

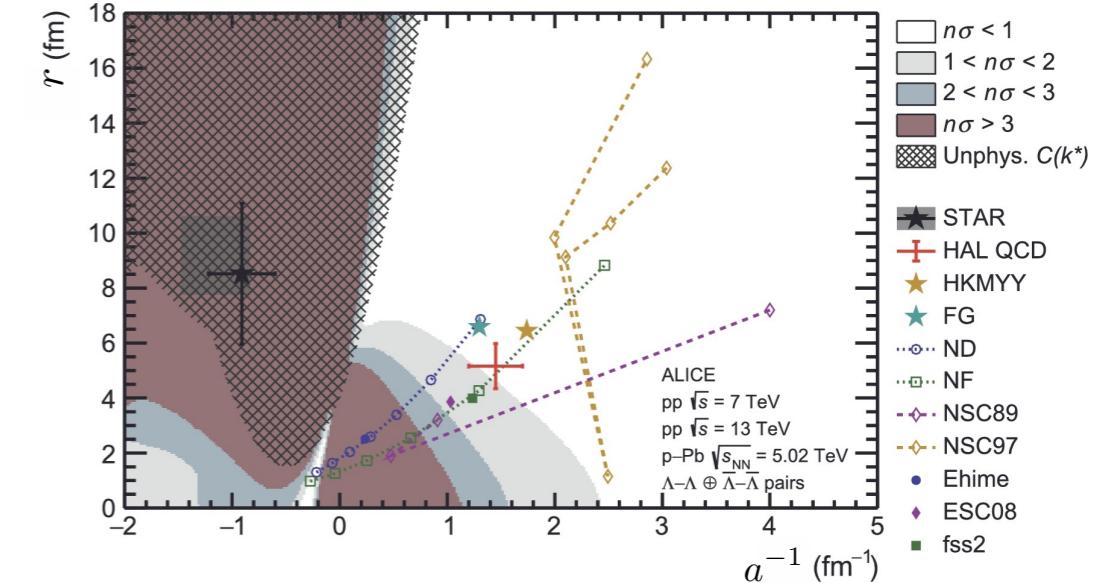
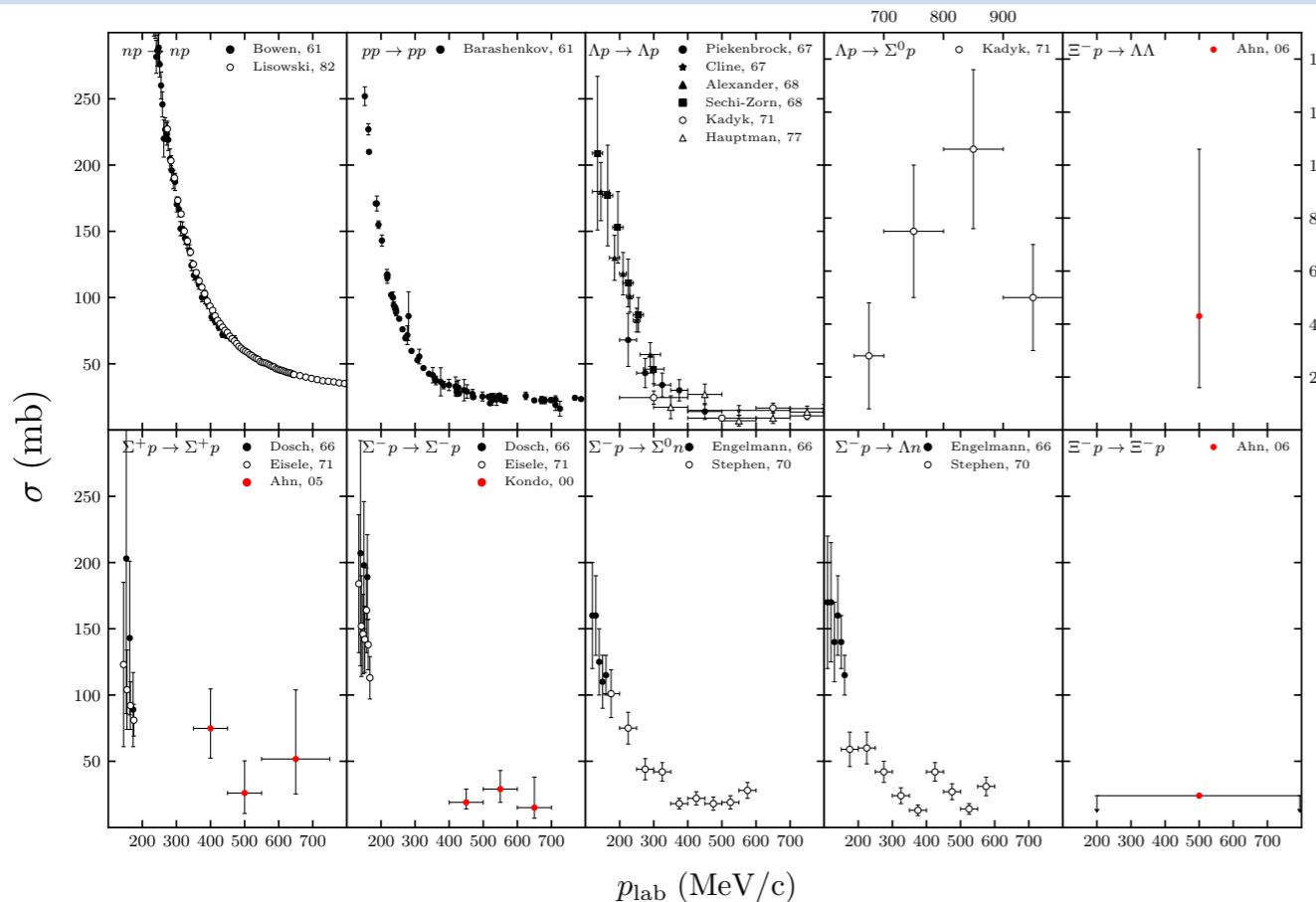
9th International Conference on Quarks and Nuclear Physics  
6th September, 2022

