

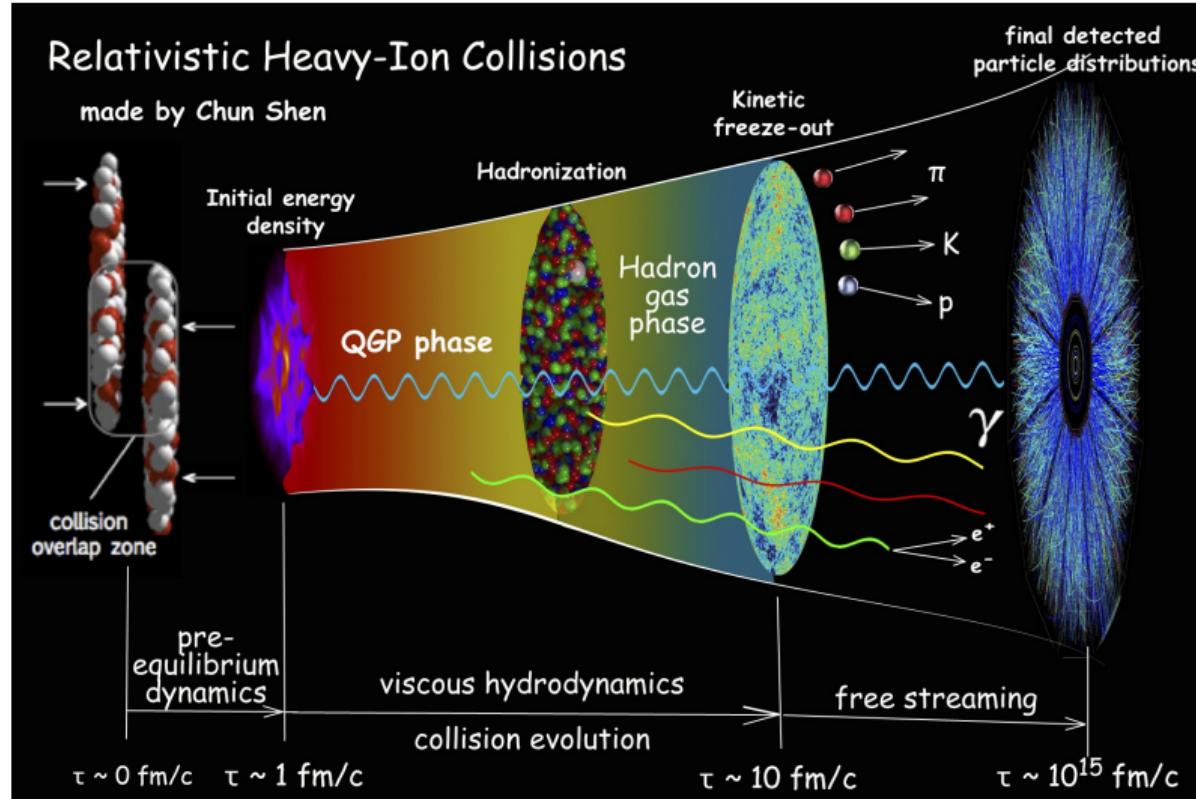
# The initial state in heavy-ion collisions

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*"Statistical analysis of initial state and final state response in heavy-ion collisions"*  
N. Borghini, M. Borrell, N. Feld, HR, S. Schlichting, C. Werthmann

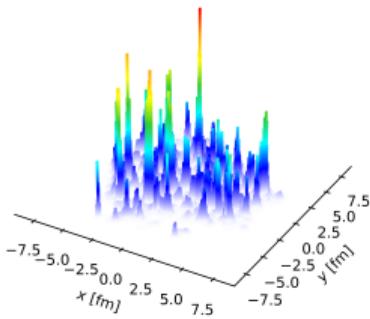
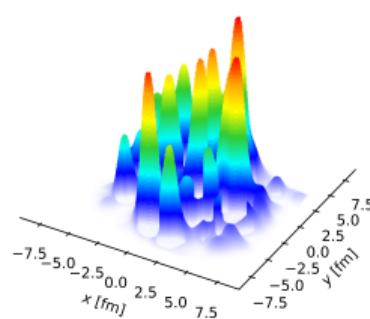
# Heavy-ion collisions — the standard model



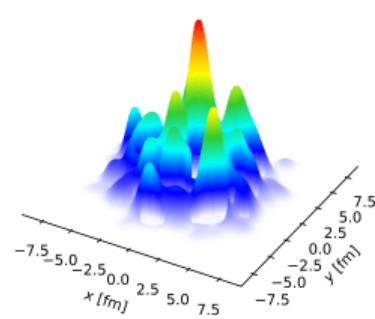
[<https://bit.ly/3zfDPMM>]

- Many different initial state models with different underlying degrees of freedom:
  - Effective theories of high-energy QCD:
    - IP-Glasma [1, 2]
    - EKRT [3, 4]
  - Parametric models:
    - MC-Glauber [5]
    - T<sub>R</sub>ENT<sub>O</sub> [6]

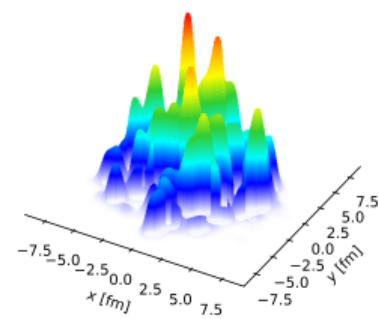
IP-Glasma:

T<sub>R</sub>ENT<sub>O</sub>:

MC-Glauber:



Saturation:



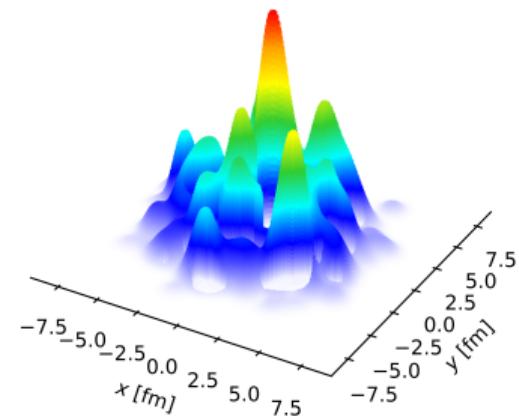
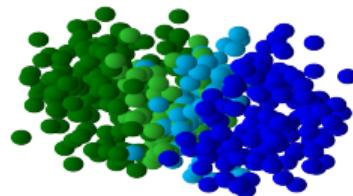
# First steps to a characterization method

