Status of Hadronic Molecules

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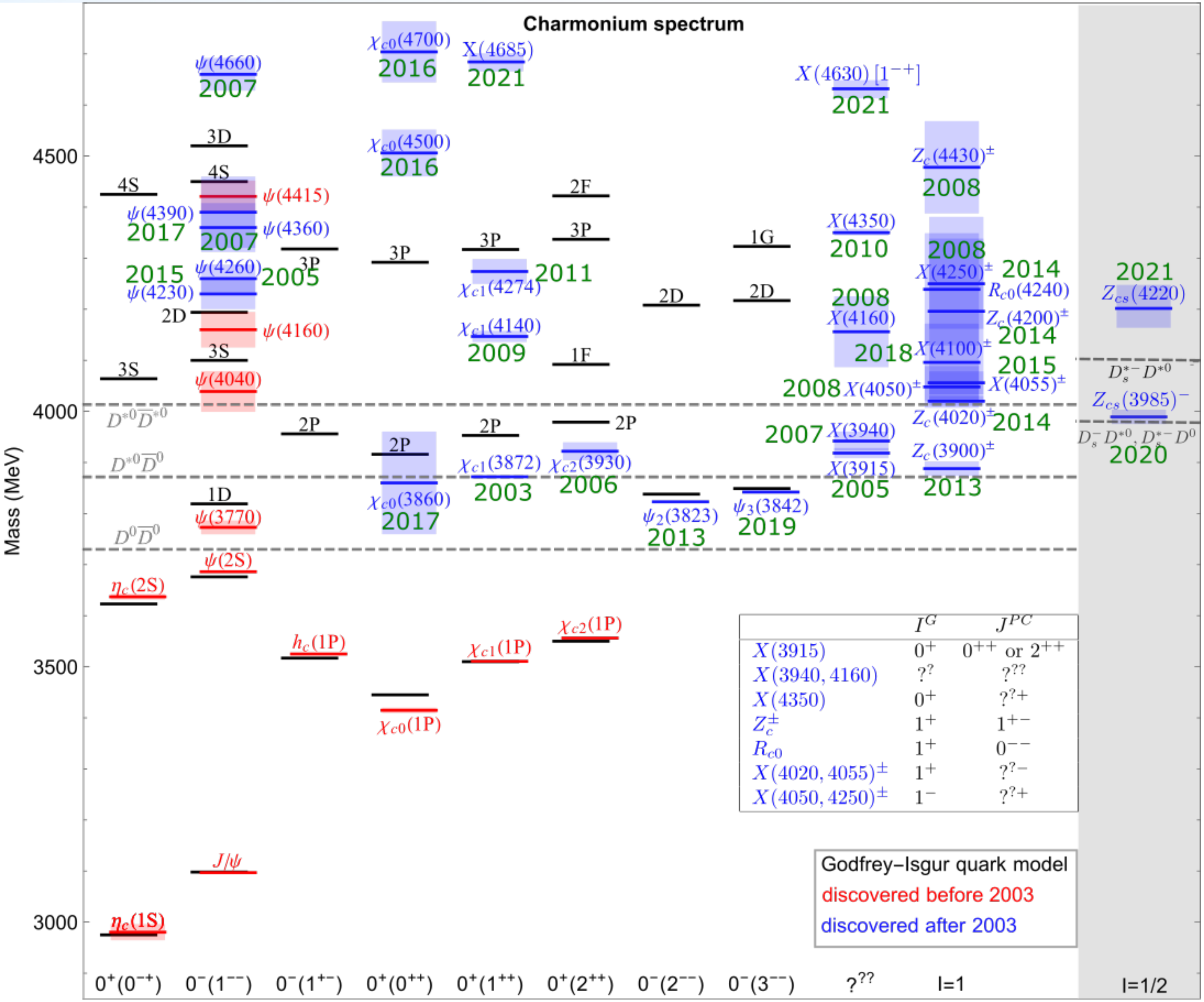
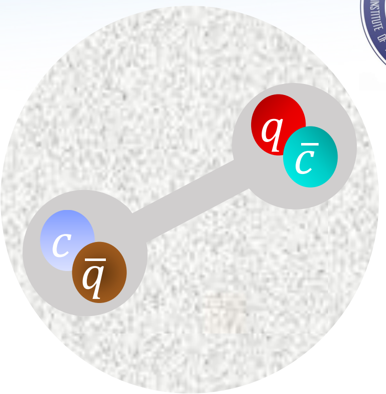
Charmonium-like structures



Hadronic molecules



- Masses of excited hadrons:
  - ➢ Radial excitations?
  - ➢ Excitation of light quark-antiquark pairs?
  - ➢ Hadron-hadron pairs? In the form of hadronic molecules
    - Indication from large N_c analysis: $\frac{V_{qq[\bar{3}]}}{V_{q\bar{q}[1]}} = \frac{1}{N_c - 1}$ Lucha et al., PPNP 120 (2021) 103867
    - Indication from functional method (DS and BS equations) Eichmann et al., 2008.10240
      See also the talk by Eichmann
    - Implication of confinement (large-size systems in favor of color-singlet clusters)?
    - Experimental evidence is accumulating

- Hadronic molecules: dominant component is a composite system of 2 or more hadrons; extended object
  - ➢ Compositeness: well-defined for S-wave loosely bound state; can be expressed in terms of low-energy observables

    S. Weinberg (1965); V. Baru et al. (2004); T. Hyodo et al. (2012); F. Aceti, E. Oset (2012); Z.-H. Guo, J. Oller (2016); I. Matuschek et al. (2021); Y. Li et al. (2022); J. Song et al. (2022); M. Albaladejo, J. Nieves (2022) …
    Talk by V. Baru

Hadronic molecules

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- Compositeness for S-wave shallow bound state as derived in Weinberg's paper, X_W, expressed in terms of scattering length and effective range

$$a = -\frac{2X_W}{1+X_W}R + O(m_\pi^{-1}), \quad r = -\frac{1-X_W}{X_W}R + O(m_\pi^{-1}) \qquad R \equiv \frac{1}{\sqrt{2\mu|E_B|}}$$