# Two-baryon interactions from lattice QCD

Marc Illa



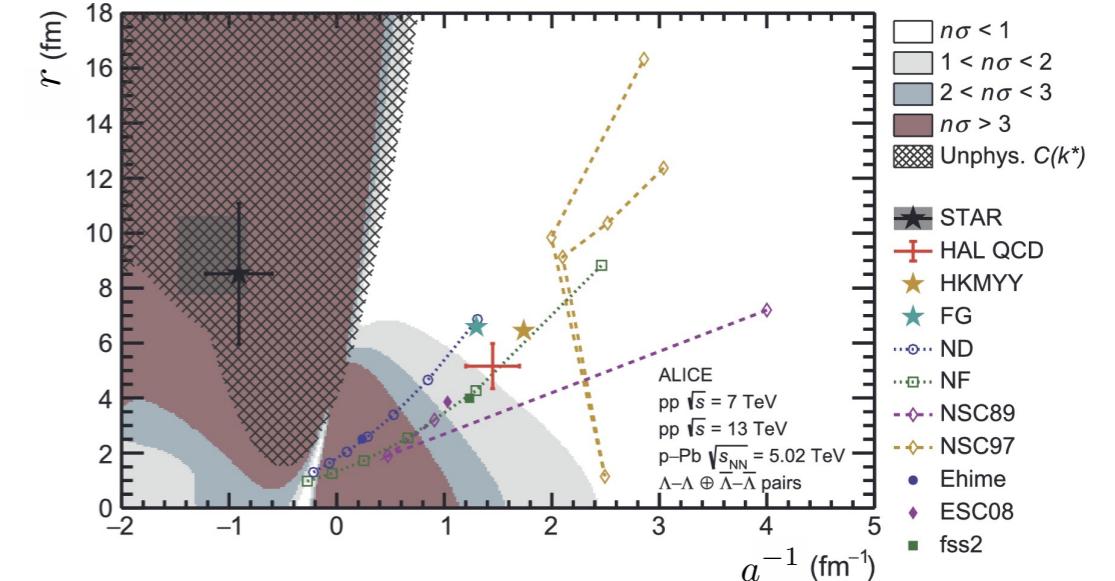
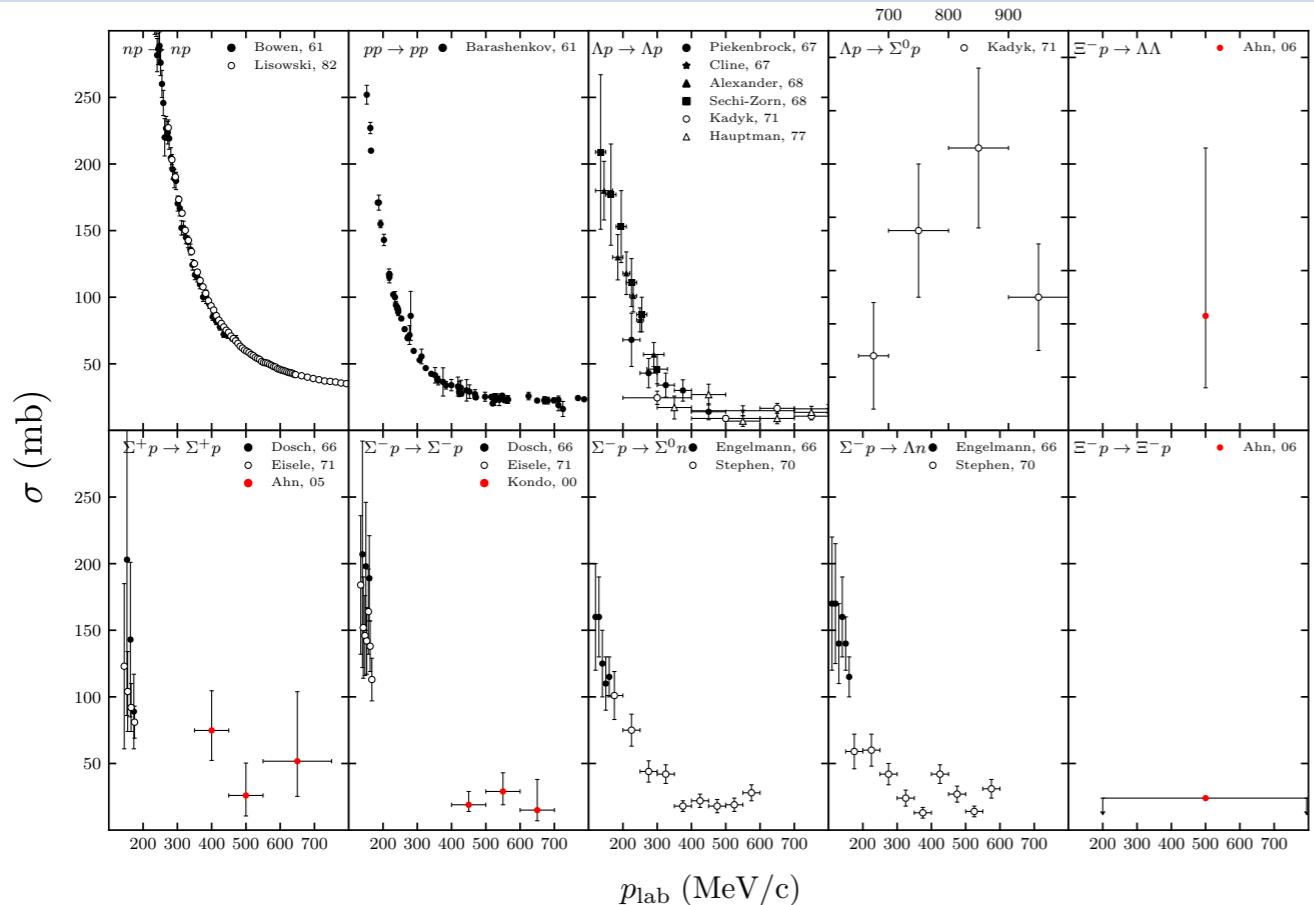
# Baryon-baryon interactions



ALICE Collaboration, PLB 797 (2019)

updated from Dover and Feshbach, Ann. Phys. 198 (1990)

# Baryon-baryon interactions



$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8}_s \oplus \mathbf{1} \oplus \overline{\mathbf{10}} \oplus \mathbf{10} \oplus \mathbf{8}_a$$

Wagman et al. [NPLQCD], PRD 96 (2017)

$$m_u = m_d = m_s$$

$$m_\pi = m_K \sim 806 \text{ MeV}$$

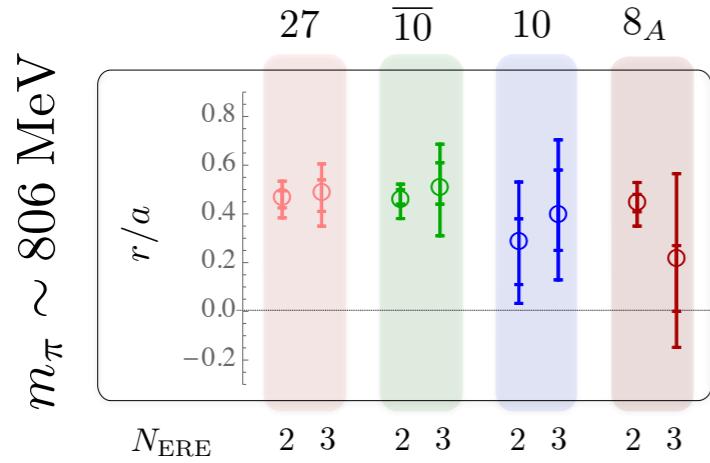
$$\mathbf{27} \quad \overline{\mathbf{10}} \quad \mathbf{10} \quad \mathbf{8}_a$$

