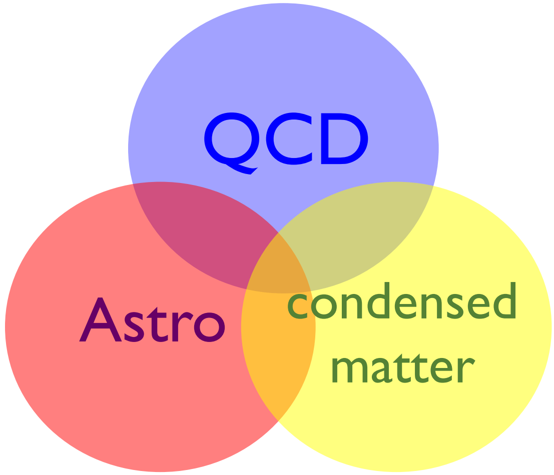
From hadrons to quarks in neutron stars



Toru Kojo

(**Tohoku Univ.**)

Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, Review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

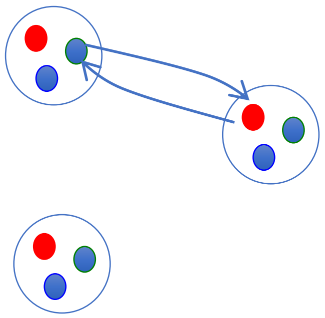
TK-Baym-Hatsuda, "QHC21" ApJ (2022)

State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$)

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

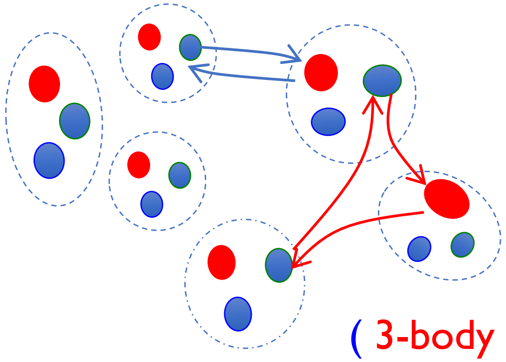


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

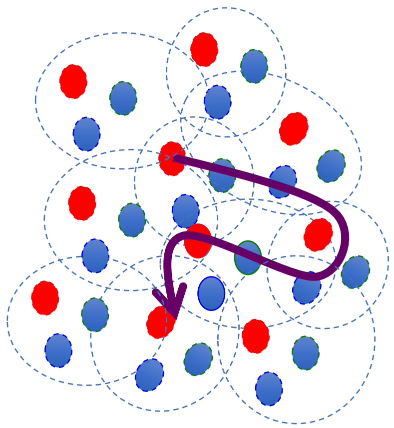


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 $M_\odot$

~ 2 $M_\odot$

n_B

~ $2n_0$

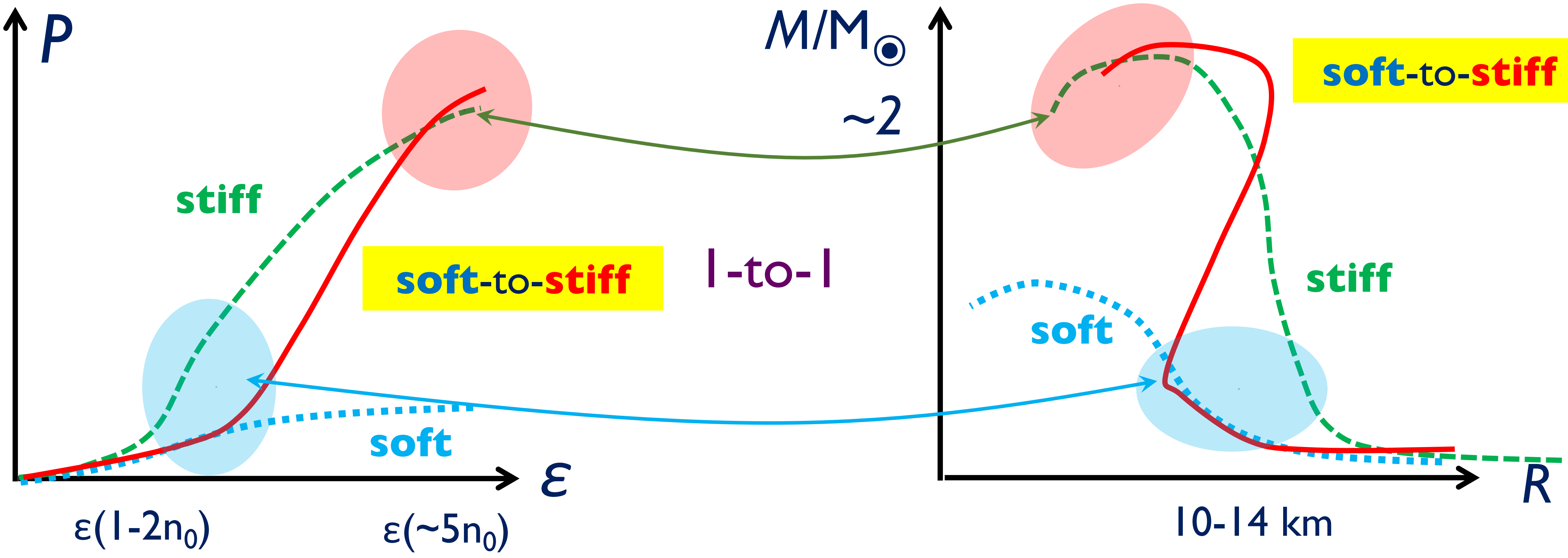
Hints from NS

~ $5n_0$

~ $40n_0$

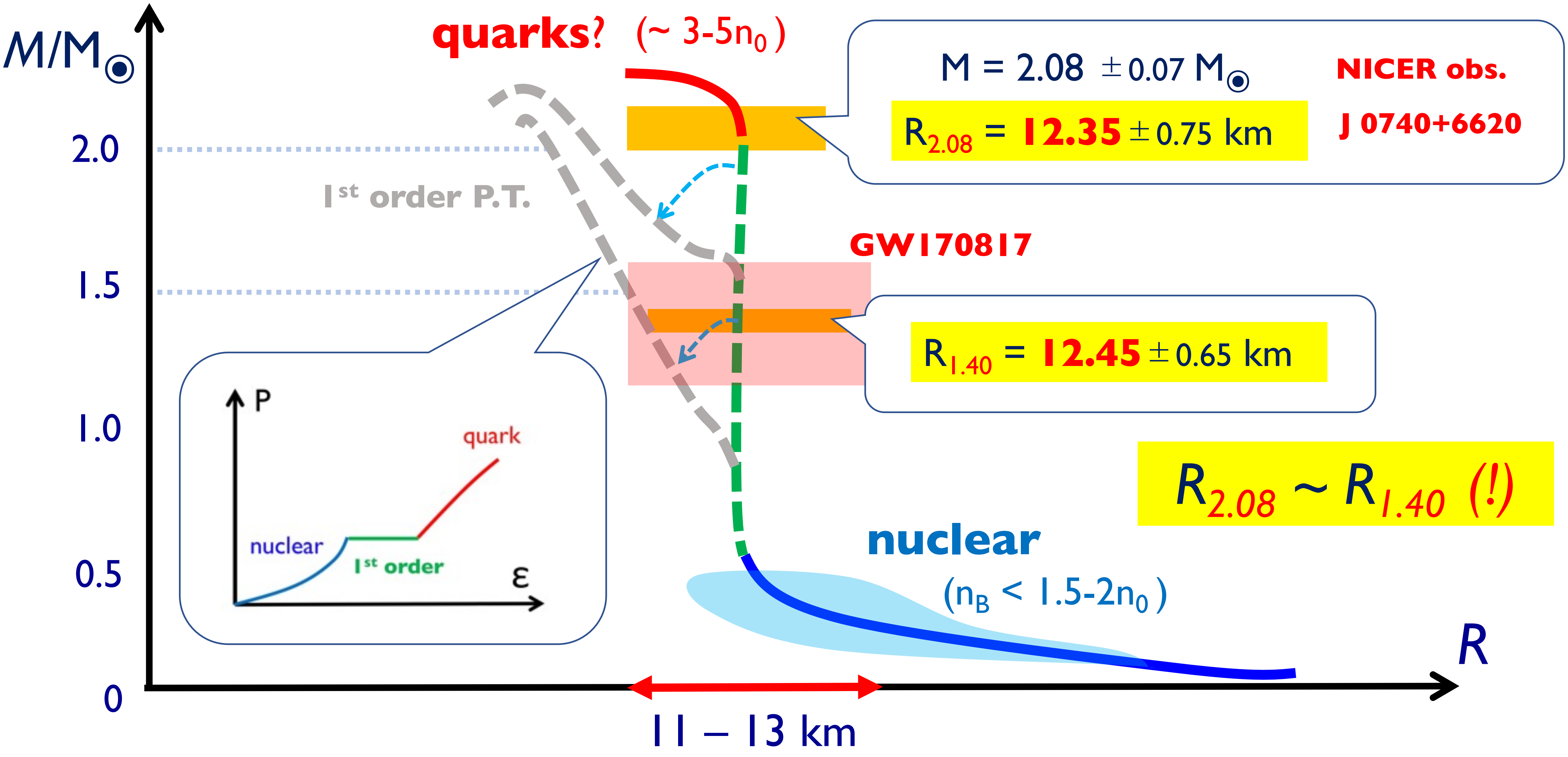
EoS & Neutron Star M-R relation

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ QCD (+EW) EoS



Observations: (NICER, GW170817, nuclear)

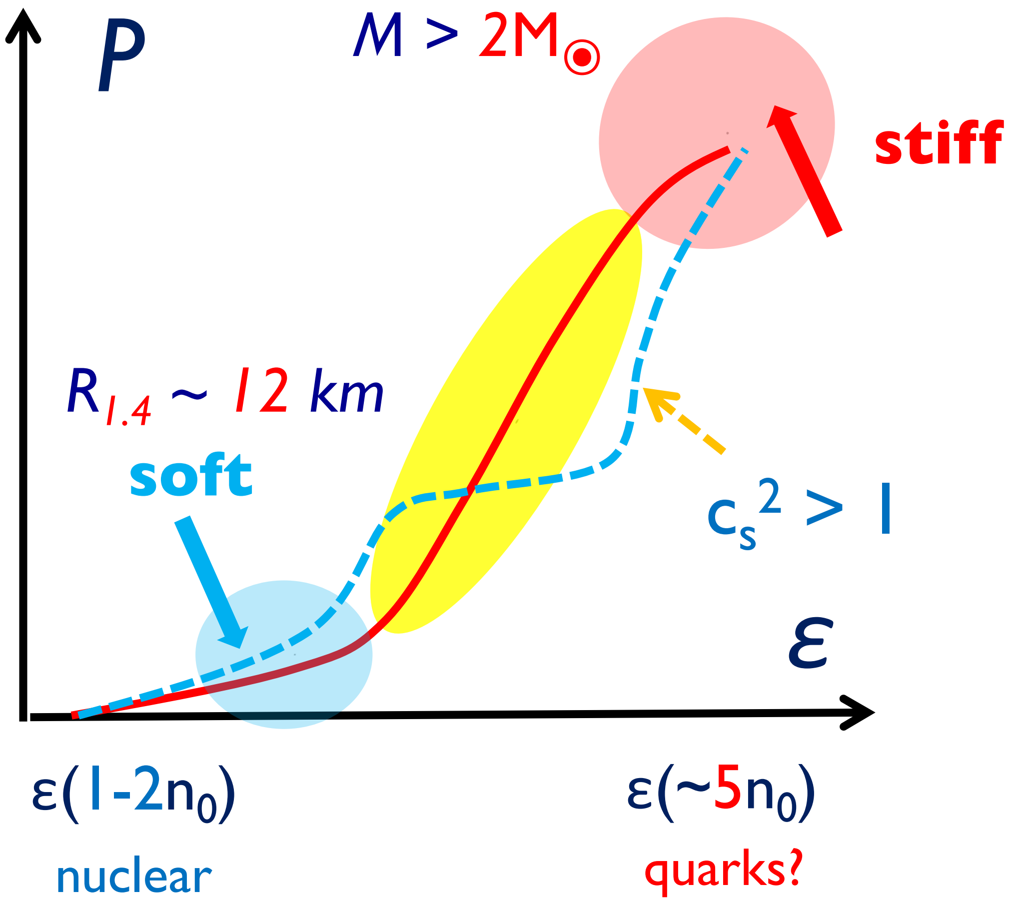
[e.g., Miller+ '21]

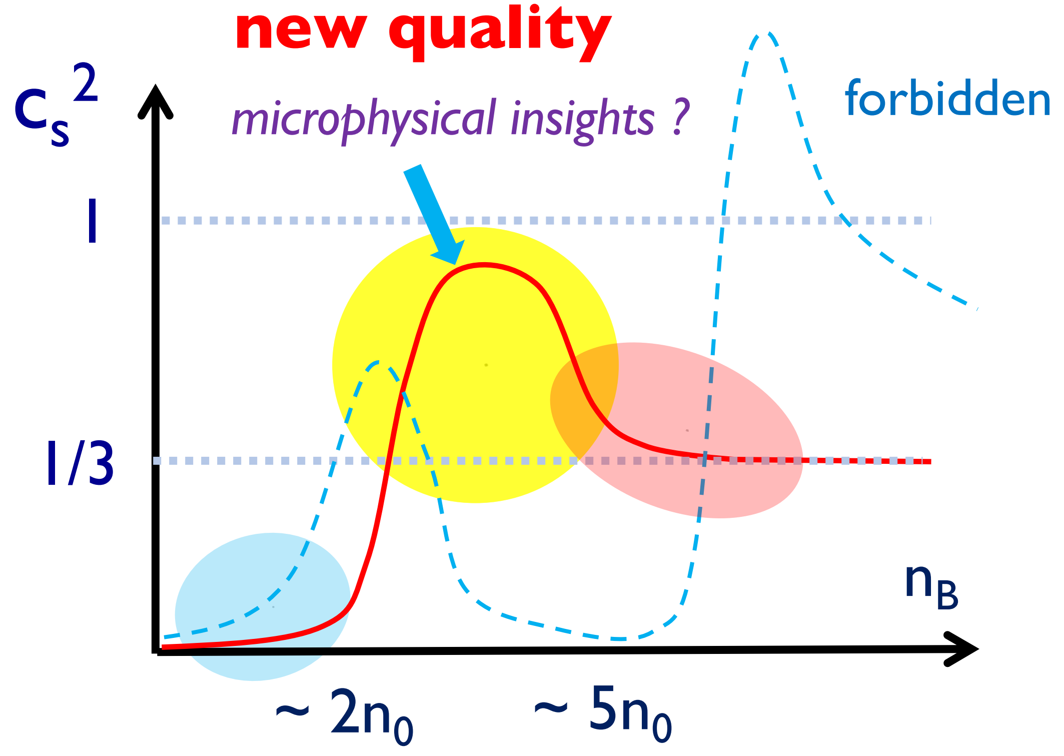


Soft to stiff is challenging:

