The initial state in heavy-ion collisions

Hendrik Roch
QNP2022 - The 9th International Conference on Quarks and Nuclear Physics
05.09.2022

“Statistical analysis of initial state and final state response in heavy-ion collisions”
N. Borghini, M. Borrell, N. Feld, HR, S. Schlichting, C. Werthmann
Heavy-ion collisions — the standard model

[https://bit.ly/3zfDPMM]
Many different initial state models with different underlying degrees of freedom:

- Effective theories of high-energy QCD:
  - IP-Glasma [1, 2]
  - EKRT [3, 4]

- Parametric models:
  - MC-Glauber [5]
  - TRENTO [6]

IP-Glasma:  \[ T_{\text{RENTO}}: \]  
MC-Glauber:  \[ \text{Saturation}: \]
First steps to a characterization method

There are some first works on this initial state characterization:
- Bessel-Fourier decomposition [7]
- Describe fluctuations around an average-state profile to single out effects of different initial-state fluctuations [8, 9]
  + reduction of computational cost
  - typically limited to linear response
  - modes are not uncorrelated
  - not yet tested against event-by-event simulations

Today’s talk based on arXiv:2209.01176
- Decompose ensemble of events into an average state and uncorrelated fluctuation modes
- Mode-by-mode tested against event-by-event simulations
- General framework

⇒ Choose two initial state models for a first exploratory study
1st example: MC-Glauber model

- Sample nucleon positions from Woods-Saxon distribution 
  \[ \rightarrow N_{\text{part}}(x, y) \text{ and } N_{\text{coll}}(x, y) \]
- \[ e_d(x, y) \propto (1 - \alpha) \frac{N_{\text{part}}(x, y)}{2} + \alpha N_{\text{coll}}(x, y) \] with \( \alpha = 0.2 \) [10]
- Smear \( e_d \) at each point with Gaussian

\[ e(x, y) \equiv \left. \frac{dE}{\tau_0 d^2x dy} \right|_{y=0} \]
2nd example: Saturation model

- Based on CGC effective field theory for QCD at high energies [11, 12]
- Use the gluon distribution calculated in the GBW model [13] to obtain the energy density:

\[
[e(x) \tau]_0 = \int dY \int d^2P \frac{dN_g}{d^2x d^2P dY dy}.
\]

- Obtain the energy density at each point analytically assuming \(|P| \simeq Q_s, A/B|

\[
[e(x) \tau]_0 = \frac{N_c^2 - 1}{4g^2 N_c \sqrt{\pi}} \frac{Q_{s,A}^2 Q_{s,B}^2}{(Q_{s,A}^2 + Q_{s,B}^2)^{5/2}} \left[ 2Q_{s,A}^4 + 7Q_{s,A}^2 Q_{s,B}^2 + 2Q_{s,B}^4 \right]
\]

- Saturation scale of the nucleus is parametrized by

\[
Q_{s,A/B}(x, x) = \underbrace{Q_{s,0}^2 x^{-\lambda}(1 - x)^{\delta}}_{=Q_{s,p}(x)} \sigma_0 \quad T_{A/B}(x) = \sum_{i \in A/B} T_p(x-x_i), \\
T_p(x) = \frac{1}{2\pi B_G} e^{-x^2/2B_G}, \quad x = \frac{Q_{s,A/B}(x,x) e^{\pm Y}}{\sqrt{s_{NN}}}
\]
The average state

- Generate sample of events $\Phi^{(i)}$ (e.g. at fixed $b$): $N_{ev} = 2^{21}$ (here $b = 0$ fm)

Average state:

$$\overline{\Psi} \equiv \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \Phi^{(i)}$$

Grid: $128 \times 128$ points $\rightarrow$ spacing $\approx 0.2$ fm, Pb-Pb collisions at 5.02 TeV
Statistical analysis of initial state

