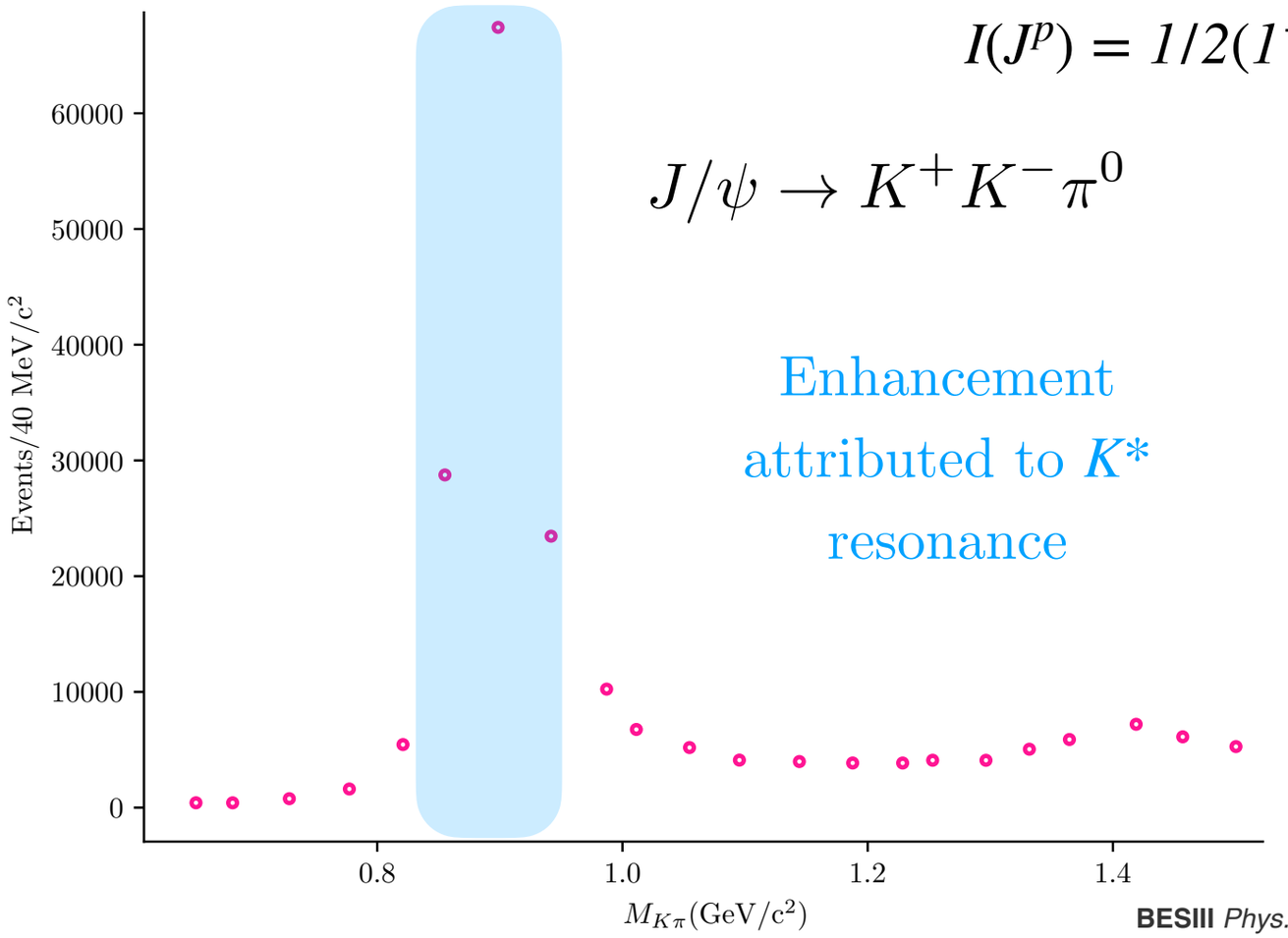
Based on arxiv: 2208.13755

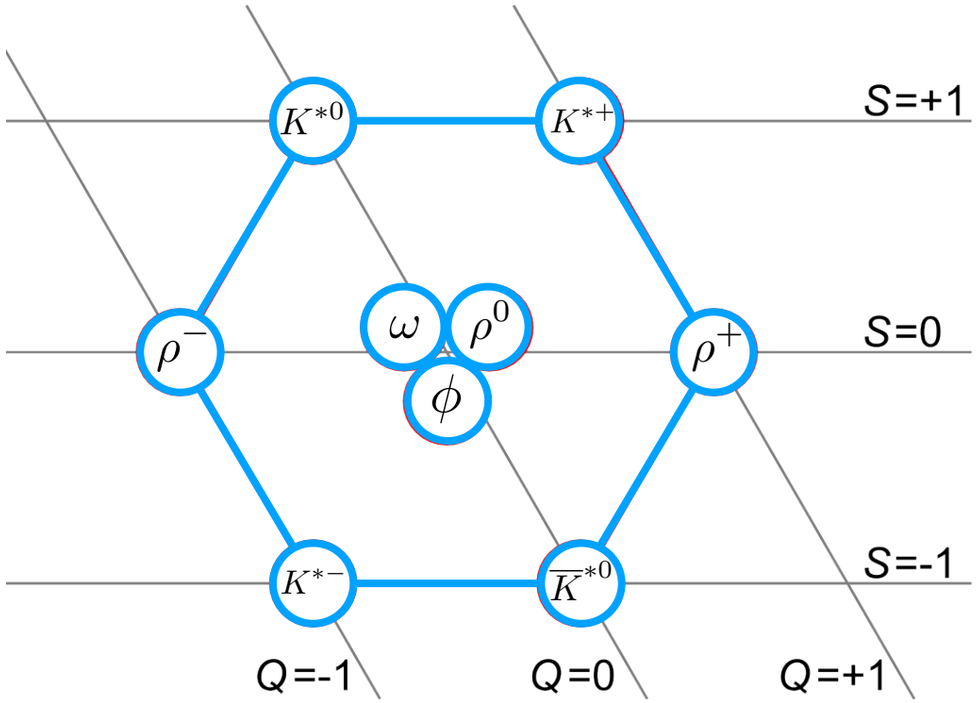
AR, Jozef Dudek, Robert Edwards
(For the HadSpec Collaboration)

Resonant K^* in $K\gamma \to K\pi$ from Lattice QCD

Archana Radhakrishnan
TIFR, Mumbai/William & Mary
QNP, 2022

$K^*(892)$





BESIII *Phys.Rev.D* 100 (2019) 3, 032004

PDG

$K^*(892)$ **MASS**	
CHARGED ONLY, HADROPRODUCED	891.67 ± 0.26 MeV

$K^*(892)$ **Decay Modes**

	Mode	*Fraction* (Γ_i/Γ)	*Scale Factor/ Conf. Level*	*P(MeV/c)*
Γ_1	$K\pi$	$(\sim 100)\%$		289
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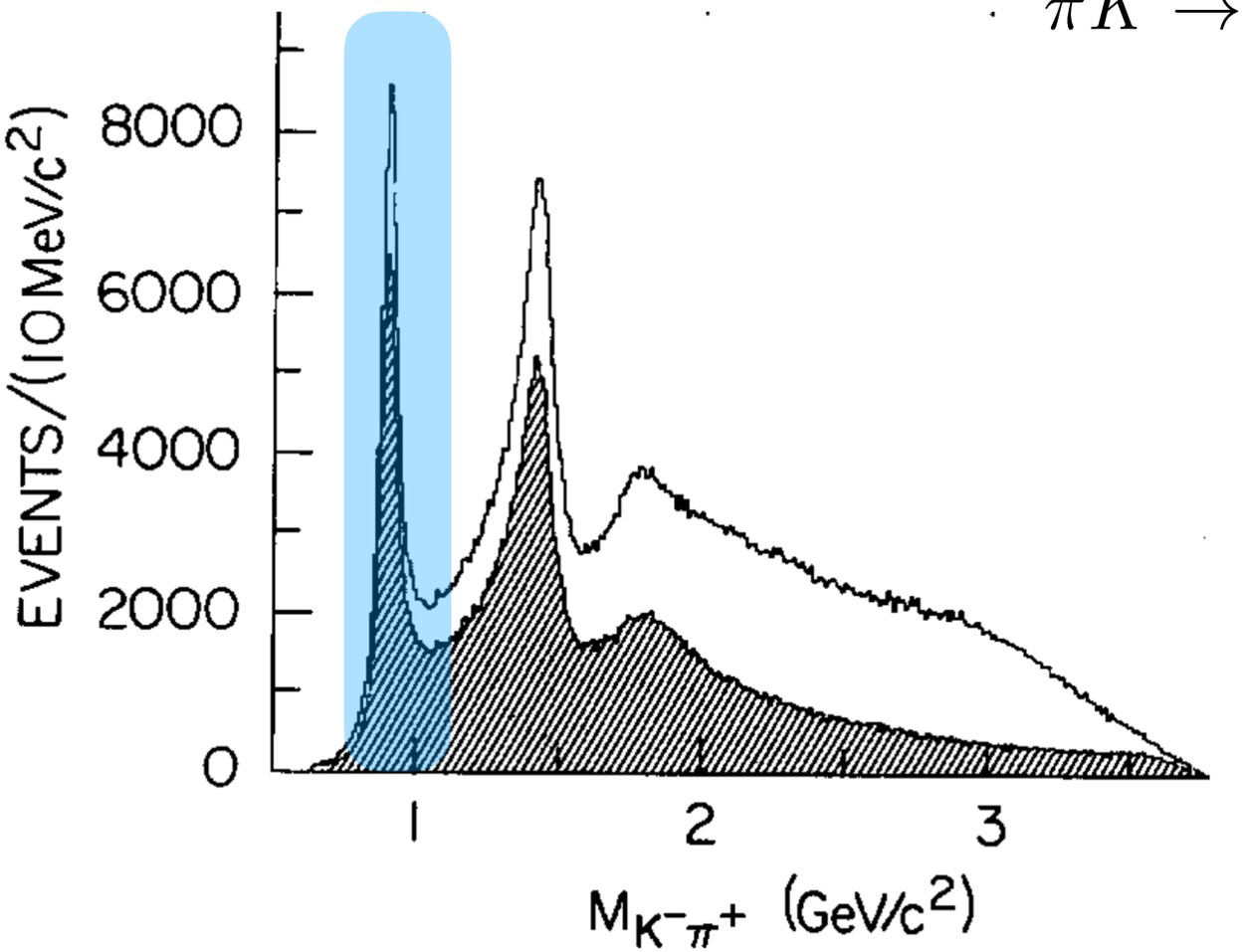
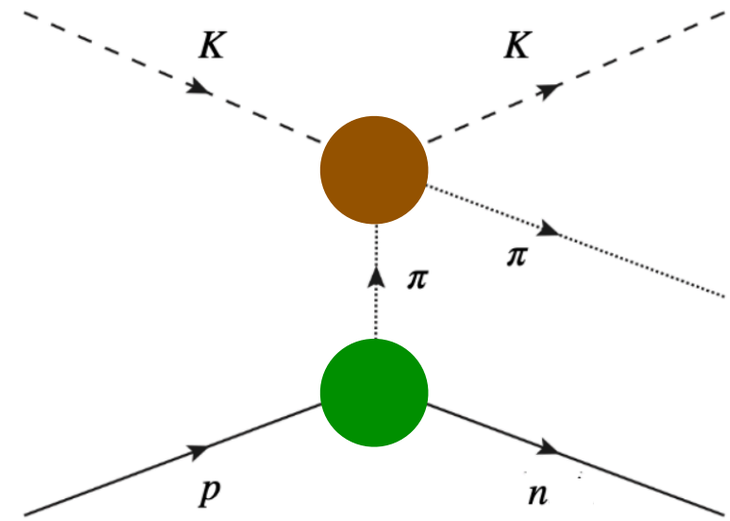
Γ_4	$K^0\gamma$	$(2.46 \pm 0.21) \times 10^{-3}$		307
Γ_5	$K^\pm\gamma$	$(9.8 \pm 0.9) \times 10^{-4}$		309

$K^*(892)$

$\pi K \to K^* \to \pi K$





LASS,SLAC 11 GeV/c

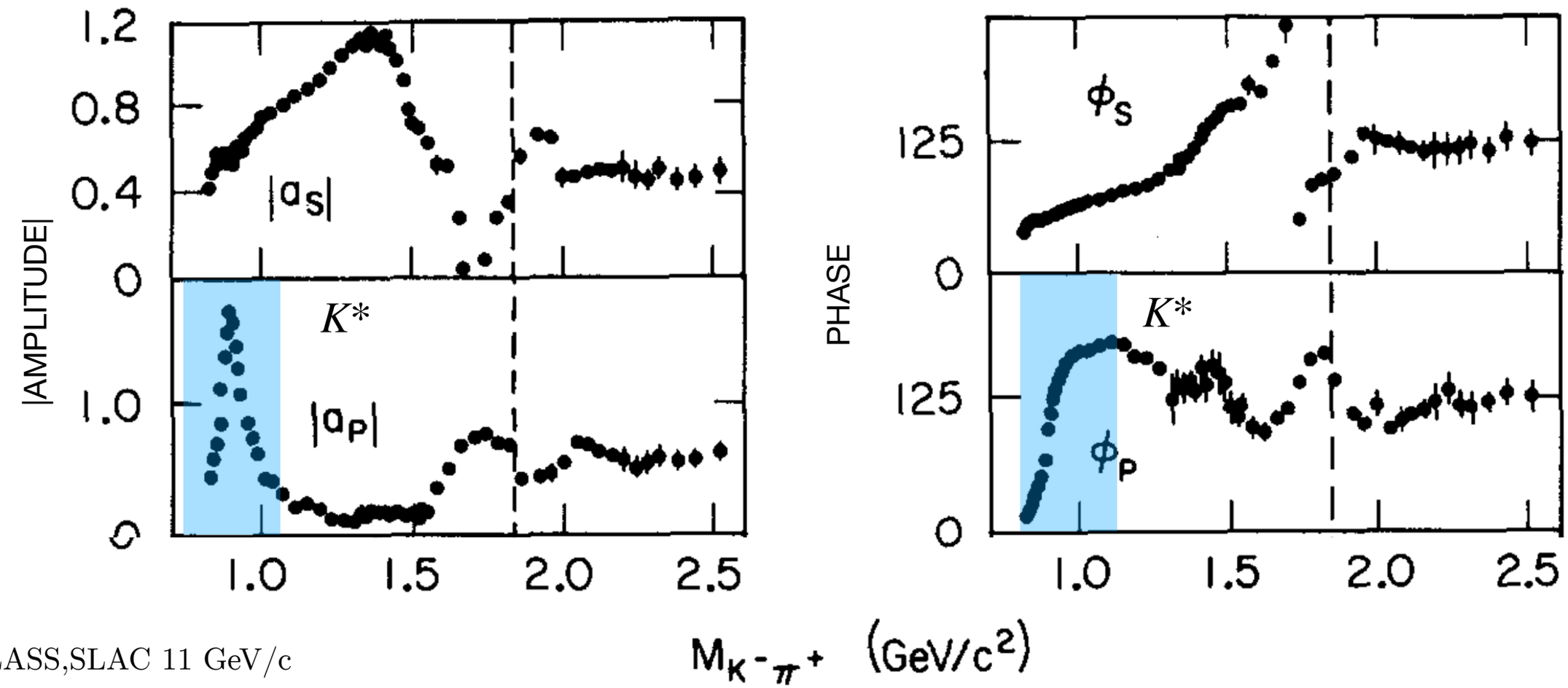
A STUDY OF $K^-\pi^+$ SCATTERING IN THE REACTION $K^-p \to K^-\pi^+n$ AT 11 GeV/c *

D. ASTON[1], N. AWAJI[2], T. BIENZ[1], F. BIRD[1], J. D'AMORE[3], W. DUNWOODIE[1], R. ENDORF[3], K. FUJII[2a], H. HAYASHII[2b], S. IWATA[2], W.B. JOHNSON[1], R. KAJIKAWA[2], P. KUNZ[1], D.W.G.S. LEITH[1], L. LEVINSON[1c], T. MATSUI[2a], B.T. MEADOWS[3], A. MIYAMOTO[2a], M. NUSSBAUM[3], H. OZAKI[2], C.O. PAK[2a], B. N. RATCLIFF[1], D. SCHULTZ[1], S. SHAPIRO[1], T. SHIMOMURA[2], P.K. SINERVO[1d], A. SUGIYAMA[2], S. SUZUKI[2], G. TARNOPOLSKY[1e], T. TAUCHI[2a], N. TOGE[1], K. UKAI[4], A. WAITE[1f] and S. WILLIAMS[1g]

[1]*Stanford Linear Accelerator Center, Stanford University, P.O. Box 4349, Stanford, California 94305, USA*
[2]*Department of Physics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464, Japan*
[3]*University of Cincinnati, Cincinnati, Ohio 45221, USA*
[4]*Institute for Nuclear Study, University of Tokyo, Midori-cho, Tanashi, Tokyo 188, Japan*