$$NN, \Sigma N, \Sigma\Sigma, \Xi\Sigma, \Xi\Xi$$

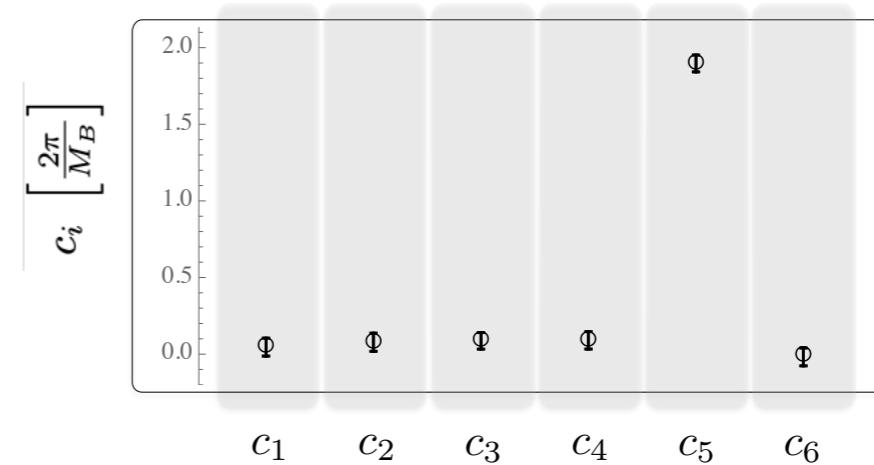
$$NN \quad \Sigma N, \Xi\Xi \quad \Xi N$$

# Baryon-baryon interactions

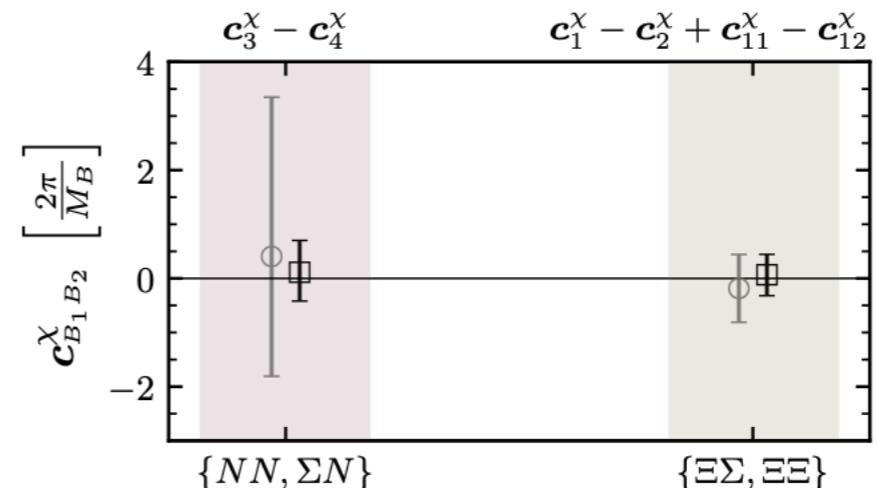
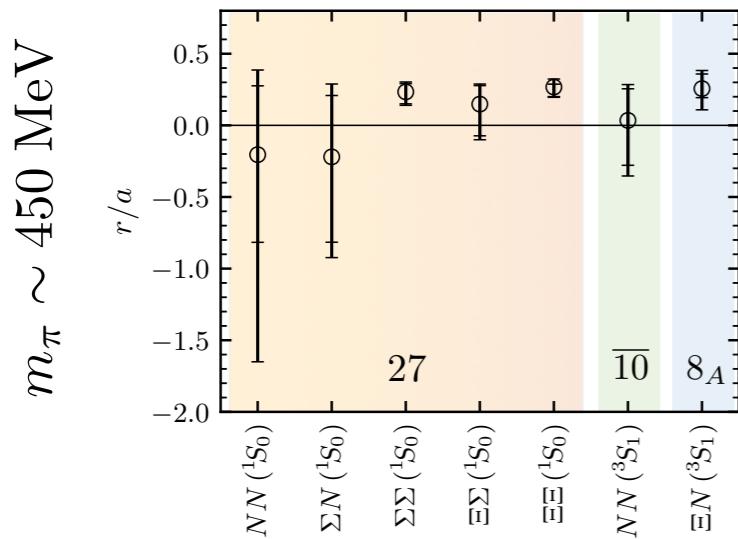
## Scattering parameters



## EFT matching



Observe accidental SU(16) symmetry at  $m_\pi \sim 800 \text{ MeV}$



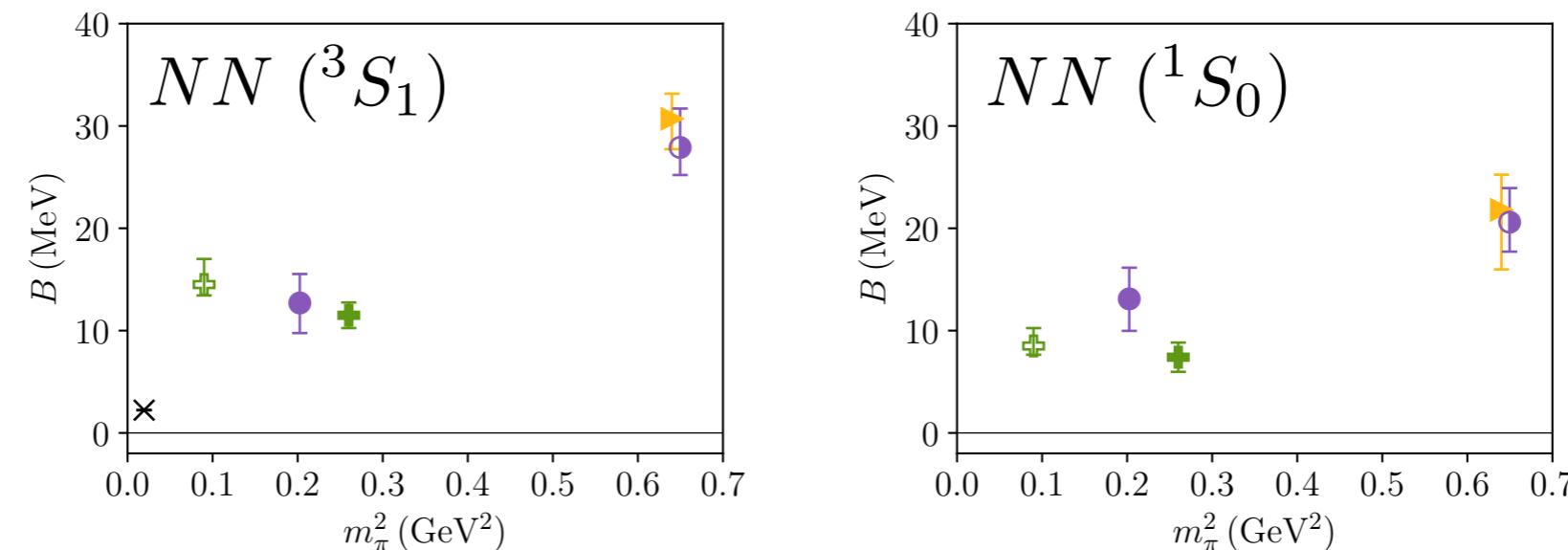
Don't observe SU(3)<sub>f</sub> symmetry breaking effects at  $m_\pi \sim 450 \text{ MeV}$