- Introduce a density matrix representing the fluctuations about $\bar{\Psi}$:

$$\rho \equiv \frac{1}{N_{\text{ev}}} \sum_i \phi(i) \phi(i)^T - \bar{\Psi} \bar{\Psi}^T$$

- Diagonalize $\rho$:

$$\rho \tilde{\Psi}_l = \lambda_l \tilde{\Psi}_l$$

- Sort eigenvectors $\{\tilde{\Psi}_l\}$ according to magnitude of eigenvalues $\lambda_l$:
  - $\lambda_l$ quantifies relative importance of mode $\tilde{\Psi}_l$ for the $N_{\text{ev}}$ events
Eigenvectors at $b = 0$ fm – Glauber model
Statistical analysis of initial state

- Each event can be decomposed into an average state and fluctuation modes:
  \[ \phi^{(i)} = \bar{\psi} + \sum_{l} \tilde{c}_l \tilde{\psi}_l \quad \text{with} \quad \langle \tilde{c}_l \rangle = 0 \]

- Fluctuations in different modes are uncorrelated: \( \langle \tilde{c}_l \tilde{c}_{l'} \rangle = \lambda_l \delta_{ll'} \)

- Rescale the eigenvectors and expansion coefficients:
  \[ \psi_l \equiv \sqrt{\lambda_l} \tilde{\psi}_l, \quad c_l \equiv \frac{\tilde{c}_l}{\sqrt{\lambda_l}} \]

- \( \{ \psi_l \} \) forms an orthogonal basis
- Coefficients have unit variance: \( \langle c_l c_{l'} \rangle = \delta_{ll'} \)
Probability distributions of the expansion coefficients

- \( \langle c_l \rangle = 0 \) and \( \langle c_l c_{l'} \rangle = \delta_{ll'} \), skewness for some modes can be related to positivity of the energy density
The average state

- Next step: $b \neq 0$, fixed orientation $\rightarrow$ rotational symmetry broken

Note: The models have not been calibrated to yield the same total energy in this exploratory study.
Eigenvectors at $b = 9$ fm – Glauber model
Typical relative weight of modes

\[ \bar{w}_l \equiv \frac{||\bar{\Psi}||}{\sum_l ||\Psi_l|| + ||\bar{\Psi}||}, \quad w_l \equiv \frac{||\Psi_l||}{\sum_l ||\Psi_l|| + ||\bar{\Psi}||} \]

- \( \bar{\Psi} \) contributes \( \approx 10\% \)
- \( w_l \) quantifies importance of \( \Psi_l \)
- Singlet and doublet structures

\( b = 0 \text{ fm} \)

\( b = 9 \text{ fm} \)

\( \bar{\Psi} \) contribution (Glauber): 0.13
\( \bar{\Psi} \) contribution (Saturation): 0.08

\( \bar{\Psi} \) contribution (Glauber): 0.13
\( \bar{\Psi} \) contribution (Saturation): 0.08
Mode energy and eccentricities

\[ \tilde{\varepsilon}_n e^{i n \Phi_n} \equiv -\frac{\int r^n e^{i n \theta} \Psi_l(r, \theta) \, r \, dr \, d\theta}{\int r^n \overline{\Psi}(r, \theta) \, r \, dr \, d\theta} \quad \text{for } n \neq 1 \]

- Singlet and doublet structure \( b = 0 \) fm (rotational symmetry)
- Radial modes contain energy \((n = 0)\)
- \( b = 9 \) fm: multiple \( \tilde{\varepsilon}_n \) for each \( l \)
(Non-)linear response theory

- Write observables as

\[ O_\alpha = \bar{O}_\alpha + \frac{\partial O_\alpha}{\partial c_l} \bigg|_{\bar{\Psi}} c_l + \frac{1}{2} \frac{\partial^2 O_\alpha}{\partial c_l \partial c_{l'}} \bigg|_{\bar{\Psi}} c_l c_{l'} + O(c_l^3) \]

\[ \equiv \bar{O}_\alpha + L_{\alpha,l} c_l + \frac{1}{2} Q_{\alpha,ll'} c_l c_{l'} + O(c_l^3). \]