- There are some first works on this initial state characterization:
    - Bessel-Fourier decomposition [7]
    - Describe fluctuations around an average-state profile to single out effects of different initial-state fluctuations [8, 9]
      - + reduction of computational cost
      - typically limited to linear response
      - modes are not uncorrelated
      - not yet tested against event-by-event simulations
  - Today's talk based on arXiv:2209.01176
    - Decompose ensemble of events into an average state and uncorrelated fluctuation modes
    - Mode-by-mode tested against event-by-event simulations
    - General framework
- ⇒ Choose two initial state models for a first exploratory study

# 1st example: MC-Glauber model

- Sample nucleon positions from Woods-Saxon distribution  
→  $N_{\text{part}}(x, y)$  and  $N_{\text{coll}}(x, y)$
- $e_d(x, y) \propto (1 - \alpha) \frac{N_{\text{part}}(x, y)}{2} + \alpha N_{\text{coll}}(x, y)$  with  $\alpha = 0.2$  [10]
- Smear  $e_d$  at each point with Gaussian

$$e(x, y) \equiv \left. \frac{dE}{\tau_0 d^2 \mathbf{x} dy} \right|_{y=0}$$



# 2nd example: Saturation model

- Based on CGC effective field theory for QCD at high energies [11, 12]
- Use the gluon distribution calculated in the GBW model [13] to obtain the energy density:

$$[e(\mathbf{x})\tau]_0 = \int dY \int d^2\mathbf{P} |\mathbf{P}| \frac{dN_g}{d^2\mathbf{x} d^2\mathbf{P} dY dy}.$$

- Obtain the energy density at each point analytically assuming  $|\mathbf{P}| \simeq Q_{s,A/B}$ :

$$[e(\mathbf{x})\tau]_0 = \frac{N_c^2 - 1}{4g^2 N_c \sqrt{\pi}} \frac{Q_{s,A}^2 Q_{s,B}^2}{(Q_{s,A}^2 + Q_{s,B}^2)^{5/2}} \left[ 2Q_{s,A}^4 + 7Q_{s,A}^2 Q_{s,B}^2 + 2Q_{s,B}^4 \right]$$

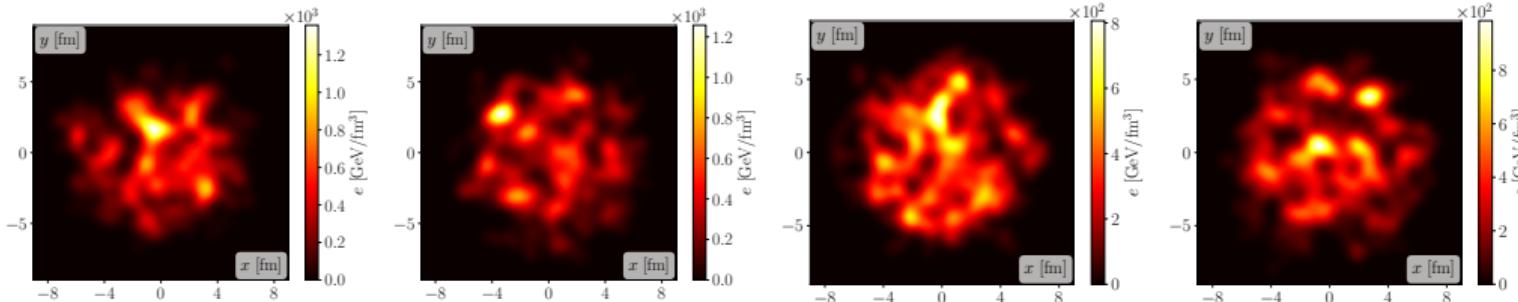
- Saturation scale of the nucleus is parametrized by

$$Q_{s,A/B}^2(x, \mathbf{x}) = \underbrace{Q_{s,0}^2 x^{-\lambda} (1-x)^\delta}_{=Q_{s,p}^2(x)} \sigma_0 \underbrace{T_{A/B}(\mathbf{x})}_{=\sum_{i \in A/B} T_p(\mathbf{x}-\mathbf{x}_i)}, \quad T_p(\mathbf{x}) = \frac{1}{2\pi B_G} e^{-\mathbf{x}^2/2B_G}$$

$$x = \frac{Q_{s,A/B}(x, \mathbf{x}) e^{\pm Y}}{\sqrt{s_{NN}}}$$

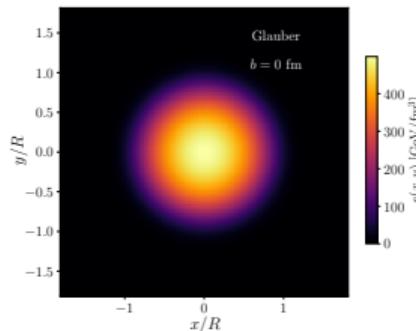
# The average state

- Generate sample of events  $\Phi^{(i)}$  (e.g. at fixed  $\mathbf{b}$ ):  $N_{\text{ev}} = 2^{21}$  (here  $b = 0 \text{ fm}$ )



- Average state:

$$\bar{\Psi} \equiv \frac{1}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} \Phi^{(i)}$$



Grid:  $128 \times 128$  points  $\rightarrow$  spacing  $\approx 0.2 \text{ fm}$ , Pb-Pb collisions at 5.02 TeV

# Statistical analysis of initial state

- Introduce a density matrix representing the fluctuations about  $\bar{\Psi}$ :