Binding energy

- Applied to the deuteron case

($E_B = -2.22$ MeV, $R = 4.31$ fm, $a = -5.42$ fm, $r = 1.77$ fm), $X_W = 1.68 > 1$

Inconsistency already pointed out in I. Matuschek et al., EPJA57, 101 (2021); see V. Baru's talk

- Assumptions used in the derivations
  - Neglecting the non-pole term from the Low equation
  - Approximating the form factor by a constant

$$T_{p,k} = V_{p,k} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q} \qquad \text{w/ } h_k \equiv k^2/(2\mu)$$

- New expression dropping the 2nd assumption

$$\boxed{X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE \frac{\delta_B(E)}{E - E_B}\right) \in [0,1]} \qquad \delta_B \approx \text{phase shift}$$

- Separable ansatz ⇒ closed form of the form factor

$$g^2(p) = \frac{8\pi^2}{\mu^2 R} \times \begin{cases} X_W + \mathcal{O}(p^4) & \text{for } a \in [-R, 0]\ \&\ r \le 0 \quad \text{constant} \\ \frac{a^2}{R^2}\frac{1}{1+(a+R)^2 p^2} + \mathcal{O}(p^4) & \text{for } a < -R\ \&\ r > 0 \end{cases}$$

Other talks on hadronic molecules

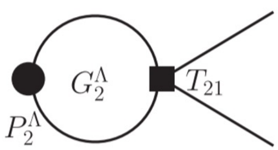
Speakers	Topics
R. Molina	Exotic flavor states in the hidden-gauge formalism (plenary)
M. Albaladejo	Multiplets of $Z_{cs}(3985)$ and $X(3960)$ states
E. Oset	D^*K^* molecular states
V. Baru	Compositeness from line shapes, T_{cc}, $X(3872)$
L. Roca	$\bar{B}^{(*)}\bar{K}^{(*)}$ molecular states
M. Bayar	$X(3960)$
N. Ikeno	$\Omega(2012)$
M.-Z. Liu	P_c pentaquarks
W.-H. Liang	T_{cc}
Related lattice talks:	
S. Prelovsek	T_{cc}
D. Wilson	$D\pi$, DK

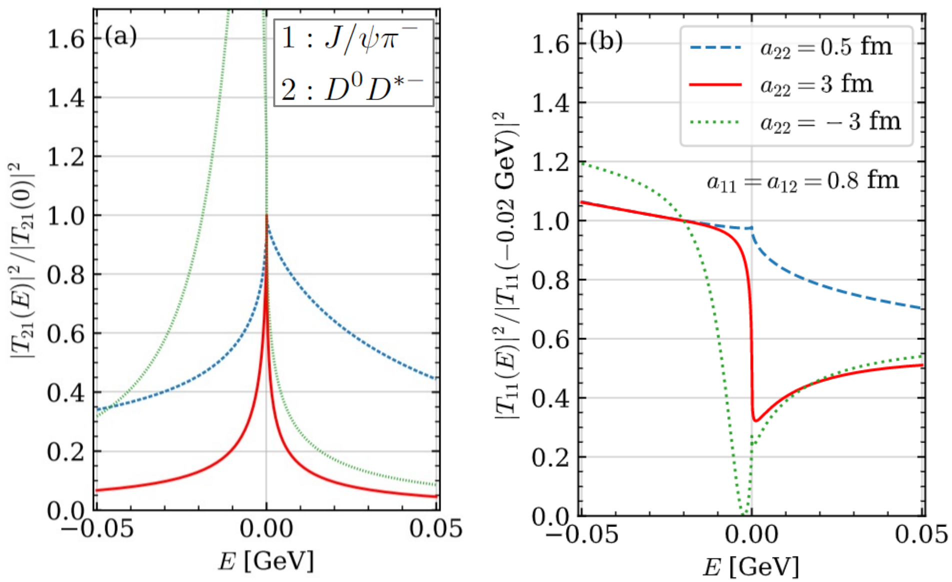
… …

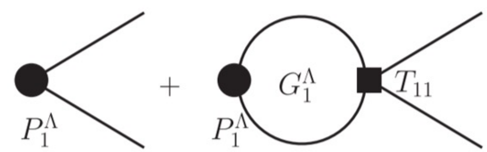
(Near-)threshold structures

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

- (Near-)threshold structures (S-wave)
  - NREFT at LO: nontrivial (near-)threshold structures for attractive S-wave interaction
  - Either threshold cusp or below-threshold peak
  - Peak more pronounced for heavier hadrons and stronger interaction
    - ✓ That's why many (near-)threshold structures were observed in hidden-charm and hidden-bottom spectra
  - Structures are process dependent







Poles in complex momentum plane:

$(-0.08 - i0.37)$GeV

$(-0.08 - i0.04)$GeV

$(-0.08 - i0.09)$GeV

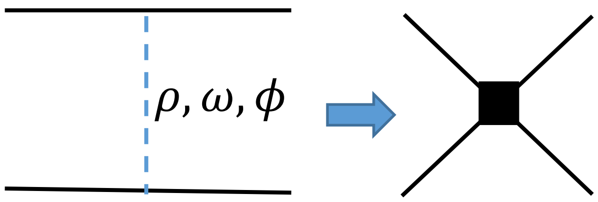
Distinct line shapes of amplitudes in the same coupled channels with the same poles

Survey of hidden-charm hadronic molecules

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

- Approximations:
  - Constant contact terms (V) saturated by light-vector-meson exchange, similar to the VMD in the resonance saturation of the low-energy constants in CHPT

