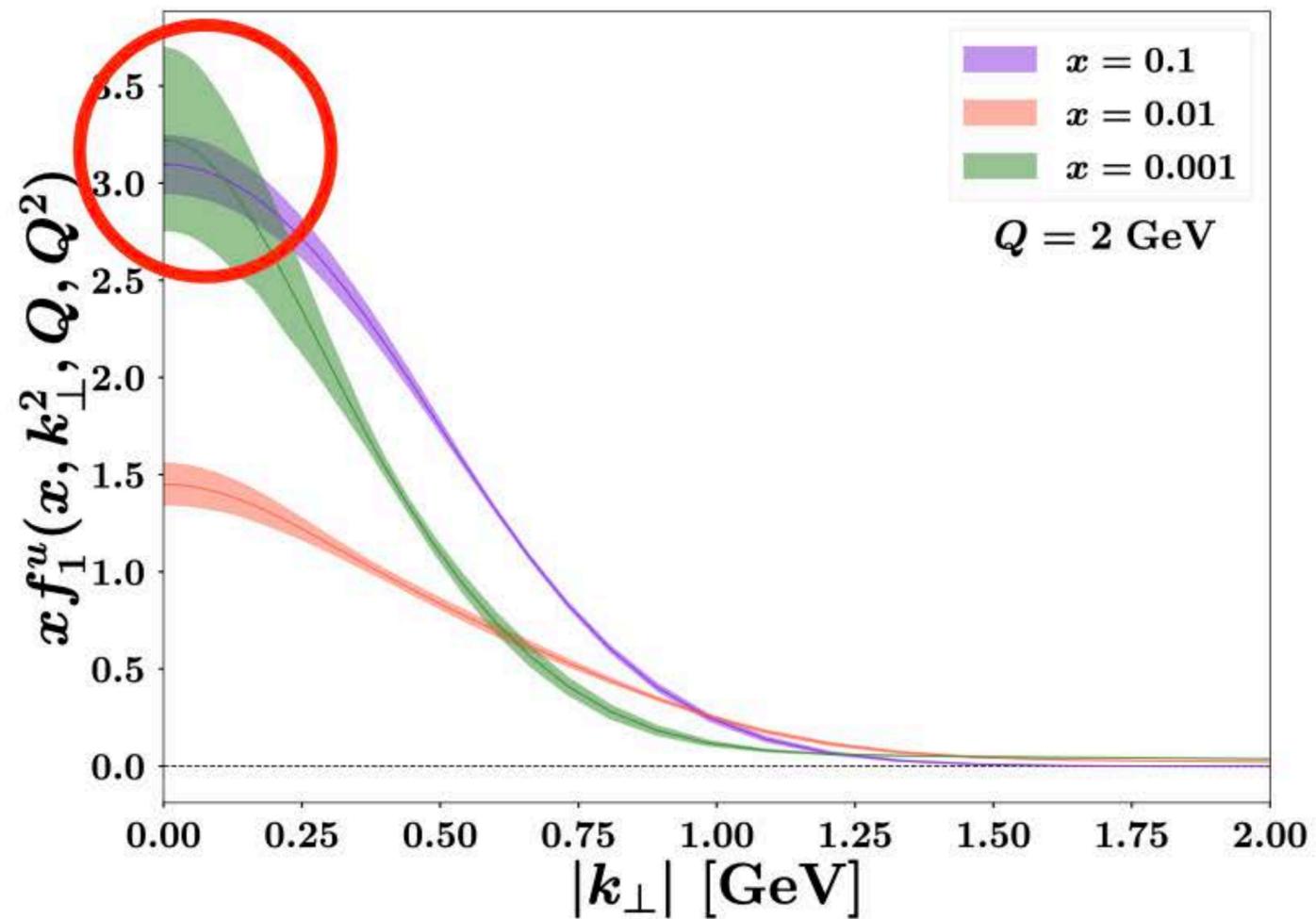
# MAPTMD22 — Output of the fit

## Visualisation of TMD PDFs



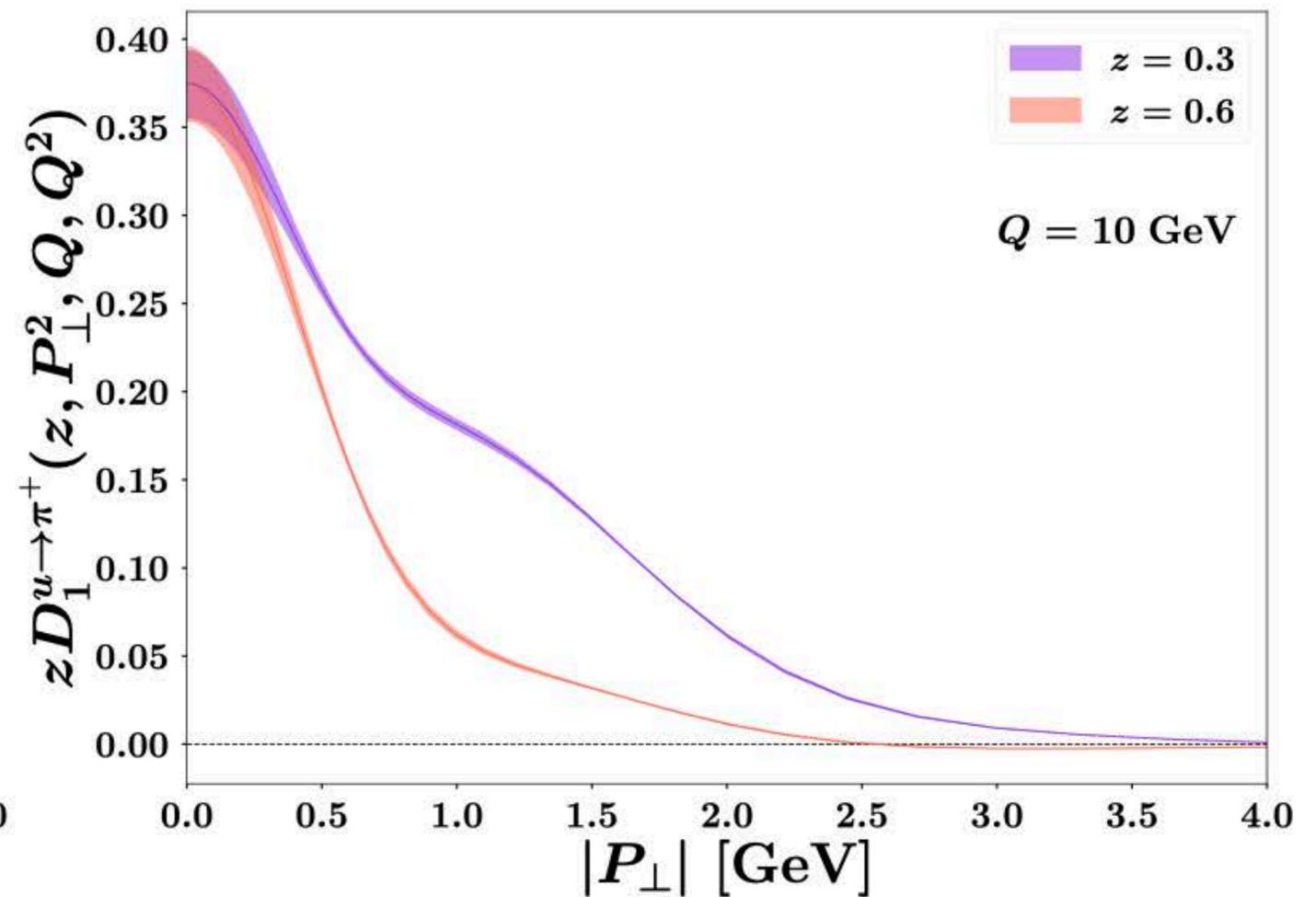
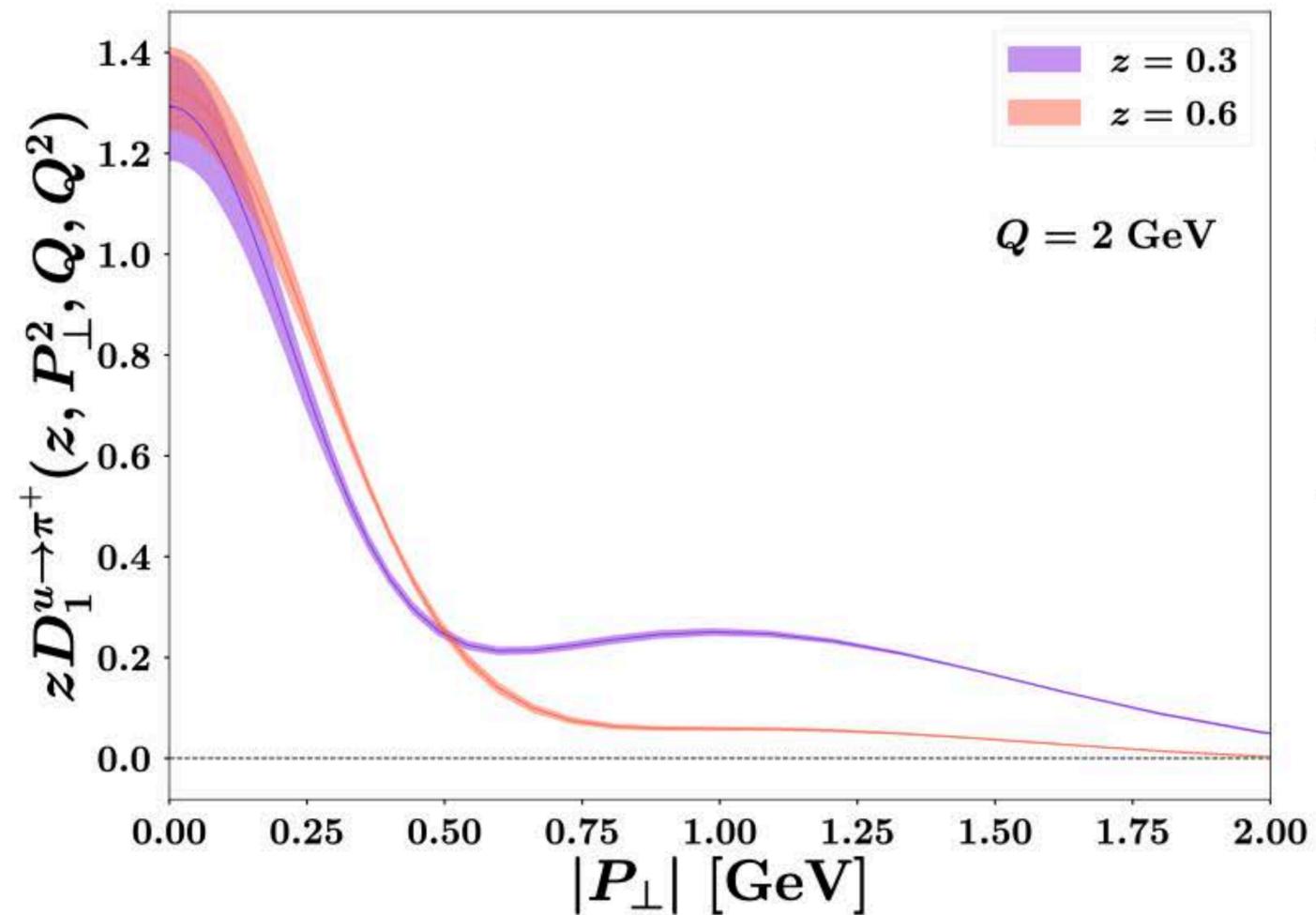
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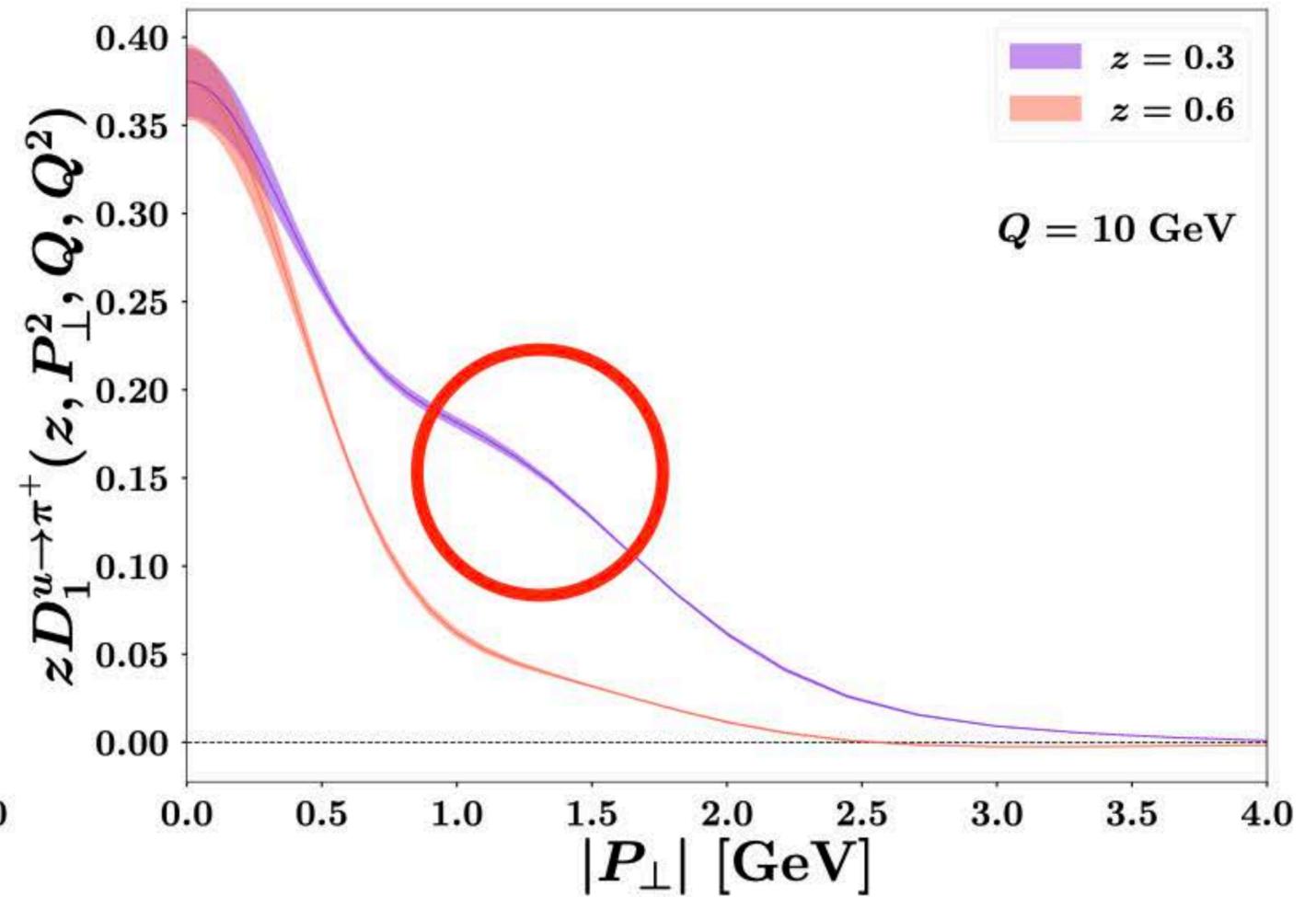
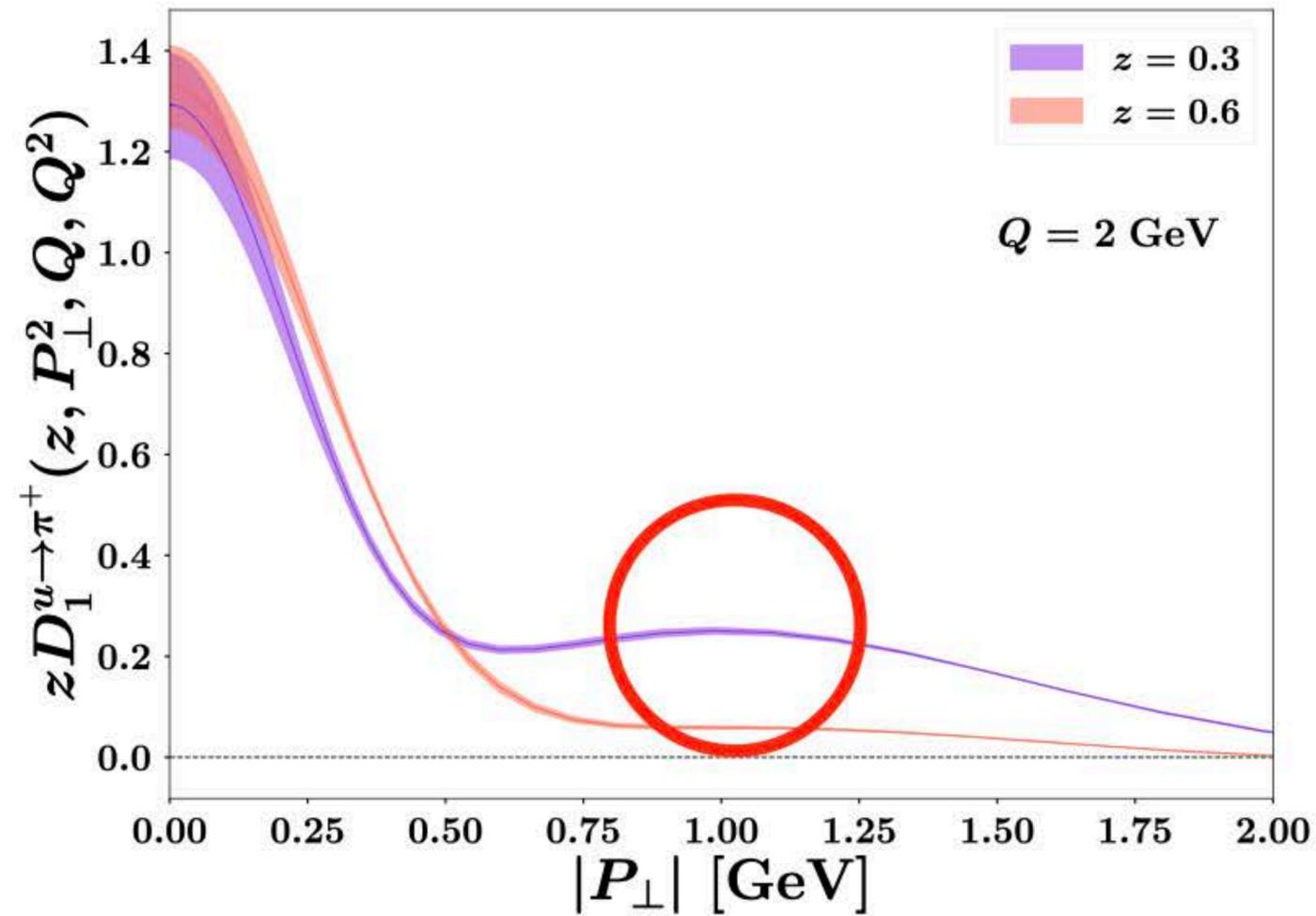
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## Visualisation of TMD FFs