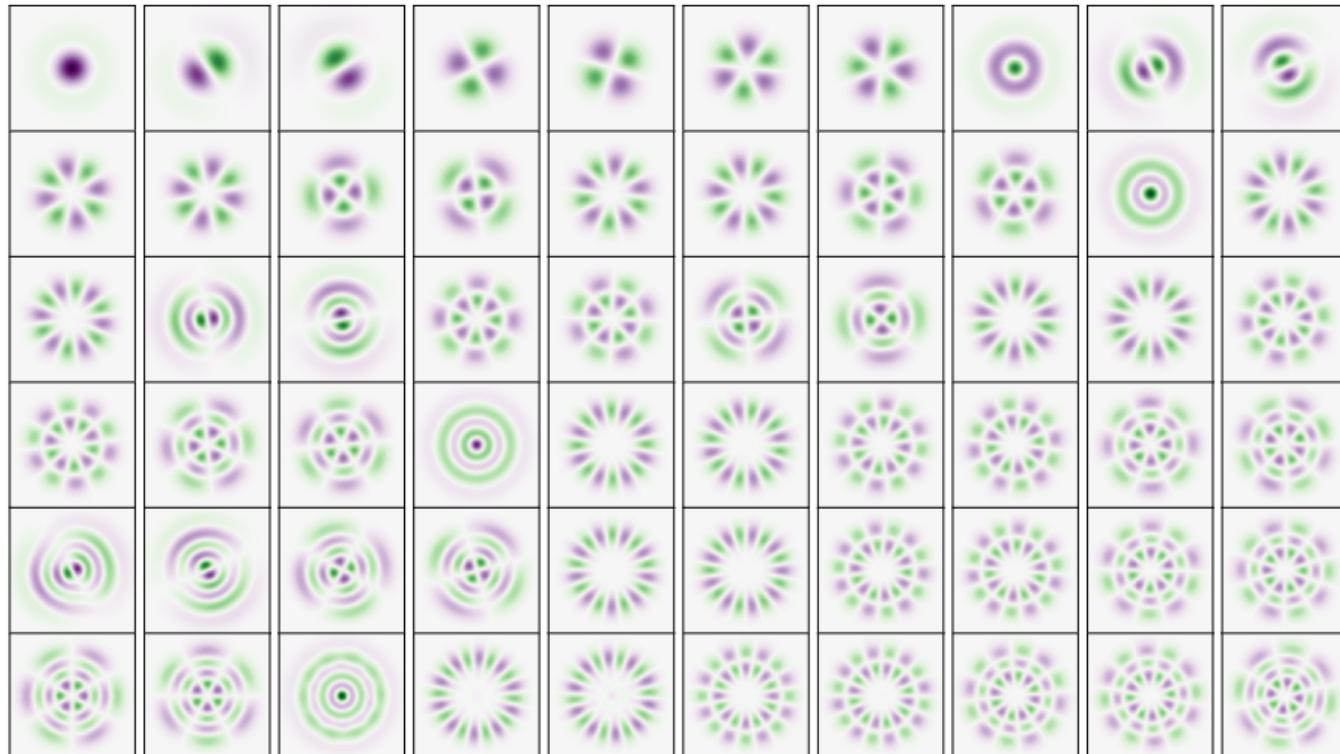
$$\rho \equiv \frac{1}{N_{\text{ev}}} \sum_i \Phi^{(i)} \Phi^{(i)\top} - \bar{\Psi} \bar{\Psi}^T$$

- Diagonalize  $\rho$ :

$$\rho \tilde{\Psi}_I = \lambda_I \tilde{\Psi}_I$$

- Sort eigenvectors  $\{\tilde{\Psi}_I\}$  according to magnitude of eigenvalues  $\lambda_I$ 
  - $\lambda_I$  quantifies relative importance of mode  $\tilde{\Psi}_I$  for the  $N_{\text{ev}}$  events

# Eigenvectors at $b = 0$ fm – Glauber model



# Statistical analysis of initial state

- Each event can be decomposed into an average state and fluctuation modes:

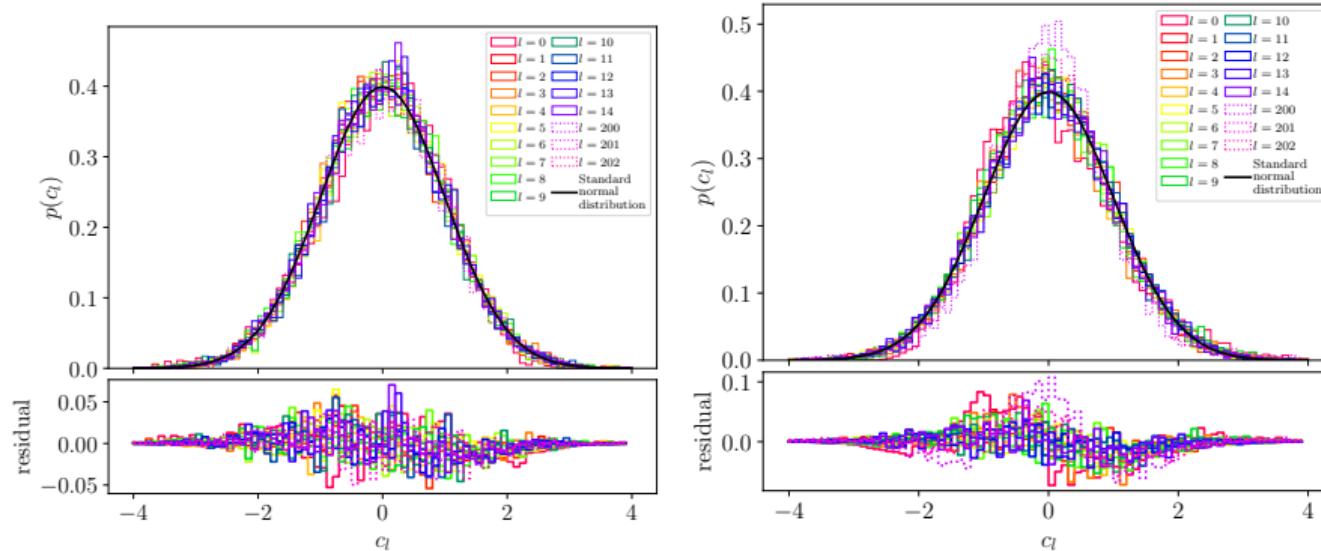
$$\Phi^{(i)} = \bar{\Psi} + \sum_I \tilde{c}_I \tilde{\Psi}_I \quad \text{with} \quad \langle \tilde{c}_I \rangle = 0$$

- Fluctuations in different modes are uncorrelated:  $\langle \tilde{c}_I \tilde{c}_{I'} \rangle = \lambda_I \delta_{II'}$
- Rescale the eigenvectors and expansion coefficients:

$$\Psi_I \equiv \sqrt{\lambda_I} \tilde{\Psi}_I, \quad c_I \equiv \frac{\tilde{c}_I}{\sqrt{\lambda_I}}$$

- $\{\Psi_I\}$  forms an orthogonal basis
- Coefficients have unit variance:  $\langle c_I c_{I'} \rangle = \delta_{II'}$

