Sound velocity beyond the High-Density Relativistic Limit from Lattice Simulation of Dense Two-Color QCD

Etsuko Itou (RIKEN/ Keio U. / Osaka U.)
Based on K.lida and EI, arXiv: 2207.01253

The 9th International Conference on Quarks and Nuclear Physics (QNP2022), 06/09/2022, Florida State University (online)
\[ c_s^2/c^2 > 1/3 \] is found by Lattice Simulation in Dense Two-Color QCD

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Introduction: finite-\(T\) transition

EoS and sound velocity at zero-\(\mu\)

Finite Temperature transition
(Nf=2+1 QCD)

EoS
\( (p \text{ and } \varepsilon) \)

Sound velocity
\( c_s^2 = \frac{dp}{d\varepsilon} \)

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Introduction: Today's talk
EoS and sound velocity at low-T and high-$\mu$

EoS
\[ p(\mu) \text{ vs } \varepsilon(\mu) \]

Sound velocity
\[ c_s^2 = \frac{\partial p}{\partial \varepsilon} \]

Low-$\mu$ ($n_B \lesssim 2n_0$): Hadronic matter

High-density relativistic limit
\[ \frac{c_s^2}{c^2} = \frac{1}{3} \]

High-$\mu$ ($5n_0 < n_B$): Quark matter

\[ \rightarrow \text{pQCD (}50n_0 < n_B)\]
EoS, $c_s$ and neutron star

Mass and radius of neutron star

Sound velocity $c_s^2 = \partial p / \partial \epsilon$

Mass-Radius of neutron star $\Leftrightarrow$ EoS in dense QCD

Prediction by phenomenology and effective models

Sound velocity has a peak?

\[ \frac{c_s^2}{c^2} \]

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests \( c_s^2 \) peaks at \( n_B = 1 - 10n_0 \)
  - Masuda, Hatsuda, Takatsuka (2013)
  - Baym, Hatsuda, Kojo (2018)

- Quarkyonic matter model
  \( c_s^2 \) peaks at \( n_B = 1 - 5n_0 \)
  - McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (color independent)
  - Kojo (2021), Kojo and Suenaga (2022)

Lattice study on 2color dense QCD
2color QCD $\approx$ 3color QCD

- 2color QCD reduced model with color d.o.f. in real QCD
  
  - Properties of 3color QCD at $\mu = 0$
    - asymptotic freedom
    - finite T transition (chiral/confinement)
    - pseudo-scalar meson is lightest (pion) cf.) QCD inequality
    - EoS(energy, pressure)
  
  - Qualitatively, 2color QCD has the same ones
  
  - Quantitatively, EoS shows very similar at least quenched QCD case

In 2color QCD at $\mu \neq 0$, the sign problem is absent. Find qualitative property of real dense 3color QCD
2color QCD phase diagram

(1) K.lida, K.Ishiguro, El, arXiv: 2111.13067
(2) K.lida, El, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) T.Furusawa, Y.Tanizaki, El: PRResearch 2(2020)033253
Phase diagram of 2color QCD

This work

<table>
<thead>
<tr>
<th></th>
<th>Hadronic</th>
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<th>Superfluid</th>
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Scaling law of order param. is consistent with ChPT.

Kogut et al., NPB 582 (2000) 477
Phase diagram of 2color QCD

This work

In high-$\mu$, $\langle n_q \rangle \approx n_q^{\text{tree}}$

number density of free particle

BEC-BCS crossover

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Equation of state

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)
  - beta=0.80 (Iwasaki gauge), $16^4$ lattice
  - T=79MeV, j->0 extrapolation is taken

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3} \left( a \frac{d\beta}{da} \left. LCP \langle \frac{\partial S}{\partial \beta} \rangle_{\text{sub.}} \right|_{\text{LCP}} + a \frac{d\kappa}{da} \left. LCP \langle \frac{\partial S}{\partial \kappa} \rangle_{\text{sub.}} \right|_{\text{LCP}} + a \left. \frac{\partial j}{\partial a} \right|_{\text{LCP}} \right)
  - No renormalization for $\mu$
  - $\langle \cdot \rangle_{\text{sub.}} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$
  - Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_0}^{\mu} n_q(\mu') d\mu'$

---

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV
Equation of state

- Fixed scale approach ($\mu \neq 0$ version)
  beta=0.80 (Iwasaki gauge), $16^4$ lattice
  T=79MeV, j->0 extrapolation is taken