Received 13 July 1987

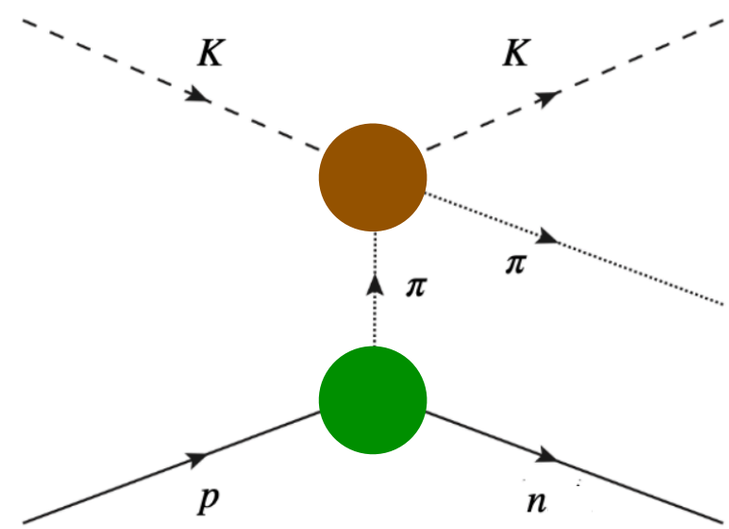
$K^*(892)$



LASS,SLAC 11 GeV/c

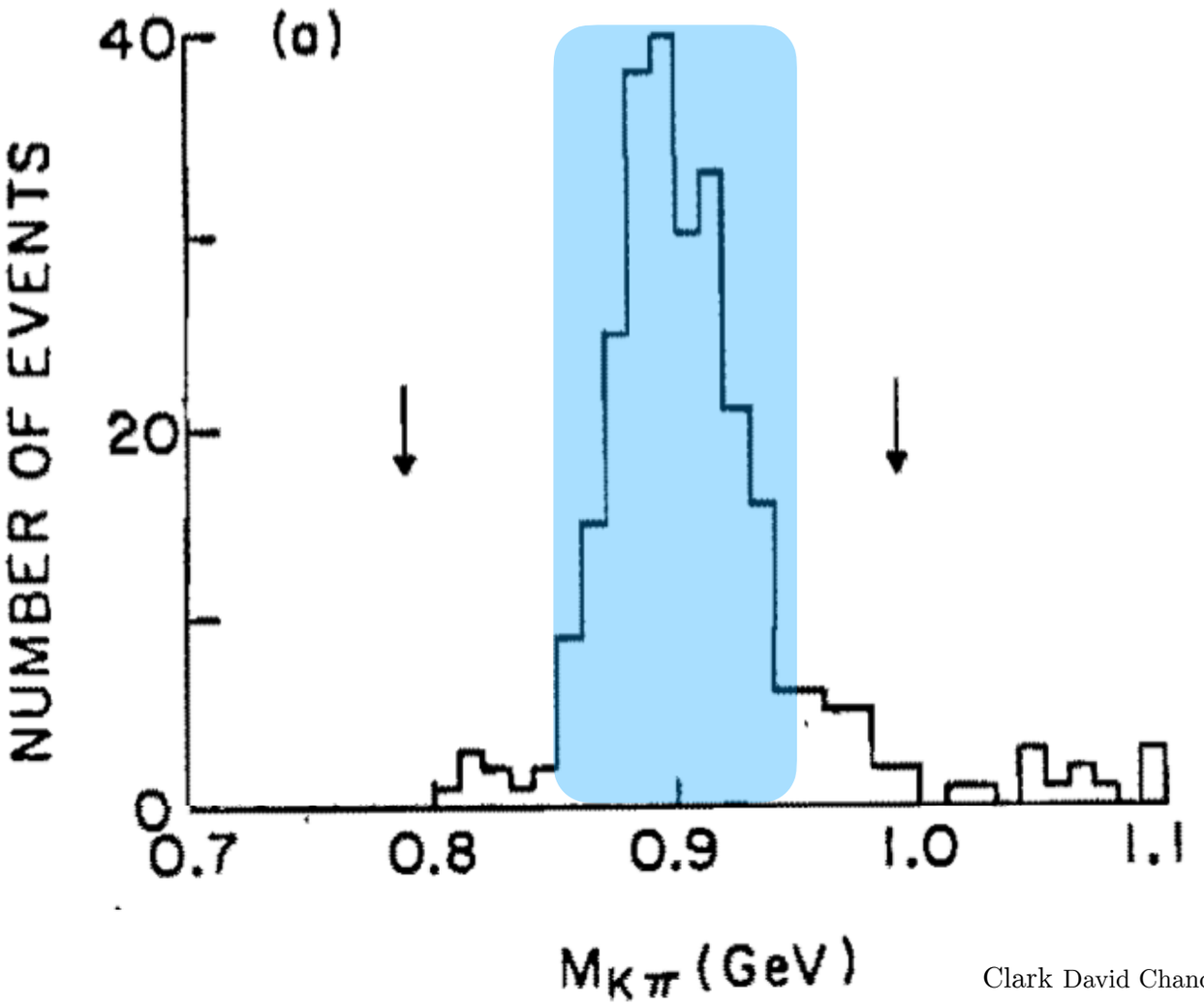
Partial wave projected amplitude:

$$\mathcal{M}_\ell(s) = \int_{-1}^{1} d(\cos\theta)\mathcal{M}(s,\theta)P_\ell(\cos\theta)$$

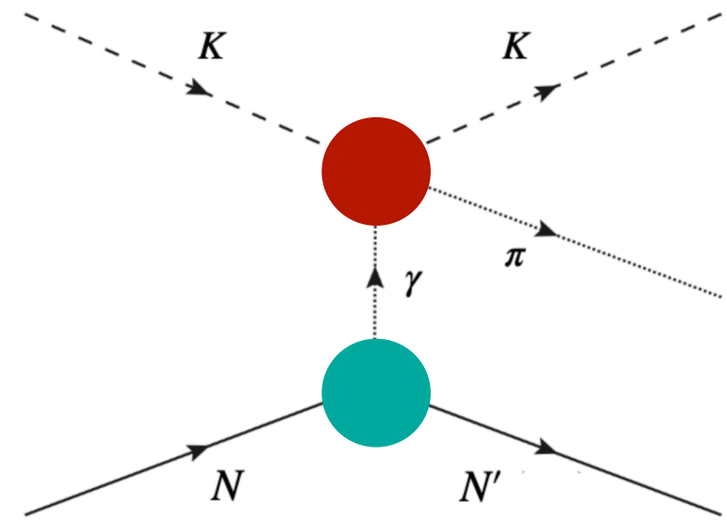


$K^*(892)$

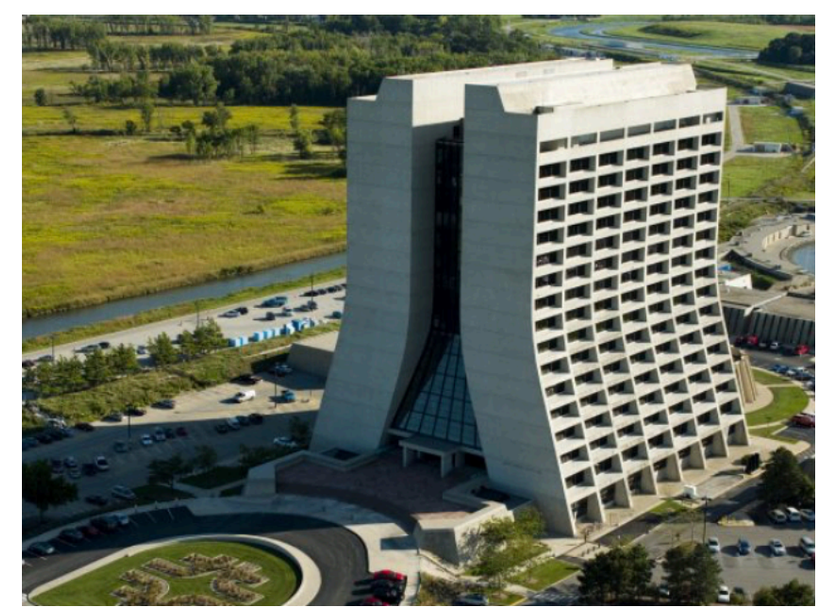
$$\gamma K \to K^* \to \pi K$$



Clark David Chandlee, Thesis



Fermilab, E272 200 GeV/c

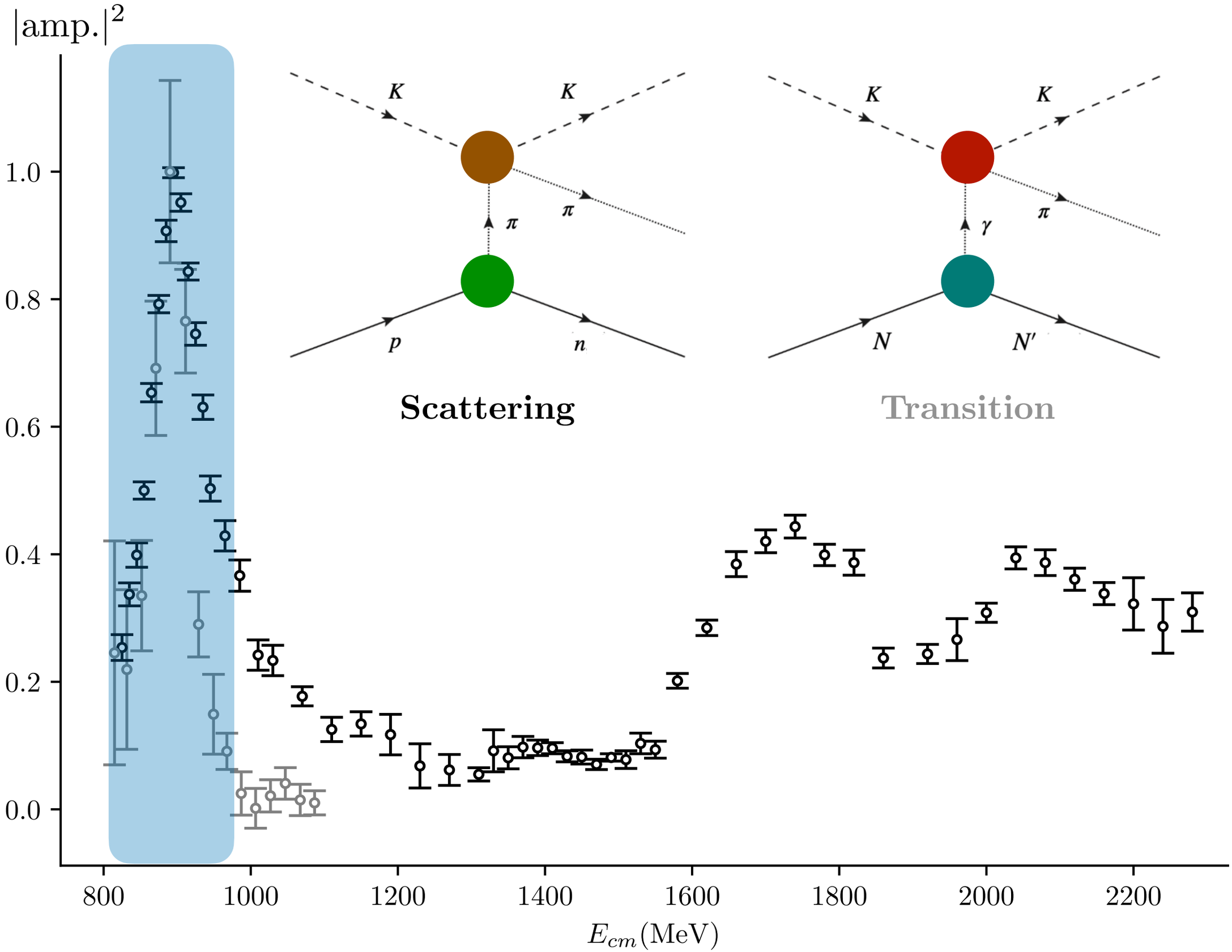


VOLUME 51, NUMBER 3 PHYSICAL REVIEW LETTERS 18 JULY 1983

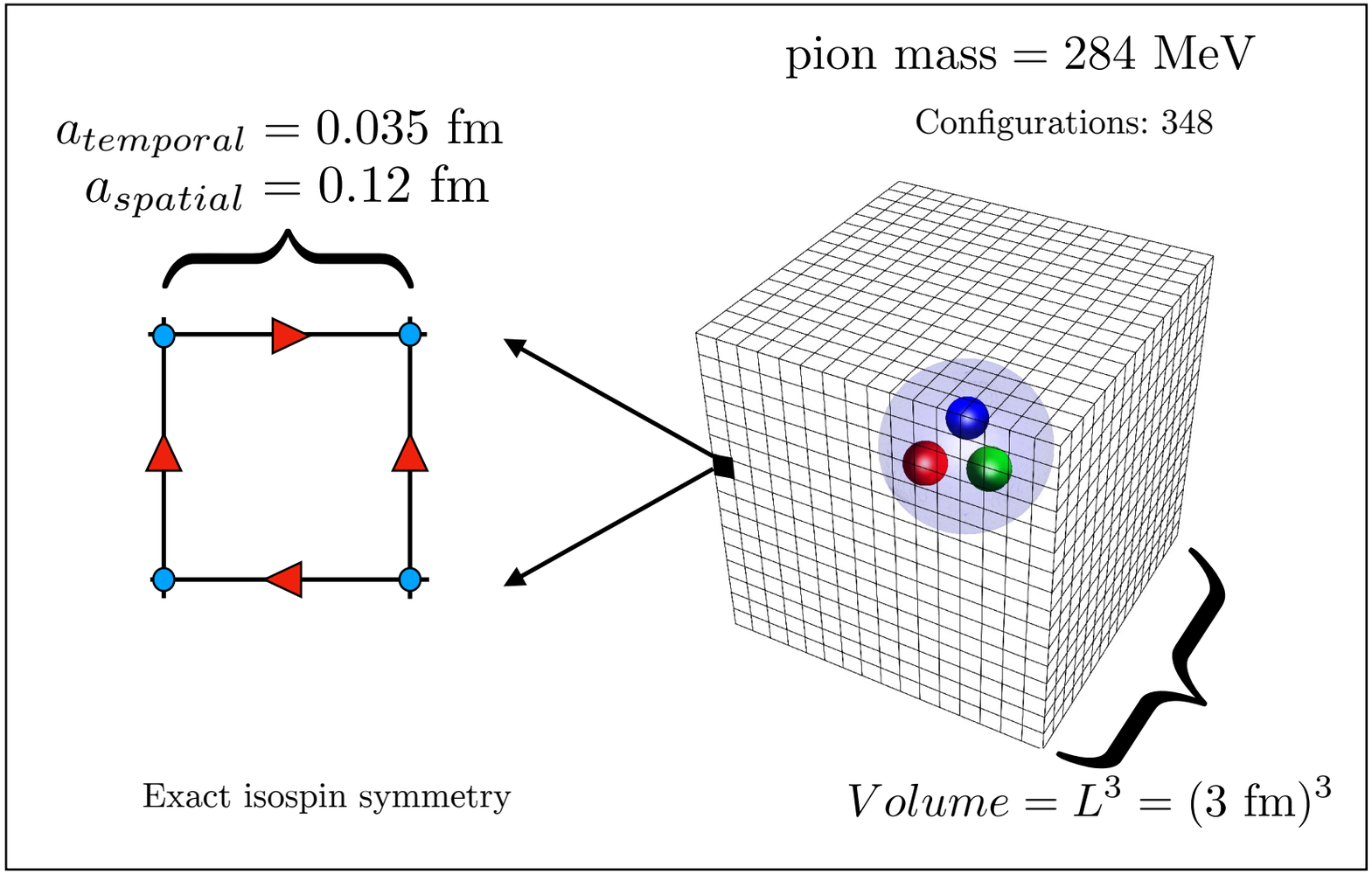
Measurement of the Radiative Width of the $K^{*+}(890)$

C. Chandlee, D. Berg, S. Cihangir, B. Collick, T. Ferbel, S. Heppelmann, J. Huston, T. Jensen, A. Jonckheere, F. Lobkowicz, Y. Makdisi, M. Marshak, M. McLaughlin, C. Nelson, T. Ohshima, E. Peterson, K. Ruddick, P. Slattery, P. Thompson, and M. Zielinski
Fermi National Accelerator Laboratory, Batavia, Illinois 60510, and University of Minnesota, Minneapolis, Minnesota 55455, and University of Rochester, Rochester, New York 14627
(Received 18 March 1983)

$K^*(892)$


|amp.|²
1.0
0.8
0.6
0.4
0.2
0.0
800
1000
1200
1400
1600
1800
2000
2200
E_cm (MeV)
K
K
π
π
p
n
Scattering
K
K
γ
π
N
N′
Transition

Lattice QCD



most of the quantities in this talk will be represented in temporal lattice units a_t