Wagman et al. [NPLQCD], PRD 96 (2017)

Illa et al. [NPLQCD], PRD 103 (2021)

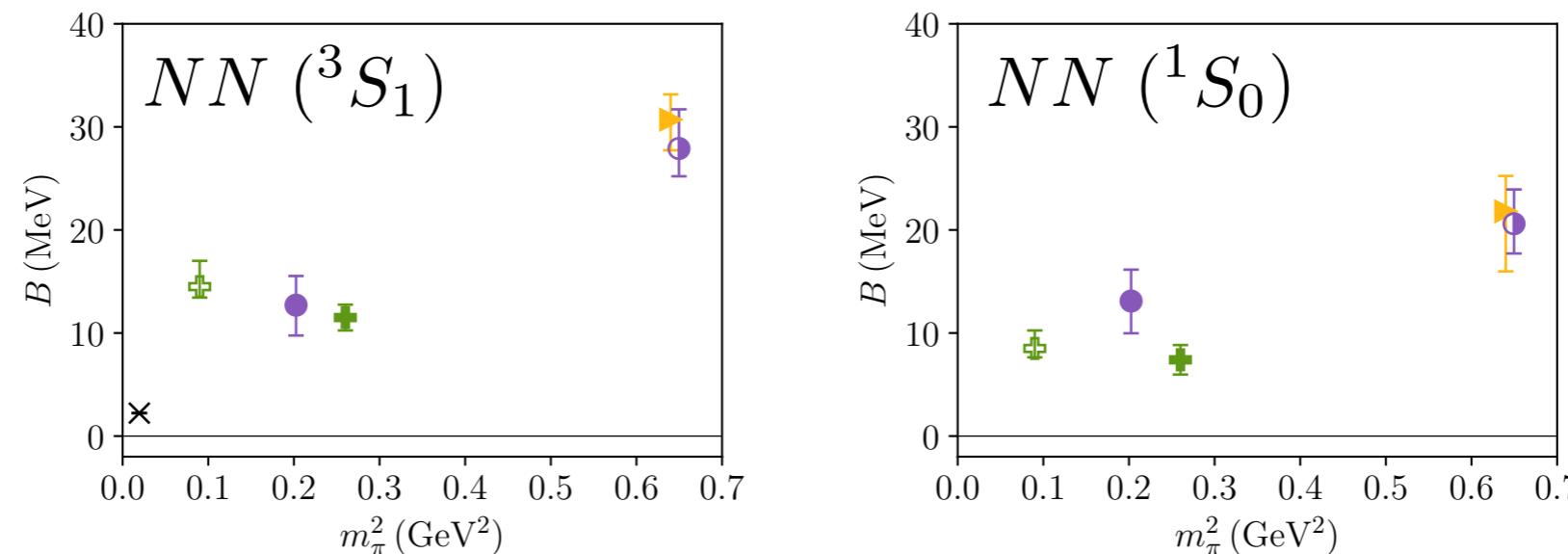
# Binding energies as a function of the pion mass

Traditionally, calculations with the direct approach were performed with asymmetrical correlators (different source and sink operators), leading to bound  $NN$  systems with unphysical quark masses

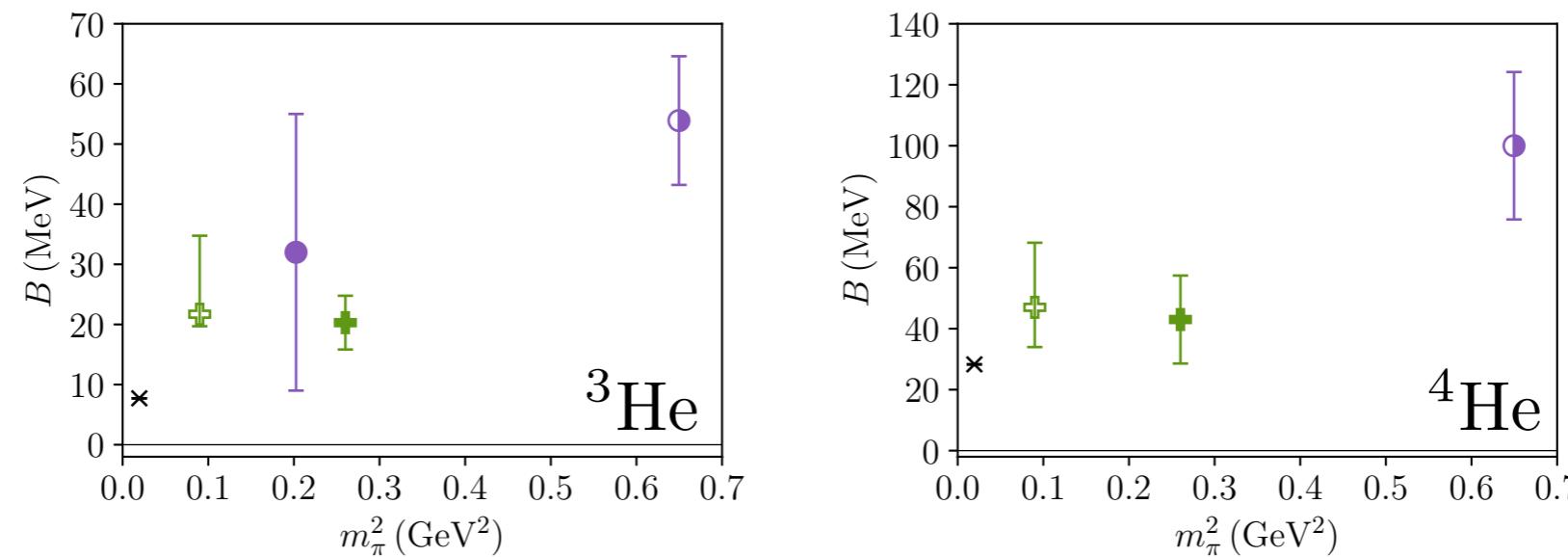


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And also 3- and 4-body bound systems

