

Doubly charm tetraquark from lattice QCD

Sasa Prelovsek University of Ljubljana & Jozef Stefan Institute, Slovenia

QNP2022 - Conference on Quarks and Nuclear Physics

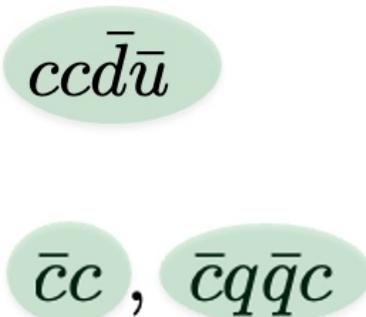
September 5-9, 2022

Florida State University

in collaboration with

Outline:

Doubly charm tetraquark



Charmonium(like) states

M. Padmanath

S. Collins, D. Mohler,
M. Padmanath, S. Piemonte

Outline and lattice QCD results

CLS ensembles: u,d,s dynamical quarks

$m_u = m_d > m_{u,d}^{phy}$, $m_\pi \approx 280$ MeV

$a \approx 0.086$ fm, $L = 2.1$ fm, 2.7 fm

Doubly charm tetraquark (T_{cc})



I=0
 $J^P=1^+$

DD^* scattering

Padmanath, S.P.: 2202.10110, PRL

- T_{cc} found as a virtual bound state ≈ 10 MeV below DD^* threshold
- likely related to T_{cc} discovered by LHCb

Charmonium(like) states



I=0
 $J^{PC}=0^{++}, 1^{--}, 2^{++}, 3^{--}$
q=u,d,s

$D\bar{D} - D_s\bar{D}_s$ scattering

S.P., Collins, Padmanath,
Mohler, Piemonte
2011.02542 JHEP,
1905.03506 PRD
2111.02934

- masses and decay widths of conventional charmonia confirmed : ground states (bound states)
first excitations (resonances)
- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds

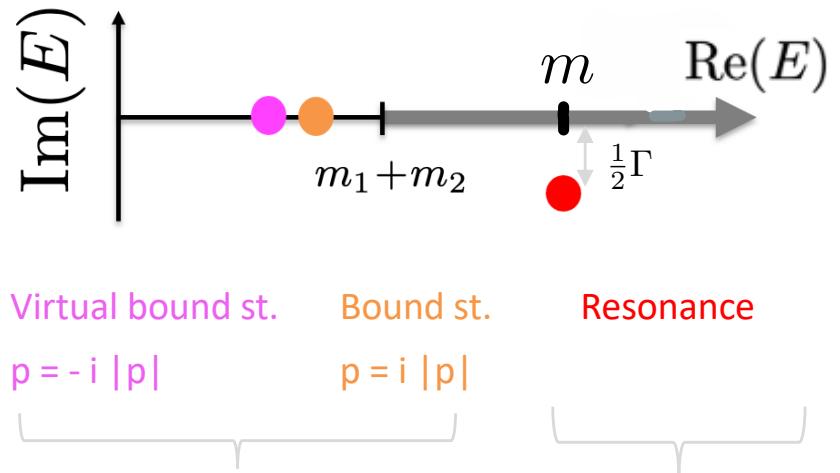
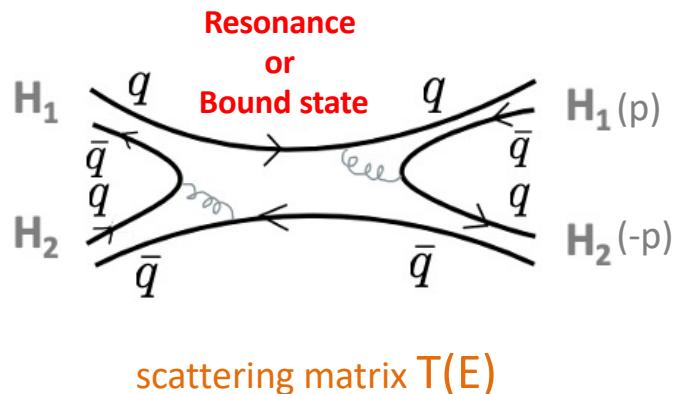


seen in dispersive re-analysis of exp.
[Danilkin et al 2111.15033]

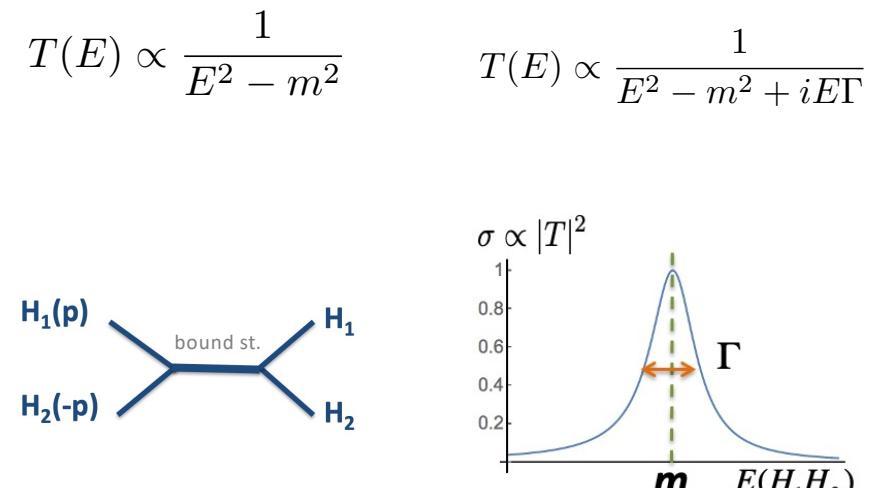
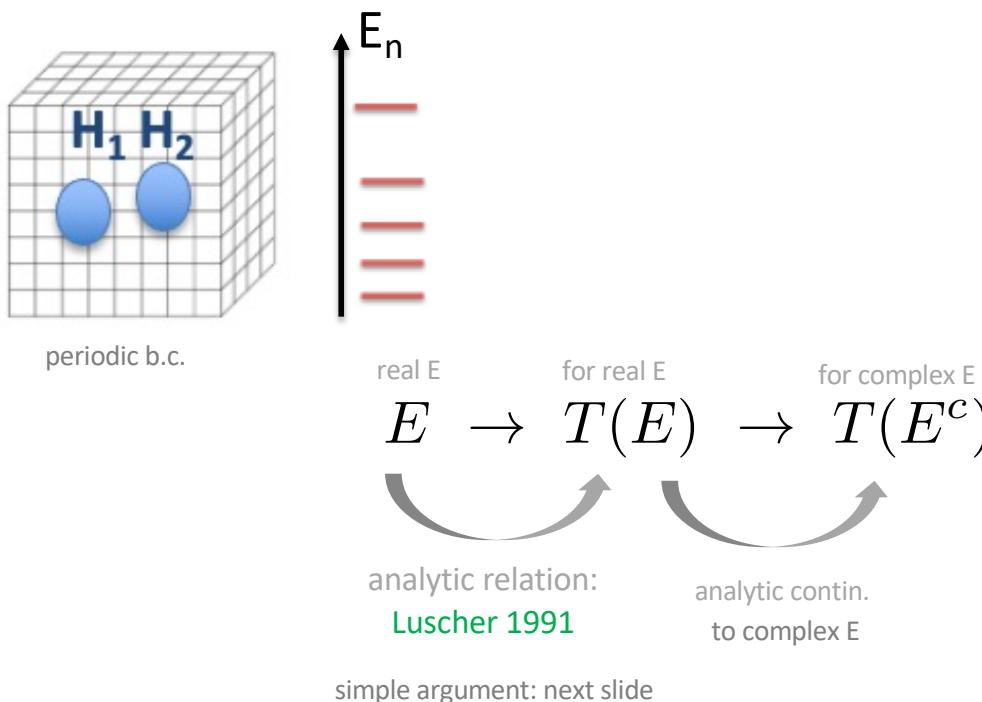


likely related to $X(3915) / \chi_{c0}(3930) / X(3960)$
LHCb2020 LHCb2022

Extract resonances and (virtual) bound states from $H_1 H_2$ scattering



Scattering matrix $T(E)$ from lattice QCD



one-channel scattering

$$S = 1 + i \frac{4p}{E} T = e^{2i\delta}$$

$$T = \frac{E}{2} \frac{1}{p \cot \delta - ip}$$

Relation between E and $\delta(E)$, $T(E)$: 1D nonrelativistic quantum mechanics

$V=0$: outside the region of potential

$$\psi(x) = A \cos(p|x| + \delta) = \begin{cases} A \cos(px + \delta) & x > R \\ A \cos(-px + \delta) & x < -R \end{cases}$$