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

nuclear & *quark* physics constrain each other



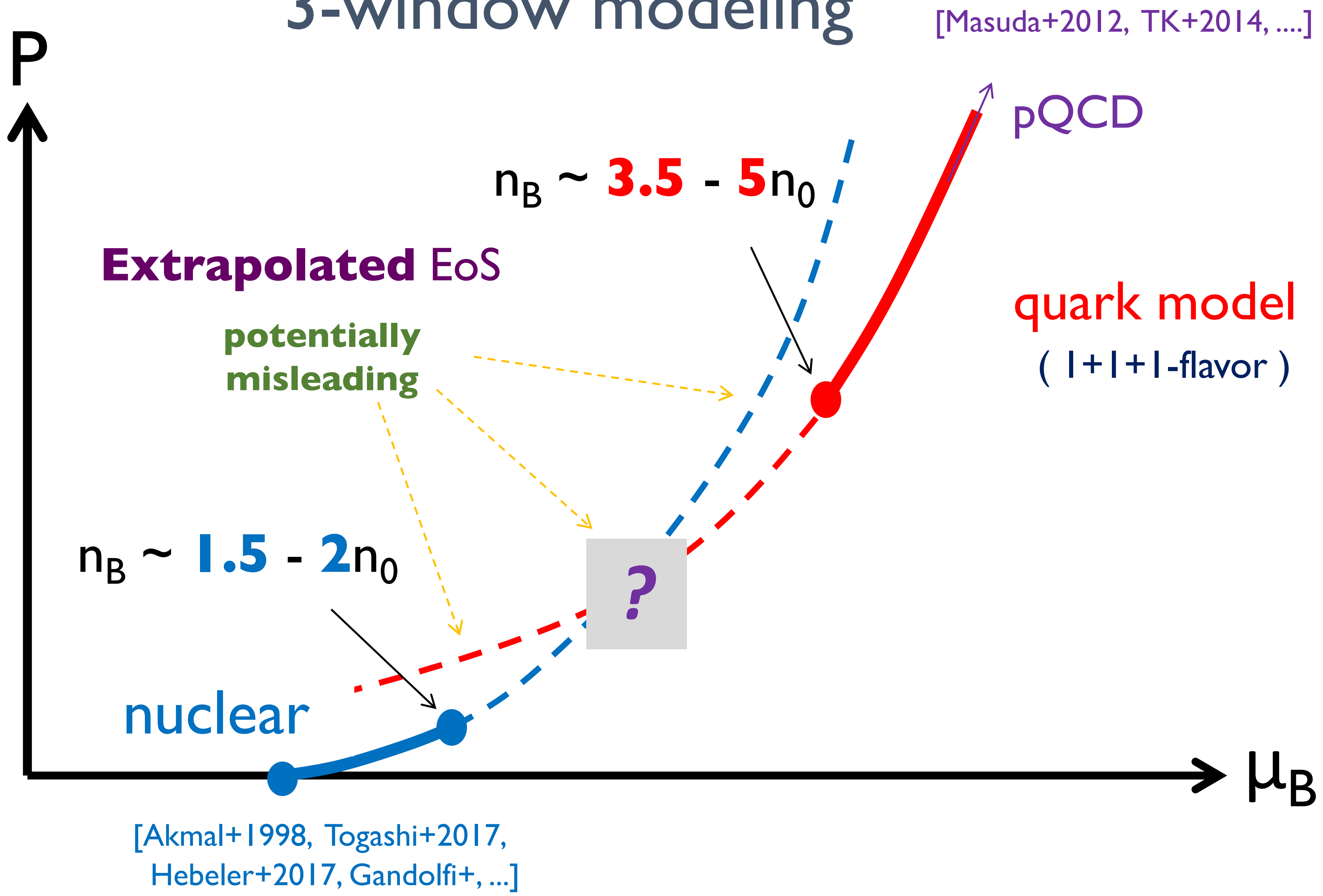


baseline: *quark-hadron continuity* **(QHC)**

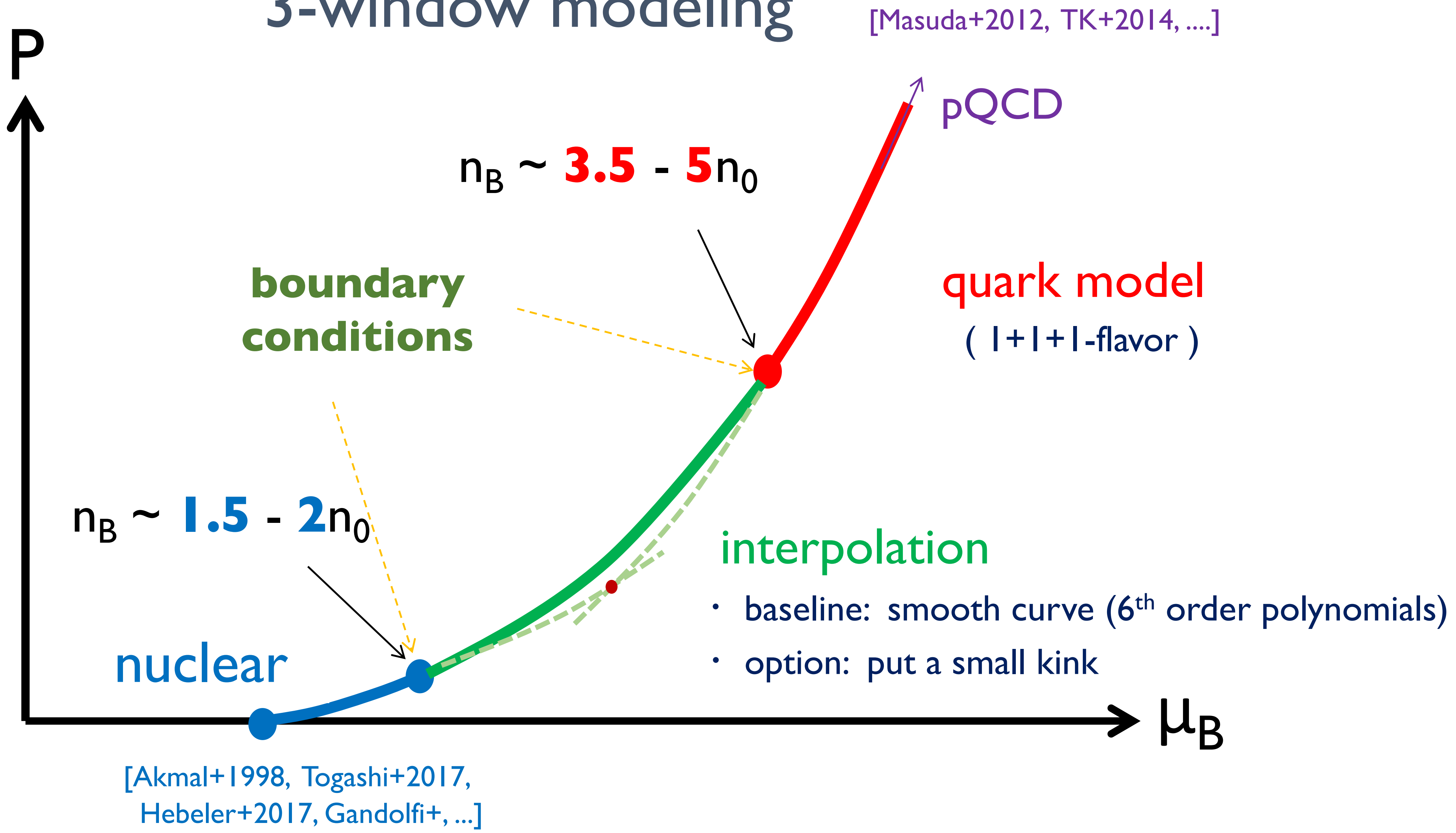
2, Phenomenological modeling: 3-window model & QHC19 & 21

- Baym-Hatsuda-TK-Powell-Song-Takatsuka (2018): QHC18
- Baym-Furusawa-Hatsuda-TK-Togashi (2019): QHC19-Togashi
- TK-Baym-Hatsuda (2021): QHC21-χ & QHC21-T

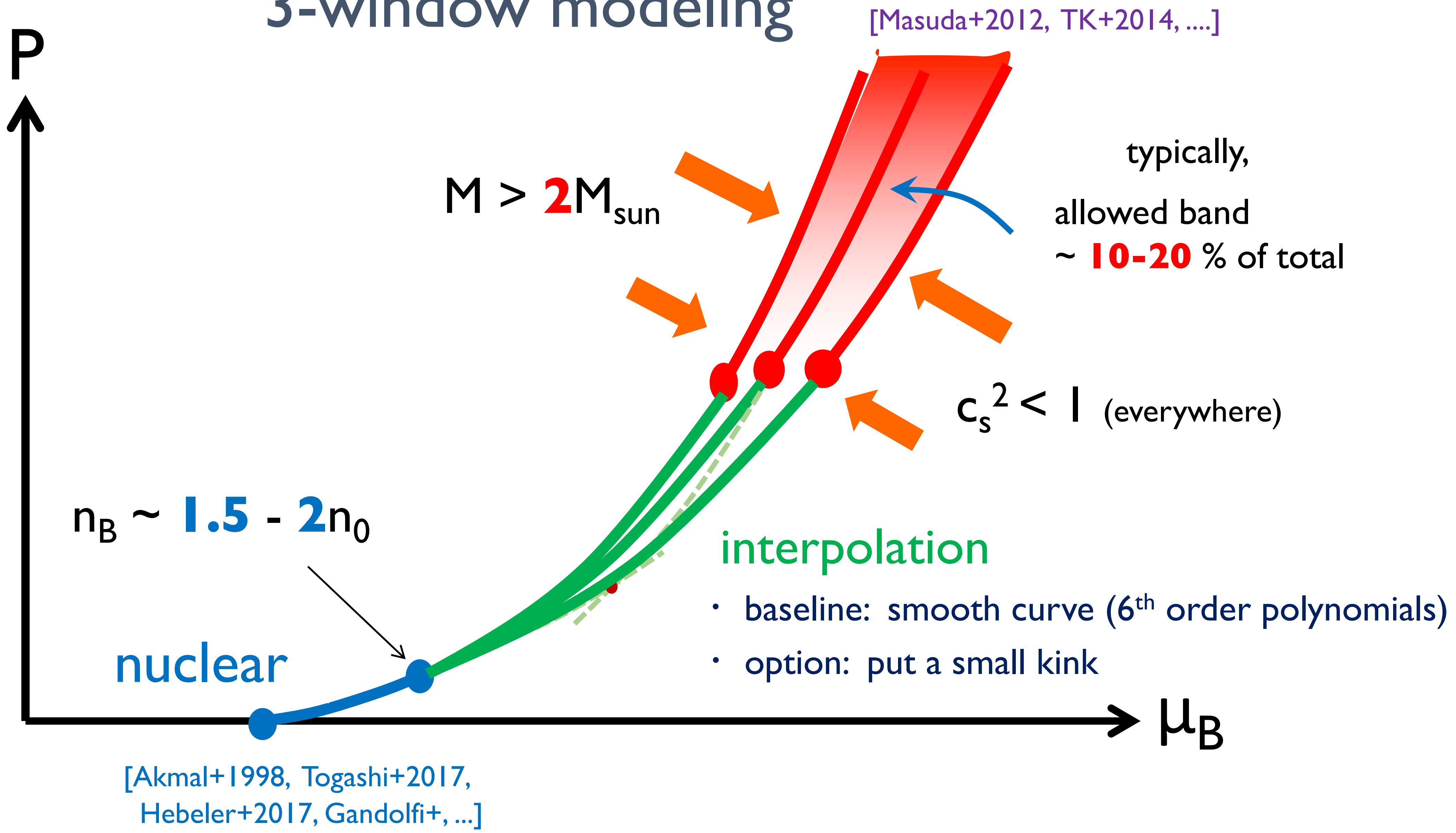
3-window modeling



3-window modeling



3-window modeling



Stiff quark EoS ? : *a guide*

rela. kin. energy   interactions

$$\varepsilon(n) = an^{4/3} + bn^{\alpha}$$

(n: quark density)

ideal gas   interactions

$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right)n^{\alpha}$$

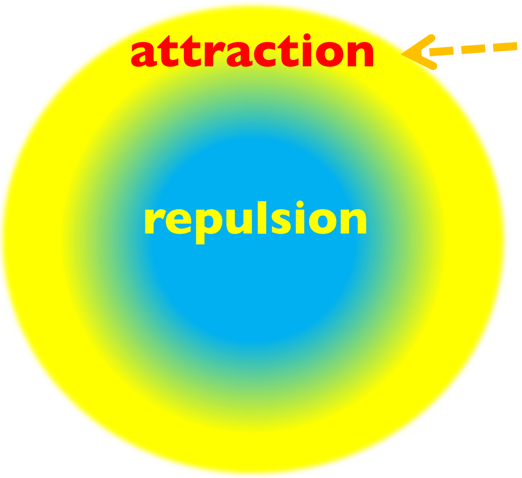
For stiff EoS:

(for large P)

for $\alpha > 4/3$:   b > 0   (e.g. bulk repulsion, ~ $+\, n_B^{2}/\Lambda^{2}$)

for $\alpha < 4/3$:   b < 0   (e.g. surface pairings, ~ $-\,\Lambda^{2} n_B^{2/3}$)

quark
Fermi sea
(ideal combo)



2- or 3-quark correlations

Stiff EOS from **attractive forces**

first discussed in [TK-Powell-Song-Baym, '14]

A nonperturbative quark model for $n_B >\sim 5n_0$ (~ 1fm^{-3})

A guide : *Quark-Hadron Continuity* : eff. Hamiltonian continuously evolves from hadron physics

"**3-window**" [Manohar-Georgi 1983, Weinberg 2010,...]

- $Q <\sim 0.2$ GeV
  - very long-range (> 1fm)
  - confinement
- 0.2 GeV < Q < 1-2 GeV
  - constituent quarks + OGE (quasi-particles)
  - **chiral** SB & color-**mag**. int. & **baryon-baryon**. int.
- ~2 GeV < Q
  - short range
  - pQCD

A template) chiral color-mag. nB-nB int.

$$\mathcal{H} = \mathcal{H}_{\rm NJL} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} \, (\bar{q}\gamma_0 q)^2$$

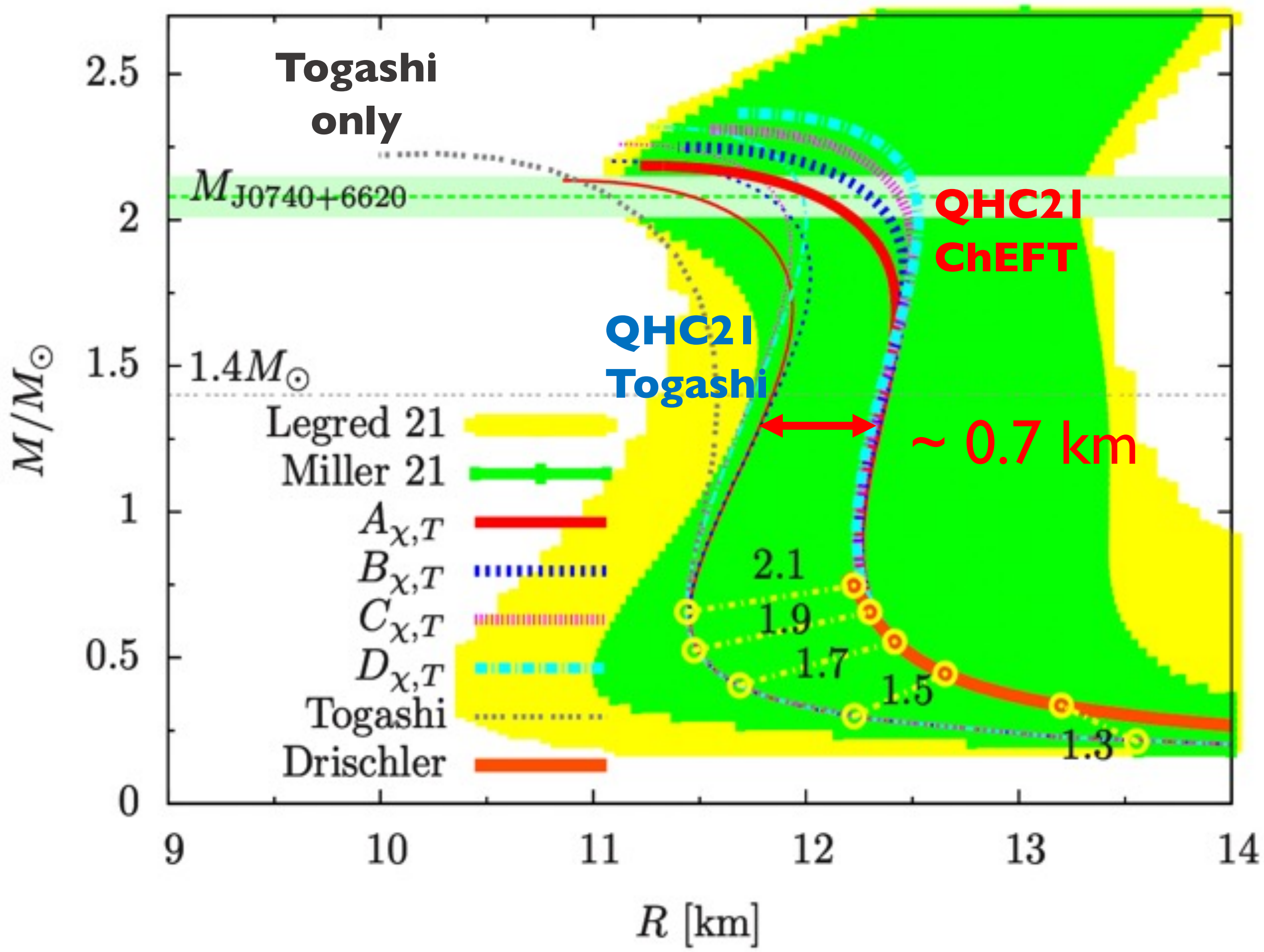
solve within **MF**
+ color- & charge- neutrality
+ β-equilibrium

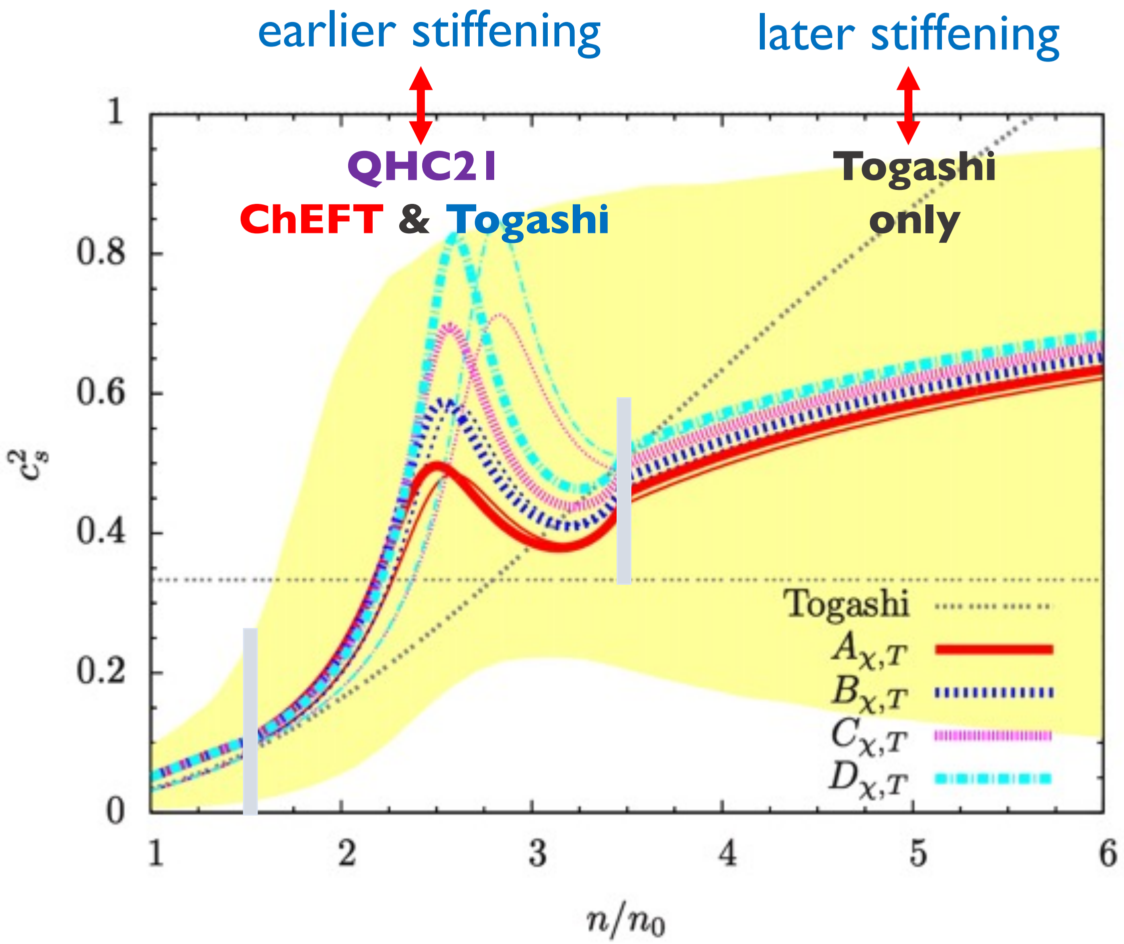
[Masuda+2015, TK+2014, Blaschke+....]

