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Coupled Channel Partial Wave Analysis for Light Meson Spectroscopy

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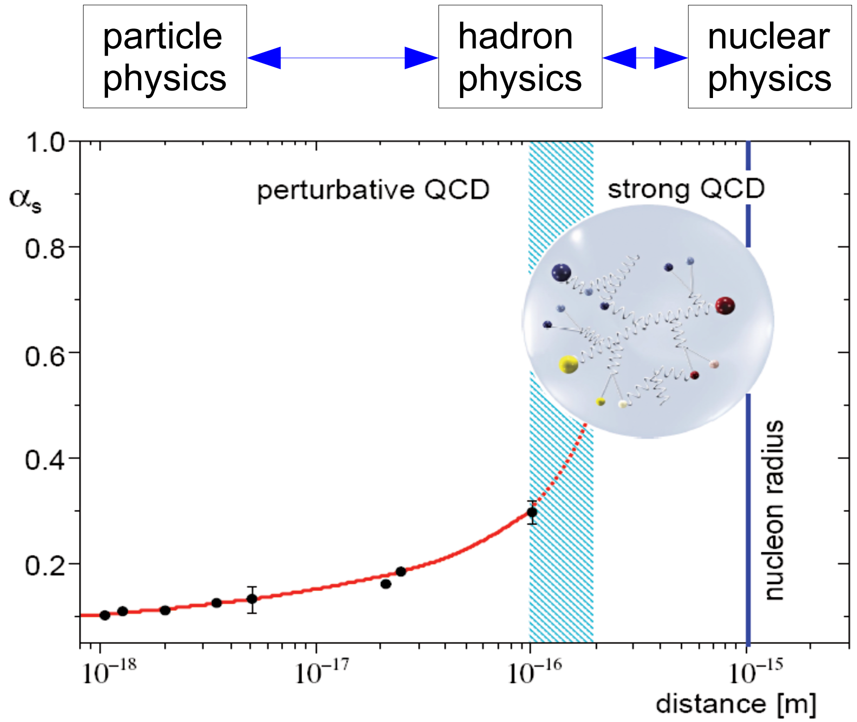
Introduction

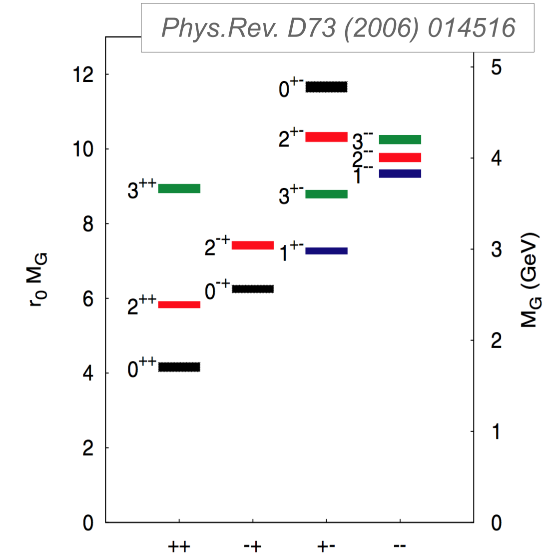
- Light mesons are bound states consisting of u-, d- and s-quarks
- Cover the non-perturbative QCD regime
- Description very challenging
  - lattice QCD
  - phenomenological models
- Observation and measurements of the resonance properties very challenging
  - many overlapping resonances with same quantum numbers
  - resonances decay in different channels
  - distinction between conventional $q\bar{q}$-mesons and exotics difficult

energy dependence of α_s



Glueballs and Hybrids

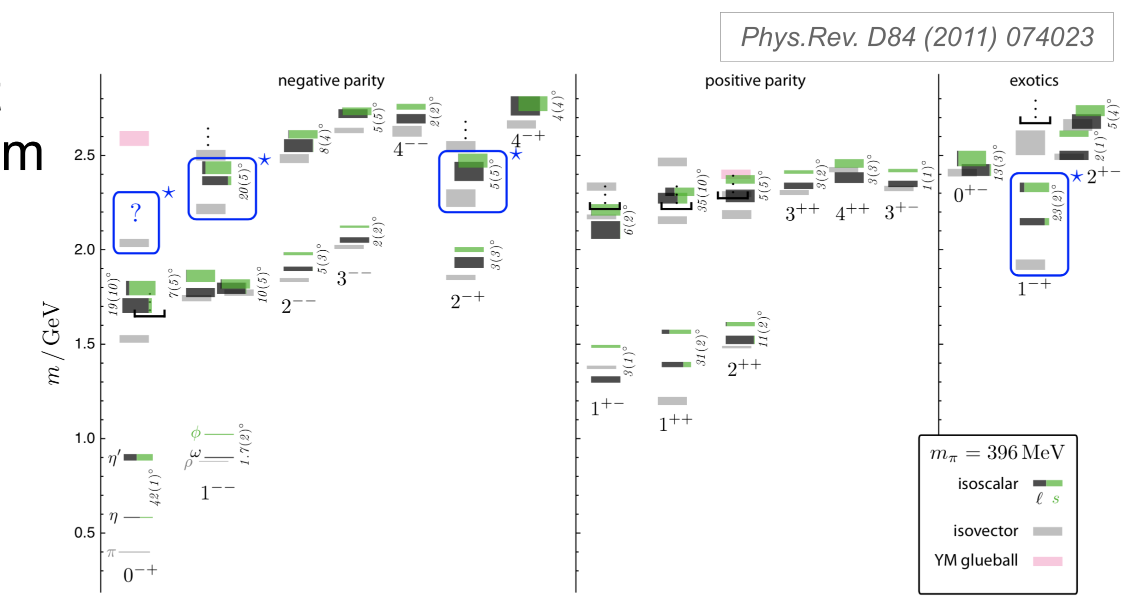
- A doubtless evidence for exotics are the observation of resonances with spin-exotic quantum numbers which are forbidden for q$\bar{q}$-mesons