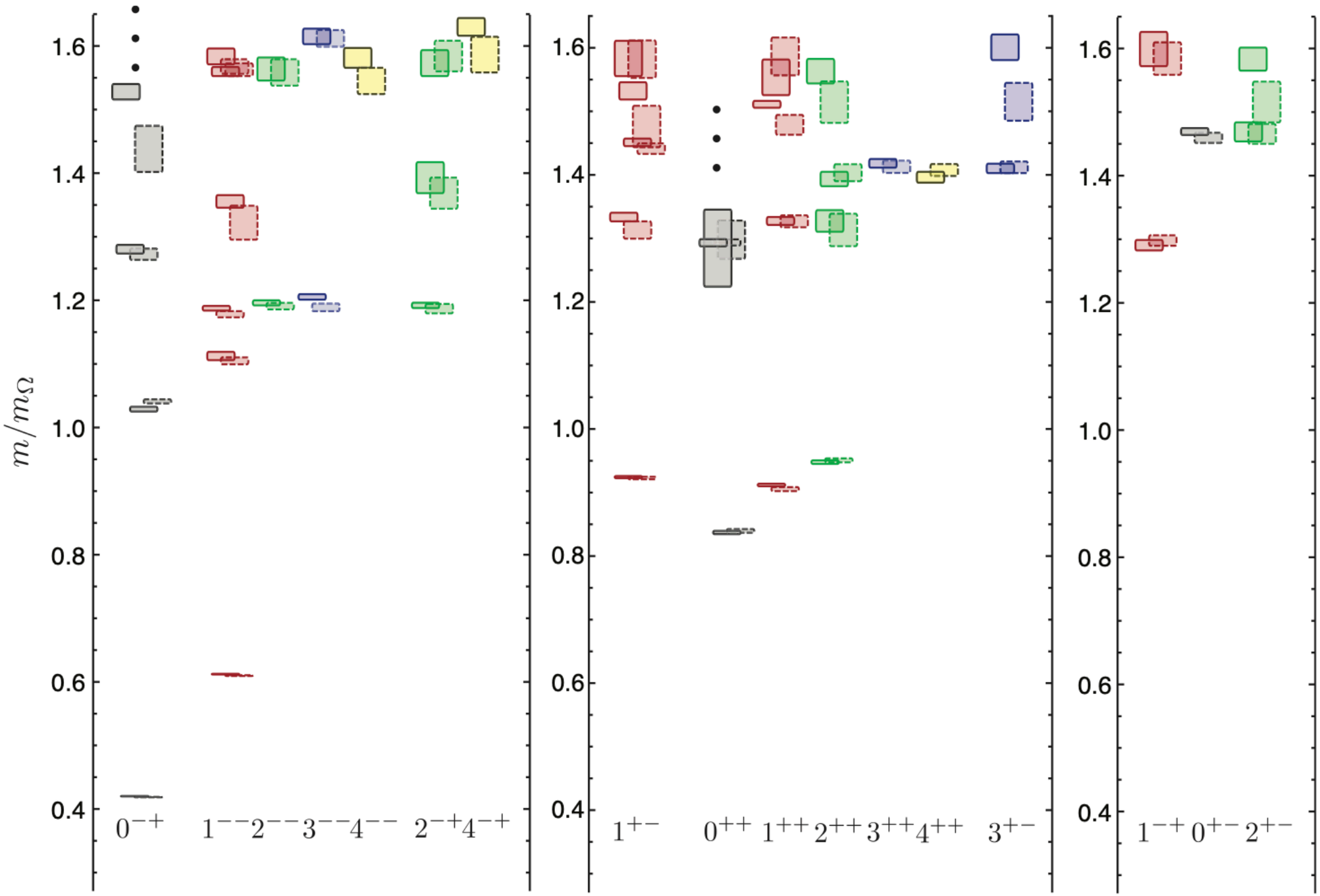
Numerical approach to solve QCD using QCD path integral

$$\int DA_\mu D\psi D\overline{\psi} f(\psi, \overline{\psi}, A_\mu) e^{-S_{\text{Euc.}}}$$

- Uses Monte-Carlo to sampling over configurations to perform the integral
  - calculate correlation functions as average over many configurations

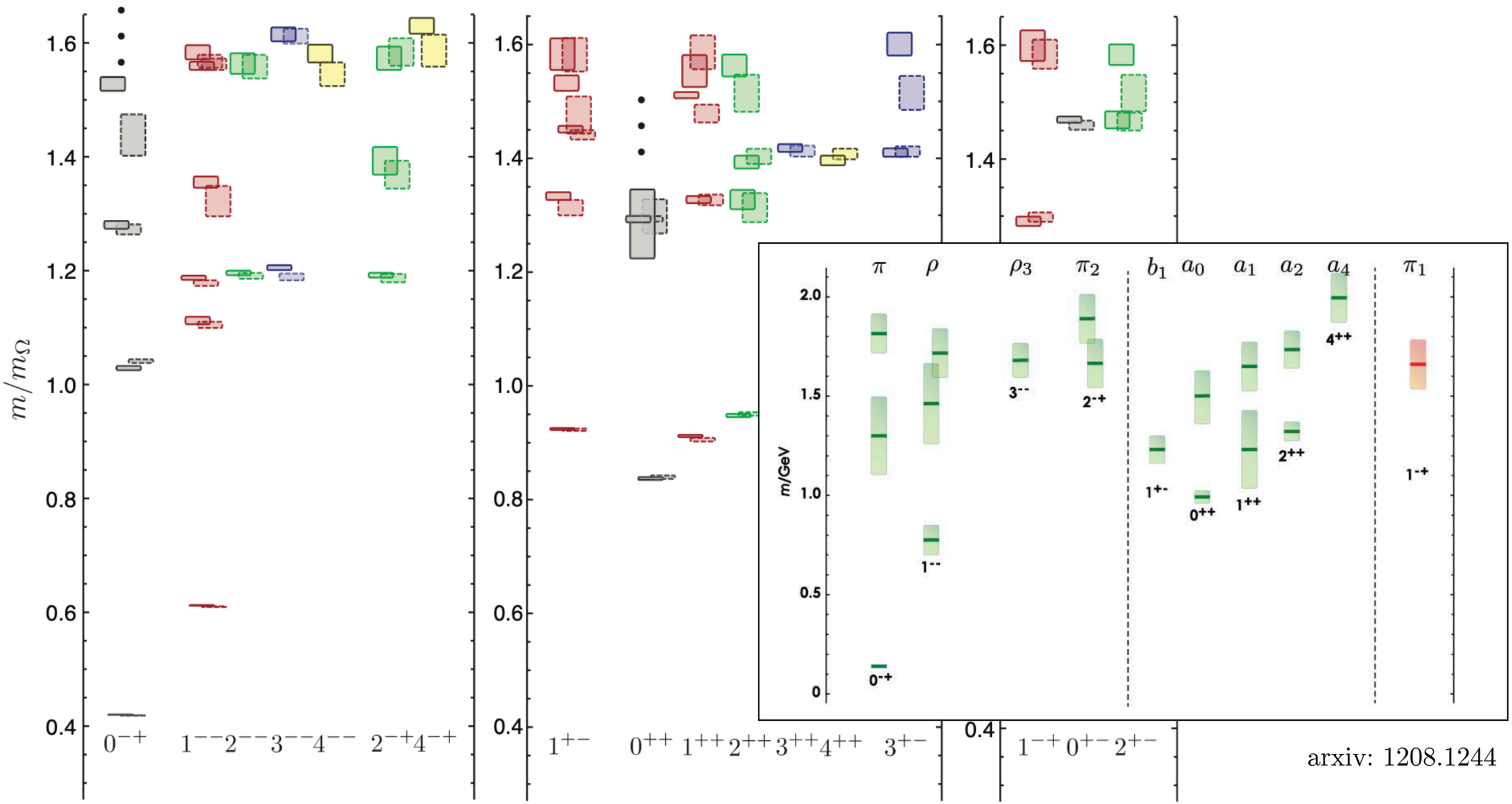
Finite volume used as a tool

Isovector meson spectrum from Lattice QCD



Phys.Rev.D82:034508,2010

Isovector meson spectrum from Lattice QCD



arxiv: 1208.1244

Phys.Rev.D82:034508,2010

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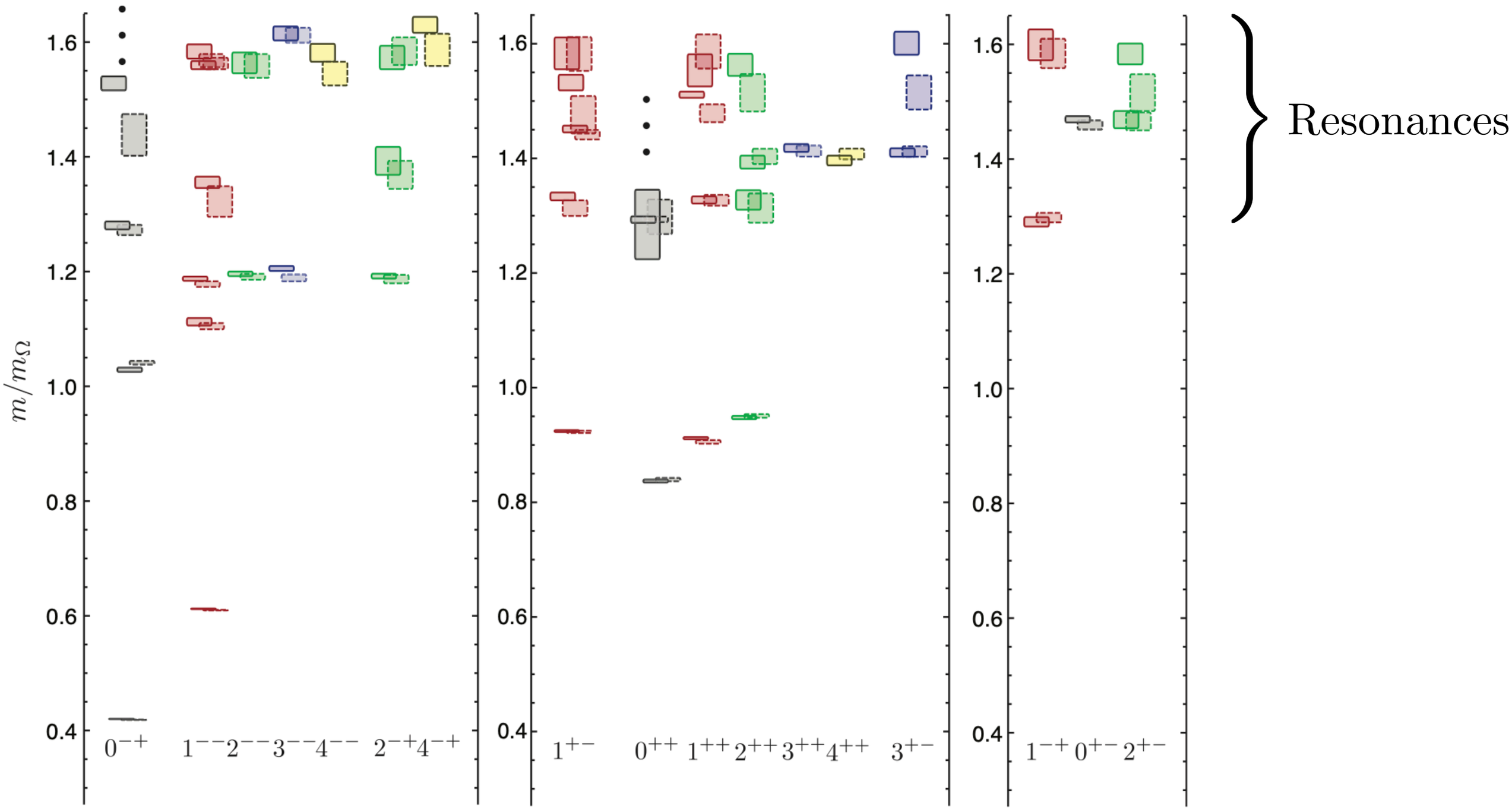
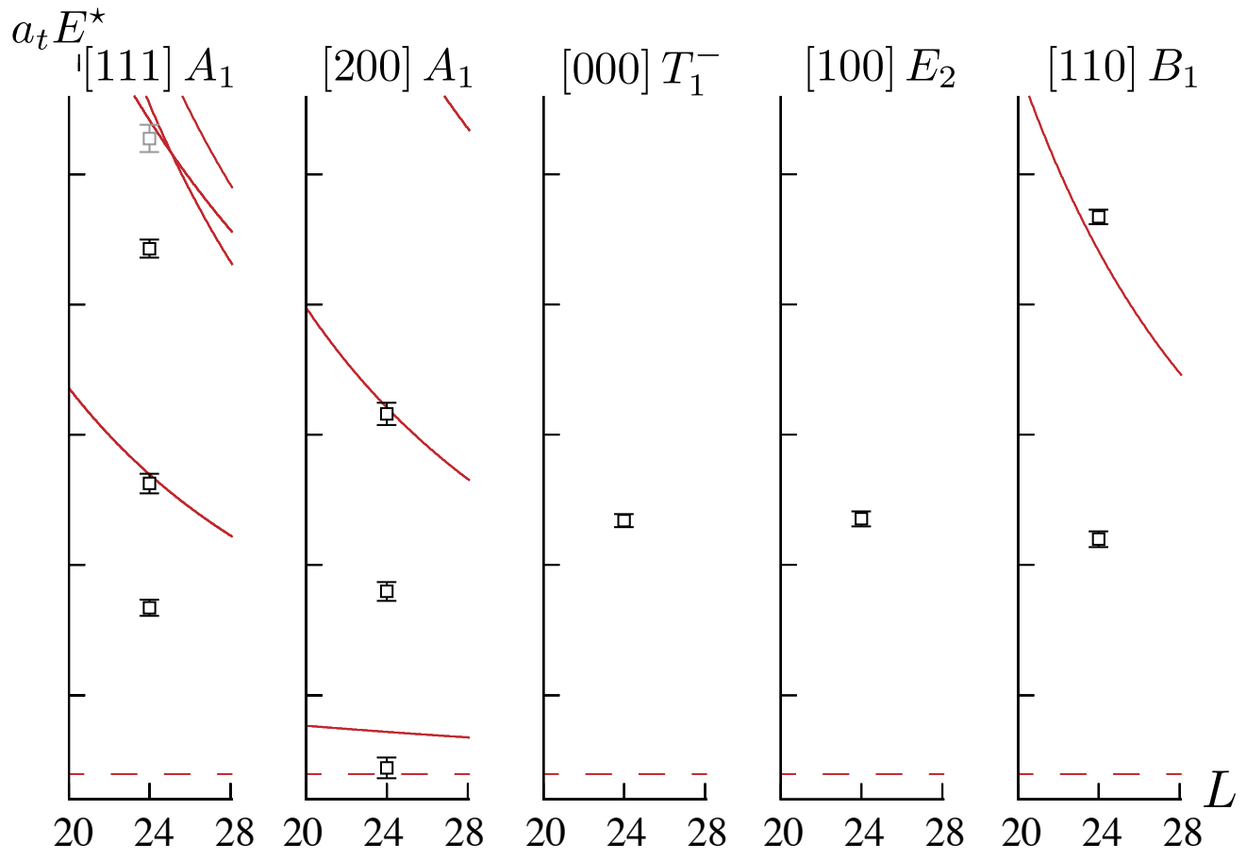
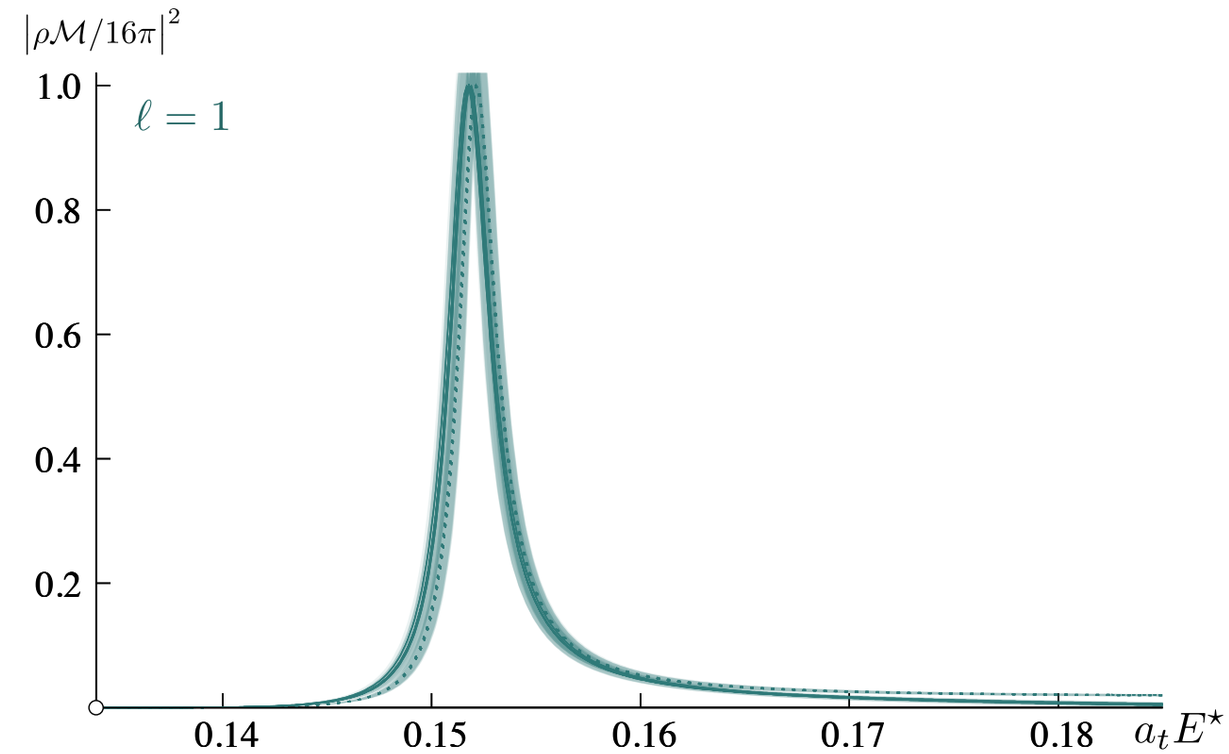