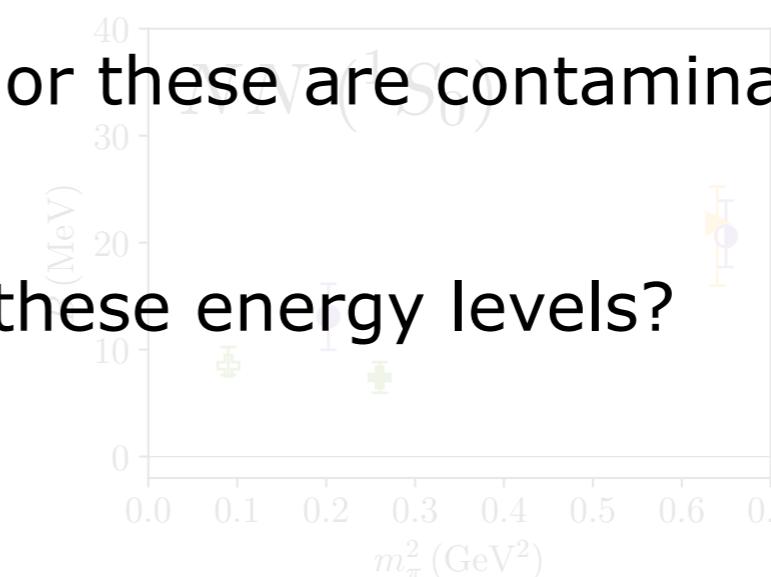
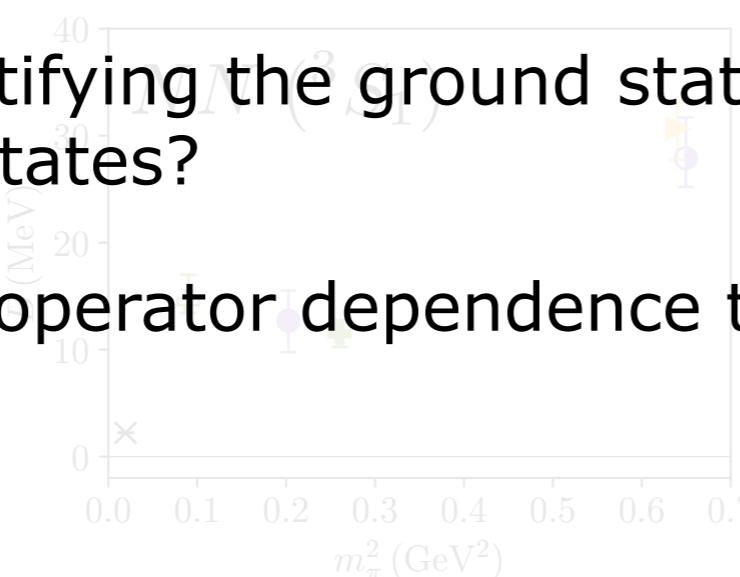


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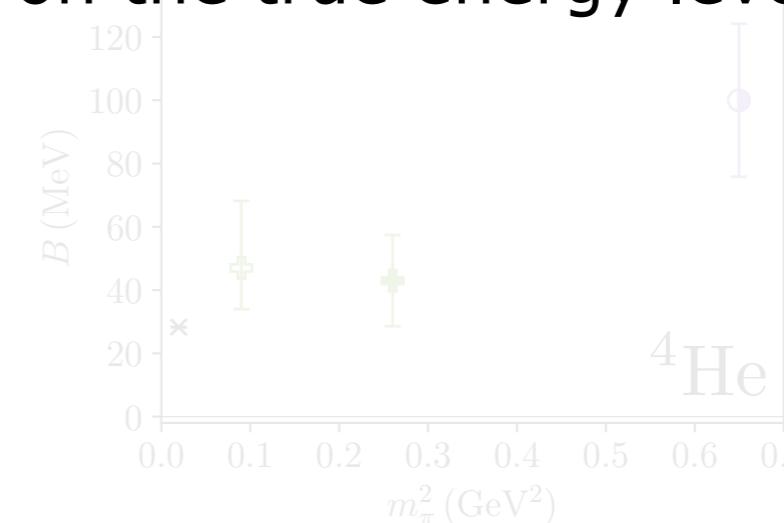
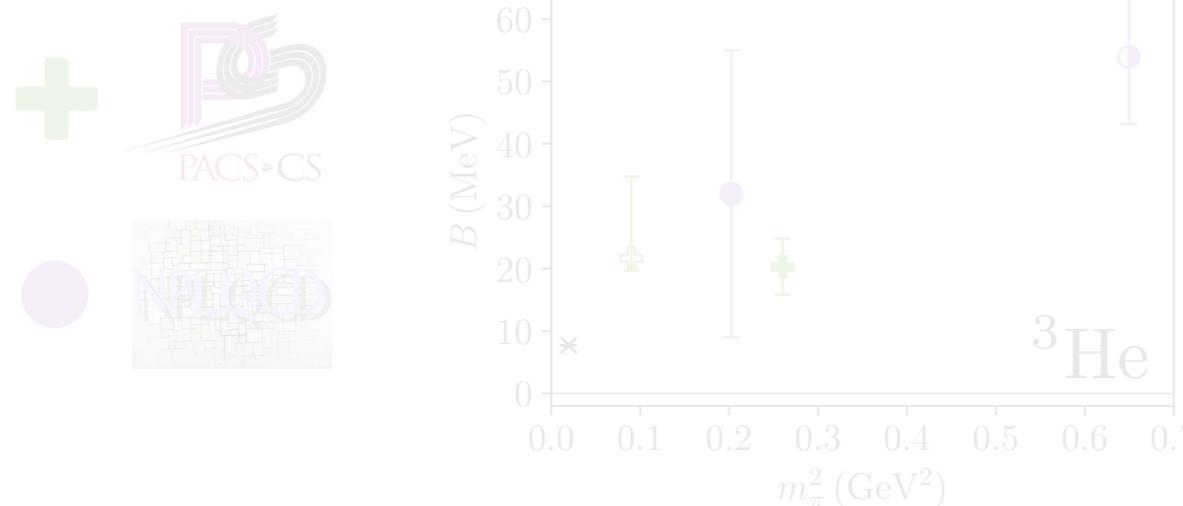
+ Are we identifying the ground state, or these are contaminated by excited states?

• Is there an operator dependence to these energy levels?