(gv, H): both inspired from color-mag. interactions

EOS → microscopic insights

[Baym+ '19, TK '21]





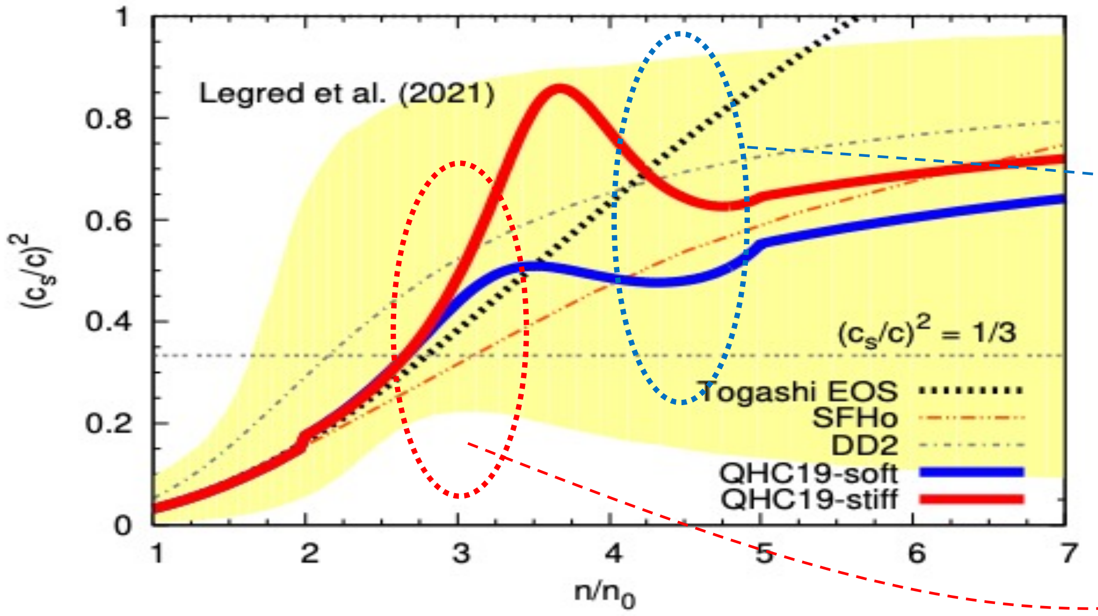
- M-R → (g_V , H) ~ G_{vac} → non-pert. effects important!
- QHC type models → earlier stiffening (!) than in pure hadronic models

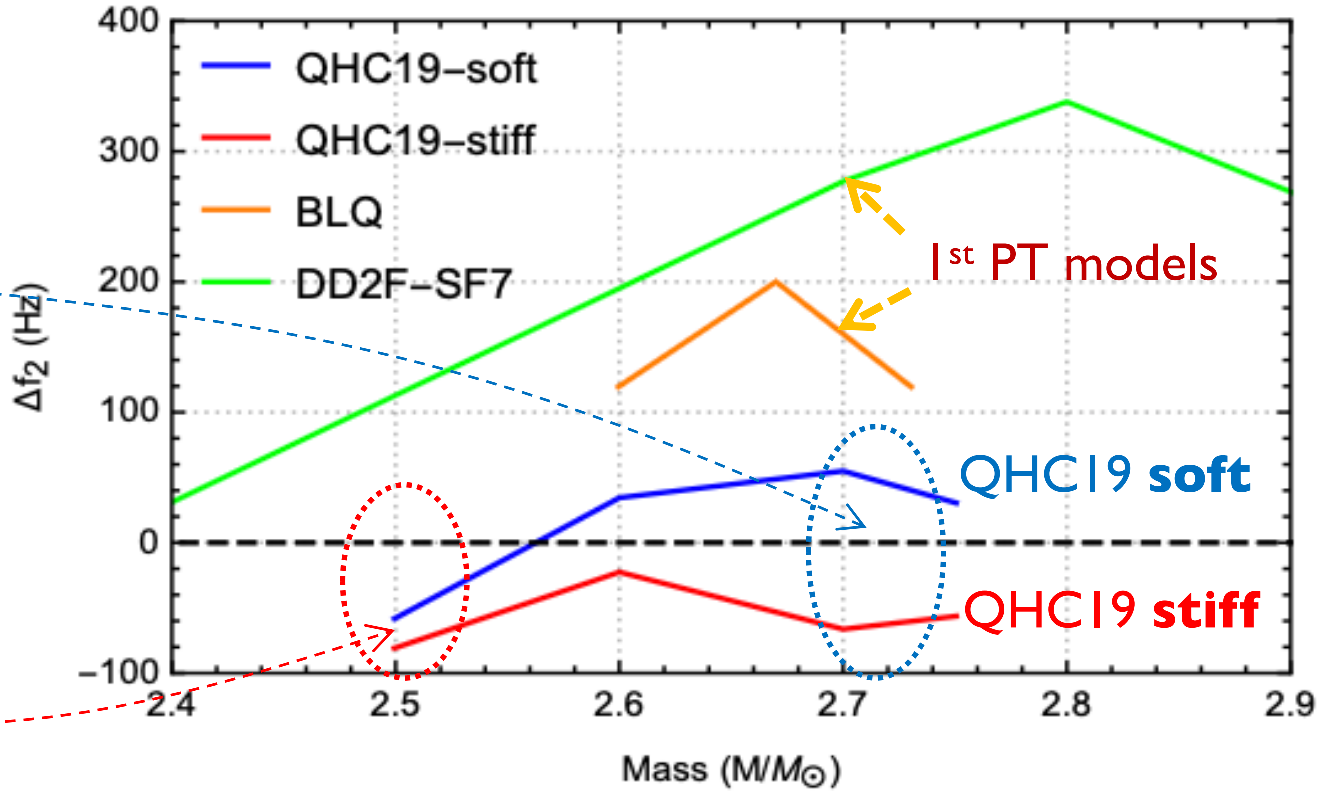
1^{st} NS merger simulations with crossover EOS

1^{st} quantitative estimates on the impact of c_s^2 peak [Y. Huang+ (2021)]

- f_2 peak ←→ compactness of NS
  - 1st PT → rapid growth in f_2
  - QHC → mild changes, even have reduction
- mass sym. mergers (1.25, 1.30, 1.35, 1.375 $M_\odot$)

$$\Delta f_2 = f_2 - f_2^{\text{nuclear only}}$$



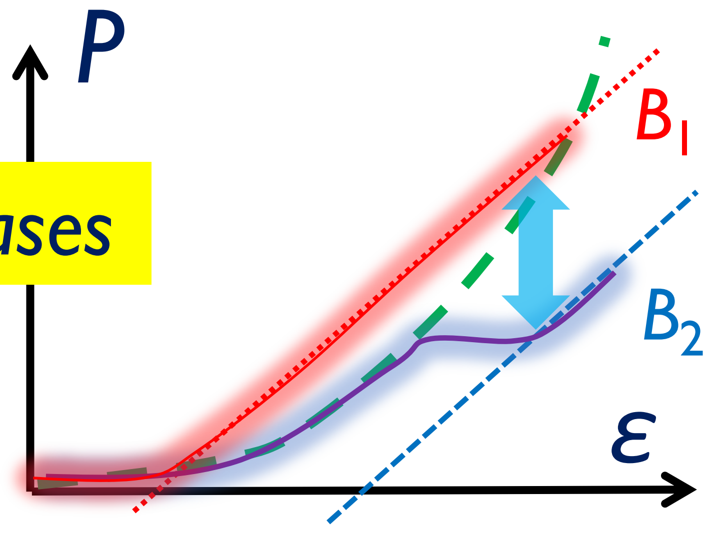


3, Sound velocity peaks

- TK (2021): "Stiffening of matter in quark-hadron continuity"
- TK-Suenaga (2021): "c_s^2 in two-color QCD"
- Fukushima-TK-Weise (2020): "percolation "

Direct descriptions for 2-5n_0 ?

confusing point:



▪ *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)

Strategy

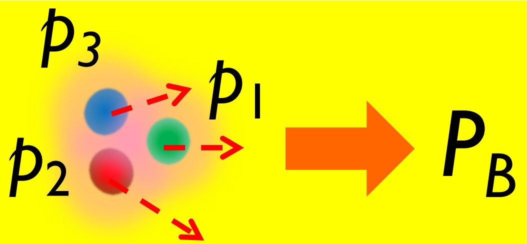
Keep track of quark states from nuclear to quark matter

(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

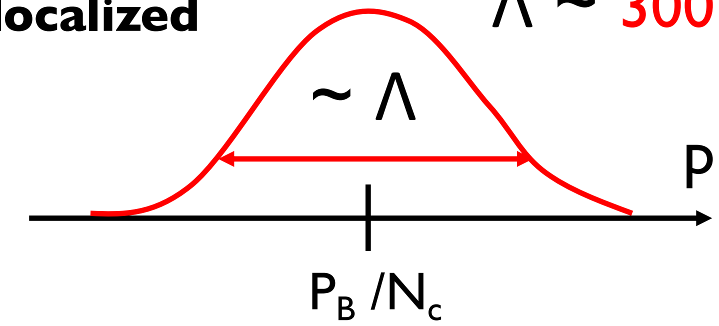
Quarks in a baryon

N_c (=3): number of colors

probability density: $$Q_{\text{in}}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2}$$



localized



Λ ~ 300 MeV

mean: $$\langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\text{in}}(\boldsymbol{p}, \boldsymbol{P}_B)$$

variance: $$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$ *energetic !*

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \underbrace{\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B = 0}}_{\times N_c} + \underbrace{\frac{1}{6} \left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B = 0} \left(\frac{\boldsymbol{P}_B}{N_c}\right)^2}_{\times N_c} + \cdots$$

average energy (quark)

$\sim N_c (M_q + E_{kin})$ ≫ $\sim P_B^2 / (N_c E_q)$

baryon mass — baryon kin. energy

(short range correlations
→ included later)

A new **unified** model for **QHC**

cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p

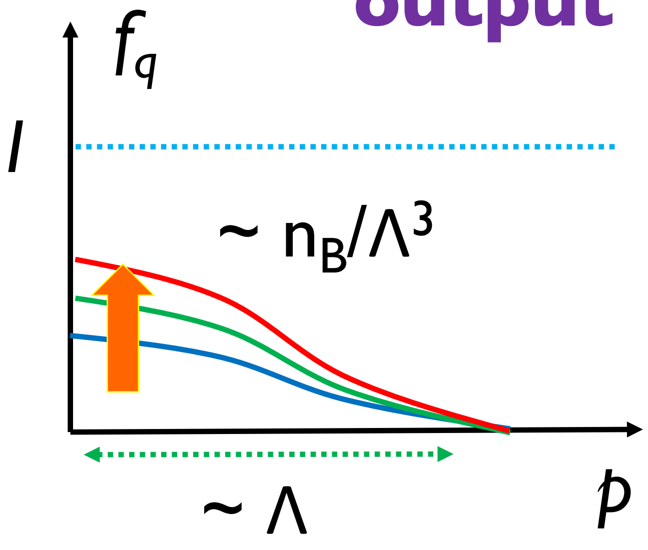
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_q(\underline{p}; n_B) = \int_{\underline{\boldsymbol{P}_B}} \mathcal{B}(\underline{P_B}; n_B) Q_{\rm in}(\underline{\boldsymbol{p}}, \underline{\boldsymbol{P}_B})$$

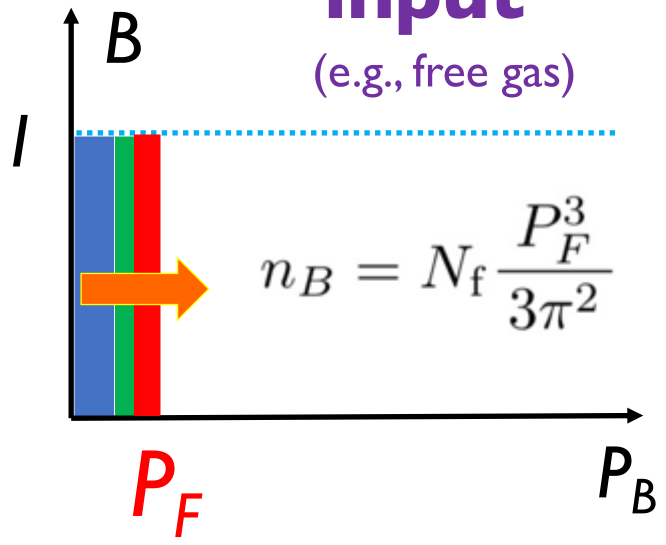
e.g.) in **ideal** baryonic matter

output



input

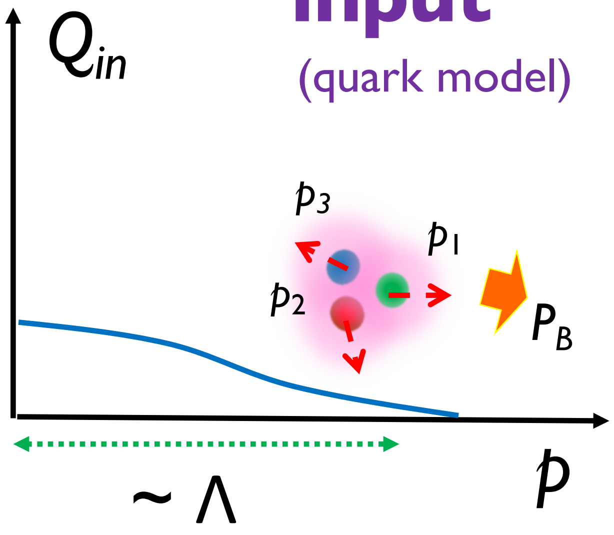
(e.g., free gas)



$\otimes$

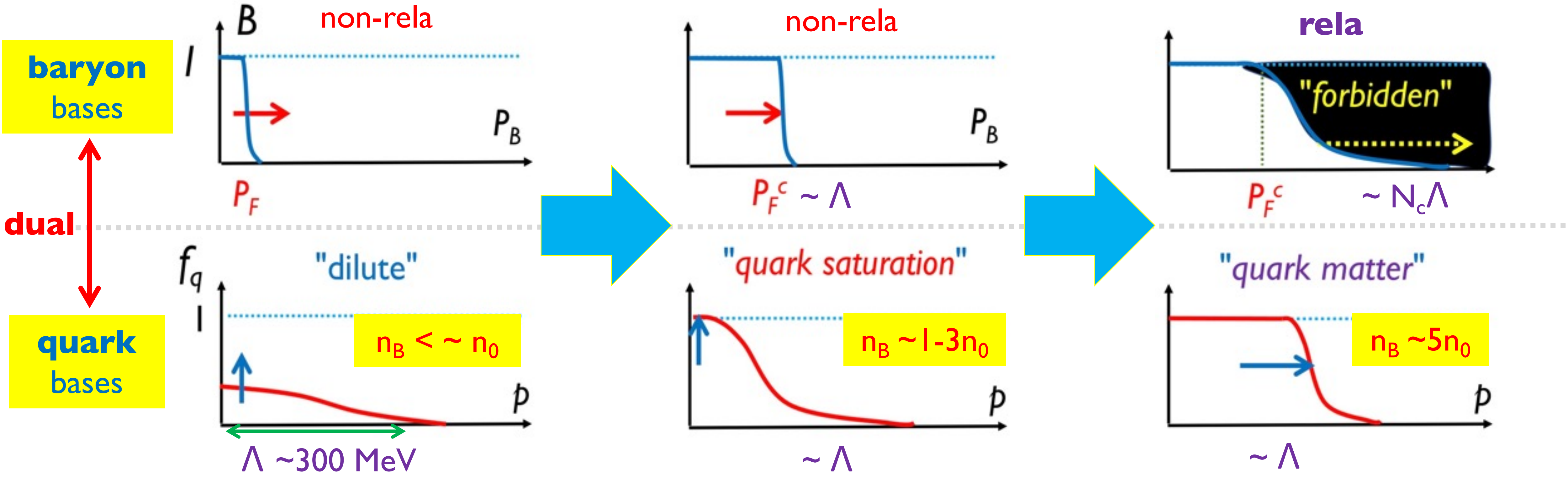
input

(quark model)



Evolution of occ. probabilities

$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B;n_B) Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$

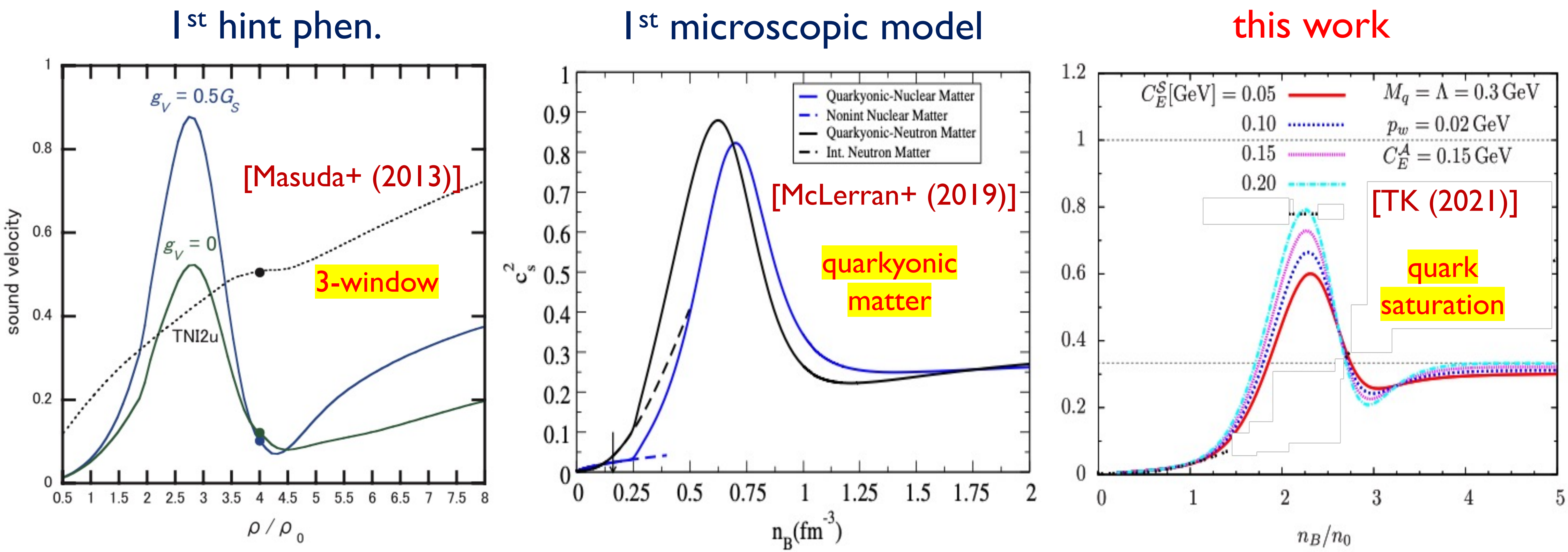


"**quark saturation**" constraint

→ **relativistic baryons at low density, $n_B \sim 1\text{-}3n_0$!**

cf) McLerran-Reddy model (2018) of quarkyonic matter

c_s^2 peak: from hadrons to quarks



& 1st evidence from lattice QCD → see Itou's talk on 2-color dense QCD

(the next session on NS & EoS)

Summary

$R_{2.08} \sim R_{1.40}$ (!) → **Quark-Hadron-Continuity** : a **baseline**

Peak in sound velocity → signature of quark matter formation

(quark saturation effects)

Stiff quark matter EoS → bulk repulsion & Fermi surface **attraction**

(disparity ← channel dep. of gluon exchanges)