Phys.Rev.D82:034508,2010

Resonances from Lattice QCD

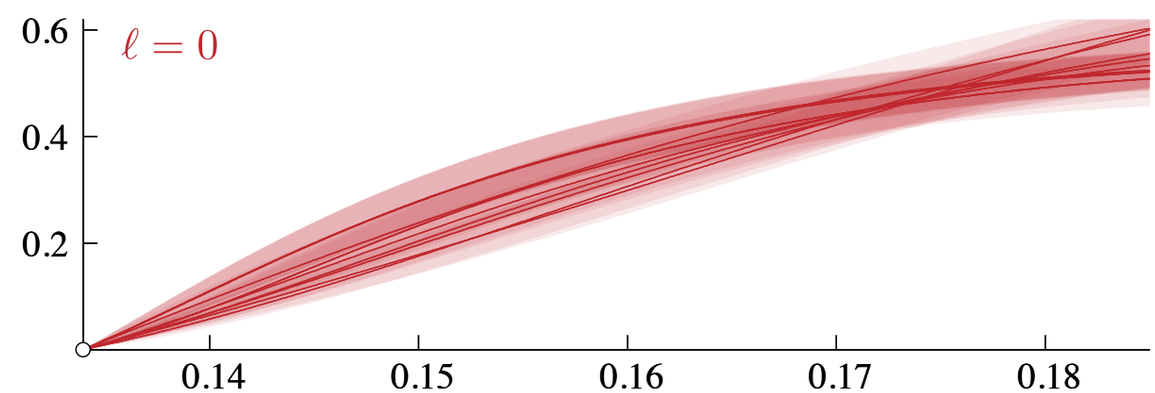
arxiv: 2208.13755





Lüscher formalism

$$\det\left[F^{-1} + \mathcal{M}\right] = 0$$

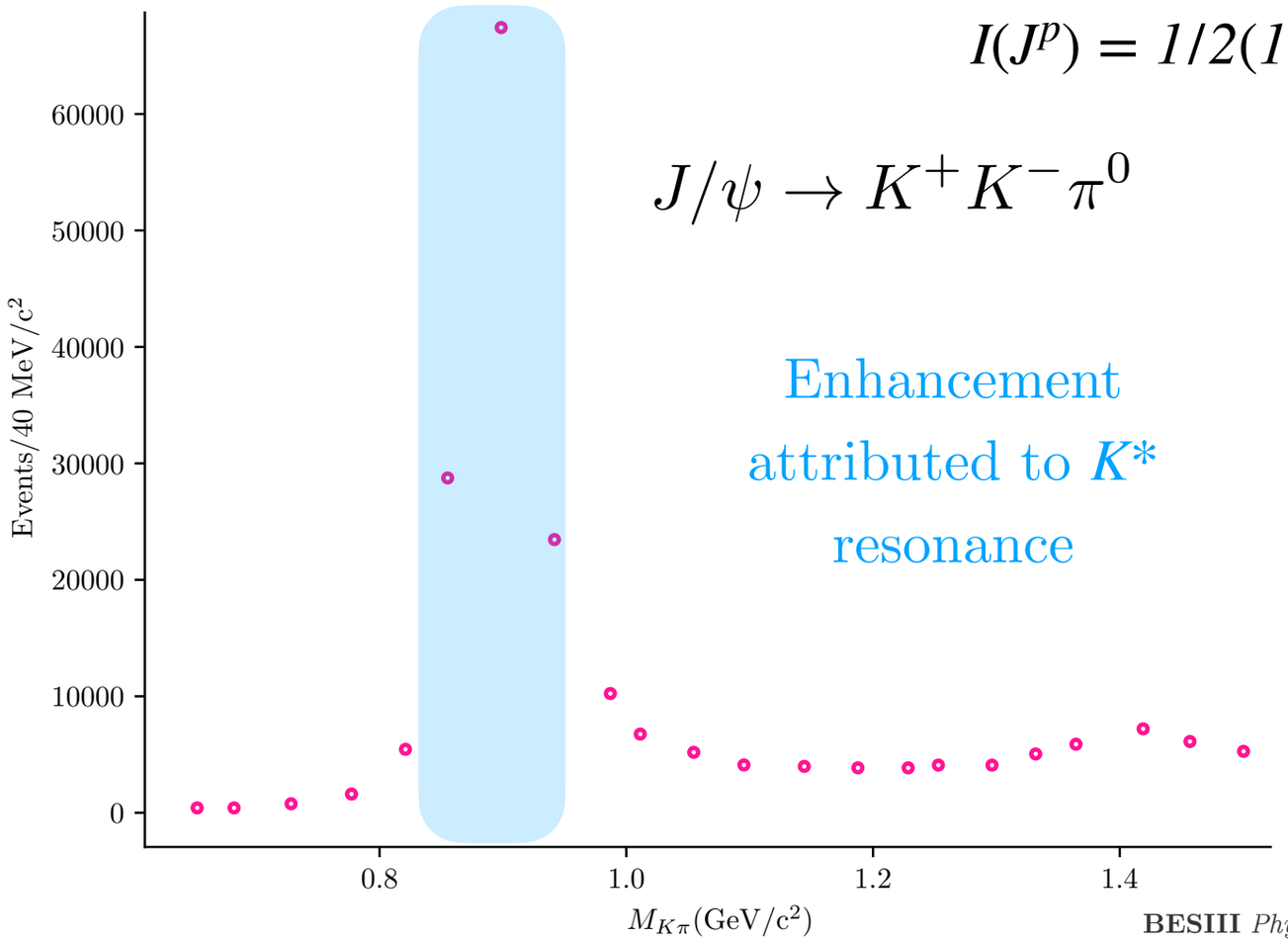


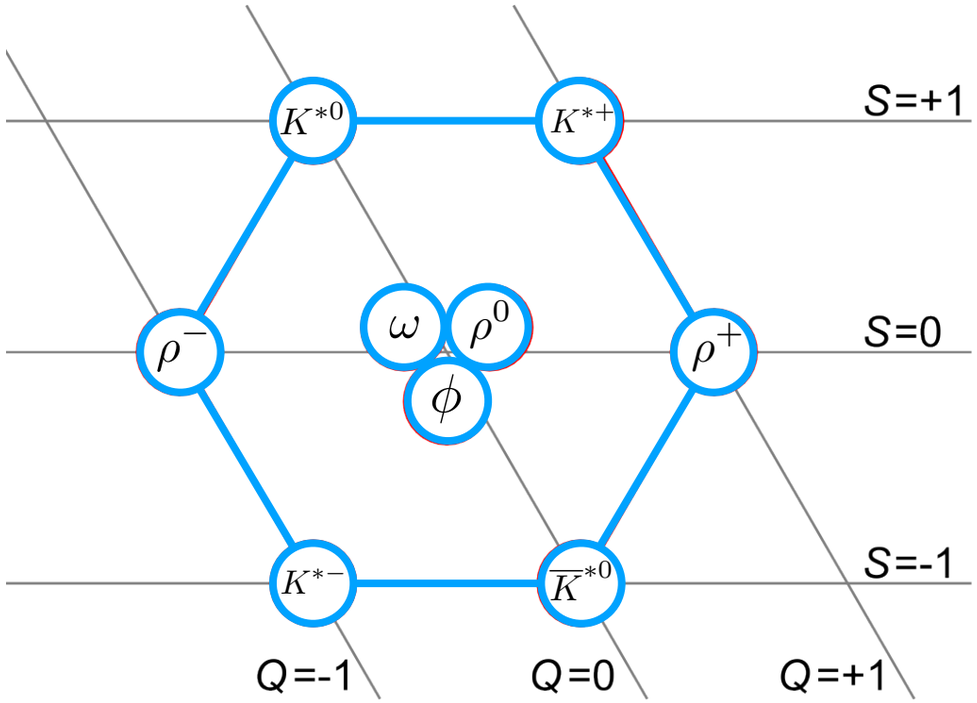
Review: RevModPhys.90.025001

Next step: Photo-couplings

from Lattice QCD

$K^*(892)$





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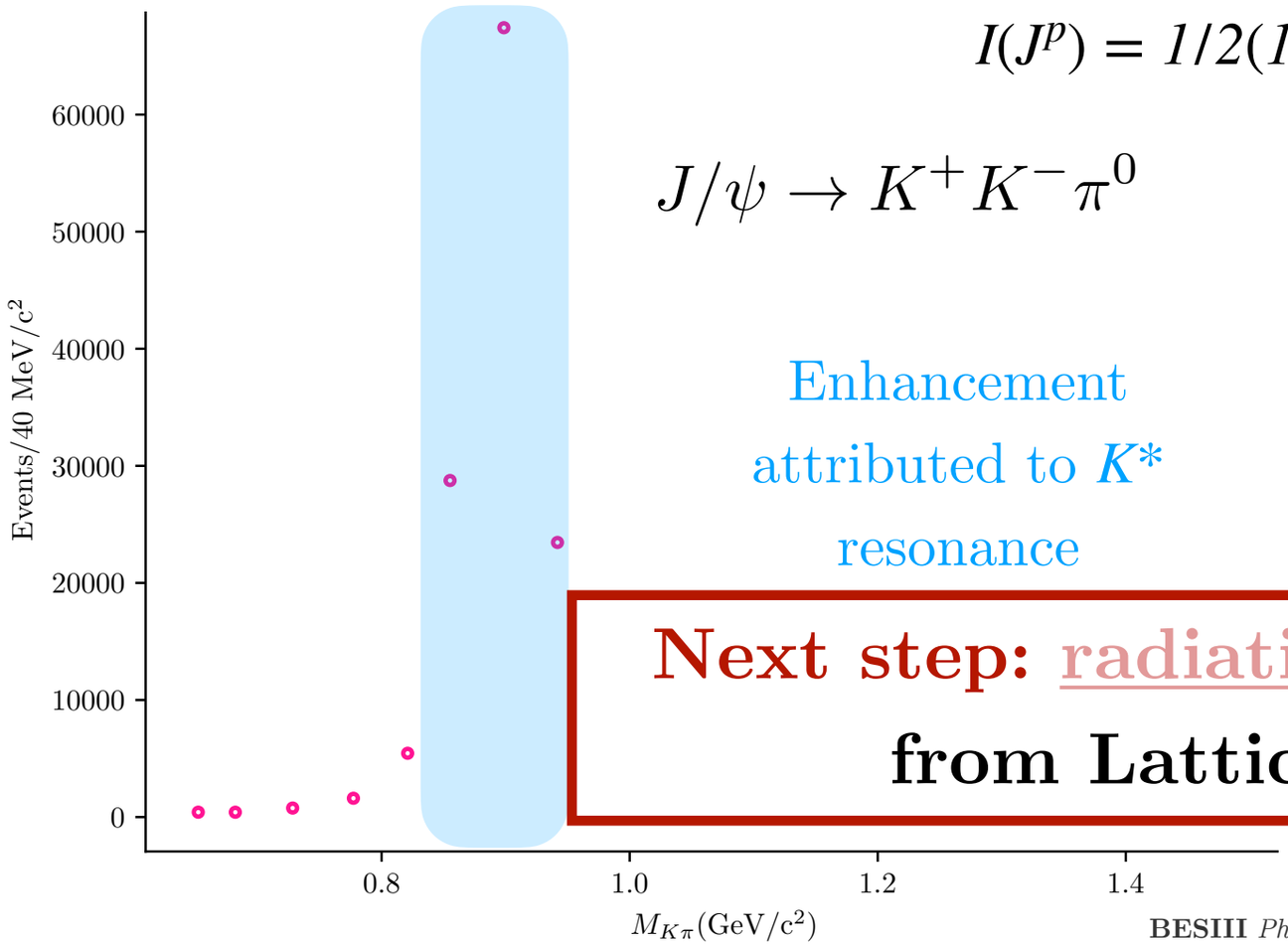
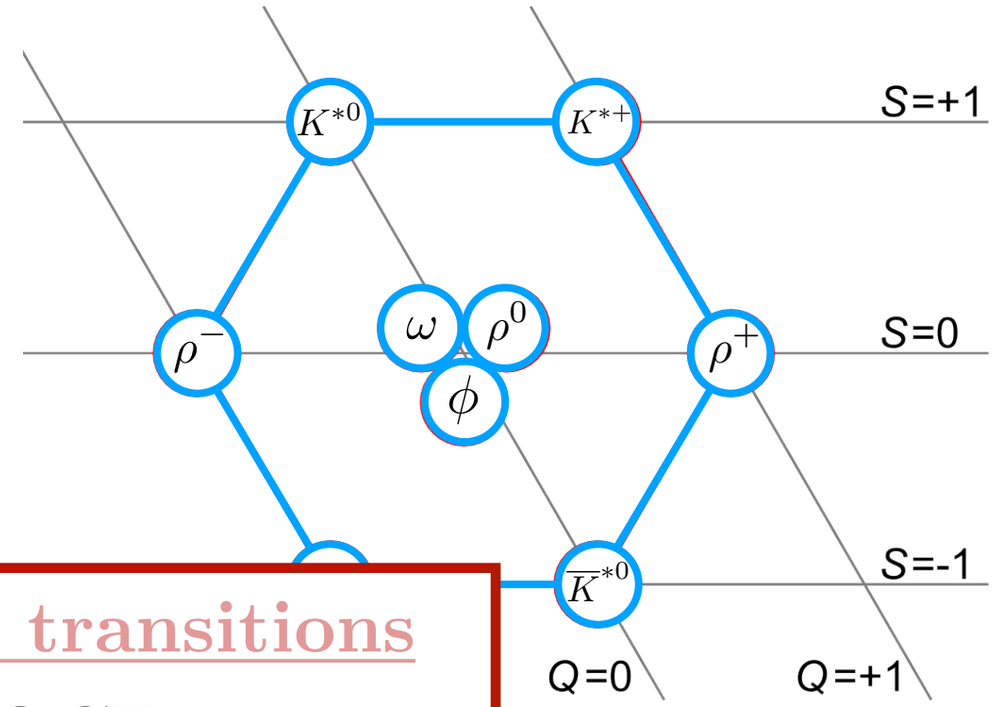


$I(J^p) = 1/2(1^-)$

$$J/\psi \to K^+K^-\pi^0$$

Enhancement attributed to K^* resonance



Next step: radiative transitions from Lattice QCD

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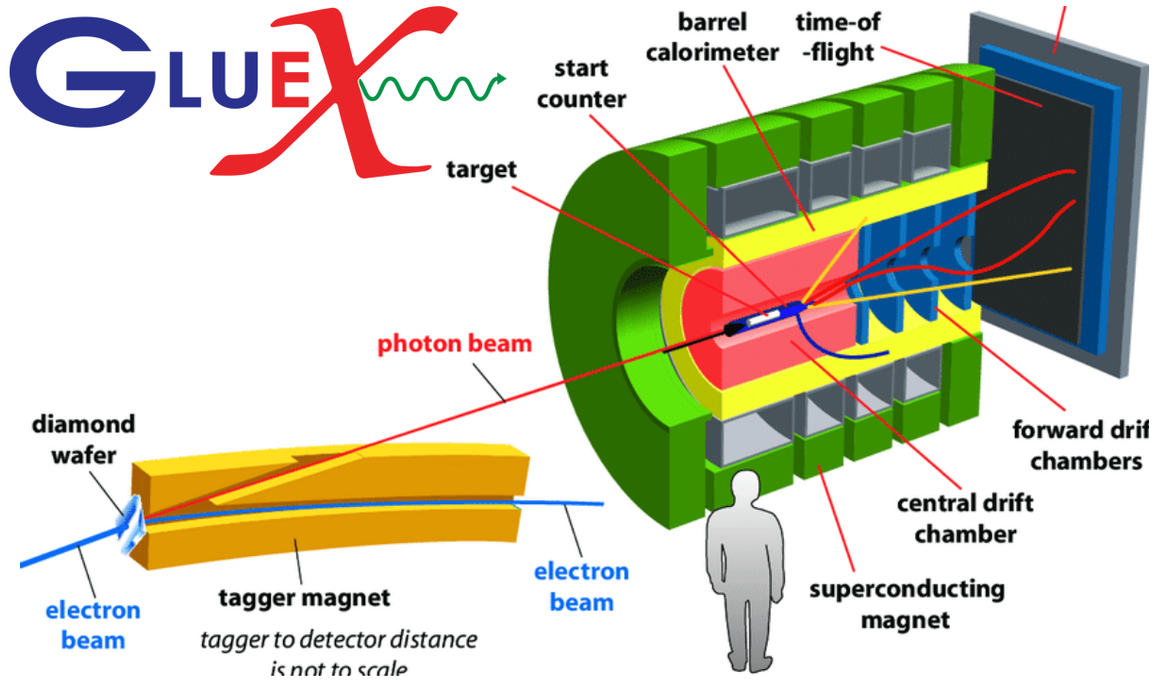
Meson spectrum from experiments