- this form already ensures
 $\psi(L/2) = \psi(-L/2)$

- the other BC:
 $\psi'(L/2) = \psi'(-L/2)$

this requires

$$Ap \sin(p(\frac{L}{2}) + \delta) = -Ap \sin(-p(\frac{L}{2}) + \delta)$$

$$\rightarrow \psi'(L/2) = 0, \sin(p\frac{L}{2} + \delta) = 0$$

$$p\frac{L}{2} + \delta = n\pi$$

$$p = n\frac{2\pi}{L} - \frac{2}{L}\delta$$

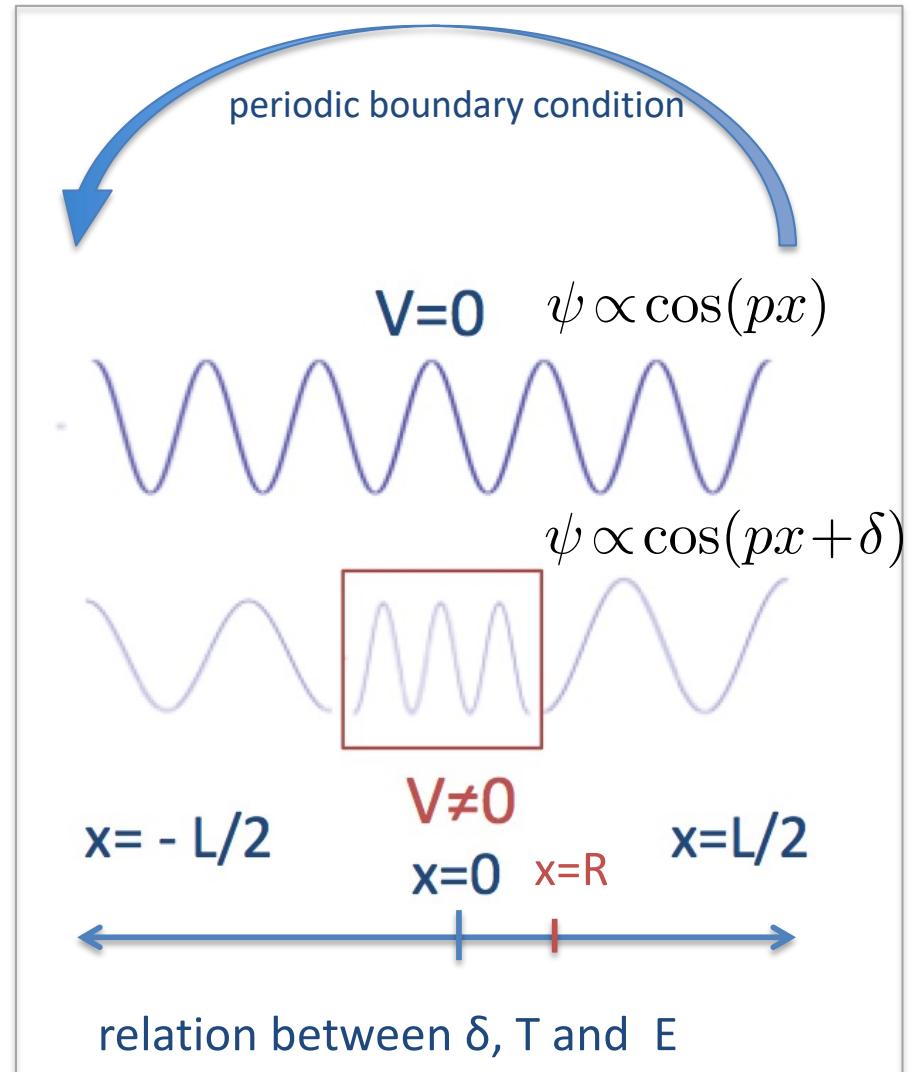
relation between n , δ , L

$$p = \frac{2\pi}{L}n$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta$$

$$E = p^2/2m$$

in both cases



relation between δ , T and E

$$cc\bar{d}\bar{u}$$

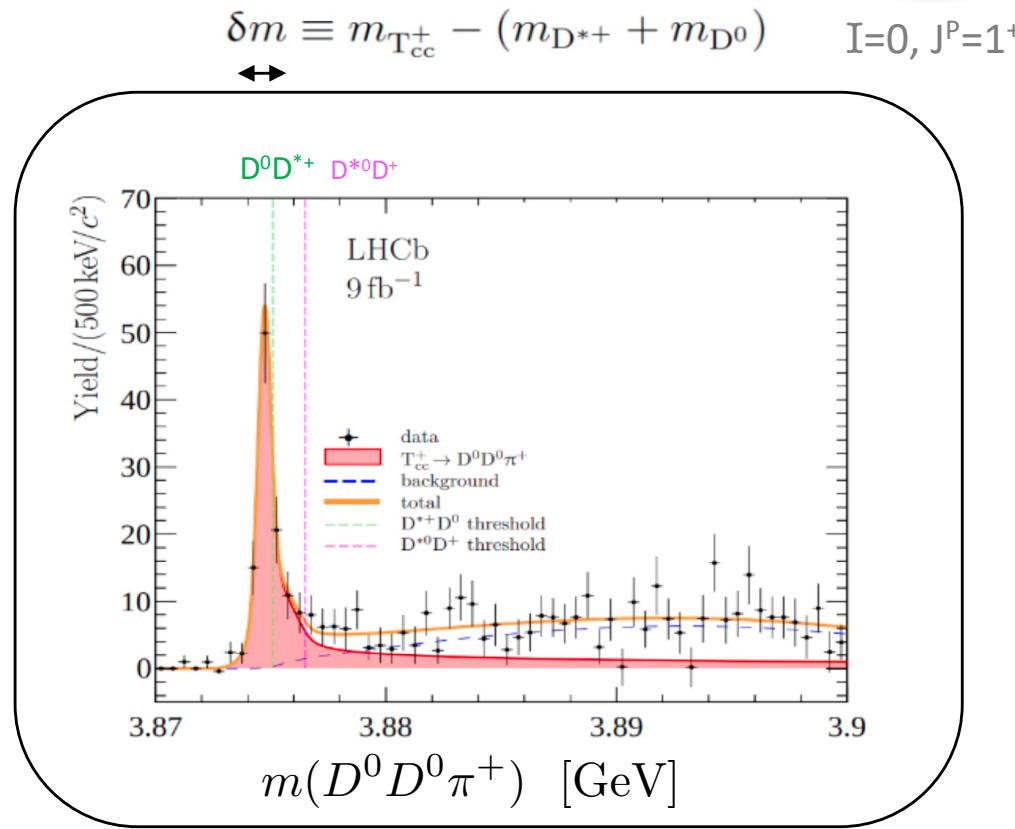
$$= T_{cc}$$

Padmanath, S.P.: 2202.10110,
Phys.Rev.Lett. 129 (2022) 3, 032002
&
subsequent studies with S. Collins

LHCb discovery of T_{cc}^+

$cc\bar{u}\bar{d}$

The longest lived exotic hadron ever discovered

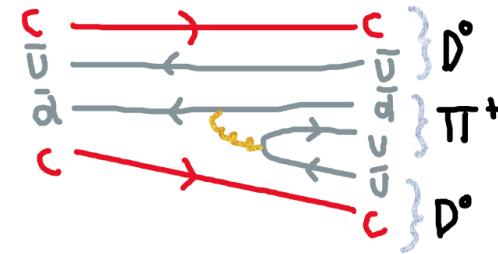


LHCb July 2021, 2109.01038, 2109.01056, Nature Physics

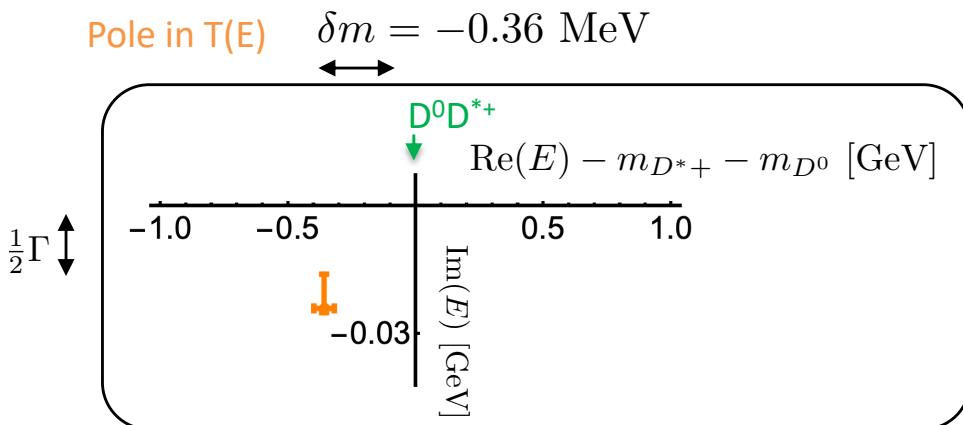
The doubly charmed tetraquark T_{cc}^+ , $I = 0$ and favours $J^P = 1^+$.

No states observed in $D^0 D^+ \pi^+$: eliminates possibility of $I = 1$.

Near-threshold state: Demands pole identification to confirm existence.



Omitting $D^* \rightarrow D\pi$, $T_{cc} \rightarrow DD\pi$
 T_{cc} would be a bound state

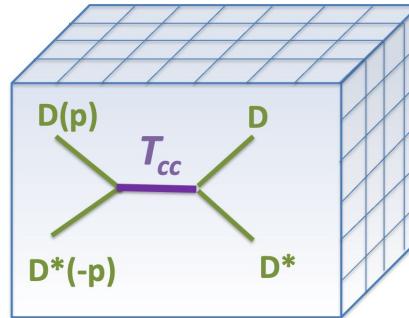


Doubly charm tetraquark from lattice

$$\begin{aligned} \delta m_{\text{pole}} &= -360 \pm 40^{+4}_{-0} \text{ keV}/c^2 \\ \Gamma_{\text{pole}} &= 48 \pm 2^{+0}_{-14} \text{ keV}, \end{aligned}$$

Lattice study

$$C \rightarrow E \rightarrow T(E)$$



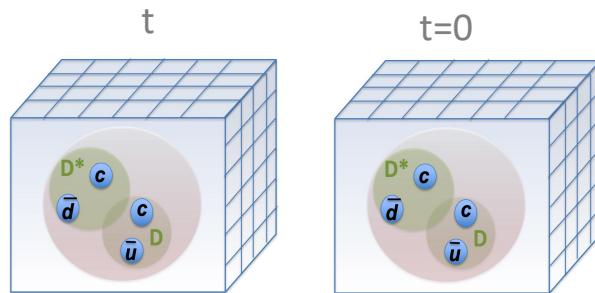
$m_\pi \simeq 280$ MeV :
 $D^* \not\rightarrow D\pi, T_{cc} \not\rightarrow DD\pi$
 $DD\pi$ above analyzed region

$$\sum_n |n\rangle\langle n|$$

↓

$$C_{ij}(t) = \langle 0 | \mathcal{Q}_i(t) \mathcal{Q}_j^\dagger(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{Q}_i | n \rangle e^{-E_n t} \langle n | \mathcal{Q}_j^\dagger | 0 \rangle$$

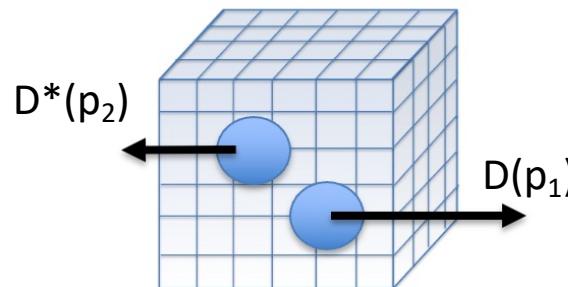
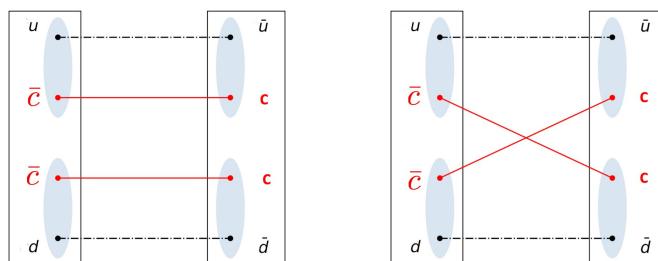
$$\langle C \rangle = \int DG Dq D\bar{q} C e^{-S_{QCD}/\hbar}$$



$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1} (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2)$$