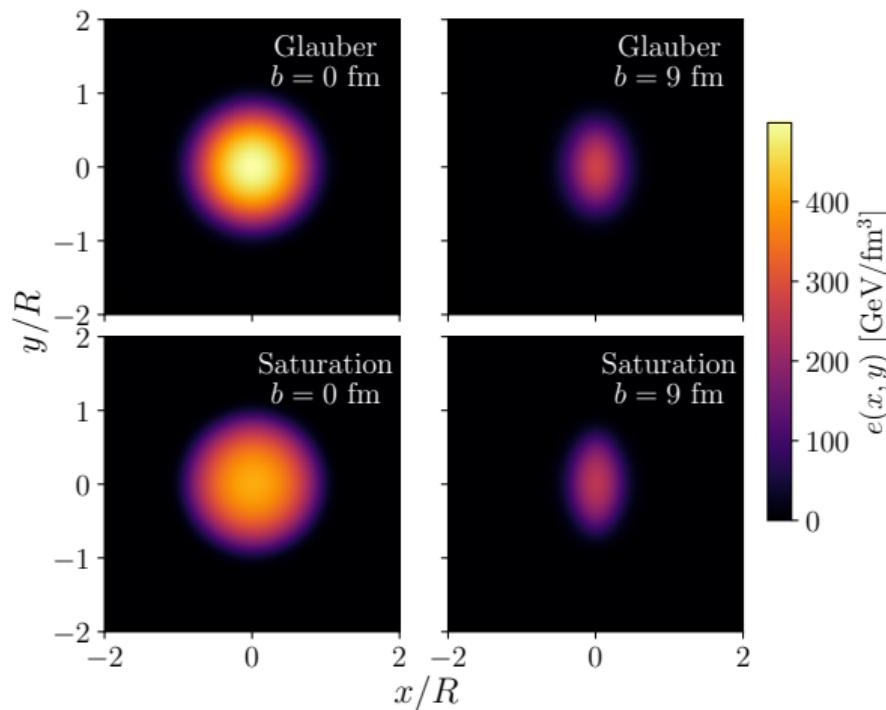
# Probability distributions of the expansion coefficients



- $\langle c_l \rangle = 0$  and  $\langle c_l c_{l'} \rangle = \delta_{ll'}$ , skewness for some modes can be related to positivity of the energy density

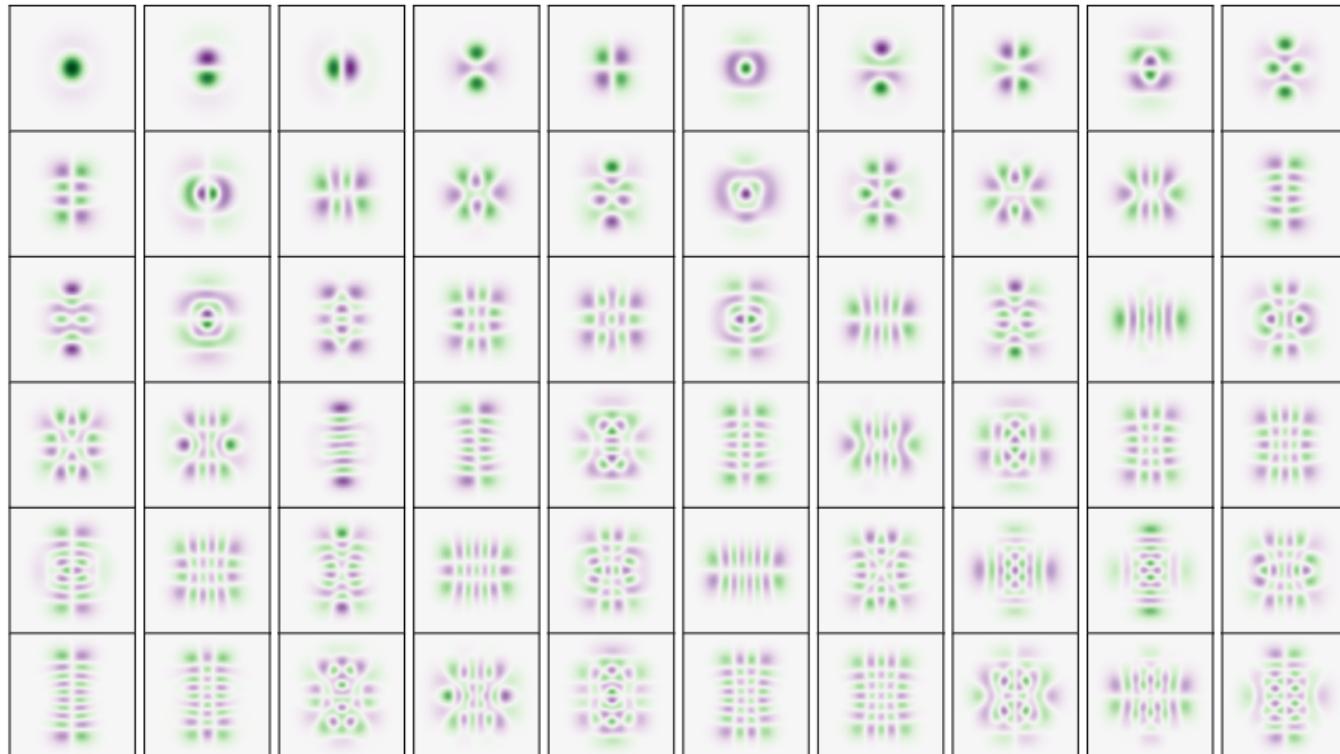
# The average state

- Next step:  $b \neq 0$ , fixed orientation  $\rightarrow$  rotational symmetry broken