    G. Ecker, J. Gasser, A. Pich, E. de Rafael, NPB321(1989)311
  - Single channels
  - Neglecting mixing with normal charmonia
- Resummation:



$$T = \frac{V}{1 - VG}$$

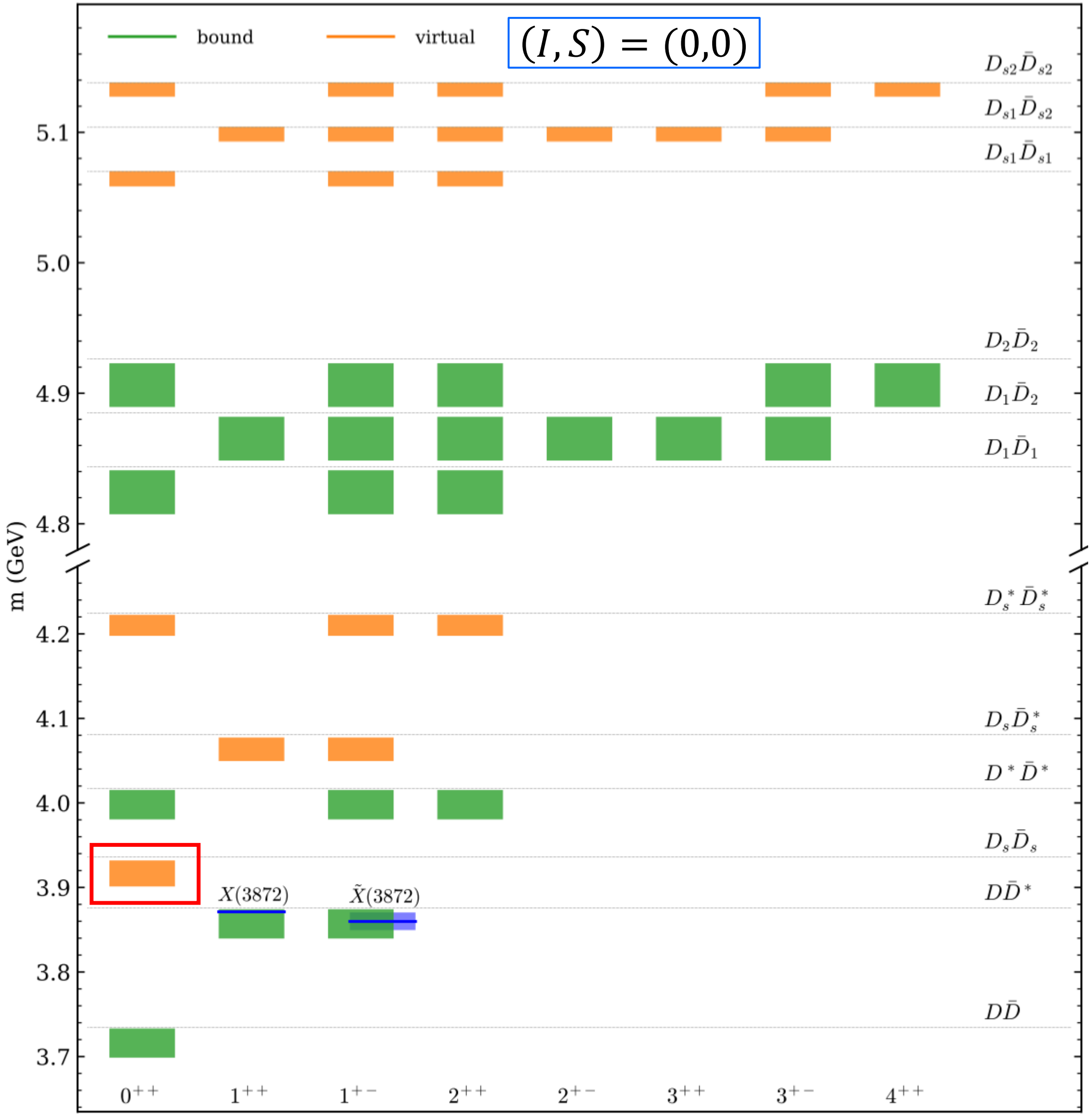
G: two-point scalar loop integral regularized using dim.reg. with a subtraction constant matched to a Gaussian regularized G at threshold

$$G(E) = \frac{1}{16\pi^2}\left\{a(\mu) + \log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\log\frac{m_2^2}{m_1^2} + \frac{k}{E}\log\frac{(2kE+s)^2 - m_1^2 + m_2^2}{(2kE-s)^2 - m_1^2 + m_2^2}\right\}$$

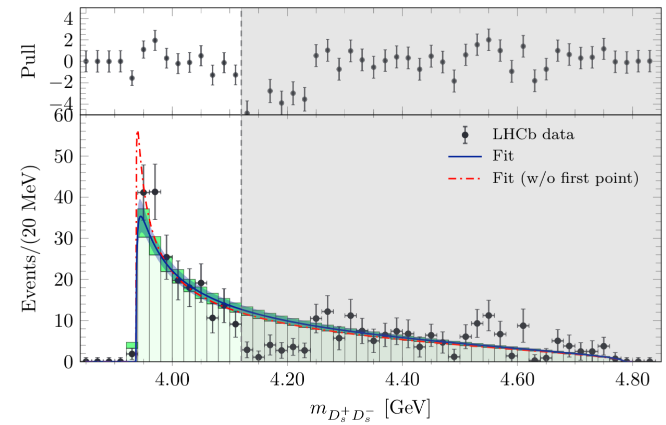
$$G(E) = \int \frac{l^2 dl}{4\pi^2}\frac{\omega_1+\omega_2}{\omega_1\omega_2}\frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1+\omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

- Hadronic molecules appear as bound or virtual state poles of the T matrix; more than 200 hidden-charm hadronic molecules were predicted

