Sound velocity beyond the High-Density Relativistic Limit from Lattice Simulation of Dense Two-Color QCD

Etsuko Itou (RIKEN/ Keio U. / Osaka U.)

Based on K.Iida and EI, arXiv: 2207.01253

The 9th International Conference on Quarks and Nuclear Physics (QNP2022), 06/09/2022, Florida State University (online)

$c_s^2/c^2 > 1/3$ is found by Lattice Simulation in Dense Two-Color QCD

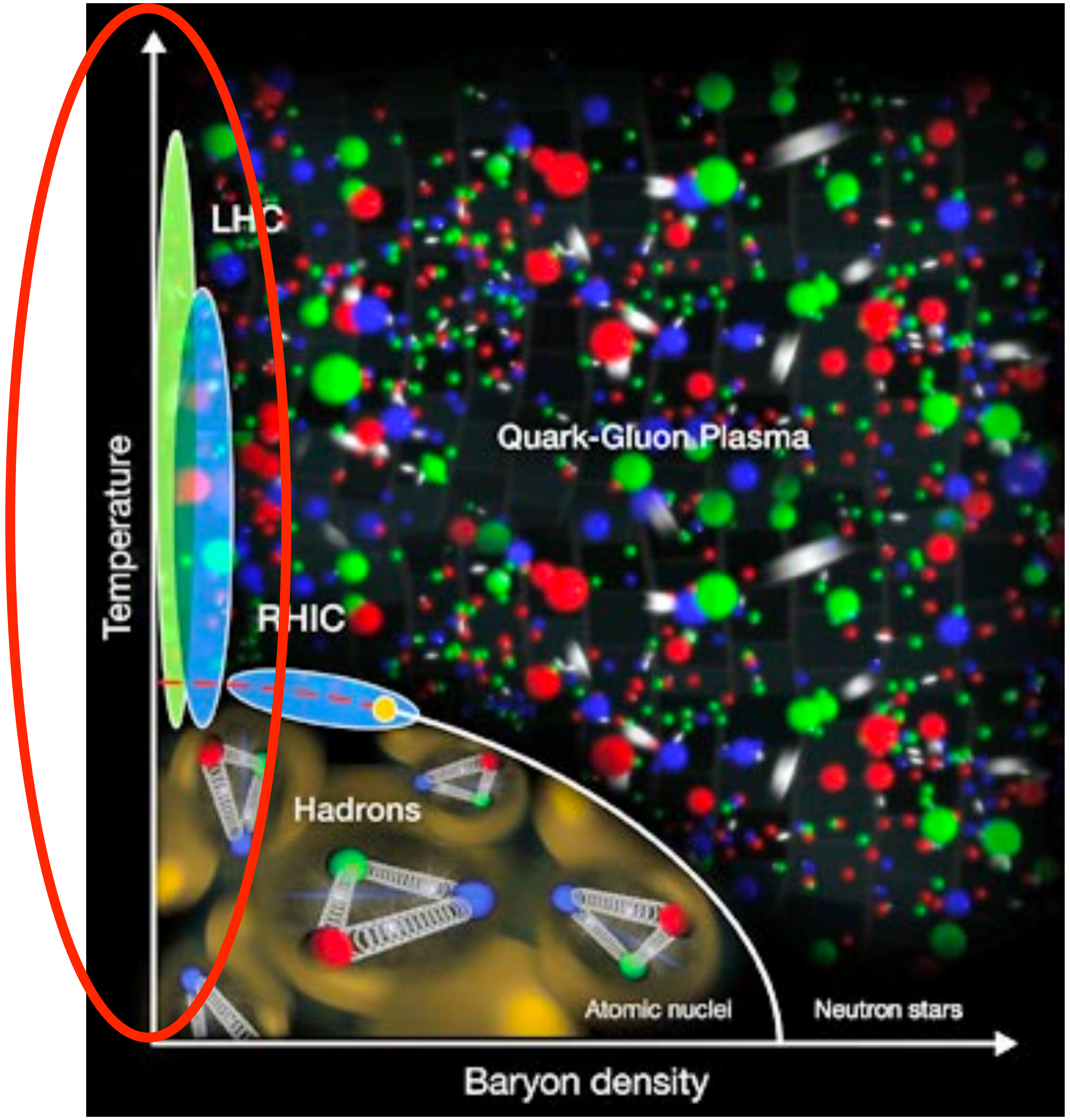
Etsuko Itou (RIKEN/ Keio U. / Osaka U.)

Based on K.Iida and EI, arXiv: 2207.01253

The 9th International Conference on Quarks and Nuclear Physics (QNP2022), 06/09/2022, Florida State University (online)

Introduction: finite-T transition

EoS and sound velocity at zero-μ

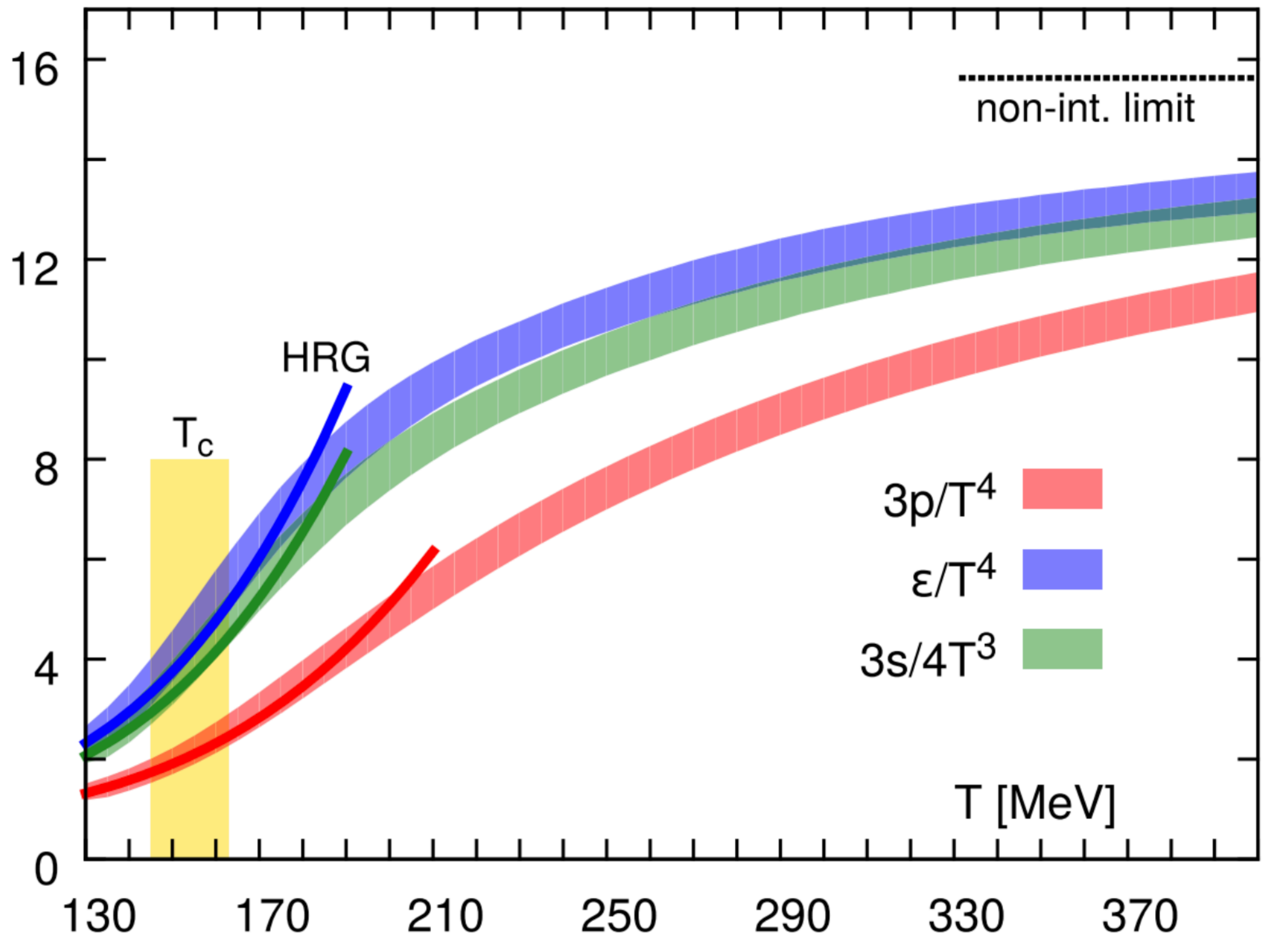


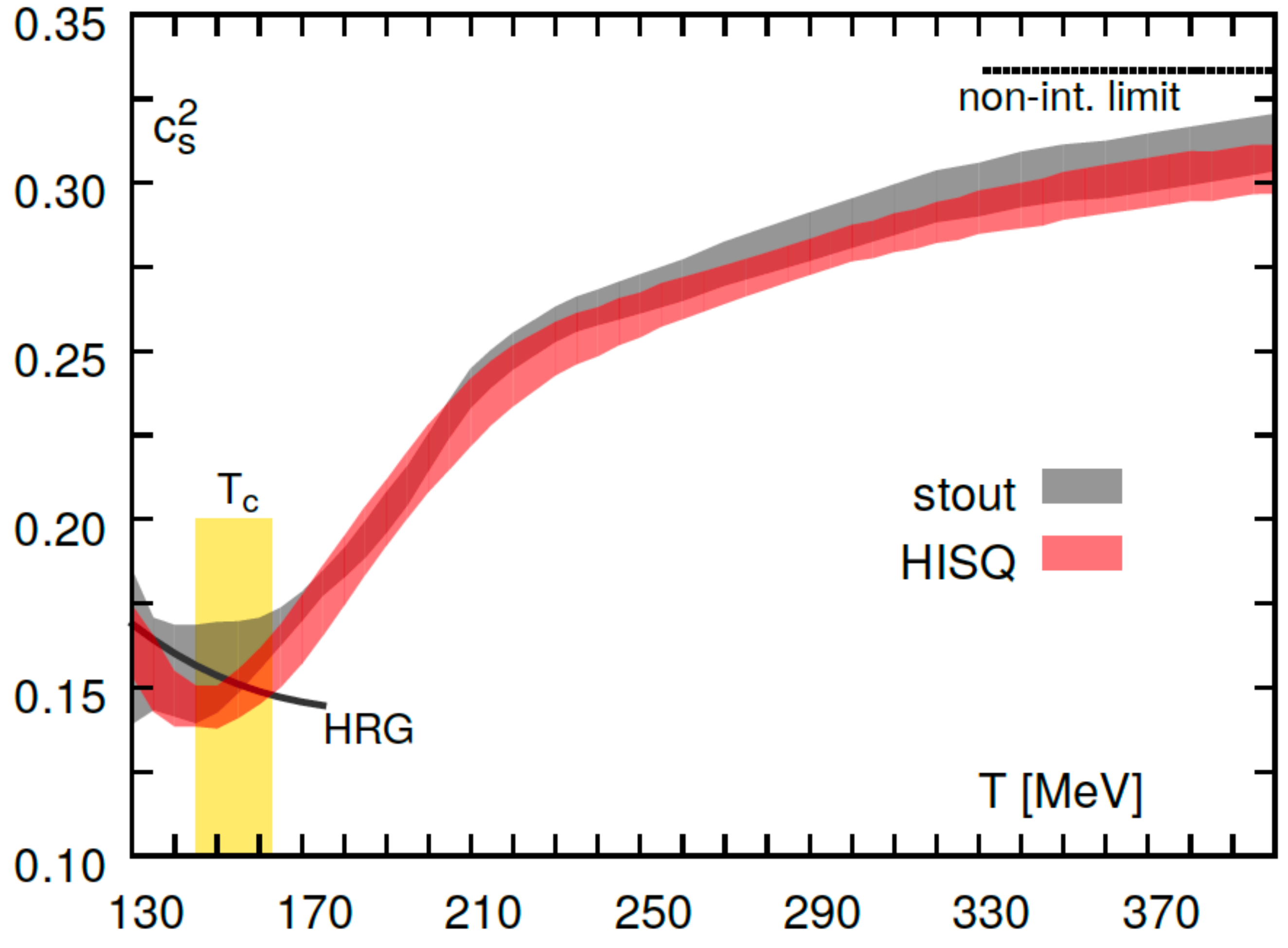
©BNL/RHIC

Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)

Sound velocity $c_s^2 = \partial p/\partial \epsilon$

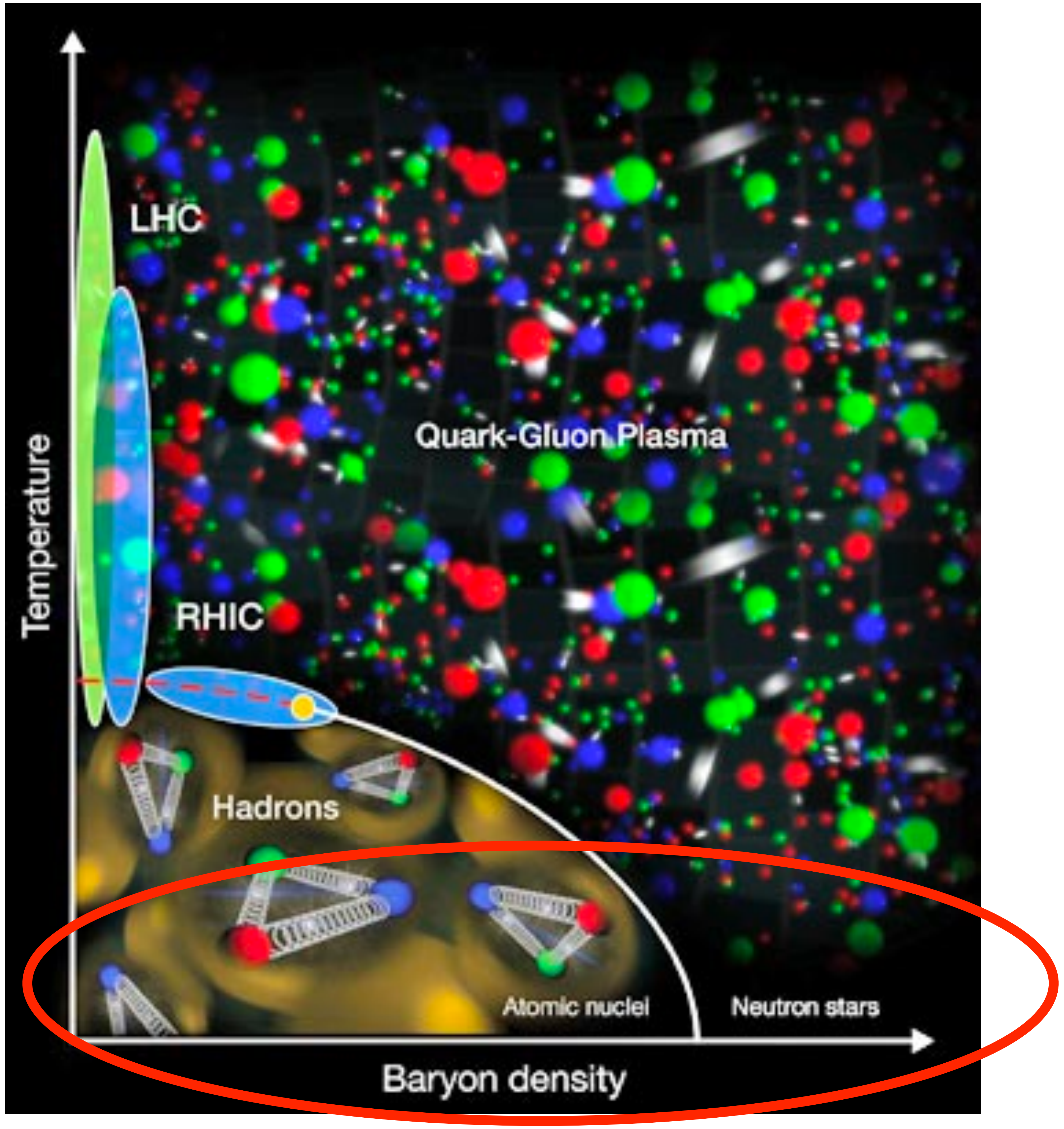




HotQCD (2014)