- LQCD: lightest glueballs with spin-exotic quantum numbers J^{PC}= 0^{+-}, 1^{-+}, 2^{+-} above 4 GeV/c^2

- Glueballs in the light meson mass range only with non exotic quantum numbers J^{PC}= 0^{++}, 0^{-+}, 2^{++} predicted

- Lightest hybrid state expected just below 2 GeV/c^2 with exotic quantum numbers $I^G(J^{PC})$ = $1^-(1^{-+})$



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Exotic π_1 below 2 GeV/c^2

- 2 π_1 candidates below 2 GeV/c^2 listed in the PDG
- $\pi_1(1400)$
  - only observed in the decay channel $\pi\,\eta$ in πp-scattering and $\bar{p}$p- and $\bar{p}$n-annihilation
- $\pi_1(1600)$
  - observed in the decay channels $\rho\pi$, $\eta'\pi$, $f_1(1285)\,\pi$, $b_1(1235)\,\pi$ in πp-scattering
  - but not observed in $\pi\eta$
- How are these observations compatible with the LQCD calculations?
  - only one π_1 predicted slightly below 2 GeV/c^2
  - is one of the candidates not a hybrid state or even not a resonance?

Coupled Channel Analysis

- Access to the inner structure of a resonance
  - determination of resonance properties like quantum numbers, pole positions and coupling strengths
  - characteristics of the production and the decay pattern
  - extractions of these properties are often not sufficient for analyses of only one single channel

- Advantages compared to single channel fits
  - common and unique description of the dynamics
  - better description of threshold effects
  - better fulfillment of the conservation of unitarity
  - more constraints due to common amplitudes

Dynamical Functions

- Breit-Wigner functions widely used
  - good approximation for isolated resonances
  - violate the unitarity
  - resonance parameters are not unique and depend on the production and decay process
- More sophisticated descriptions needed for
  - resonances decaying into multiple channels
  - several resonances with the same quantum numbers appearing in the same channel
  - resonances located at thresholds → distortion of the line shape

Approaches with an adequate consideration
of unitarity and analyticity needed
(K-matrix, N/D-method, Two-potential decomposition)

K-Matrix with P-Vector Approach

- Two-body scattering process can be described by the S-matrix: $S = I + 2i\sqrt{\rho}\,T\sqrt{\rho}$
- T-matrix can be expressed by K-matrix: $T = (I - i\,K\,C(s))^{-1}\,K$

C(s): Chew-Mandelstam function guaranties analyticity
$\rho(s)$= -Im(C(s)

bare coupling to channel i an j of resonance α

- Elements of the K-matrix: $K_{ij} = \sum_{\alpha} \frac{g_{\alpha_i} g_{\alpha_j}}{m_\alpha^2 - s} + \sum_k c_{kij} s^k$

K-matrix element for channel i (initial) and j (final)

bare mass of resonance α

energy dependent background term

- Generalization of the K-matrix formalism to the case of production of resonances in more complex reaction

Aitchison: Nucl Phys A189 (1972) 417

- Dynamical function for P-vector approach: $F = (I - i\,K\,C(s))^{-1}\,P$

bare coupling strength to the production

with: $P_i = \sum_{\alpha} \frac{\beta_\alpha g_{\alpha_i}}{m_\alpha^2 - s} + \sum_k c_{ki} s^k$