Calculations from first principles

QCD can be useful



$\langle \text{Meson } \gamma | \text{Resonance} \rangle$

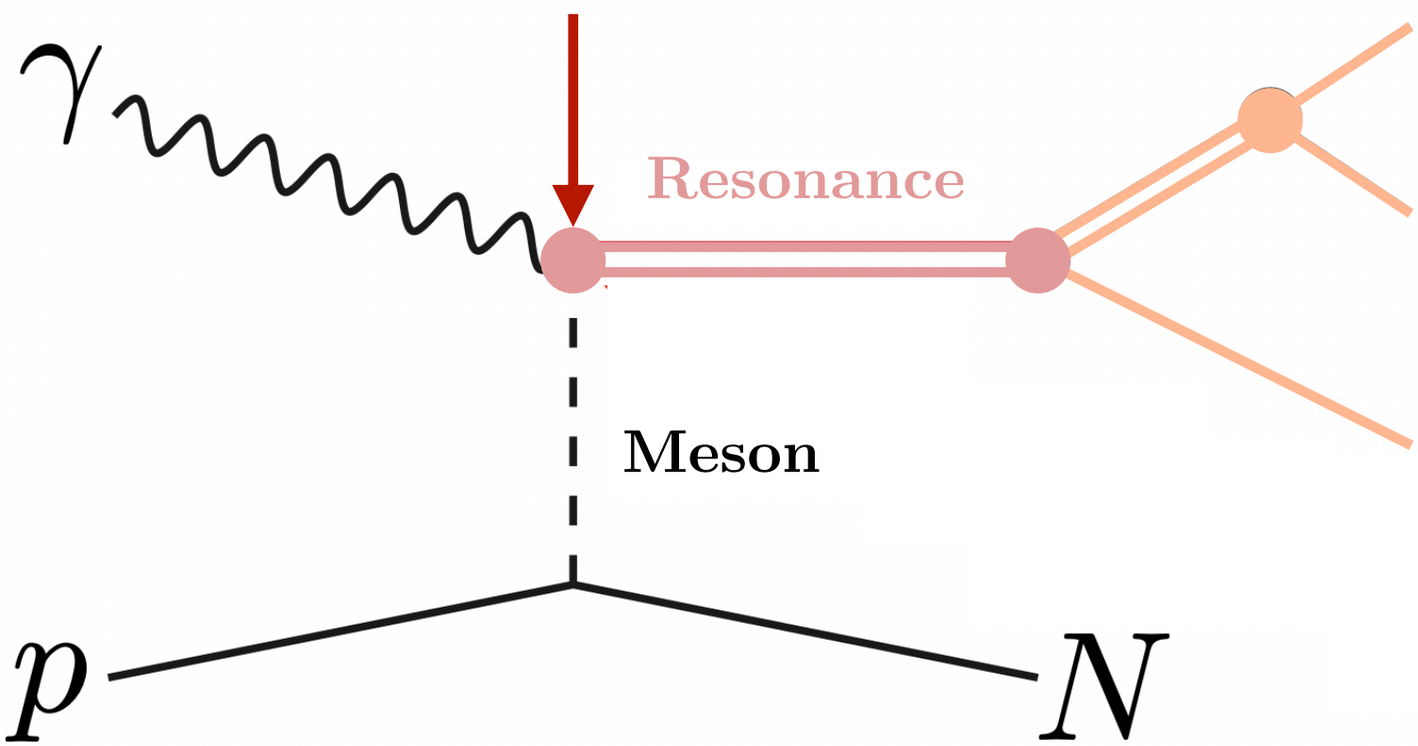
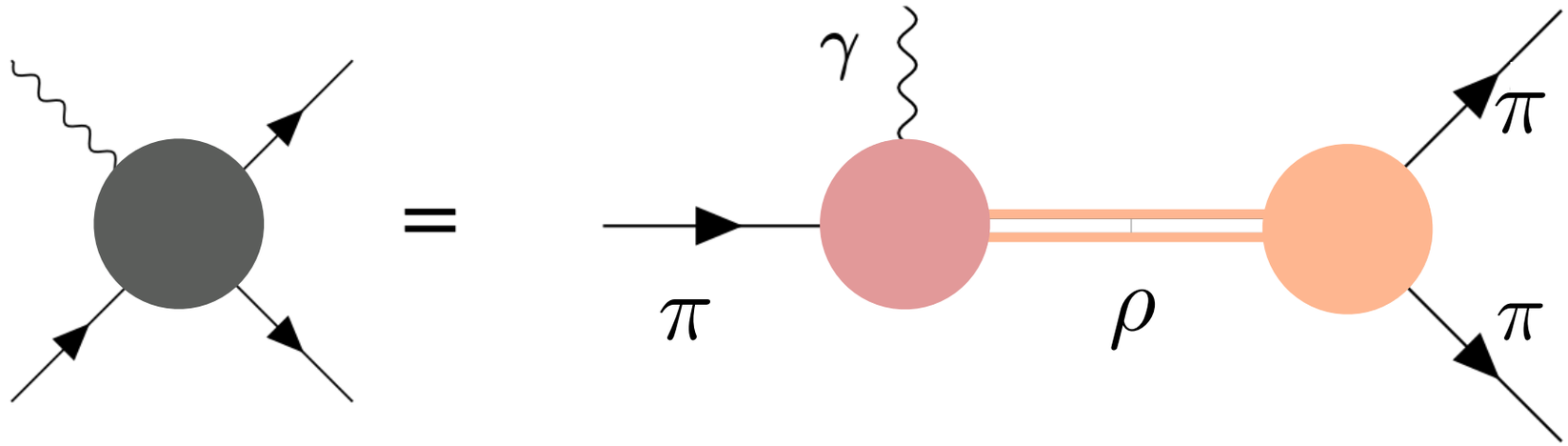
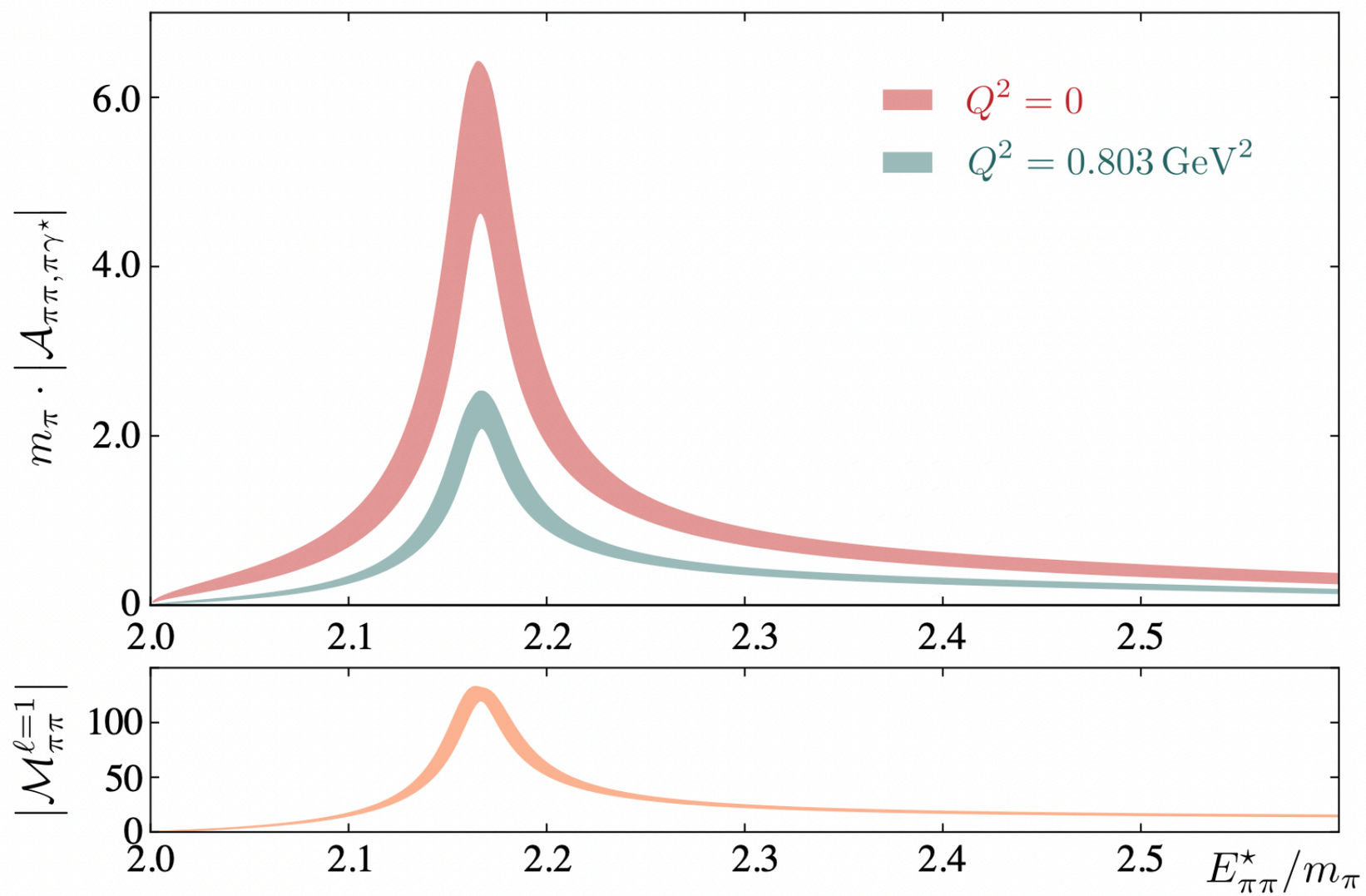


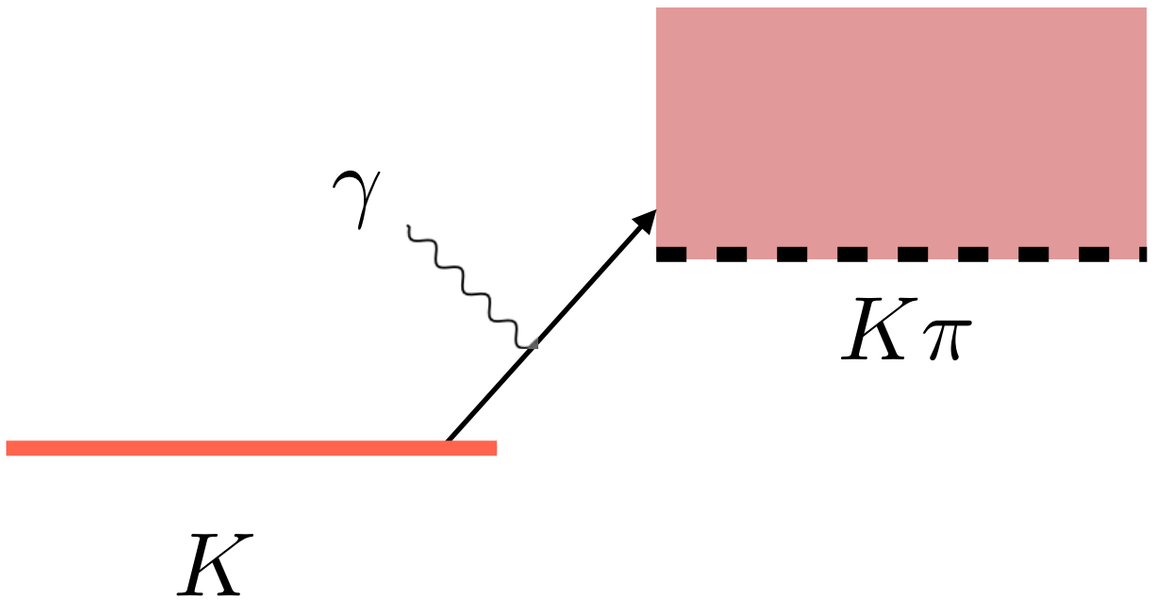
Photo-coupling from Lattice QCD

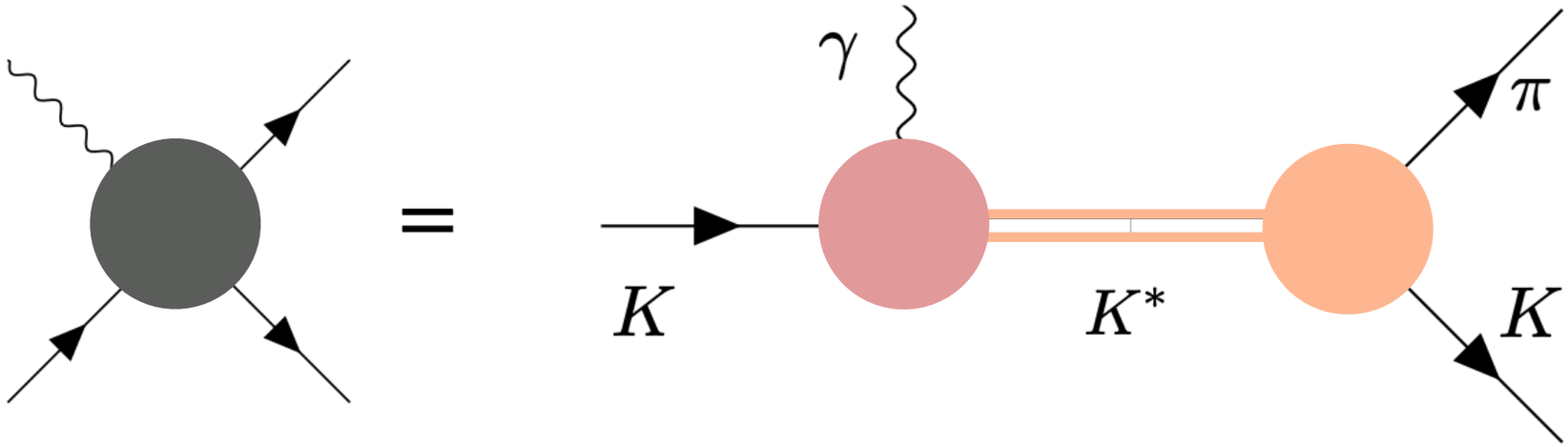




Phys. Rev. D 93, 114508 (2016)

$K\pi$ radiative transition from QCD





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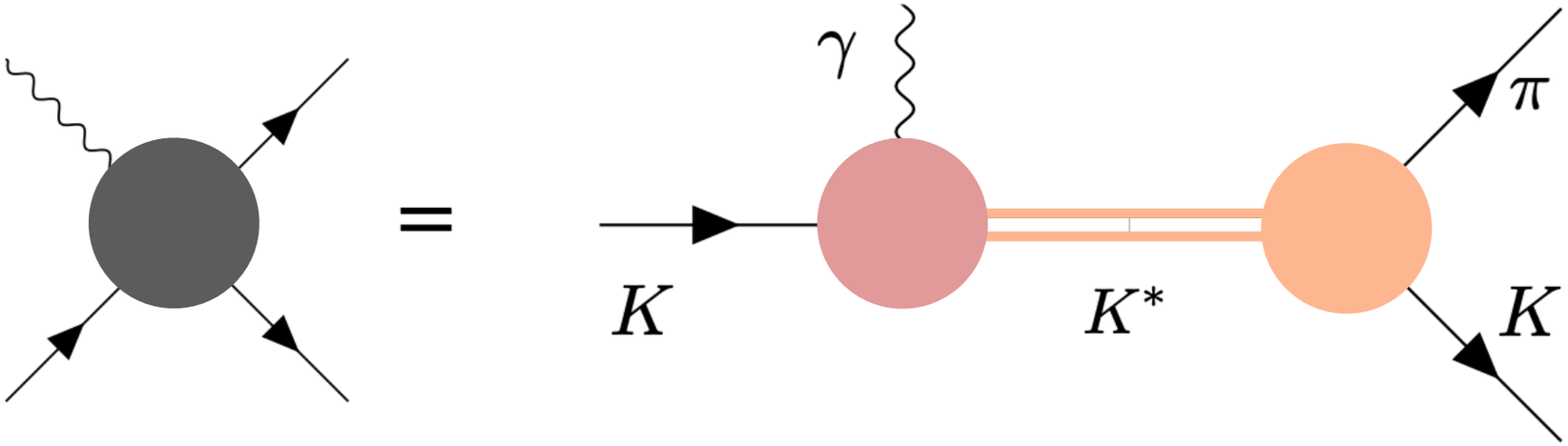


strong re-scattering in the final state

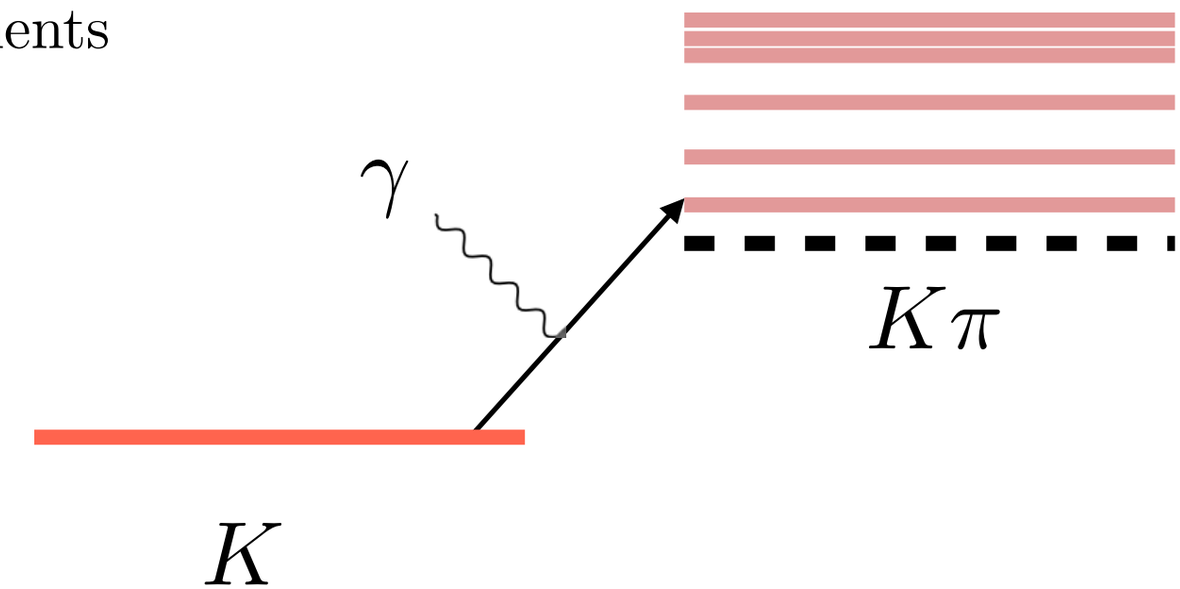
$$\mathcal{H}(Q^2, E^\star) = \mathcal{A}(Q^2, E^\star) \cdot \frac{1}{k^\star} \cdot \mathcal{M}(E^\star)$$

Watson's theorem

The transition amplitude contains information about the scattering amplitude

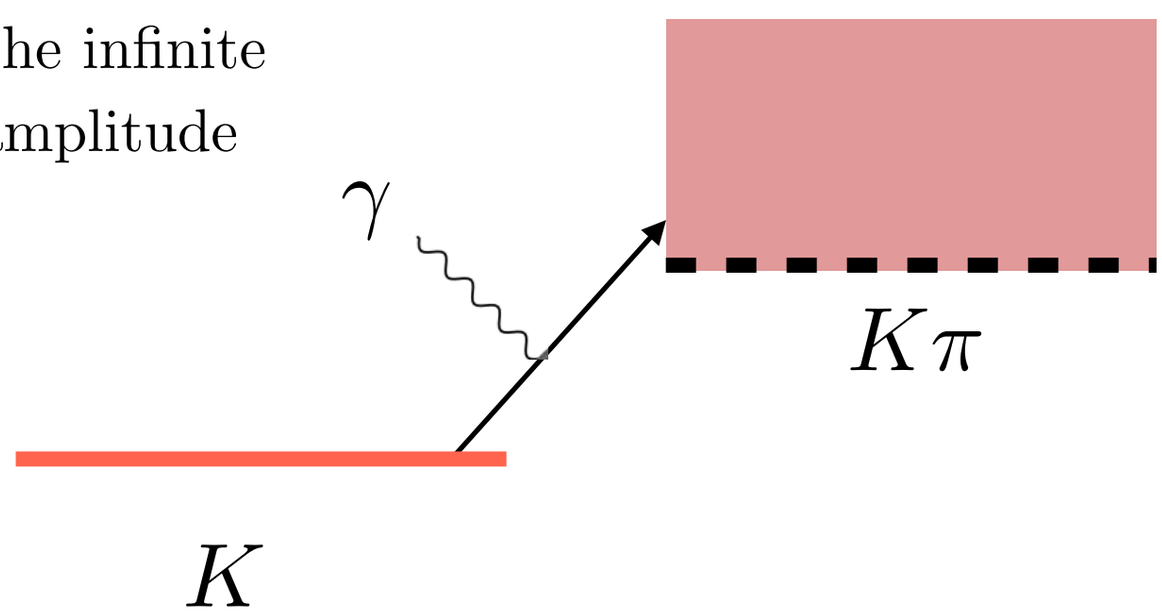
$K\pi$ radiative transition from Lattice QCD