Introduction: Today's talk

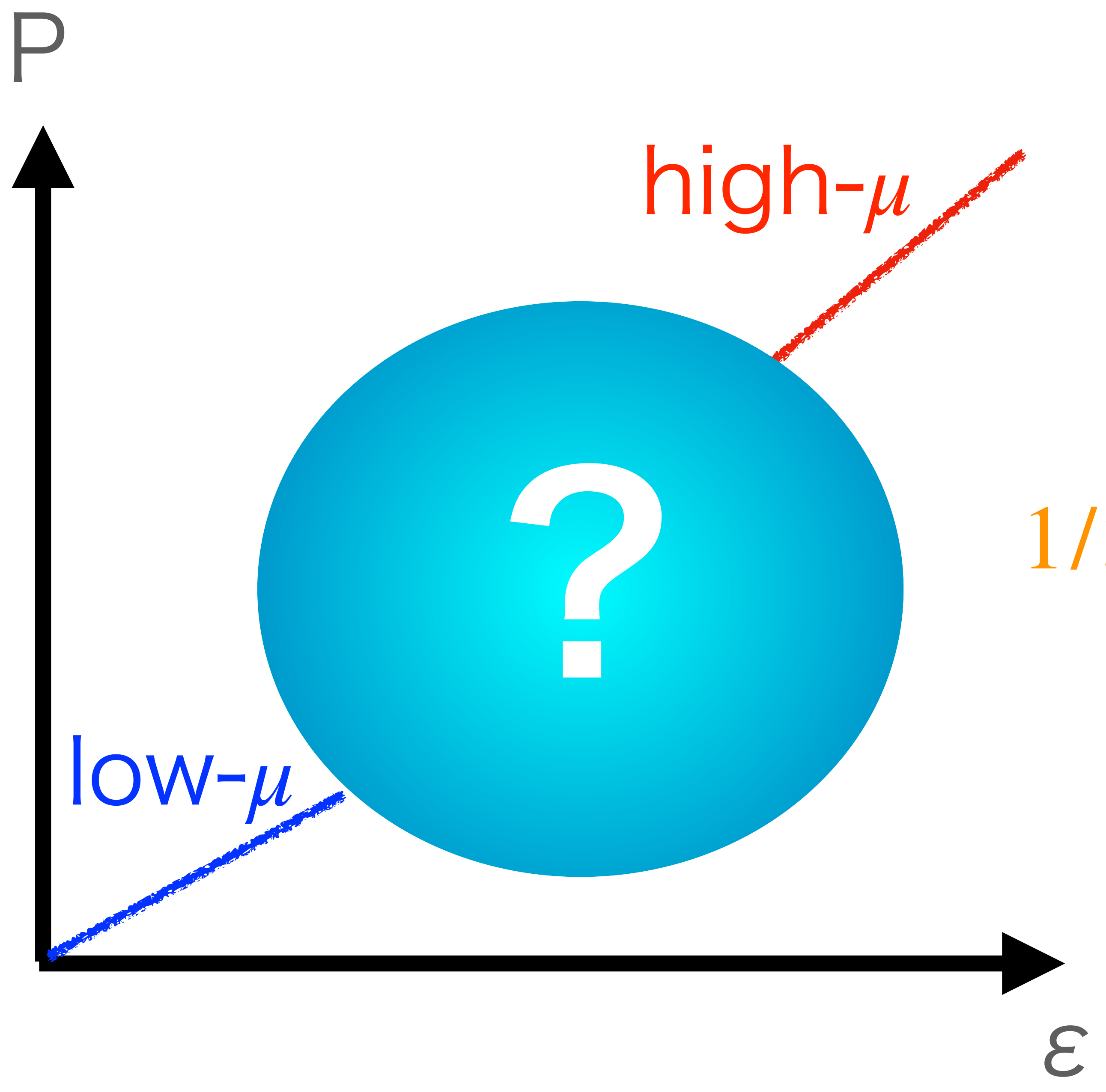
EoS and sound velocity at low-T and high-μ



©BNL/RHIC

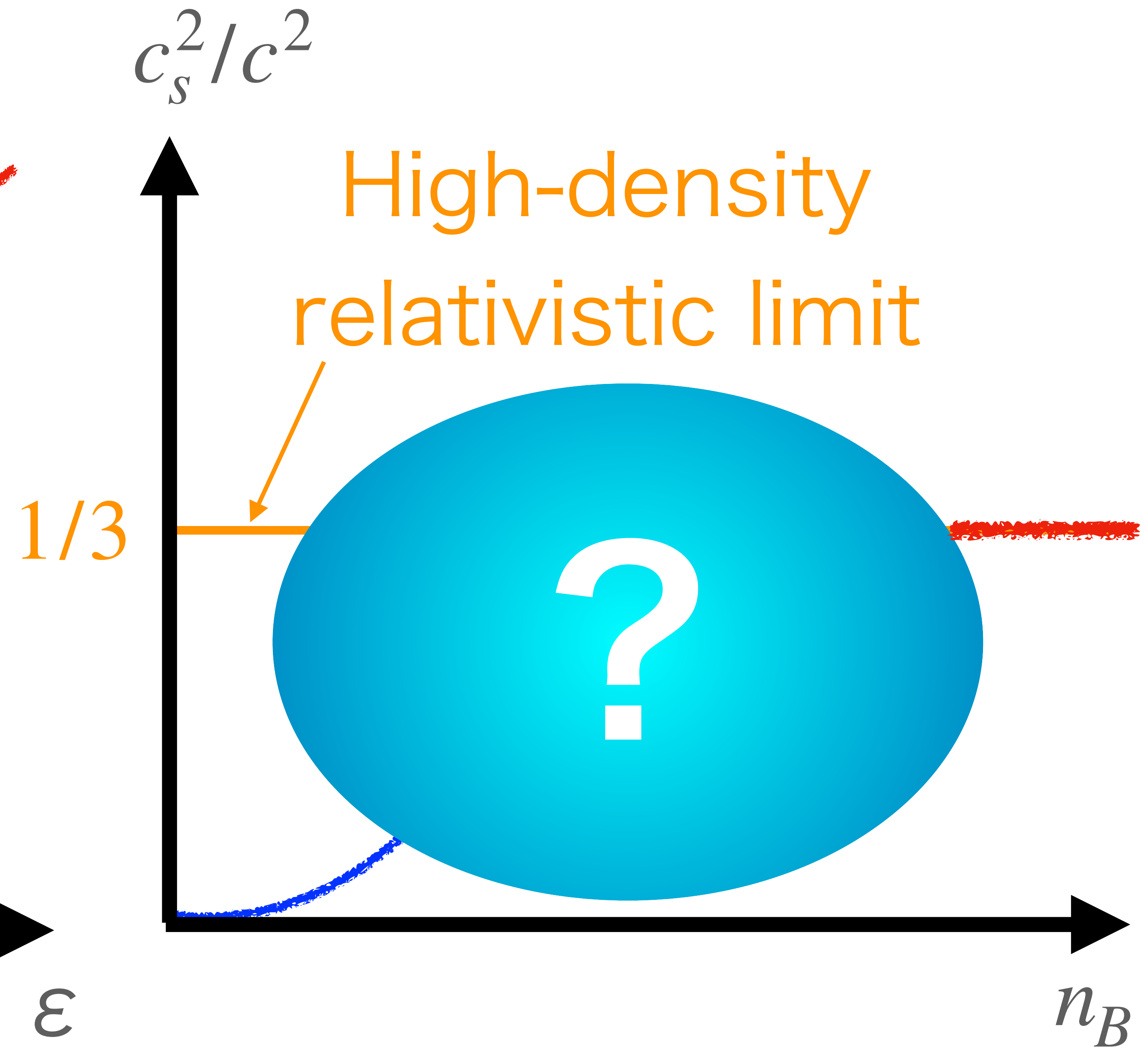
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



low -μ ($n_B \lesssim 2n_0$): Hadronic matter

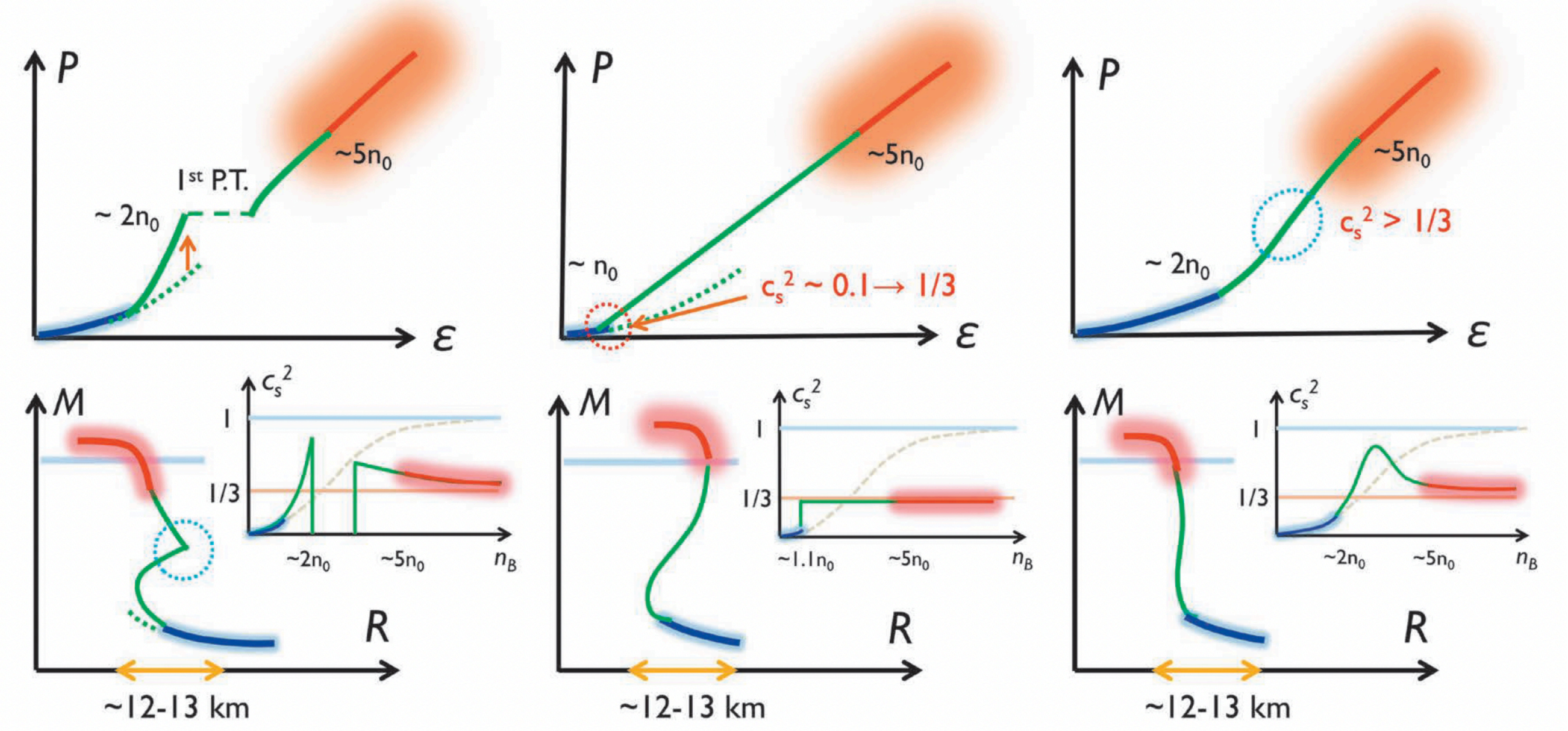
high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

EoS, c_s and neutron star

Mass and radius of neutron star

T. Kojo, arXiv:2011.10940



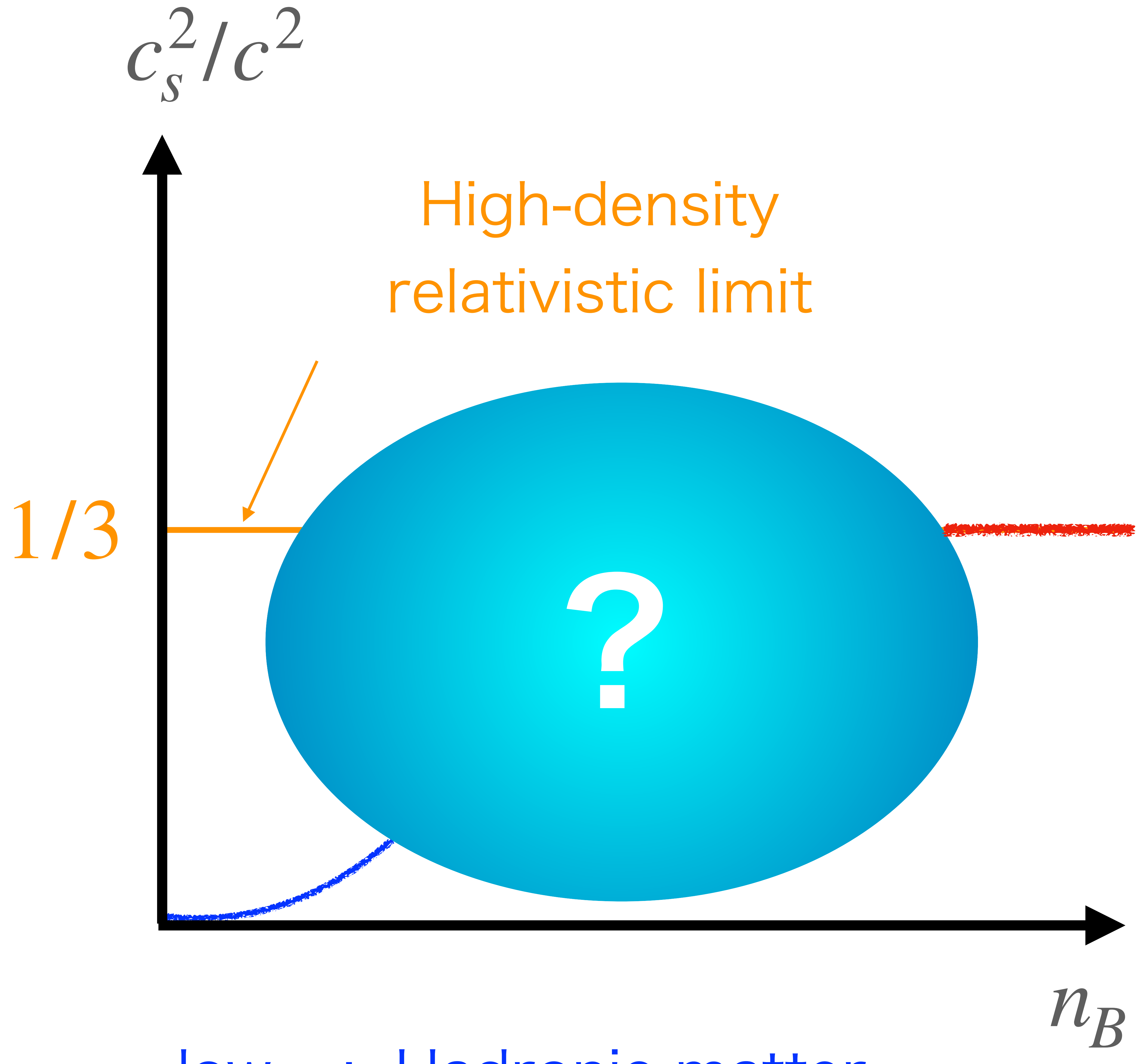
Sound velocity

$c_s^2 = \partial p / \partial \epsilon$

Mass-Radius of neutron star <=> EoS in dense QCD

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (color independent) Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD

2color QCD ≈ 3color QCD

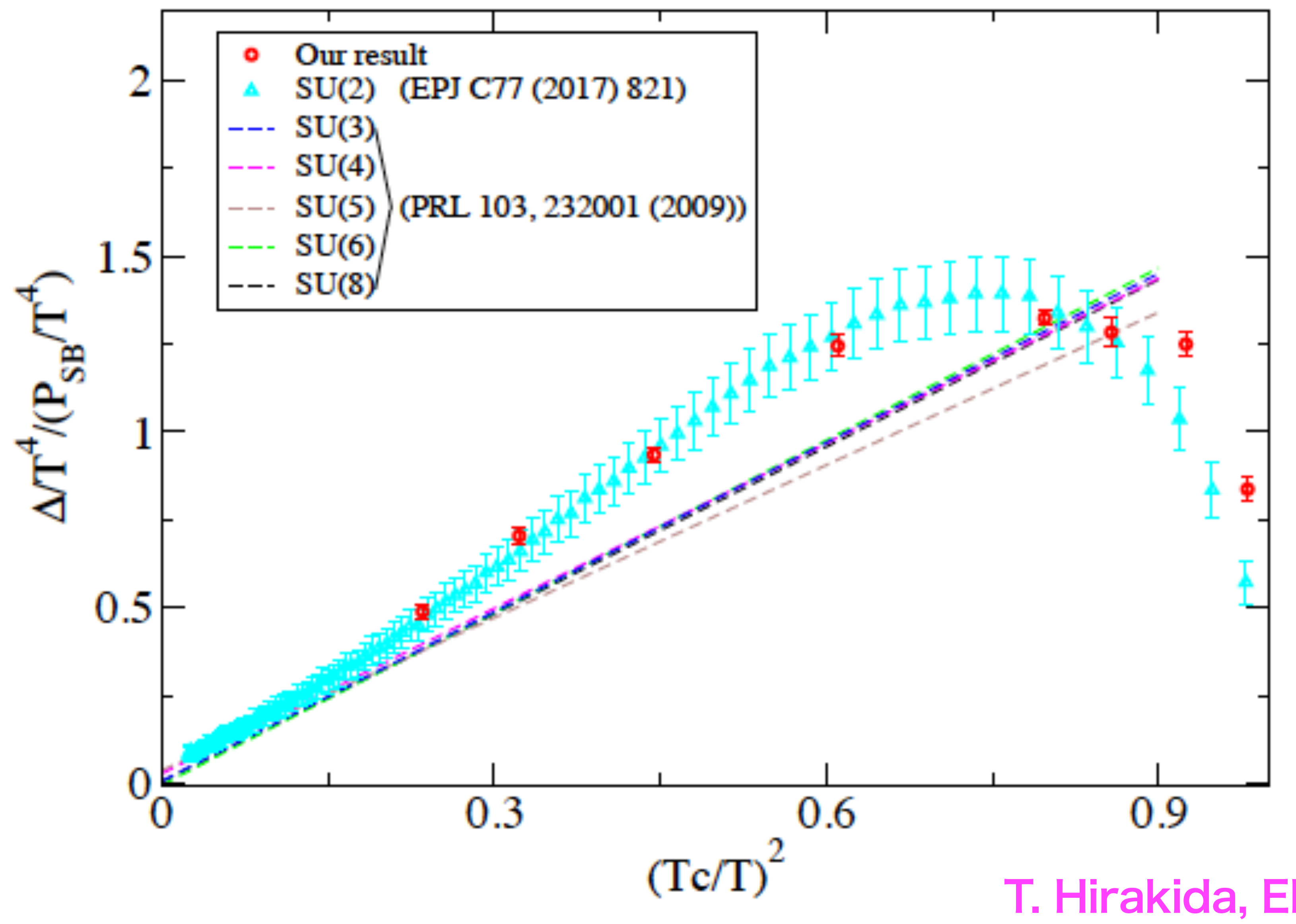
- 2color QCD
  reduced model with color d.o.f. in real QCD