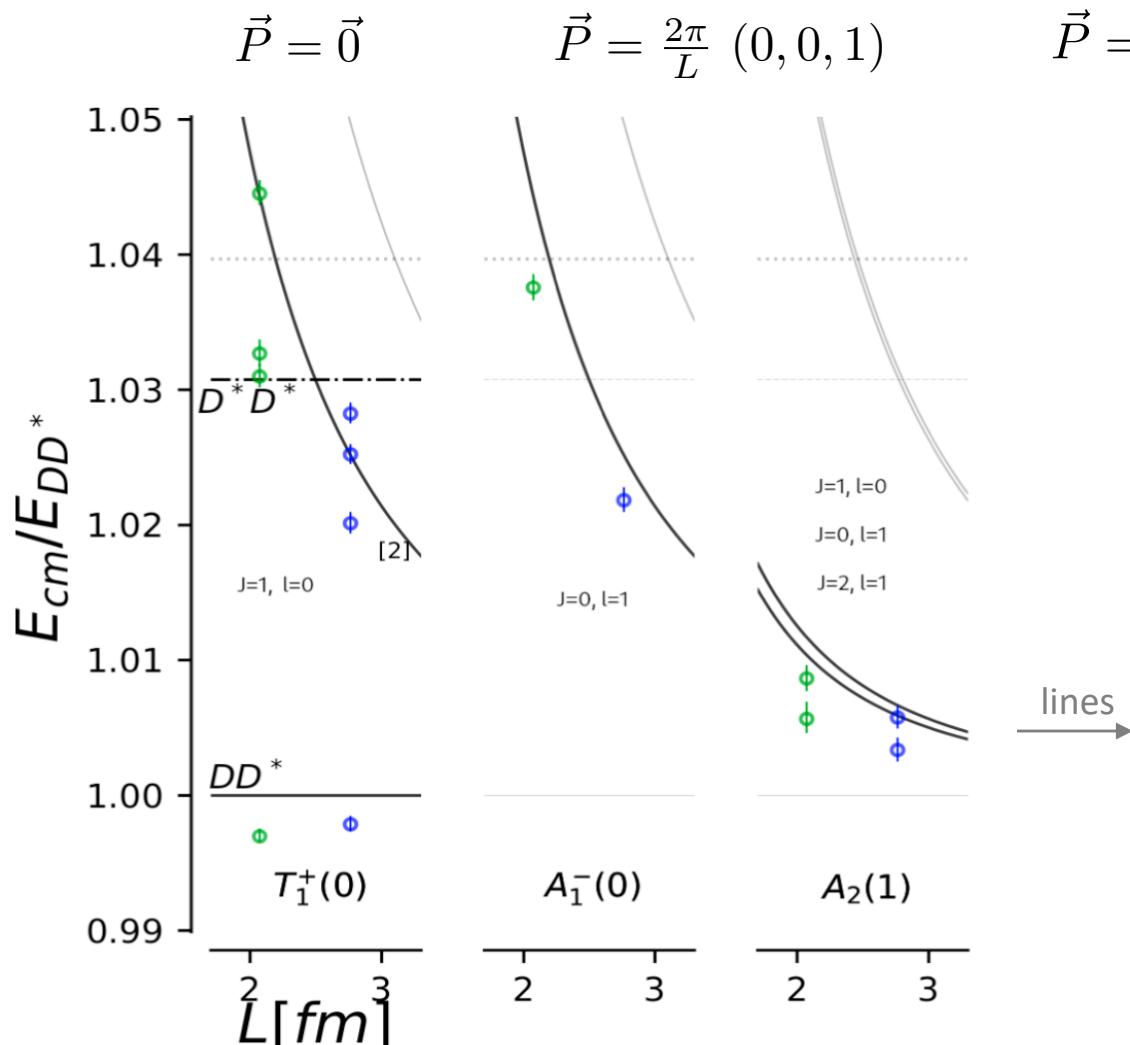
$$(\bar{u}\gamma_5 \gamma_t c)_{\vec{p}_1} (\bar{d}\gamma_i \gamma_t c)_{\vec{p}_2}$$

$$\vec{p}_{1,2} = \vec{n}_{1,2} \frac{2\pi}{L}$$



Eigen-energies on the lattice

at $m_\pi \approx 280 \text{ MeV}$



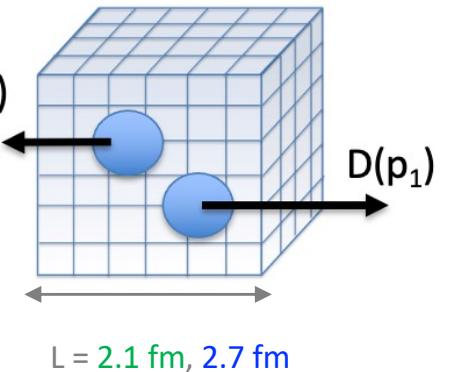
lines →

non-interacting energies

$$E^{n.i.} = \sqrt{m_D^2 + \vec{p}_1^2} + \sqrt{m_{D^*}^2 + \vec{p}_2^2}$$

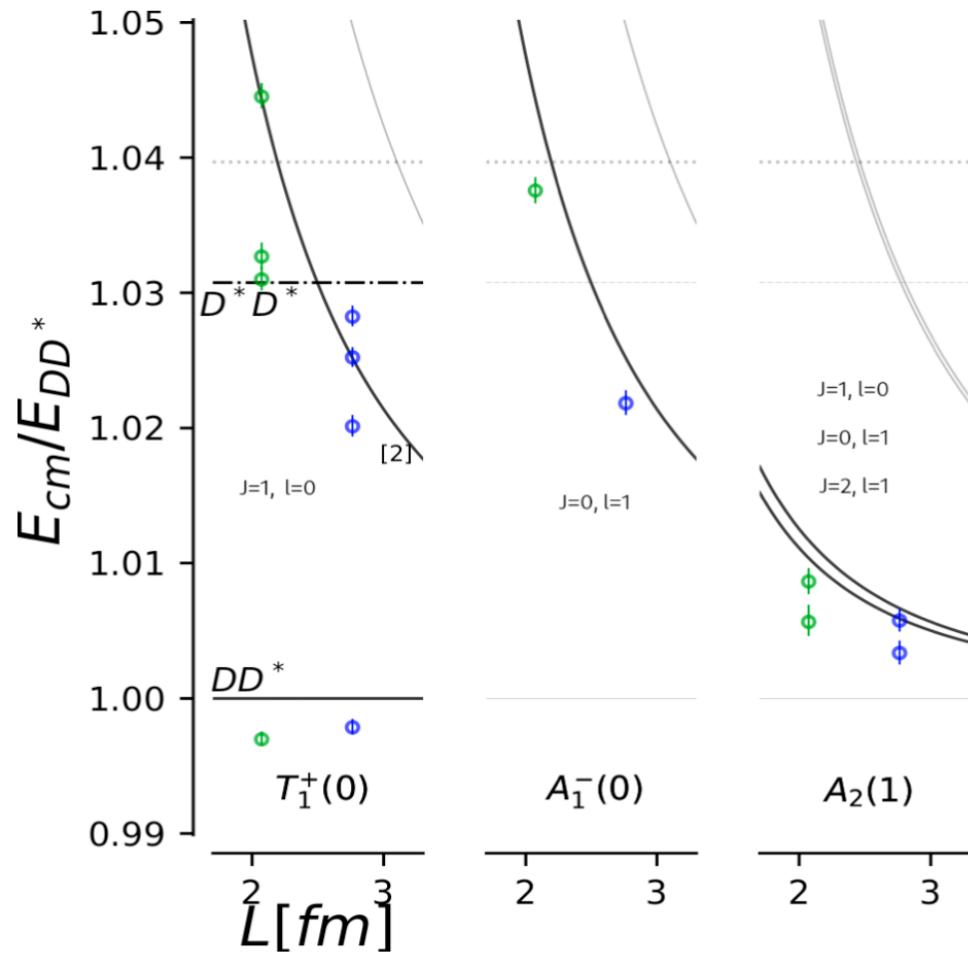
$$\vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$

$$E_{DD^*} \equiv m_D + m_{D^*}$$



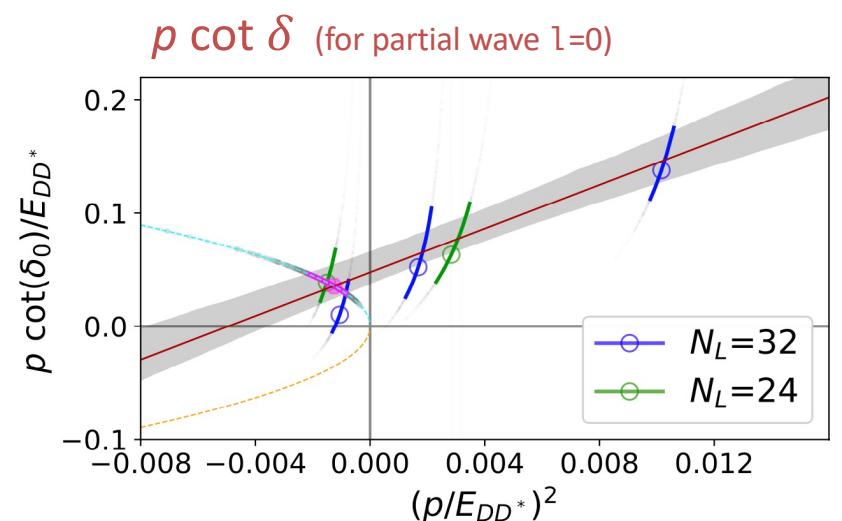
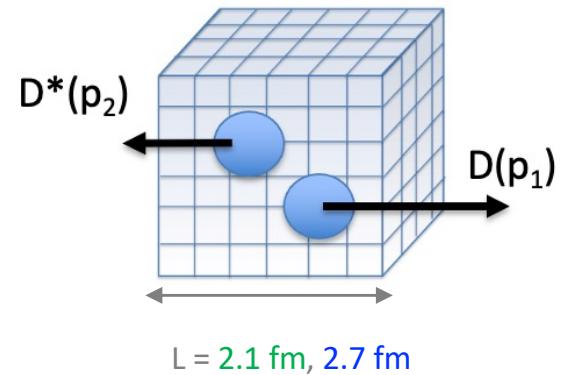
Eigen-energies and scattering amplitude

at $m_\pi \approx 280 \text{ MeV}$



$$E_{DD^*} \equiv m_D + m_{D^*}$$

Luscher's relation
 $E \rightarrow T(E), \delta(E)$



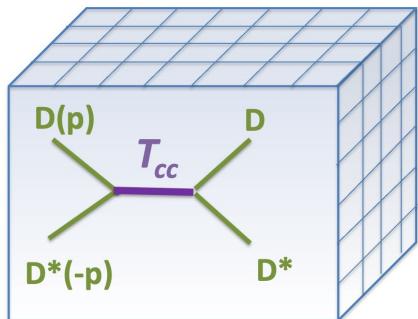
$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = 1.04(0.29) \text{ fm} \quad \& \quad r_0 = 0.96^{(+0.18)}_{(-0.20)} \text{ fm}$$