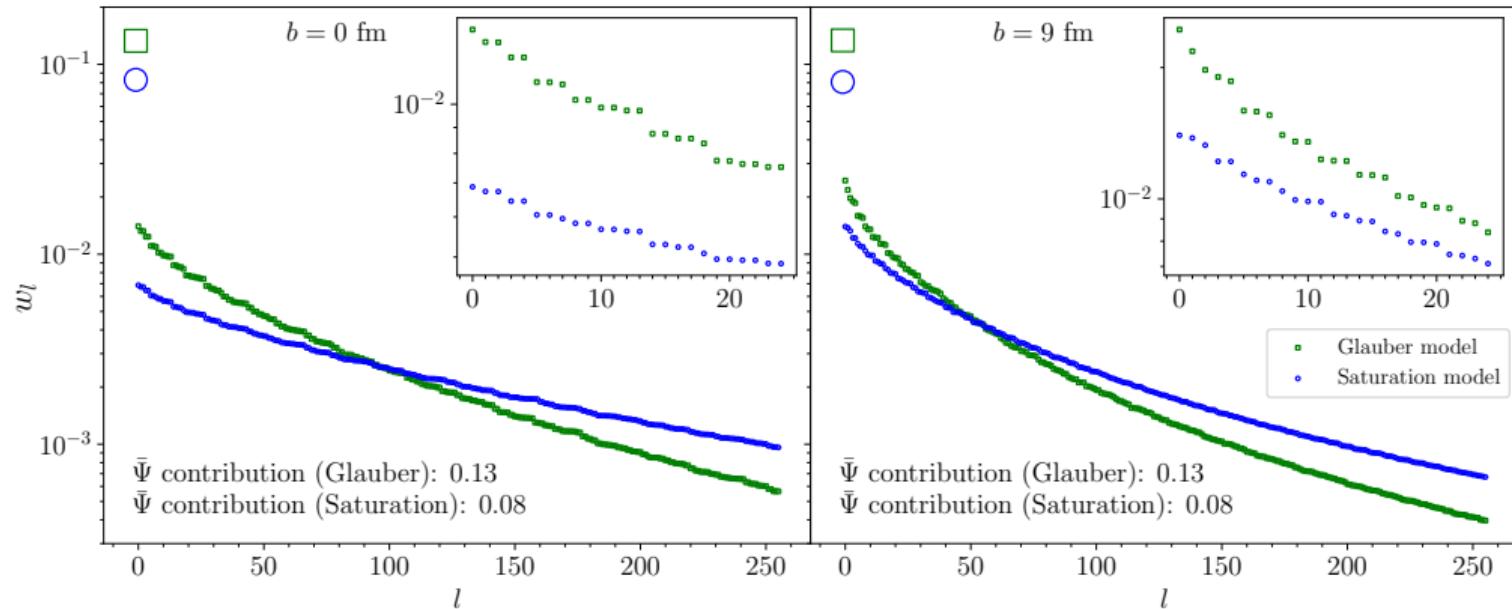


Note: The models have not been calibrated to yield the same total energy in this exploratory study.

# Eigenvectors at $b = 9$ fm – Glauber model



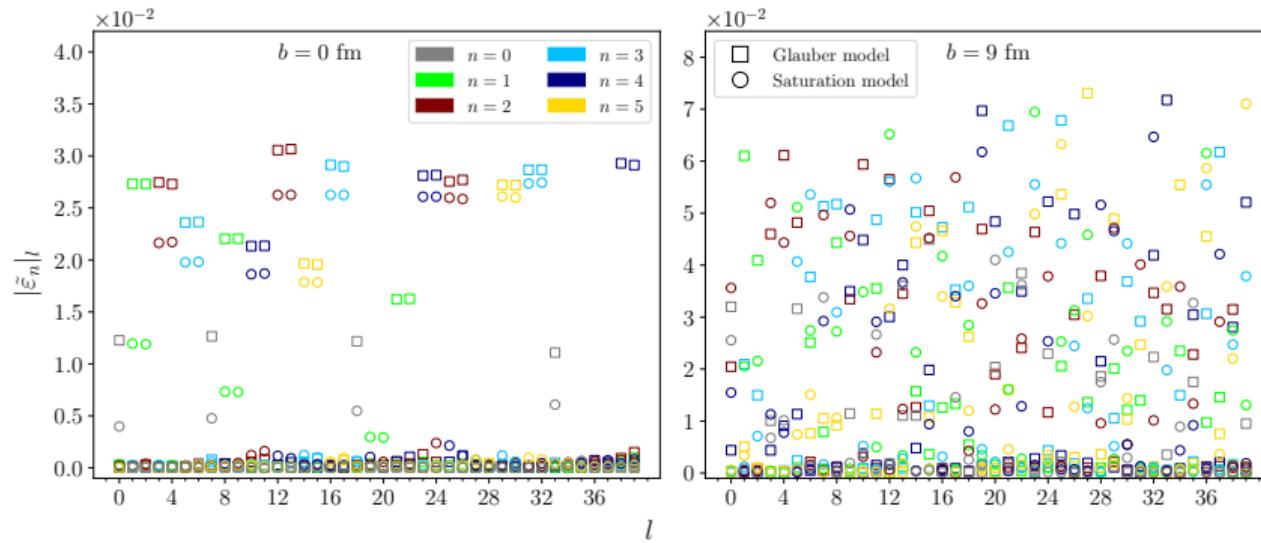
# Typical relative weight of modes



$$\bar{w}_I \equiv \frac{||\bar{\Psi}||}{\sum_I ||\Psi_I|| + ||\bar{\Psi}||}, \quad w_I \equiv \frac{||\Psi_I||}{\sum_I ||\Psi_I|| + ||\bar{\Psi}||}$$

- $\bar{\Psi}$  contributes  $\approx 10\%$
- $w_I$  quantifies importance of  $\Psi_I$
- Singlet and doublet structures

# Mode energy and eccentricities



$$\tilde{\epsilon}_n e^{in\Phi_n} \equiv - \frac{\int r^n e^{in\theta} \Psi_l(r, \theta) r dr d\theta}{\int r^n \bar{\Psi}(r, \theta) r dr d\theta} \quad \text{for } n \neq 1$$

- Singlet and doublet structure  $b = 0 \text{ fm}$  (rotational symmetry)
- Radial modes contain energy ( $n = 0$ )
- $b = 9 \text{ fm}$ : multiple  $\tilde{\epsilon}_n$  for each  $l$

# (Non-)linear response theory

- Write observables as

$$\begin{aligned} O_\alpha &= \bar{O}_\alpha + \frac{\partial O_\alpha}{\partial c_I} \Big|_{\bar{\Psi}} c_I + \frac{1}{2} \frac{\partial^2 O_\alpha}{\partial c_I \partial c_{I'}} \Big|_{\bar{\Psi}} c_I c_{I'} + \mathcal{O}(c_I^3) \\ &\equiv \bar{O}_\alpha + L_{\alpha,I} c_I + \frac{1}{2} Q_{\alpha,II'} c_I c_{I'} + \mathcal{O}(c_I^3). \end{aligned}$$