- Properties of 3color QCD at $\mu = 0$
  - asymptotic freedom
  - finite T transition (chiral/confinement)
  - pseudo-scalar meson is lightest(pion)
    cf.) QCD inequality
  - EoS(energy, pressure)

- Qualitatively, 2color QCD has the same ones

- Quantitatively, EoS shows very similar
  at least quenched QCD case

Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc) gauge theories with several Nc



T. Hirakida, EI, H. Kouno
PTEP 2019 (2019) 033B01

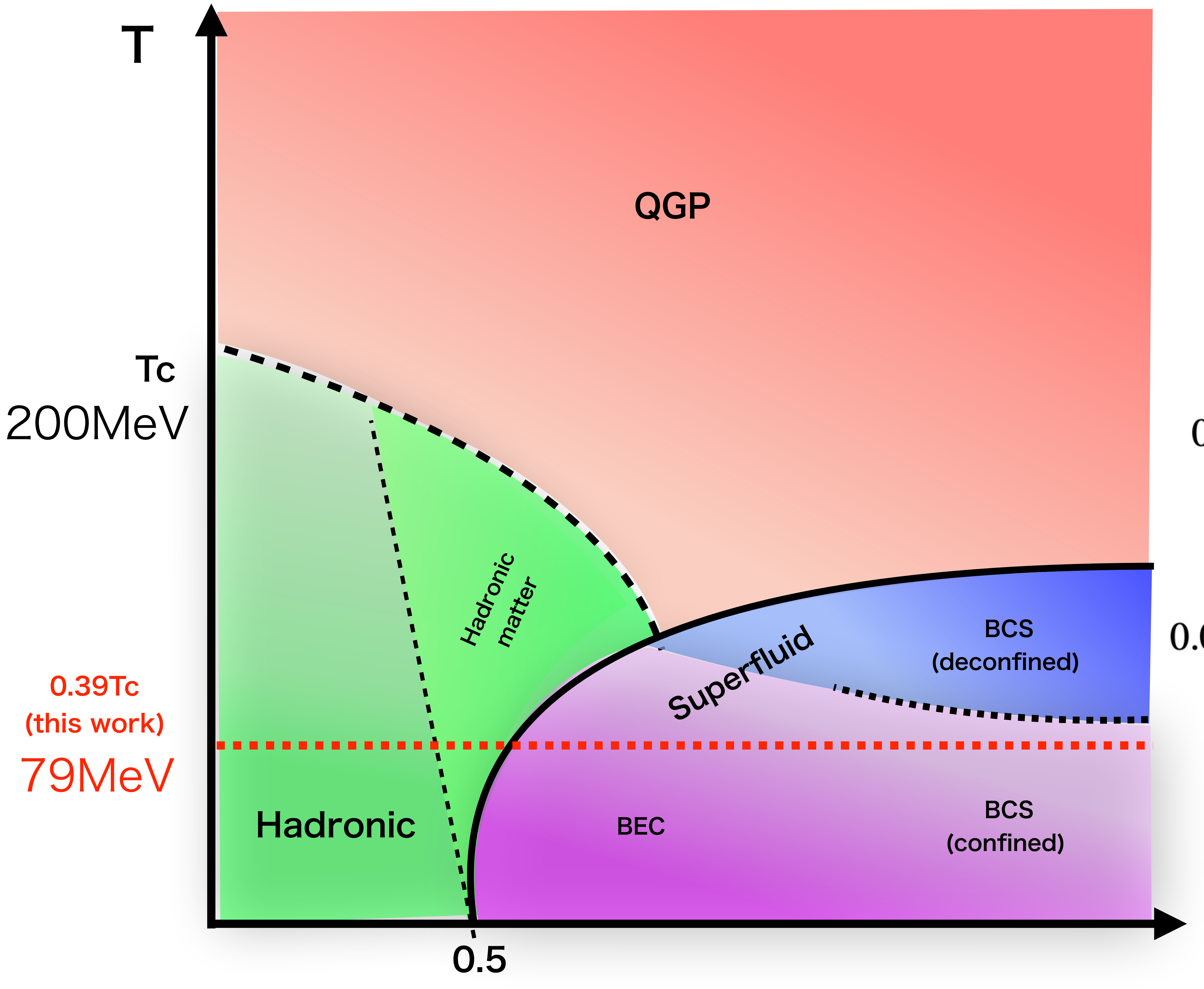
In 2color QCD at $\mu \neq 0$,
the sign problem is absent.
Find qualitative property of real dense 3color QCD

2color QCD phase diagram

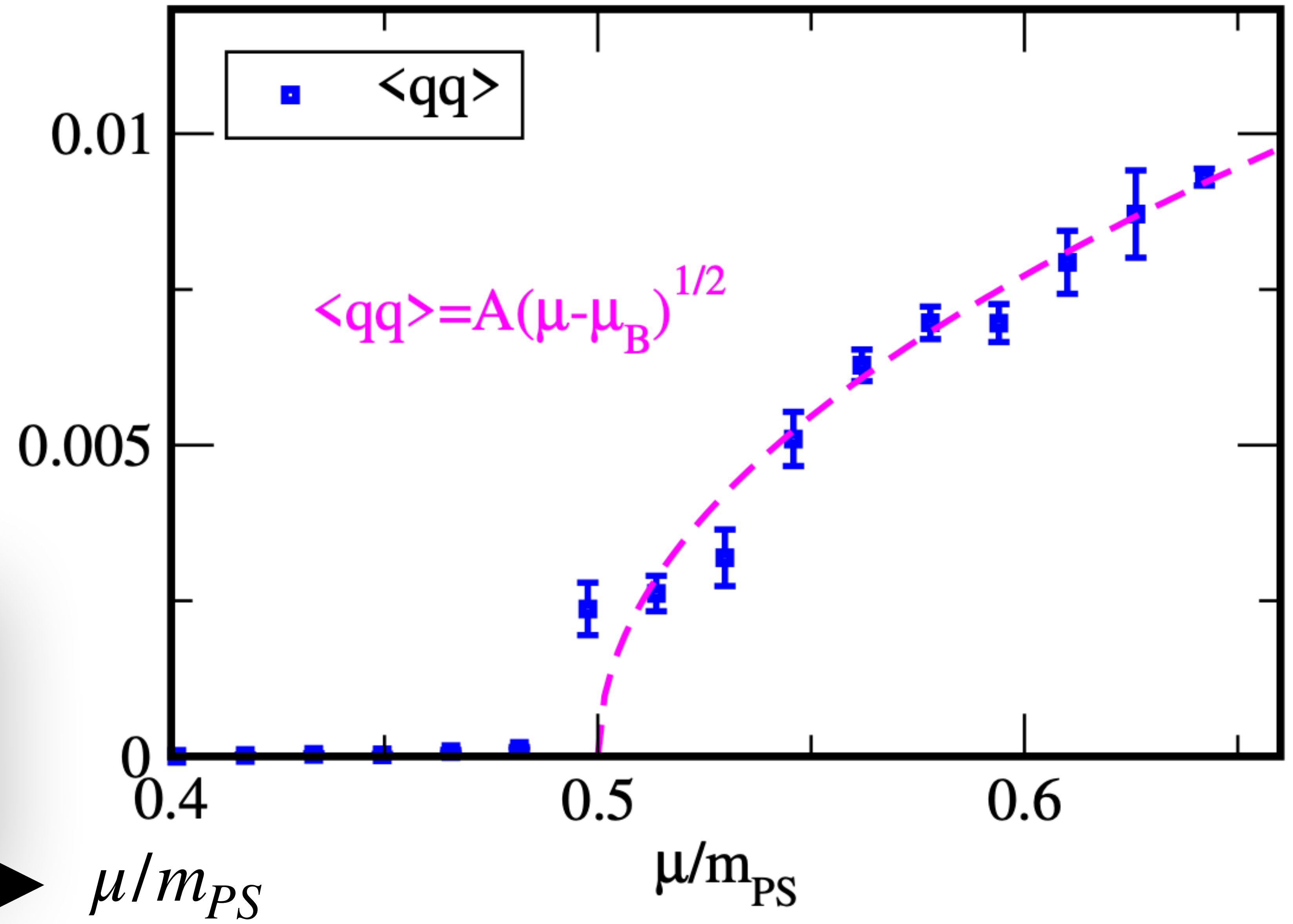
(1) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(2) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(3) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(4) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Phase diagram of 2color QCD

This work



	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\rm tree} \approx 1$

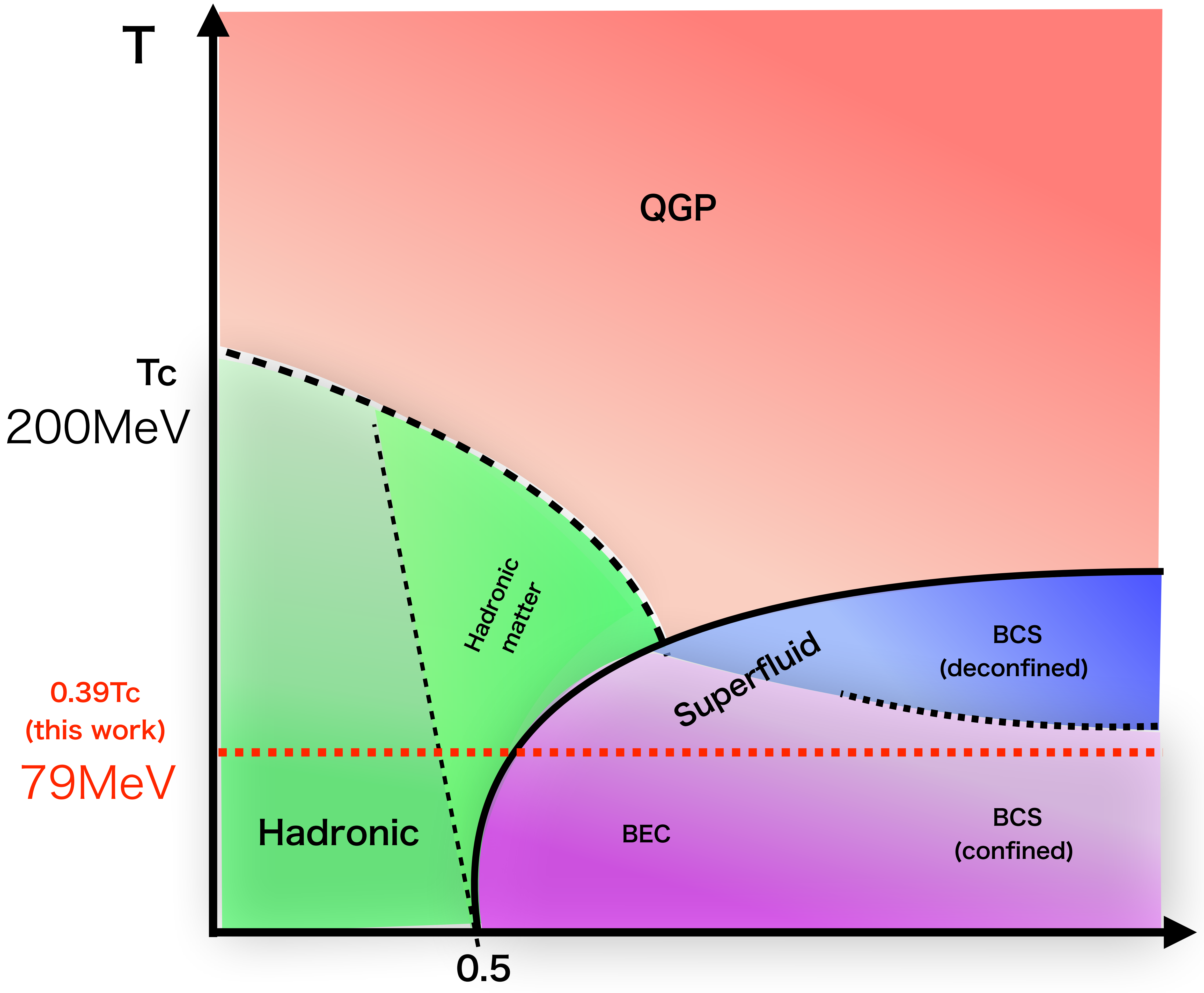


Scaling law of order param. is consistent with ChPT.

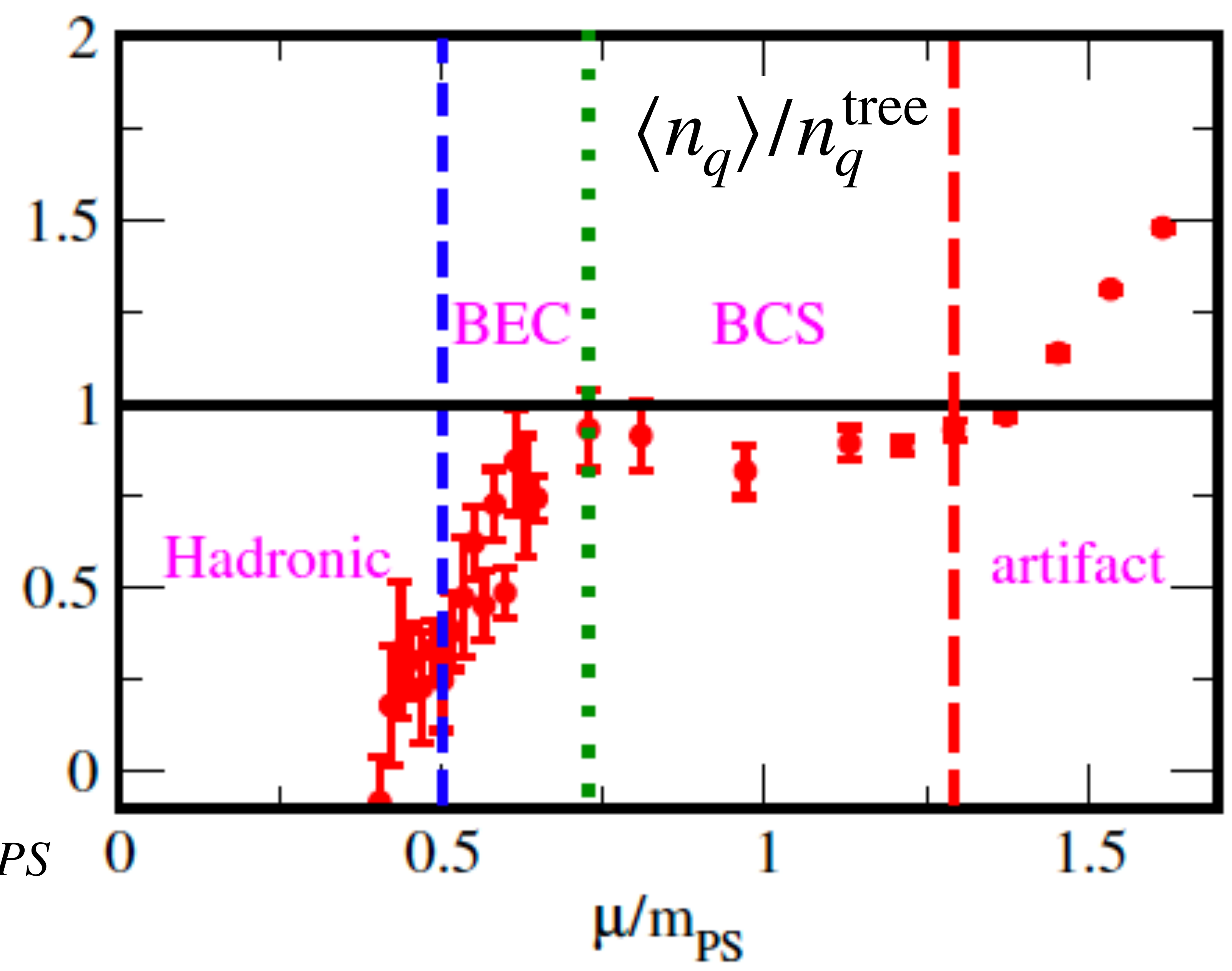
Kogut et al., NPB 582 (2000) 477

Phase diagram of 2color QCD

This work



	Hadronic	Hadronic-matter	QGP	Superfluid BEC	BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\mathrm{tree}} \approx 1$



In high-μ, $\langle n_q \rangle \approx n_q^{\mathrm{tree}}$

number density of free particle

BEC-BCS crossover

Equation of state

K.Iida and EI, arXiv: 2207.01253

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge), 16^4lattice

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\underline{\langle\frac{\partial S}{\partial \beta}\rangle_{sub.}} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + \underline{\cancel{a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle}} \right)$

  No renormalization for μ

  $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge), 16^4lattice

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + \cancel{a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle}\right)$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Technical steps

(1) Measure the gauge action and chiral cond.