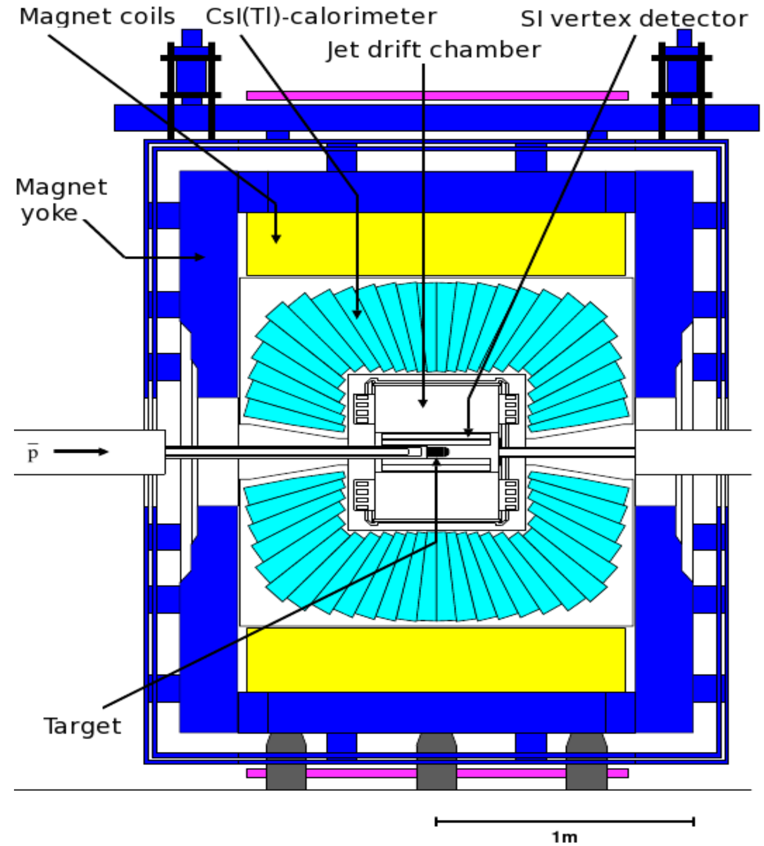
same pole structure as for K-matrix

PAWIAN

- **PA**rtial **W**ave **I**nteractive **AN**alysis software package
- Supports $\bar{p}p$- and e^+e^--annihilation, $\gamma\gamma$-fusion and πp- and $\pi\pi$- scattering
- Hypothesis and other settings defined via configuration files
  - spin formalisms: e.g. canonical, helicity, ...
  - dynamics: Breit-Wigner, K-matrix, ...
- Channels with arbitrary number of final state particles
- Event based maximum likelihood fit using MINUIT2
- Support for parallelization
- Analysis tools: extraction of pole positions, branching fractions, ...
- Event generator, histogramming, efficiency correction, ...

PWA with p̄p Data from Crystal Barrel at LEAR

- Fixed target experiment at CERN
- In operation between 1989 and 1996
- p̄p annihilation at rest and in flight
  - highest beam momentum 1.94 GeV/c
- Physics program
  - spectroscopy of light mesons and search for exotic states



Eur. Phys. J. C (2020) 80:453 *Crystal Barrel Collaboration*

Coupled channel analysis of $\bar{p}p \rightarrow \pi^0\pi^0\eta, \pi^0\eta\eta$ and $K^+K^-\pi^0$ at 900 MeV/c and of $\pi\pi$-scattering data

$\bar{p}p \rightarrow K^+ K^- \pi^0, \pi^0 \pi^0 \eta, \pi^0 \eta \eta$

Why $\bar{p}p$ data?

- Many a_0, a_2, f_0 and f_2 resonances appear in two or all three channels
  - constraints due to common production amplitudes
  - disentangling of the isospin components in the K^+K^- system possible
  - description of the dynamics via K-matrices (unitarity and analyticity)
- Exotic spin wave π_1 so far only seen in $\bar{p}p$ data at rest
  - also visible in $\bar{p}p$ in flight data

Why scattering data?

- Process only characterized by elasticity and phase motion
  - good and easy access to resonance properties
- Considered for I=0 S- and D-wave and I=1 P-wave
- Good constraints for f_0, f_2 and ρ resonance

$\bar{p}p \to K^+ K^- \pi^0,\ \pi^0 \pi^0 \eta,\ \pi^0 \eta\ \eta$

Best Fit Result achieved for

- K-matrix description for
  - f_0 with 5 poles and 5 channels
  - f_2 with 4 poles and 4 channels
  - ρ with 2 poles and 3 channels
  - a_0 and a_2 with 2 poles and 2 channels, each
  - $\pi_1^0 \to \pi^0\eta$ in $\pi^0\pi^0\eta$ with 1 pole and 2 channels

all pole positions and coupling strengths are free parameters

  - $(K\pi)_S$-wave: fixed parameterization from FOCUS-experiment *Phys. Lett. B653 (2007) 1-11*
- Breit-Wigner description for
  - $\Phi(1020) \to K^+ K^-$
  - $K^{*\pm}(892) \to K^\pm \pi^0$