X(3872) and related states



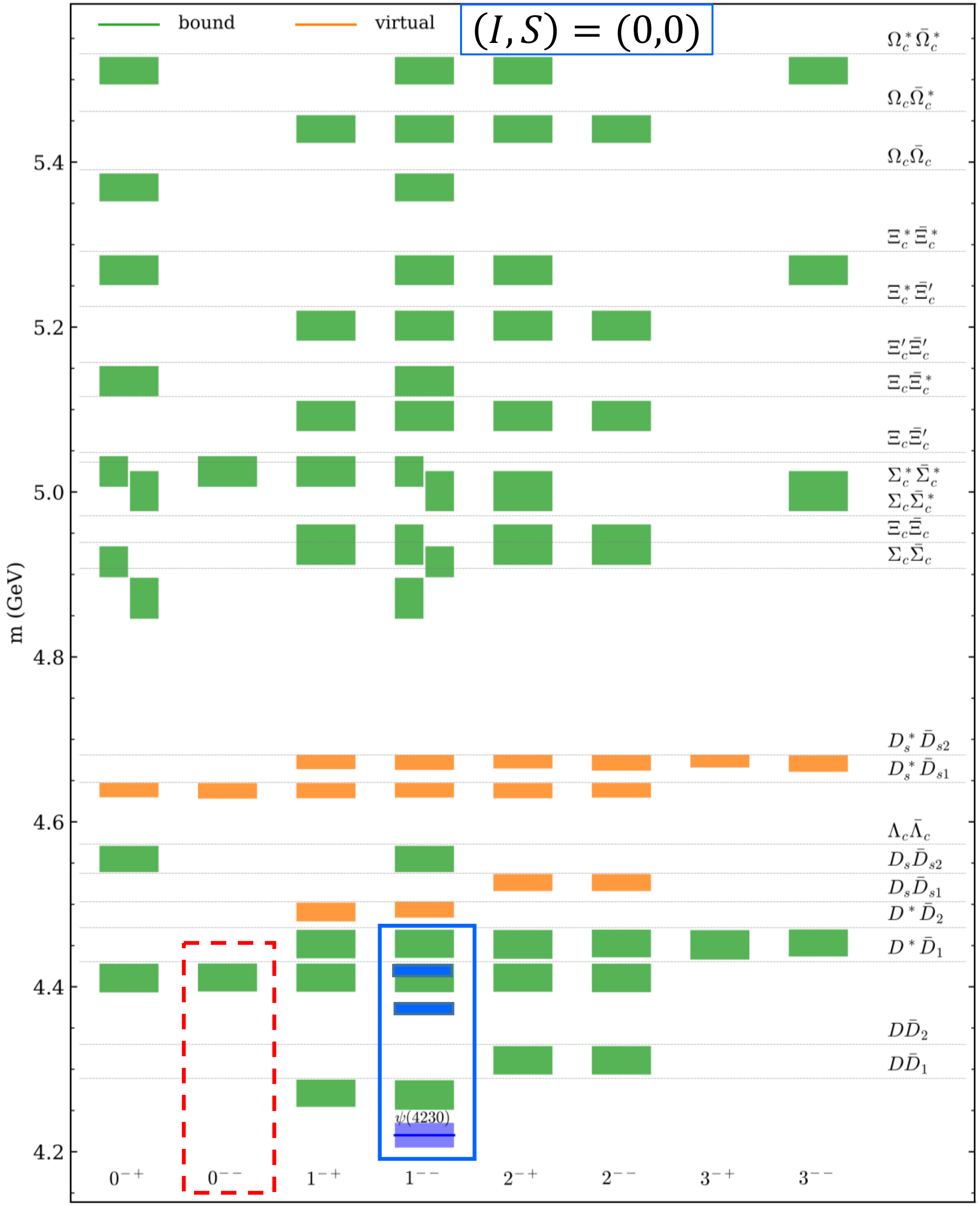
- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ $\tilde{X}(3872)$ COMPASS, PLB783(2018)334
- ✓ $\bar{D}D$ bound state predicted with lattice Prelovsek et al., JHEP2106,035
- ✓ $X(3960)$



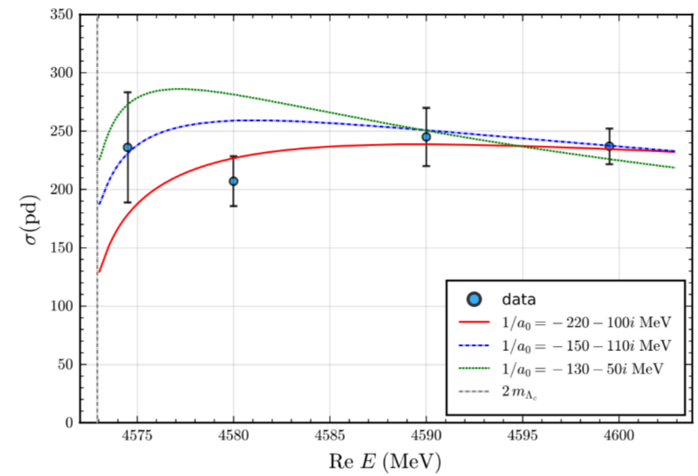
Data from
LHCb seminar by E. Spadaro Norella & Chen Chen, July 05, 2022
Fit in
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, FKG, J. Nieves, arXiv:2207.08563
See the talk by Miguel Albaladejo

Virtual state may become bound with mixing from the $D^*\bar{D}^*$ state

Isoscalar vectors and related states



- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ $\psi(4360), \psi(4415)$: $D^*\bar{D}_1, D^*\bar{D}_2$?
- ✓ Evidence for 1^{--} $\Lambda_c\bar{\Lambda}_c$ bound state in BESIII data
  - Sommerfeld factor
  - Near-threshold pole
  - Different from $Y(4630/4660)$



Data taken from BESIII, PRL120(2018)132001; Fitted with a virtual state pole in Q.-F. Cao et al., PRD100(2019)054040

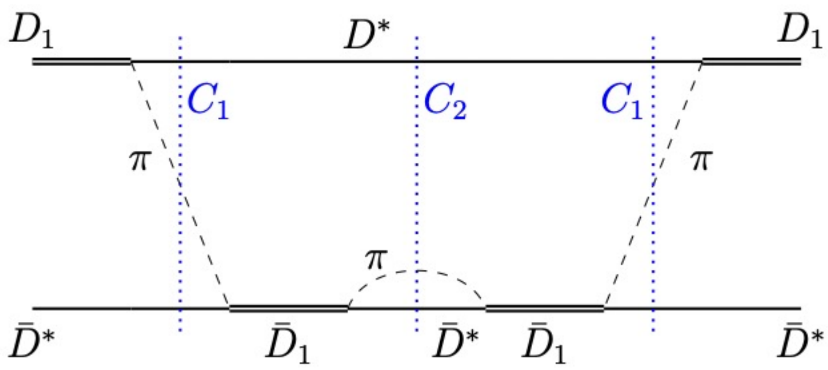
- ✓ Many 1^{--} states in [4.8, 5.6] GeV