(2) Calculate the beta fn. at $\mu = 0$

(3) Numerical integration of n_q

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge), 16^4lattice

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + \cancel{a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle} \right)$

  Zero at $j \to 0$

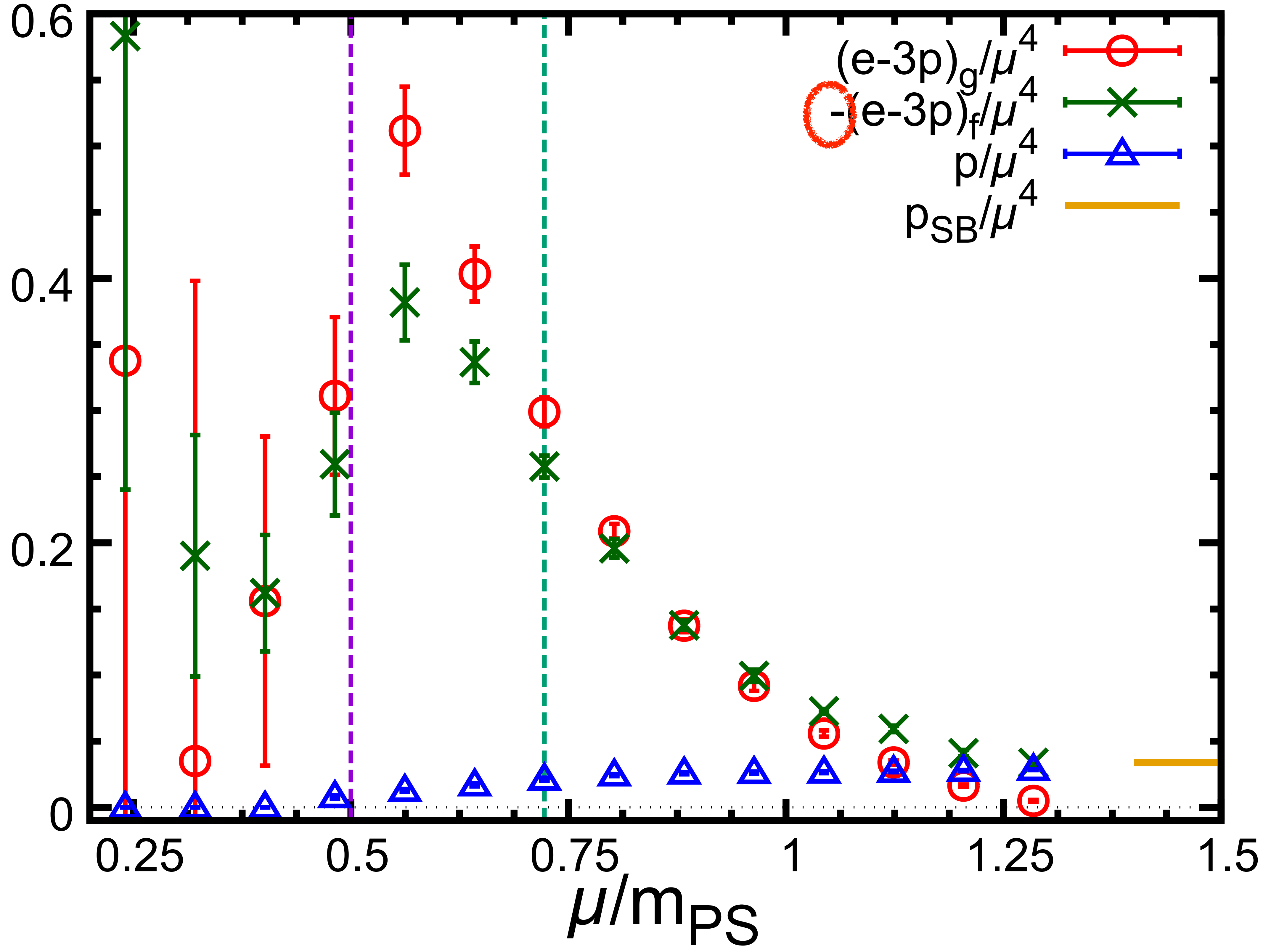
- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Nonperturbative beta-fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Trace anomaly and pressure



- Sum of trace anomaly, $(e-3p)_g + (e-3p)_f$

  zero in Hadronic phase

  positive in BEC phase

  positive -> negative in BCS phase

  Finally, fermions give the larger contribution

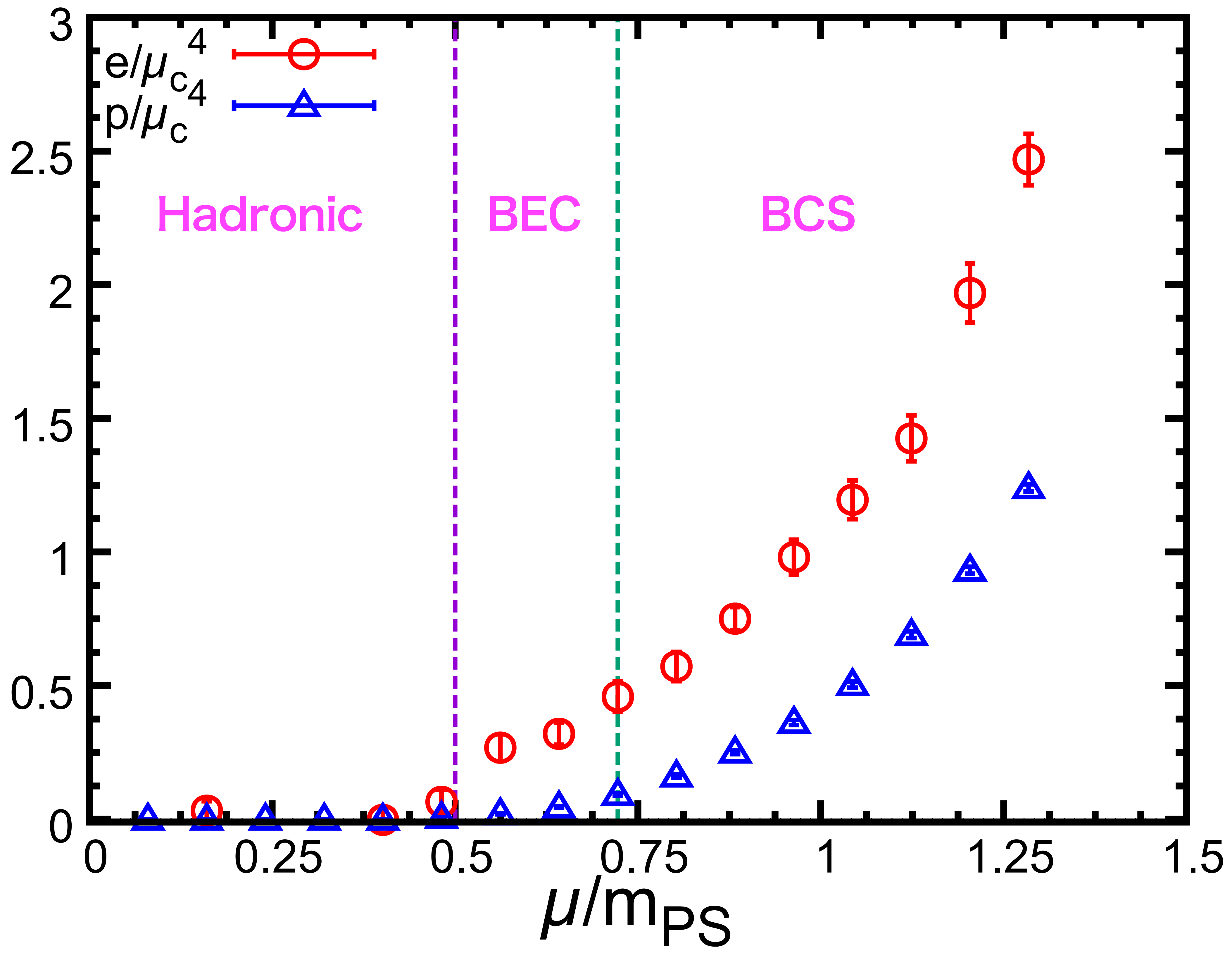
- Pressure increase monotonically

  In high density, it approaches

  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$

P and e as a function of μ

(Normalized by $1/\mu_c^4$ to be dim-less)

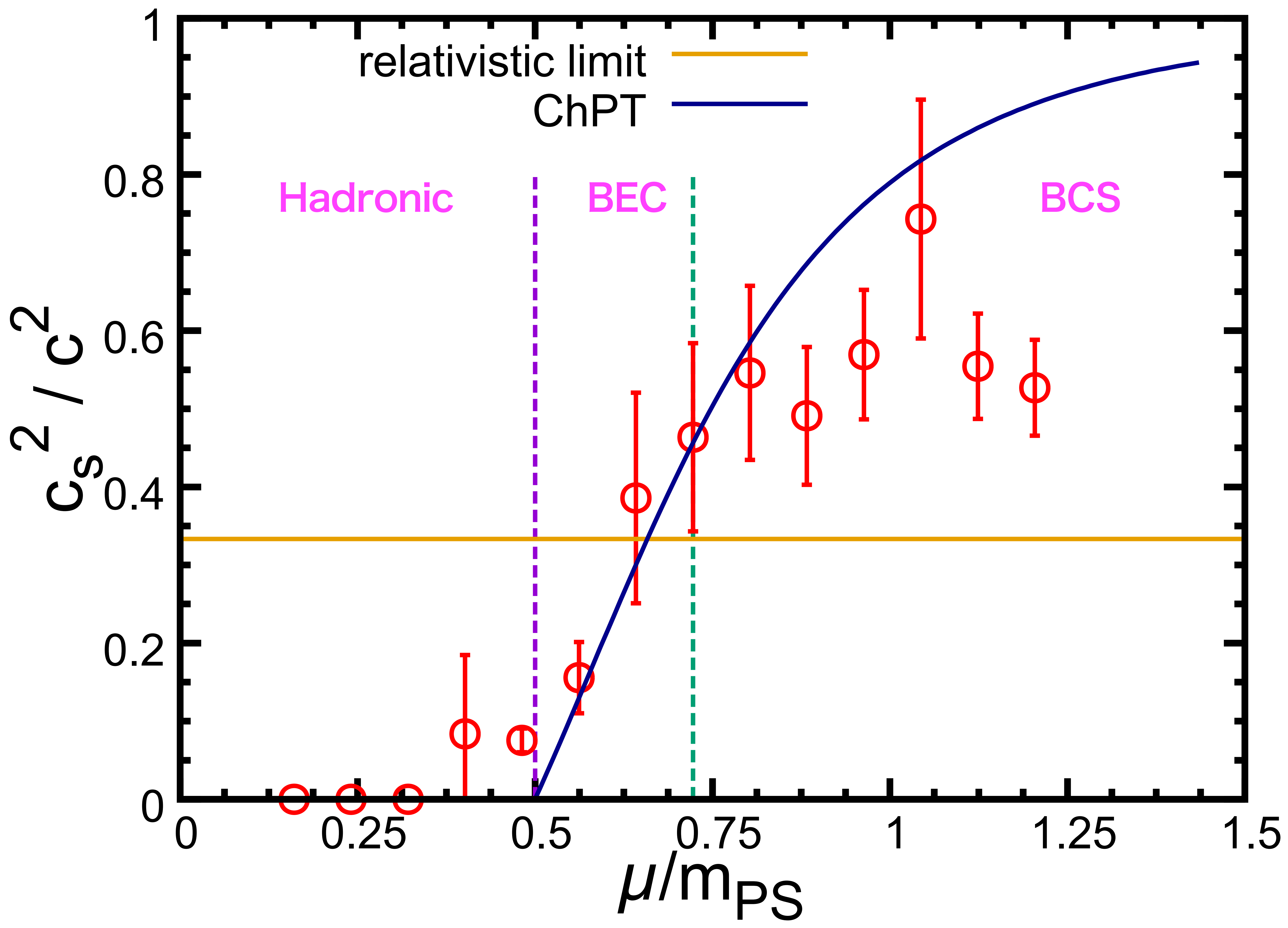


- P is zero in Hadronic phase since $n_q = 0$
- e is also zero in Hadronic phase by the cancelation between $(e-3p)_g$ and $(e-3p)_f$

From these data, the sound velocity is obtained

$$c_s^2/c^2 = \frac{\Delta p}{\Delta e} = \frac{p(\mu+\Delta\mu) - p(\mu-\Delta\mu)}{e(\mu+\Delta\mu) - e(\mu-\Delta\mu)}$$

Sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



Chiral Perturbation Theory (ChPT)

$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!!

Son and Stephanov (2001) : 3color QCD with isospin μ

Hands, Kim, Slullerud (2006) : 2color QCD with real μ

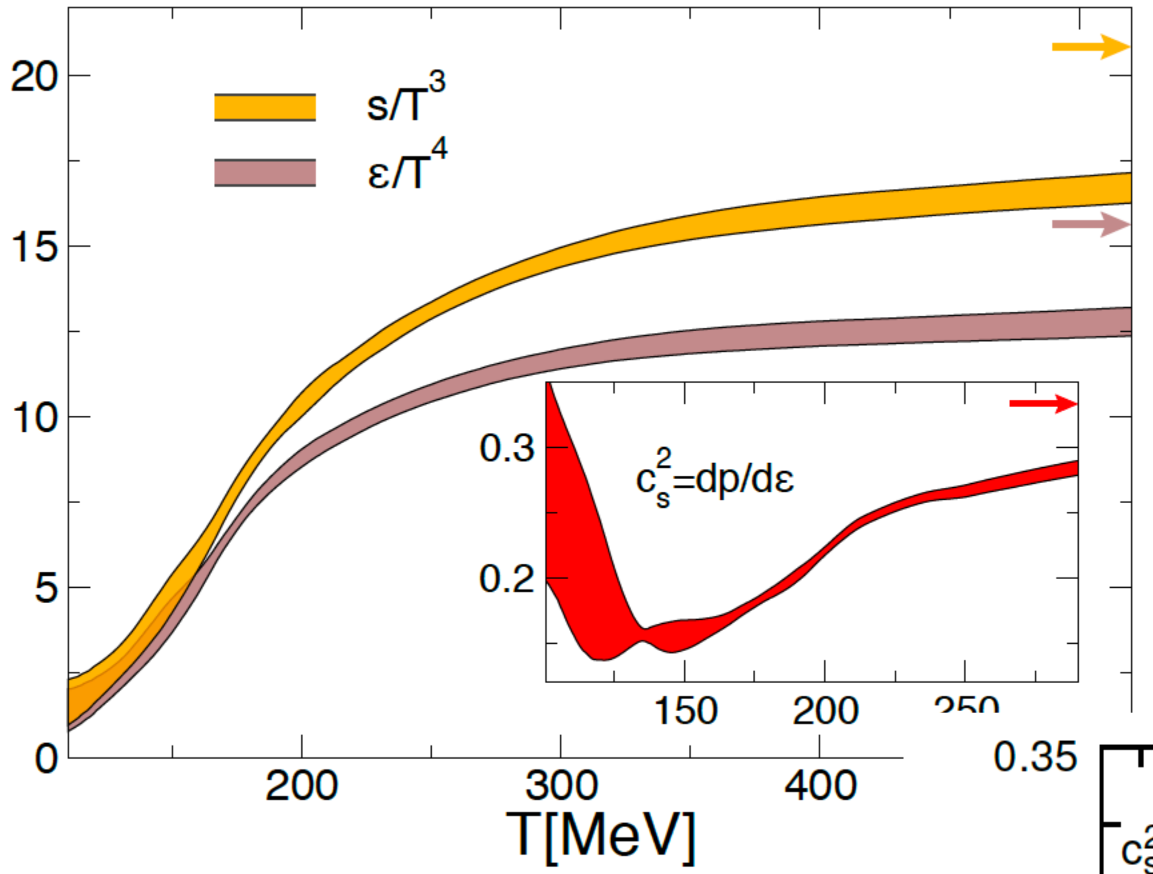
- In BEC phase, our result is consistent with ChPT.
- c_s^2/c^2 exceeds the relativistic limit
- In high-density, it peaks around $\mu \approx m_{PS}$.