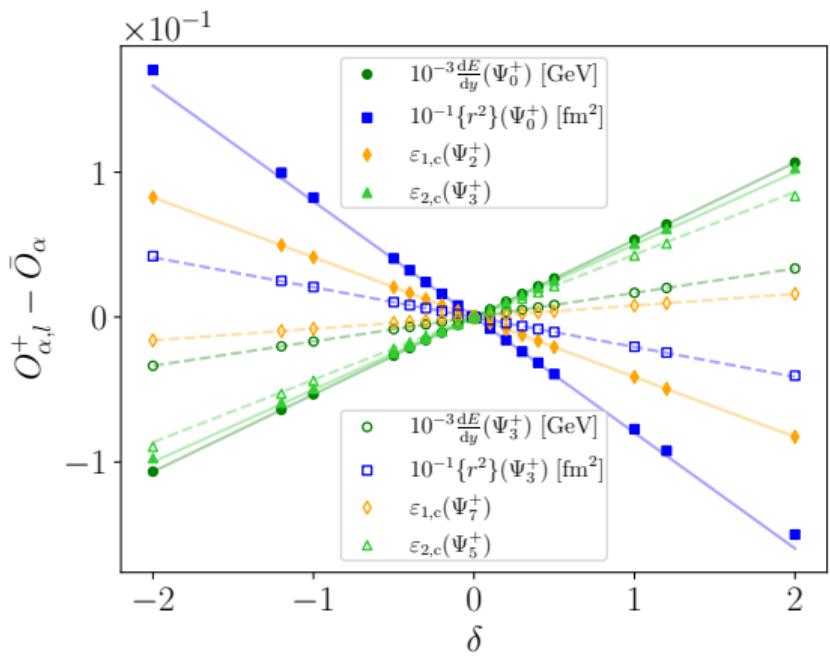
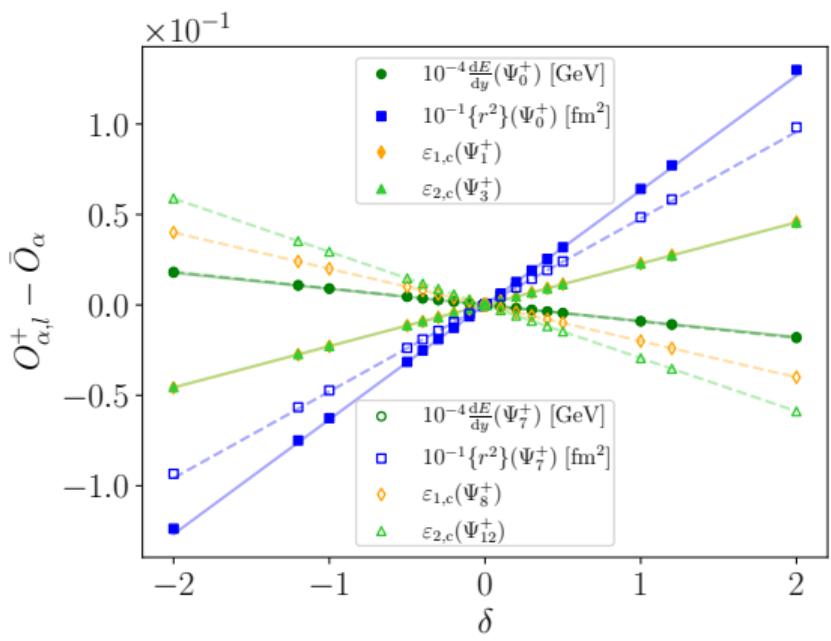
- Define states

$$\Psi_I^+ \equiv \bar{\Psi} + \delta\Psi_I, \quad \Psi_I^- \equiv \bar{\Psi} - \delta\Psi_I$$

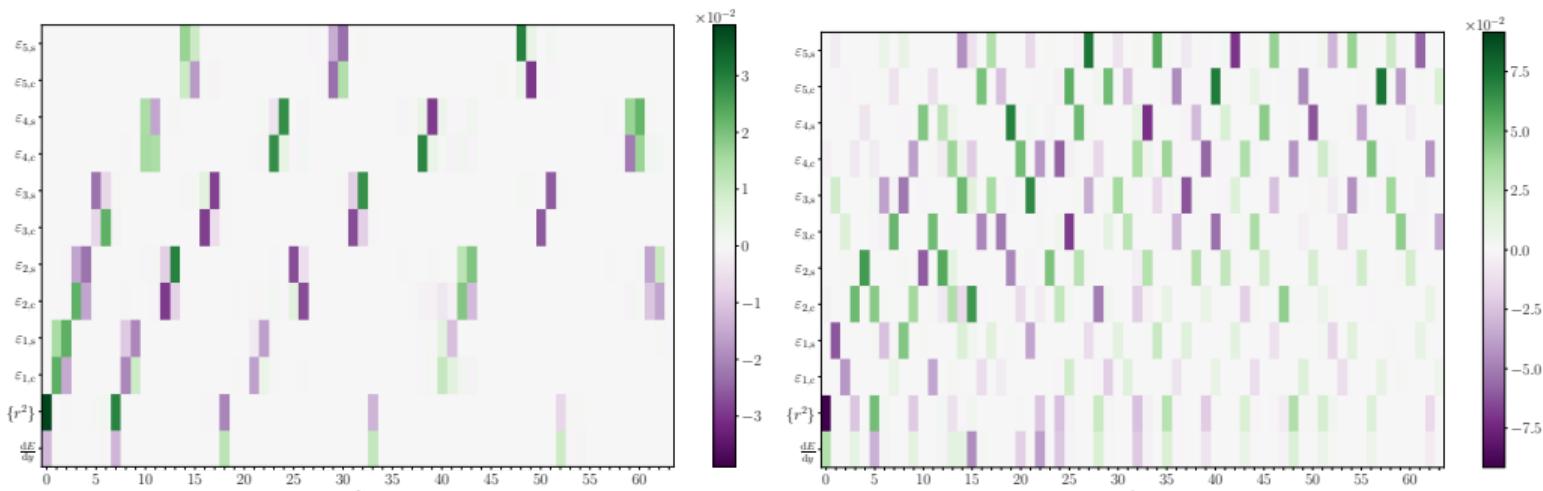
- Compute the linear and quadratic response:

$$L_{\alpha,I} = \frac{O_{\alpha,I}^+ - O_{\alpha,I}^-}{2\delta}, \quad Q_{\alpha,II} = \frac{O_{\alpha,I}^+ + O_{\alpha,I}^- - 2\bar{O}_\alpha}{\delta^2}$$

# Linearity check ( $b = 0, 9$ fm) – Glauber model



# Mode-by-mode perturbation ( $b = 0, 9$ fm) $L_{\alpha,I}$



- Mode-by-mode perturbation of  $\bar{\Psi} \rightarrow$  linear response of initial state
- $L_{\alpha,I}$  sufficient to estimate the (co-)variances of the characteristics

## Conclusion

- Found an optimal basis with uncorrelated fluctuation modes on top of an average state to decompose the initial-state profiles
- Difference between models: relative weight of modes → steeper fall off in Glauber model
  - Density in Saturation model has more detailed structure

## Outlook

- Perform the study with centrality dependence and make a more quantitative comparison of the two models
- Approach could be used for other models (3D, conserved charges,...)