Closer look at the spin partners of $\psi(4230)$

T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

- Prediction of an exotic 0^{--} spin partner $\psi_0(4360)$ $[D^*\bar{D}_1]$ of $\psi(4230), \psi(4360)$, $\psi(4415)$ as $D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_2$ hadronic molecules
- Robust against the inclusion of coupled channels and three-body effects

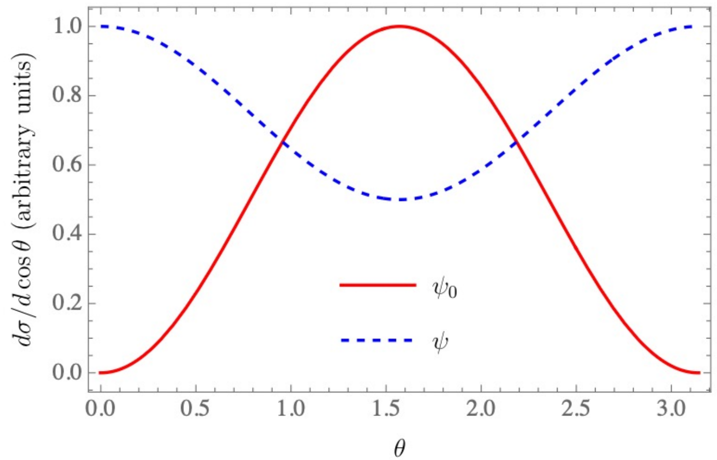
Molecule	Components	J^{PC}	Threshold	E_B
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	1^{--}	4287	67 ± 15
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	1^{--}	4429	62 ± 14
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2 - \bar{D}^*D_2)$	1^{--}	4472	49 ± 4
ψ_0	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 + \bar{D}^*D_1)$	0^{--}	4429	63 ± 18



- May be searched for using $e^+e^- \to \psi_0\eta, \psi_0 \to J/\psi\eta, D\bar{D}^*, D^*\bar{D}^*\pi, \ldots$