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3} \left( a \frac{d\beta}{da} \langle \frac{\partial S}{\partial \beta} \rangle_{sub.} + a \frac{dk}{da} \langle \frac{\partial S}{\partial k} \rangle_{sub.} + a \frac{dj}{da} \langle \frac{\partial S}{\partial j} \rangle_{sub.} \right) - 3p$
  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_0}^{\mu} n_q(\mu') d\mu'$

Technical steps

1) Measure the gauge action and chiral cond.
2) Calculate the beta fn. at $\mu = 0$
3) Numerical integration of $n_q$
Equation of state

- Fixed scale approach ($\mu \neq 0$ version)
  - beta=0.80 (Iwasaki gauge), $16^4$ lattice
  - $T=79\text{MeV}$, $j \to 0$ extrapolation is taken

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3} \left( a \frac{d \beta}{d a} \langle \frac{\partial S}{\partial \beta} \rangle_{\text{sub.}} + a \frac{d \kappa}{d a} \langle \frac{\partial S}{\partial \kappa} \rangle_{\text{sub.}} + a \frac{\partial j}{d a} \langle \frac{\partial S}{\partial j} \rangle \right)$
  - Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu') d\mu'$

Nonperturbative beta-fn.

- $a \frac{d \beta}{d a} = -0.3521$, $a \frac{d \kappa}{d a} = 0.02817$

K.lida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0
Trace anomaly and pressure

- Sum of trace anomaly, \((e - 3p)_g + (e - 3p)_f\)
  - zero in Hadronic phase
  - positive in BEC phase
  - positive -> negative in BCS phase
  - Finally, fermions give the larger contribution

- Pressure increase monotonically
  - In high density, it approaches
  \[ p_{SB}/\mu^4 = N_cN_f/(12\pi^2) \approx 0.03 \]
P and e as a function of $\mu$

(Normalized by $1/\mu_c^4$ to be dim-less)

- P is zero in Hadronic phase since $n_q = 0$
- e is also zero in Hadronic phase by the cancelation between $(e - 3p)_g$ and $(e - 3p)_f$

From these data, the sound velocity is obtained

$$c_s^2/c^2 = \frac{\Delta p}{\Delta e} = \frac{p(\mu + \Delta \mu) - p(\mu - \Delta \mu)}{e(\mu + \Delta \mu) - e(\mu - \Delta \mu)}$$
Sound velocity \( \left( \frac{c_s^2}{c^2} = \frac{\Delta p}{\Delta e} \right) \)

Chiral Perturbation Theory (ChPT)

\[
\frac{c_s^2}{c^2} = \frac{1 - \mu_c^4/\mu^4}{1 + 3\mu_c^4/\mu^4} : \text{no free parameter!!}
\]

Son and Stephanov (2001) : 3color QCD with isospin \( \mu \)
Hands, Kim, Slullerud (2006) : 2color QCD with real \( \mu \)

- In BEC phase, our result is consistent with ChPT.
- \( \frac{c_s^2}{c^2} \) exceeds the relativistic limit
- In high-density, it peaks around \( \mu \approx m_{PS} \).

"Stiffen" and then "soften" picture as density increases
Sound velocity and phase transition

Finite Temperature transition
(Nf=2+1 QCD)

Finite Density transition
(Nf=2 2color QCD)

- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$
- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

Borsanyi et al. (2013)
lida and El arXiv: 2207.01253

HotQCD (2014)
Further high density?

Kojo, Baym, Hatsuda (2021)

**pQCD prediction**
(Ultra high-density regime)

- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$
  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

\[ c_s^2/c^2 = \frac{1 - 5\beta_0\alpha_s^2/(48\pi^2)}{3} \]
Summary and future work

• In BEC phase, our result is consistent with ChPT. Sound velocity exceeds the relativistic limit and has a peak after BEC-BCS crossover cf.) cond-mat model study also find it
  Tajima and Liang (2022)

• Find a mechanism of a peak structure
  - quark saturation? (Kojo, Suenaga), strong coupling with trace anomaly? (McLerran, Fukushima et al.), others?
  - attractive or repulsive force between hadrons?
    => extended HAL QCD method in finite density
  - independent of the color dof?

• This finding might have a possible relevance to the EoS of neutron star matter revealed by recent measurements of neutron star masses and radii.
Backup
Two problems at low-T high-$\mu$ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)
  
  Reduce the color dof, 2color QCD
  quarks becomes pseudo-real reps.
  The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-$\mu$ (e.g. $\mu_q > m_\pi/2, \ m_N/3$),
  
  It comes from the phase transition to superfluid phase (SSB of baryon sym.)
  
