TMD distributions at the next-to-leading power.

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Outline

Why?

Tools: background field approach,
position space computation and
multipole expansion

Results and discussion:
NLO matching for quark TMDPDFs and LO matching for gluon TMDPDFs

What is the matching?

Selection of first few terms in the light-cone OPE for the TMD operator

Small-b matching schematically

$$F(x,b) = \underbrace{C(x,\ln(\mu b)) \otimes f(x,\mu)}_{\text{Contains quark-gluon mixing!}} + \mathcal{O}(b^2)$$

Why is it necessary?

It greatly increase agreement between theory and experiment

Reduces parametric freedom in model building

Quark TMD distributions

$$\Phi^{[\Gamma]}(x,b)=\frac{1}{2}\int\frac{dz}{2\pi}e^{-ixzp^+}\langle p,S|\bar{q}(zn+b)[zn+b,-\infty n+b]\Gamma[-\infty n,0]q(0)|p,S\rangle$$

$$\Gamma\in\{\gamma^+,\gamma^+\gamma_5,i\sigma^{\alpha+}\gamma_5\}$$

	U	H	T
U	f_1 (tw2)		$h_1^\perp$ (tw3)
L		g_1 (tw2)	$h_{1L}^\perp$ (tw2 & tw3)
T	$f_{1T}^\perp$ (tw3)	g_{1T} (tw2 & tw3)	h_1 (tw2) $h_{1T}^\perp$ (tw3 & tw4)

Quark pol.

Proton pol.

Collinear twist of the matching distribution

TMD twist

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Collinear distributions

Twist-2

Quark case

$$\Phi^{[\Gamma]}(x)=\frac{1}{2}\int\frac{dz}{2\pi}e^{-ixzp^+}\langle p,S|\bar q(zn)[zn,0]\Gamma q(0)|p,S\rangle$$

Gluon case

$$\int\frac{dz}{2\pi}e^{-ixzp^+}\langle p,S|F_{\mu+}(zn)[zn,0]F_{\nu+}(0)|p,S\rangle$$
$$=\frac{xp^+}{2}\left(-\frac{g_T^{\mu\nu}f_g(x)}{1-\epsilon}-\frac{i\epsilon_T^{\mu\nu}\Delta f_g(x)}{(1-\epsilon)(1-2\epsilon)}\right)$$

ε-factors to ensure
same result in any dimension

Twist-3

$$\langle p,S|g\bar q(z_1n)F^{\mu+}(z_2n)\gamma^+q(z_3n)|p,S\rangle$$
$$=2\epsilon_T^{\mu\nu}s_\nu(p^+)^2M\int[dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}T(x_1,x_2,x_3)$$
$$\langle p,S|g\bar q(z_1n)F^{\mu+}(z_2n)\gamma^+\gamma^5q(z_3n)|p,S\rangle$$
$$=2is_T^{\mu}(p^+)^2M\int[dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\Delta T(x_1,x_2,x_3)$$
$$\langle p,S|g\bar q(z_1n)F^{\mu+}(z_2n)i\sigma^{\nu+}\gamma^5q(z_3n)|p,S\rangle$$
$$=2(p^+)^2M\int[dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\left(\epsilon_T^{\mu\nu}E(x_1,x_2,x_3)+i\lambda g_T^{\mu\nu}H(x_1,x_2,x_3)\right)$$

$$\langle p,S|igf^{ABC}F_A^{\mu+}(z_1n)F_B^{\nu+}(z_2n)F_C^{\rho+}(z_3n)|p,S\rangle$$
$$=(p^+)^3M\int[dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\sum_i t_i^{\mu\nu\rho}F_i^+(x_1,x_2,x_3)$$
$$\langle p,S|gd^{ABC}F_A^{\mu+}(z_1n)F_B^{\nu+}(z_2n)F_C^{\rho+}(z_3n)|p,S\rangle$$
$$=(p^+)^3M\int[dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\sum_i t_i^{\mu\nu\rho}F_i^-(x_1,x_2,x_3)$$

$$\begin{aligned}t_2^{\mu\nu\rho}&=s_T^\alpha\epsilon_T^{\mu\alpha}g_T^{\nu\rho}+s_T^\alpha\epsilon_T^{\nu\alpha}g_T^{\rho\mu}+s_T^\alpha\epsilon_T^{\rho\alpha}g_T^{\mu\nu}\\t_4^{\mu\nu\rho}&=-s_T^\alpha\epsilon_T^{\mu\alpha}g_T^{\nu\rho}+2s_T^\alpha\epsilon_T^{\nu\alpha}g_T^{\rho\mu}-s_T^\alpha\epsilon_T^{\rho\alpha}g_T^{\mu\nu}\\t_6^{\mu\nu\rho}&=s_T^\alpha\epsilon_T^{\mu\alpha}g_T^{\nu\rho}-s_T^\alpha\epsilon_T^{\rho\alpha}g_T^{\mu\nu}\end{aligned}$$

+ evanescent structures!