# MAPTMD22 — Output of the fit

## Visualisation of TMD FFs



# MAPTMD22 — Output of the fit

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Collins-Soper kernel

# MAPTMD22 — Output of the fit

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## Collins-Soper kernel

Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

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perturbatively calculable

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perturbatively calculable

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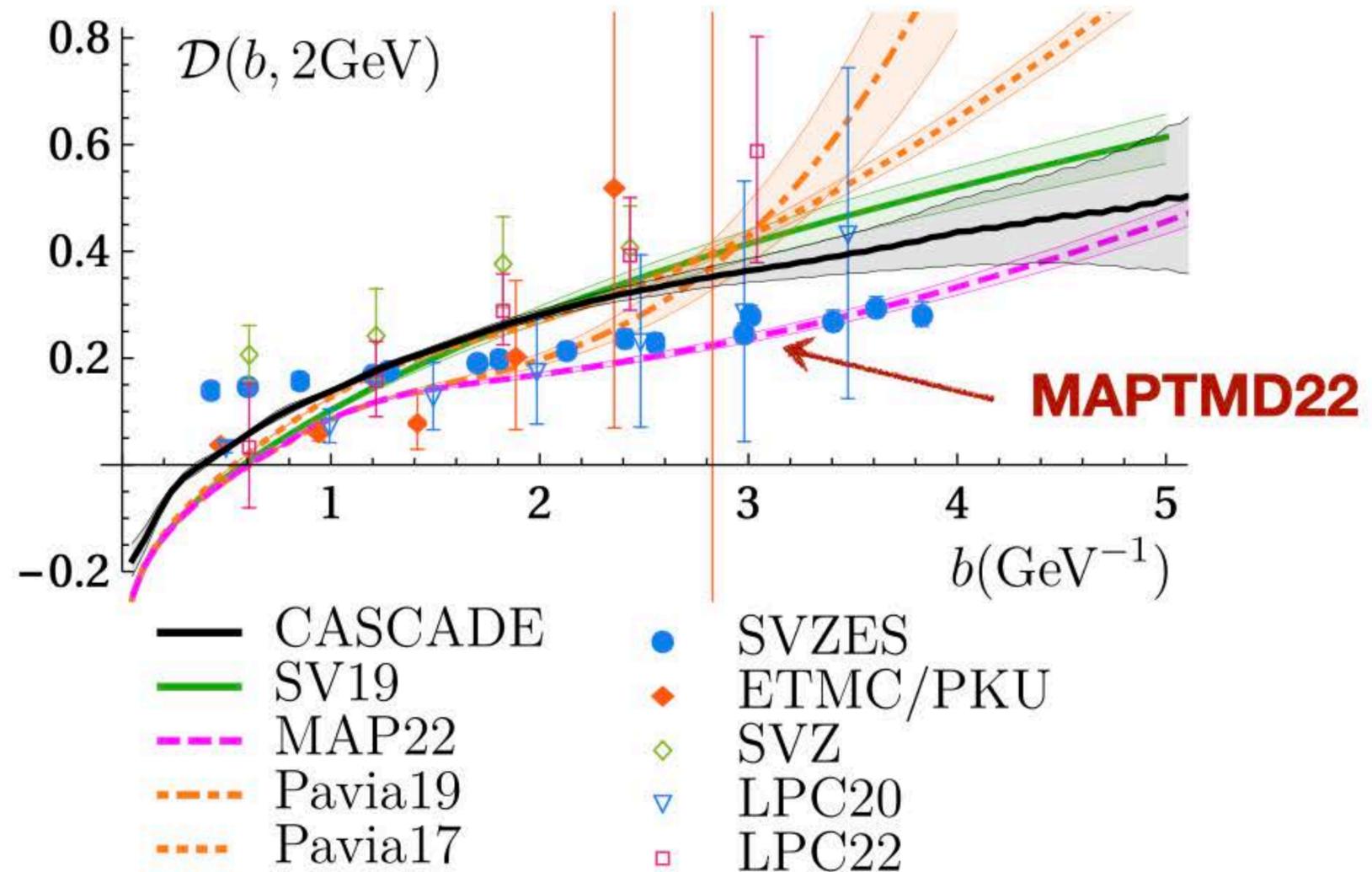
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# Summary

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# Summary

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- ***MAPTMD22***: simultaneous extraction of unpol. quark TMD PDFs and FFs

# Summary

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- Global analysis of **2031** SIDIS and DY experimental data at  **$N^3LL^-$**

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# Future Plans

# Summary

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# Future Plans

- Code and TMD grids available at the MAP Collaboration GitHub page (ask us!!)

# Summary

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- Extremely good description (except for ATLAS dataset):  **$\chi^2/N_{data} = 1.06$**

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- Code and TMD grids available at the MAP Collaboration GitHub page (ask us!!)
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- Matching with fixed-order
- Flavor dependence

---

**BACKUP SLIDES**

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# Logarithmic Accuracy

	Sudakov form factor	Matching coefficient
LL	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right)$	$\tilde{C}^0$
NLL	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$	$\tilde{C}^0$
NLL'	$\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$	$\left( \tilde{C}^0 + \alpha_S \tilde{C}^1 \right)$

the difference between the two is NNLL:  $\alpha_S^n \ln^{2n-2} \left( \frac{Q^2}{\mu_b^2} \right)$

# Logarithmic Accuracy

$$S_{\text{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left( \frac{\alpha_S(Q)}{4\pi} \right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n, 2n-k)} \quad L = \ln \left( \frac{Q^2}{\mu_b^2} \right)$$

Sudakov form factor

Matching coefficient

LL  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right)$

$\tilde{C}^0$

NLL  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$

$\tilde{C}^0$

NLL'  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$

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the difference between the two is NNLL:  $\alpha_S^n \ln^{2n-2} \left( \frac{Q^2}{\mu_b^2} \right)$

# Structure of a TMD - NP content

$$\mu_b = \frac{2e^{-\gamma E}}{b_T}$$

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0 \quad \alpha_S(\mu_b) \rightarrow +\infty$$

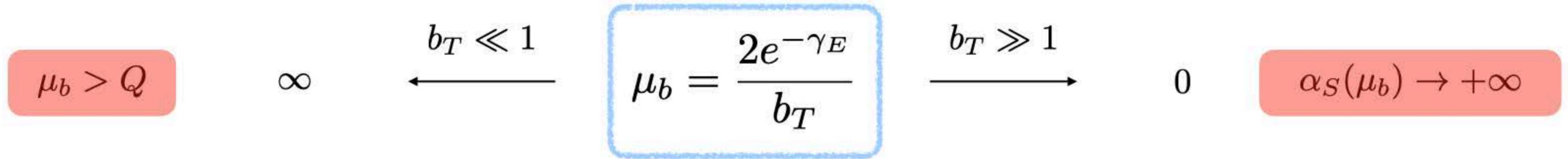
# Structure of a TMD - NP content

$$\infty \xleftarrow{b_T \ll 1} \boxed{\mu_b = \frac{2e^{-\gamma_E}}{b_T}} \xrightarrow{b_T \gg 1} 0 \quad \alpha_S(\mu_b) \rightarrow +\infty$$

# Structure of a TMD - NP content

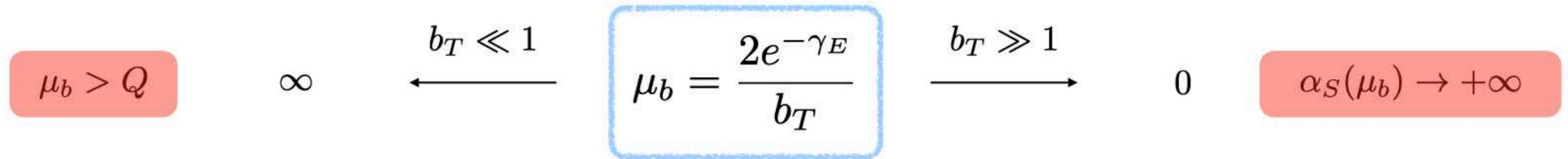
$$\begin{array}{ccccccc} \boxed{\mu_b > Q} & \infty & \xleftarrow{b_T \ll 1} & \boxed{\mu_b = \frac{2e^{-\gamma_E}}{b_T}} & \xrightarrow{b_T \gg 1} & 0 & \boxed{\alpha_S(\mu_b) \rightarrow +\infty} \end{array}$$

# Structure of a TMD - NP content



$b_*$ -prescription

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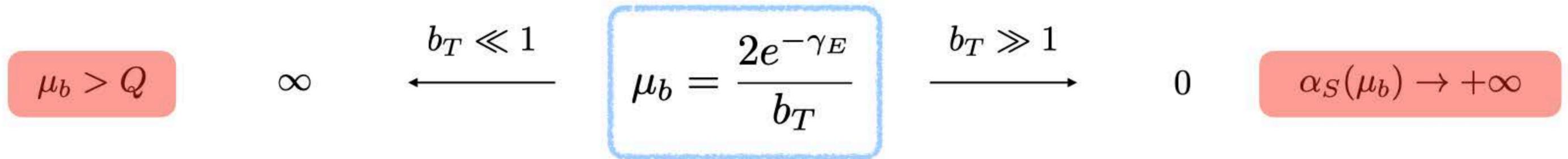
$$b_*(b_T) = b_{\max} \left( \frac{1 - \exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b_T^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

Bacchetta, Delcarro, Pisano, et al., JHEP 06 (2017)