Scattering amplitude for $\ell=0$

at $m_\pi \approx 280 \text{ MeV}$

$$T = \frac{E}{2} \frac{1}{p \cot \delta - ip}$$



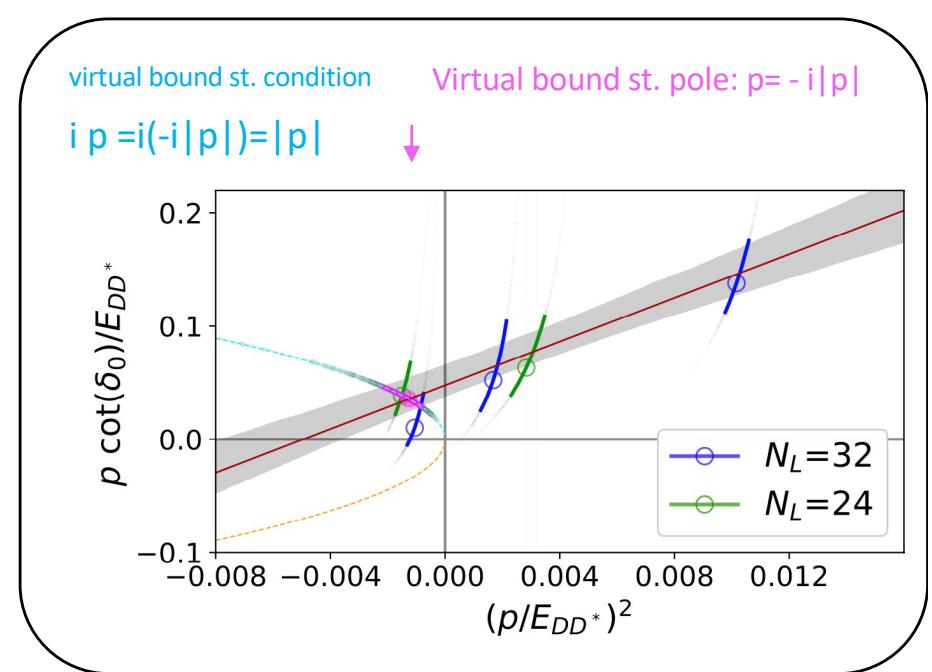
Lattice: virtual bound st. pole

Binding energy:

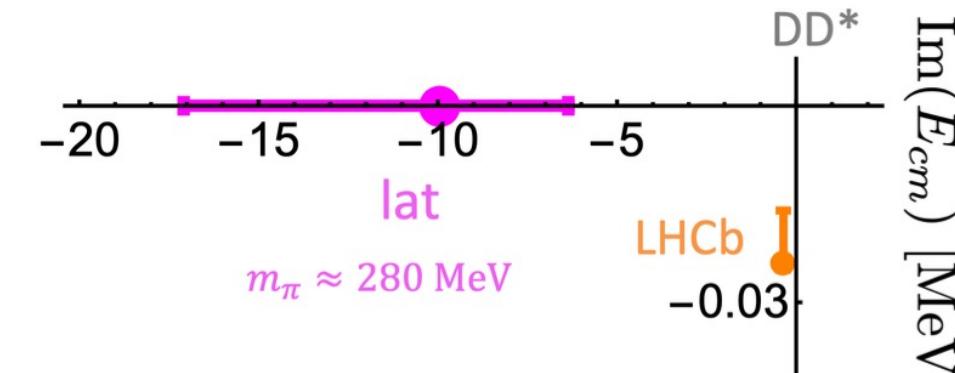
$$\delta m_{T_{cc}} = -9.9^{(+3.6)}_{(-7.2)} \text{ MeV}$$

Nature (LHCb): bound st. pole

omitting $D^* \rightarrow D\pi$, $T_{cc} \rightarrow DD\pi$

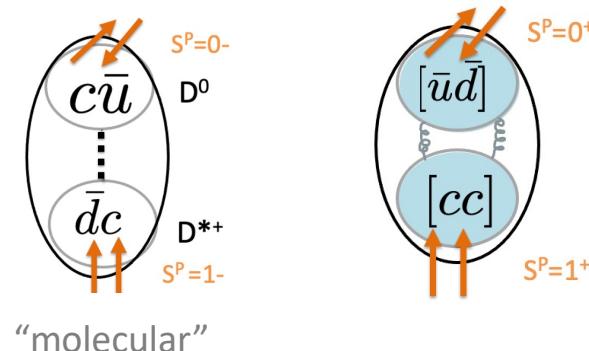


$$\delta m_{T_{cc}} = \text{Re}(E_{cm}) - m_{D^0} - m_{D^{*+}} \text{ [MeV]}$$



Possible binding mechanisms of T_{cc}

molecular
likely dominant
[e.g. Janc, Rosina 2003]



Molecular component: dependence on $m_{u/d}$

exchanged particles:
light mesons π, ρ, \dots

increasing $m_{u/d}$
increasing m_{ex}
decreasing R or
decreasing attraction $|V|$

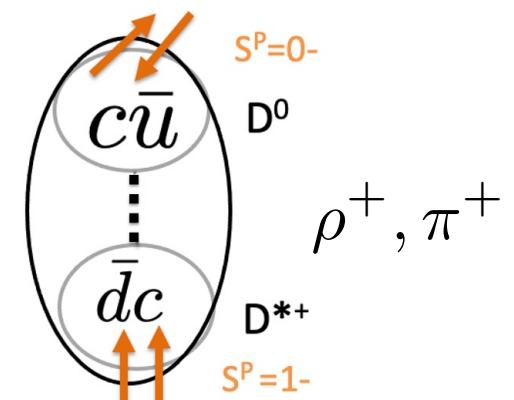
Yukava-like potential

$$V(r) \propto -\frac{e^{-m_{ex}r}}{r}$$

analogous conclusion for any
fully attractive

$$V(r) = -V_0 f(r/R)$$

$$f = e^{-r/R}, e^{-r^2/R^2}, \theta(R-r), \dots$$



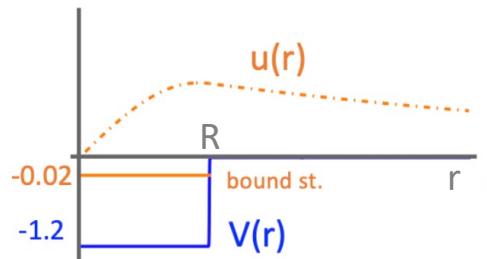
subsequent lattice study:
CLQCD, Chen et al. 2206.06185
comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ρ exchange

Simplest Example: scattering in square-well potential in QM

$$\delta = \arctan[\tan(qR) \frac{p}{q}] - pR$$

$$u(r) = A \sin(qr) \quad u(r) = B \sin(pr + \delta)$$

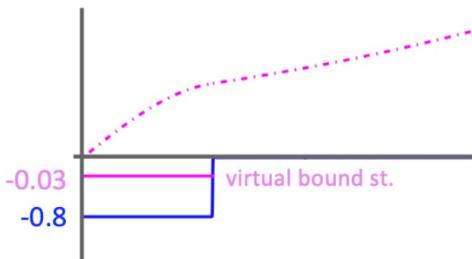
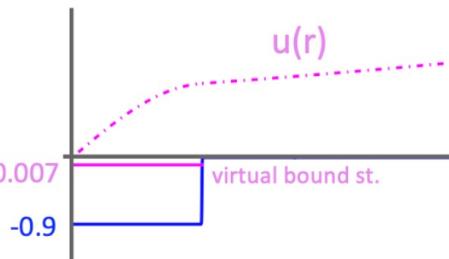
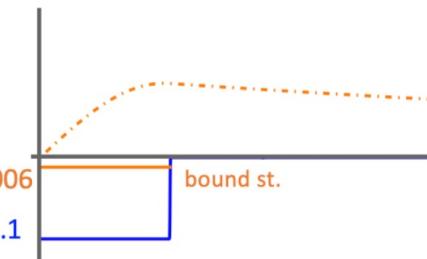
\downarrow



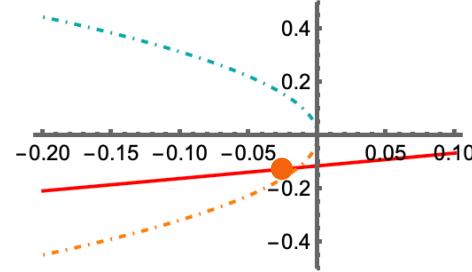
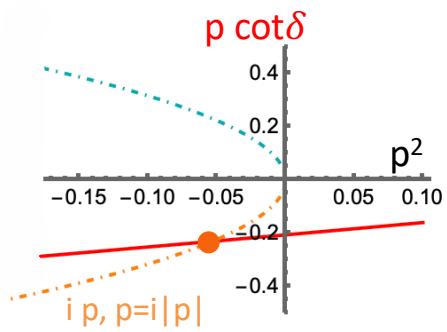
$$p=i|p| \\ e^{ipr} = e^{-|p|r}$$

$$p=-i|p| \\ e^{ipr} = e^{|p|r}$$

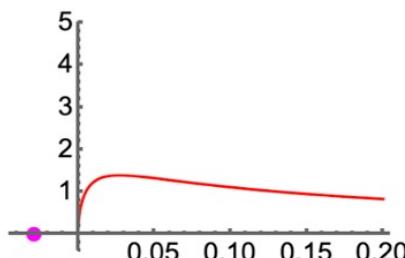
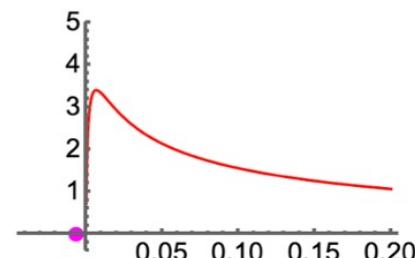
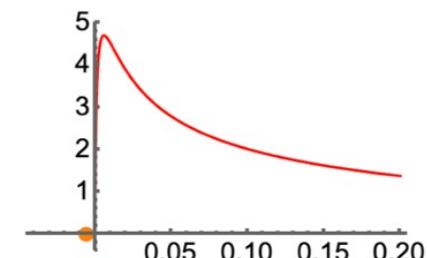
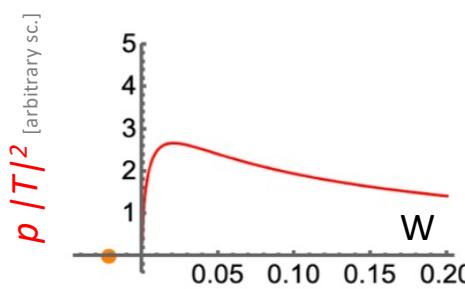
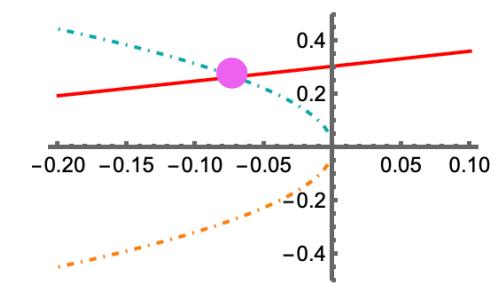
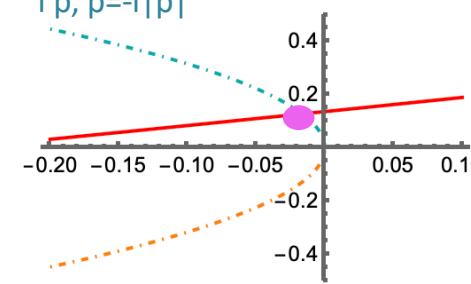
partial wave $l=0$
 $T \propto (p \cot \delta - ip)^{-1}$