$\bar{p}p \rightarrow K^+ K^- \pi^0,\ \pi^0 \pi^0 \eta,\ \pi^0 \eta \eta$

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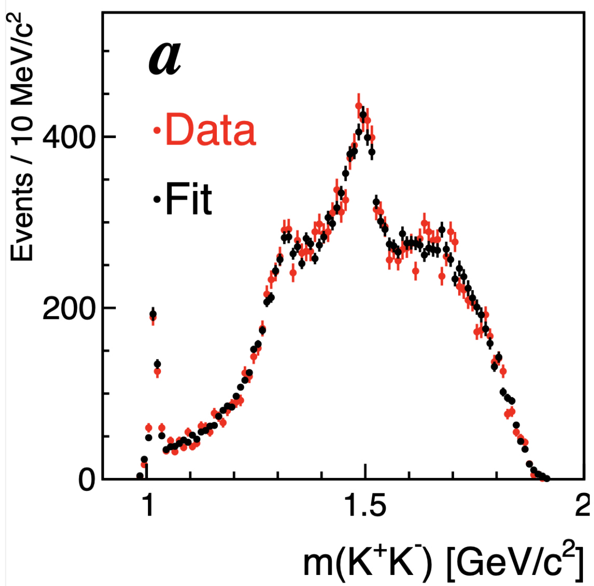
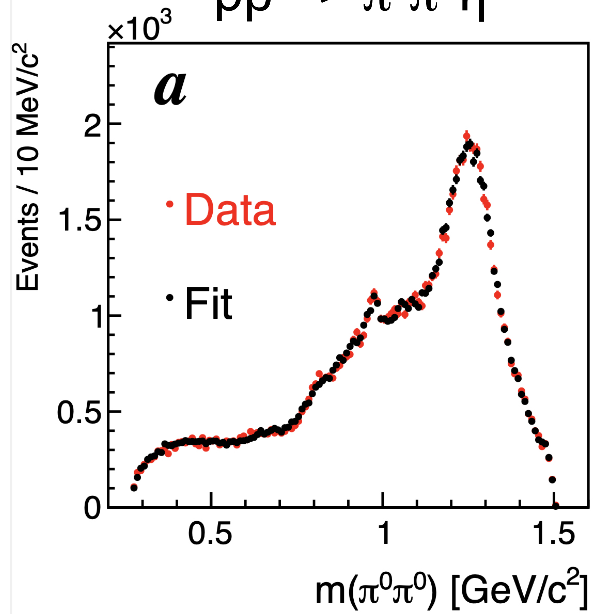
$\bar{p}p \rightarrow K^+K^-\pi^0$



channel	contribution %
K*(892) K	45.0 ±1.3 ±11.0
$\rho\ \pi^0$	17.2 ±1.0 ±4.0
$f_2\ \pi^0$	17.1 ±0.7 ±10.0
$f_0\ \pi^0$	7.4 ±0.3 ±4.1
$(K\pi)_S$ K	6.1 ±0.4 ±4.9
$a_2\ \pi^0$	6.4 ±0.2 ±2.9
$\phi\ \pi^0$	2.5 ±0.3 ±0.3
$a_0\ \pi^0$	6.1 ±0.2 ±2.8
Σ	107.8 ±1.9 ±12.5

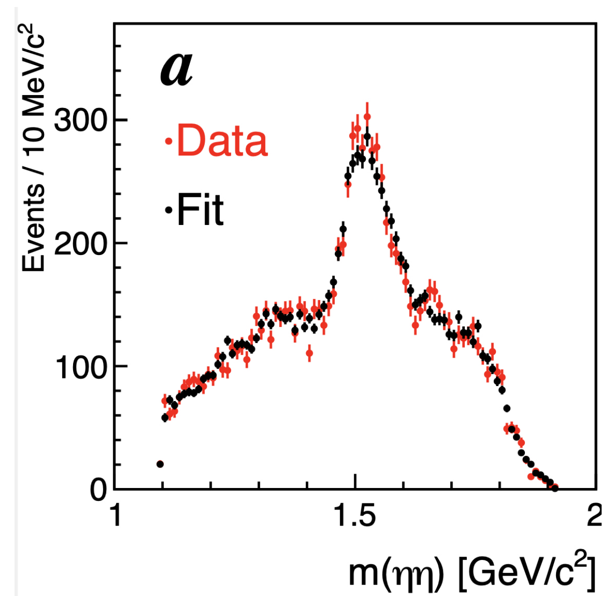
$\bar{p}p \rightarrow \pi^0\pi^0\eta$



channel	contribution %
$f_2\ \eta$	52.3 ±0.8 ±5.0
$a_2\ \pi^0$	33.0 ±0.6 ±2.9
$f_0\ \eta$	10.7 ±0.4 ±1.8
$a_0\ \pi^0$	22.4 ±0.4 ±1.0
$\pi_1\ \pi^0$	16.7± 0.5± 3.0
Σ	135.0 ±1.2 ±8.7

spin-exotic 1^{-+} contribution

$\bar{p}p \rightarrow \pi^0\eta\eta$



channel	contribution %
$f_0\ \pi^0$	23.7 ±1.2 ±2.3
$a_2\ \eta$	18.8 ±1.1 ±5.6
$a_0\ \eta$	28.6 ±1.1 ±7.5
$f_2\ \pi^0$	30.1 ±1.3 ±2.7
Σ	101.2 ±2.4 ±11.7