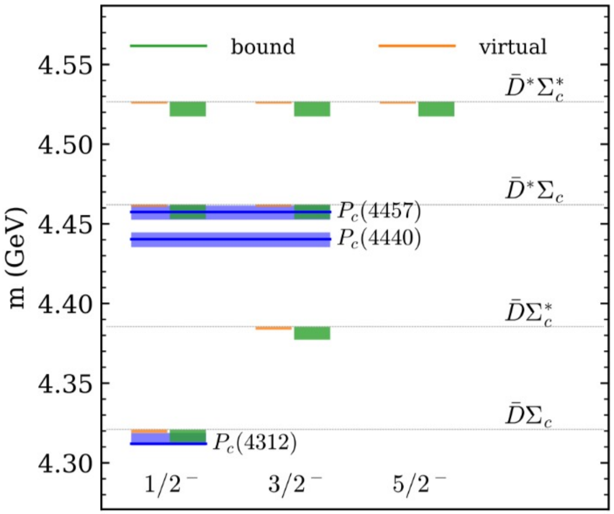
$$M = (4366 \pm 18)\ \text{MeV},$$

$$\Gamma < 10\ \text{MeV}$$

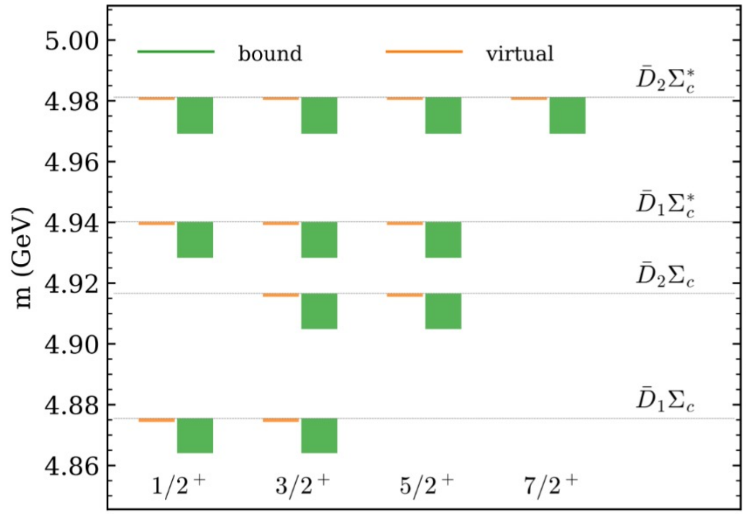


Hidden-charm pentaquarks

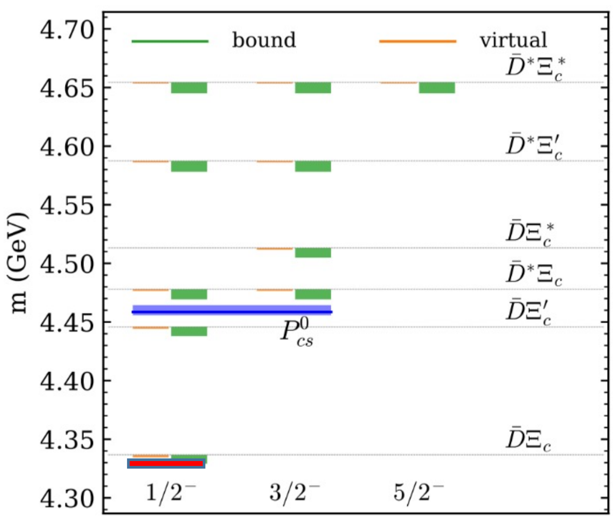
$(I,S) = (1/2,0)$



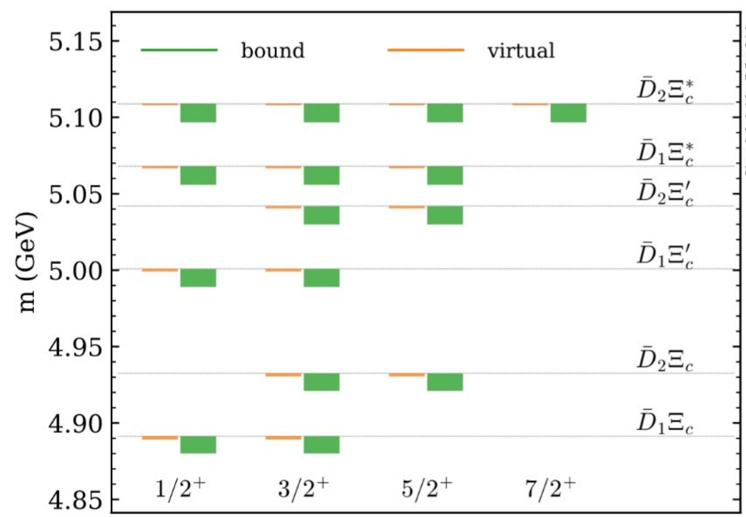
$(I,S) = (1/2,0)$



$(I,S) = (0,1)$

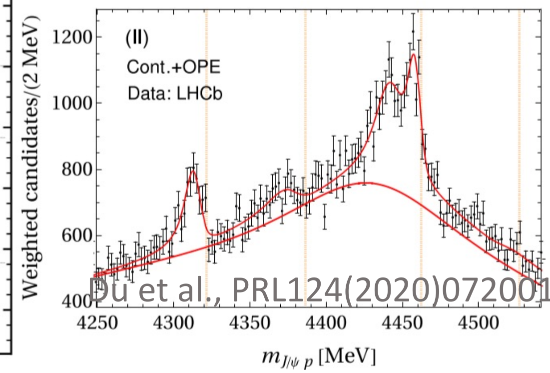


$(I,S) = (0,1)$



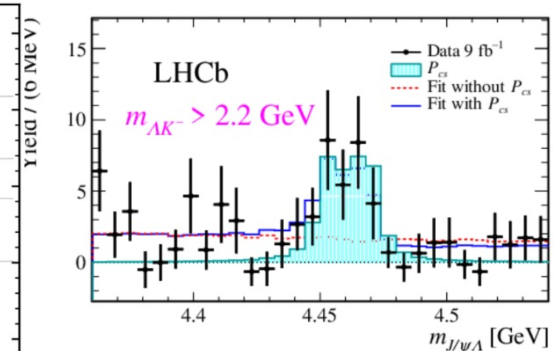
$e^+e^- \to J/\psi p\bar{p}, \Lambda_c\bar{D}^{(*)}p, J/\psi\Lambda\bar{\Lambda}, \Sigma_c^{(*)}\bar{D}^{(*)}p, \ldots$

- ✓ The LHCb P_c states as $\bar{D}^{(*)}\Sigma_c$ molecules
- ✓ $\bar{D}\Sigma_c^*$ molecule: hint in the LHCb data