Three-point functions on the lattice - get the finite volume matrix elements



$$|_L\langle K|j^\mu(0)|K\pi\rangle_L|$$

have to map the finite volume matrix elements to the infinite volume transition amplitude



$\mathcal{H}$

$K\pi$ radiative transition from Lattice QCD

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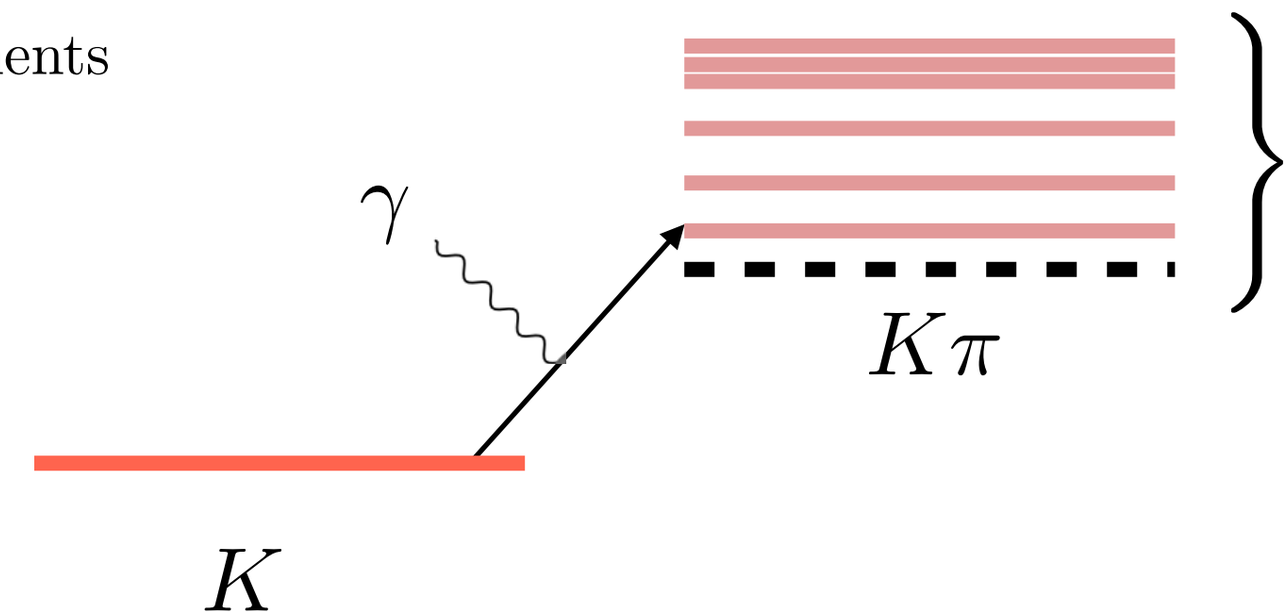
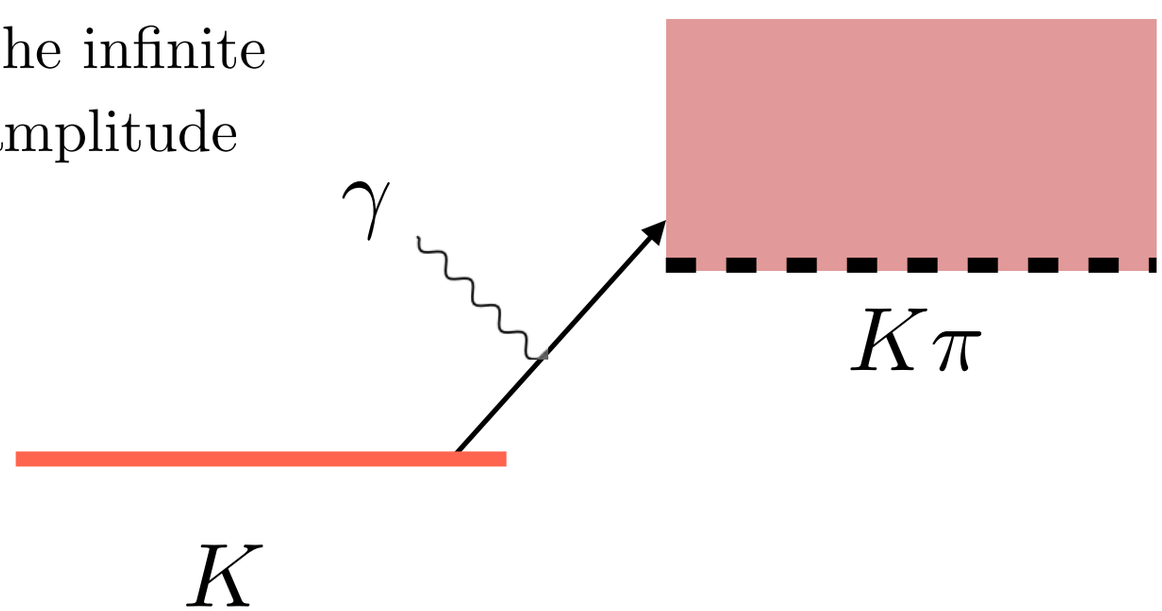


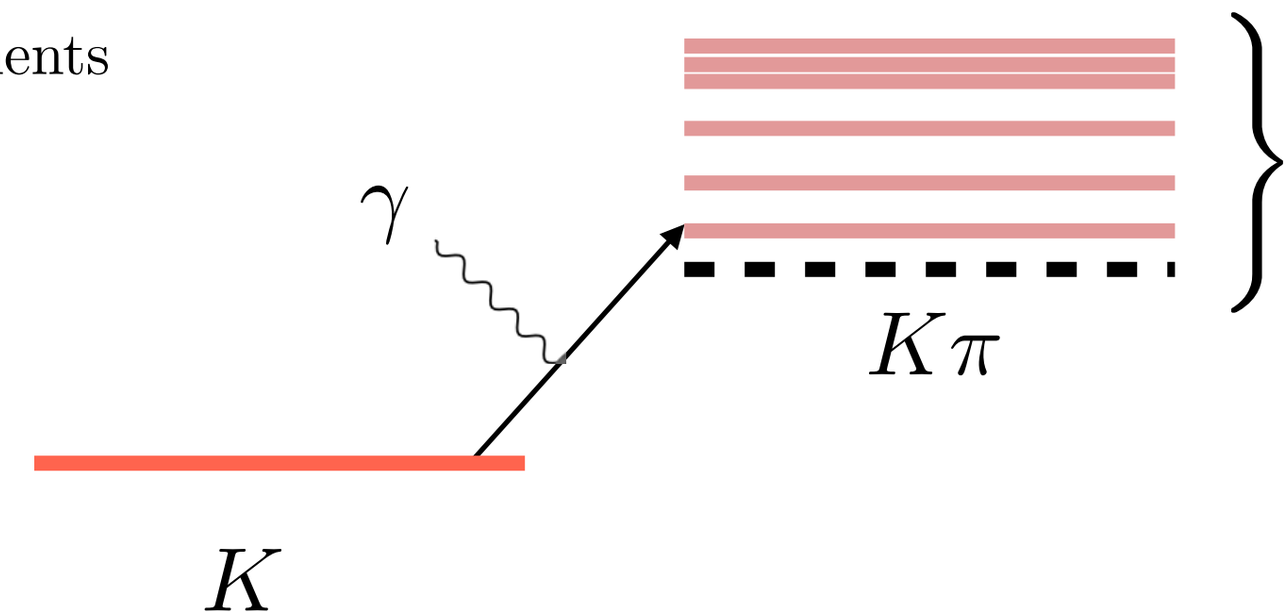
The mapping to the infinite volume involves accounting for the correct normalization of the discrete $K\pi$ energy levels

have to map the finite volume matrix elements to the infinite volume transition amplitude



$K\pi$ radiative transition from Lattice QCD

Three-point functions on the lattice - get the finite volume matrix elements



The mapping to the infinite volume involves accounting for the correct normalization of the discrete $K\pi$ energy levels

Lellouch-Lüscher formalism

$$\left| {}_L\langle K | j \ (0) | K\pi \rangle_L \right| = \frac{1}{L^3} \frac{1}{\sqrt{2E_K}} \frac{1}{\sqrt{2E_n}} \left(\mathcal{H} \ \cdot \widetilde{R}_n \cdot \mathcal{H} \ \right)^{1/2}$$

$$\widetilde{R}_n(\mathbf{p}_{K\pi}, L) \equiv 2E_n \cdot \lim_{E \to E_n} (E - E_n) \left(F^{-1}(E^\star, \mathbf{p}_{K\pi}; L) + \mathcal{M}(E^\star) \right)^{-1}$$

two body scattering matrix

When multiple partial waves are projected into a specific lattice irrep, the discrete energy levels receive contributions from all non-negligible partial waves

$K\pi$ irreps

$\vec{p}$	LG($\vec{p}$)	Λ^P	J^P
$[0,0,0]$	O_h^D	$A_1^\pm$	$0^\pm,\ 4^\pm,\ \ldots$
		$T_1^\pm$	$1^\pm,\ 3^\pm,\ 4^\pm,\ \ldots$

$|\lambda|^{\tilde{\eta}} = 0^+$ contains S,P,...
Assuming higher partial waves do not contribute - there is S&P wave mixing in A_1 irreps in boosted frames

$\vec{p}$	LG($\vec{p}$)	Λ	$\lambda^{(\tilde{\eta})}$
$[0,0,n]$	Dic_4	A_1	$0^+,\ 4,\ \ldots$
		E_2	$1,\ 3,\ \ldots$
$[0,n,n]$	Dic_2	A_1	$0^+,\ 2,\ 4,\ \ldots$
		B_1	$1,\ 3,\ \ldots$
		B_2	$1,\ 3,\ \ldots$
$[n,n,n]$	Dic_3	A_1	$0^+,\ 3,\ \ldots$
		E_2	$1,\ 2,\ 4,\ \ldots$

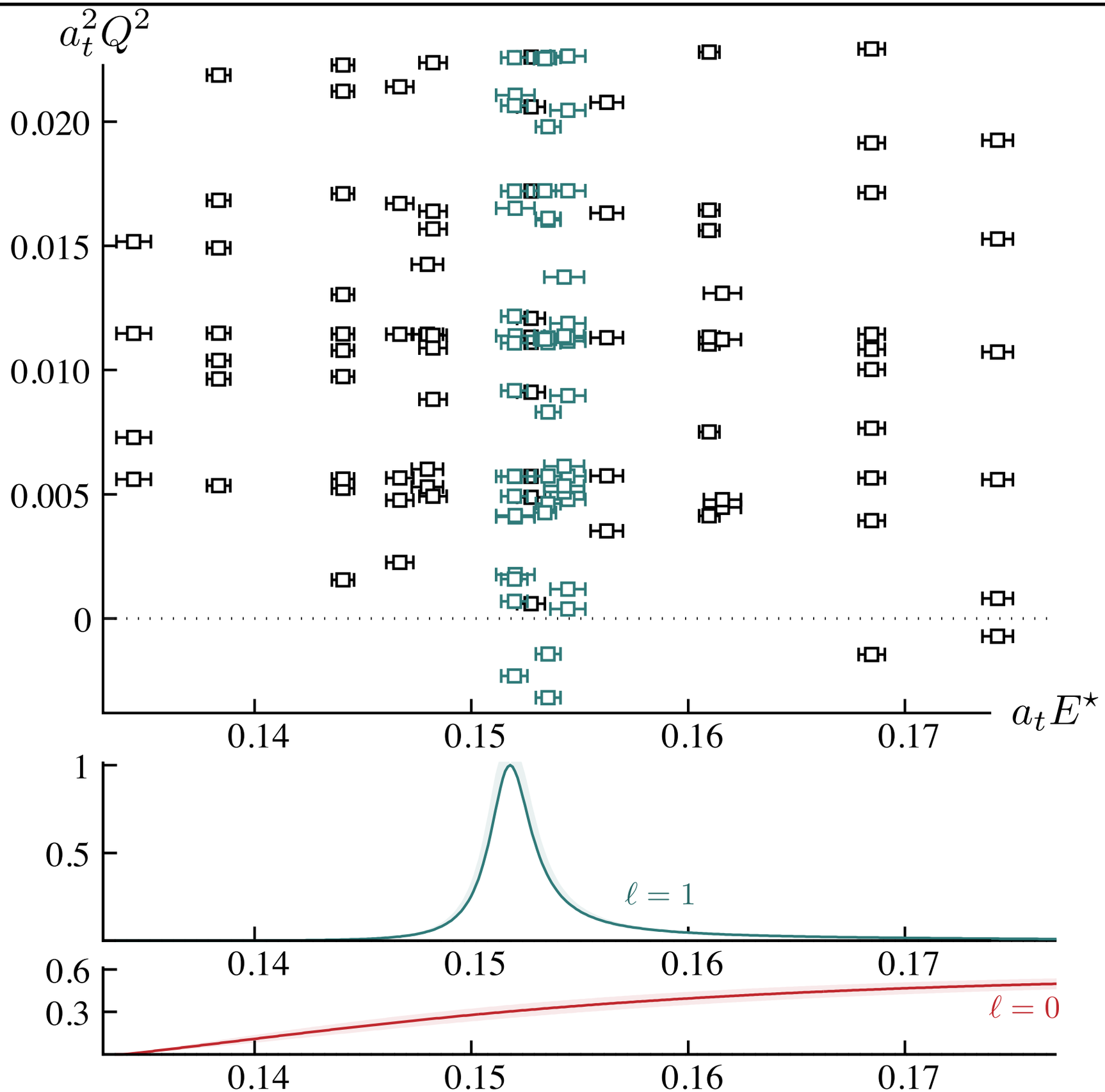
for equal particles $\pi\pi$ instead of πK
Bose symmetry prevents this mixing