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

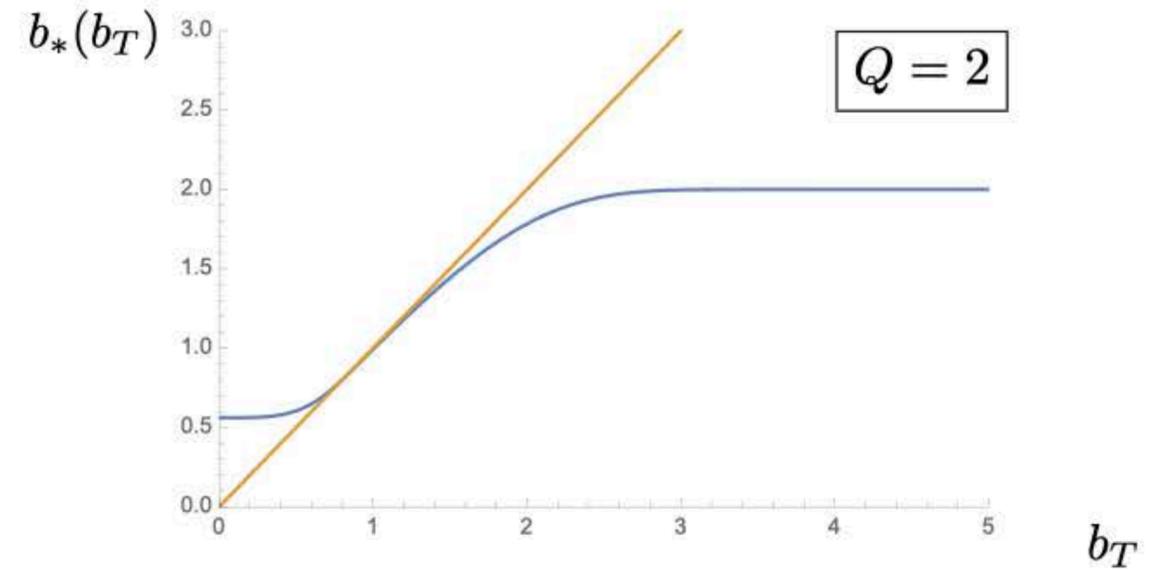
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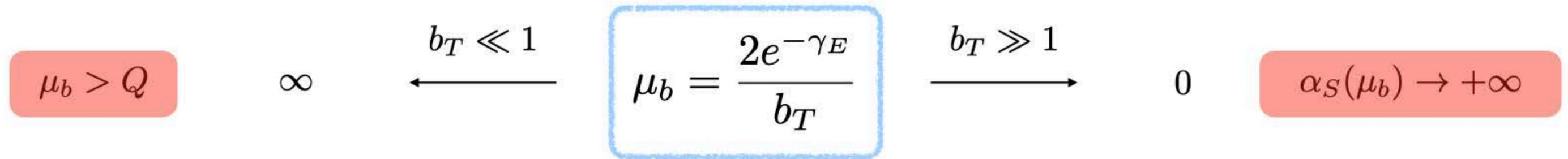
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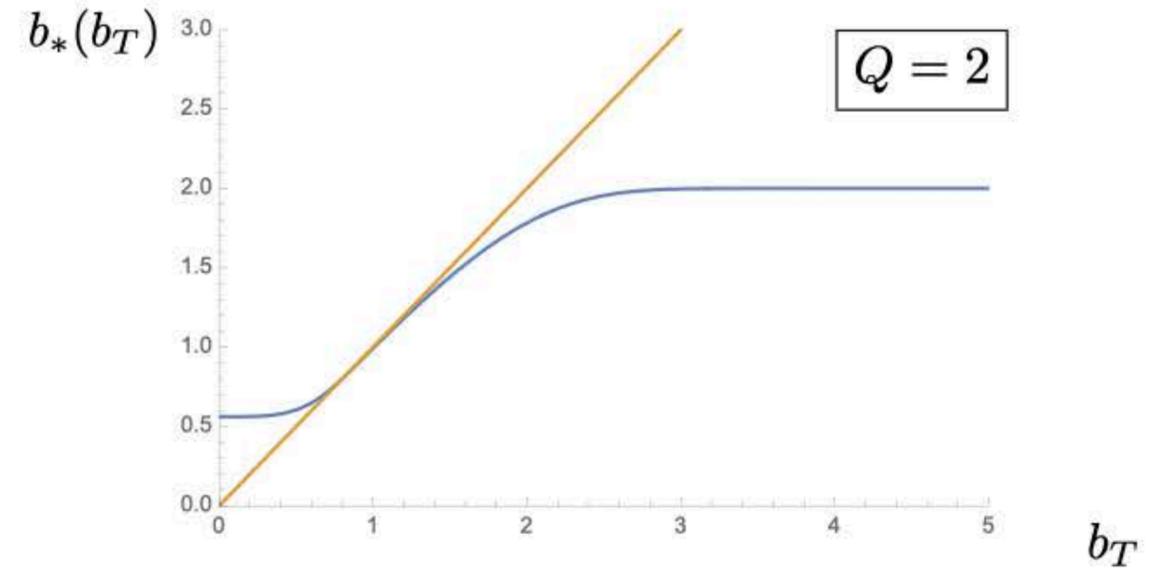
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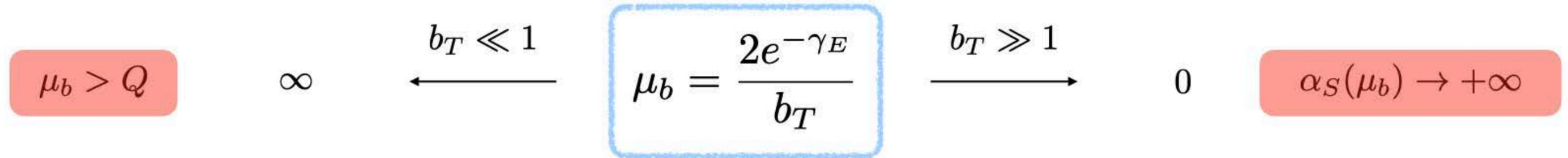


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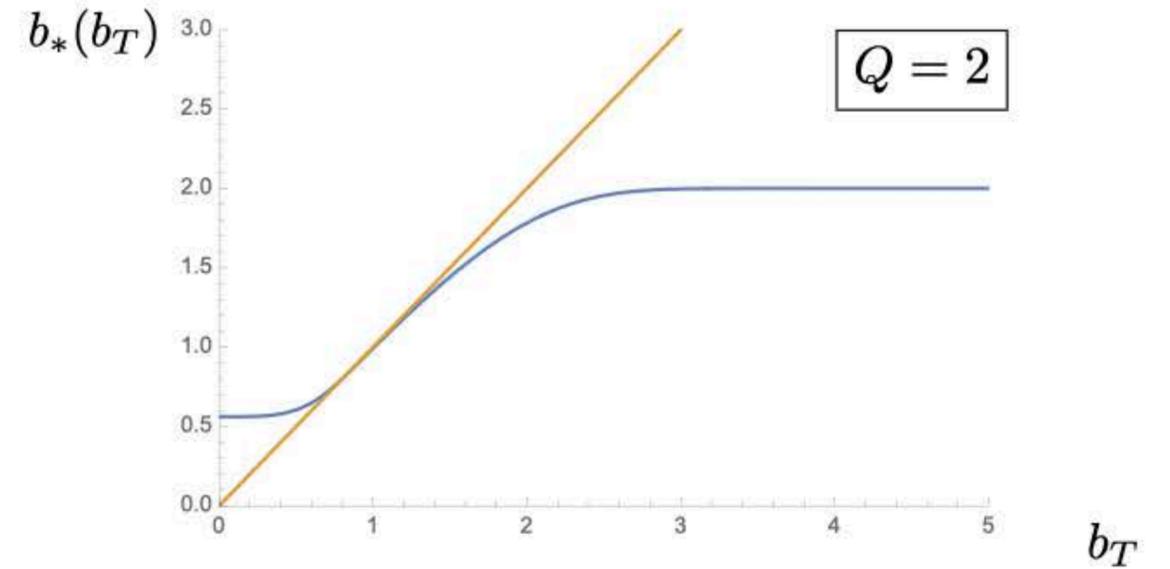
$$\hat{f}_1(x, b_T; \mu, \zeta) = \left[ \frac{\hat{f}_1(x, b_T; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T); \mu, \zeta) \equiv f_{\text{NP}}(x, b_T; \zeta) \hat{f}_1(x, b_*(b_T); \mu, \zeta)$$

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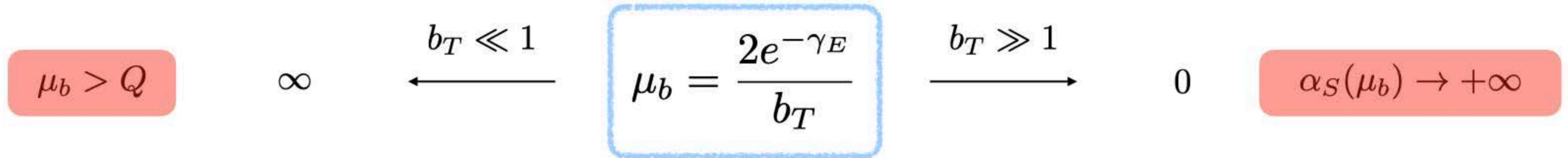


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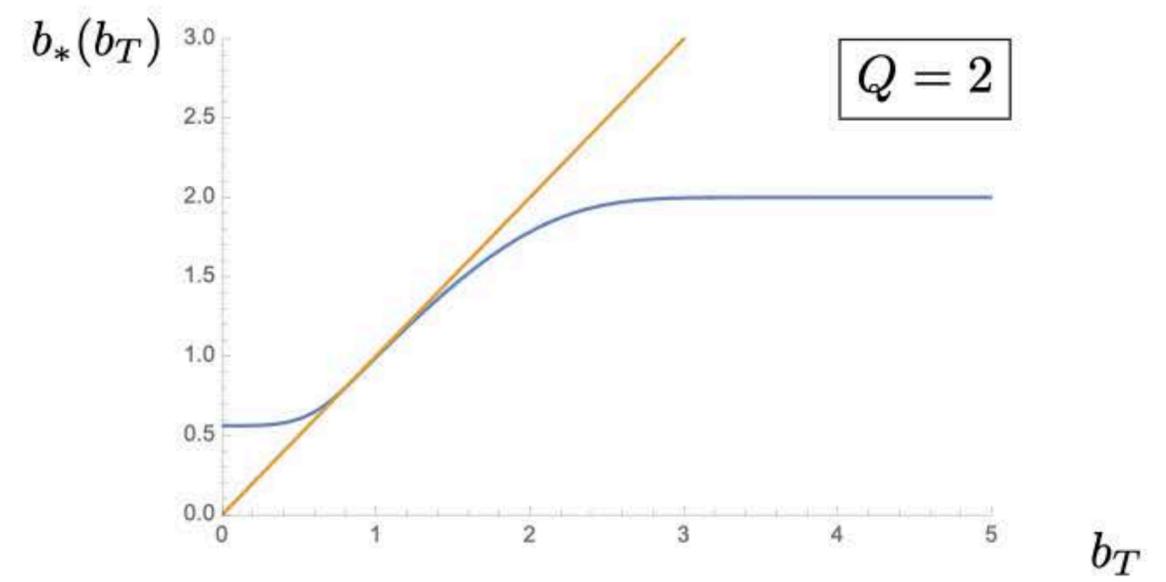
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➔ GLOBAL FIT

# Non-mixed terms in collinear SIDIS cross section

$$\begin{aligned} \frac{d\sigma^h}{dx dQ^2 dz} \Big|_{O(\alpha_s^1)} &= \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[ D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ &\quad \left. + \frac{1-y}{1+(1-y)^2} \left[ D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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$$C_1^{qq} = \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ \left. + \delta(1-x) \left[ P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \right. \\ \left. + \delta(1-z) \left[ P_{qq}(x) \ln \frac{Q^2}{\mu^2} + L_1(x) - L_2(x) + (1-x) \right] \right. \\ \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\},$$

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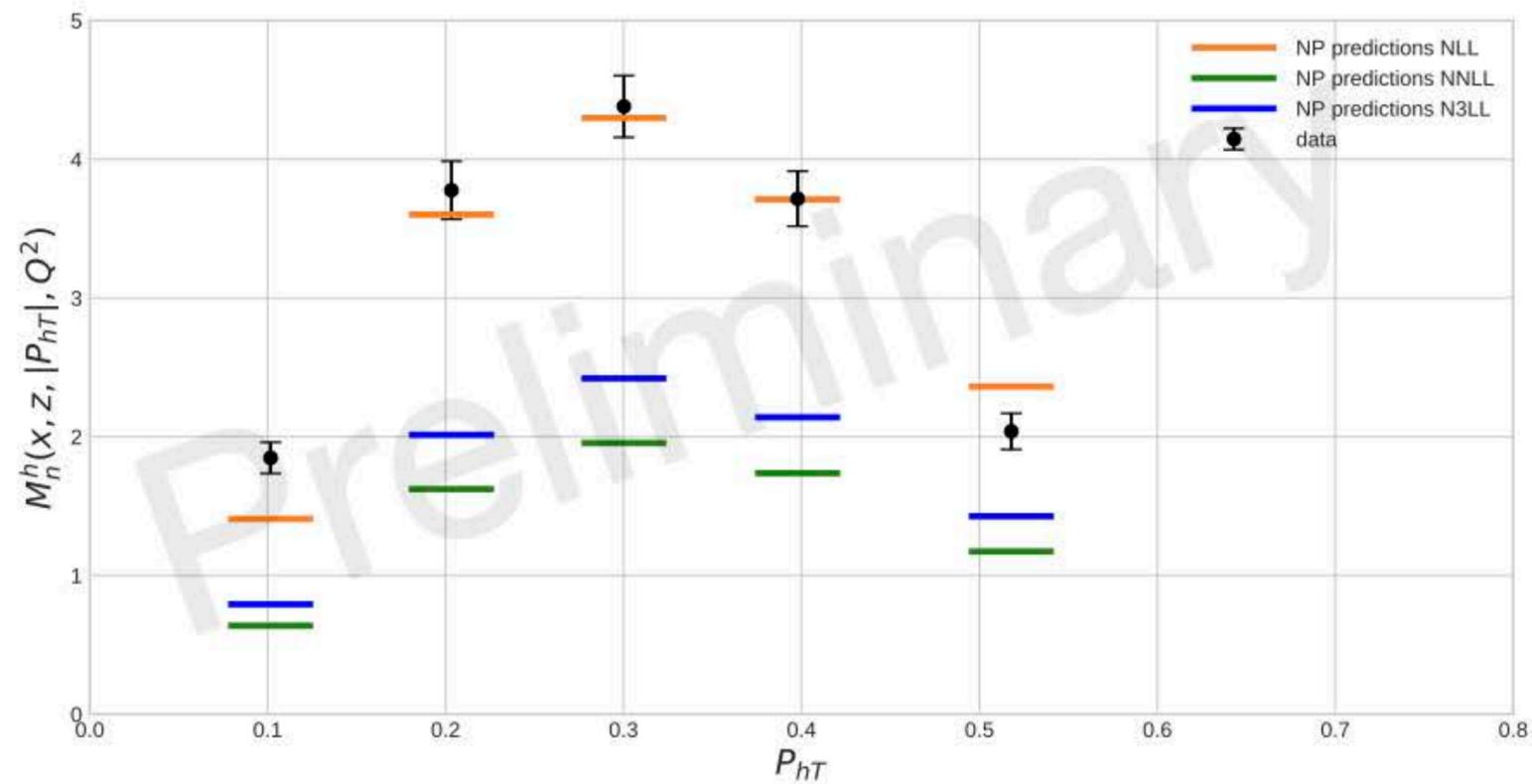
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# Source of W-term suppression