Du et al., PRL124(2020)072001

- ✓ $P_{cs}(4459)$: 2 $\bar{D}^*\Xi_c$ states



LHCb, 2012.10380

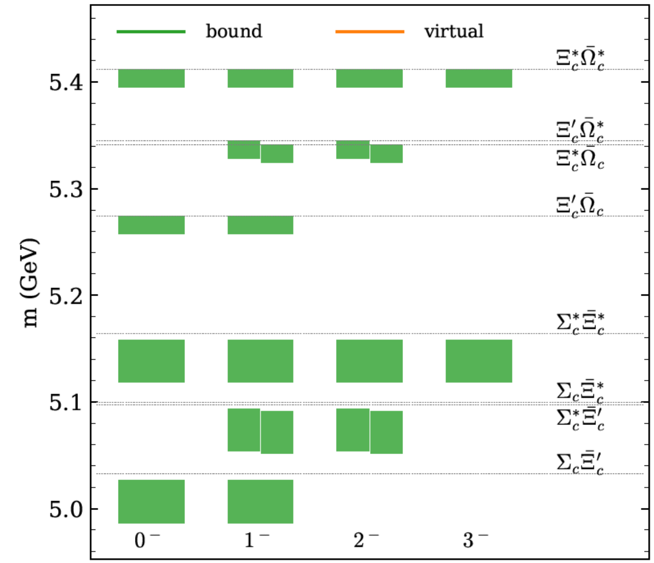
- ✓ $P_{cs}(4338)$: $\bar{D}\Xi_c$

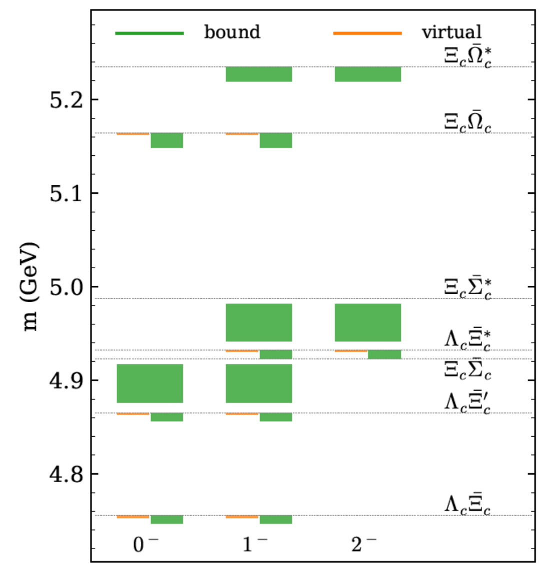
LHCb (2022); talk by M. Mikhasenko

More states with exotic quantum numbers

$(I, S) = (1,0)$

$(I, S) = (1/2,1)$



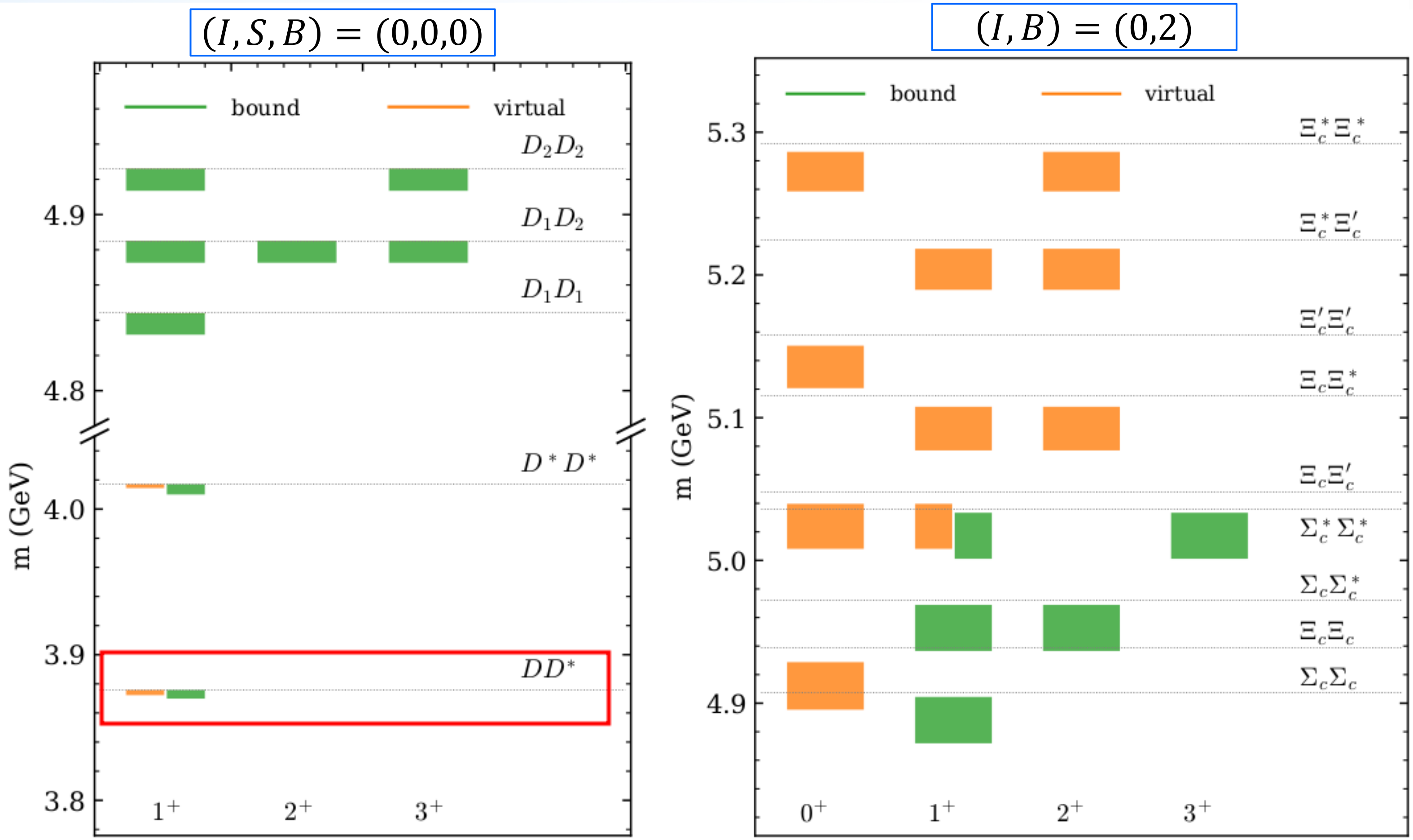


✓ Many baryon-antibaryon molecular states above 4.7 GeV, possible decay channels:

$J/\psi\pi, J/\psi K, \ldots$

Double-charm

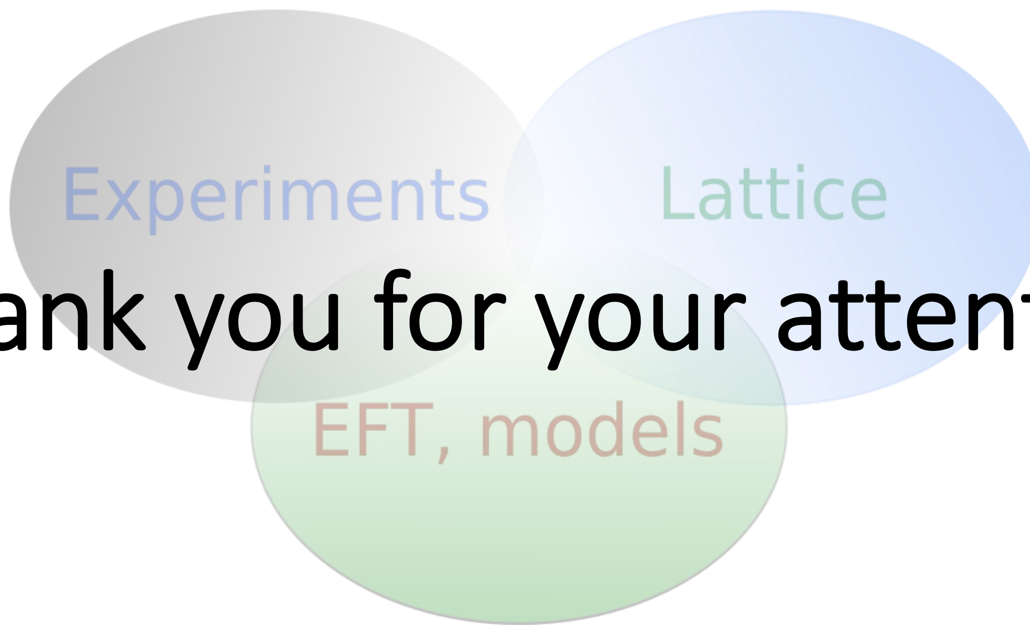
X.-K. Dong, FKG, B.-S. Zou, CTP73(2021)125201



- ✓ There is an isoscalar DD^* molecular state
- ✓ It has a spin partner 1^+ D^*D^* state
- ✓ Many (>100) other similar double-charm molecular states in other sectors

Conclusion and outlook

- Many hadronic molecular candidates in the heavy hadron spectrum
  - (Near-)threshold structures expected for hadron pairs with S-wave attraction
  - Many hidden-charm molecules will be detected at LHCb, BEPCII-U, Belle-II, PANDA, EIC etc. Some example processes in e^+e^- :
    - Meson-meson pairs via emission of light meson: $e^+e^- \to M_1 M_2(\phi, \omega, \rho, \pi, \eta, K^{(*)})$
    - Many baryon-antibaryon pairs above 4.8 GeV: $e^+e^- \to B_1 \bar{B}_2$



Thank you for your attention!