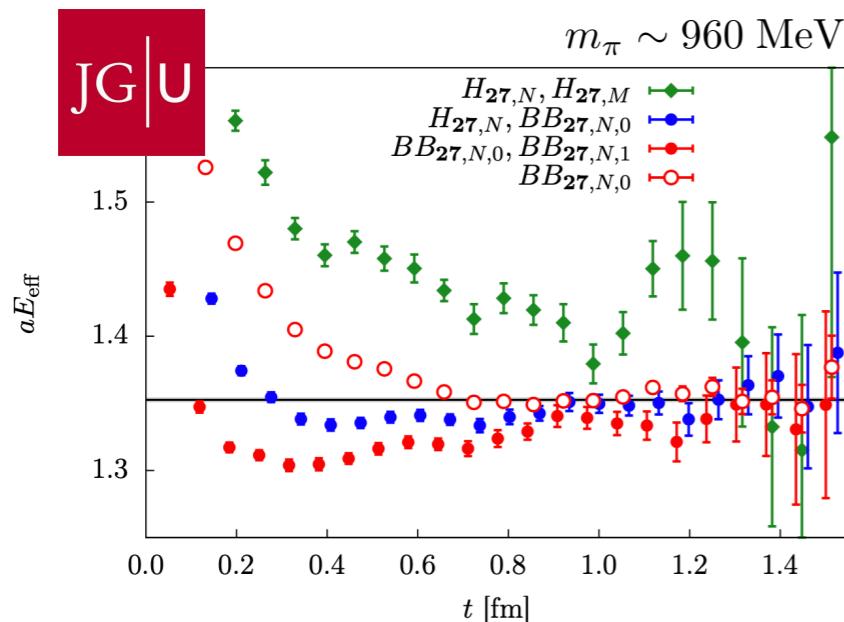
And also 3- and 4-body bound systems

→ A possible answer to these questions might be found through a variational analysis, where the lattice results can provide upper bounds on the true energy levels



# First variational calculations

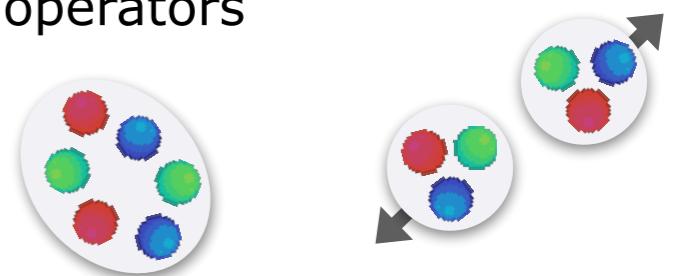
The first variational calculations appeared in 2018 by the Mainz group, and additional studies were performed in 2020-21 by CalLat and NPLQCD



Francis et al., PRD 99 (2019)

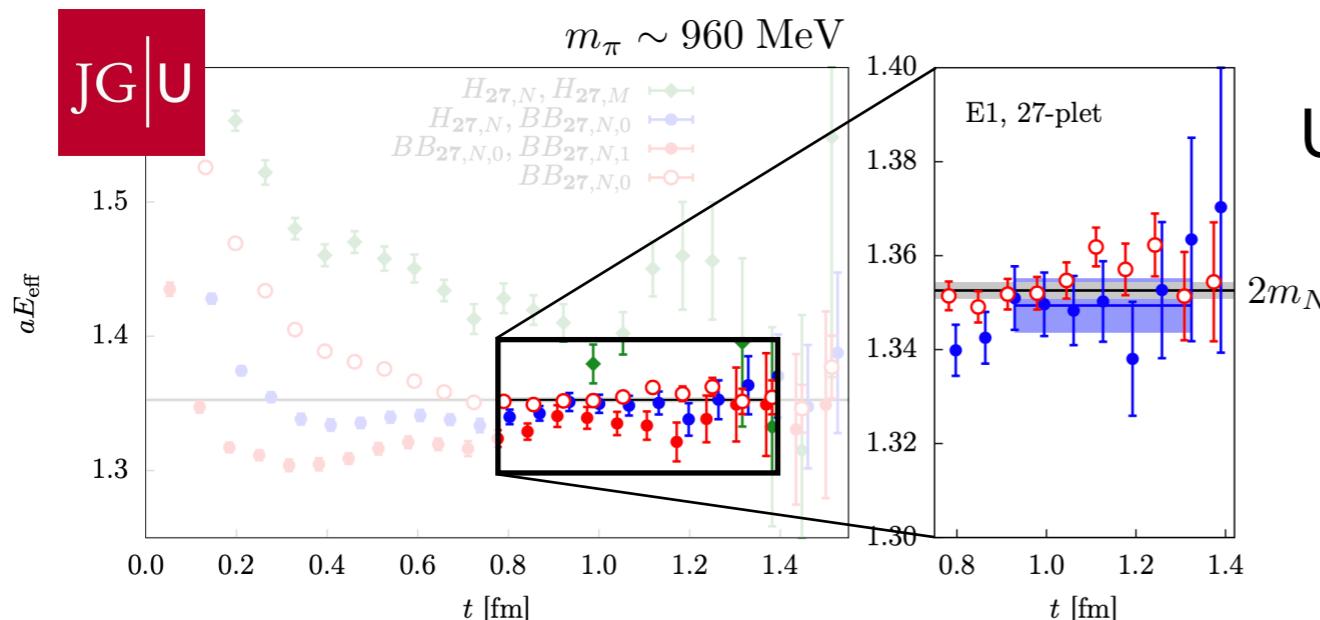
Non-hermitian matrices with hexaquark and dibaryon-like operators

Hermitian matrices with only hexaquark or dibaryon-like operators



# First variational calculations

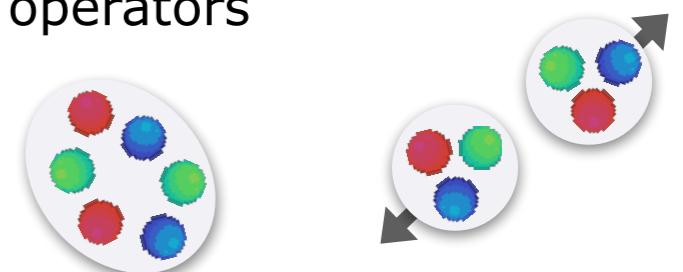
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## Unbound $NN$ ( ${}^1S_0$ )