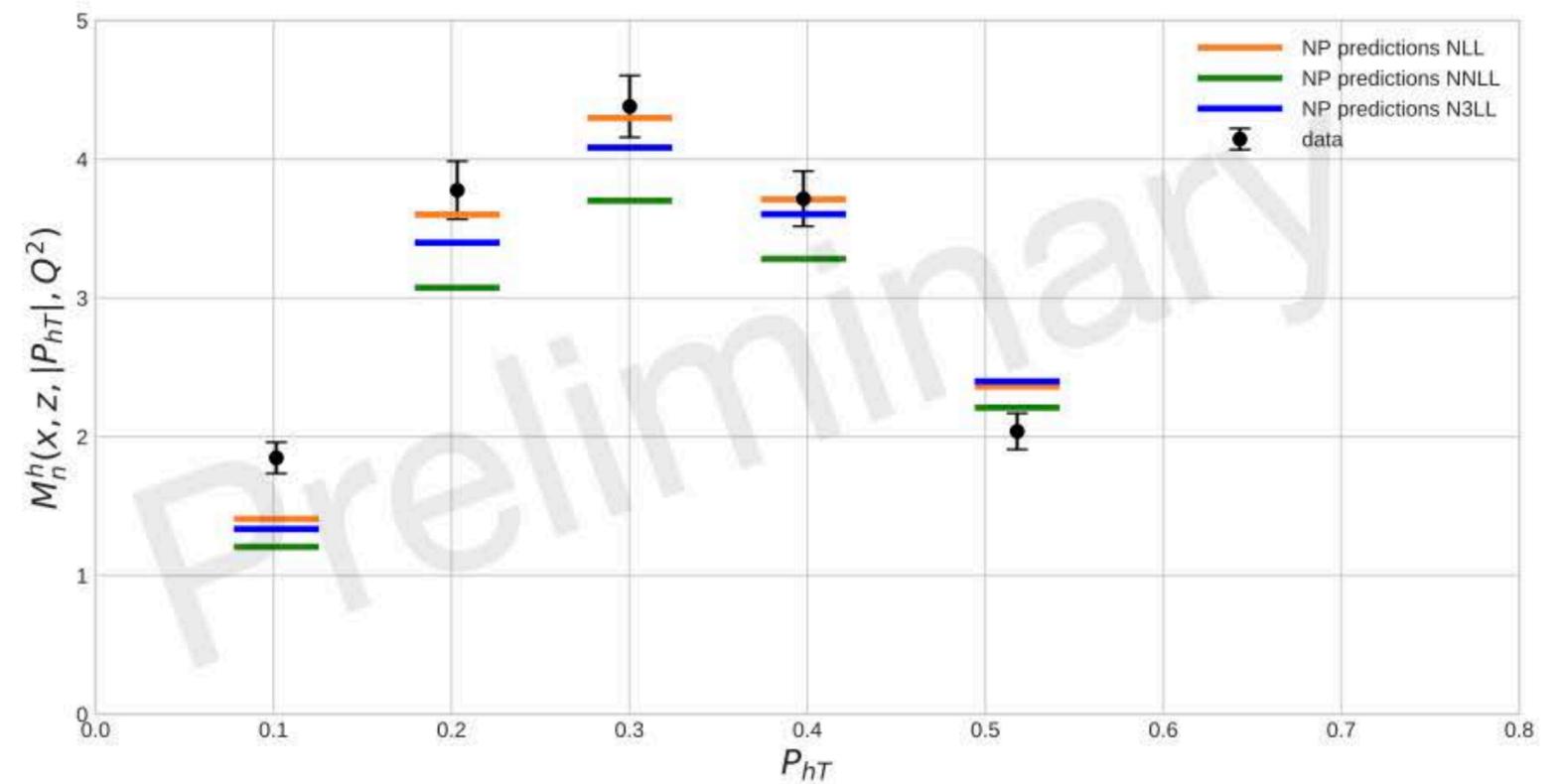
Present situation at low Q

HERMES multiplicity

Full Hard Factor



Hard Factor = 1

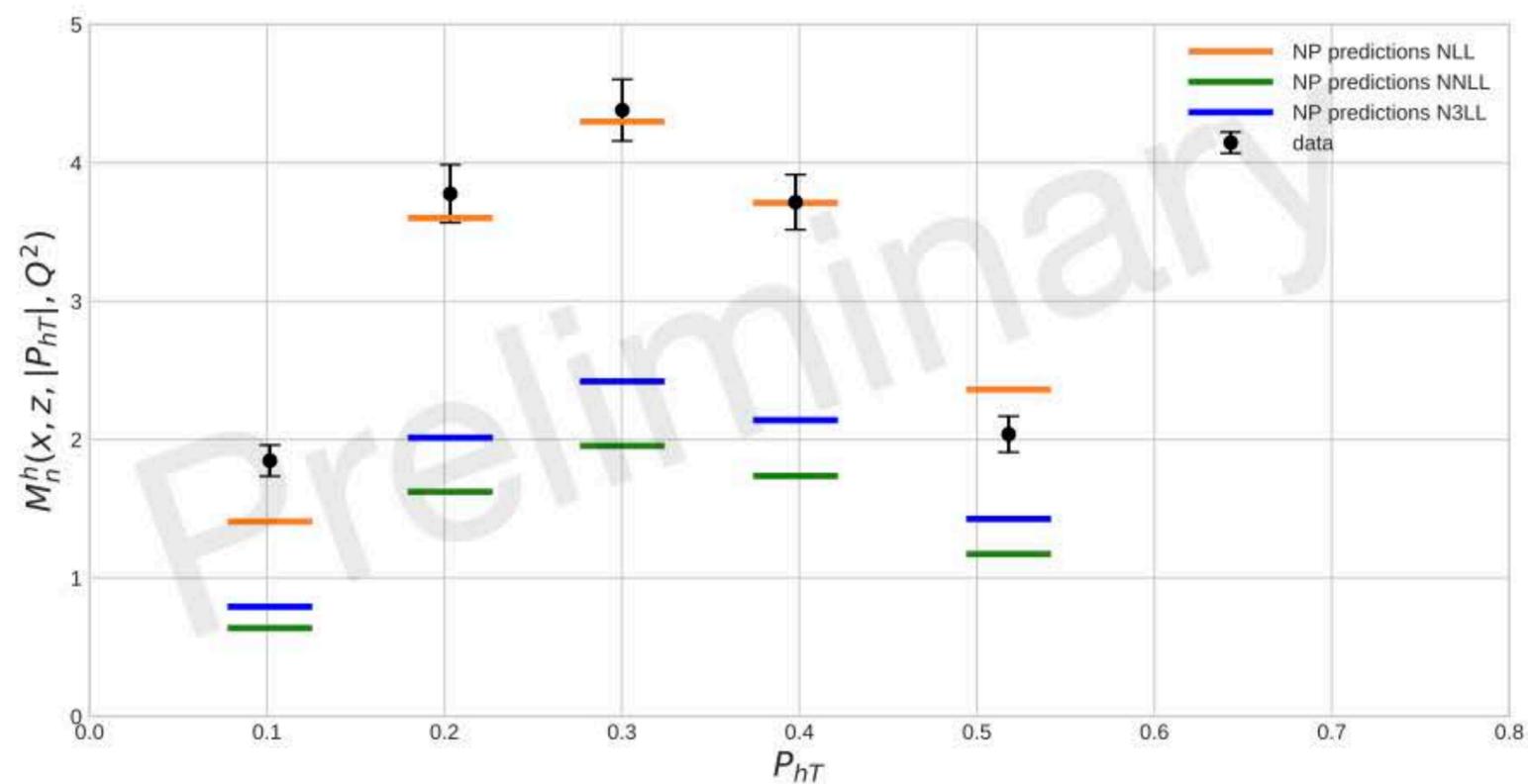


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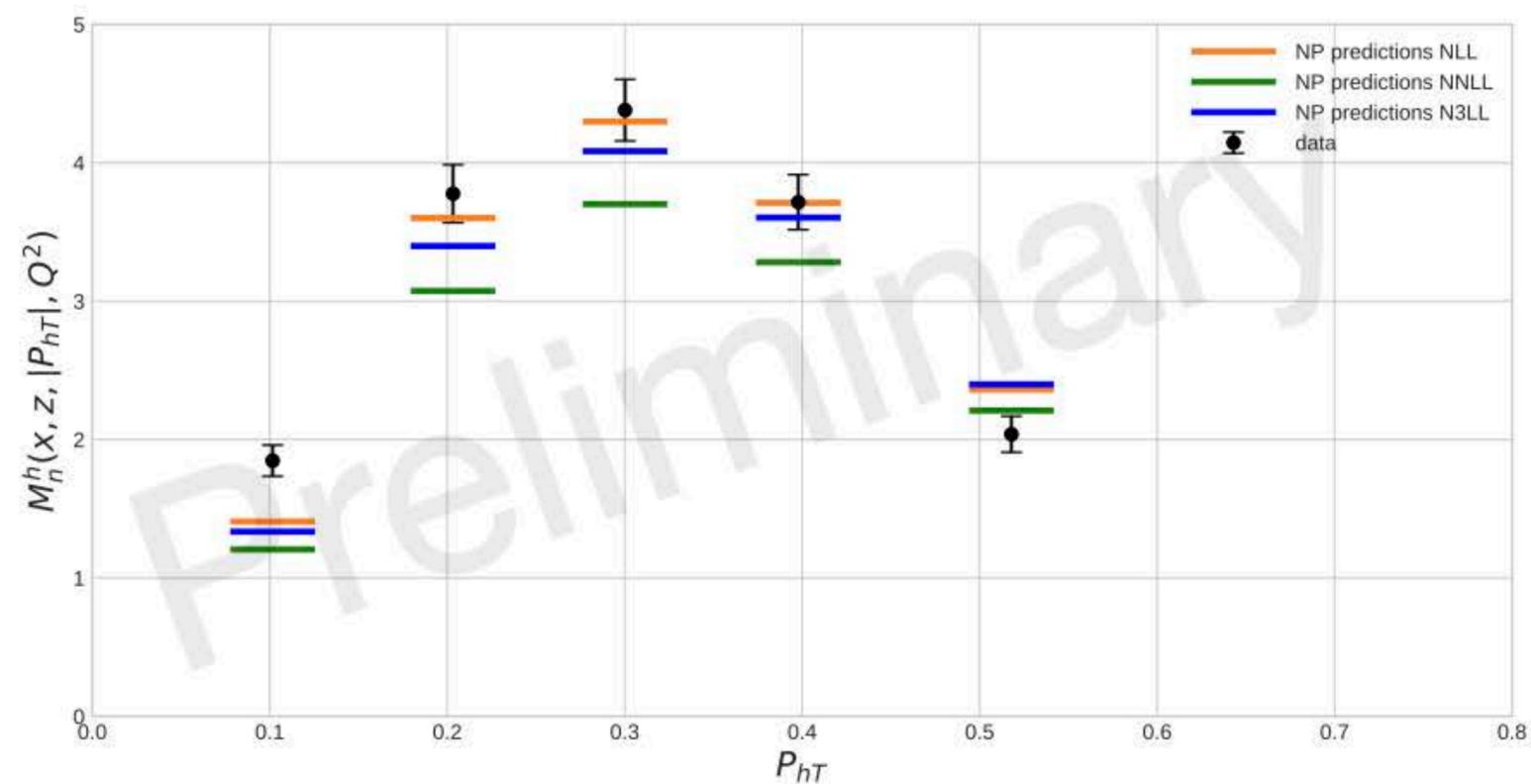
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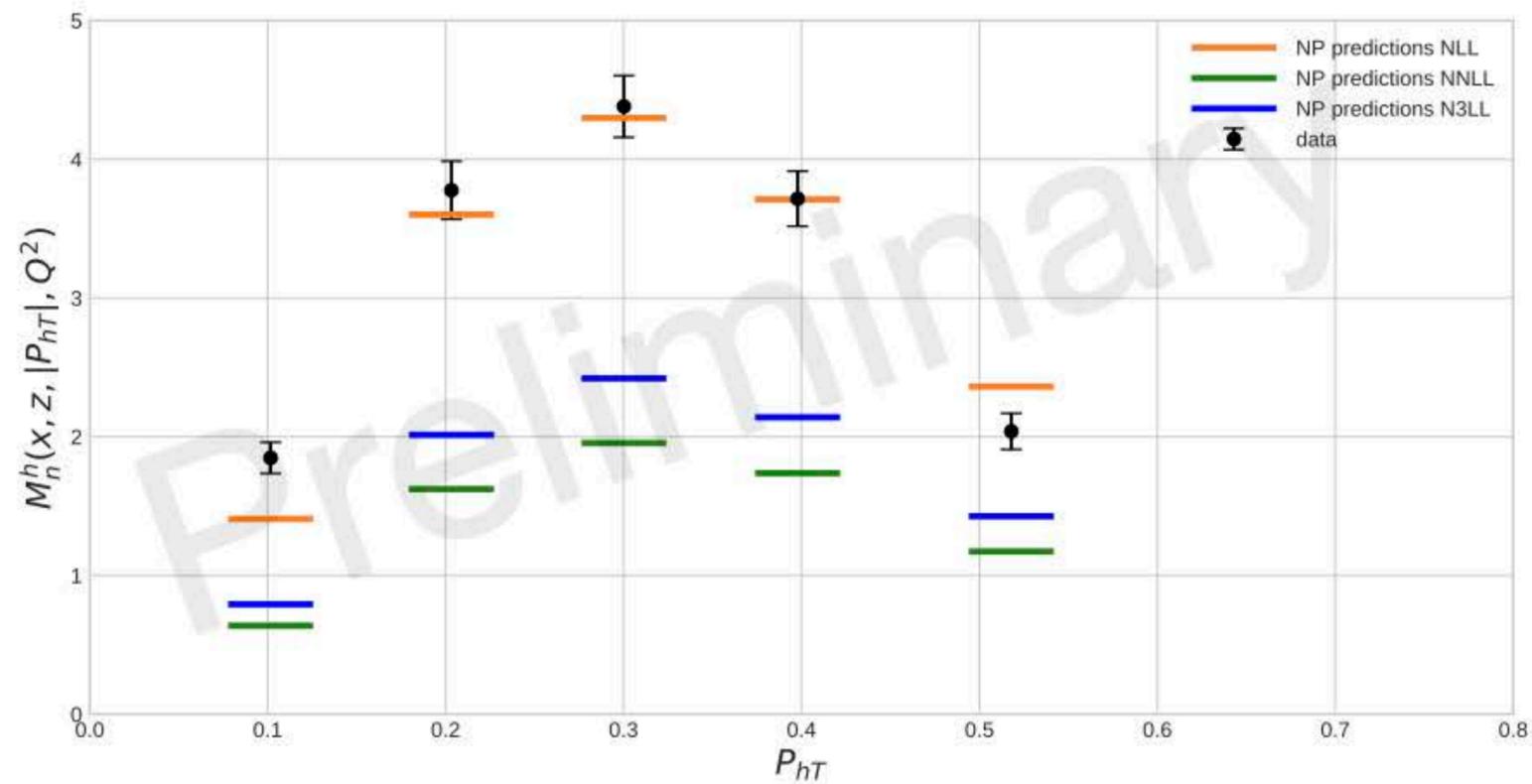