boosted frames are important to constrain on amplitude

$$\tilde{\eta} = P(-1)^J$$

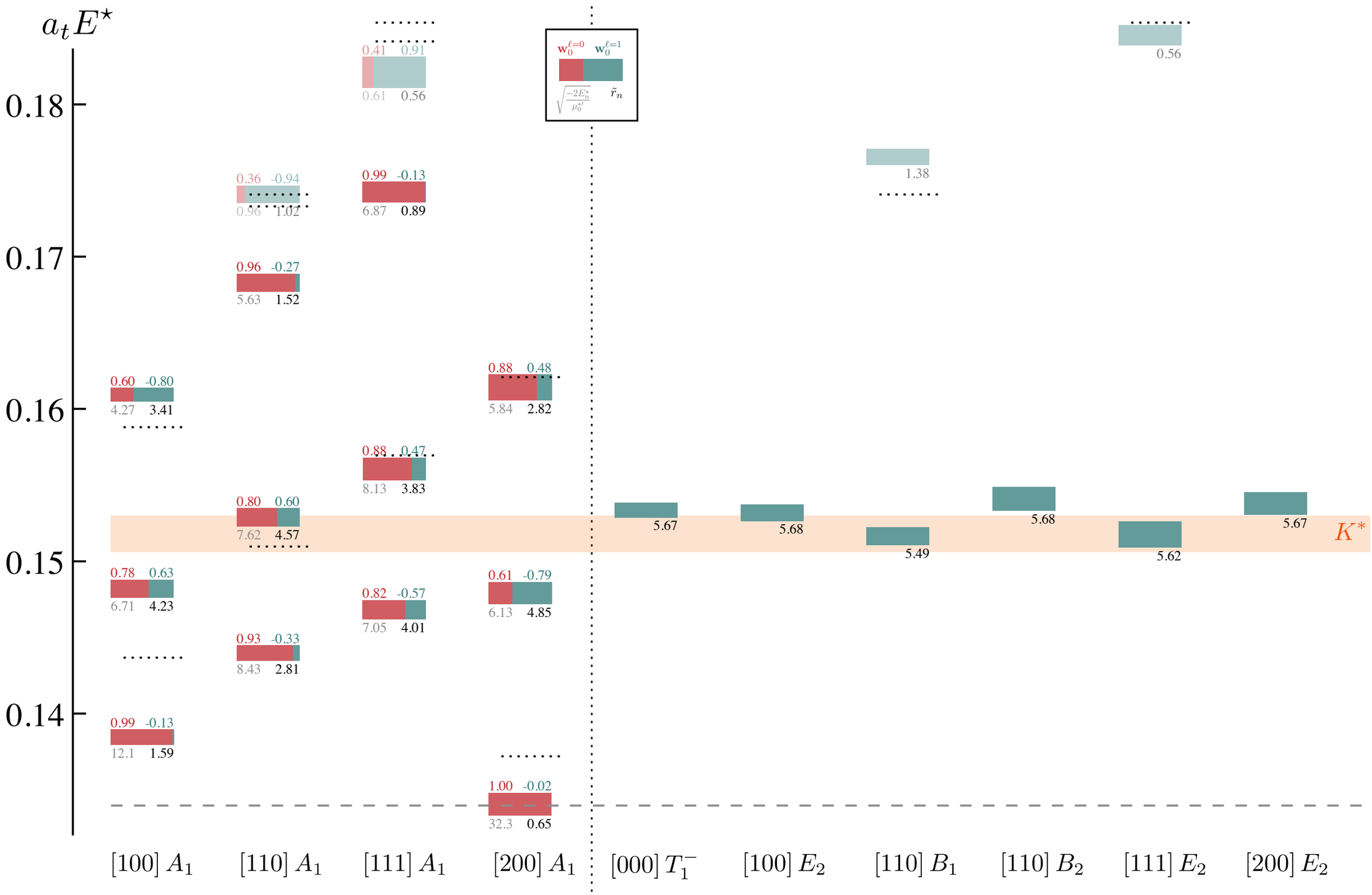
Kinematic Coverage

arxiv: 2208.13755



The S&P wave contribution to each $K\pi$ energy level

arxiv: 2208.13755



Lellouch-Lüscher factors

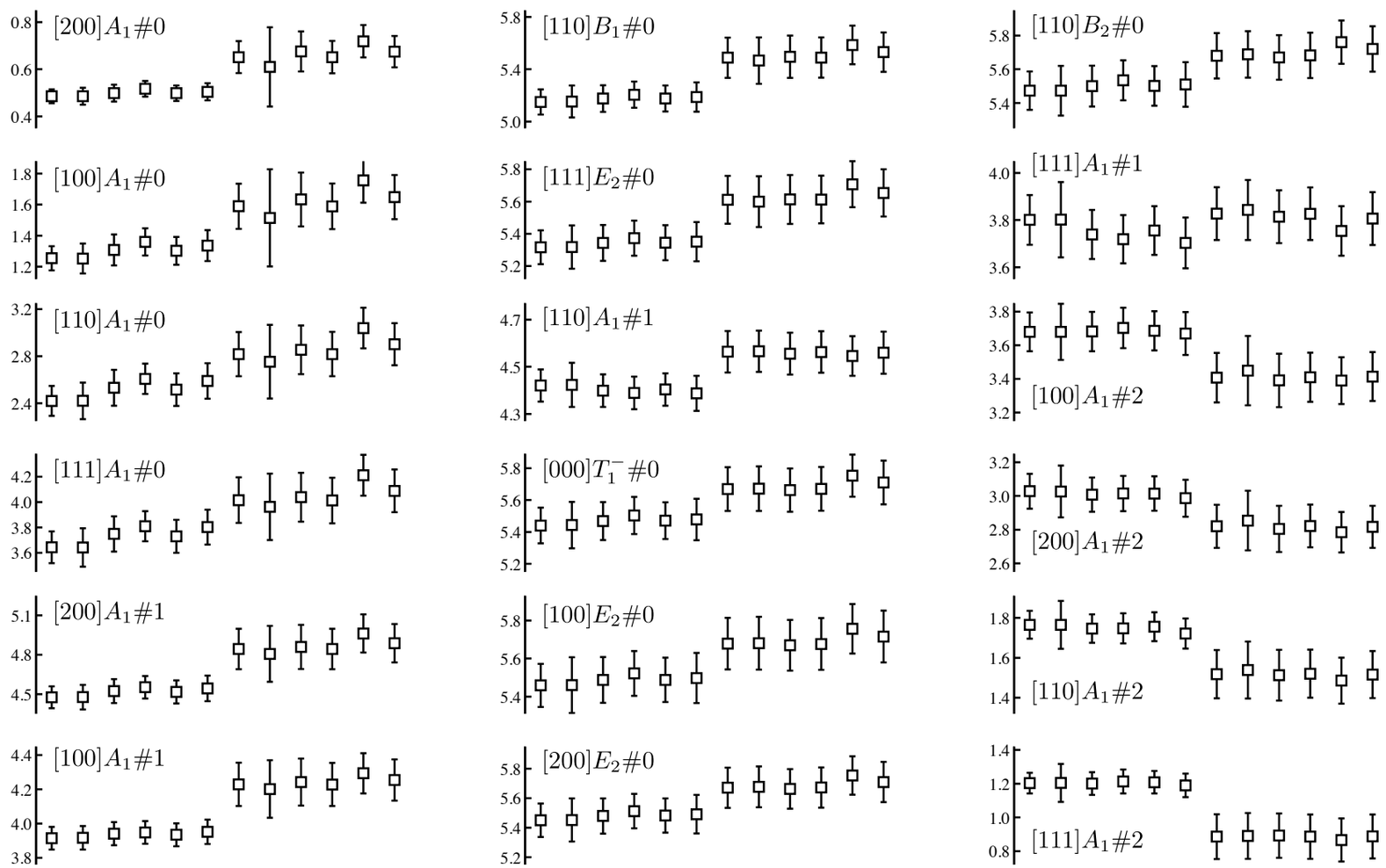
arxiv: 2208.13755

Lellouch-Lüscher formalism

$$\left|{}_L\langle K|j\ \ (0)|K\pi\rangle_L\right| = \frac{1}{L^3}\frac{1}{\sqrt{2E_K}}\frac{1}{\sqrt{2E_n}}\left(\mathcal{H}\ \cdot\widetilde{R}_n\cdot\mathcal{H}\ \right)^{1/2}$$

$$\widetilde{R}_n(\mathbf{p}_{K\pi}, L) \equiv 2E_n\cdot\lim_{E\to E_n}(E-E_n)\left(F^{-1}(E^\star,\mathbf{p}_{K\pi};L)+\mathcal{M}(E^\star)\right)^{-1}$$

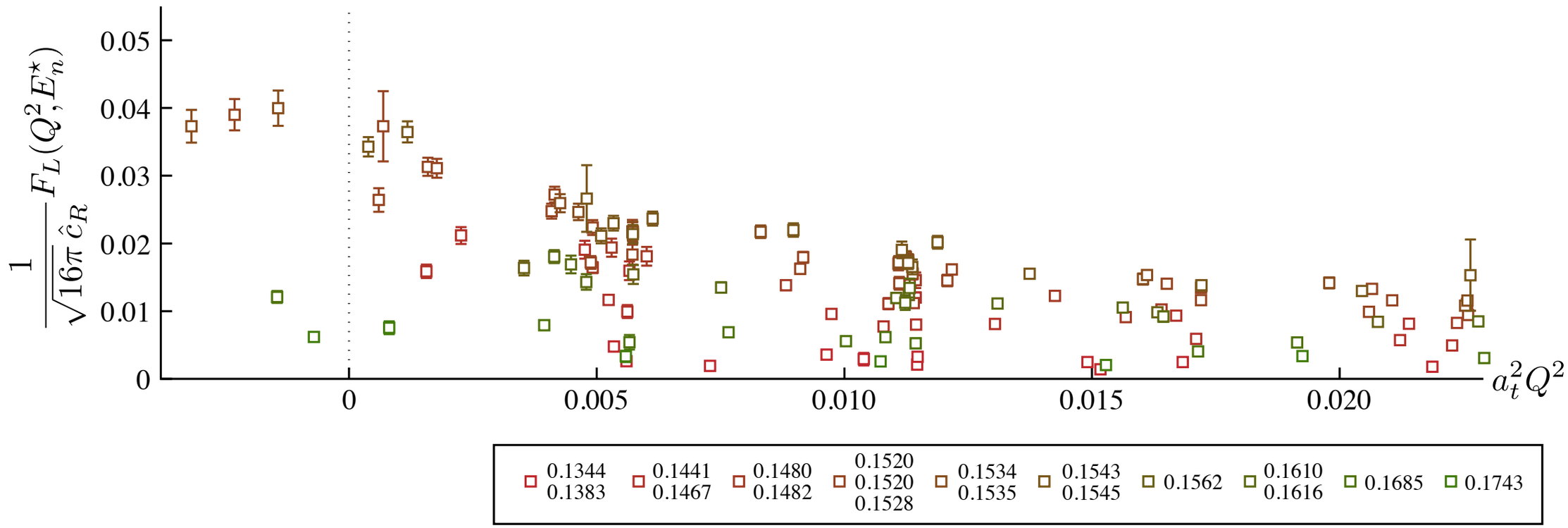
two body scattering matrix



Lellouch-Lüscher factors for different parametrization of $\mathscr{M}(E^\star)$

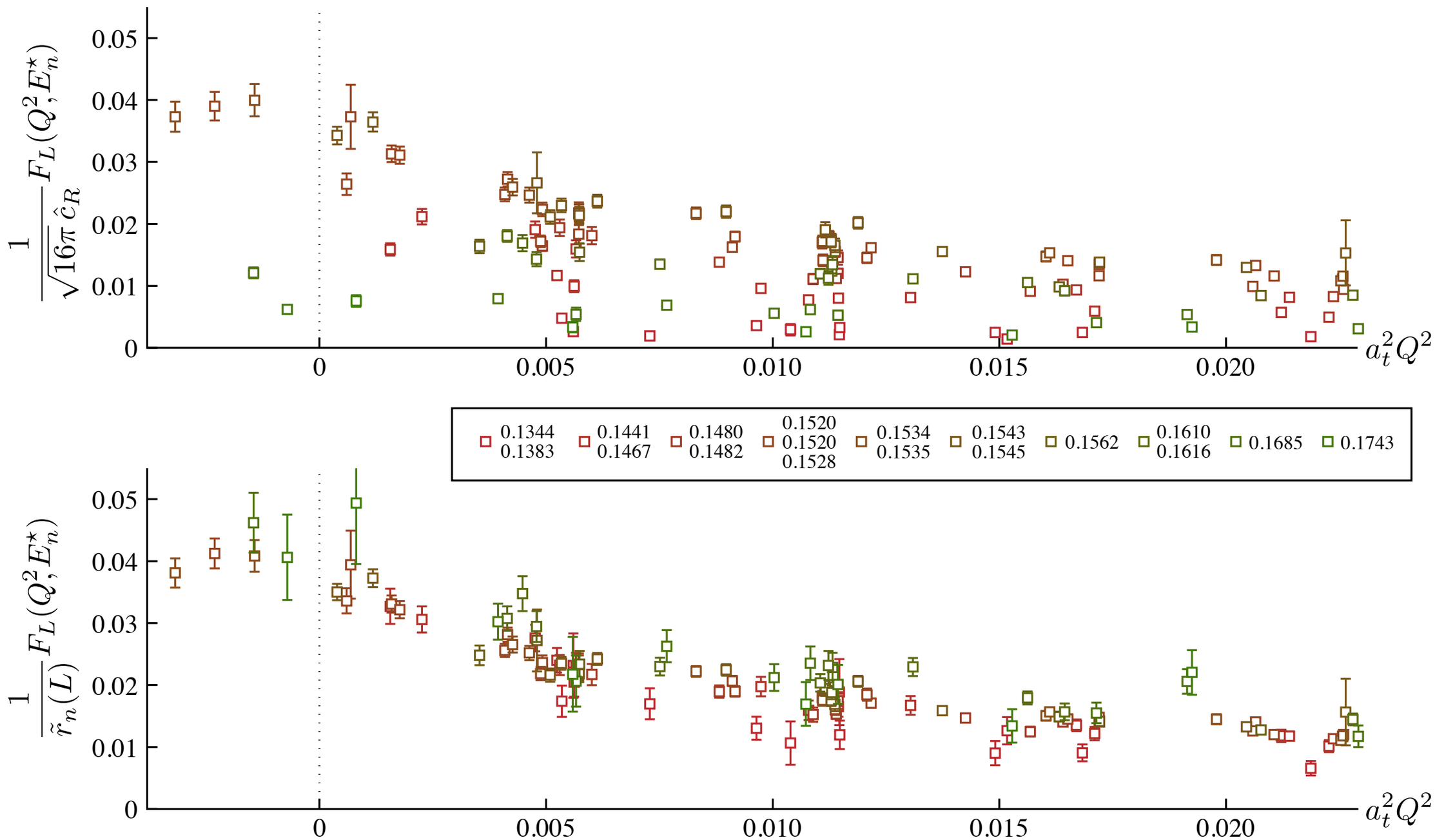
Finite volume matrix elements

arxiv: 2208.13755



Lellouch-Lüscher corrected matrix elements

arxiv: 2208.13755



$K^*(892)$ photo-coupling and $K\gamma \to K\pi$ amplitude

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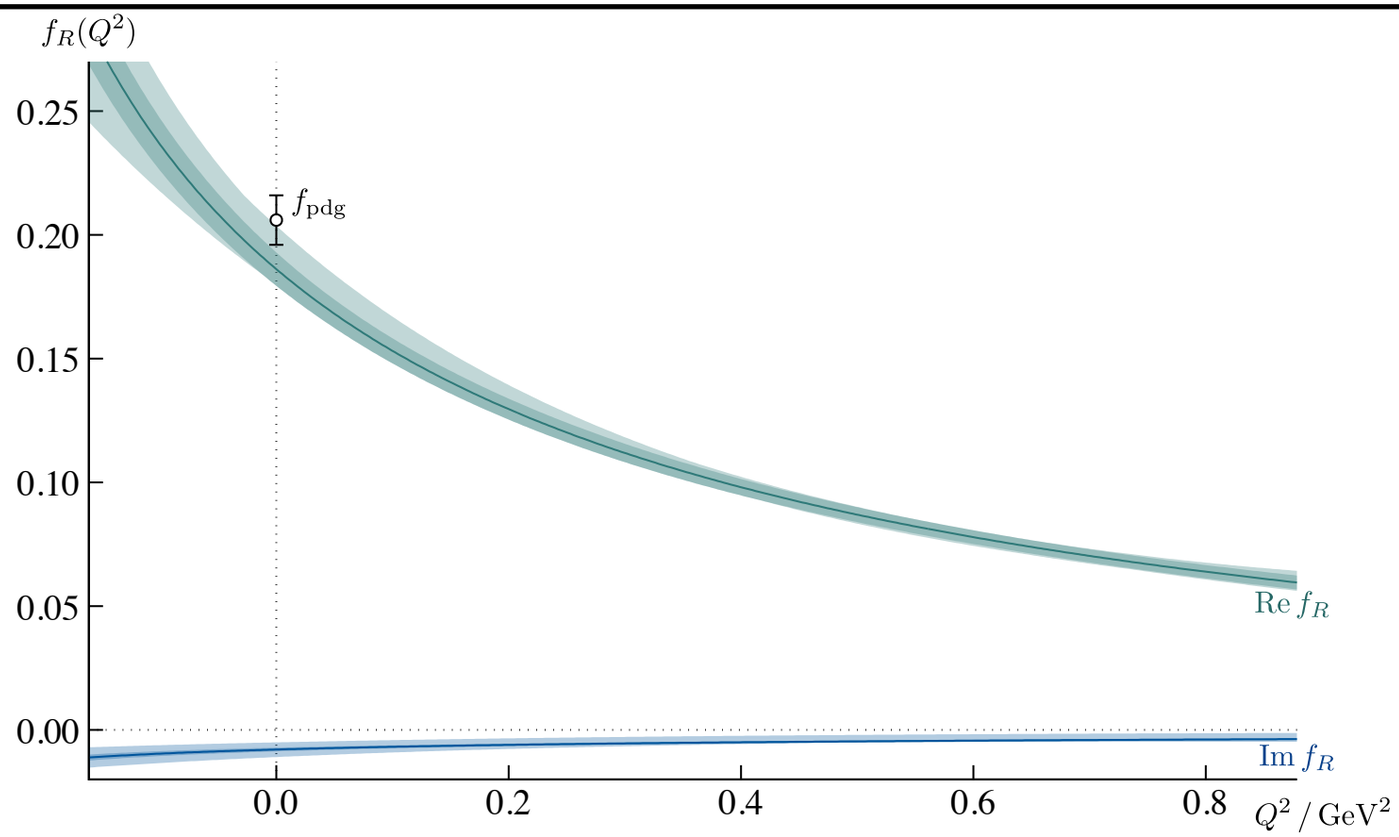
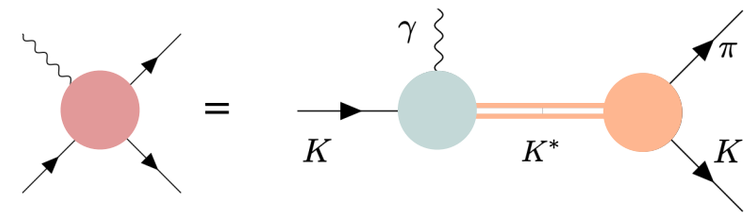
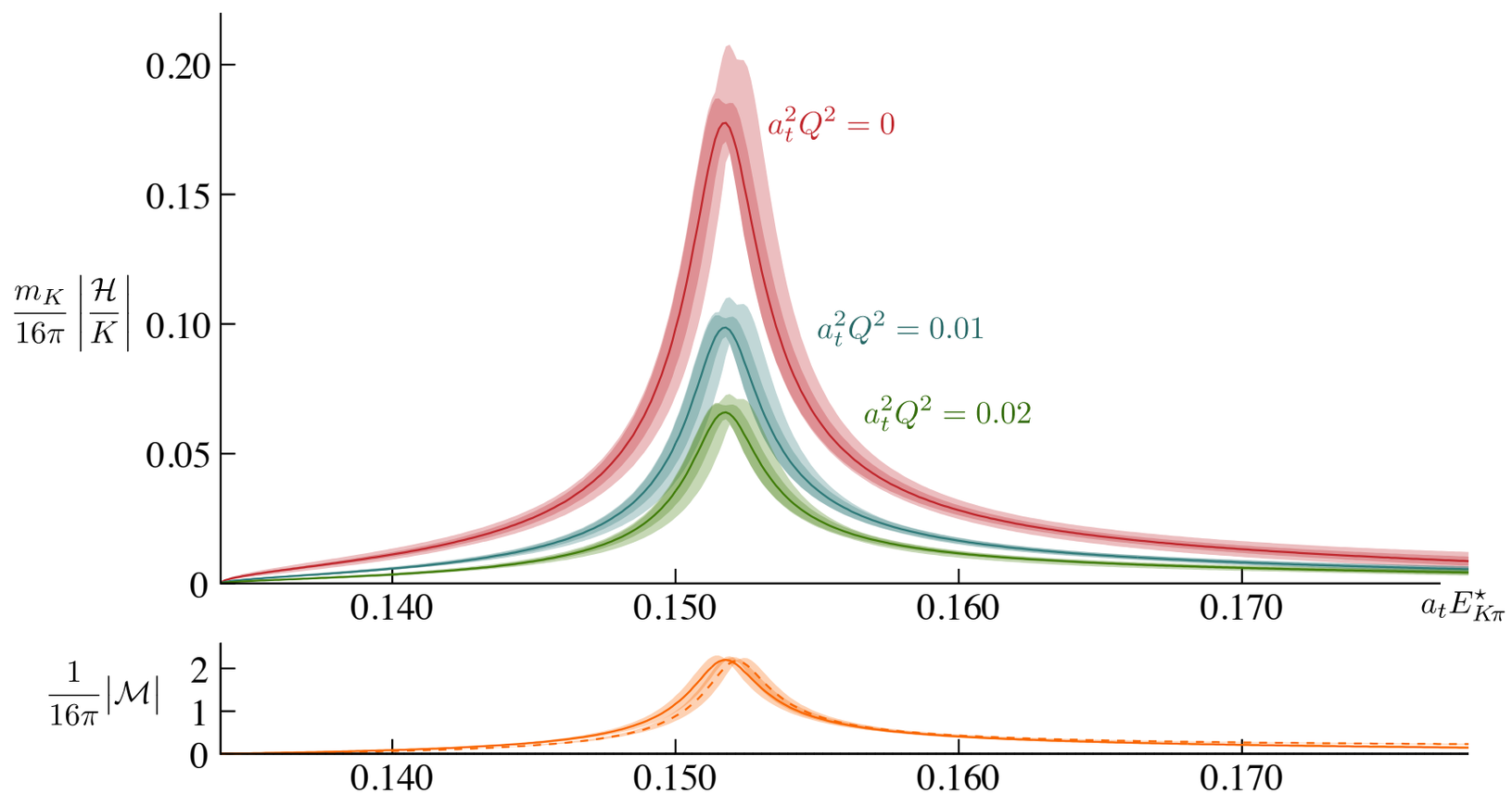