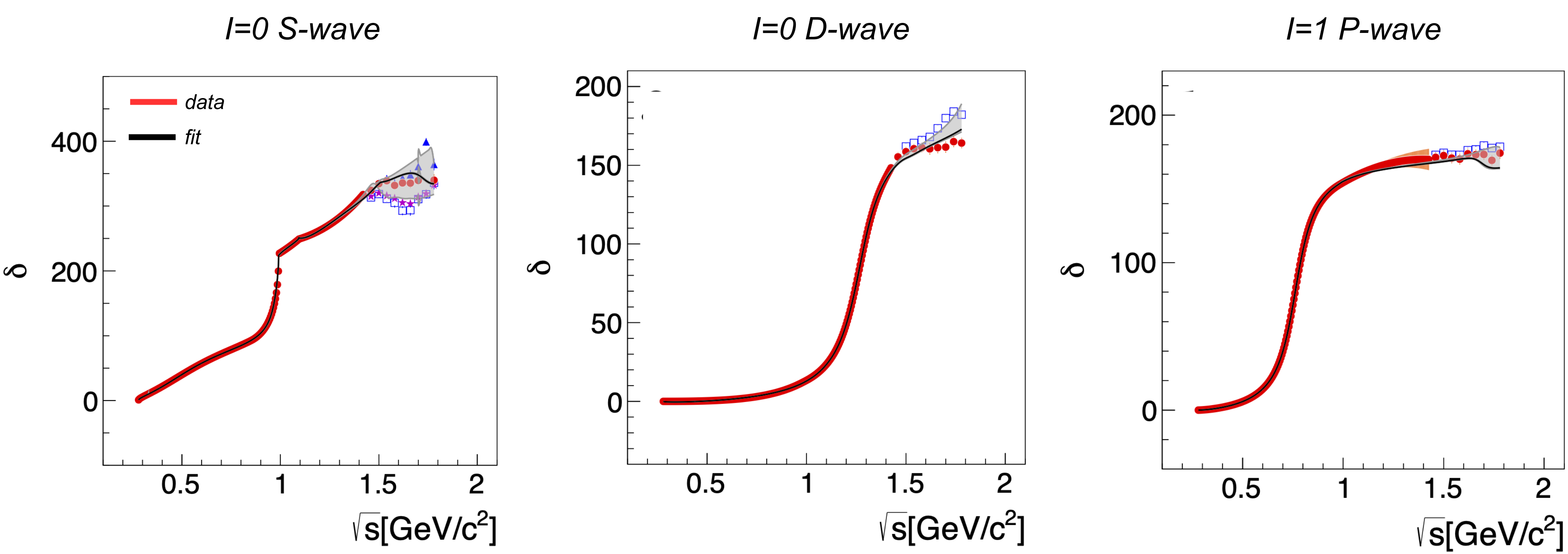
$\pi\pi$ Scattering Data

- All scattering data are well described
- $\pi\pi \to \pi\pi$ elasticity and $\pi\pi \to K\bar{K}$, $\eta\eta$, $\eta\eta'$ are not shown here

used data

Phys. Rev. D83(2011) 074004
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B100 (1975) 205-224
J. Phys G40 (2013) 043001
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B269 (1986) 485
Nouvo Cimento A80 (1984) 363

Phases for $\pi\pi \to \pi\pi$

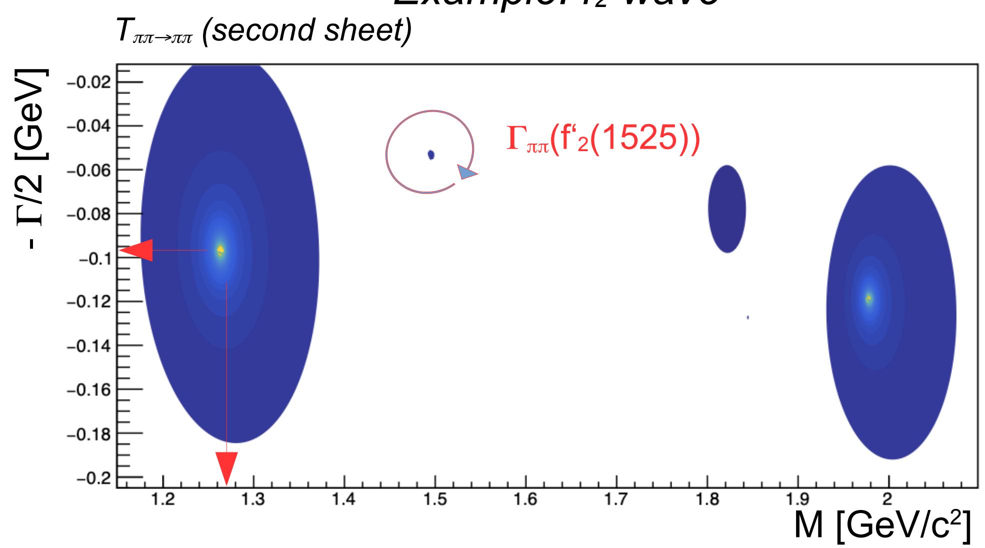


Extraction of Resonance Properties

- K-matrix contains all resonance parameters
- Masses and widths defined by the pole position in the complex energy plane of the T-matrix sheet closest to the physical sheet
- Related partial decay width can be extracted via the residues:

$$Res^{\alpha}_{k\to k} = \frac{1}{2\pi i} \oint_{C_{z\alpha}} \sqrt{\rho_k} \cdot T_{k\to k}(z) \cdot \sqrt{\rho_k}\, dz$$

Example: f_2-wave



More than 50 different resonance properties extracted on the relevant Riemann-sheets for f_0, f_2, a_0, a_2 and ρ resonances