# Backup

# Saturation model

- Start with  $k_T$ -factorization formula [14, 15]:

$$\frac{dN_g}{d^2\mathbf{x} d^2\mathbf{P} dy} = \frac{g^2 N_c}{4\pi^5 \mathbf{P}^2 (N_c^2 - 1)} \delta(Y - y) \int \frac{d^2\mathbf{k}}{(2\pi)^2} \Phi_A \left( \mathbf{x} + \frac{\mathbf{b}}{2}, \mathbf{k} \right) \Phi_B \left( \mathbf{x} - \frac{\mathbf{b}}{2}, \mathbf{P} - \mathbf{k} \right)$$

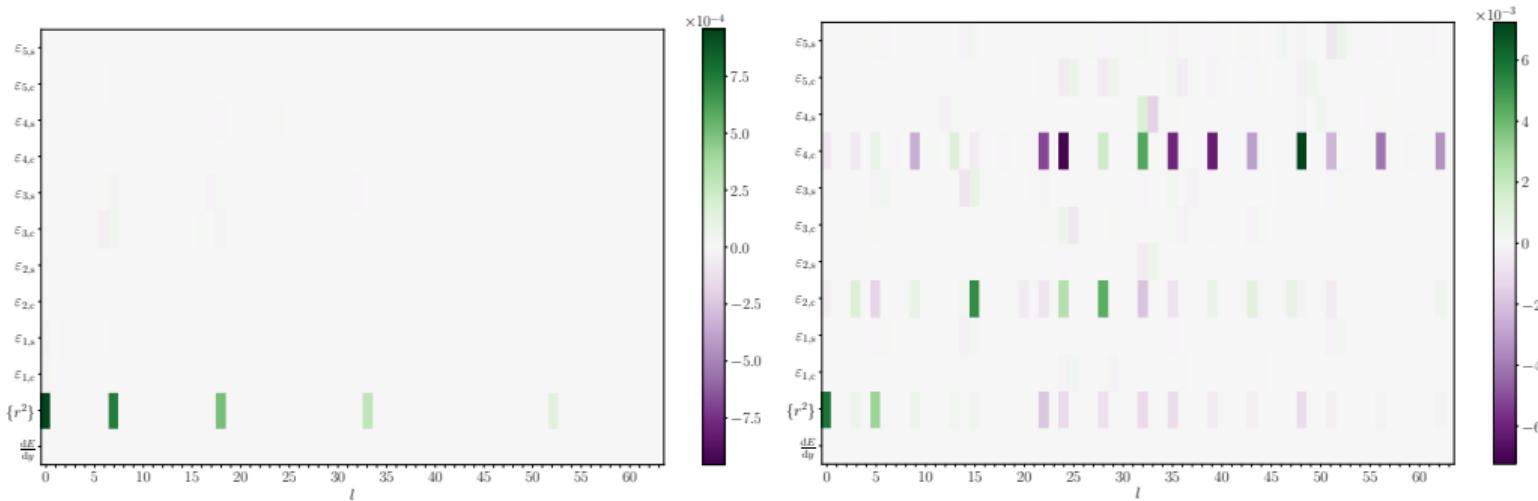
- In GBW model [13] the unintegrated gluon distribution is parametrized as:

$$\Phi_A(\mathbf{x}, \mathbf{k}) = 4\pi^2 \frac{N_c^2 - 1}{g^2 N_c} \frac{\mathbf{k}^2}{Q_{s,A/B}^2} e^{-\mathbf{k}^2/Q_{s,A/B}^2}$$

- Self consistent solution for  $x \ll 1$ :

$$x = \left( \frac{Q_{s,0}^2 \sigma_0 T_{A/B}(\mathbf{x}) e^{\pm 2Y}}{s_{NN}} \right)^{\frac{1}{2+\lambda}}$$

# Mode-by-mode perturbation ( $b = 0, 9$ fm) $Q_{\alpha,II}$



- Mode-by-mode perturbation of  $\bar{\Psi} \rightarrow$  non-linear response small

# Bessel-Fourier decomposition

- Basis functions:

$$\chi_{n,k}(r, \theta) = \frac{1}{J_{|n|+1}(j_{n,k})} J_n \left( \frac{r}{r_0} j_{n,k} \right) e^{in\theta}$$

- Scalar product:

$$\langle a, b \rangle \equiv \frac{1}{\pi r_0^2} \int_0^{2\pi} \int_0^{r_0} a(r, \theta) b^*(r, \theta) r dr d\theta$$

- Expand function in transverse plane:

$$f(r, \theta) = \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} A_{n,k} \chi_{n,k}(r, \theta)$$