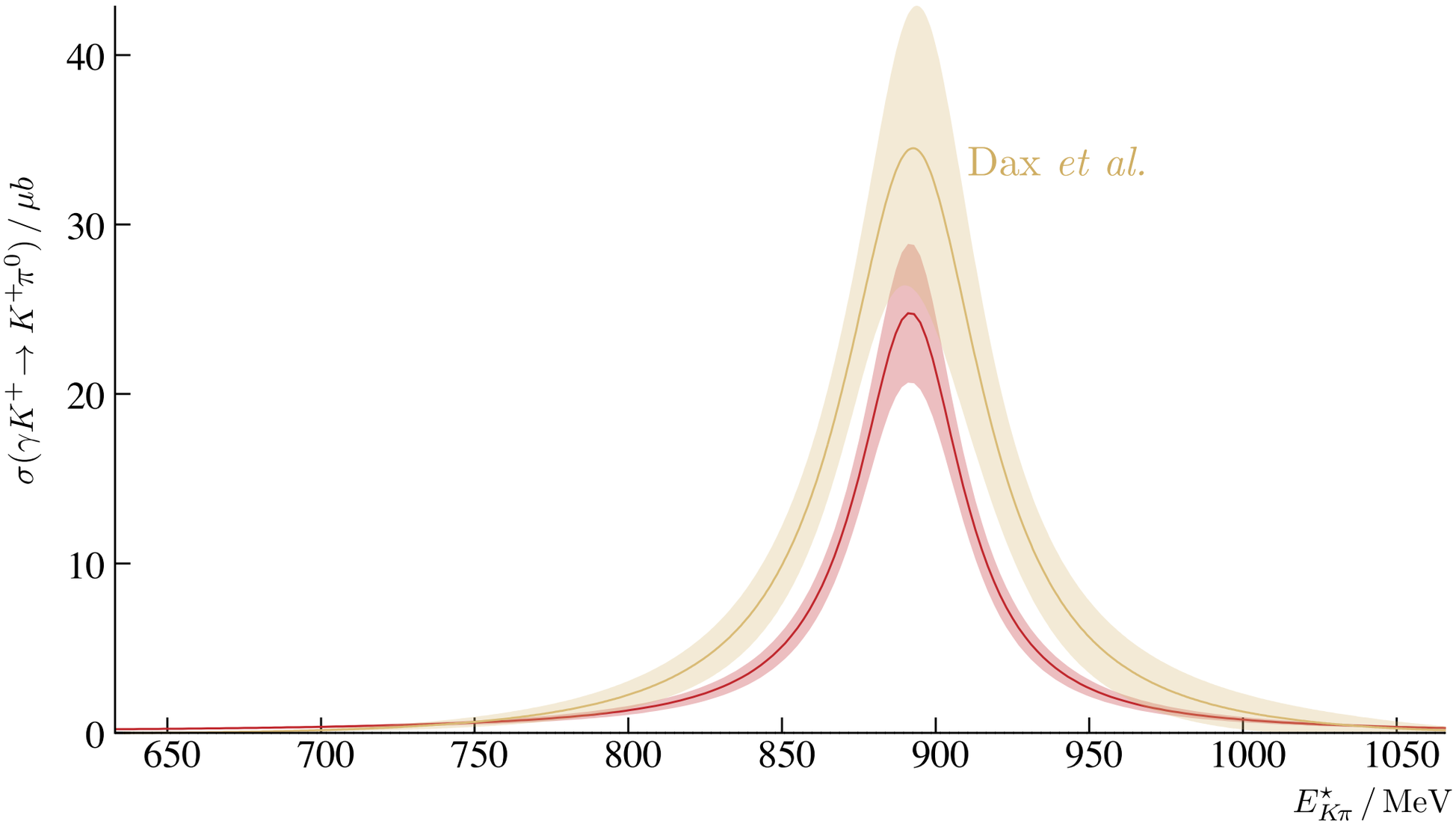
$$\mathcal{H}(Q^2, E^\star) = \mathcal{A}(Q^2, E^\star) \cdot \frac{1}{k^\star} \cdot \mathcal{M}(E^\star)$$



Crude extrapolation

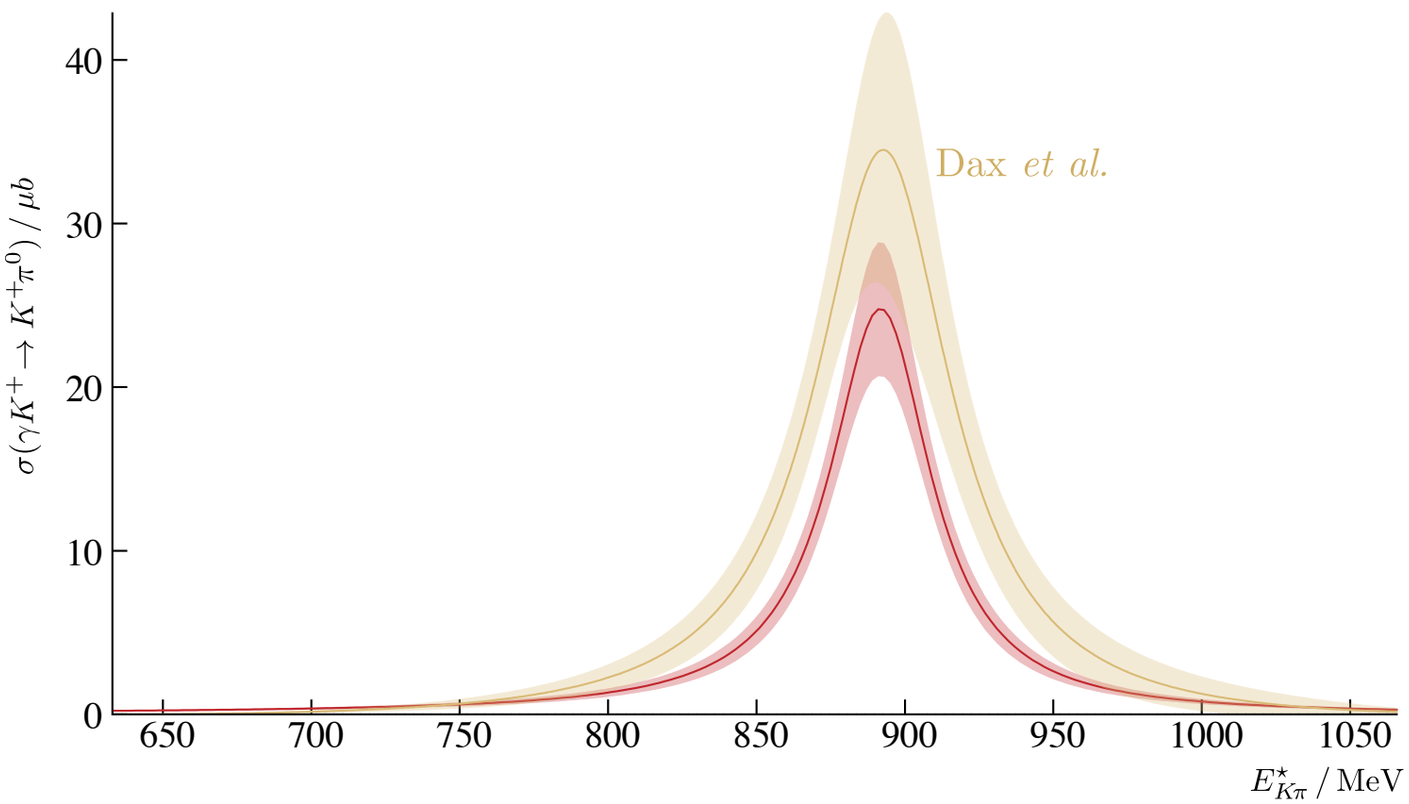
arxiv: 2208.13755

Eur.Phys.J.C 81 (2021)

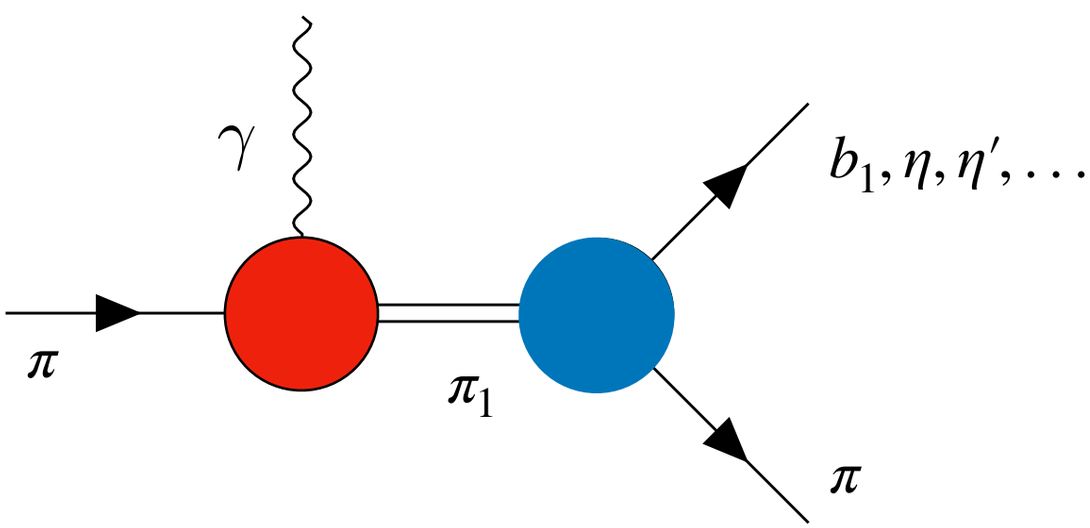


To extrapolate to the physical point by keeping the "couplings" fixed to the lattice results

Summary

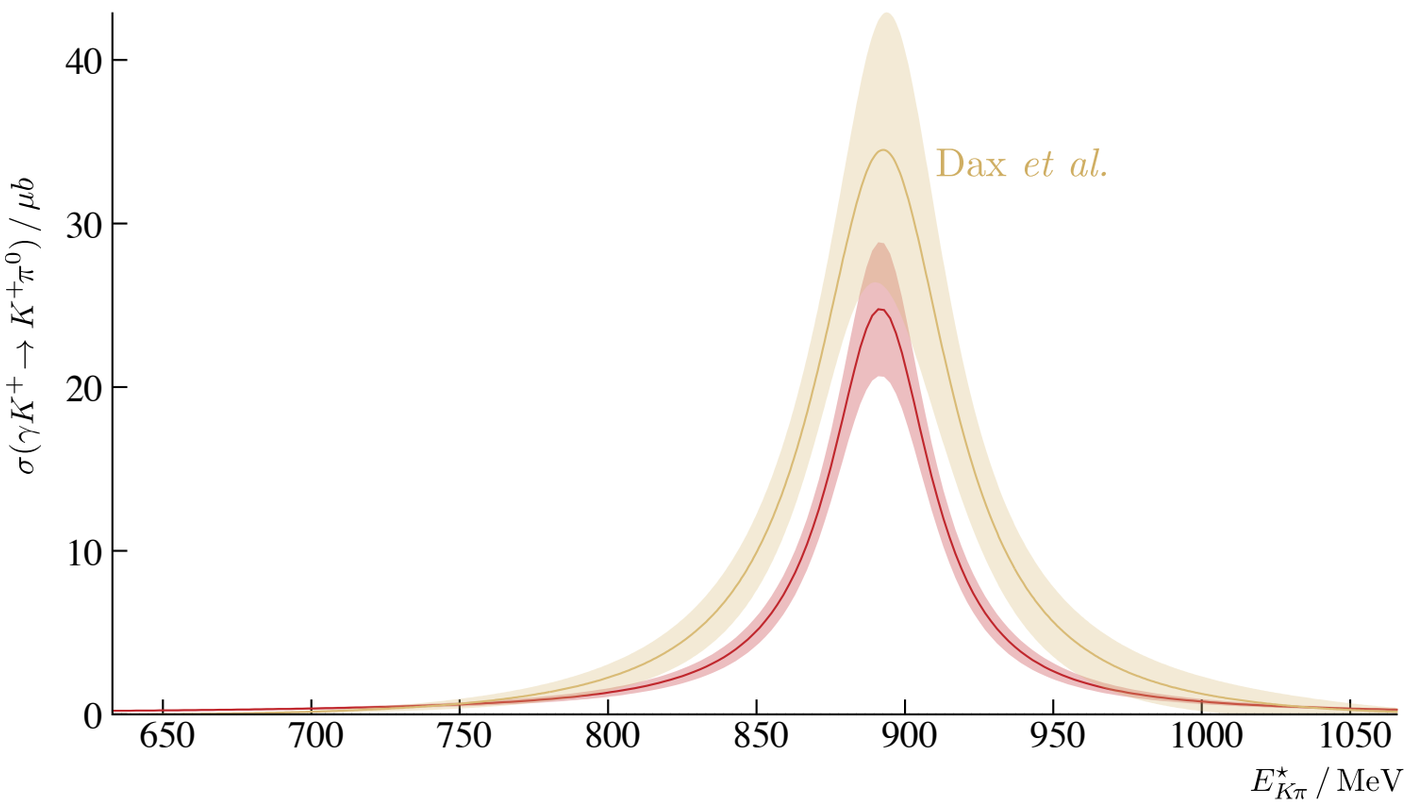


- Lellouch-Lüscher formalism was implemented for the first time for $\gamma K^+ \to \pi^0 K^+$ transition where there is **substantial** S & P-wave mixing in finite volume
- The resonance form factor and decay width were obtained by analytically continuing to the K^* pole
- Can be extended to many other interesting calculations!



Can be extended to the study of other resonance form factors

Summary



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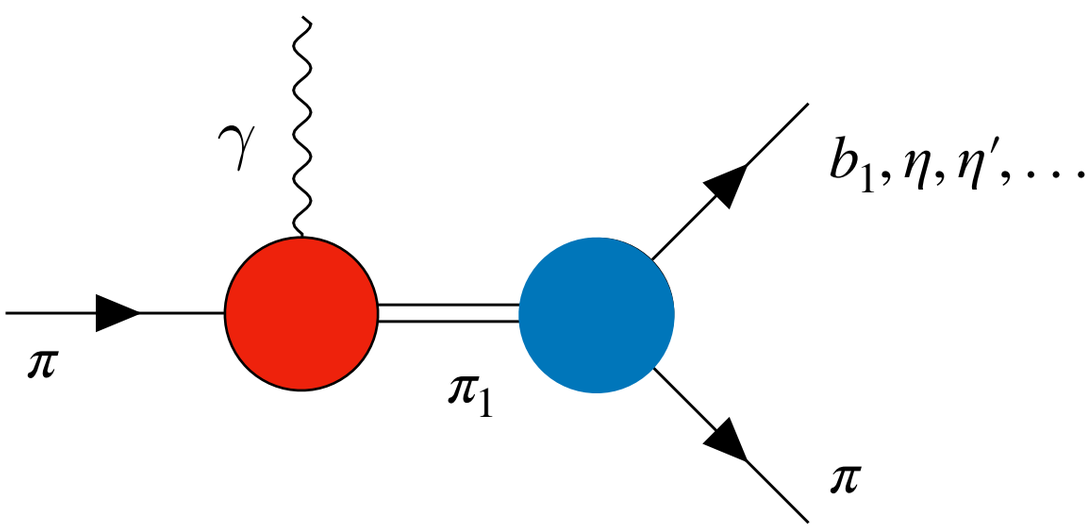

Can be extended to the study of other resonance form factors like the exotic π_1