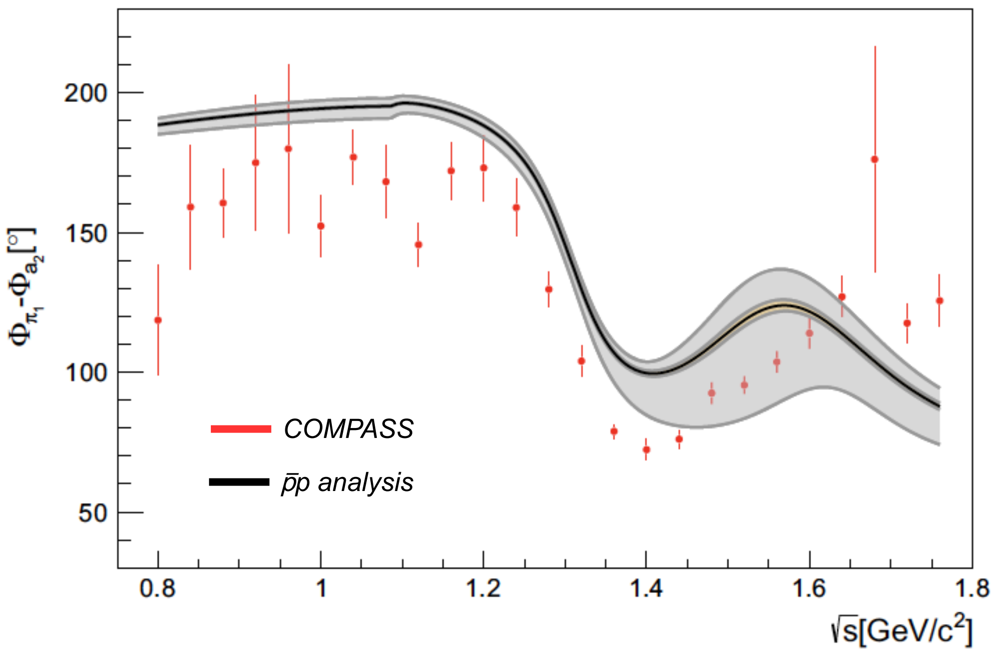
1^{-+} Wave in $\bar{p}p \rightarrow \pi^0 \pi^0 \eta$

- 1^{-+} wave seen in the decay $\pi^0\eta$
- K-matrix description with 1 pole and two channels $\pi\eta$ and $\pi\eta'$
  - no data for $\pi\eta'$ and only used for unitarity
- Phase difference between the π_1 and a_2 wave from $T_{\pi\eta\rightarrow\pi\eta}$ in good agreement with COMPASS measurement
- Obtained pole parameters consistent with $\pi_1(1400)$

Phys. Lett. B740 (2015) 303-311
Phys. Lett. B811 (2020) 135913 (erratum)

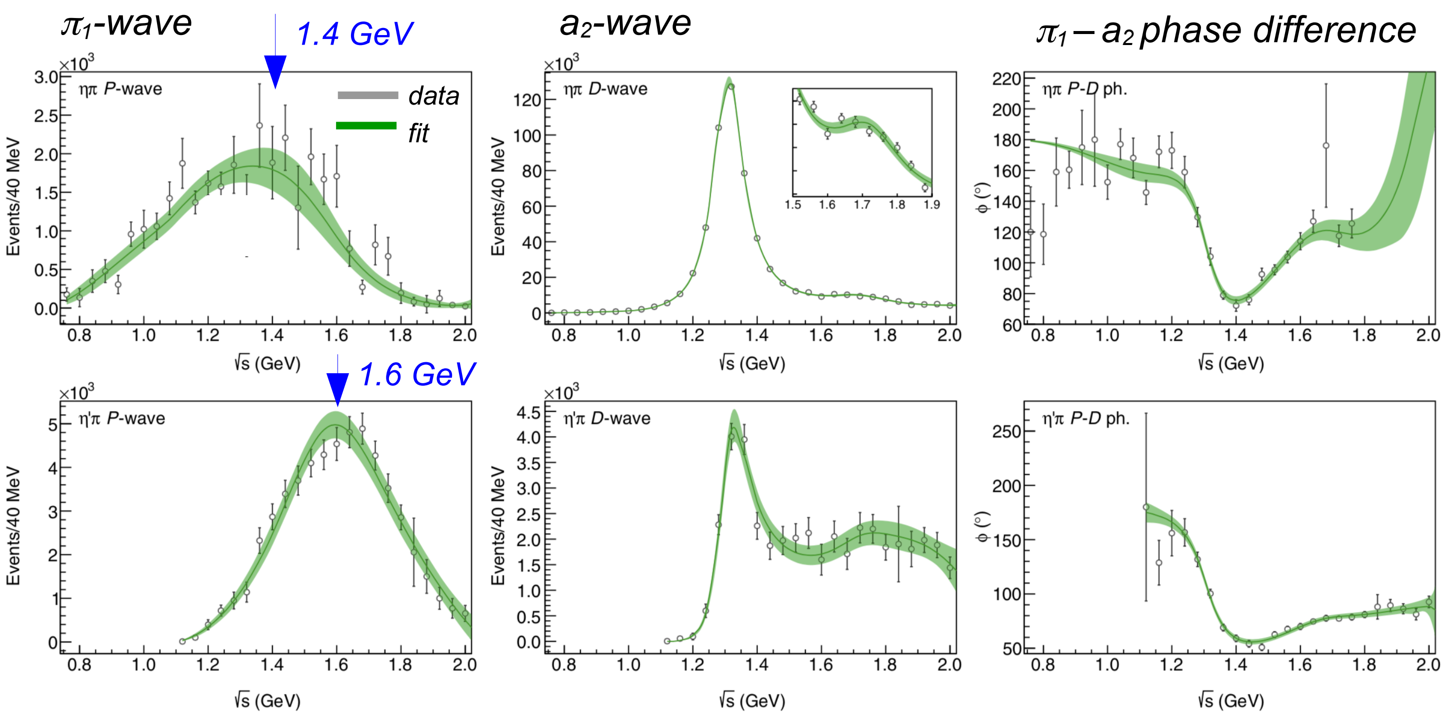
M = $(1404.7 \pm 3.5\ (\text{stat.})\ ^{+9.0}_{-17.3}\ (\text{sys.}))\ \text{MeV}/c^2$