  Add an explicit breaking term of the sym., then take $j \rightarrow 0$ limit

  $S_F^{cont.} = \int d^4x \bar{\psi}(x)(\gamma_{\mu}D_{\mu} + m)\psi(x) + \mu \hat{N} - \frac{j}{2}(\bar{\psi}_1 K \psi_2 - \psi_2^T K \psi_1)$

  Number op. diquark source

  HMC simulations for whole $T-\mu$ regime are doable!
  (j-$\rightarrow$0 extrapolation is taken in all plots today)
In massive fermion theory, the trace anomaly does not vanish because the mass term breaks the scale invariance. The mass term will give a negative contribution, so that we expect \( e/\mu^4 < e_{SB}/\mu^4 = N_c N_f/(4\pi^2) \).
Scheme dependence of pressure

\[ \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_0}^{\mu} n_q(\mu') d\mu'}{\int_{\mu_0}^{\mu} n_{SB}^{\text{cont}}(\mu') d\mu'}; \quad (28) \]

\[ \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_0}^{\mu} n_{SB}^{\text{cont}}(\mu') n_q(\mu') d\mu'}{\int_{\mu_0}^{\mu} n_{SB}^{\text{cont}}(\mu') d\mu'}; \quad (29) \]
Sound velocity (ratio $\Delta p/\Delta e$) vs energy

\[ \frac{c_s^2}{c^2} \]

relativistic limit

\[ p/e \]

\[ e/\mu_c \]

\[ e/\mu_c \]
Holography bound?

A bound on the speed of sound from holography

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University of Maryland, College Park, MD 20742-4111

Abhinav Nelor
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound $v_s^2$ is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which $v_s^2$ exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1$ is an upper bound for a broad class of four-dimensional theories.

$\frac{c_s^2}{c^2} \leq \frac{1}{3}$ at high $T$

Counterexample for $N=4$ SYM at finite density
Phase diagram
Current status on 2color QCD phase diagram

At least three independent group studying the phase diagram

(1) S. Hands group: Wilson-Plaquette gauge + Wilson fermion
(2) Russian group: tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group: Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale

<table>
<thead>
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<th>Temperature (MeV)</th>
<th>Description</th>
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<tr>
<td>158</td>
<td>Deconfined, hadron -&gt; QGP phase transition occurs</td>
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<tr>
<td>130</td>
<td>Deconfined? QGP phase?, 2019</td>
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<td>140</td>
<td>Deconfined in high mu, &lt;qq&gt; is not zero, 2017, 2018, 2020</td>
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<td>93</td>
<td>Deconfined in high mu ?, also &lt;qq&gt; is not zero?, 2017</td>
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<td>87</td>
<td>Confined in 2019</td>
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<td>Confined even in high mu</td>
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<td>55</td>
<td>Confined in high mu, 2016</td>
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<tr>
<td>47</td>
<td>Deconfined coarse lattice in 2012, but confined in 2019</td>
</tr>
<tr>
<td>45</td>
<td>Confined in 2019</td>
</tr>
</tbody>
</table>

- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- 2color QCD phase diagram has been determined by independent works!
Scale setting at $\mu = 0$

- $T_c$ at $\mu = 0$ from chiral susceptibility

Scale setting at \( \mu = 0 \)

- Tc at \( \mu = 0 \) from chiral susceptibility
- Assume Tc=200MeV
  Tc is realize Nt=10, \( \beta = 0.95 \) (a=0.1[fm])
- Find relationship between \( \beta \) (lattice bare coupling) and \( a \) (lattice spacing)
  In finite density simulation, a=0.1658[fm]
Order parameters in $j=0$ limit

At $T=0.39T_c$, we find the BCS with confined phase until $\mu \lesssim 1152\text{MeV}$. 

Scaling law of order param.
is consistent with ChPT.
Ref: Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky
NPB 582 (2000) 477
**BEC/BCS crossover**

![Graph](image)

**BEC phase**

**BCS phase**

Distance between quarks $\gg \Delta^{-1}$

Quarks behave free particles

Distance between quarks $\ll \Delta^{-1}$

Number density of free particle

$$n_q^{\text{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_T} \sum_k \frac{i \sin \tilde{k}_0 [\sum_i \cos k_i - \frac{1}{2\pi}]}{[\frac{1}{2\pi} - \sum_\nu \cos \tilde{k}_\nu]^2 + \sum_\nu \sin^2 \tilde{k}_\nu}$$
J->0 extrapolation
Diquark condensate has a strong j dependence

Figure 5. The $j$-dependence of the diquark condensate for several $\mu/m_{PS}$. 
J->0 extrapolation

Chiral condensate and $n_q$ have a mild j-dependence