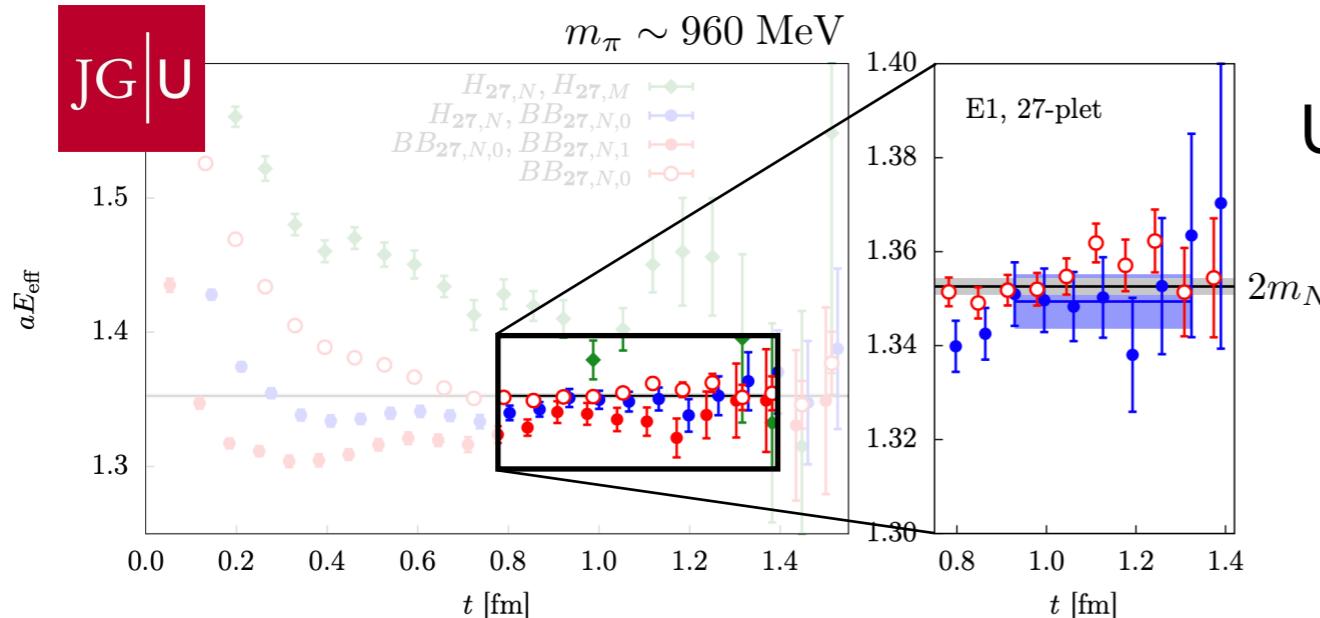
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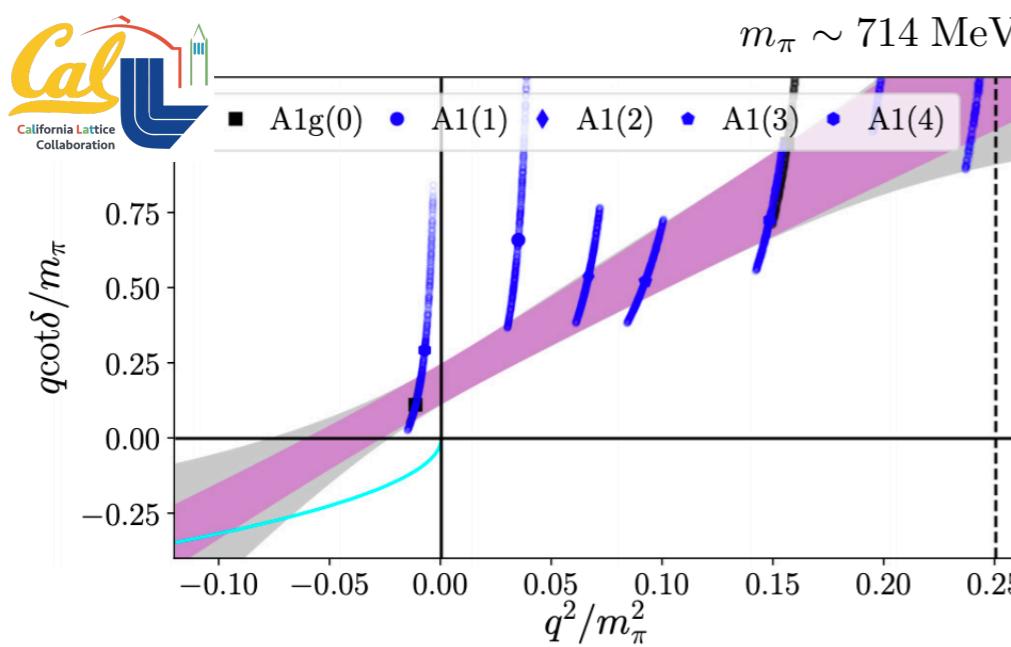
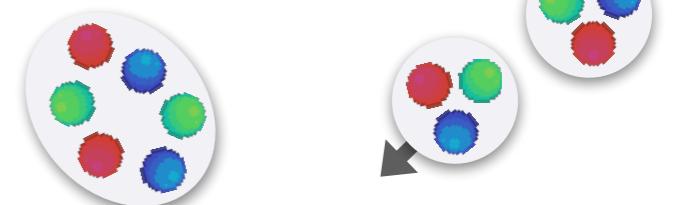
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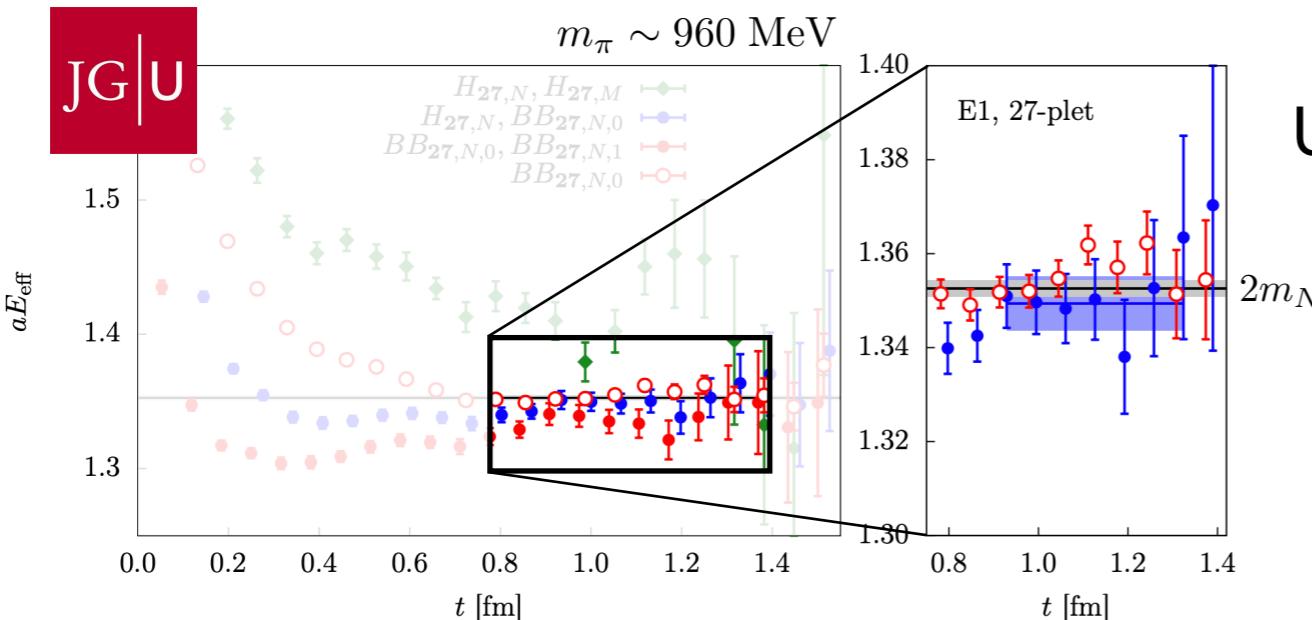