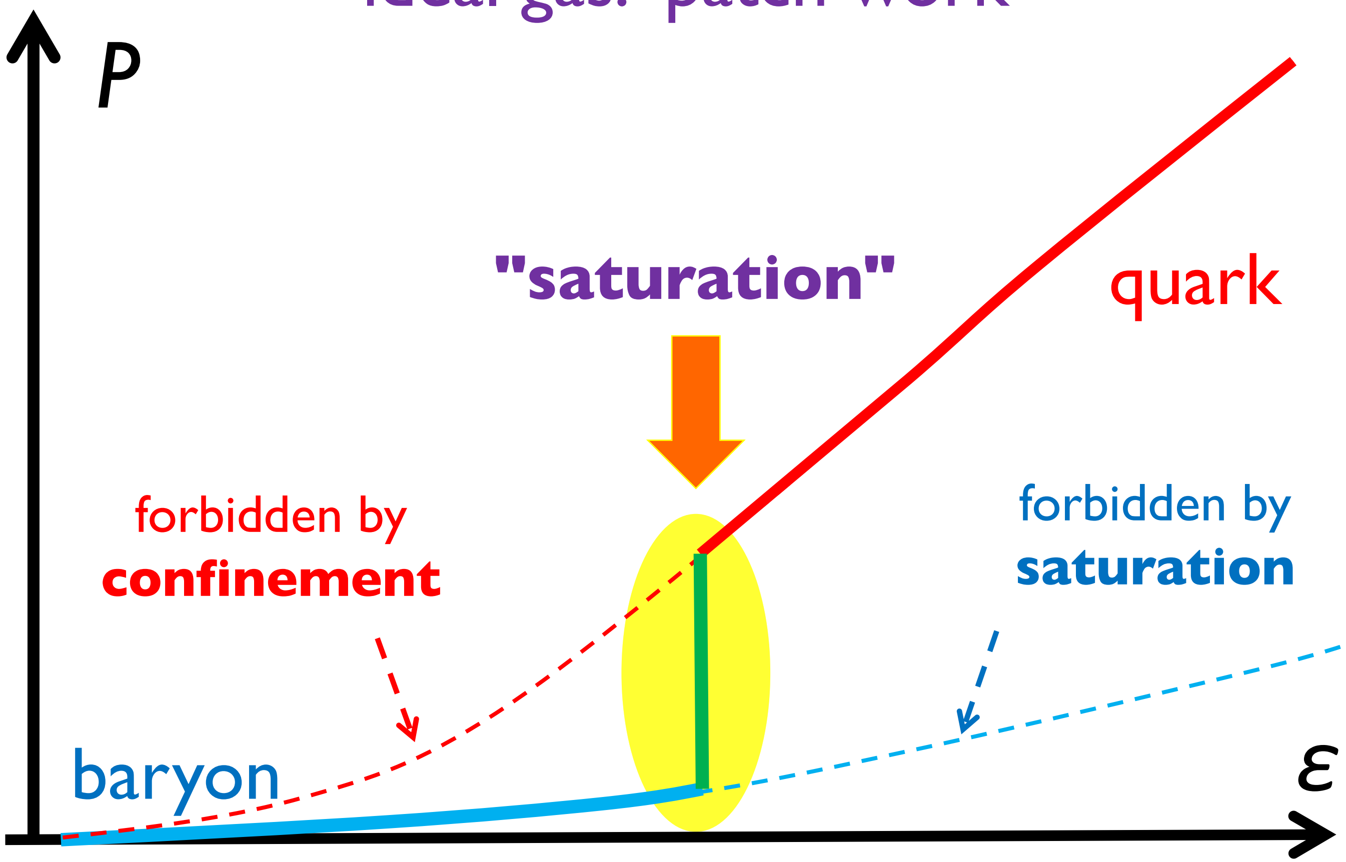
Outlook

Hadron physics for dense QCD

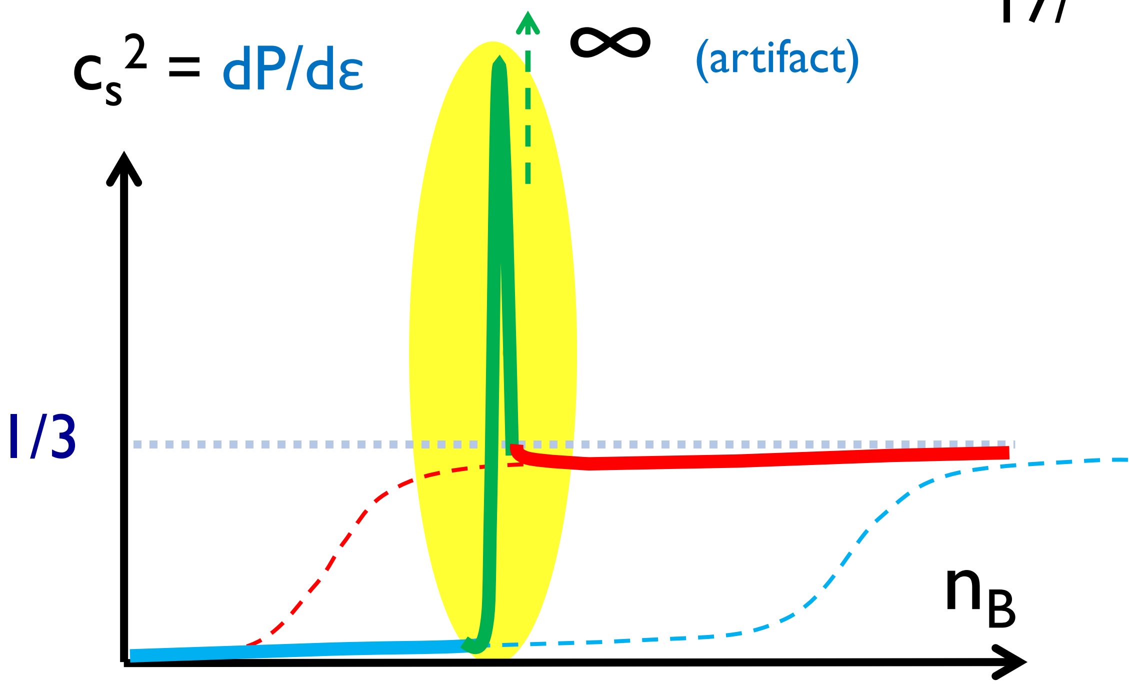
Back up

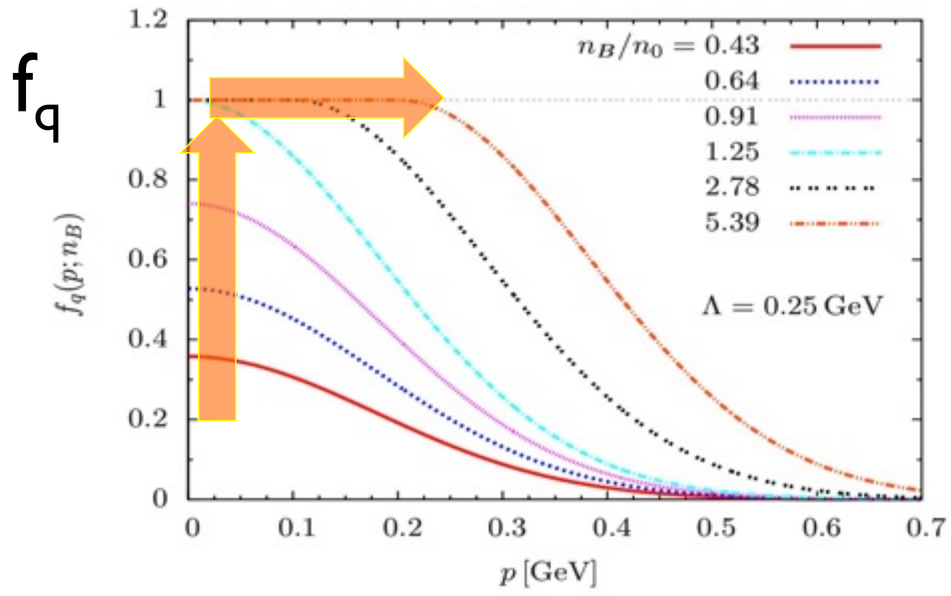
Peak in sound velocity

ideal gas: patch work



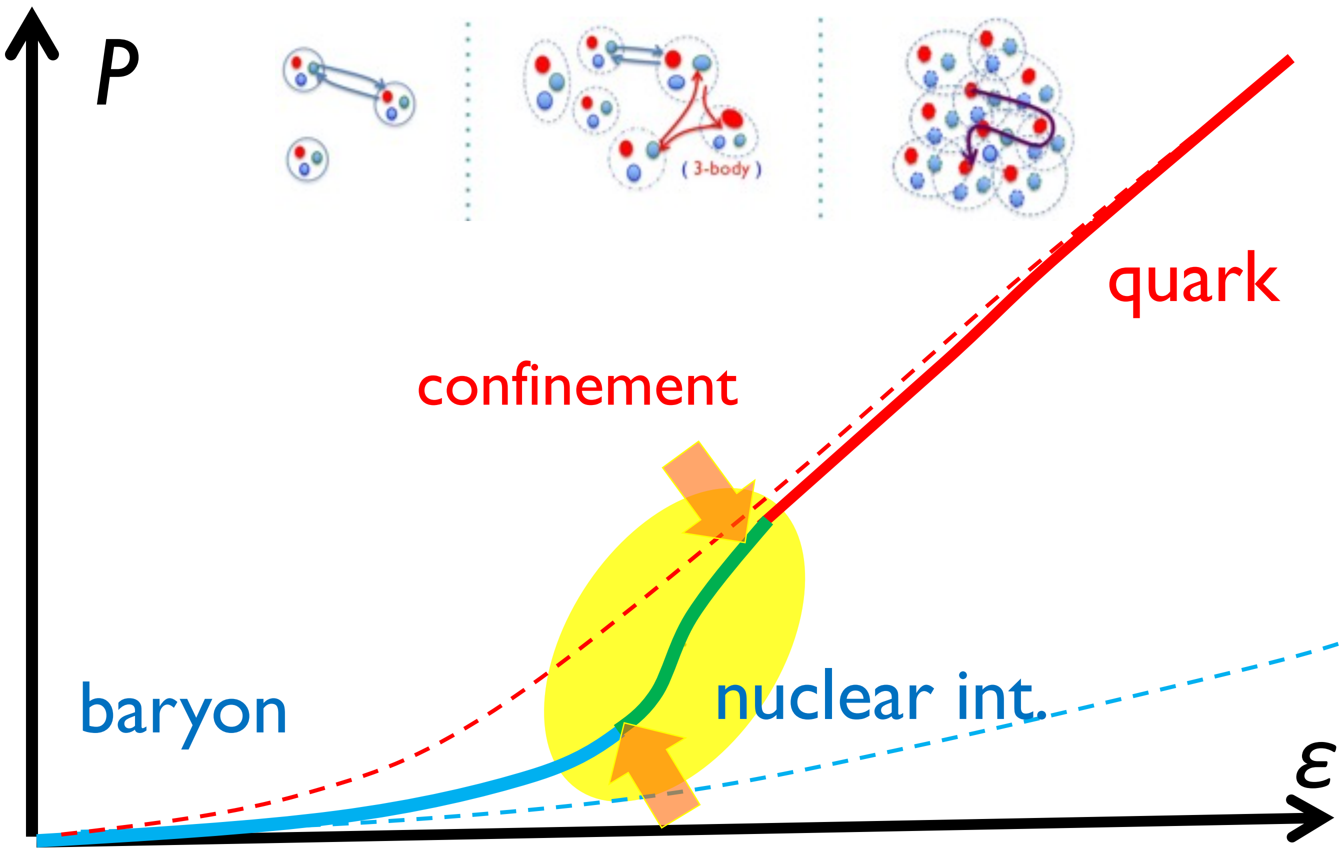
"inevitable" stiffening





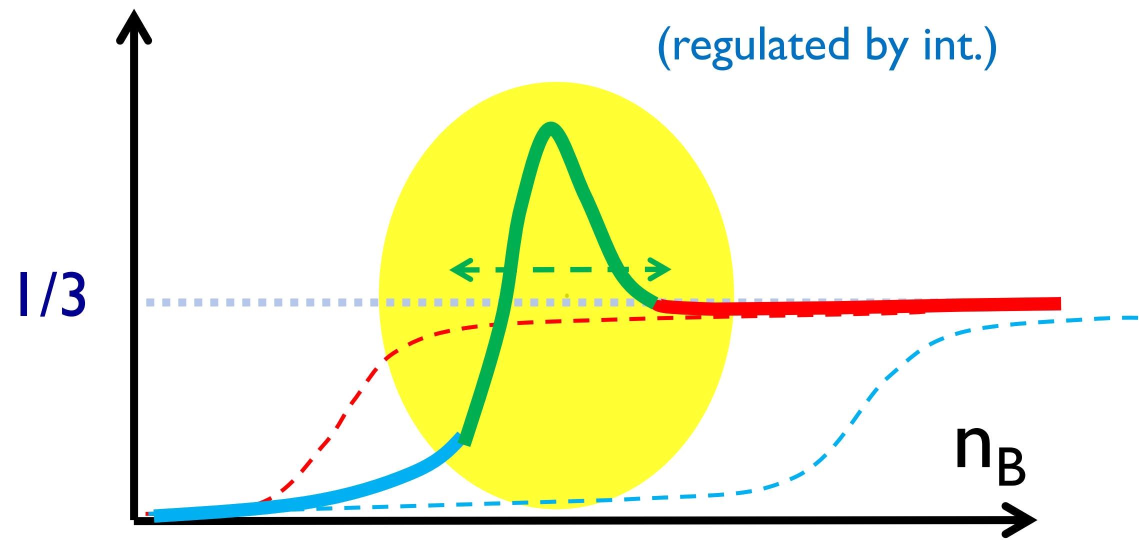
Peak in sound velocity

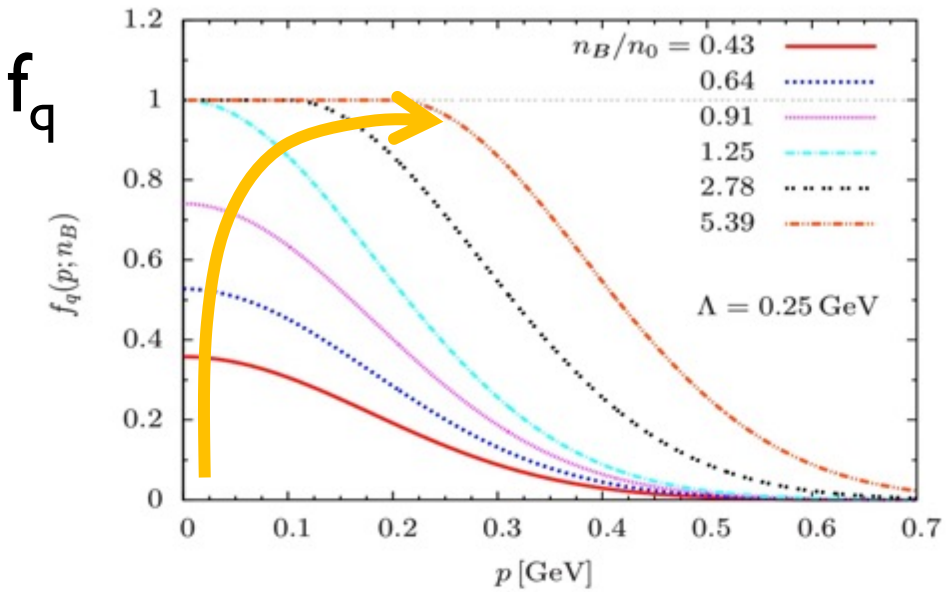
more realistic picture

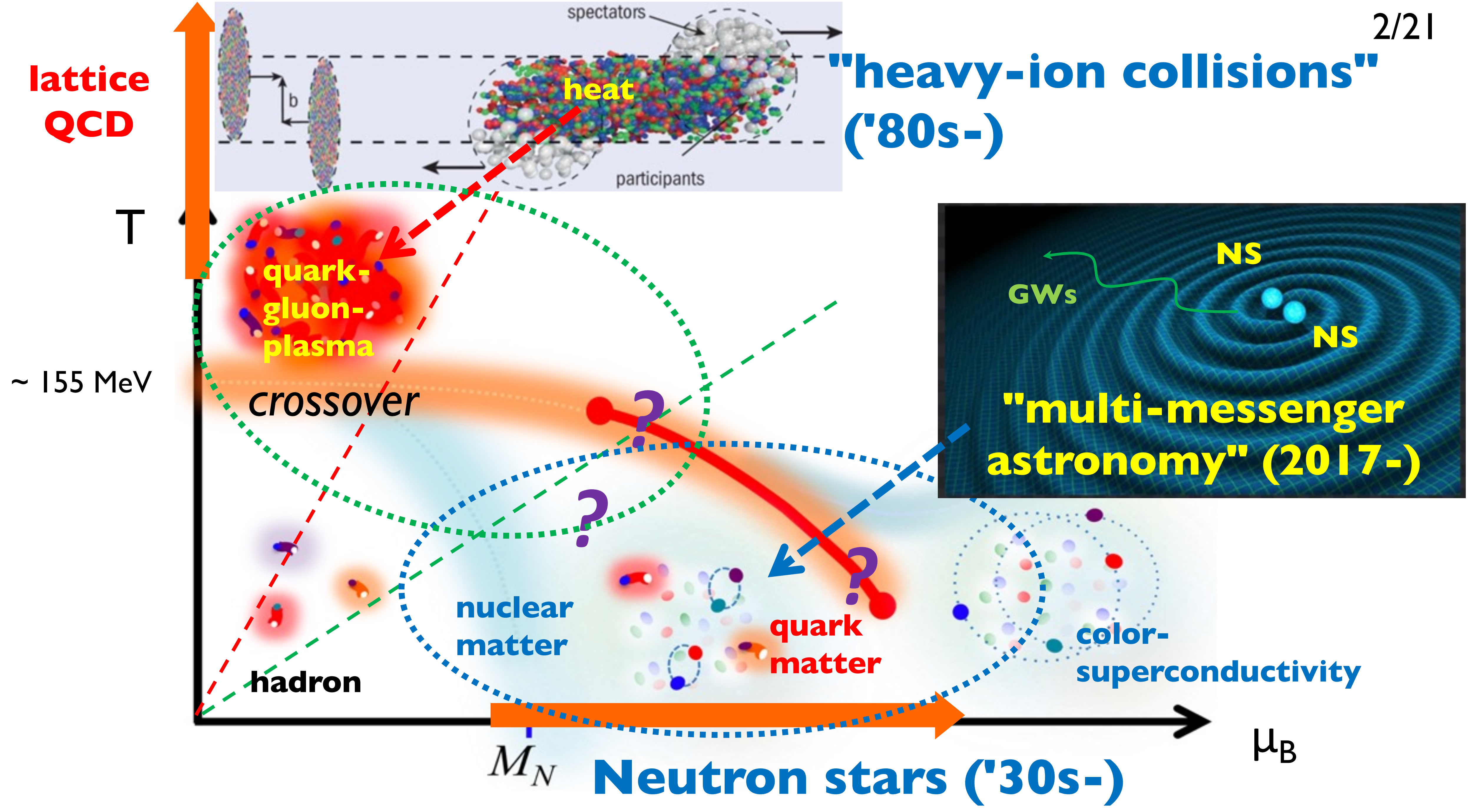


nuclear interactions as
"perturbations"

$c_s^2 = dP/d\varepsilon$







lattice
QCD
spectators
b
heat
participants
"heavy-ion collisions"
('80s-)
T
quark-
gluon-
plasma
~ 155 MeV
crossover
?
?
?
GWs
NS
NS
"multi-messenger
astronomy" (2017-)
nuclear
matter
quark
matter
color-
superconductivity
hadron
M_N
μ_B
Neutron stars ('30s-)

Trends found in this exercise (for quark matter part)

for quark EoS consistent with all constraints

- bottom line: $(g_V\,, H)_{@3.5\text{-}5n0} \sim (G_s)_{@vac}$

  interactions remain non-perturbative (!)

- Slow chiral restoration

  at $5n_0$: $M_u \sim M_d \sim 50$ MeV >> ~ 5 MeV, $M_s \sim 300$ MeV >> ~100 MeV

- Pairing effects important

  at $5n_0$: $\Delta_{CFL} \sim$ **200** MeV (!)