Small-b expansion, tree level

No special technique needed

Expand the operator and use equation of motion to remove `bad' components

$$\Phi^{[\Gamma]}(x,b) = \phi^{[\Gamma]}(x) + b^\mu \phi^{[\Gamma]}_\mu(x) + b^\mu b^\nu \phi^{[\Gamma]}_{\mu\nu}(x) + \ldots$$

$$\begin{aligned}
\phi^{[\Gamma]}(x) &= \frac{1}{2}\int \frac{dz}{2\pi} e^{-ixzp_+} \langle p,S|\bar{q}(z,n)[zn,0]\Gamma q(0)|p,S\rangle \\
\phi^{[\Gamma]}_\mu(x) &= \frac{1}{2}\int \frac{dz}{2\pi} e^{-ixzp_+} \langle p,S|\bar{q}(z,n)[zn,-\infty n]\overleftarrow{D_\mu}[-\infty n,0]\Gamma q(0)|p,S\rangle
\end{aligned}$$

Reduction to standard collinear twist-3 distributions is tricky. One approach:

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It keeps track of T-evenness/oddness of the `parent' TMD distribution

Small-b expansion, tree level

Twist-2 is trivial

$$f_1(x,b) \sim f_1(x) + \mathcal{O}(b^2)$$
$$g_1(x,b) \sim g_1(x) + \mathcal{O}(b^2)$$
$$h_1(x,b) \sim h_1(x) + \mathcal{O}(b^2)$$

Twist-2 T-odd TMDs have `simple' matching to pure twist-3 collinear distributions

$$f_{1T}^{\perp}(x,b) = \pm \pi T(-x,0,x)$$
$$h_1^{\perp}(x,b) = \mp \pi E(-x,0,x)$$

Upper (lower) sign for Drell-Yan (SIDIS)-like Gauge link structure

Twist-2 T-even TMDs have a more complex matching with both a twist-2 and twist-3 collinear contributions

$$g_{1T}^{\mathrm{tw2}}(x,b) = x \int_x^1 \frac{dy}{y} g_1(y)$$

$$h_{1L}^{\perp,\mathrm{tw2}}(x,b) = -x^2 \int_x^1 \frac{dy}{y} h_1(y)$$

$$g_{1T}^{\mathrm{tw3}}(x,b) = 2x \int [dy] \int_0^1 d\alpha \delta(x - \alpha y_3) \left(\frac{\Delta T(y_{1,2,3})}{y_2^2} + \frac{T(y_{1,2,3}) - \Delta T(y_{1,2,3})}{2 y_2 y_3} \right)$$

$$h_{1L}^{\perp,\mathrm{tw3}}(x,b) = -2x \int_0^1 d\alpha \int [dy] \alpha \delta(x - \alpha y_3) H(y_1,y_2,y_3) \frac{y_3 - y_2}{y_2^2 y_3}$$

One loop computation

Background field approach

Why? It allows to, up to a certain degree, ignore the specific external state and focus only on the operator
It is also extremely handy for higher-twist operators

Basic idea: mode separation

$$\langle S_1|\mathcal{U}|S_2\rangle = \int D\Phi\ \Psi^*_{S_1}[\Phi]\mathcal{U}[\Phi]\Psi_{S_2}[\Phi]e^{iS[\Phi]} \qquad \Phi = \phi(\mu) + \psi(\mu)$$

$\mathrm{mom} < \mu$ Slow modes
aka background modes

$\mathrm{mom} > \mu$ Fast modes
aka dynamical modes

Suppose that hadron contains only slow modes, i.e. $\Psi_{S_i}[\phi+\psi] \sim \Psi_{S_i}[\psi]$

$$\langle S_1|\mathcal{U}|S_2\rangle \sim \int D\psi\ \Psi^*_{S_1}[\psi]\Psi_{S_2}[\psi]e^{iS[\psi]} \underbrace{\left[\int D\phi U[\phi+\psi]e^{iS[\psi+\phi]-iS[\psi]}\right]}_{\text{Perturbative computable}}$$

How do we separate the fast and slow modes in TMD physics?

$$\{(n\cdot\partial),(\bar{n}\cdot\partial),\partial_T\}\psi \lesssim p^+\{1,\lambda^2,\lambda\}\psi \qquad \lambda \ll 1$$

The hadron (external state) defines with its momentum a direction
The fields scale depending on their momentum w.r.t. the external hadron

Background field approach preserves gauge invariance at each step

One can define two different gauges for dynamical and background fields
Common choice: Feynman-like gauges for dynamical sector and light-cone gauge for background

One can also derive scaling for the “good” and “bad” components of the fields

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Flow of the computation

The matrix element for a TMD is presented in a functional-integral form.
QCD fields are split into the dynamical and background, with corresponding momentum counting.