$p \cot \delta$



$i p, p=-i|p|$

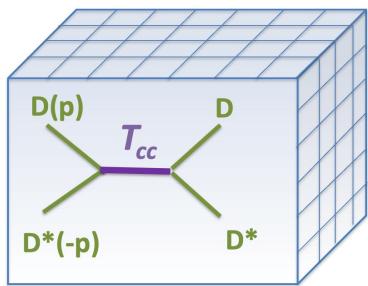


increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

Conclusions on T_{cc}

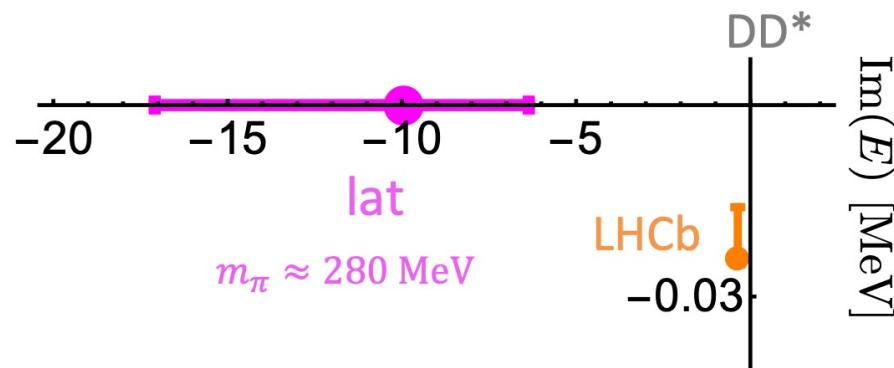
$cc\bar{d}\bar{u}$

The longest lived exotic hadron ever discovered



Pole of $T(E)$ at $m_c^{(h)}$

$$\delta m_{T_{cc}} = \text{Re}(E) - m_{D^0} - m_{D^{*+}} \text{ [MeV]}$$



	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \approx 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \approx 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0^{(+4.6)}_{(-9.3)}$	virtual bound st.
exp.	1864.85(5)	-0.36(4)	bound st.

closer-to physical m_c

$T_{cc} \rightarrow DD\pi$ $D^* \rightarrow D\pi$ omitting

Simple arguments within molecular picture:

$m_{u/d}$ increases :

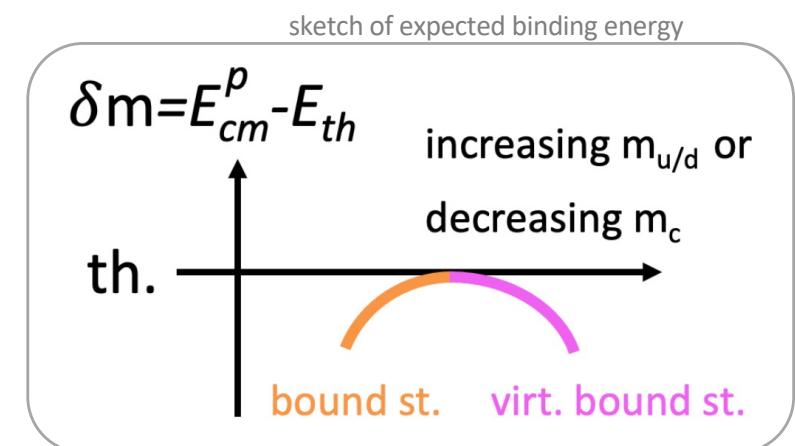
$$m_{u/d}^{phy} \rightarrow m_{u/d}^{lat}$$

(LHCb) would-be **bound st.** \rightarrow **virtual bound st.**

m_c decreases

$|\delta m_{T_{cc}}|$ increases for **virtual bound st.**
(see backup slides)

Both in agreement with the lattice result



Hypothesis to be verified by future simulations

$\bar{c}c$, $\bar{c}q\bar{q}c$

I=0

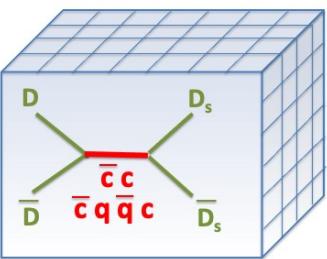
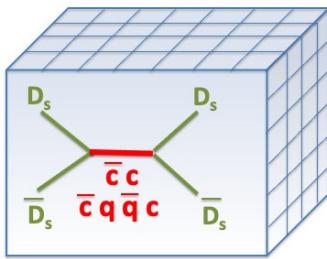
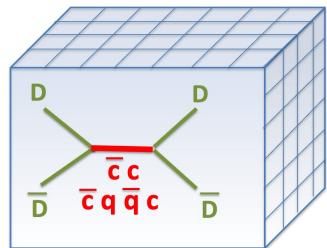
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP, 1905.03506 PRD, 2111.02934

Charmonium(like) resonances and bound states

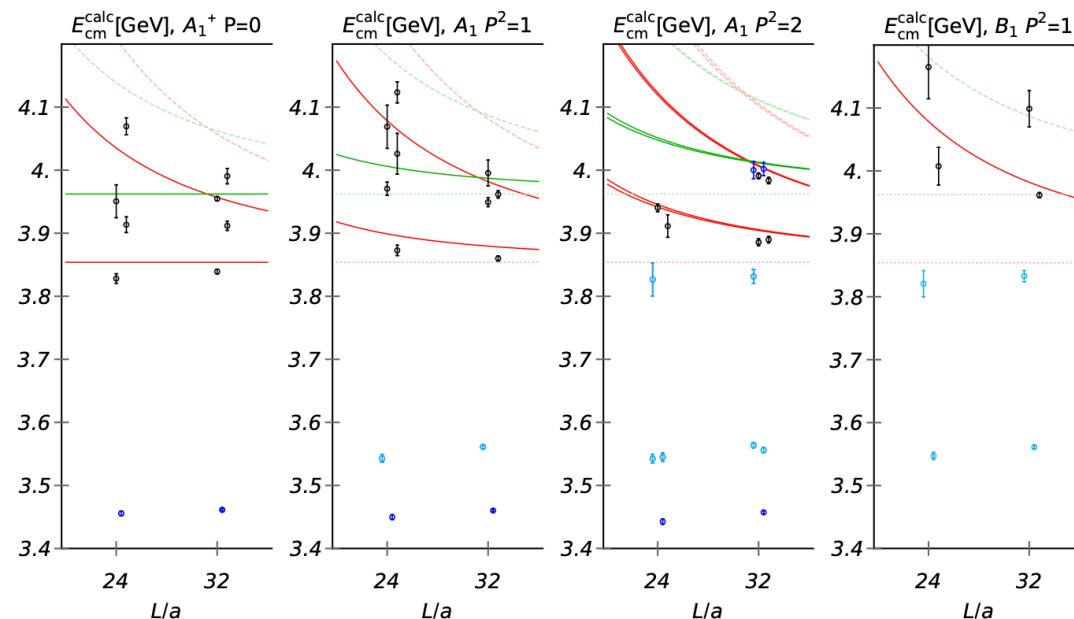
$$D\bar{D} - D_s\bar{D}_s$$

$\bar{c}c$, $\bar{c}q\bar{q}c$

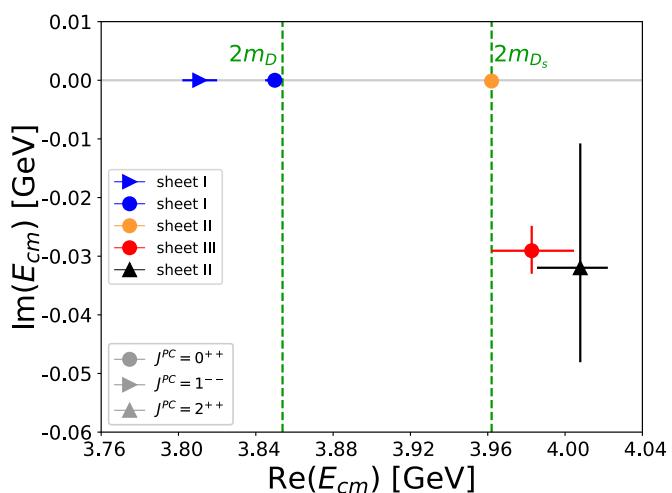
$q=u,d,s$ $I=0$



Eigen-energies



Luscher formalism



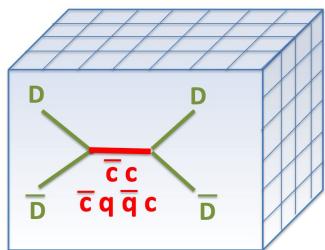
$T_{ij}(E)$

Doubly charm tetraquark from lattice

Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$

$q=u,d,s$ $I=0$



$\bar{D}_s D_s$

$J^P=0^+$ state

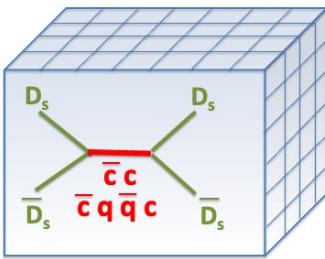
likely related to $X(3915)$ / $\chi_{c0}(3930)$ / $X(3960)$
[BaBar, LHCb 2009.00026, LHCb 2022 indico..../1176505/]

explaining why it has narrow width to $D\bar{D}$.