"Stiffen" and then "soften" picture as density increases

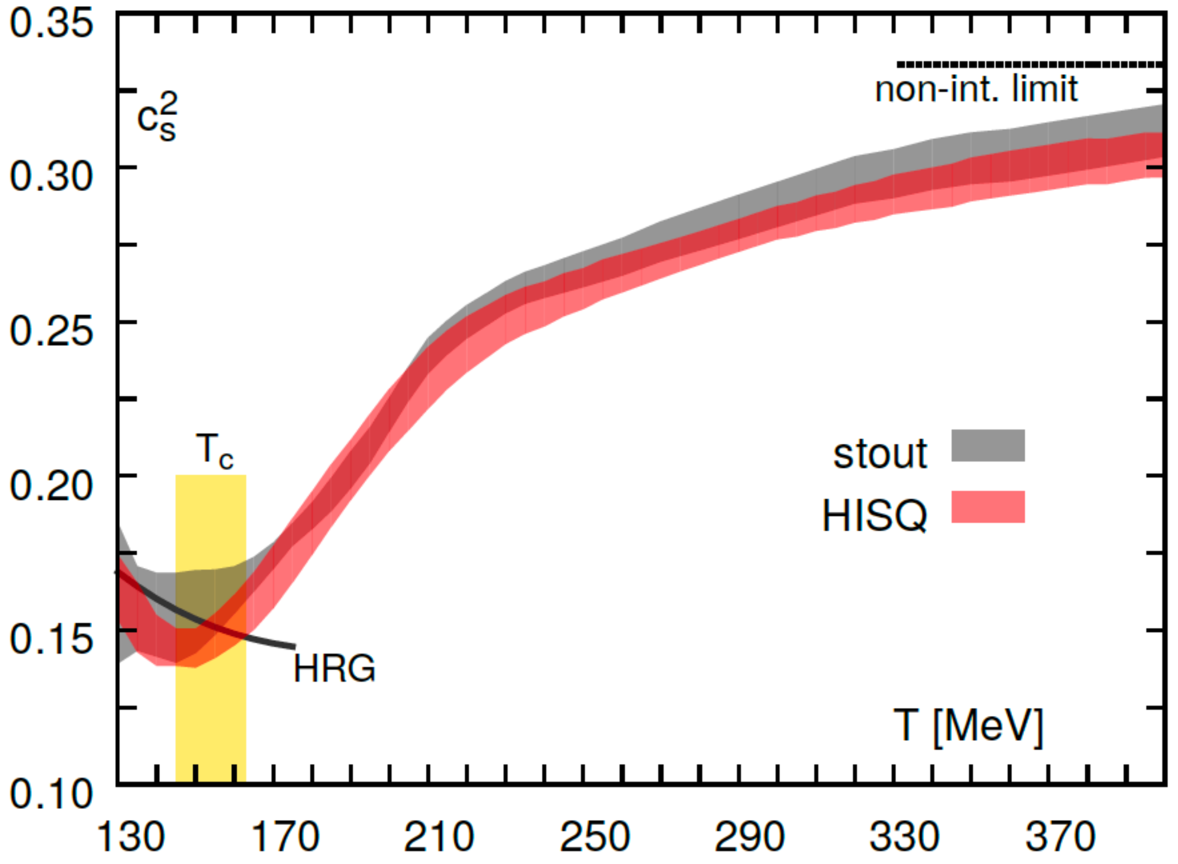
Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)

Borsanyi et al. (2013)



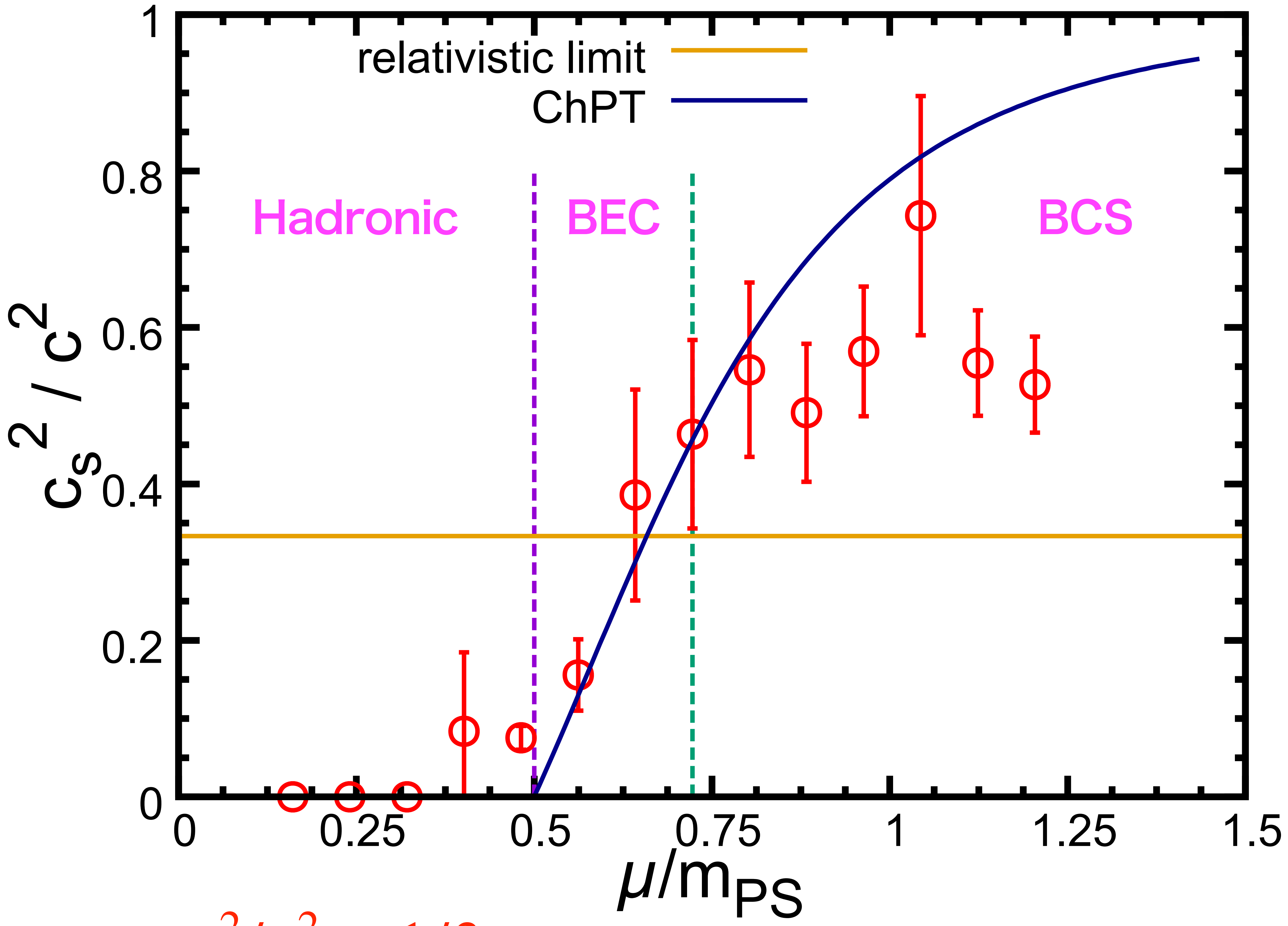
HotQCD (2014)



- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

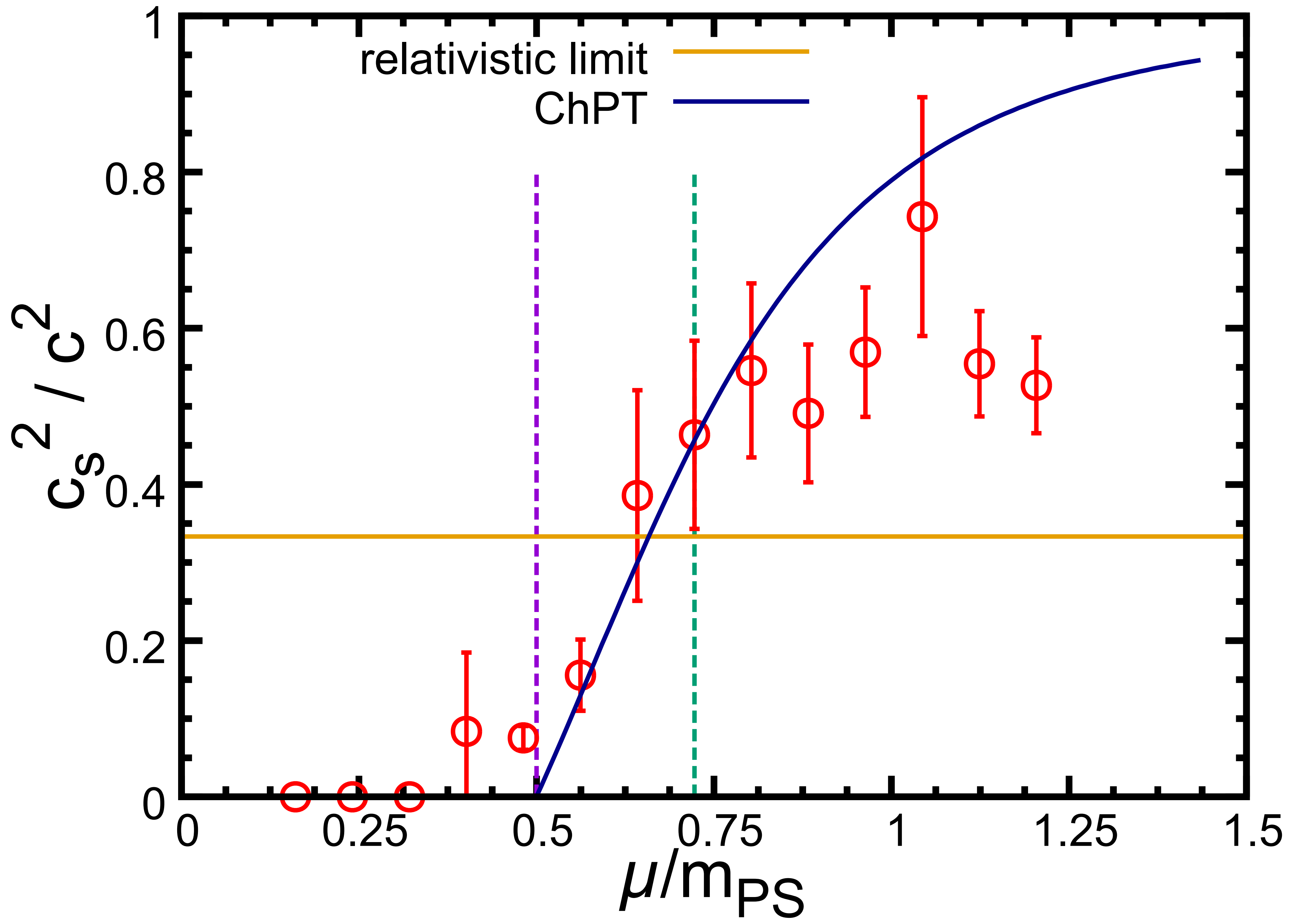
Iida and El arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

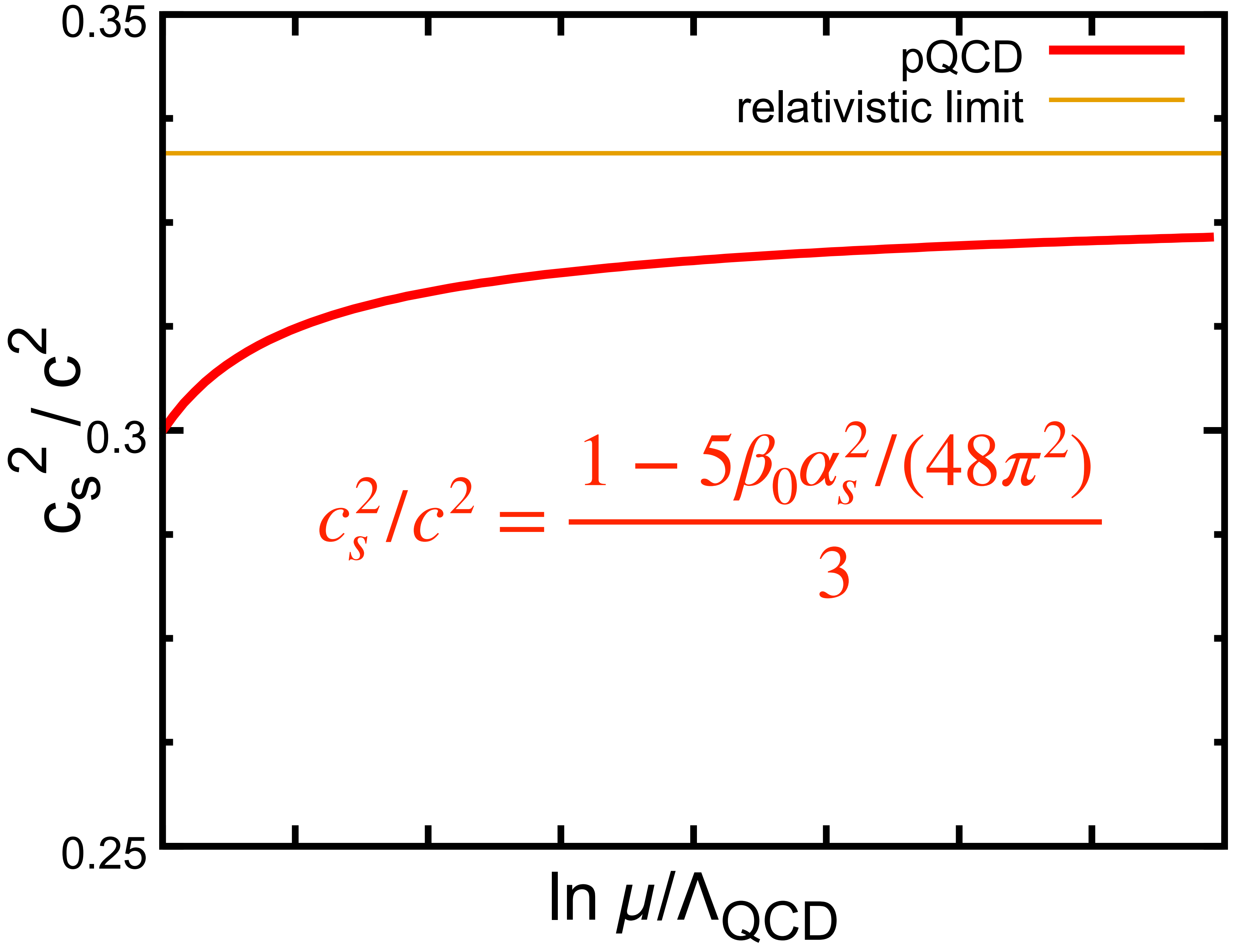
Further high density?

Kojo, Baym, Hatsuda (2021)



pQCD prediction

(Ultra high-density regime)



- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$ (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

Summary and future work

- In BEC phase, our result is consistent with ChPT.
  Sound velocity exceeds the relativistic limit and has a peak after BEC-BCS crossover
  cf.) cond-mat model study also find it Tajima and Liang (2022)

- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), strong coupling with trace anomaly?(McLerran,Fukushima et al.), others?
  - attractive or repulsive force between hadrons?
    => extended HAL QCD method in finite density
  - independent of the color dof?

- This finding might have a possible relevance to the EoS of neutron star matter revealed by recent measurements of neutron star masses and radii.

Backup

Two problems at low-T high-μ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)

➡ Reduce the color dof, 2color QCD
quarks becomes pseudo-real reps.
The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-μ (e.g. $\mu_q > m_\pi/2, \ \ m_N/3$),

It comes from the phase transition to superfluid phase(SSB of baryon sym.)