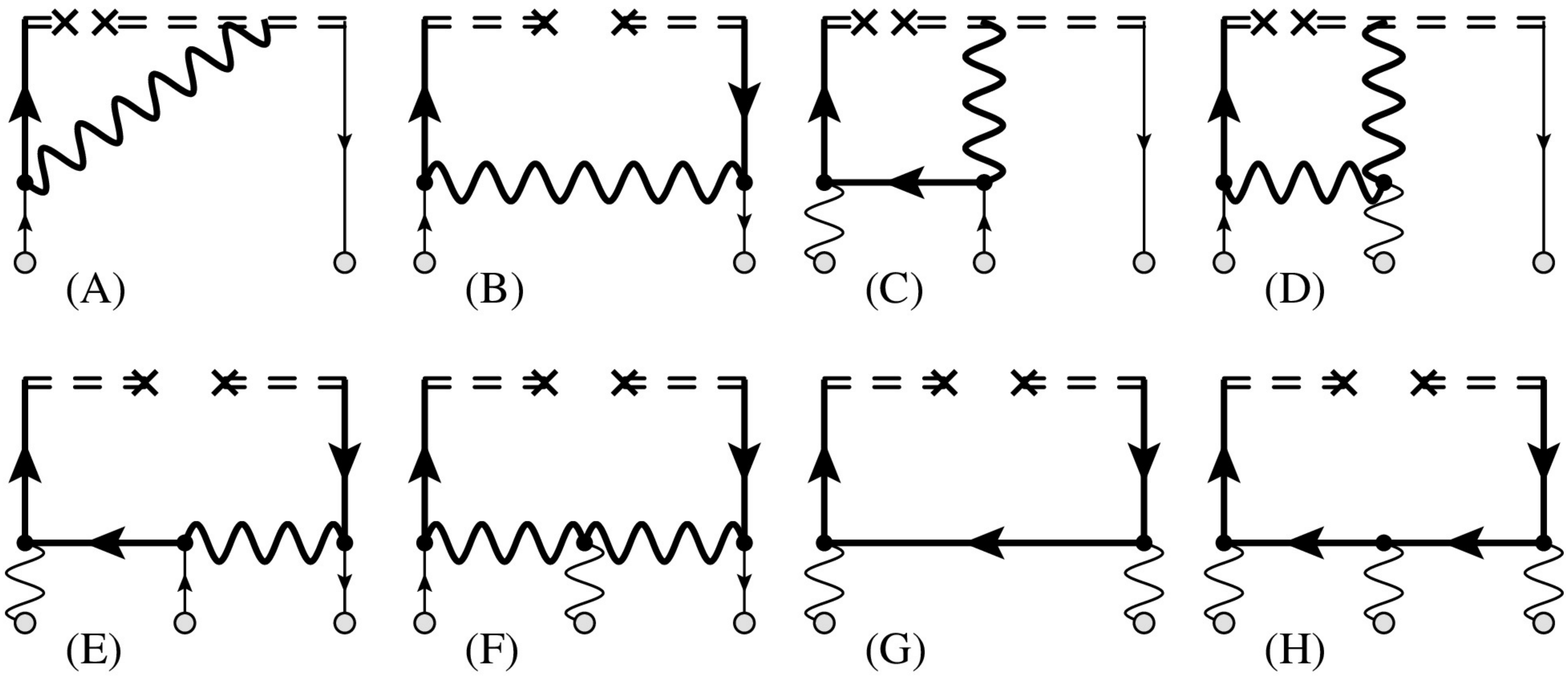
Expand in the coupling and in the number of fields
Integration of the dynamical modes to obtain the effective operator