Supported by some pheno studies:

Lebed, Polosa 1602.08421, Oset et al . 2207.08490,

Guo et al, 2101.01021,

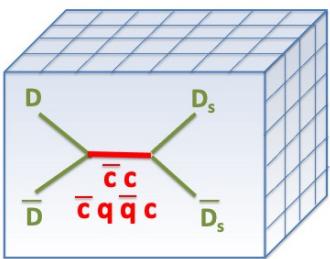


$\bar{D}D$

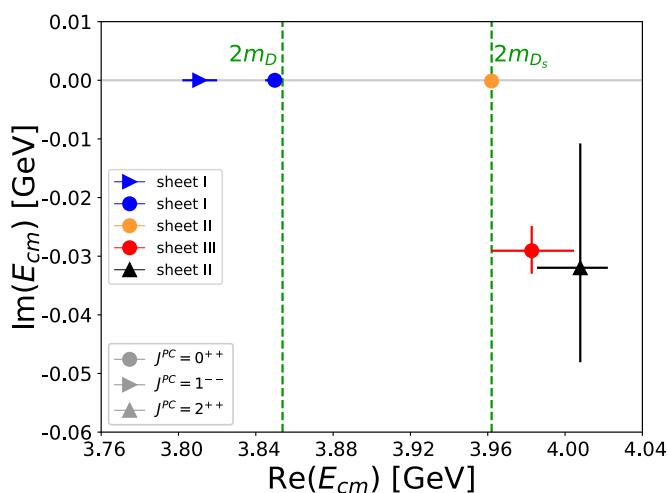
$J^P=0^+$ state

predicted in models [Oset et al,
0612179 PRD, Hildago Duque et al
1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive re-analysis of exp.
[Danilkin et al 2111.15033]



+ expected conventional charmonia

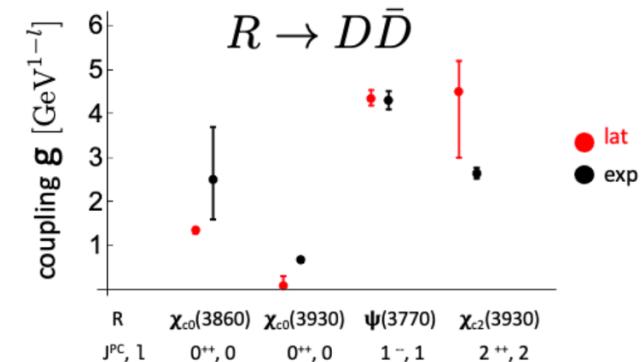
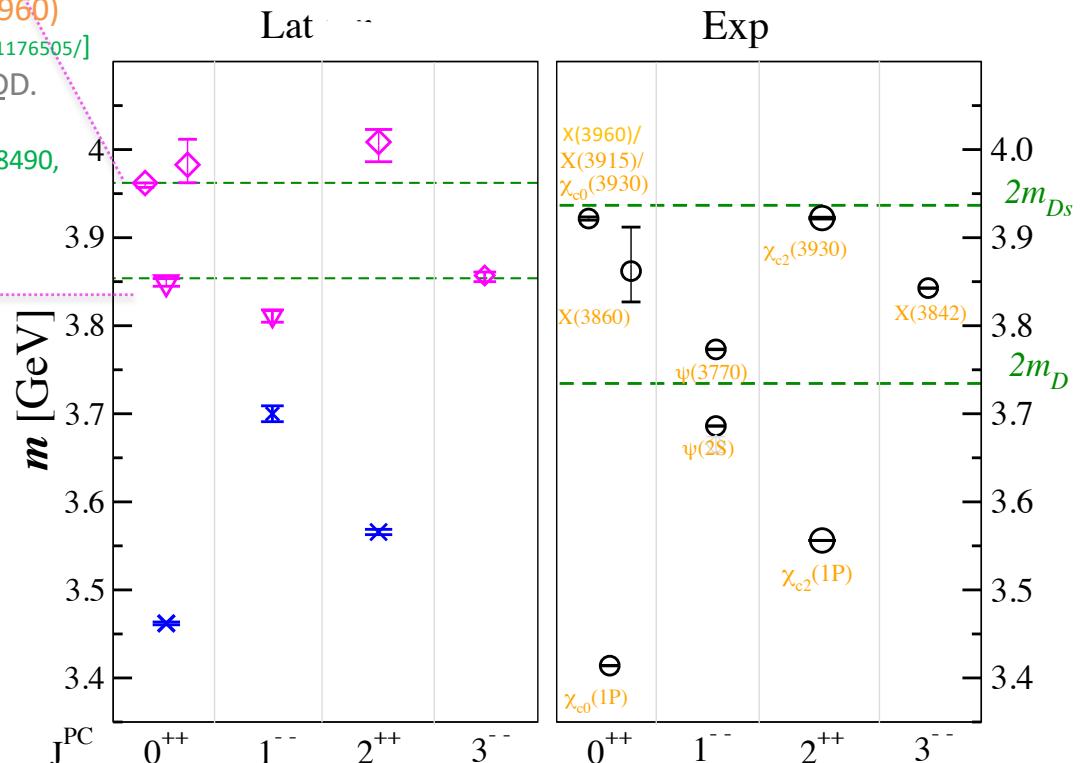


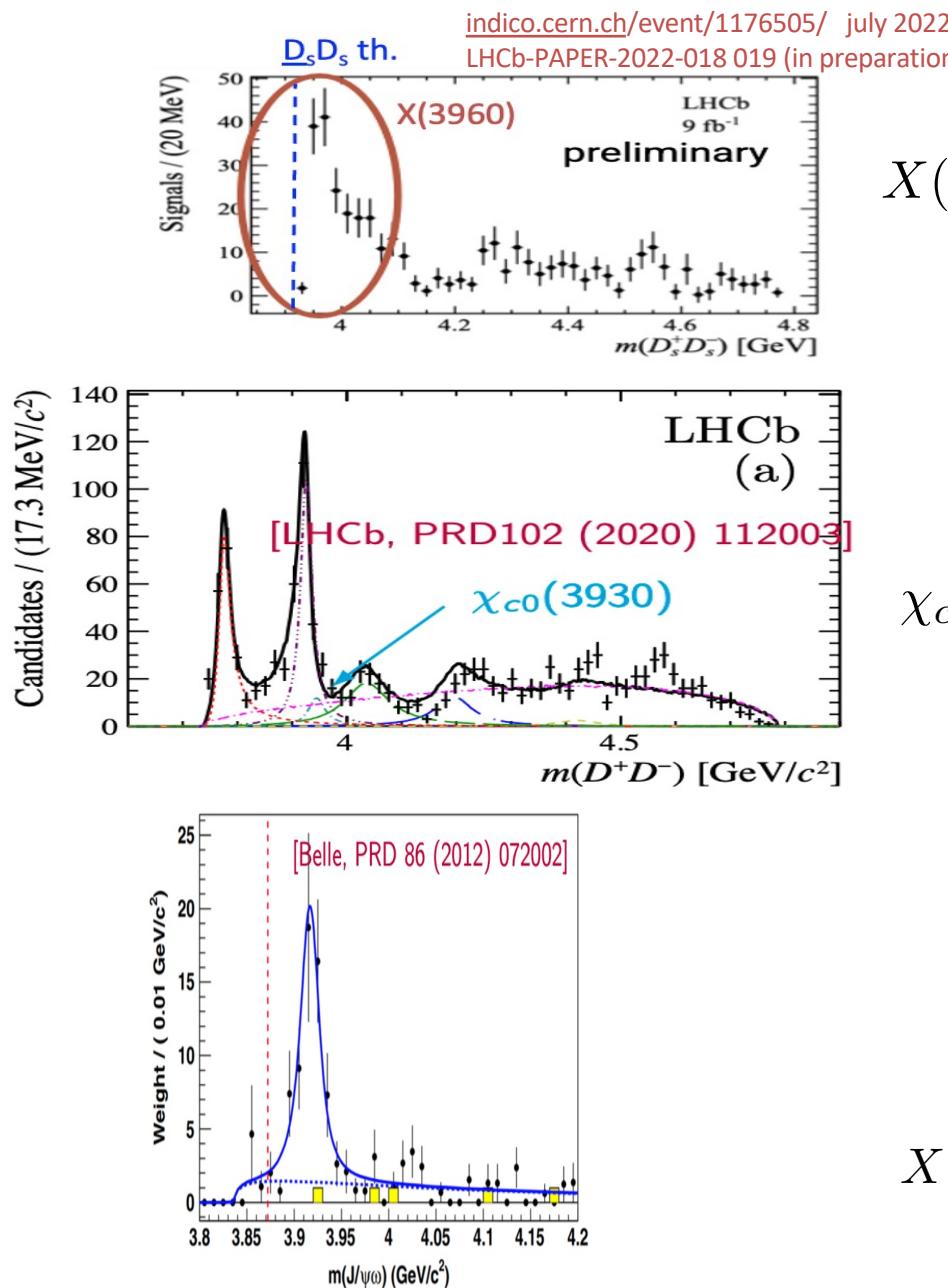
$$\Gamma \equiv g^2 \frac{p_D^{2l+1}}{m^2}$$

$m_\pi \simeq 280$ MeV

Lat

Exp



$\bar{D}_s D_s$ likely related to $X(3915)$ / $\chi_{c0}(3930)$ / $X(3960)$ all three likely the same state
currently named $\chi_{c0}(3914)$ in PDG $J^{PC}=0^{++}$ 

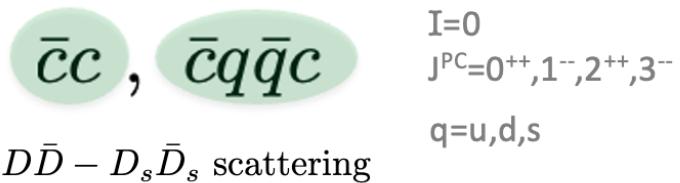
$$X(3960) \rightarrow D_s \bar{D}_s$$

$$\frac{Br(D\bar{D})}{Br(D_s \bar{D}_s)} \simeq 0.3$$

$$\chi_{c0}(3930) \rightarrow D\bar{D}$$

$$X(3915) \rightarrow J/\psi \omega$$

Conclusions on charmonium(like) states



- masses and decay widths of conventional charmonia confirmed : ground states (bound states)
first excitations (resonances)
- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds



seen in dispersive re-analysis of exp.
[Danilkin et al 2111.15033]



likely related to $X(3915) / \chi_{c0}(3930) / X(3960)$
LHCb2020 LHCb2022

Backup

Lattice details

CLS ensembles with u/d, s dynamical quarks

$a \approx 0.086$ fm

$N_L = 24, 32$

lat	exp
$m_{u/d}$	$> m_{u/d}^{\text{exp}}$
m_s	$< m_s^{\text{exp}}$
$m_u + m_d + m_s$	$= m_u^{\text{exp}} + m_d^{\text{exp}} + m_s^{\text{exp}}$

$$m_c \gtrsim m_c^{\text{exp}}$$

m [MeV]	lat	exp
m_π	280(3)	137
m_D	1927(2)	1867
m_{D_s}	1981(1)	1968
M_{av}	3103(3)	3068