Kogut et al. NPB642 (2002)18

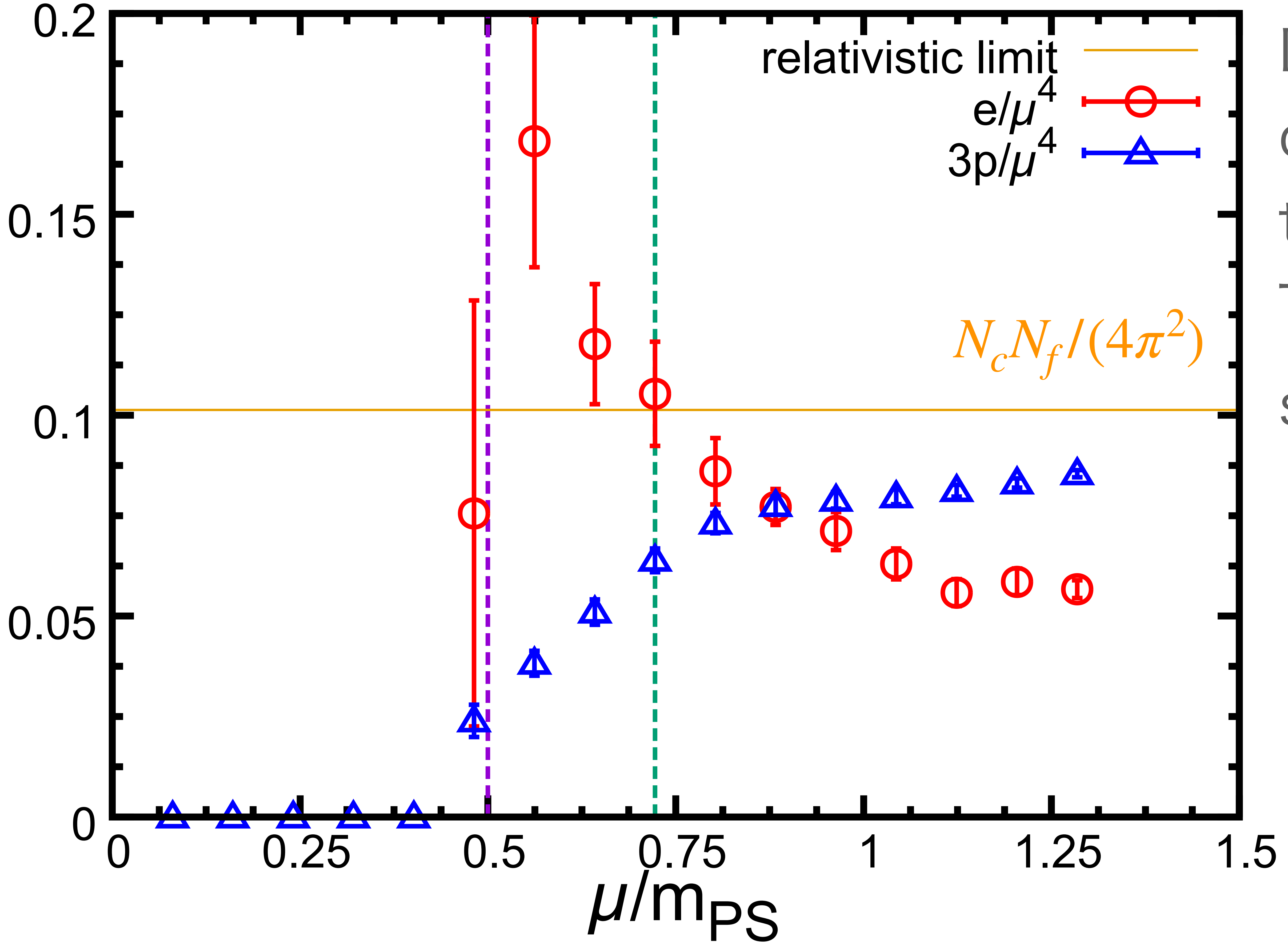
➡ Add an explicit breaking term of the sym., then take $j \to 0$ limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

HMC simulations for whole T-μ regime are doable!

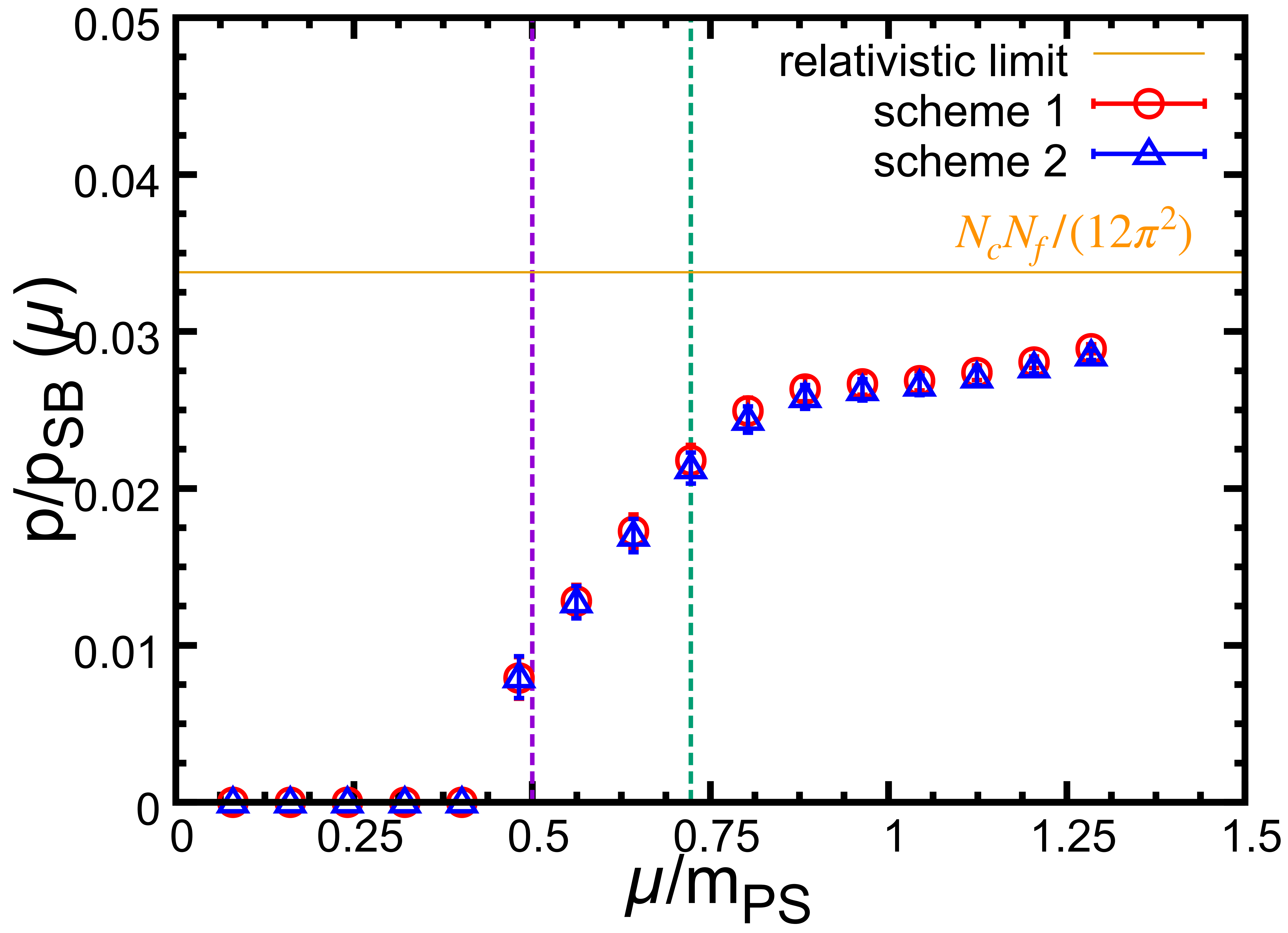
(j->0 extrapolation is taken in all plots today)

scaling of p and e in high density



In massive fermion theory, the trace anomaly does not vanish because the mass term breaks the scale invariance.
The mass term will give a negative contribution, so that we expect $e/\mu^4 < e_{SB}/\mu^4 = N_cN_f/(4\pi^2)$

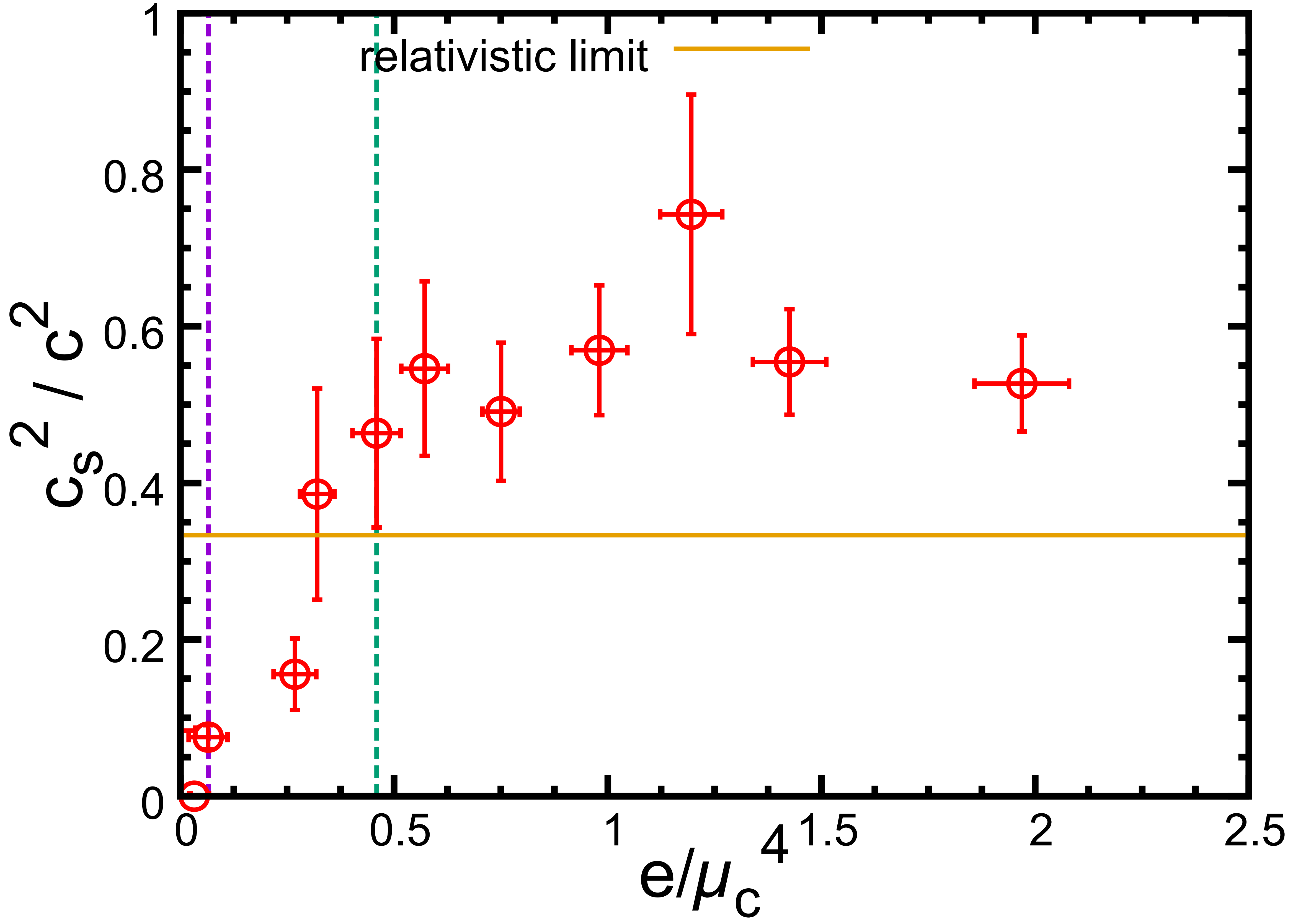
Scheme dependence of pressure

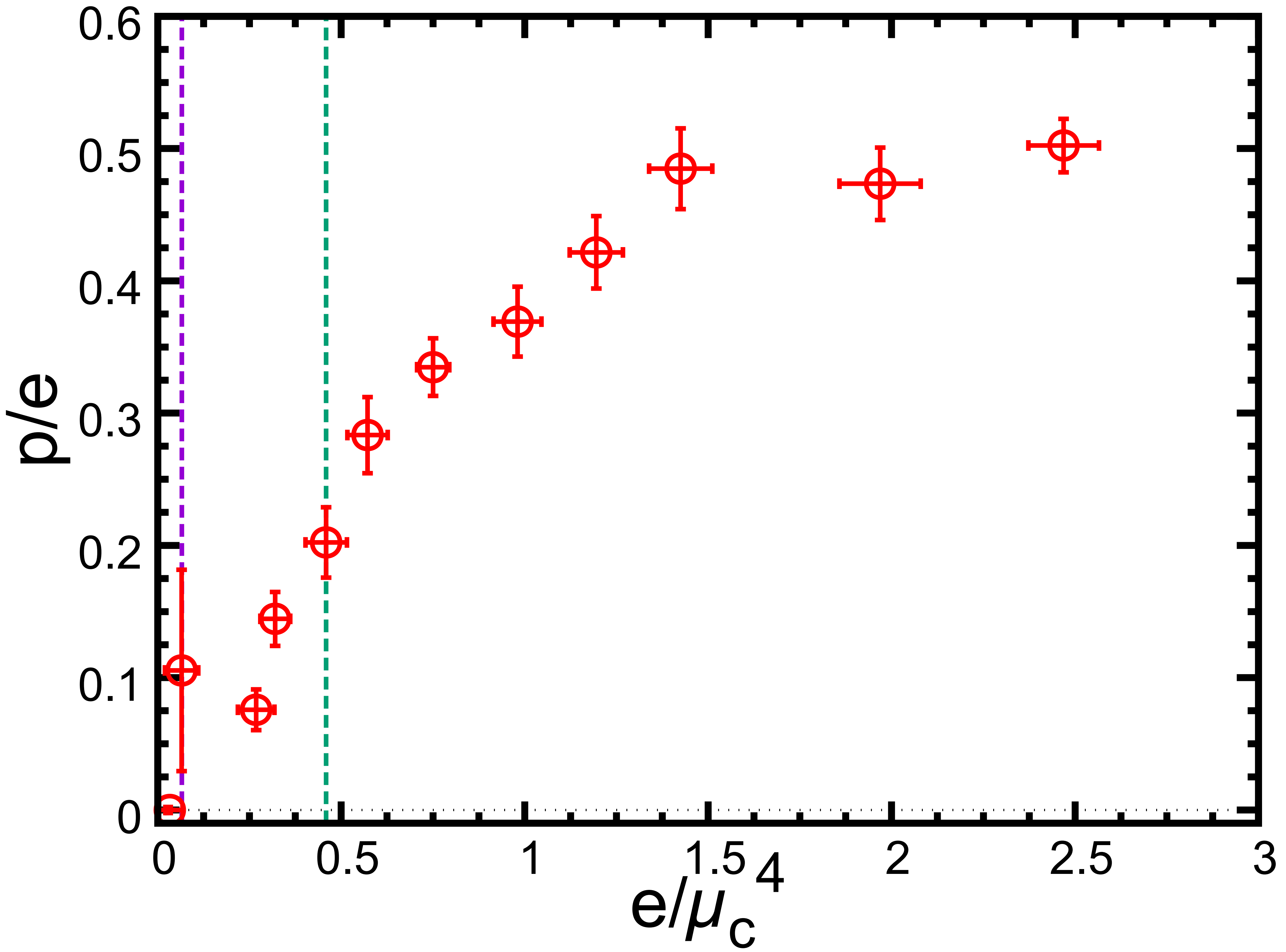


$$\mathrm{I}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{lat}}(\mu')d\mu'}\,; \tag{28}$$

$$\mathrm{II}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} \frac{n_{SB}^{\mathrm{cont}}}{n_{SB}^{\mathrm{lat}}}(\mu')n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{cont}}(\mu')d\mu'}\,, \tag{29}$$

Sound velocity (ratio $\Delta p/\Delta e$) vs energy





Holography bound?

A bound on the speed of sound from holography

Aleksey Cherman* and Thomas D. Cohen†
Center for Fundamental Physics, Department of Physics,
University of Maryland, College Park, MD 20742-4111

Abhinav Nellore‡
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1$
upper bound for a broad class of four-dimensional theories.