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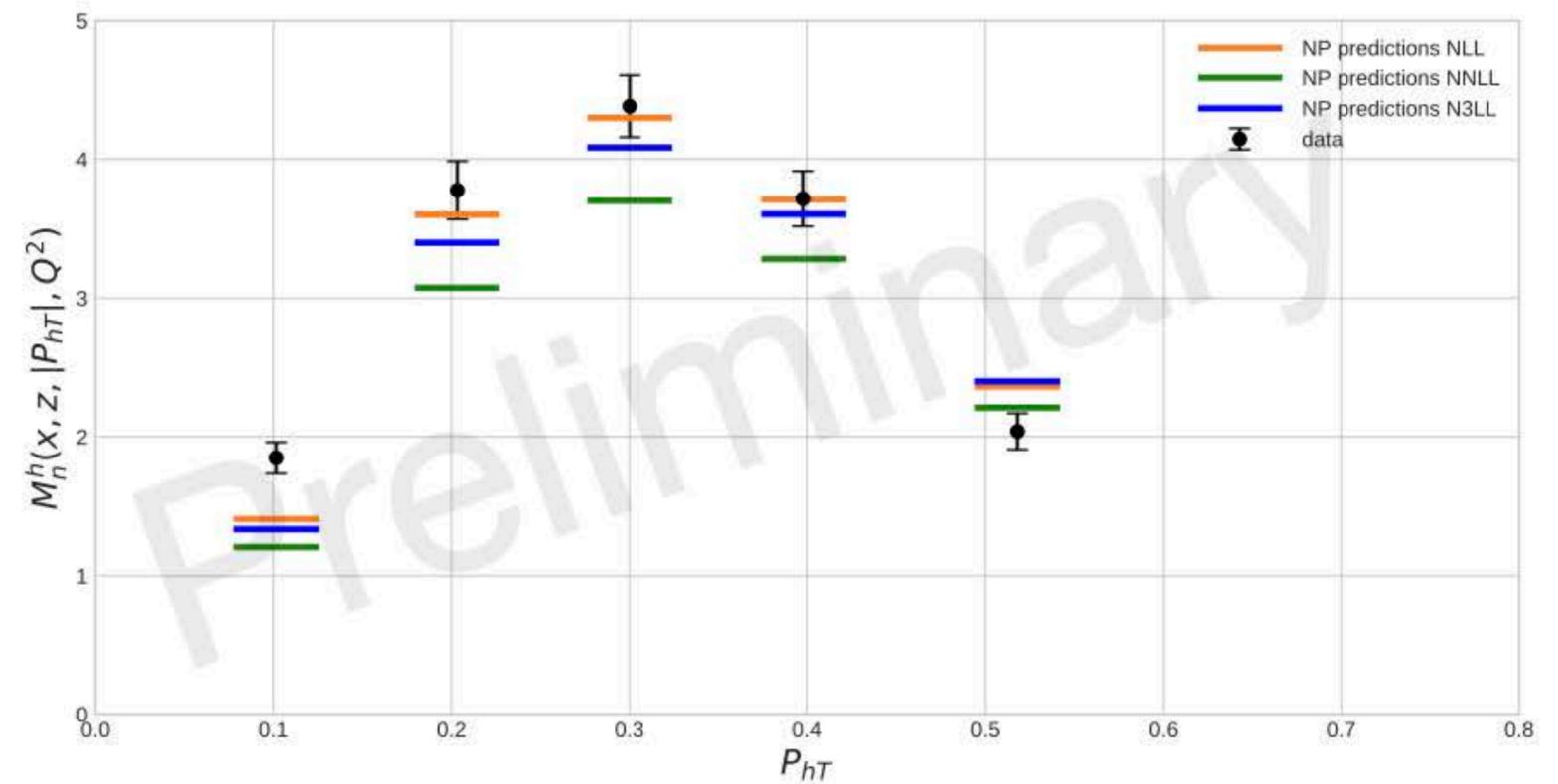
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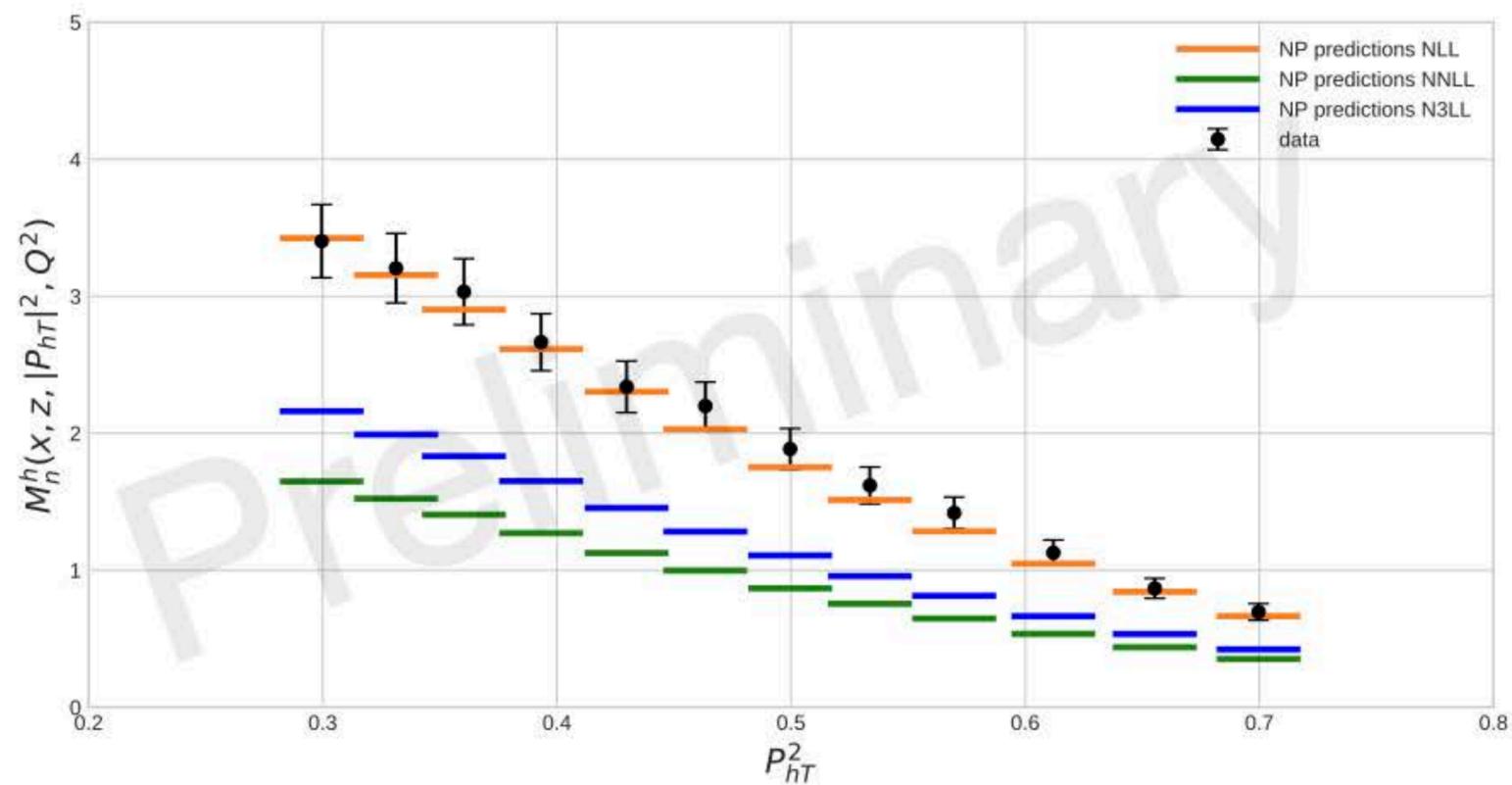


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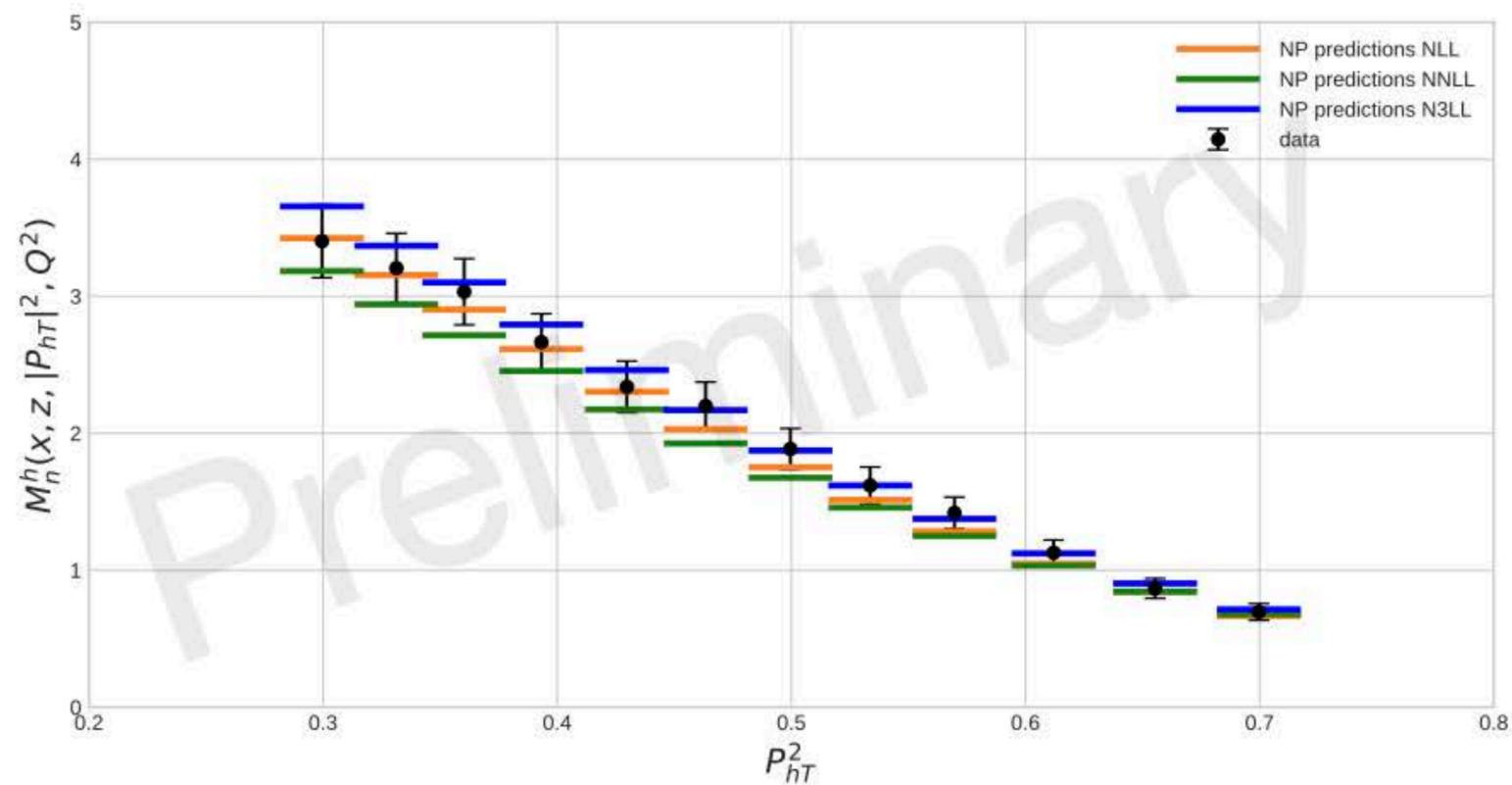
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