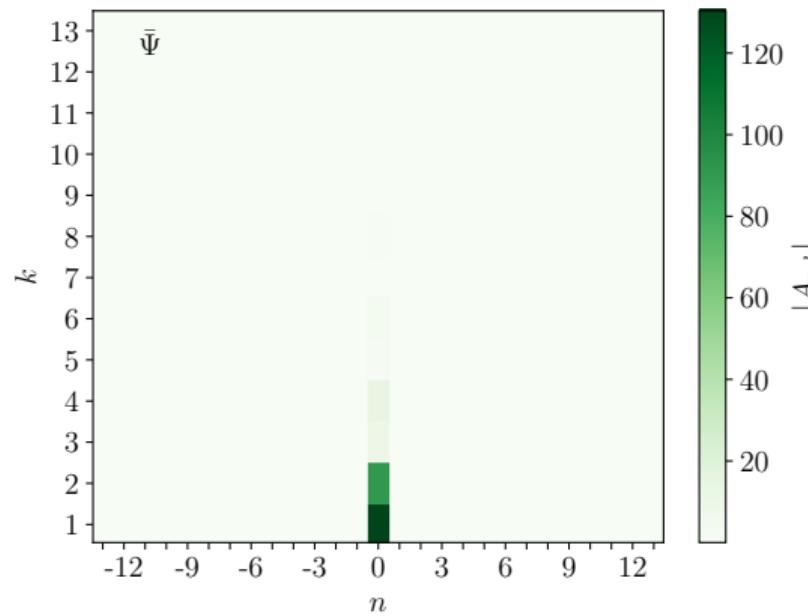
- Expansion coefficients:

$$A_{n,k} = \langle f(r, \theta), \chi_{n,k}(r, \theta) \rangle = \frac{1}{\pi r_0^2} \iint f(r, \theta) \chi_{n,k}^*(r, \theta) r dr d\theta$$

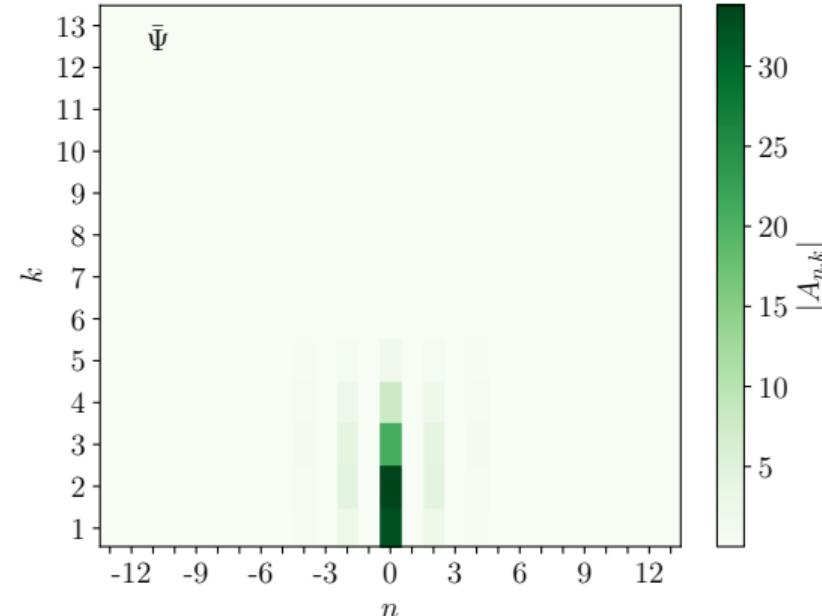
with  $A_{n,k} = A_{-n,k}^*$

# Bessel-Fourier decomposition - Example: Average states

Glauber b=0 fm:

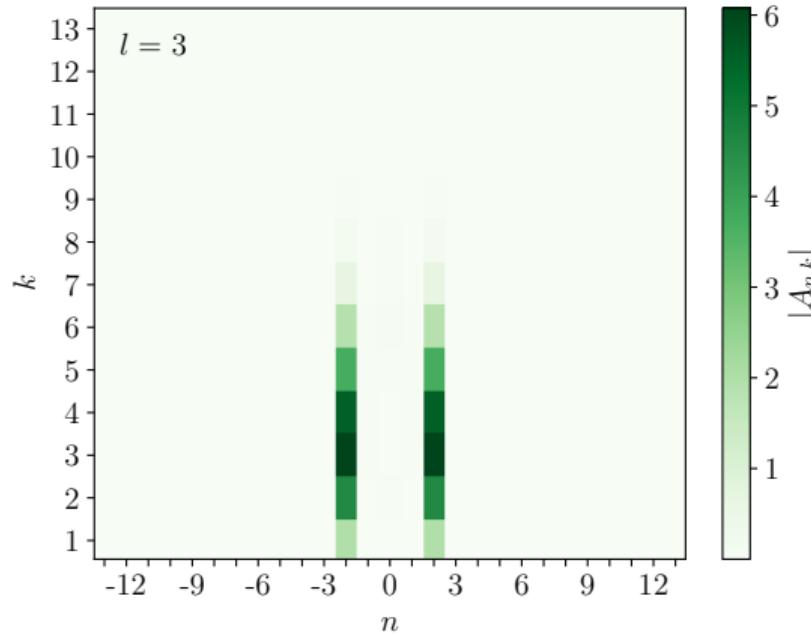


Glauber b=9 fm:

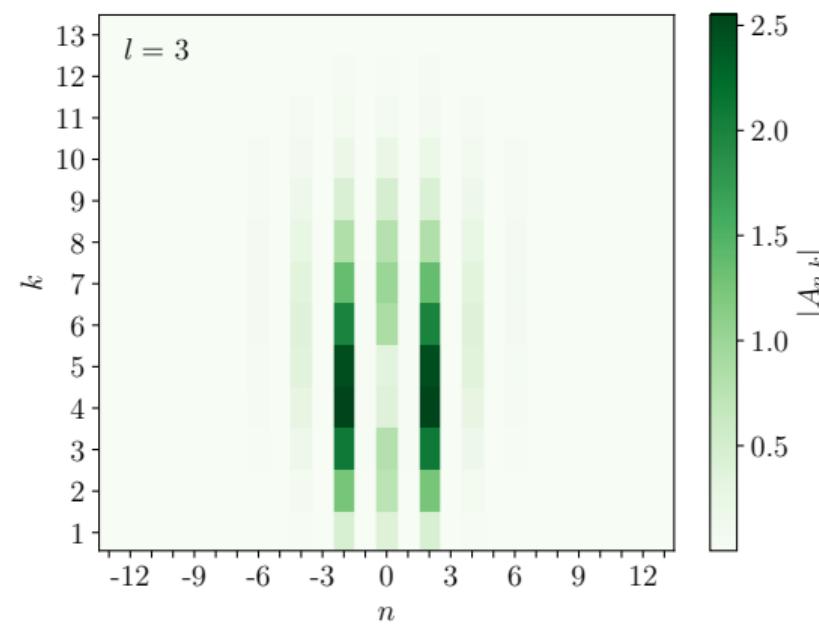


## Bessel-Fourier decomposition - Example: $\varepsilon_2$ modes

## Glauber b=0 fm:



Glauber b=9 fm:



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