$$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$

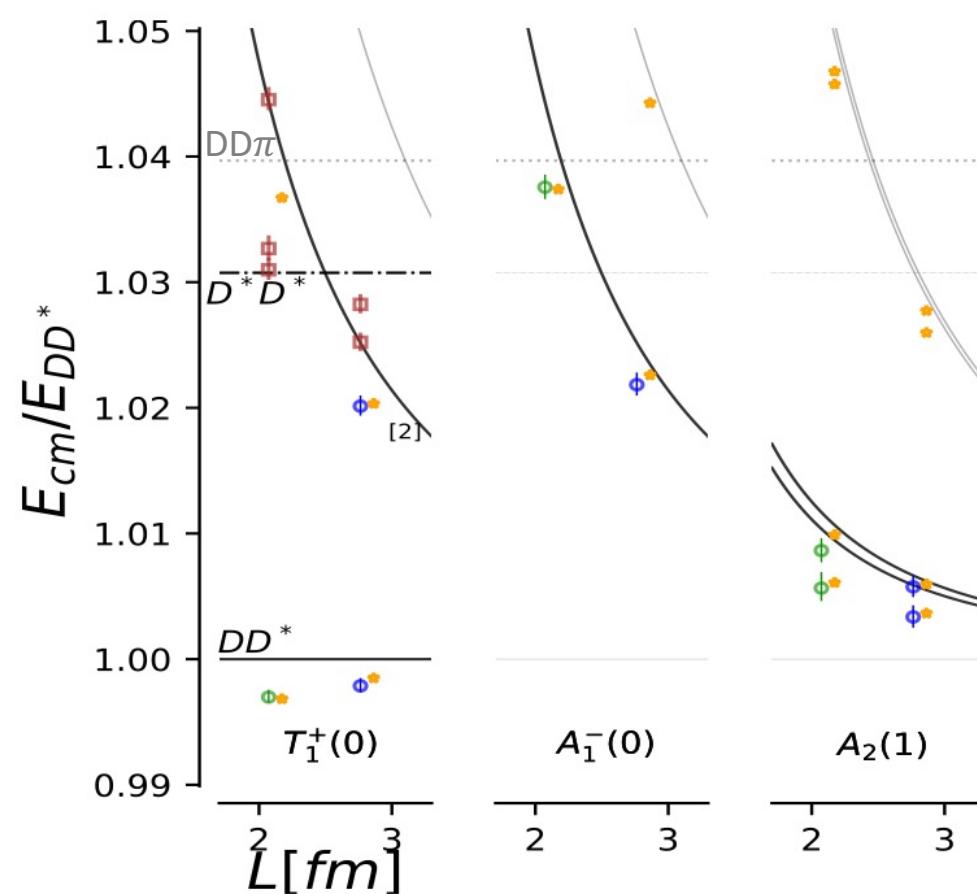
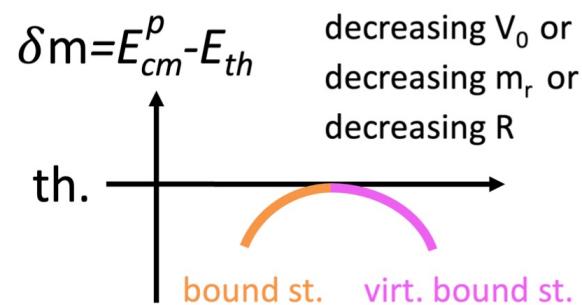
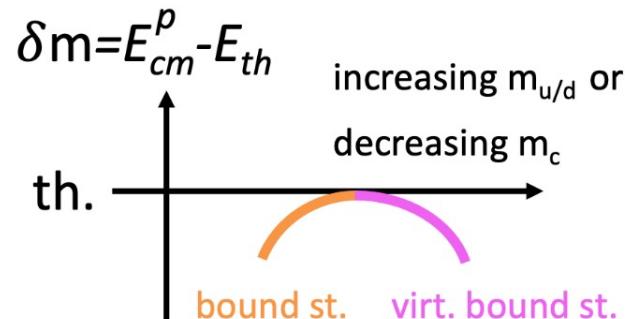
separation between DD and DsDs threshols smaller than in exp

Wick contractions evaluated with
distillation or stochastic distillation method.

Lattice results on Tcc

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96^{(+0.18)}_{(-0.20)}$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92^{(+0.17)}_{(-0.19)}$	$-15.0^{(+4.6)}_{(-9.3)}$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9), 0]	-0.36(4)	bound st.

$$V(r) = -V_0 f(r/R)$$



Interpolators for Tcc

Example: P=0

$J^P=1^+$ \rightarrow cubic irrep T_1^+

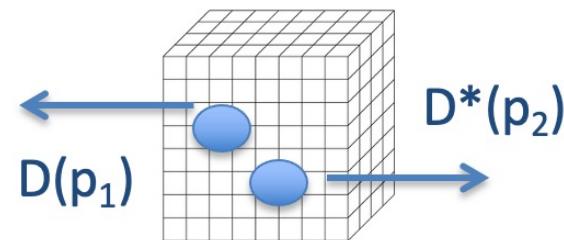
$$O^{l=0} = P(\{0, 0, 0\}) V_z(\{0, 0, 0\})$$

$$\begin{aligned} O^{l=0} = & P(\{1, 0, 0\}) V_z(\{-1, 0, 0\}) + P(\{-1, 0, 0\}) V_z(\{1, 0, 0\}) \\ & + P(\{0, 1, 0\}) V_z(\{0, -1, 0\}) + P(\{0, -1, 0\}) V_z(\{0, 1, 0\}) \\ & + P(\{0, 0, 1\}) V_z(\{0, 0, -1\}) + P(\{0, 0, -1\}) V_z(\{0, 0, 1\}) \end{aligned}$$

$$\begin{aligned} O^{l=2} = & P(\{1, 0, 0\}) V_z(\{-1, 0, 0\}) + P(\{-1, 0, 0\}) V_z(\{1, 0, 0\}) \\ & + P(\{0, 1, 0\}) V_z(\{0, -1, 0\}) + P(\{0, -1, 0\}) V_z(\{0, 1, 0\}) \\ & - 2[P(\{0, 0, 1\}) V_z(\{0, 0, -1\}) + P(\{0, 0, -1\}) V_z(\{0, 0, 1\})] \end{aligned}$$

$$O^{l=0} = V_{1x}[0, 0, 0] V_{2y}[0, 0, 0] - V_{1y}[0, 0, 0] V_{2x}[0, 0, 0]$$

P=D, V=D*



$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \quad (1) \\ \mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) .$$

Here

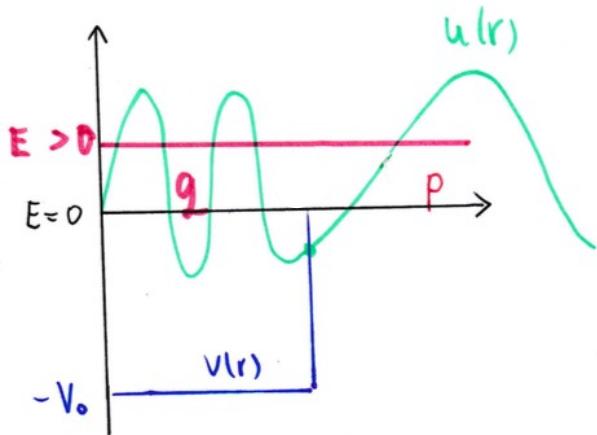
$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i \frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \quad (5)$$

We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix}. \quad (6)$$

S-wave scattering on spherical potential well



$$A \sin qr \quad B \sin(pr + \delta_0)$$

$$\left. \begin{aligned} u(R) &= A \sin qR = B \sin(pr + \delta) \\ u'(R) &= q A \cos qR = p B \cos(pr + \delta) \end{aligned} \right\}$$

dividing both eqs

$$\frac{1}{q} \tan qR = \frac{1}{p} \tan(pr + \delta)$$

$$\delta_0(p) = \arctan\left(\frac{p}{q} \tan(qR)\right) - pr + n\pi$$

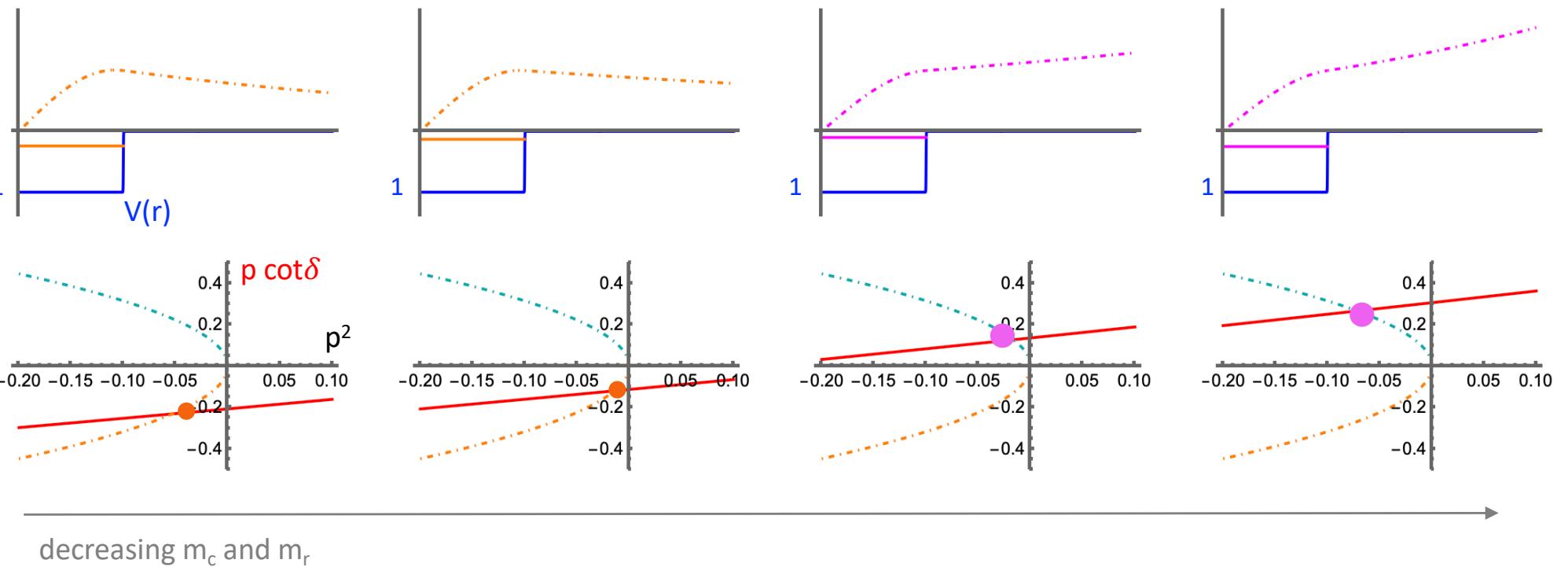
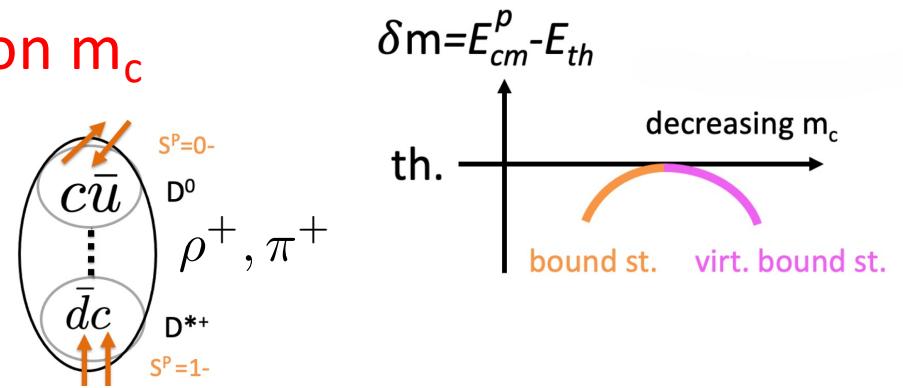
$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{2\mu V_0 + p^2}$$