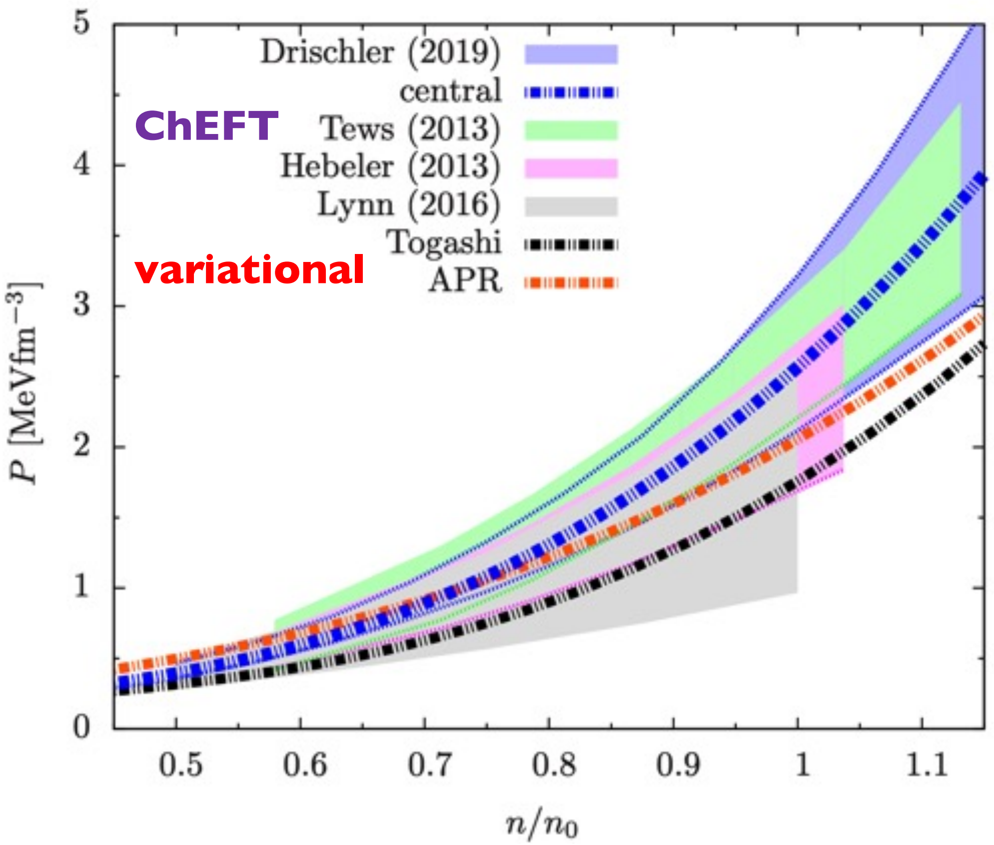
- For allowed range of (g_v, H), $M_{max} \sim 2.4\, M_{sun}$

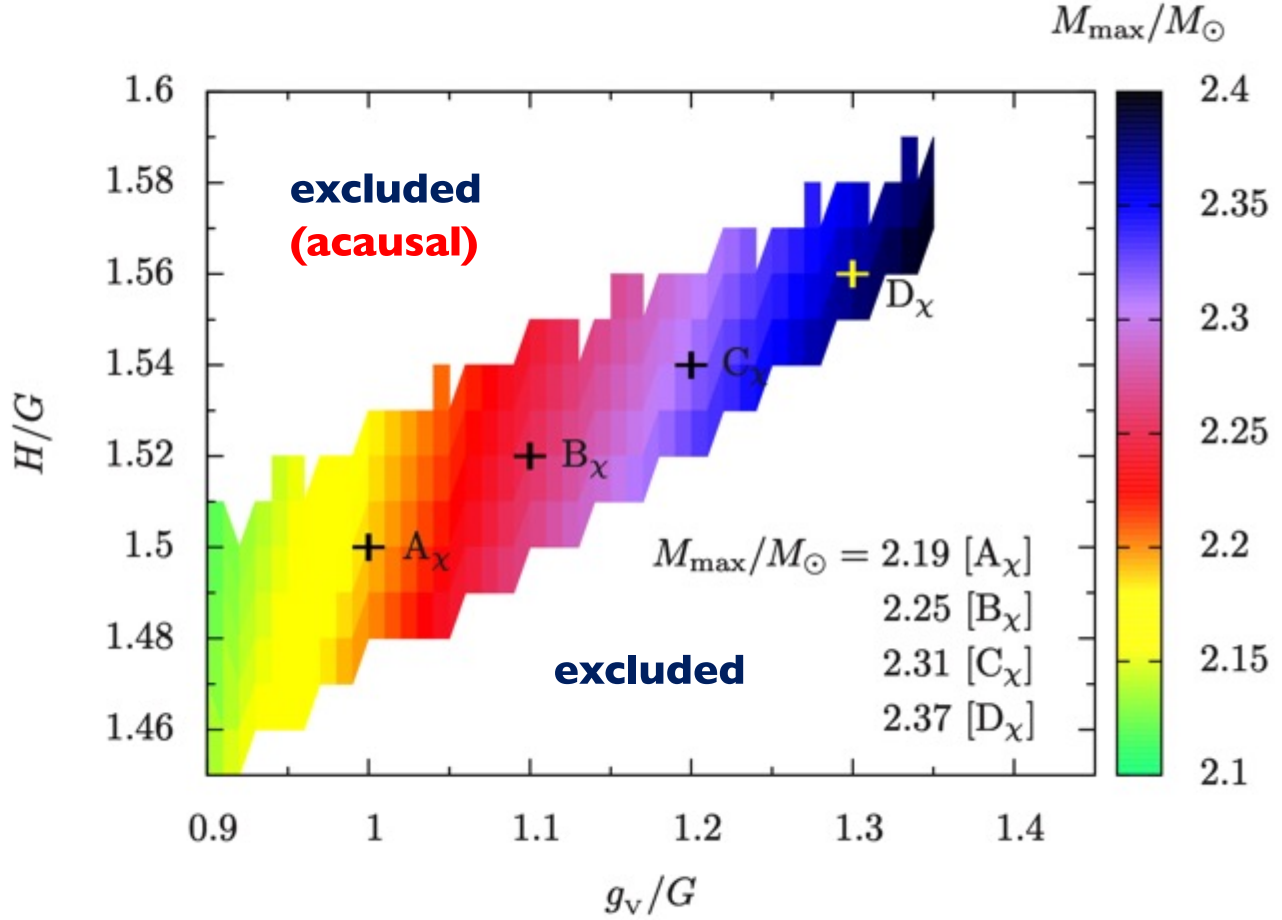
An exercise: survey for (g_V , H)@3.5-5n0 [Baym+ '19, TK '21]

Step1) Prepare **realistic** nuclear EoS up to **1.5-2**n_0

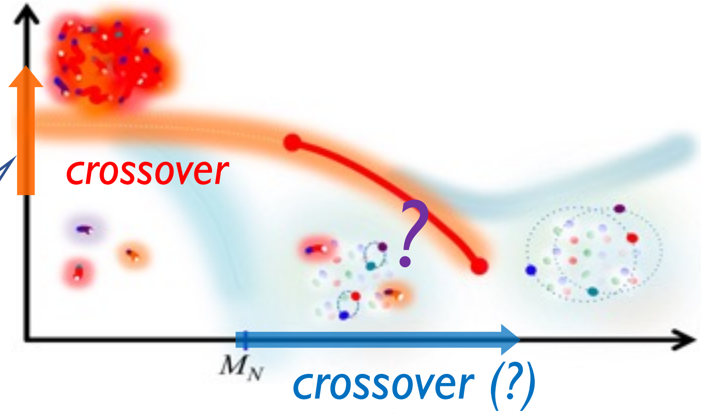
[e.g. Akmal+1998, **Togashi+2017**, **ChEFT**, ...] → 30-40% uncertainties in P @ ~ n_0

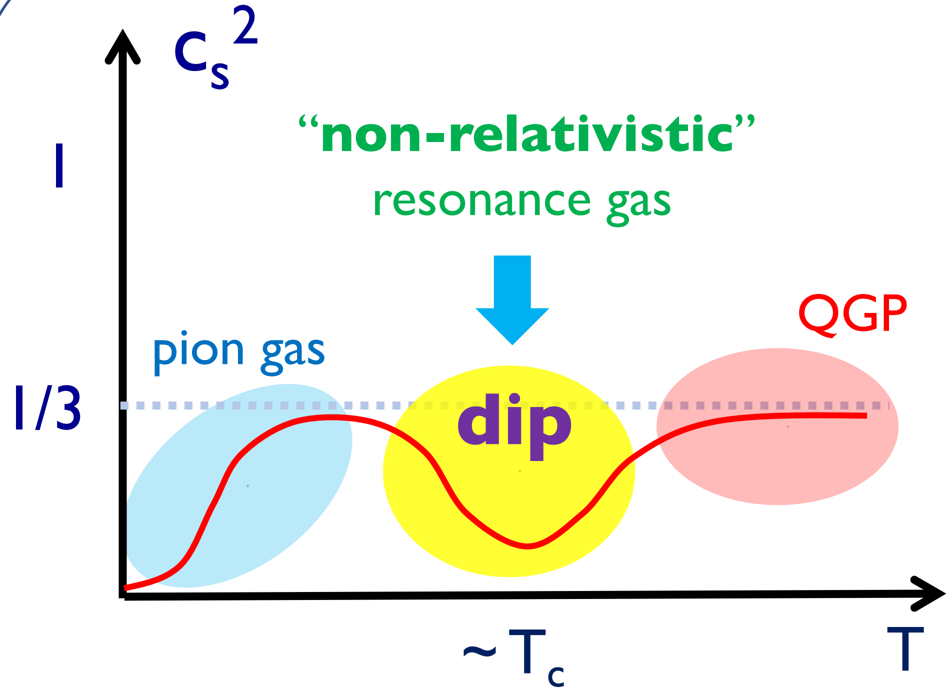
Step2) Survey the range of (g_V , H) consistent with **causality & stability**





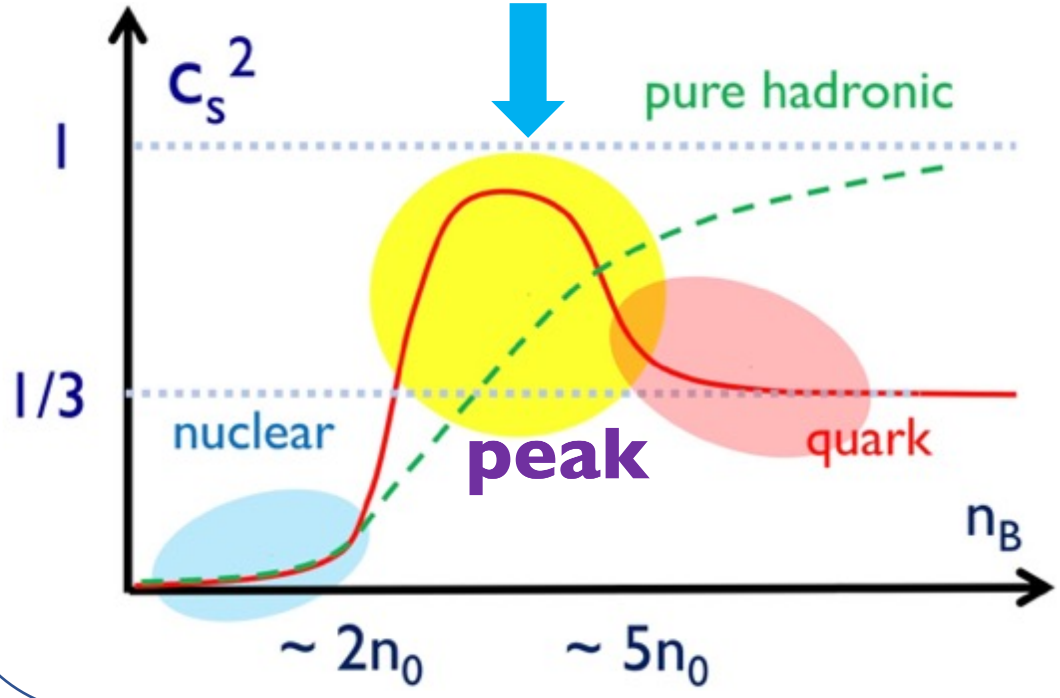
Crossovers & $c_s^2 = dP/d\varepsilon$





hadrons with **m >> T** dominate
(**entropic** effects)

microphysics ?



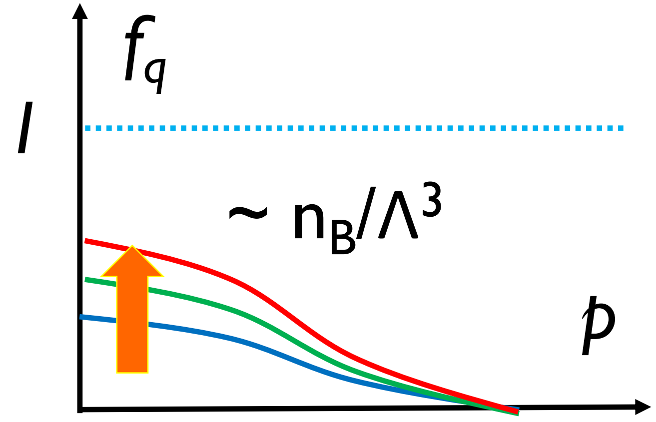
e.g., ideal baryon gas

~ 10% effect

$$f_q(p;n_B) = \int_{P_B} \mathcal{B}(P_B;n_B)\left[\, Q_{\rm in}(\boldsymbol{p},0) + O(P_B^2/N_c^2)\,\right]$$

$$= Q_{\rm in}(\boldsymbol{p},0)\int_{P_B} \mathcal{B}(P_B;n_B) + \cdots$$

$$= \underline{n_B} Q_{\rm in}(\boldsymbol{p},0) + \cdots$$



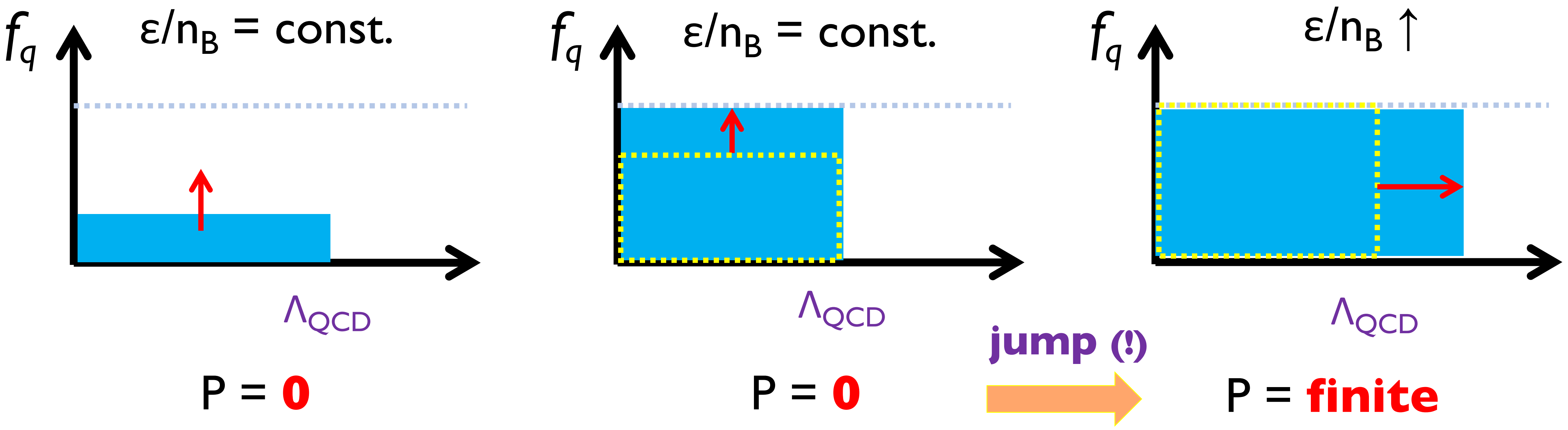
$$\varepsilon = \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) f_q(p;n_B) = \int_{\boldsymbol{p}} E_q(\boldsymbol{p})\left[\, \underline{n_B} Q_{\rm in}(\boldsymbol{p},0) + \cdots\,\right]$$

$$= \underline{n_B} M_B + \cdots$$

$\rightarrow$ **non-relativistic**

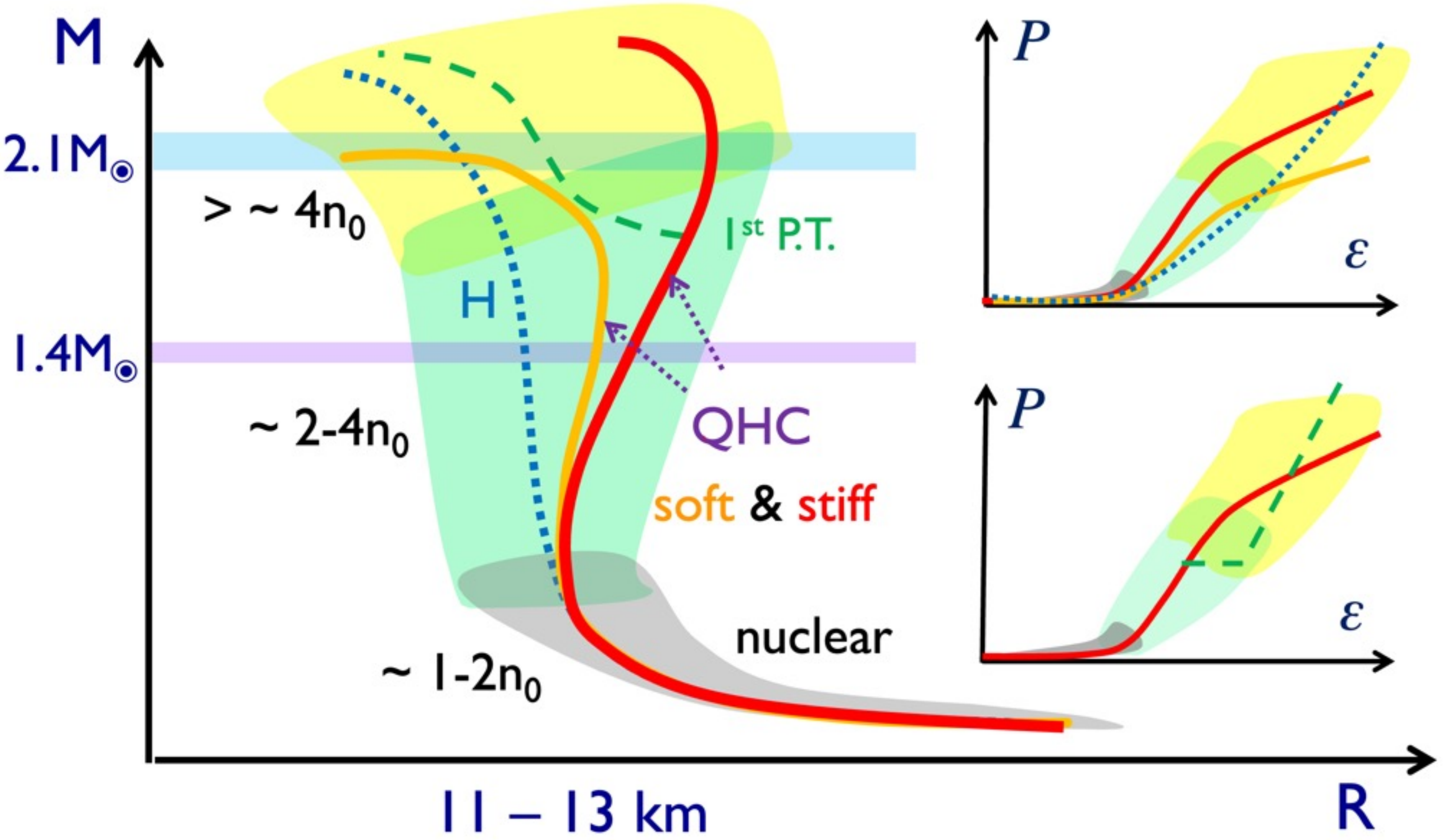
Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\underbrace{\left(\frac{\varepsilon}{n_B}\right)}_{\text{energy per particle}}$$



ε, n_B are continuous (f_q continuous)

Earlier stiffening in QHC vs later stiffening in hadronic EOS



Color-magnetic interaction play **many** roles

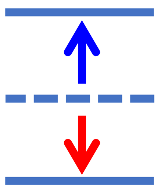
1) **Coupling** $\propto$ **velocity** ~ **p/E**

become important in relativistic regime & high density

cf) Δ (1232)

$3M_q$ + ...