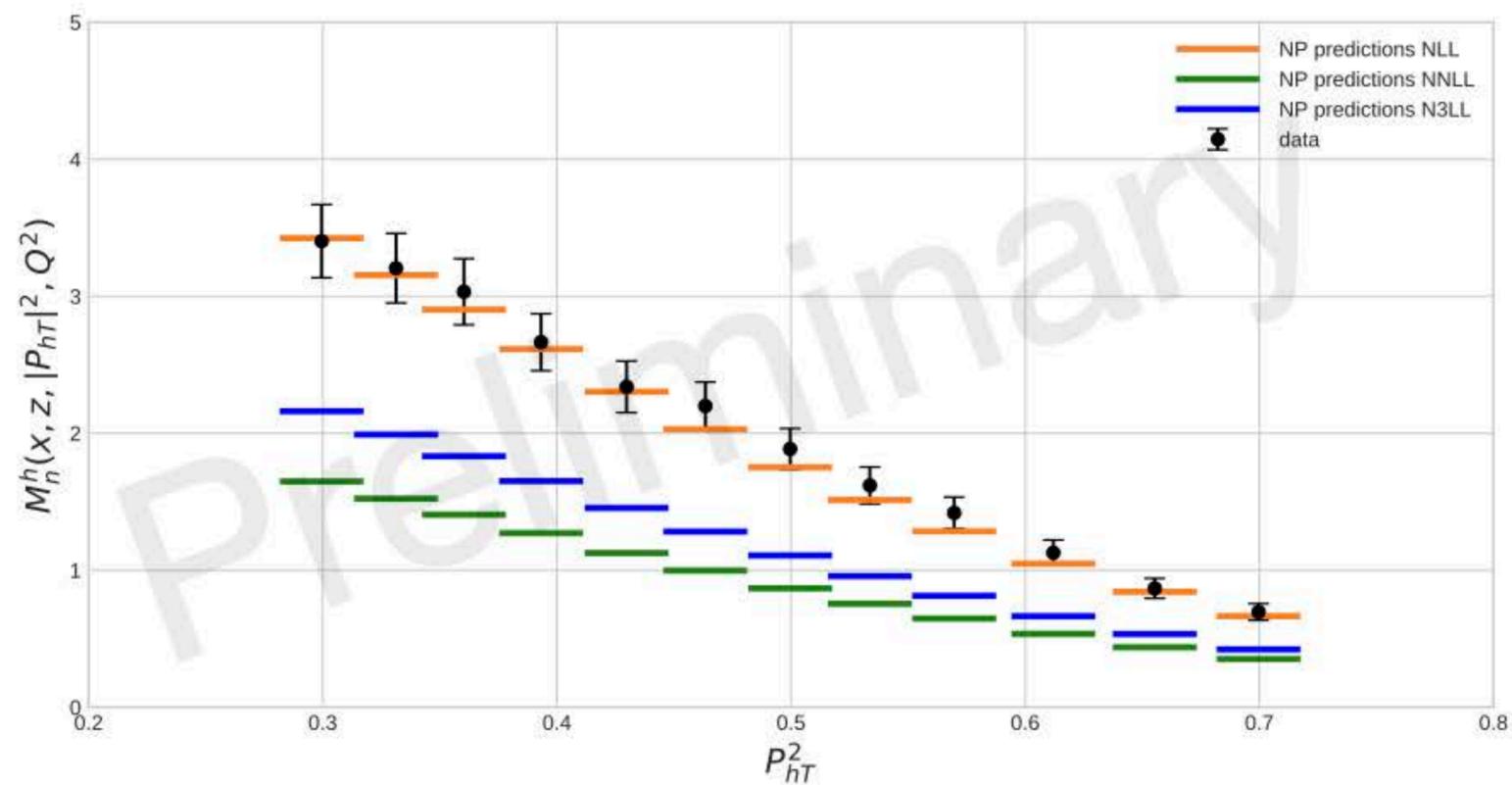


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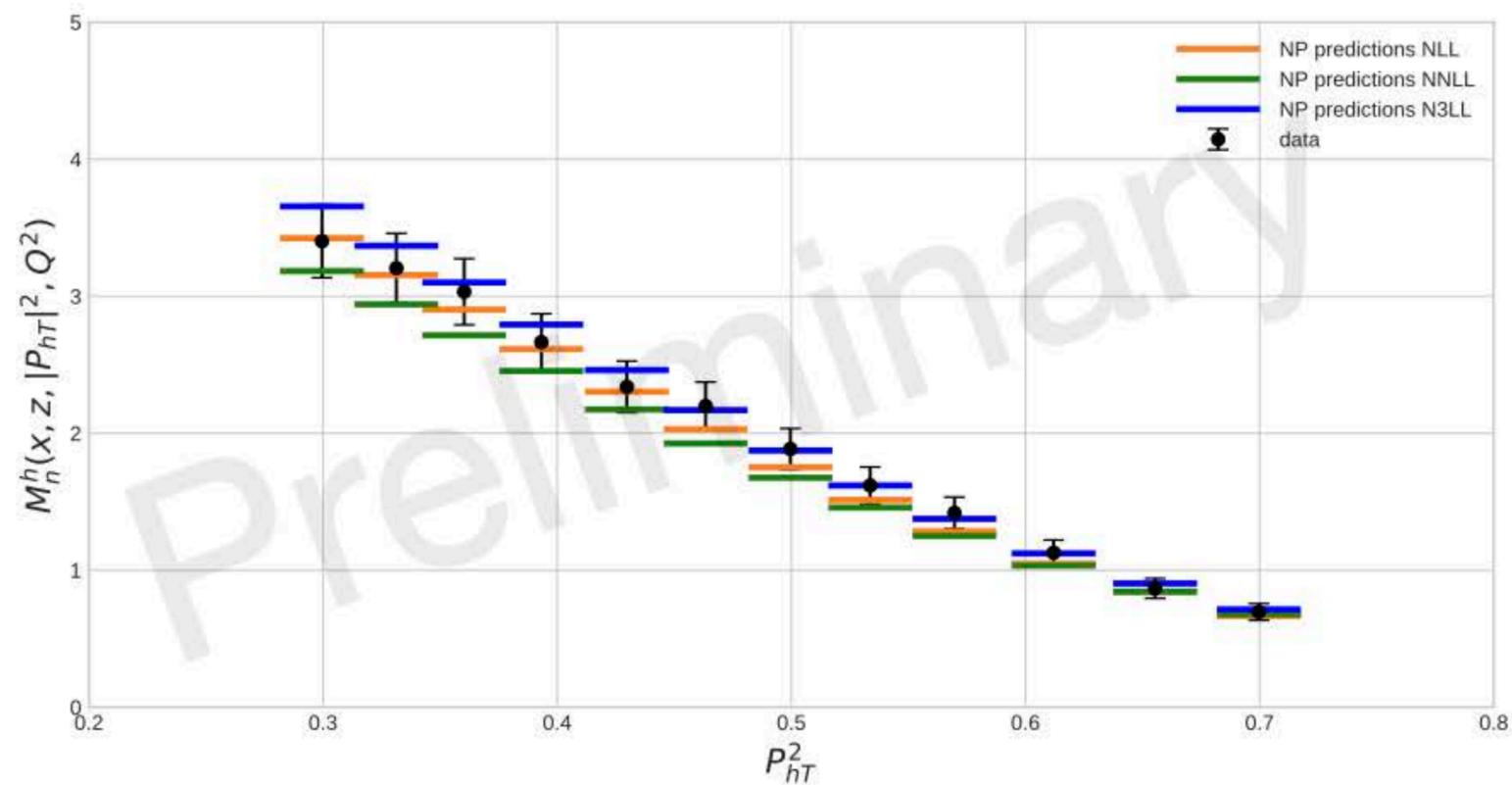
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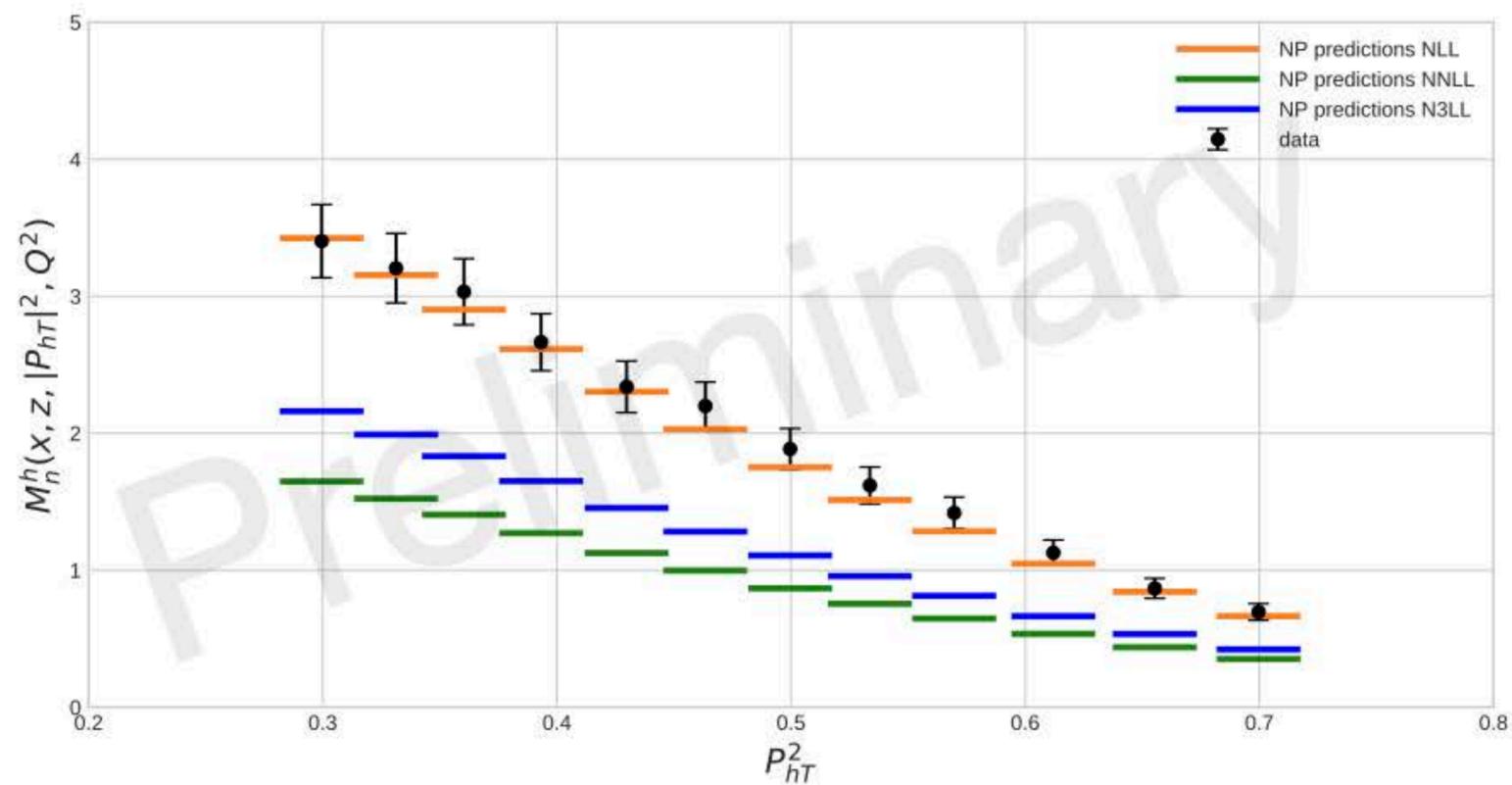