Γ = $(628.3 \pm 27.1\ (\text{stat.})\ ^{+35.8}_{-138.2}\ (\text{sys.}))\ \text{MeV}$



JPAC Analysis of COMPASS Data

- Coupled channel analysis of the 1^{-+} and 2^{++} wave in $\pi^- p \rightarrow \pi^- \eta^{(\prime)} p$
- Enforcing analyticity and unitarity utilizing N/D method
- Mass shapes and phase shifts between 1^{-+} and 2^{++} are considered
- Peak at 1.4 GeV/c^2 in $\pi\eta$ and 1.6 GeV/c^2 in $\pi\eta'$ are described by one pole at (1564 ± 24 ± 86) – i(246 ± 27 ± 51) MeV
- Cannot be described by only one resonance with Breit-Wigner description

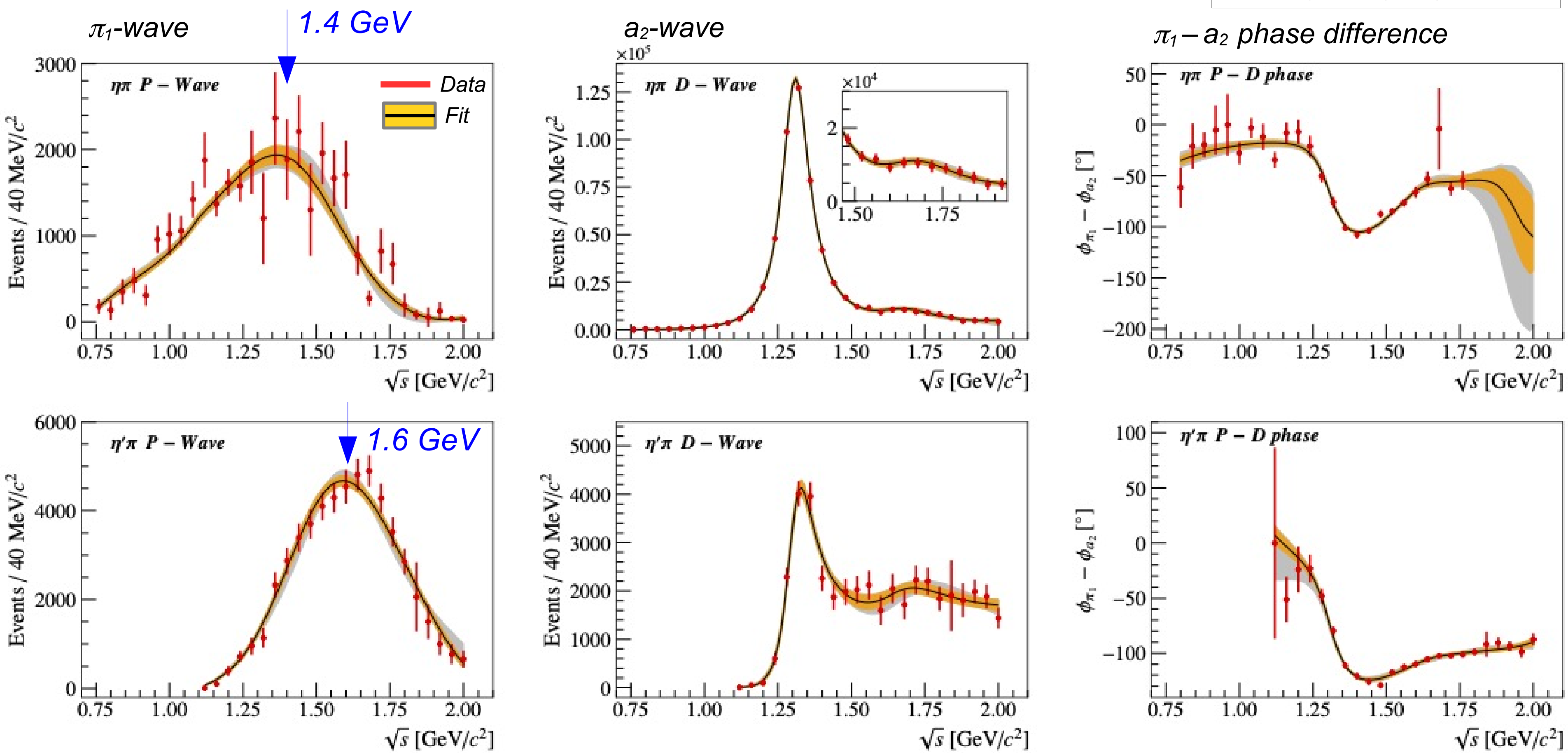


Phys. Rev. Lett. 122 (2019) 4, 042002

Coupled Channel Analysis with $\bar{p}p$ & COMPASS Data

- Extension: simultaneous fit of $\pi\pi$-scattering data, $\bar{p}p \to K^+ K^- \pi^0$, $\pi^0 \pi^0 \eta$, $\pi^0 \eta \eta$ and $\pi^- p \to \pi^- \eta^{(')} p$
- Good description with one pole scenario for the 1^{-+} wave using K-matrix
  - confirmation of the JPAC analysis based on N/D-method