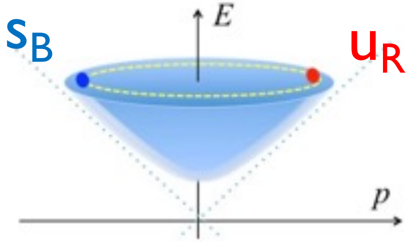
N (938)



2) **Pairing** : strongly channel dependent

hadron mass ordering: N-Δ, etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer,... 1998-]



3) **Baryon-Baryon int.** : **short-range** correlation

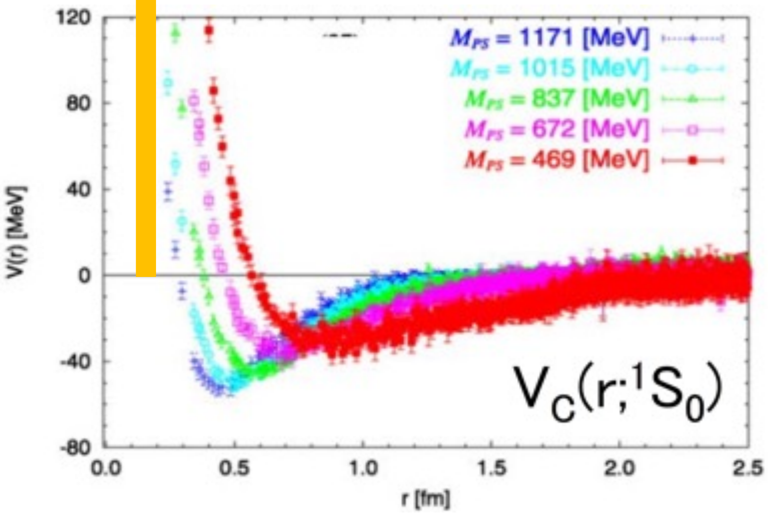
(**Pauli + color-mag.**) [Oka-Yazaki (1980),...]

channel dep. → non-universal hard core (some are attractive!)

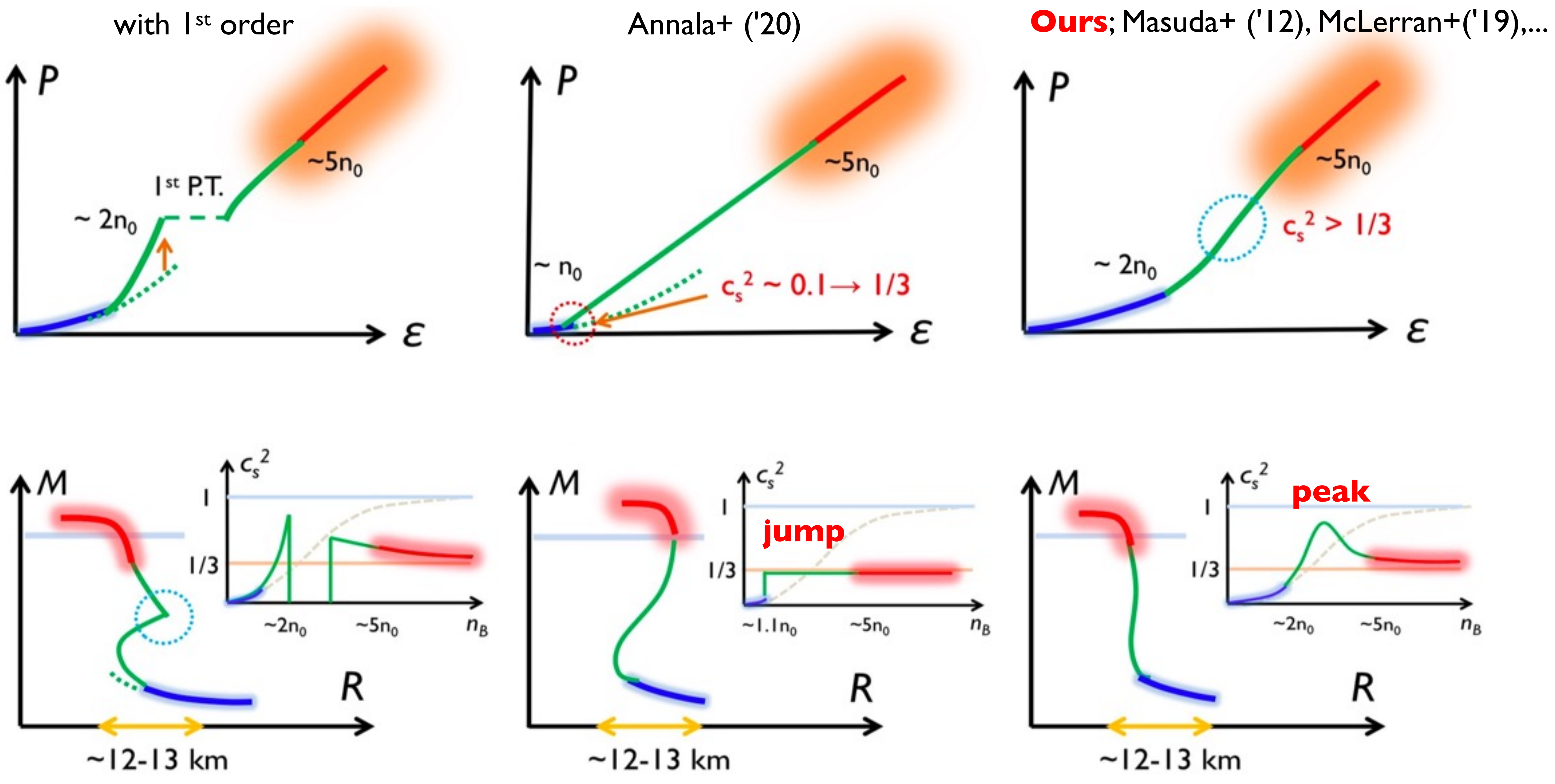
mass dep. → stronger hard core in relativistic quarks

→ **consistent with the lattice QCD** [HAL-collaboration]

lighter quark mass



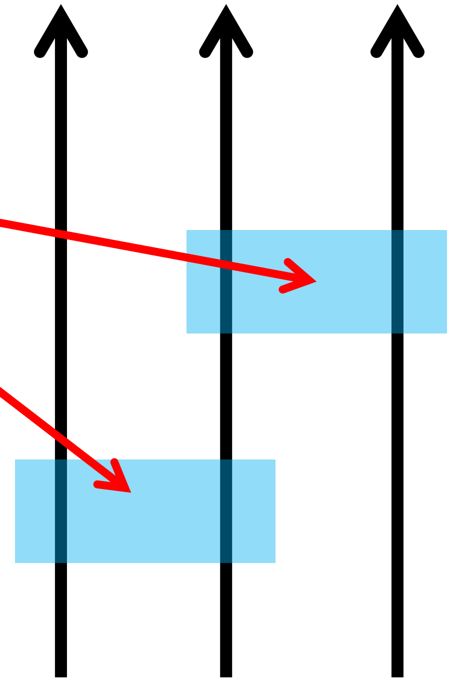
Comparisons with other scenarios



a baryon in dilute regime

(color-singlet)

(always) color-
antisymmetric



(attractive
electric int.)

e.g., **nucleons**

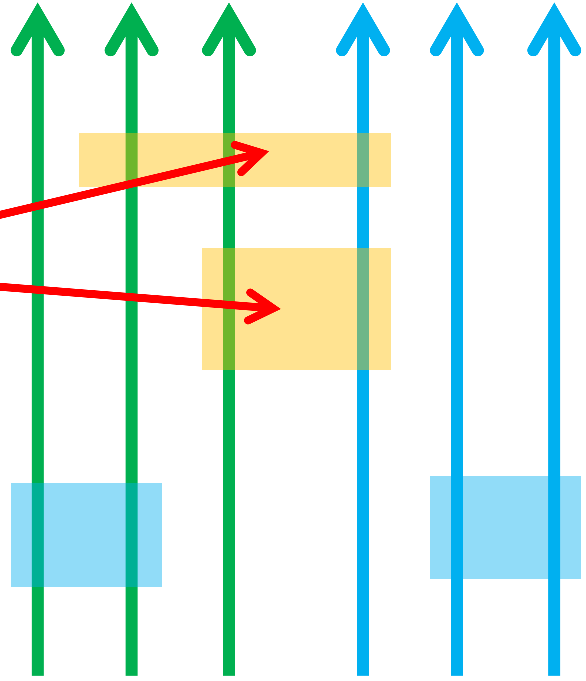
$M_N \sim 3M_{q.} \quad + \quad \text{kin.} \quad + \quad \text{color-EM}$

~ 940MeV ~ 1100MeV - 150-200MeV

in dense regime

sometimes
color-
symmetric

(repulsive)



more chances to feel repulsion

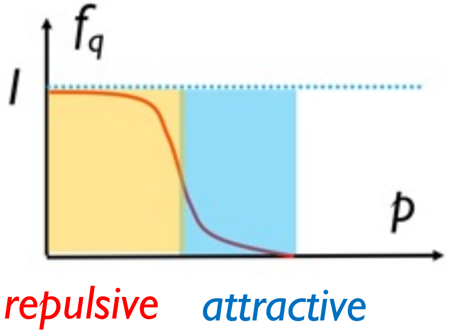
quark energy; *parameterization of MF*

$$E_{\mathrm{CQM}}(\boldsymbol{k}) = \sqrt{M_q^2 + \boldsymbol{k}^2} - C_A + C_S\underline{[f_{\boldsymbol{q}}(k)]^{\beta}}$$

for f_q (p) << 1

$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq -C_E^{\mathcal{A}}$$

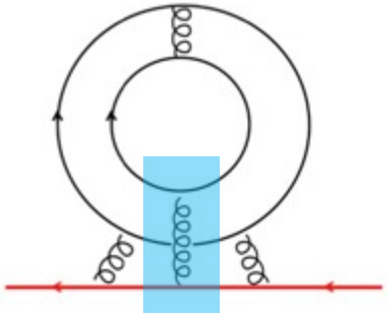
dilute in momentum space



for f_q (p) ~ 1

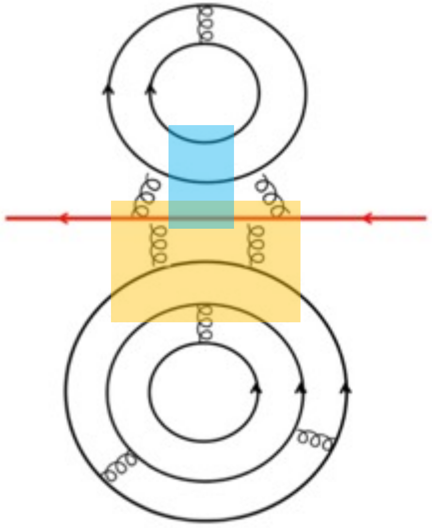
$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq C_E^{\mathcal{S}}$$

for *saturated levels*



color-*antisym.* channels dominate

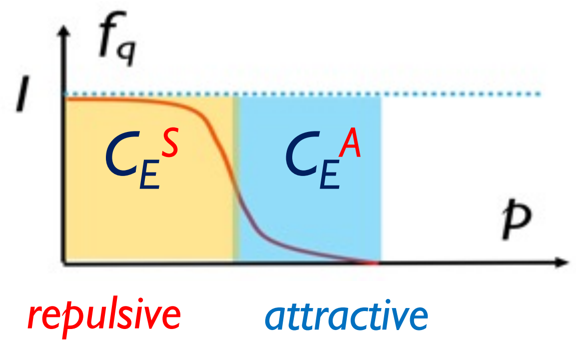
→ the quark feels *attractive* correlations

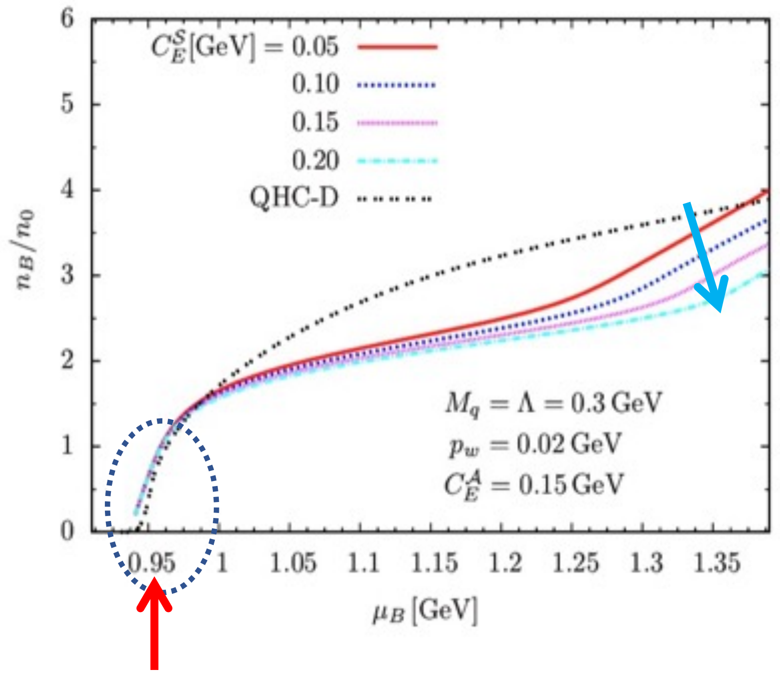


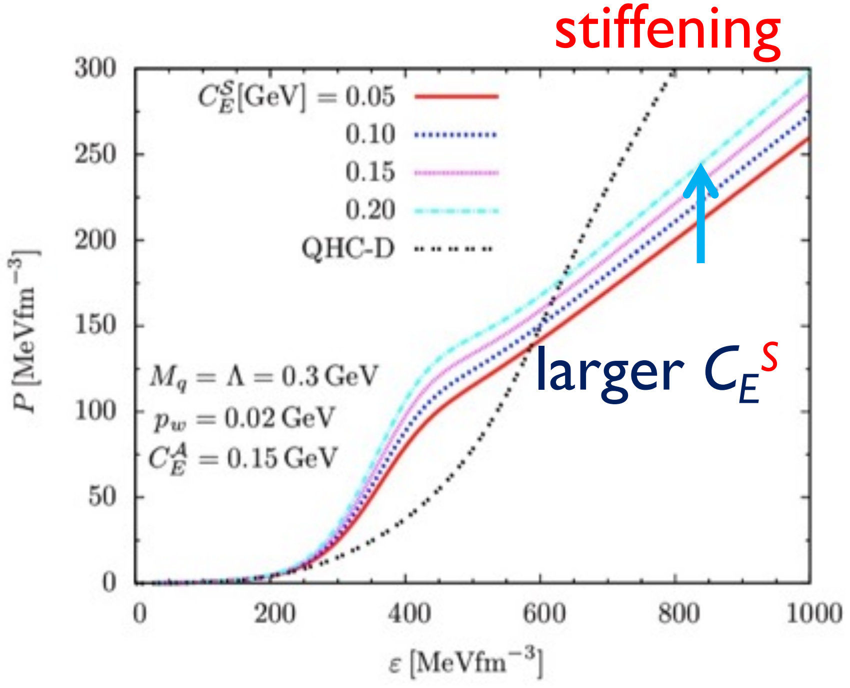
color-*sym.*
channels also enter

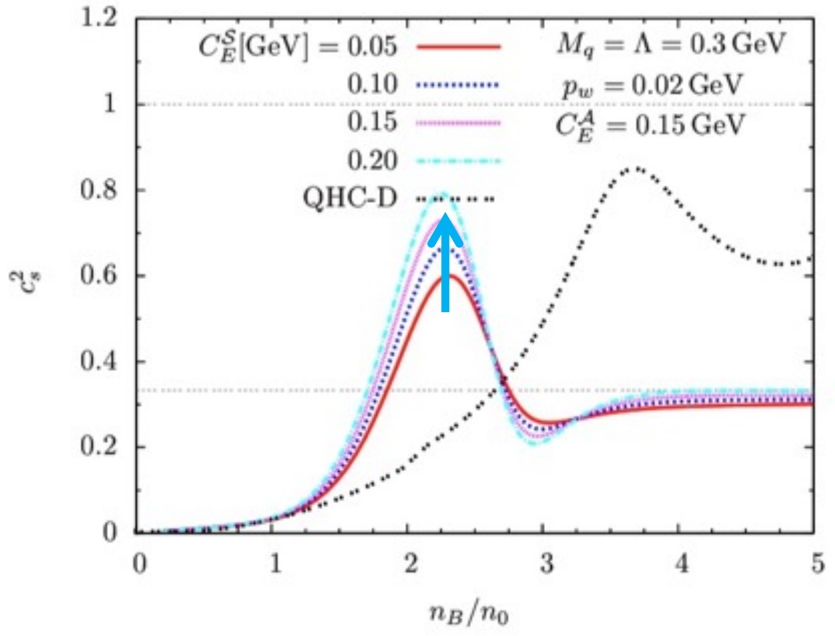
→ the quark feels
repulsive correlations