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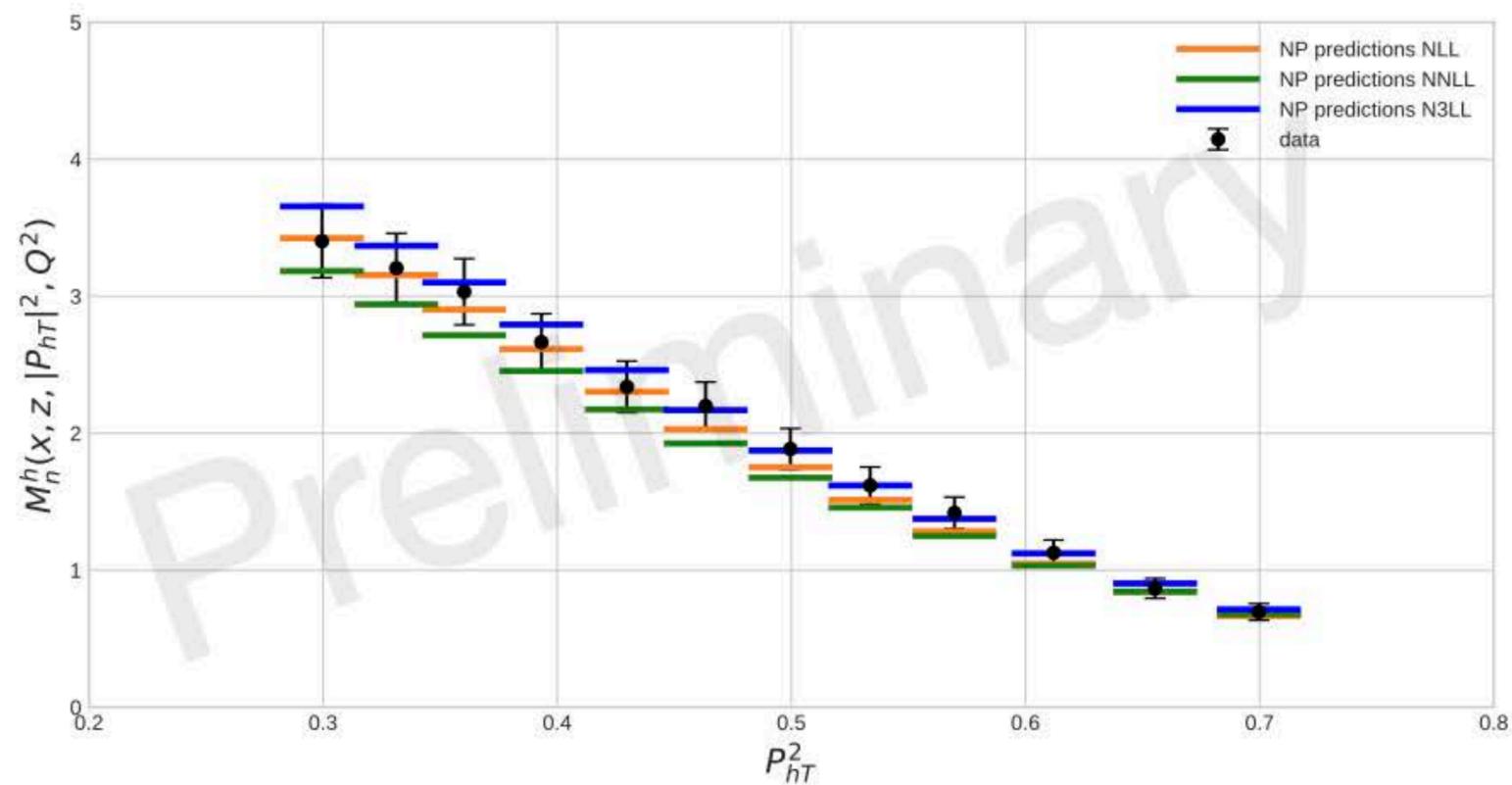
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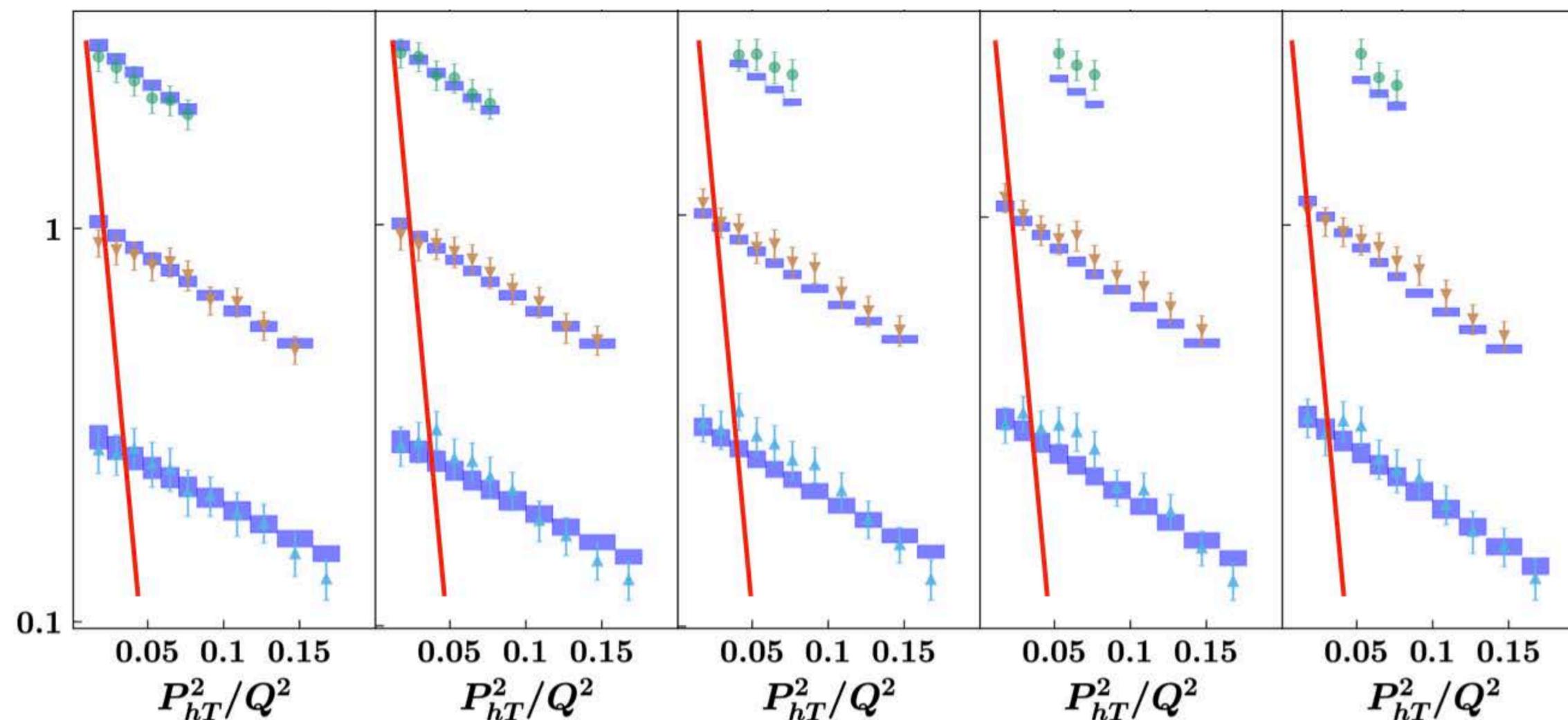


Hard Factor = 1



# MAPTMD22 – SIDIS data selection

$$\left. \frac{P_{hT}}{zQ} \right|_{\max} = 0.25$$



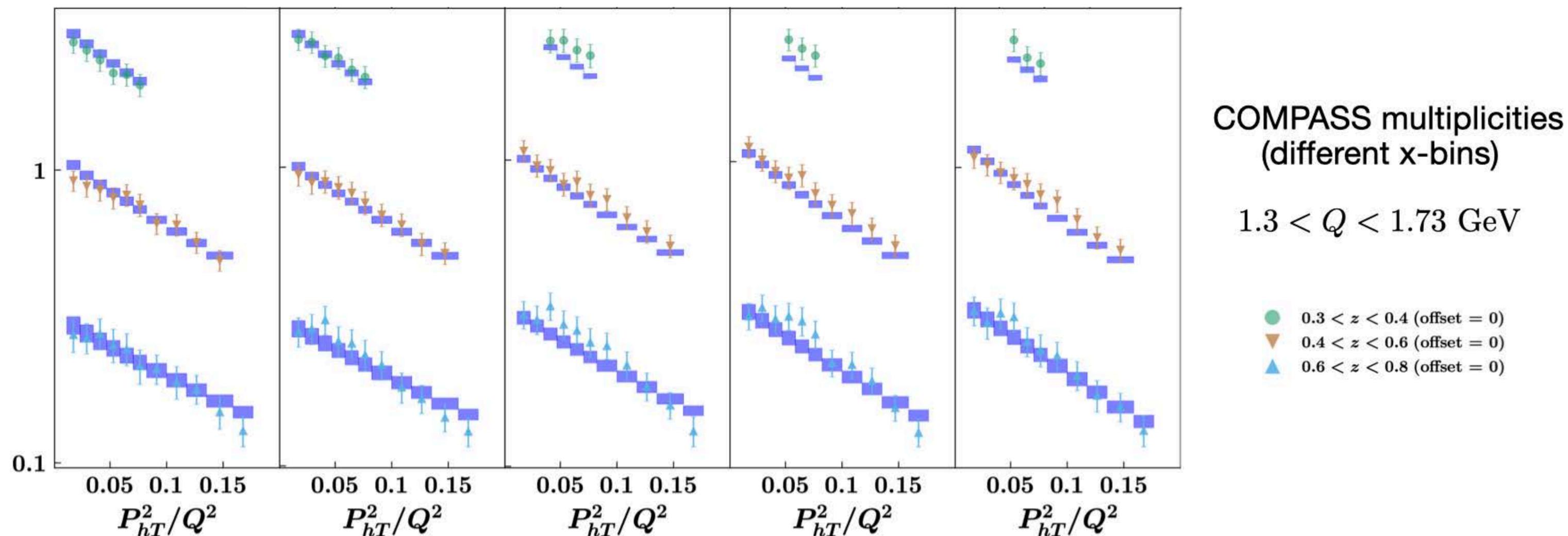
COMPASS multiplicities  
(different x-bins)

$1.3 < Q < 1.73$  GeV

- $0.3 < z < 0.4$  (offset = 0)
- ▼  $0.4 < z < 0.6$  (offset = 0)
- ▲  $0.6 < z < 0.8$  (offset = 0)

# MAPTMD22 – SIDIS data selection

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

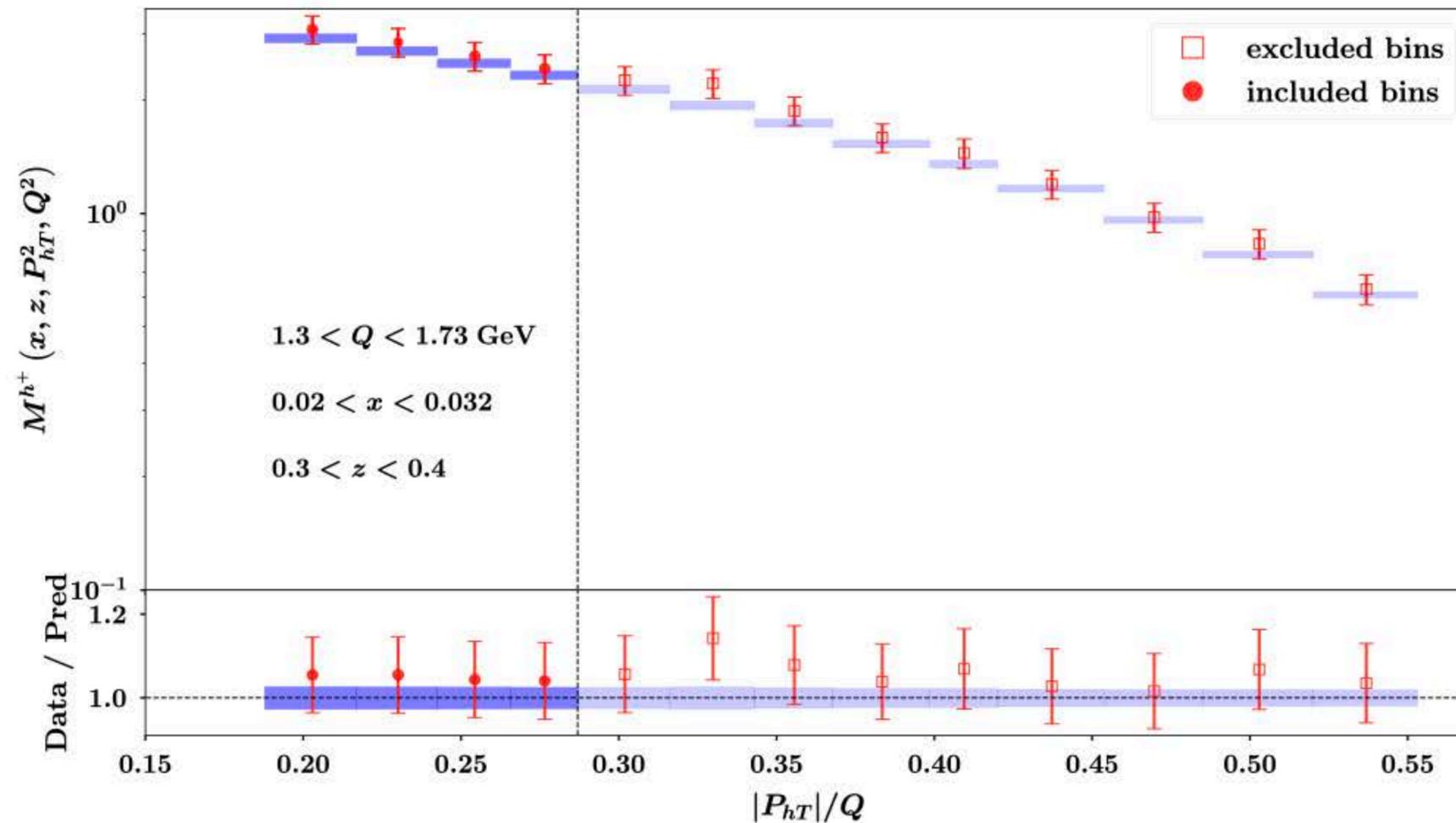


# MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

$$P_{hT}|_{\max} = \min[0.2Q, 0.7zQ] + 0.5 \text{ GeV}$$

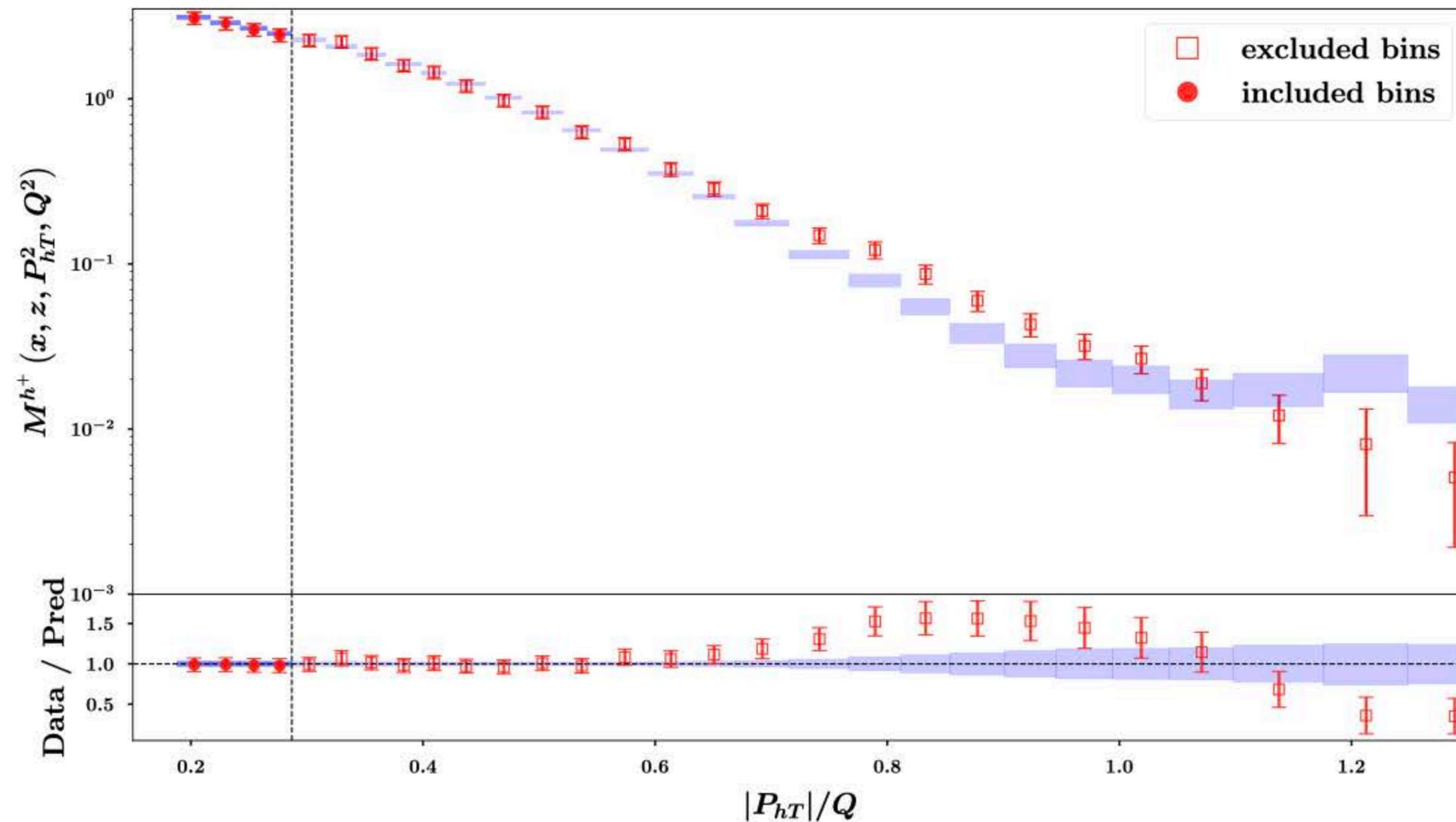


# MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Total number of points



# Cut $q_T/Q$ for SIDIS dataset

$$P_{hT}|_{\max} = \min[\min[c_1 Q, c_2 z Q] + c_3 \text{ GeV}, c_4 z Q]$$

baseline  
(c)

$$\begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 1 \end{cases}$$

$q_T/Q = 0.4$   
(a)

$$\begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 0.4 \end{cases}$$

< baseline  
(b)