$c_s^2/c^2 \leq 1/3$ at high T

conjecture it is valid for a broad class of 4-dim. theories

PHYSICAL REVIEW D **94,** 106008 (2016)

Breaking the sound barrier in holography

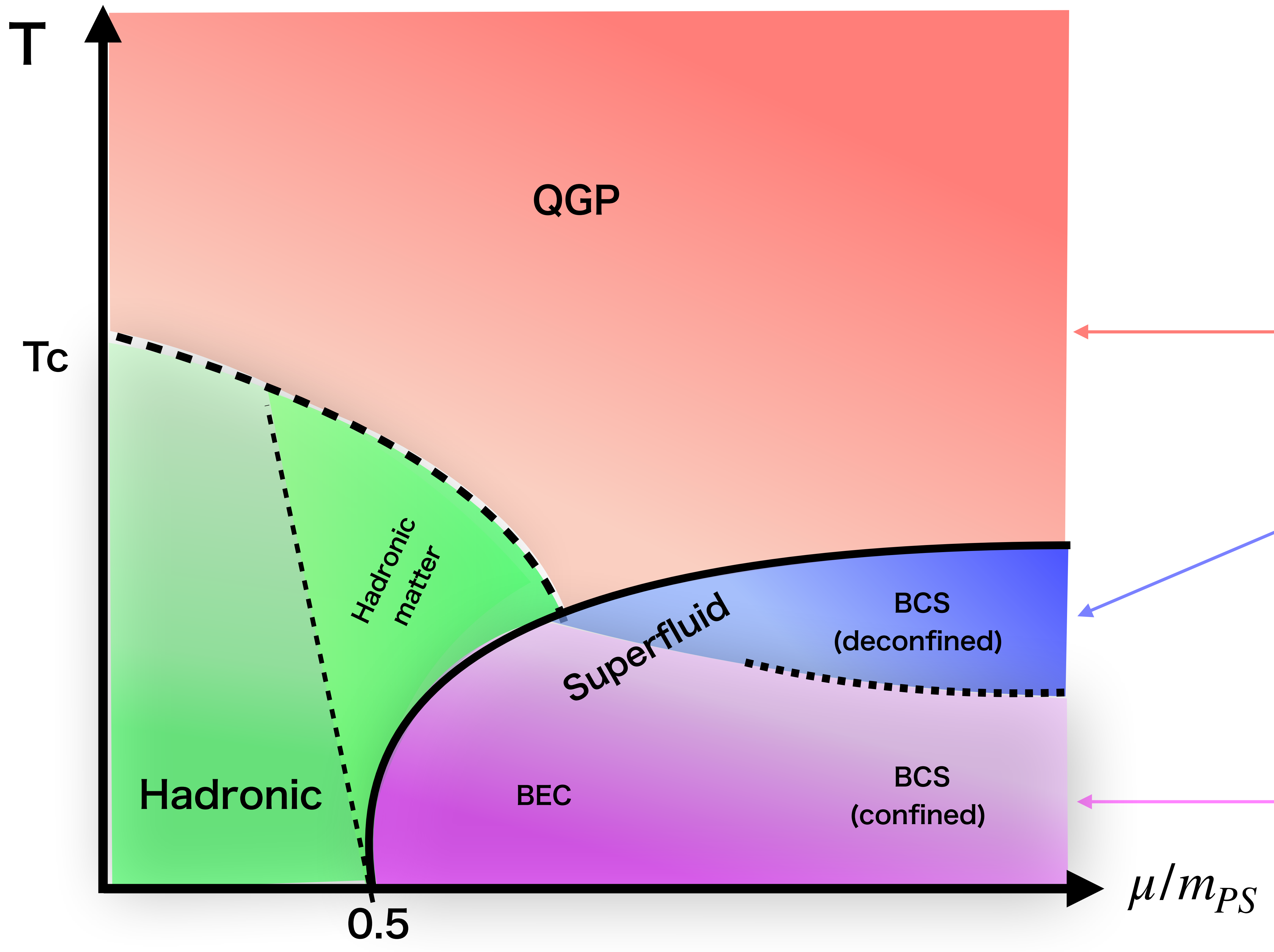
Carlos Hoyos,[1,*] Niko Jokela,[2,†] David Rodríguez Fernández,[1,‡] and Aleksi Vuorinen[2,§]
[1]*Department of Physics, Universidad de Oviedo, Avda. Calvo Sotelo 18, ES-33007 Oviedo, Spain*
[2]*Department of Physics and Helsinki Institute of Physics, P.O. Box 64,*
FI-00014 University of Helsinki, Finland
(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

Counterexample
for N=4 SYM at finite density

Phase diagram

Current status on 2color QCD phase diagram



At least three independent group studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

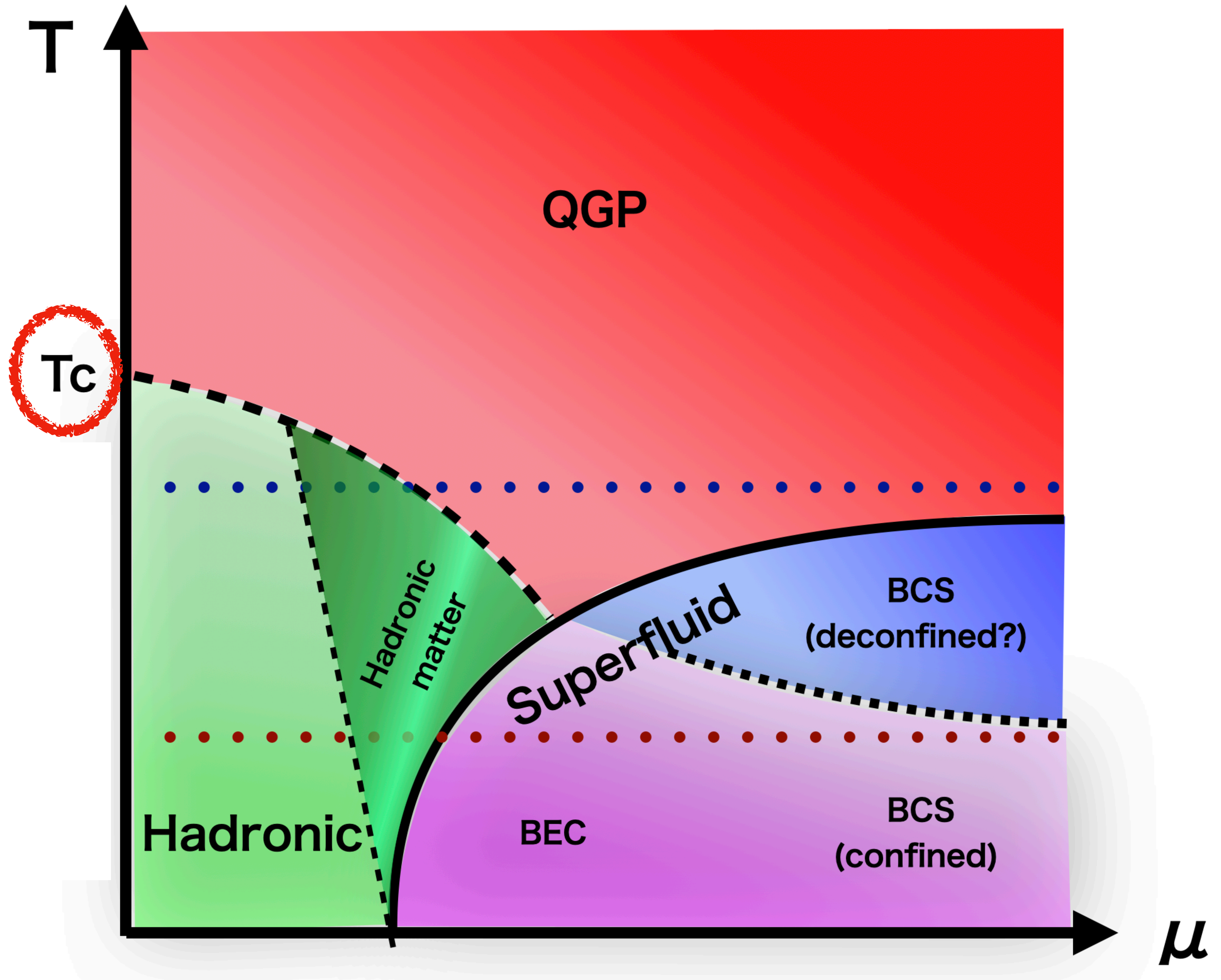
T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

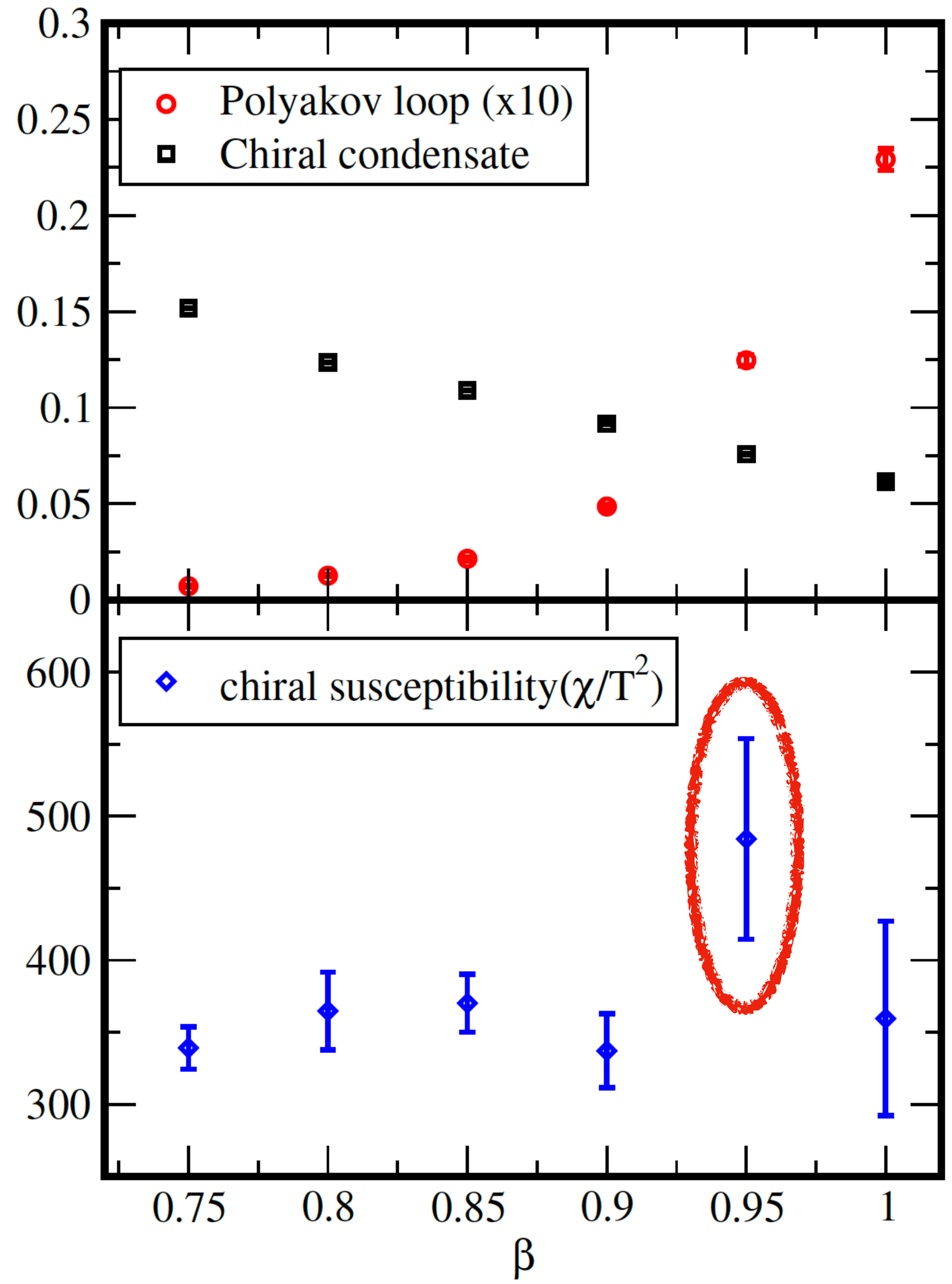
- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- 2color QCD phase diagram has been determined by independent works!

Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

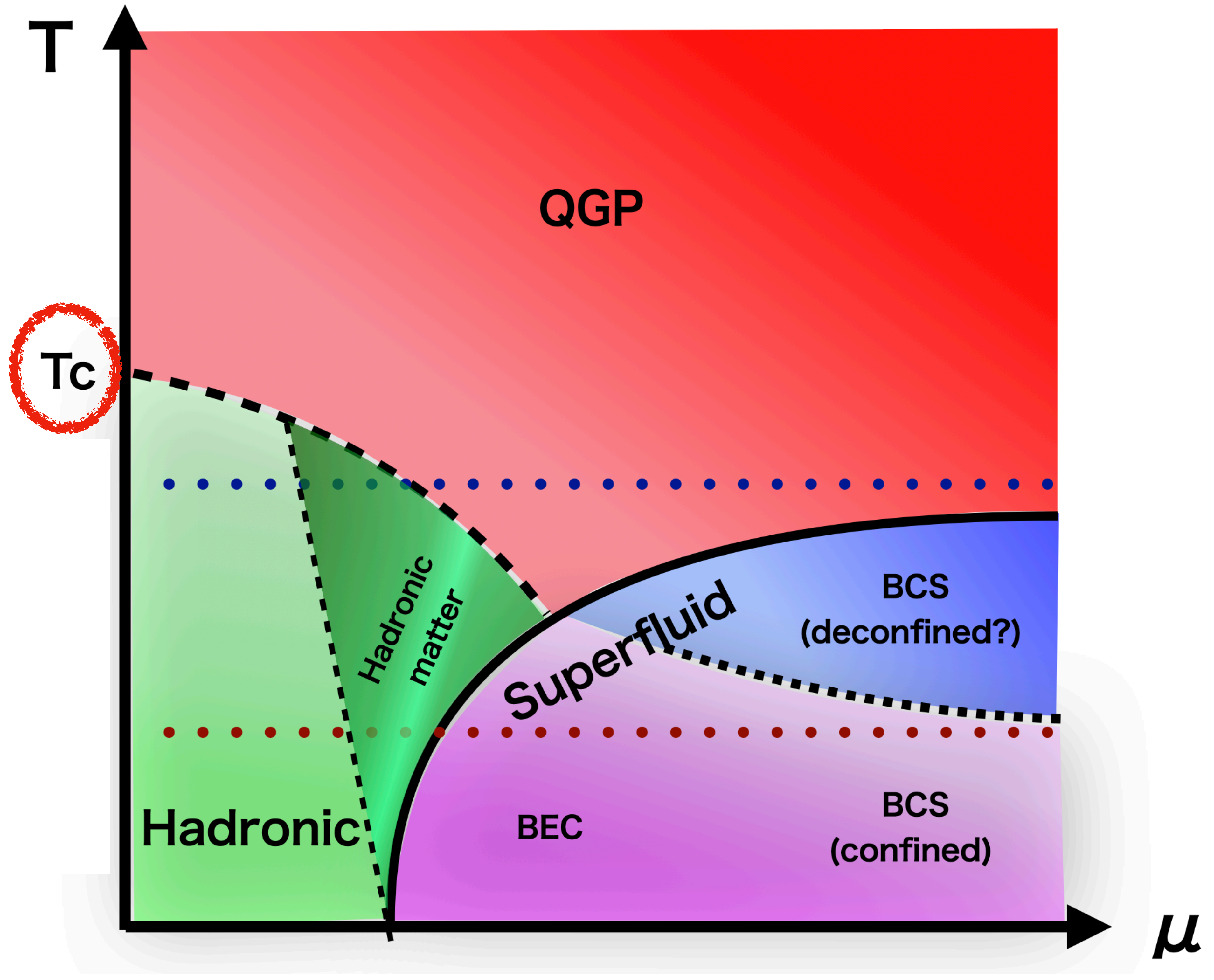
- Tc at $\mu = 0$ from chiral susceptibility





Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

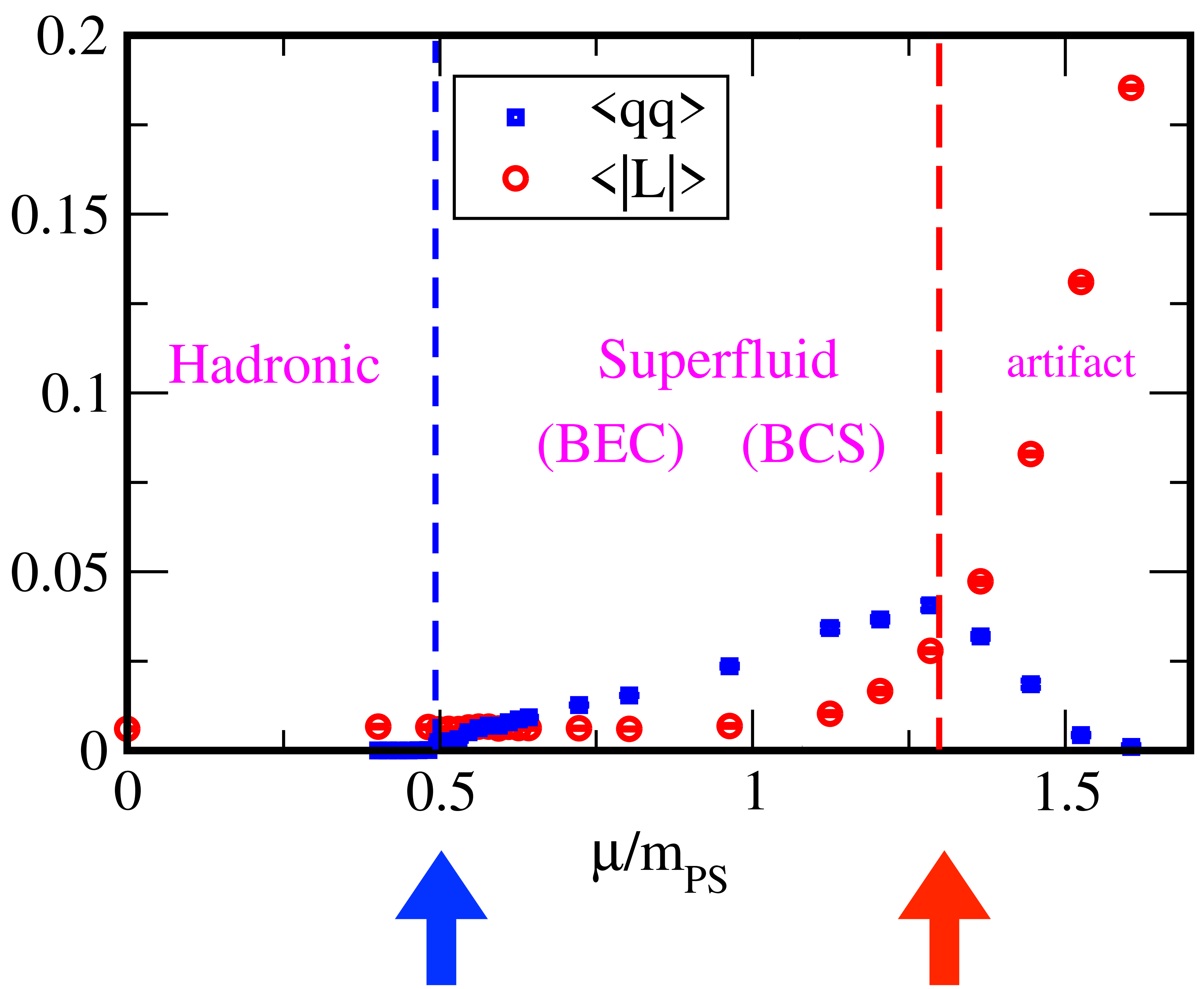


- Tc at $\mu = 0$ from chiral susceptibility
- Assume Tc=200MeV

  Tc is realize Nt=10, $\beta = 0.95$ (a=0.1[fm])
- Find relationship between β (lattice bare coupling) and a (lattice spacing)

  In finite density simulation, a=0.1658[fm]

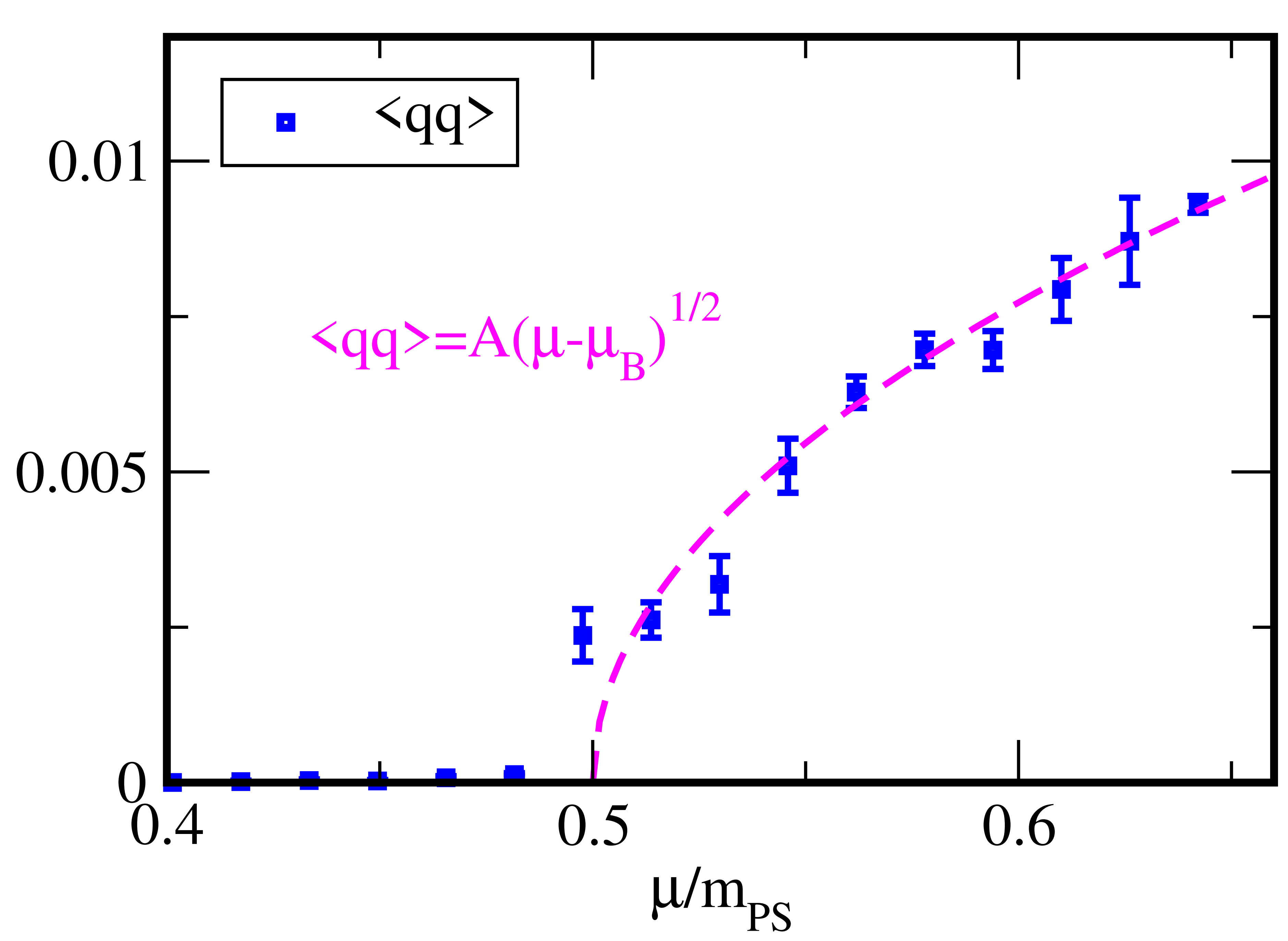
Order parameters in j=0 limit



$\mu_B/m_{\rm PS} \simeq 0.50$

$\mu/m_{\rm PS} \simeq 1.28$

$(\mu_D/m_{\rm PS} \simeq 1.44)$



Scaling law of order param.

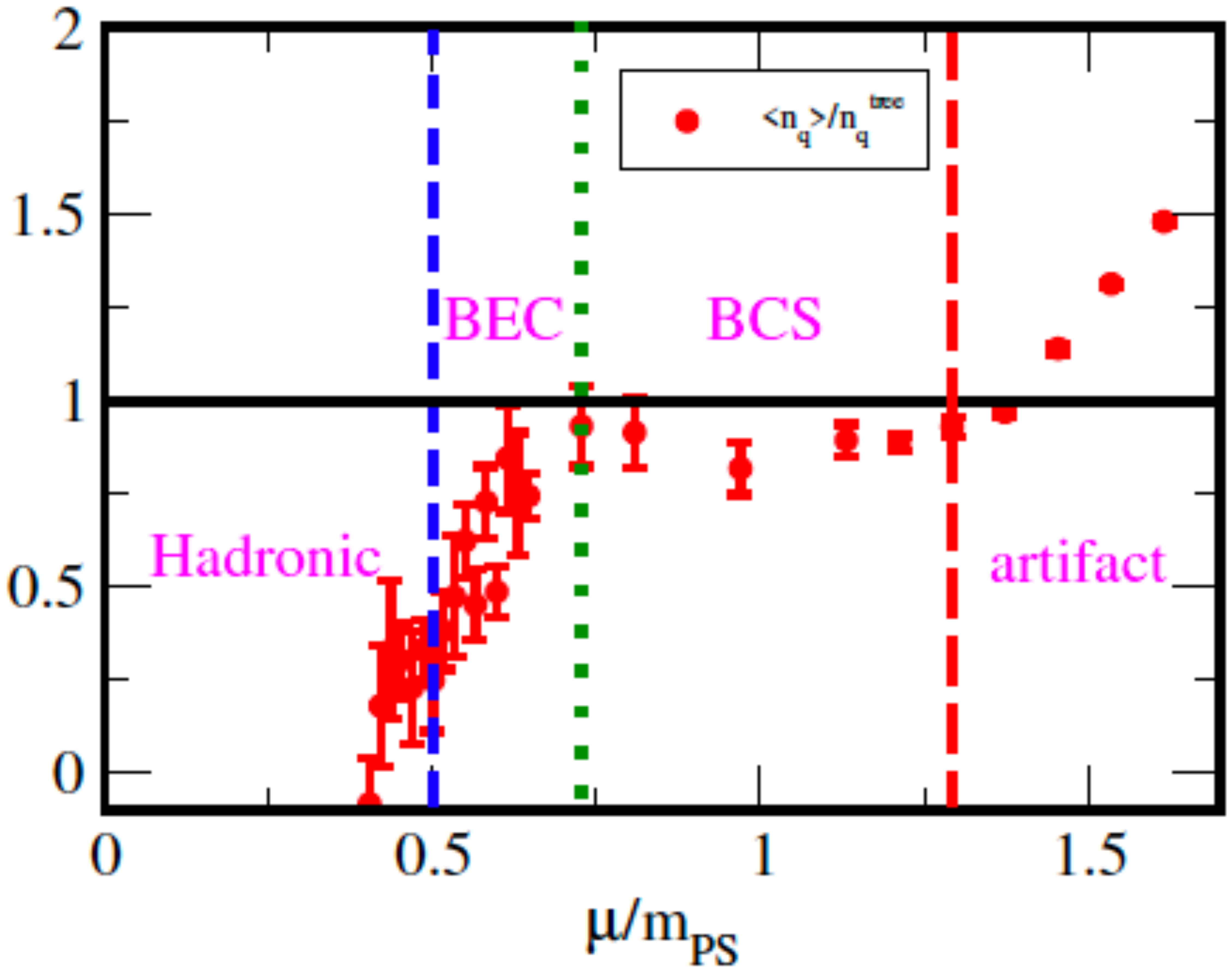
is consistent with ChPT.

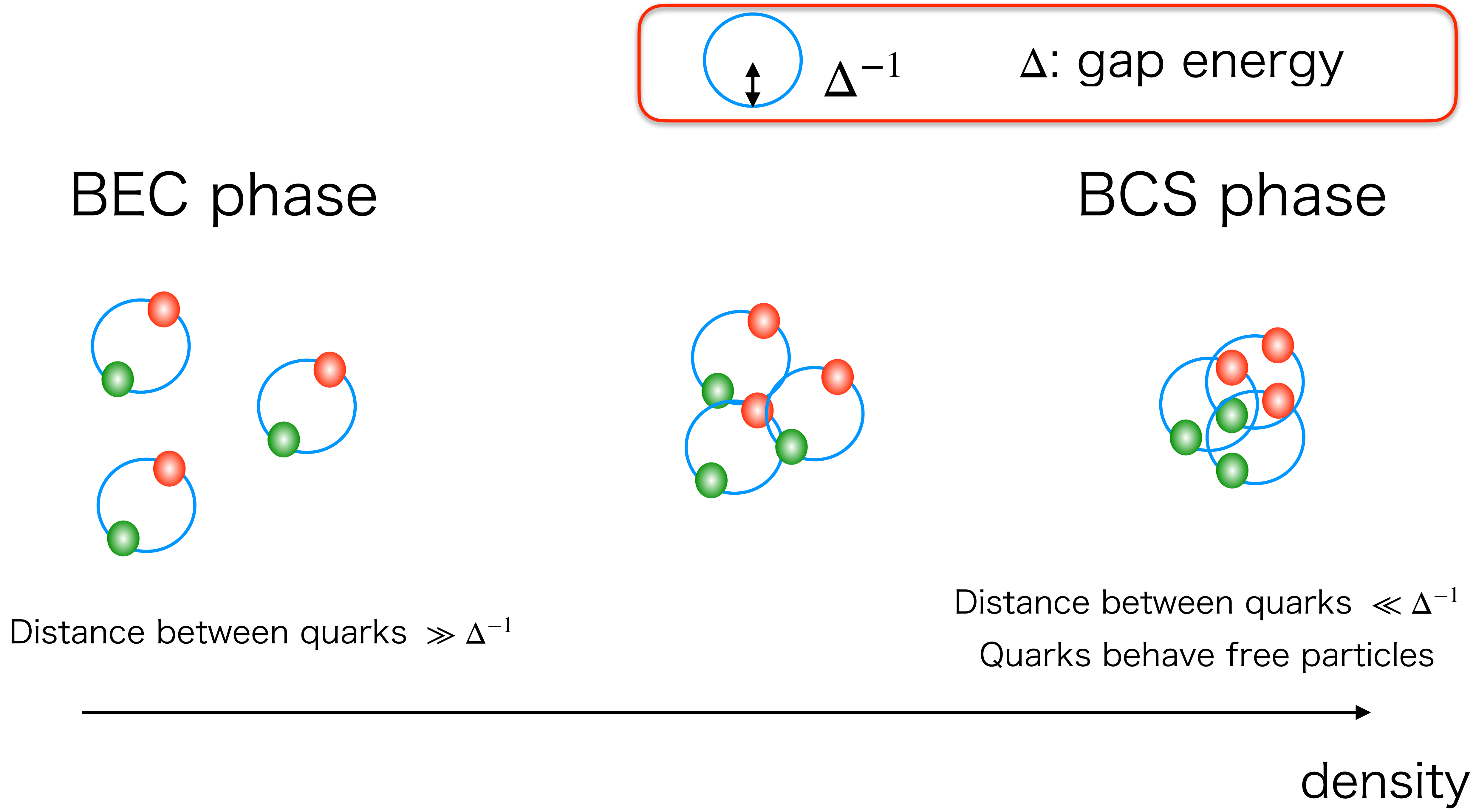
Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly

NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

BEC/BCS crossover





Number density of free particle

$$n_q^{\mathrm{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$$

J->0 extrapolation

Diquark condensate has a strong j dependence

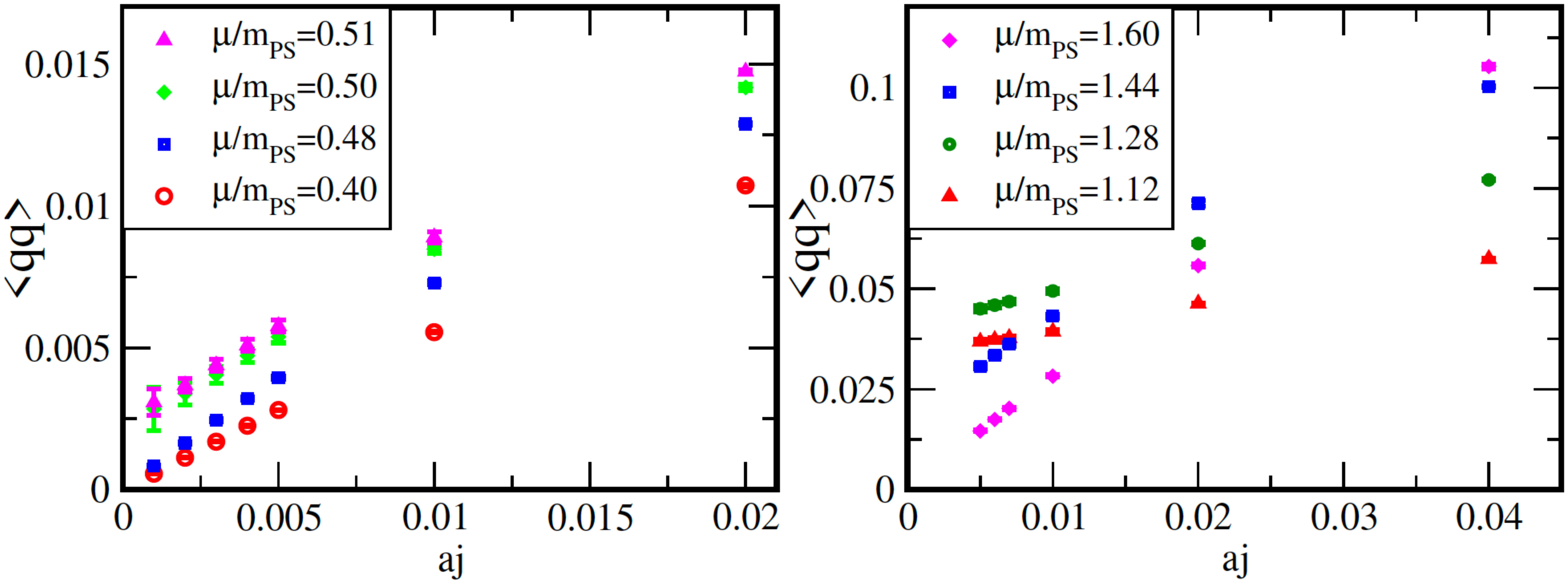


Figure 5. The j-dependence of the diquark condensate for several $\mu/m_{\rm PS}$.

J->0 extrapolation

Chiral condensate and n_q have a mild j-dependence