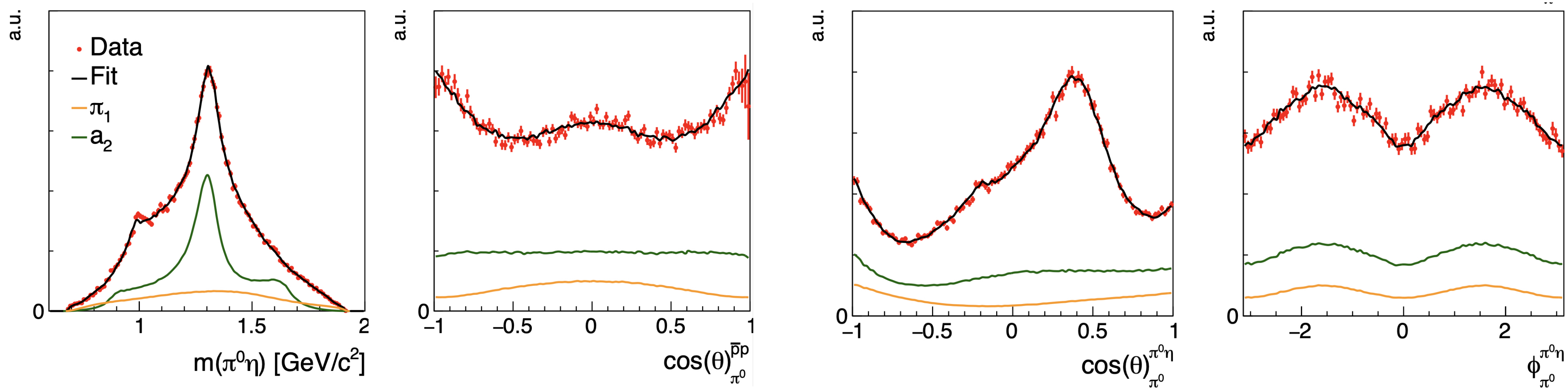
Eur. Phys.J. C (2021) 81, 1056



Coupled Channel Analysis with $\bar{p}p$ & COMPASS Data

$\bar{p}p \rightarrow \pi^0 \pi^0 \eta$

Eur. Phys.J. C (2021) 81, 1056



- π_1 mass is moving from 1.4 GeV/c² to 1.6 GeV/c² and consistent with $\pi_1(1600)$ with $\pi\eta'$ data

Table 1 Obtained masses, total widths and ratios of partial widths for the pole of the spin-exotic π_1-wave and for the two poles in the a_2-wave, the $a_2(1320)$ and the $a_2(1700)$. The first uncertainty is the statistical and the second the systematic one

Name	Pole mass (MeV/c^2)	Pole width (MeV)	$\Gamma_{\pi\eta'}/\Gamma_{\pi\eta}$ (%)	$\Gamma_{KK}/\Gamma_{\pi\eta}$ (%)
$a_2(1320)$	$1318.7 \pm 1.9^{+1.3}_{-1.3}$	$107.5 \pm 4.6^{+3.3}_{-1.8}$	$4.6 \pm 1.5^{+7.0}_{-0.6}$	$31 \pm 22^{+9}_{-11}$
$a_2(1700)$	$1686 \pm 22^{+19}_{-7}$	$412 \pm 75^{+64}_{-57}$	$3.5 \pm 4.4^{+6.9}_{-1.2}$	$2.9 \pm 4.0^{+1.1}_{-1.2}$
π_1	$1623 \pm 47^{+24}_{-75}$	$455 \pm 88^{+144}_{-175}$	$554 \pm 110^{+180}_{-27}$	–

In agreement with LQCD calculations for the lightest hybrid, but uncertainties are large

Phys. Rev. D 103, 05402 (2021)

Summary and Conclusion

- Determination of the resonance parameters and coupling strengths to different production and decay processes important for classifying states
- Coupled channel analyses of data from different production mechanisms and decay systems are needed
- Sophisticated descriptions of the dynamics needed by taking into account analyticity and unitarity
- Coupled channel PWA in the light meson sector are shown with data from $\bar{p}p$-annihilation, πp-reactions and $\pi\pi$-scattering processes
  - peaks at 1.4 GeV/c^2 in $\pi\eta$ and at 1.6 GeV/c^2 in $\pi\eta'$ of the spin exotic 1^{-+} wave can be described by only one pole using K-matrix formalism and is consistent with the $\pi_1(1600)$
- PWA of the π_1 wave with more data and additional decay channels could shed more light on the knowledge of the lightest hybrid below 2 GeV/c^2