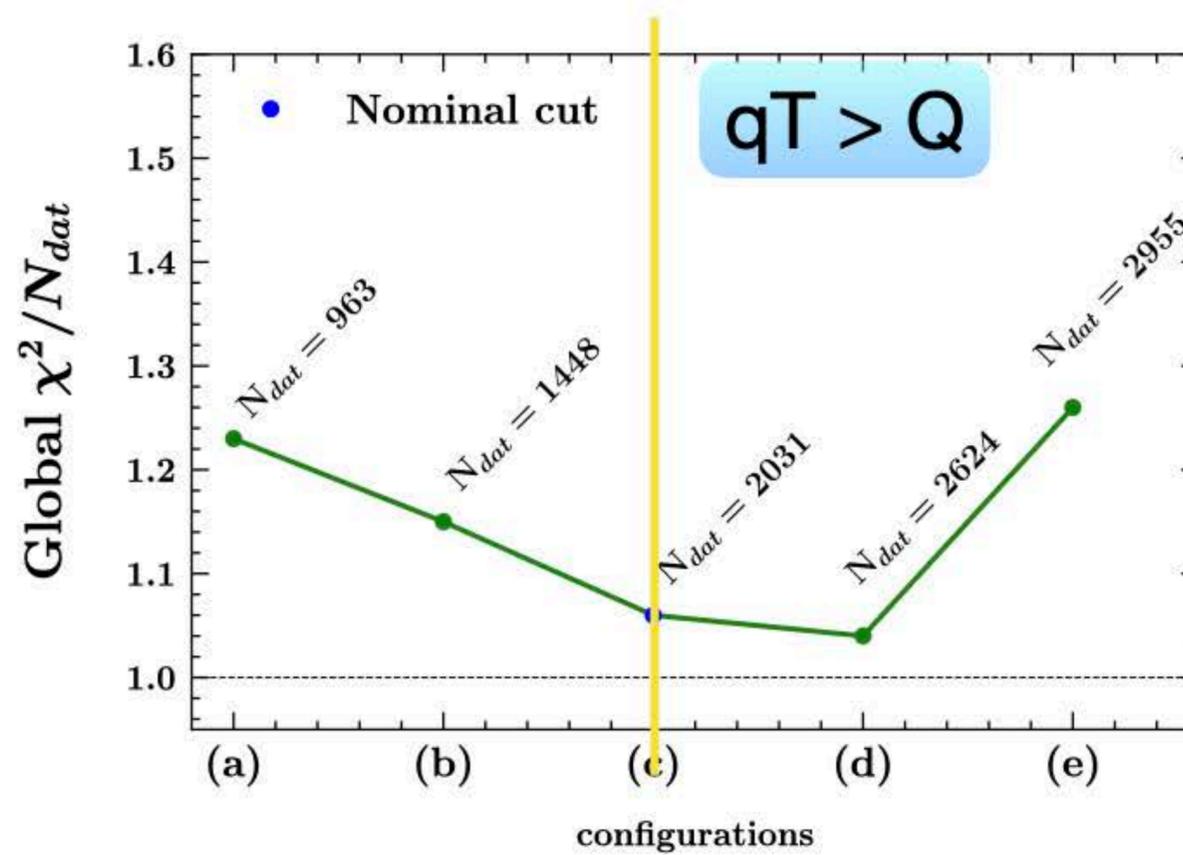
$$\begin{cases} c_1 = 0.15 \\ c_2 = 0.4 \\ c_3 = 0.2 \\ c_4 = 1 \end{cases}$$

> baseline  
(d)

$$\begin{cases} c_1 = 0.2 \\ c_2 = 0.6 \\ c_3 = 0.4 \\ c_4 = \infty \end{cases}$$

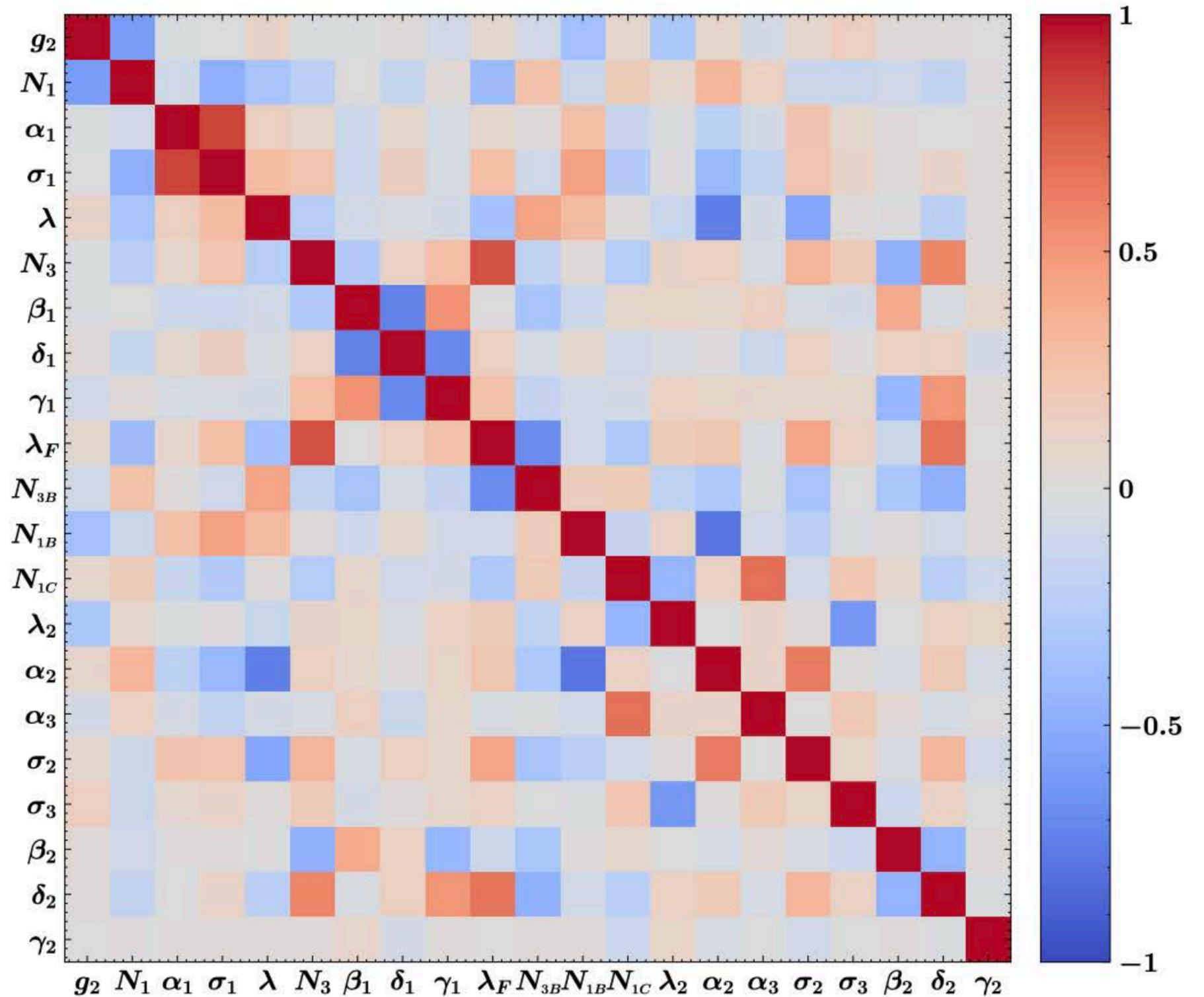
PV17  
(e)

$$\begin{cases} c_1 = 0.2 \\ c_2 = 0.7 \\ c_3 = 0.5 \\ c_4 = \infty \end{cases}$$



# Results of the baseline fit

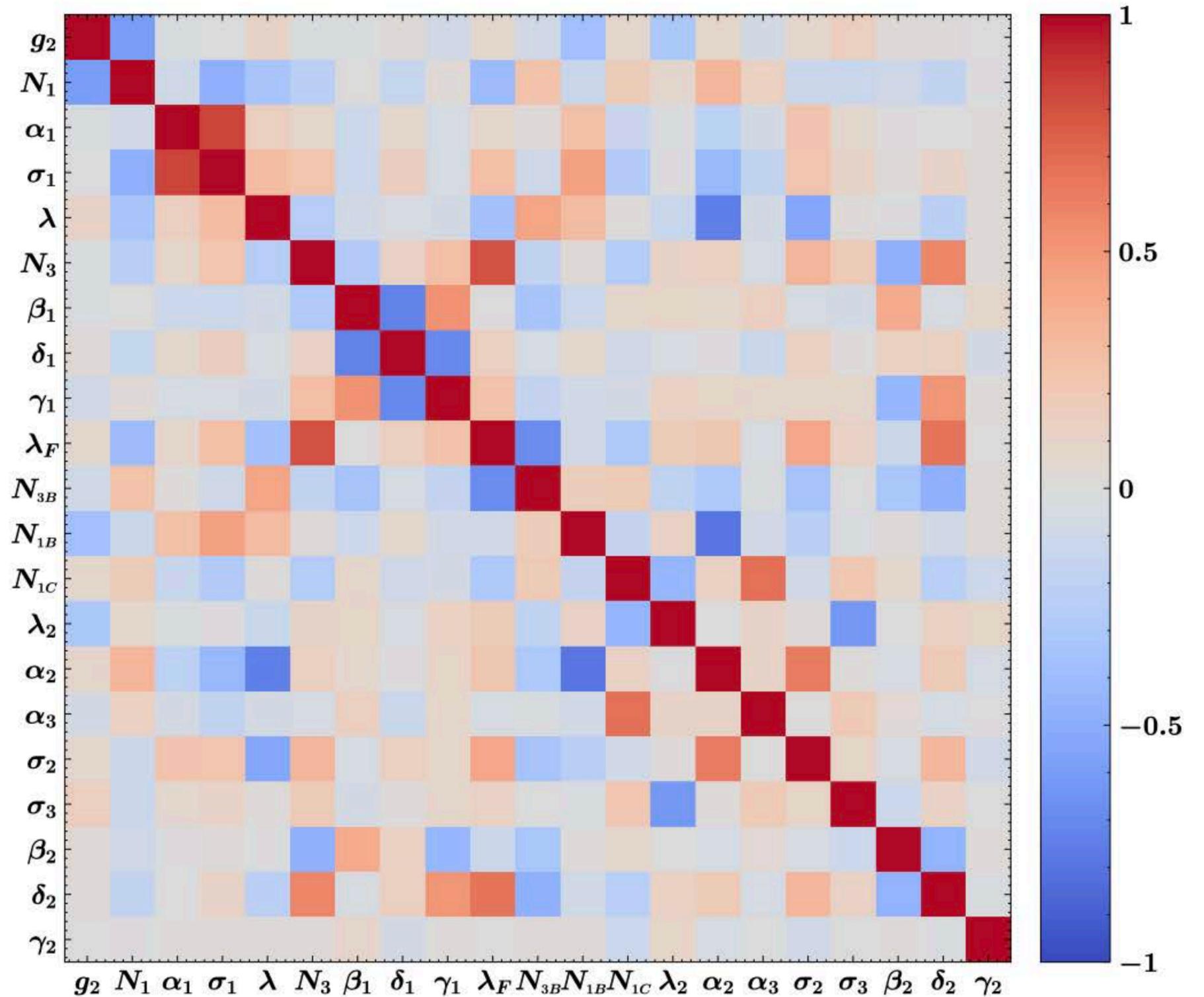
Error propagation  
↓  
250 Montecarlo replicas



# Results of the baseline fit

Error propagation  
↓  
250 Montecarlo replicas

Correlation matrix  
↓  
Hints of the  
appropriateness of the  
chosen functional form



# Results of the baseline fit

Parameter	Average over replicas
$g_2$ [GeV]	$0.248 \pm 0.008$
$N_1$ [GeV <sup>2</sup> ]	$0.316 \pm 0.025$
$\alpha_1$	$1.29 \pm 0.19$
$\sigma_1$	$0.68 \pm 0.13$
$\lambda$ [GeV <sup>-1</sup> ]	$1.82 \pm 0.29$
$N_3$ [GeV <sup>2</sup> ]	$0.0055 \pm 0.0006$
$\beta_1$	$10.23 \pm 0.29$
$\delta_1$	$0.0094 \pm 0.0012$
$\gamma_1$	$1.406 \pm 0.084$
$\lambda_F$ [GeV <sup>-2</sup> ]	$0.078 \pm 0.011$
$N_{3B}$ [GeV <sup>2</sup> ]	$0.2167 \pm 0.0055$
$N_{1B}$ [GeV <sup>2</sup> ]	$0.134 \pm 0.017$
$N_{1C}$ [GeV <sup>2</sup> ]	$0.0130 \pm 0.0069$
$\lambda_2$ [GeV <sup>-1</sup> ]	$0.0215 \pm 0.0058$
$\alpha_2$	$4.27 \pm 0.31$
$\alpha_3$	$4.27 \pm 0.13$
$\sigma_2$	$0.455 \pm 0.050$
$\sigma_3$	$12.71 \pm 0.21$
$\beta_2$	$4.17 \pm 0.13$
$\delta_2$	$0.167 \pm 0.006$
$\gamma_2$	$0.0007 \pm 0.0110$