Compositeness

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- The constant form factor assumption can be replaced by a more general separable ansatz

$$T_{p,k} = t_k\, g(p)\, g^*(k)$$

Twice-subtracted dispersion relation ⇒

$$t^{-1}(W) = (W - E_B) + (W - E_B)^2 \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|g(q)|^2}{(h_q - E_B)^2 (h_q - W)}$$

Then, we get

$$t(W) = \frac{1}{1 - F(W)} \frac{1}{W - E_B}, \quad F(W) \equiv (W - E_B) \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|g(q)|^2}{(h_q - E_B)^2 (W - h_q)}$$

- Compositeness emerges

$$F(\infty) = \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|\langle q|\hat{V}|B\rangle|^2}{(h_q - E_B)^2} = \int_0^\infty \frac{q^2 dq}{(2\pi)^3} |\langle q|B\rangle|^2 = X$$

- Introducing

$$F_1(W) \equiv \frac{\ln[1 - F(W)]}{W - E_B}, \qquad \mathrm{Im}\, F_1(E + i\varepsilon) = -\frac{\delta_B(E)}{E - E_B} \theta(E)$$

here δ_B is the phase of the T-matrix with the nonpole term neglected (convention: $\delta_B(0) = 0$)

$$\delta_B(E = h_p) \equiv \arg T_{p,p} = -\arg\left(1 - F(E + i\varepsilon)\right) \qquad \delta_B \in [-\pi, 0]$$

$F(0) \leq 0$, $\mathrm{Im}\, F(E + i\varepsilon) \leq 0$ for $E \geq 0$

Compositeness

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- From the dispersion relation for $F_1(W)$, we obtain a solution:

$$F(W) = 1 - \exp\left(\frac{W - E_B}{\pi}\int_0^\infty dE \frac{-\delta_B(E)}{(E-W)(E-E_B)}\right)$$

and an expression for the compositeness

$$\boxed{X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE \frac{\delta_B(E)}{E - E_B}\right) \in [0,1]}$$

- Using $\mathrm{Im}\, F\left(h_p + i\epsilon\right) = -\frac{\pi p \mu}{(2\pi)^3}\frac{|g(p)|^2}{h_p - E_B}$, we get

$$|g(p)|^2 = -\frac{(2\pi)^3}{\pi\, p\mu}(h_p - E_B)\, \sin\delta_B(E) \exp\left[\frac{h_p - E_B}{\pi} \fint_0^\infty dE \frac{-\delta_B(E)}{(E-h_p)(E-E_B)}\right]$$

- Consider ERE $p\cot\delta_B \approx -\frac{8\pi^2}{\mu}\mathrm{Re}T^{-1}(h_p) = \frac{1}{a} + \frac{r}{2}p^2 + \mathcal{O}(p^4)$, we finally get

$$g^2(p) = \frac{8\pi^2}{\mu^2 R} \times \begin{cases} X_W + \mathcal{O}(p^4) & \text{for } a \in [-R, 0]\ \&\ r \le 0 \\ \frac{a^2}{R^2}\frac{1}{1+(a+R)^2p^2} + \mathcal{O}(p^4) & \text{for } a < -R\ \&\ r > 0 \end{cases}$$

constant

contains $\mathcal{O}(p^2)$ terms, thus not self-consistent if using a constant g^2 but still work up to $\mathcal{O}(p^2)$ in ERE. Weinberg's relations do not hold in this case

Compositeness

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- Poles of the T-matrix with ERE up to $\mathcal{O}(p^2)$: $\frac{1}{a}+\frac{r}{2}p^2 - i\,p = \frac{r}{2}(p-p_+)(p-p_-)$

$$p_- = \frac{i}{R}, \qquad p_+ = -\frac{i}{R+a}$$

with $R = \frac{1}{\sqrt{2\mu|E_B|}}$; r is expressed as $r = \frac{2R}{a}(R+a)$

- For $a \in [-R, 0]$, then $r < 0$, one bound state and one virtual state pole

$$g^2(p) = \frac{8\pi^2}{\mu^2 R} X_W + \mathcal{O}(p^4), \qquad X = X_W \simeq \sqrt{\frac{1}{1+2r/a}}$$

- For $a < -R$, then $r > 0$, two bound state poles (the remote one $\sim i/\beta$ is unphysical)

$$g^2(p) = \frac{8\pi^2}{\mu^2 R}\frac{a^2}{R^2}\frac{1}{1+(a+R)^2p^2} + \mathcal{O}(p^4), \qquad X \simeq 1 - e^{-\infty} = 1$$

For the deuteron, $R = 4.31$ fm, $a = -5.42$ fm, $a + R \sim \beta^{-1} \sim m_\pi^{-1}$

$$\boxed{\begin{aligned} &X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\delta_B(E)}{E-E_B}\right) \in [0,1] \\ &p\cot\delta_B = \frac{1}{a} + \frac{r}{2}p^2 \Rightarrow \delta_B(\infty) = 0 \text{ for } r<0, \text{ and } \delta_B(\infty) = -\pi \text{ for } r>0 \end{aligned}}$$

- For extension of the Weinberg's relations to virtual state and near-threshold resonance, see I. Matuschek et al., EPJA57, 101 (2021); Christoph's talk