EoS with interactions









adjust C_E^A (fit M_B = 939 MeV)

high density stiffening

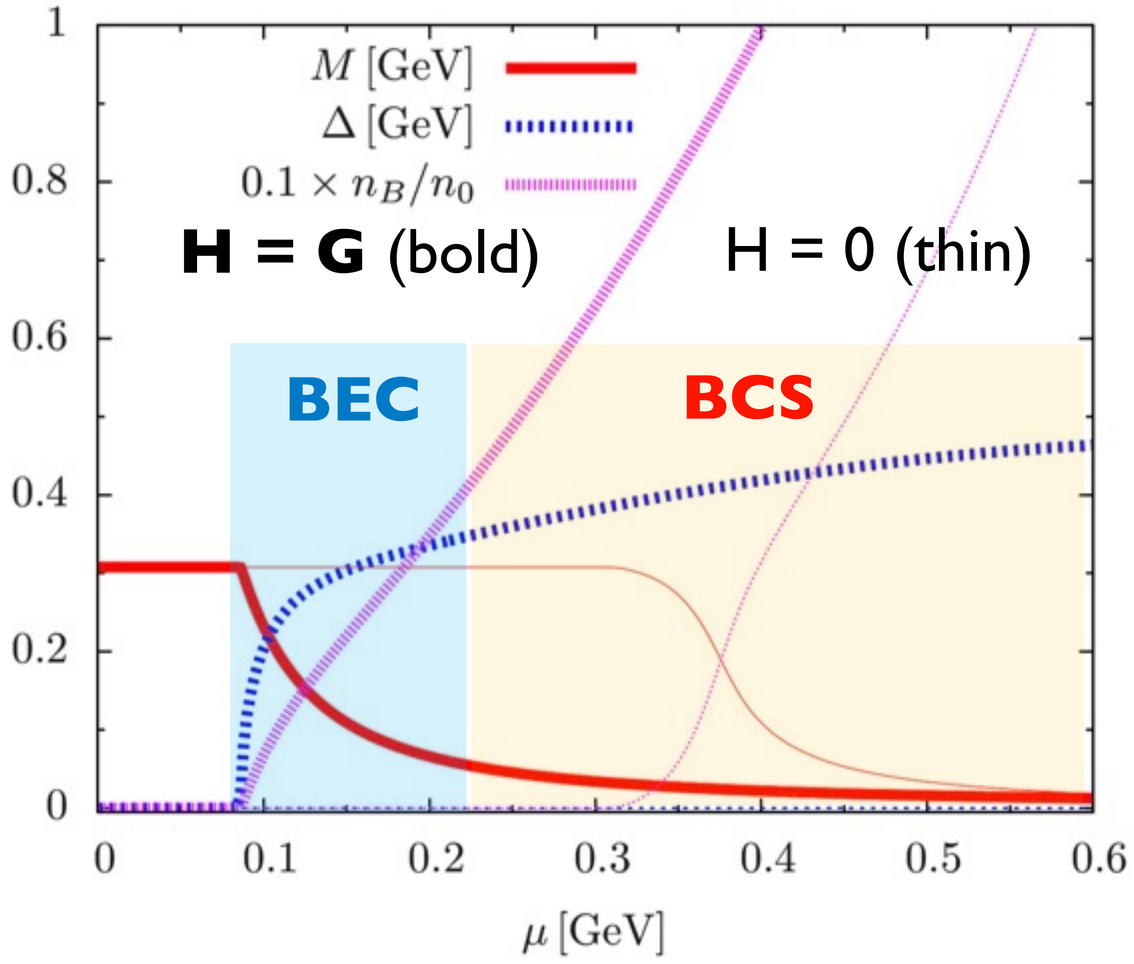
peak in c_s

Example) 2-color NJL model

[TK-Suenaga '21]

- baryons = diquarks
- diquark mass = m_π << M_q
- BEC-BCS crossover (diquark condensate)

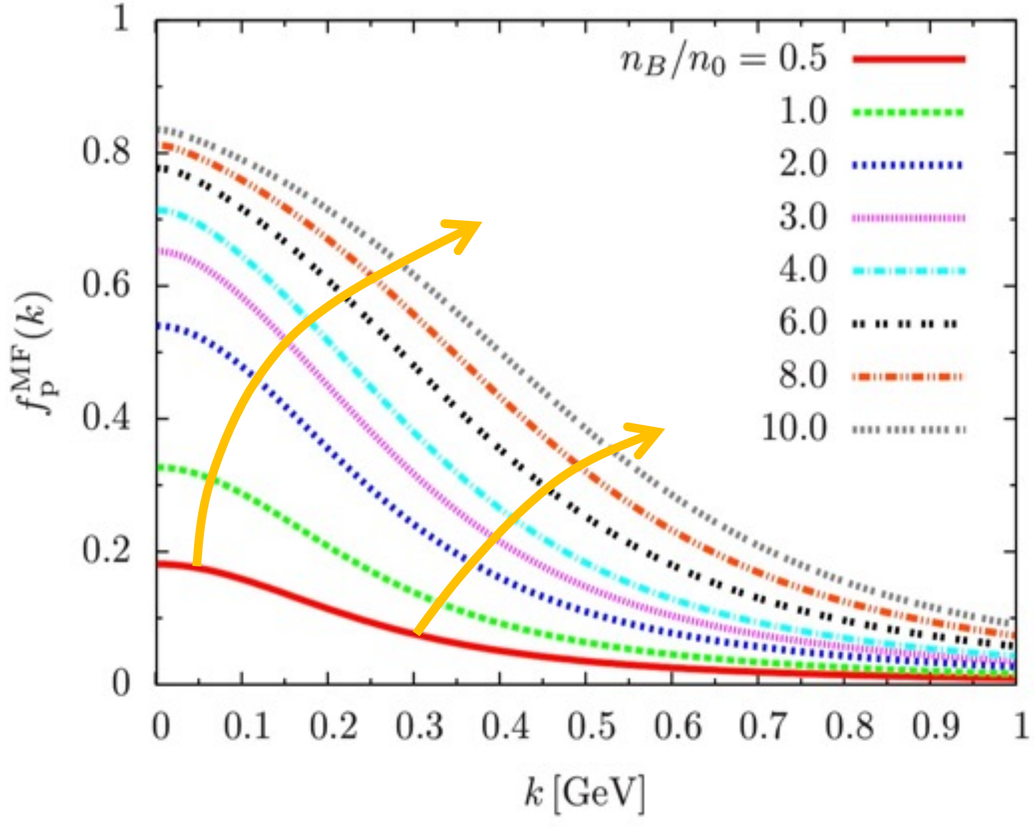
$$\mathcal{L}_4 = G\left[\,(\bar{q}\tau_a q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau_a q)^2\,\right] + H\left[\,|\bar{q}\mathrm{i}\gamma_5\tau_2\sigma_2 q_C|^2 + |\bar{q}\tau_2\sigma_2 q_C|^2\,\right]$$



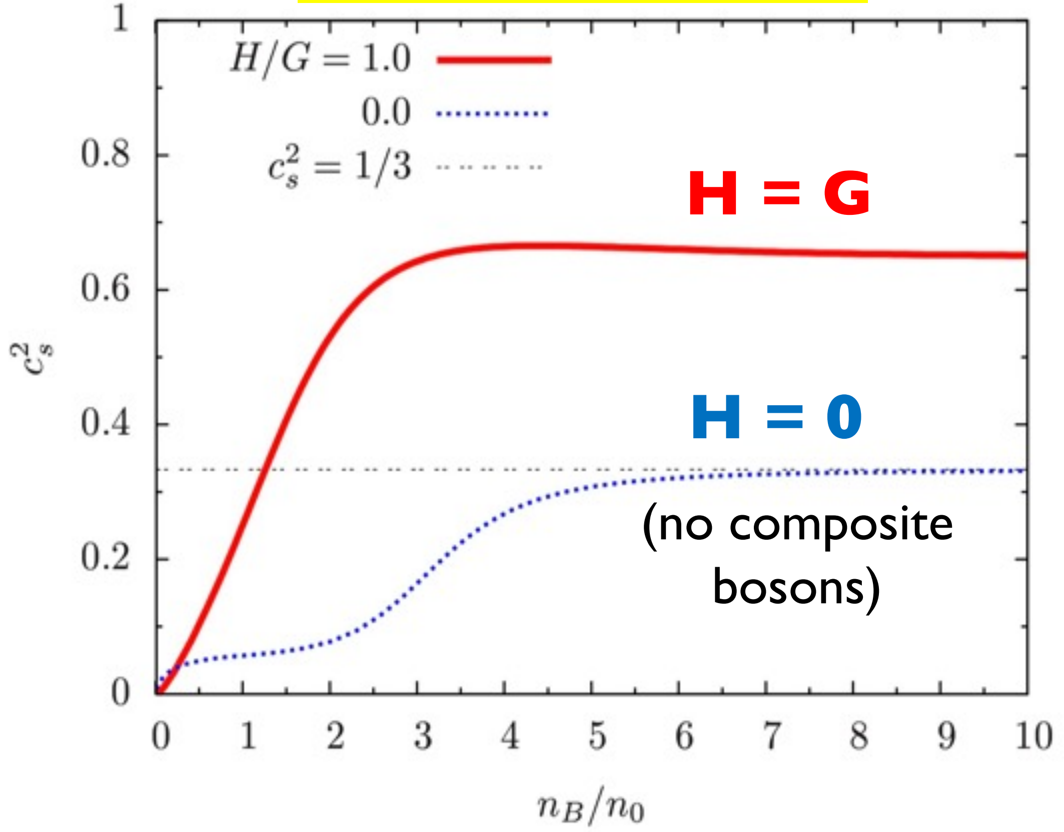
Example) 2-color NJL model

[TK-Suenaga '21]

occupation probability



sound velocity



BEC → BCS & c_s^2↑ occur at 0.5-1n_0 (early stiffening)

Inversion problem: motivations to study *B*

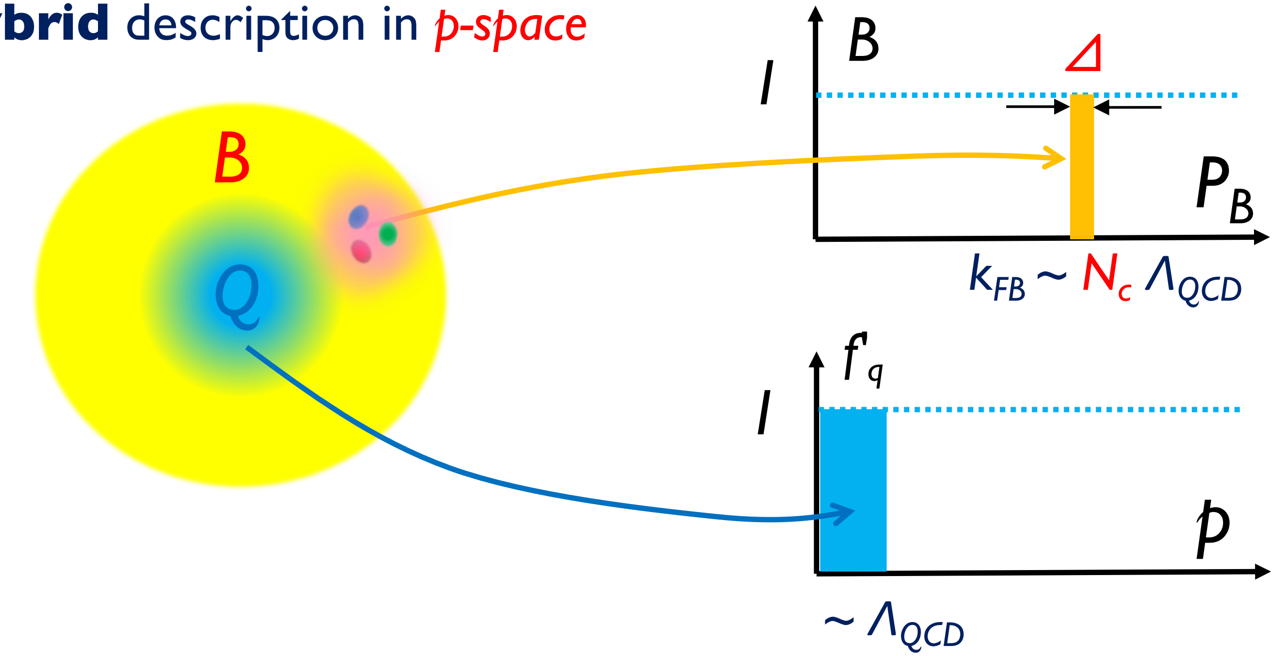
- perhaps convenient to *use the baryonic bases* for *low E* physics

$$P(\mu_B)|_{\beta\text{-eq}} \longrightarrow P(\mu_B, \mu_Q, T, \dots)$$

extensions of
the *quark-hadron continuity*

- relations to the *McLerran-Reddy* (*MR*) model

hybrid description in *p-space*



important parameter

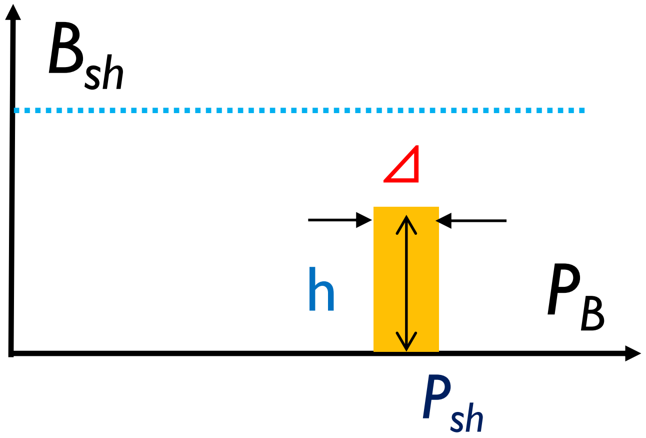
$$\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

why this form?

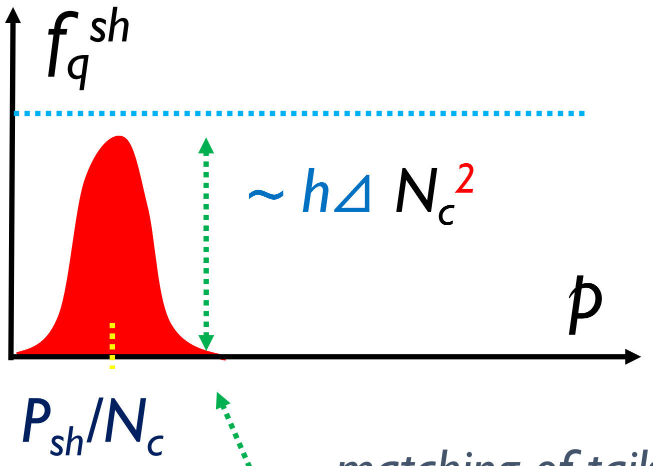
- phenomenological
  [McLerran-Reddy, PRL '19]
- derivation in excluded vol. model
  [Jeong-McLerran-Sen, '19]

A trial: *shell form*

$$\mathcal{B}^{\rm sh}(P_B;P_{\rm sh}) = \underline{h}\,\theta(P_{\rm sh}-P_B)\,\theta(P_B-P_{\rm sh}-\underline{\Delta})$$

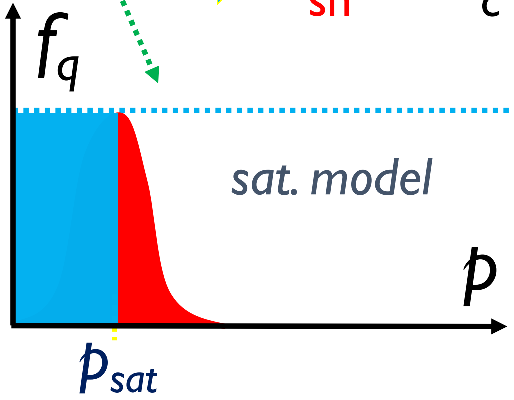


Q_{in}



matching of tails

$P_{sh} \sim N_c\, p_{sat}$



$$f_q^{\rm sh}(p) \simeq h\Delta\,\frac{N_c^3}{\sqrt{\pi}}\,\frac{\tilde{P}_{\rm sh}}{\tilde{p}}\,e^{-\tilde{p}^2-\tilde{P}_{\rm sh}^2}\left(e^{2\tilde{p}\tilde{P}_{\rm sh}}-e^{-2\tilde{p}\tilde{P}_{\rm sh}}\right)$$

$$f_q^{\rm sh}(p) \sim \underline{h\Delta}\,\underline{N_c^2}\,e^{-(\tilde{p}-\underline{\tilde{P}_{\rm sh}})^2}$$

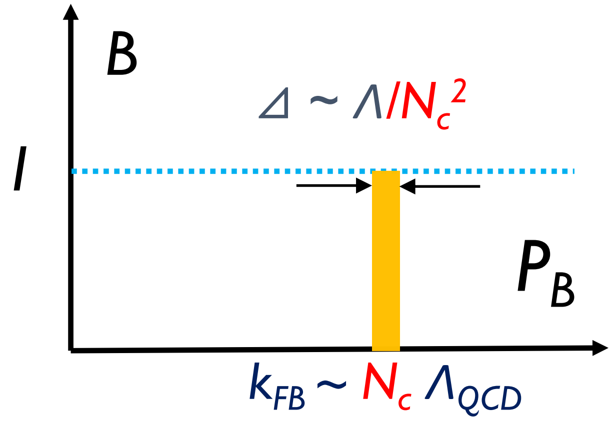
$P_{sh} \sim N_c \Lambda$

Constraints from f_q (for $P_{sh} \sim N_c \Lambda$)

$$f_q^{\rm sh}(p) \sim \underline{h}\,\underline{\Delta N_{\rm c}^2}\, e^{-(\tilde{p}-\underline{\tilde{P}_{\rm sh}})^2}$$

constraint : $f_q^{sh} < 1$ ➡ $h\Delta < \Lambda/N_c^{\,2}$

a possible scaling form: $$[h\Delta](P_{\rm sh}) \sim c_0 \Lambda \left(\frac{\Lambda^2}{\underline{P_{\rm sh}^2}} + \frac{c_1}{N_{\rm c}} \frac{\Lambda}{P_{\rm sh}} + \frac{c_2}{\underline{N_{\rm c}^2}} \right)$$



MR-model (thin shell model)

$h = 1$ & $$\Delta = \frac{\Lambda^3}{\underline{k_{\rm FB}^2}} + \kappa \frac{\Lambda}{\underline{N_c^2}}$$ ($c_1 = 0$)

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

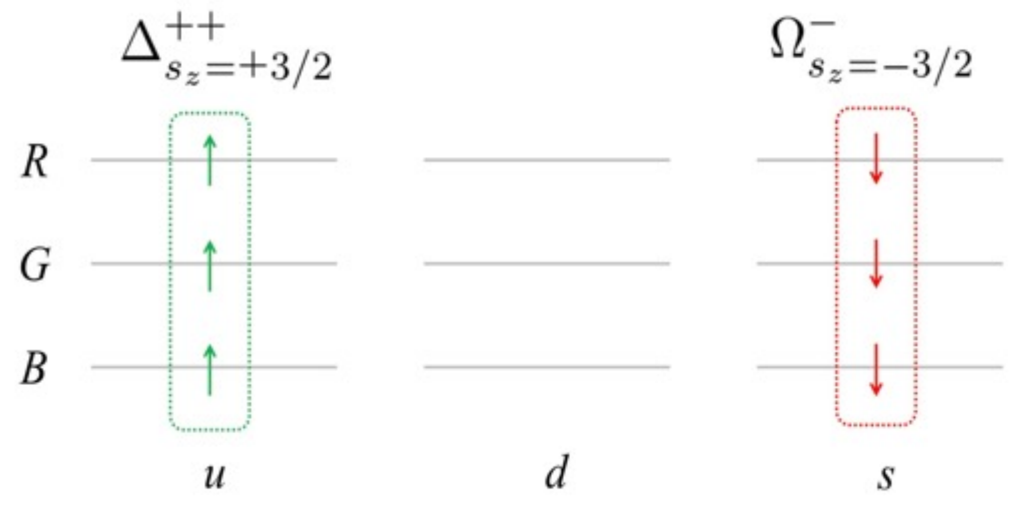
$\rightarrow$ we need only $\mathbf{2N_f = 6}$ species for $N_f = 3$

(full members of singlet, octet, decuplet are NOT necessary)

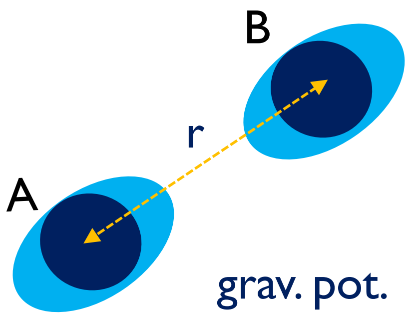
convenient **color-flavor-spin** bases

[neglect N-⊿ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\!\uparrow u_G\!\uparrow u_B\!\uparrow], \quad [u_R\!\downarrow u_G\!\downarrow u_B\!\downarrow],$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\!\uparrow d_G\!\uparrow d_B\!\uparrow], \quad [d_R\!\downarrow d_G\!\downarrow d_B\!\downarrow],$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\!\uparrow s_G\!\uparrow s_B\!\uparrow], \quad [s_R\!\downarrow s_G\!\downarrow s_B\!\downarrow],$$