Reduce the effective operator to combination of definite-twist operators via EOM

Renormalization $$\Phi_{\text{renor.}}(\mu,\zeta)=\underbrace{\Big(Z_{UV}^{-1}(\mu,\zeta)R^{-1}(\zeta)C_{\text{bare}}\otimes Z_\phi(\mu_{\text{OPE}})\Big)}_{C_{\text{renor.}}(\mu,\zeta,\mu_{\text{OPE}})}\otimes\phi_{\text{renor.}}(\mu_{\text{OPE}})$$

$$C^{\text{NLO}}_{\text{renorm}} = \mu^{2\epsilon}e^{\epsilon\gamma_E}C^{\text{NLO}}_{\text{bare}}+\Big[\mu^{2\epsilon}e^{\epsilon\gamma_E}2\left(\frac{-b^2}{4}\right)^{\epsilon}C_F\Gamma(-\epsilon)\left(\mathbf{L}_b-\mathbf{l}_\zeta+2\ln\left(\frac{\delta^+}{p^+}\right)-\psi(-\epsilon)-\gamma_E\right)-C_F\left(\frac{2}{\epsilon^2}+\frac{3+2\mathbf{l}_\zeta}{\epsilon}\right)-\frac{a_s}{\epsilon}\mathbb{H}\otimes\Big]C^{\text{LO}}$$

NLO diagrams



Disclaimer: we work with massless quark fields

For all the distributions, the final result has the following form

$$F(x,b;\mu,\zeta) = \underbrace{F^{(0)}(x)}_{\text{Tree-level}} + a_s \Bigg\{ C_F \left(-\mathbf{L}_b^2 + 2\mathbf{L}_b \mathbf{l}_\zeta + 3\mathbf{L}_b - \frac{\pi^2}{6} \right) F^{(0)}(x) \underbrace{-2\mathbf{L}_b \mathbb{H} \otimes F^{(0)}(x)}_{\text{Evolution kernel}} + \underbrace{F^{(1)}(x)}_{\text{Finite part of the coefficient function}} \Bigg\} + \mathcal{O}(a_s^2, b^2)$$

$$\mu^2 \frac{dF^{(0)}(x)}{d\mu^2} = 2a_s \mathbb{H} \otimes F^{(0)}(x)$$

$$\mathbf{L}_b = \ln\left(\frac{(-b^2)\mu^2}{4e^{-2\gamma_E}} \right)$$

`Bonus' of the computation

In the gluon sector it was needed the tree-level matching of the gluon TMDs at twist-3

$$\mathbb{O}^{\mu\alpha\nu}(z) = F^{\mu+}(zn+b)[zn, \pm\infty n]\overleftarrow{D}^{\alpha}[\pm\infty n, 0]F^{\nu+}(0)$$

$$\langle p, S| \left[\mathbb{O}^{\mu\alpha\nu}(z)\right]_{\text{tw2}} |p, S\rangle \quad = \quad \frac{\varepsilon_T^{\mu\nu} s_T^{\alpha} M}{2(1-\varepsilon)(1-2\varepsilon)} \int_0^1 d\alpha \int_{-\infty}^{\infty} dy e^{iy\alpha p^+ z} (\alpha p^+ y)^2 \Delta f_g(y),$$

$$\langle p, S| \left[\mathbb{O}^{\mu\alpha\nu}(z)\right]_{\text{tw3}} |p, S\rangle = t_2^{\mu\alpha\nu} M \ \text{FDF}_2^{\text{tw3}}(z) + t_4^{\mu\alpha\nu} M \ \text{FDF}_4^{\text{tw3}}(z) + t_6^{\mu\alpha\nu} M \ \text{FDF}_6^{\text{tw3}}(z)$$

$$\text{FDF}_2^{\text{tw3}}(z) = \mp i p_+^2 \pi \int_{-1}^{1} dy F_2^+(-y, 0, y) e^{iyp_+ z}$$

$$\text{FDF}_4^{\text{tw3}}(z) = \mp i p_+^2 \pi \int_{-1}^{1} dy F_4^+(-y, 0, y) e^{iyp_+ z}$$

$$\text{FDF}_6^{\text{tw3}}(z) = p_+^2 \int [dx](2F_2^+ + F_4^+ + F_6^+) \int_0^1 du \left(\frac{3x_1 + 2x_3}{x_2^2} u^2 e^{-iux_1 p^+ z} + \frac{x_3}{x_2^2} u^2 e^{iux_3 p^+ z} \right)$$

$$+ p_+^2 \sum_q \int [dx] 2T_q(x_1, x_2, x_3) \int_0^1 du u^2 e^{-ip_+ z u x_2}$$

Upper sign for SIDIS
Lower sign for DY

Conclusions

Complete NLO small-b matching for all quark TMDPDFs up to collinear twist-3 accuracy

Complete LO small-b matching for all gluon TMDPDFs up to collinear twist-3 accuracy

Background field approach: versatile and powerful tool
to disentangle the operator loop structure from the external states

For the future

Complete the NLO for the gluon distributions

Explore NLO for TMDFFs

Explore matching to collinear twist-4 (pretzelocity)

Small-b matching for TMDs of TMD twist-3