- Define states

\[ \psi_+^l \equiv \bar{\psi} + \delta \psi_l, \quad \psi_-^l \equiv \bar{\psi} - \delta \psi_l \]

- Compute the linear and quadratic response:

\[ L_{\alpha,l} = \frac{O_+^{\alpha,l} - O_-^{\alpha,l}}{2\delta}, \quad Q_{\alpha,ll'} = \frac{O_+^{\alpha,l} + O_-^{\alpha,l} - 2\bar{O}_\alpha}{\delta^2} \]
Linearity check \((b = 0, 9 \text{ fm})\) – Glauber model
Mode-by-mode perturbation ($b = 0, 9 \text{ fm}$) $L_{\alpha,l}$

- Mode-by-mode perturbation of $\bar{\Psi} \rightarrow$ linear response of initial state
- $L_{\alpha,l}$ sufficient to estimate the (co-)variances of the characteristics
Conclusion

- Found an optimal basis with uncorrelated fluctuation modes on top of an average state to decompose the initial-state profiles
- Difference between models: relative weight of modes → steeper fall off in Glauber model
  - Density in Saturation model has more detailed structure

Outlook

- Perform the study with centrality dependence and make a more quantitative comparison of the two models
- Approach could be used for other models (3D, conserved charges,...)
Backup
Saturation model

- Start with $k_T$-factorization formula \[14, 15\]:

\[
\frac{dN_g}{d^2xd^2PdYdy} = \frac{g^2N_c}{4\pi^5P^2(N_c^2 - 1)} \delta(Y - y) \int \frac{d^2k}{(2\pi)^2} \Phi_A(x + \frac{b}{2}, k) \Phi_B(x - \frac{b}{2}, P - k)
\]

- In GBW model \[13\] the unintegrated gluon distribution is parametrized as:

\[
\Phi_A(x, k) = 4\pi^2 \frac{N_c^2 - 1}{g^2N_c} \frac{k^2}{Q_{s,A/B}^2} e^{-k^2/Q_{s,A/B}^2}
\]

- Self consistent solution for $x \ll 1$:

\[
x = \left( \frac{Q_{s,0}^2 \sigma_0 T_{A/B}(x) e^{\pm2Y}}{s_{NN}} \right)^{\frac{1}{2+\lambda}}
\]
Mode-by-mode perturbation of $\bar{\Psi} \rightarrow$ non-linear response small
Bessel-Fourier decomposition

- Basis functions:
  \[
  \chi_{n,k}(r, \theta) = \frac{1}{J_{|n|+1}(j_{n,k})} J_n \left( \frac{r}{r_0} j_{n,k} \right) e^{in\theta}
  \]

- Scalar product:
  \[
  \langle a, b \rangle \equiv \frac{1}{\pi r_0^2} \int_0^{2\pi} \int_0^{r_0} a(r, \theta) b^*(r, \theta) \, r \, dr \, d\theta
  \]

- Expand function in transverse plane:
  \[
  f(r, \theta) = \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} A_{n,k} \chi_{n,k}(r, \theta)
  \]

- Expansion coefficients:
  \[
  A_{n,k} = \langle f(r, \theta), \chi_{n,k}(r, \theta) \rangle = \frac{1}{\pi r_0^2} \int \int f(r, \theta) \chi_{n,k}^*(r, \theta) \, r \, dr \, d\theta
  \]
  with \( A_{n,k} = A_{-n,k}^* \)
Bessel-Fourier decomposition - Example:
Average states

Glauber b=0 fm:

Glauber b=9 fm:
Bessel-Fourier decomposition - Example: $\varepsilon_2$ modes

Glauber $b=0$ fm:

![Diagram for Glauber $b=0$ fm]

Glauber $b=9$ fm:

![Diagram for Glauber $b=9$ fm]
References I


References III


References IV


References V