Molecular component: dependence on m_c

$V(r)$ independent on m_c ,

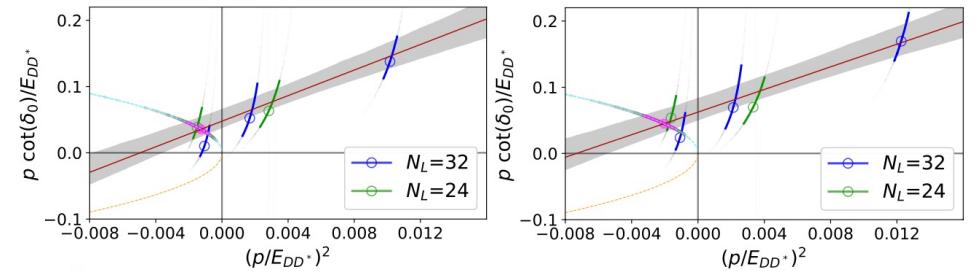
m_c decreases : reduced mass m_r of D, D^* system decreases

Square well potential (analogous conclusion for other shapes)



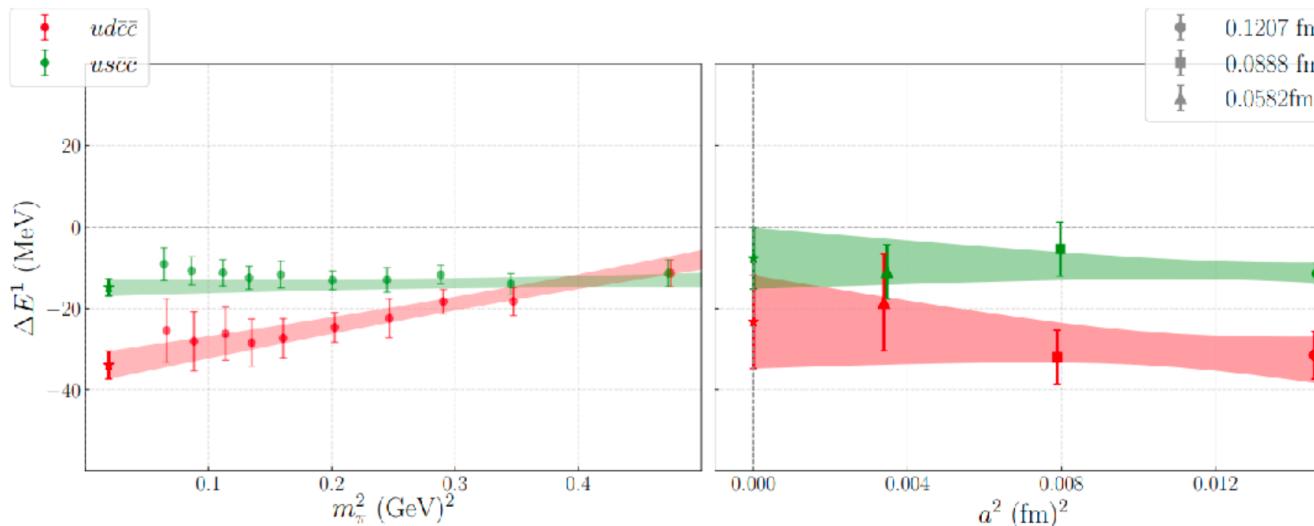
	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0^{(+4.6)}_{(-9.3)}$	virtual bound st.

lattice results



Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



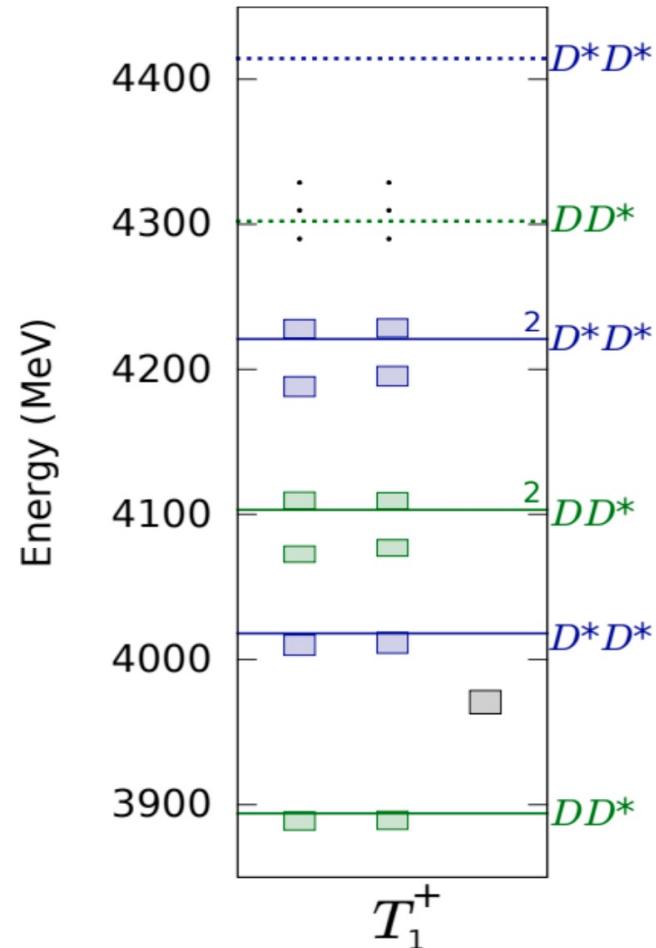
lowest finite-volume
eigen-energy for
 $P=0, J^P=1^+, I=0$

- ✿ Study performed on LQCD ensembles with different lattice spacings.
Single volume and only rest frame finite-volume irreps considered.
- ✿ Including a meson-meson and diquark-antidiquark interpolator.
Diquark-antidiquark interpolators do not influence the low energy spectrum.
- ✿ The ground state energy subjected to chiral and continuum extrapolations.
- ✿ A finite-volume energy level 23(11) MeV below DD^* threshold.
No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume
eigen-energies for
 $P=0, J^P=1^+, I=0$

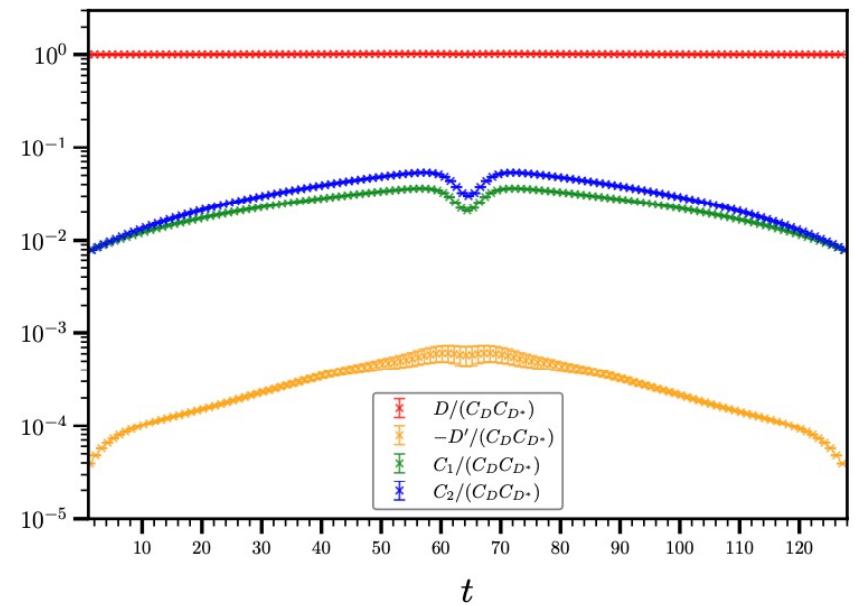
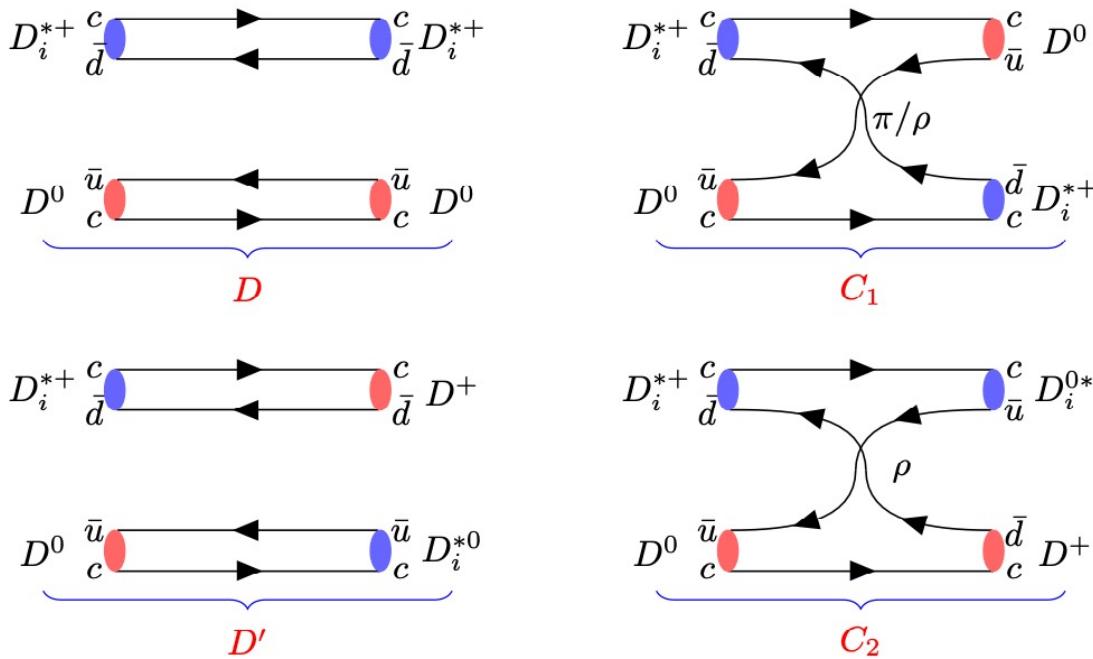


- ✿ Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- ✿ Large basis of meson-meson and diquark-antidiquark interpolators.
- ✿ Diquark-antidiquark interpolators do not influence the low energy spectrum.
- ✿ No statistically significant energy shifts observed near DD^* threshold.
⇒ No scattering amplitude extraction.

Subsequent lattice QCD study of T_{cc} channel

CLQCD, Chen et al. 2206.06185

comparison of $I=0,1$:
 attraction in $I=0$ channel arises
 mainly from ρ exchange



$$C^{(I)}(p, t) = D - C_1(\pi/\rho) + (-)^{I+1} (D' - C_2(\rho))$$