Tidal deformation → accelerated phase evolution



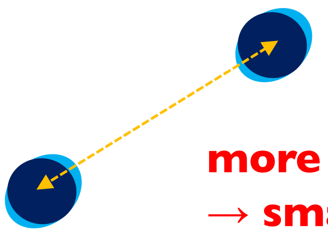
polarizability

quadrupole moment $Q_{ij} = -\lambda(M) E_{ij}$ external field $E_{ij} = -\dfrac{\partial^2 V}{\partial x_i \partial x_j}$

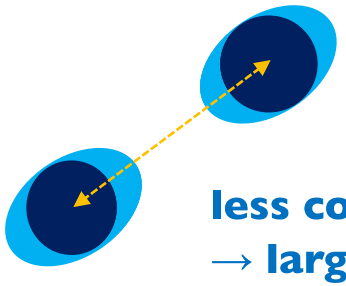
grav. pot. from the star A $V_A(r) \simeq -\dfrac{GM_A}{r} - \dfrac{GQ_{AB}}{r^3}$ deformation of A by B

additional **attraction**

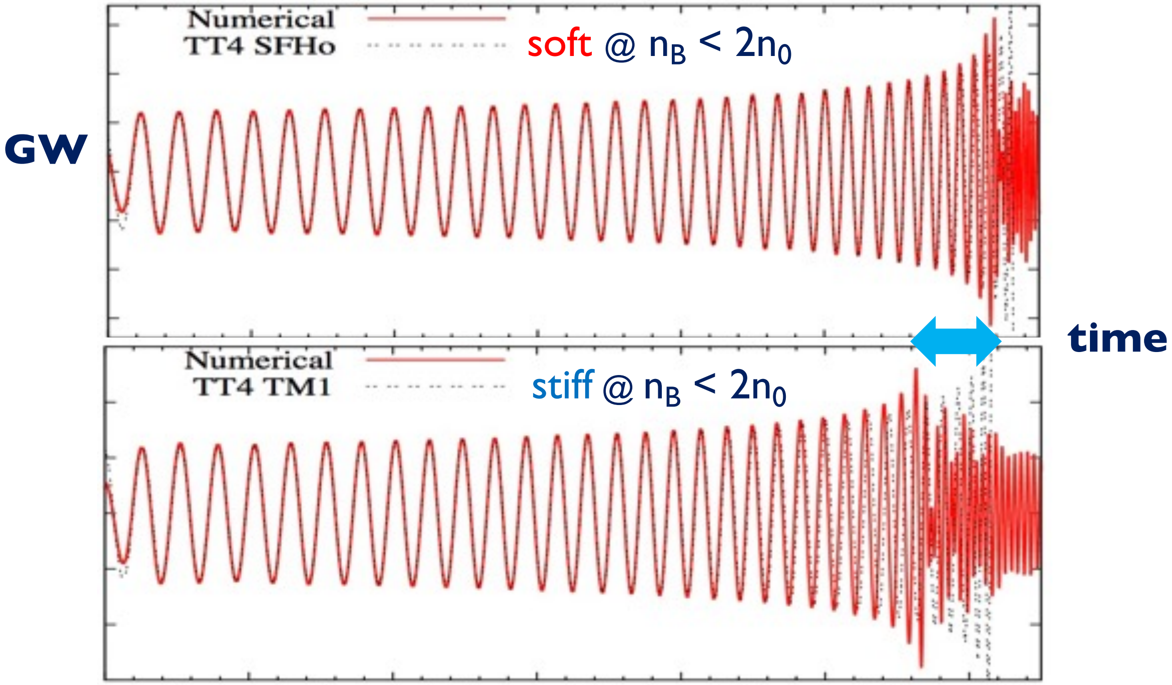
→ **NSs approach faster**



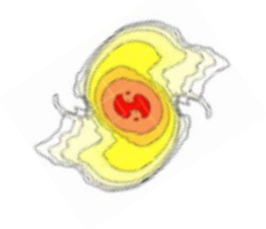
more compact
→ smaller Q



less compact
→ larger Q

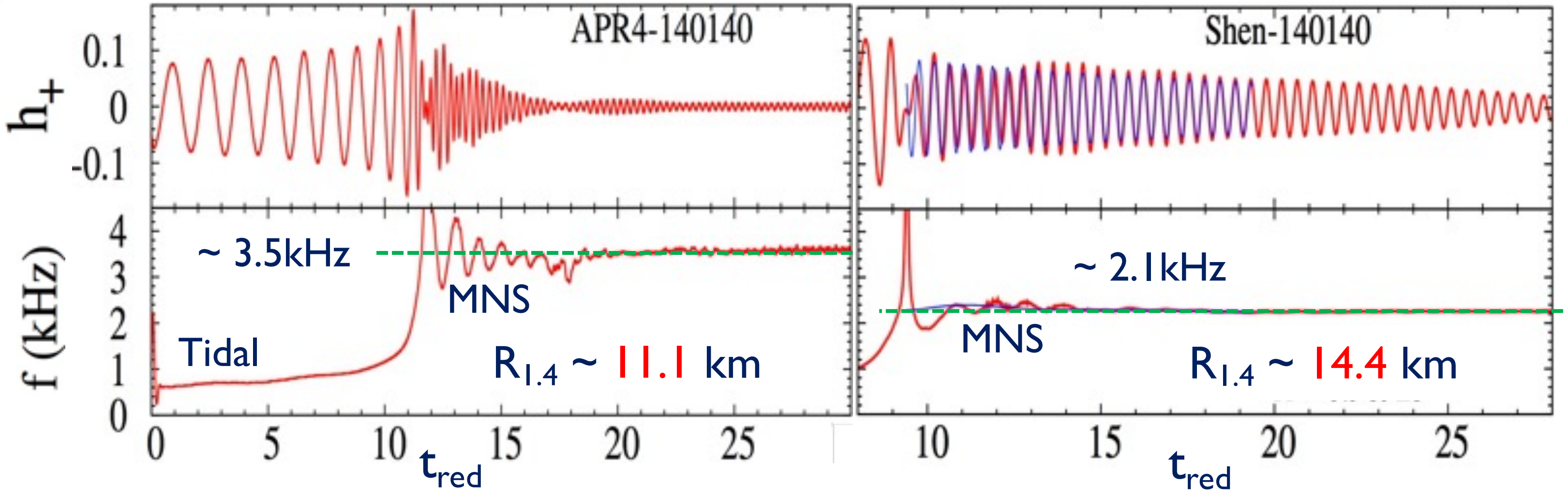


MNS



Merger & HMNS: $f_{GW} \rightarrow R_{NS}$

Figs from Hotokezaka+ 2013



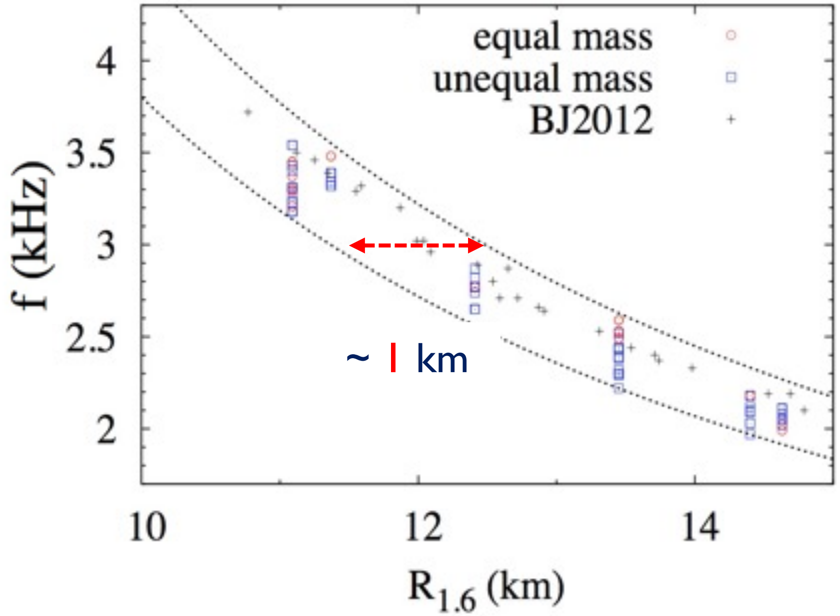
compact stars → high frequency GW

smaller R_{NS} → larger f_{GW}

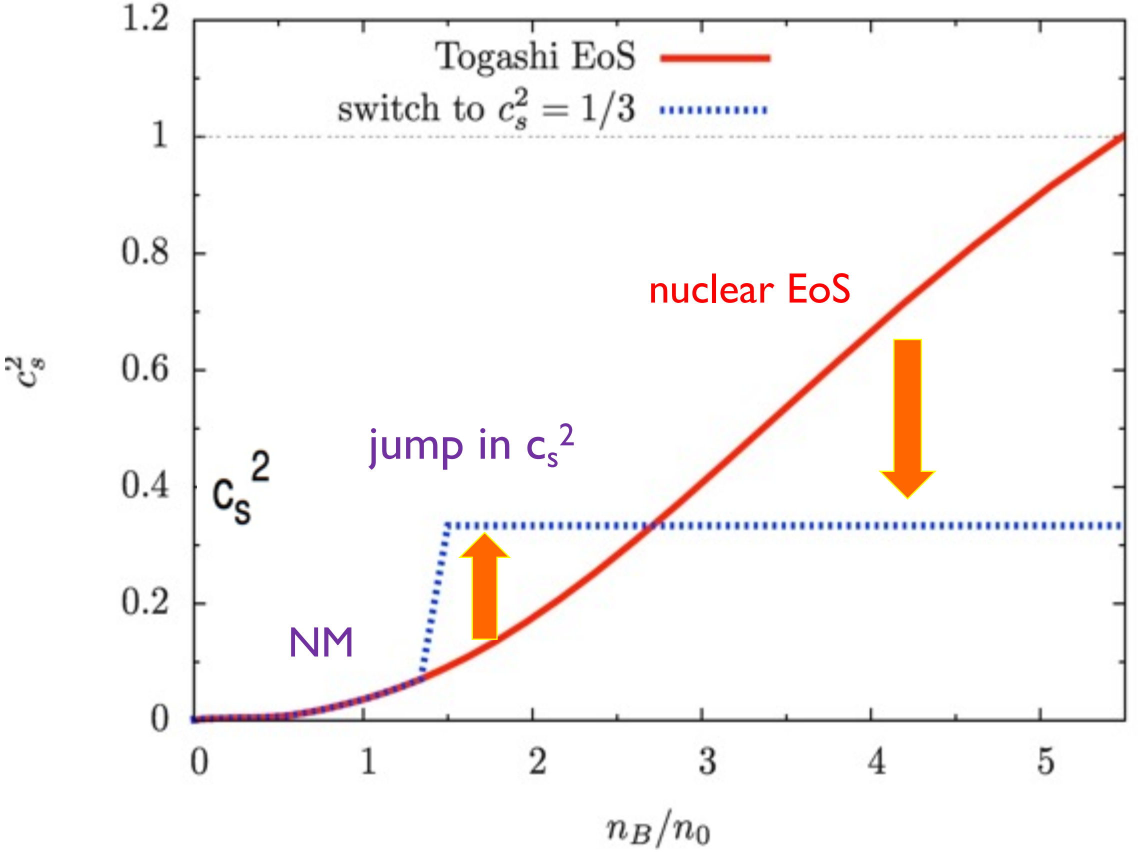
(Bauswein and Janka 2012)

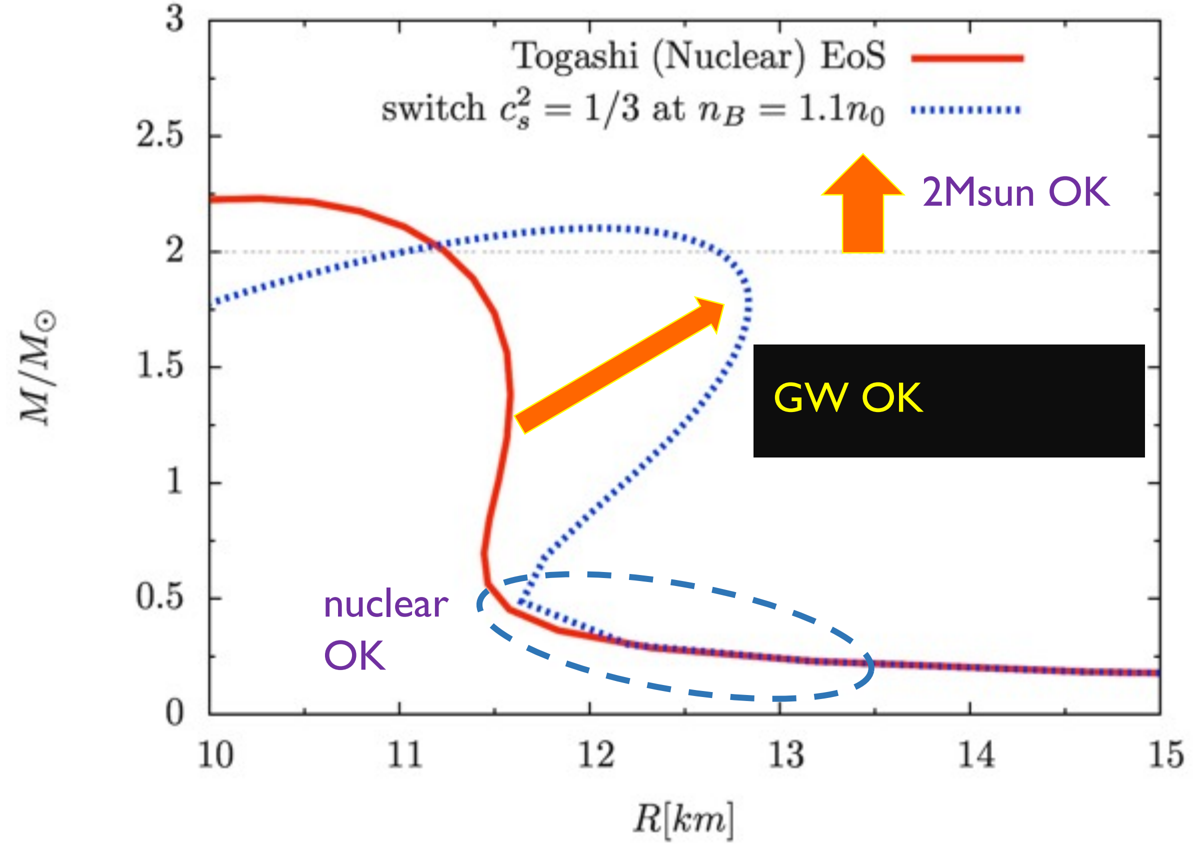
For **GW170817** :

f_{GW} is **NOT measured yet**;
high frequency region → smaller S/N



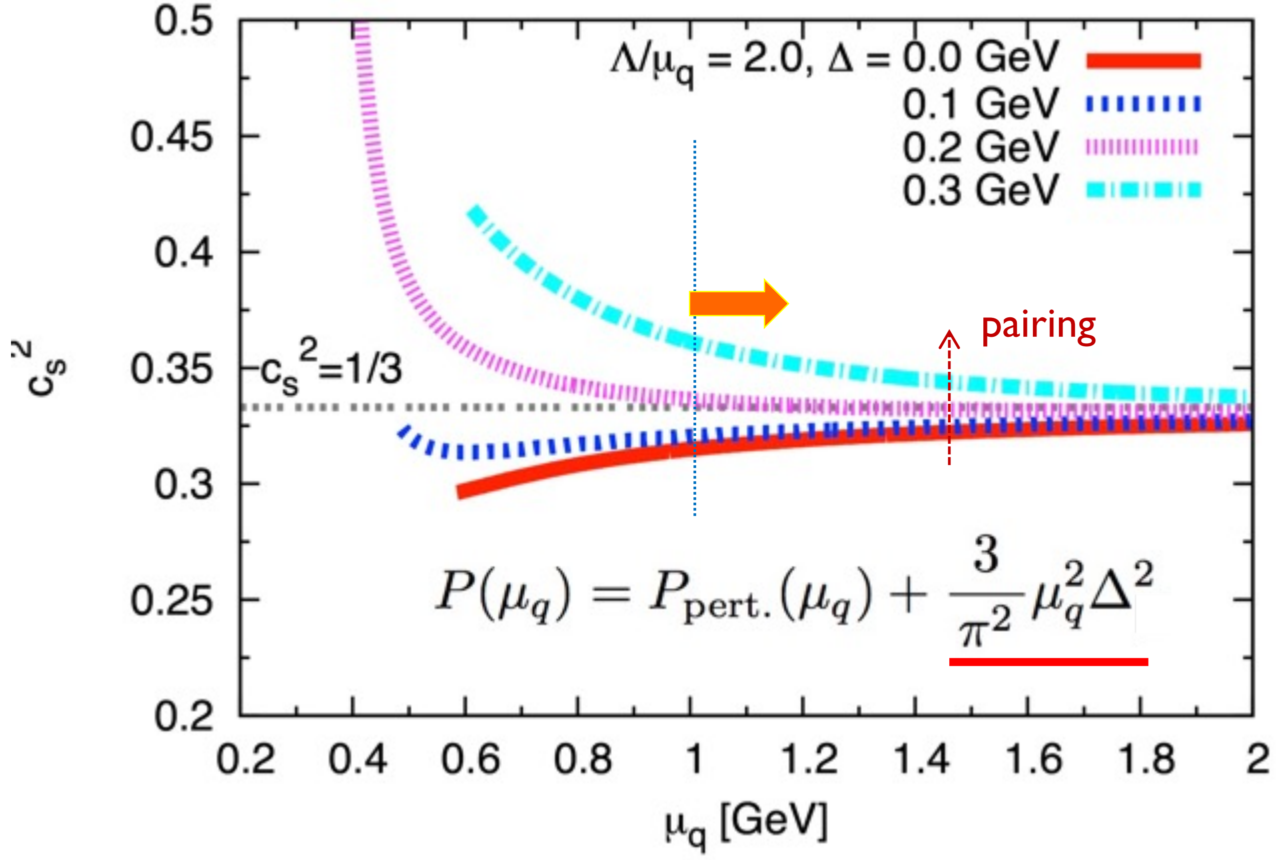
Caveat 1: $c_s^2 > 1/3$ generic ?





Depends on EoS at 1-2n_0

Remark) pQCD + pairing vs speed of sound



[pQCD: Kurkela + (2009)]

For $\Delta >\sim$ **0.2** GeV

$(\Delta / \mu_q)^2 \sim$ **4 %**

nevertheless,

c_s^2 approach 1/3 from **above**

$P \sim \mu^4 \rightarrow c_s^2 = 1/3$

$P \sim \mu^2 \rightarrow c_s^2 = 1$

If μ^2 dominates $\rightarrow c_s^2 > 1/3$

(perhaps Fermi surface effects)

[see also McLerran-Reddy (2018)]