## Unbound $NN$ ( ${}^1S_0$ )

Hermitian matrices with only dibaryon-like operators

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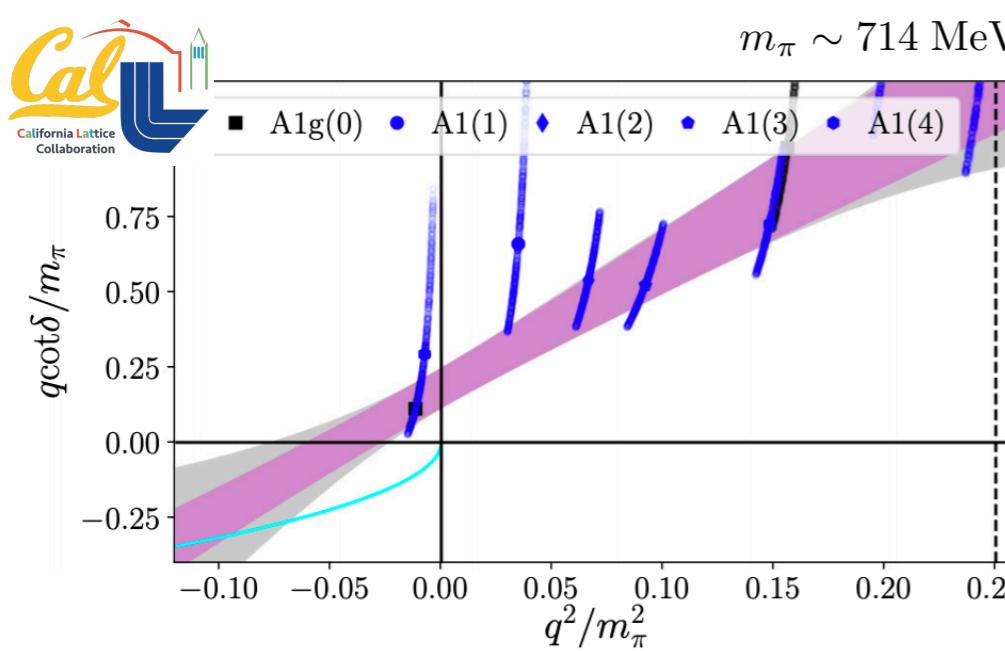
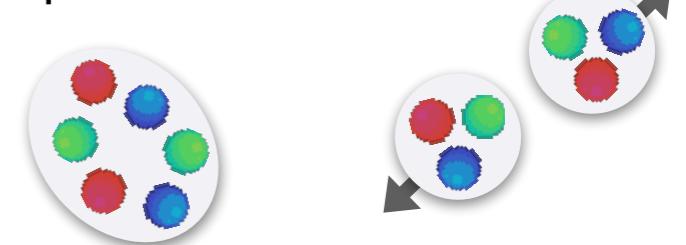


Francis et al., PRD 99 (2019)

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Hörz et al., PRC 103 (2021)

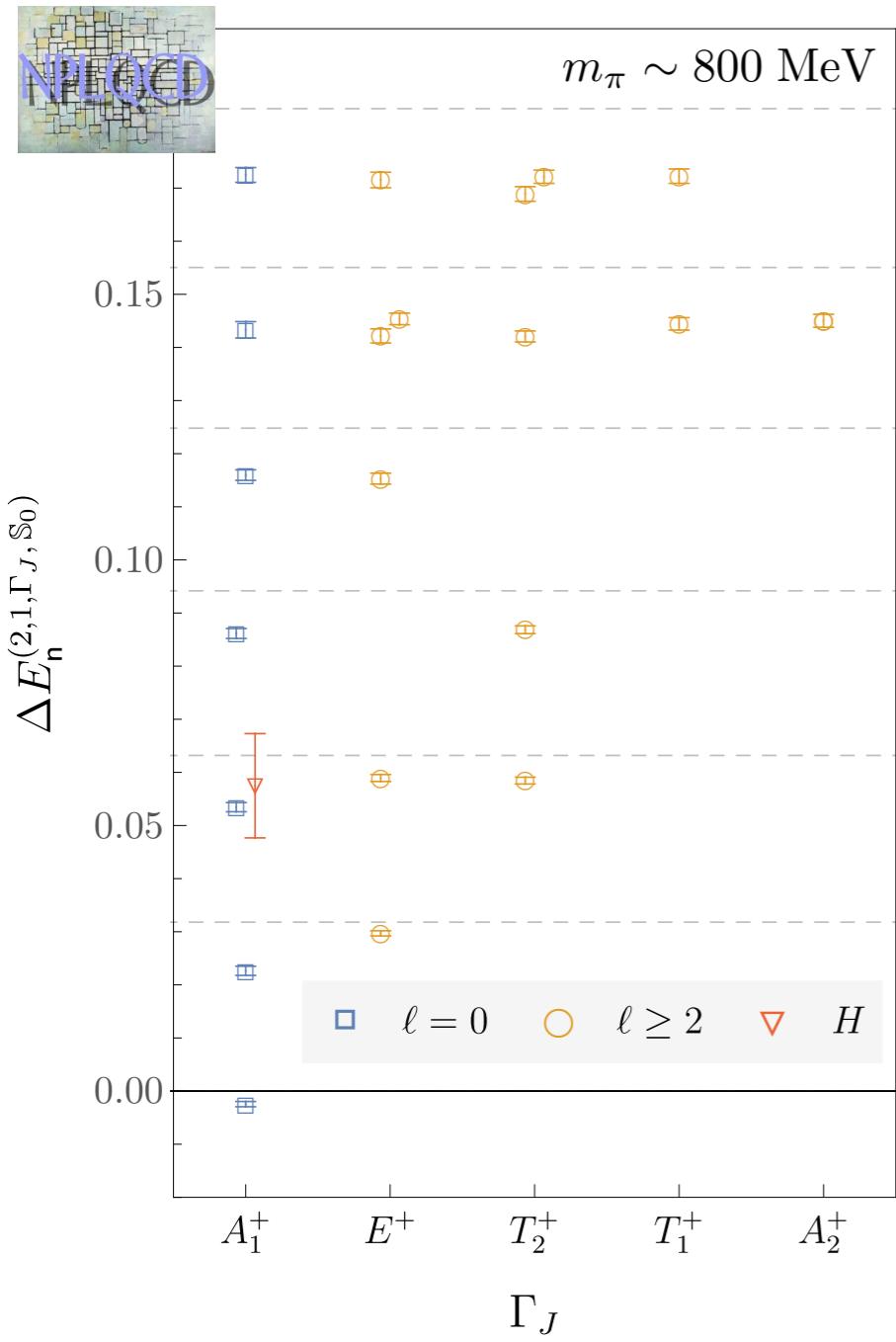
## Unbound $NN$ ( ${}^1S_0$ )

Hermitian matrices with only dibaryon-like operators

What about the hexaquark operators, which were used to find the bound states?

# First variational calculations

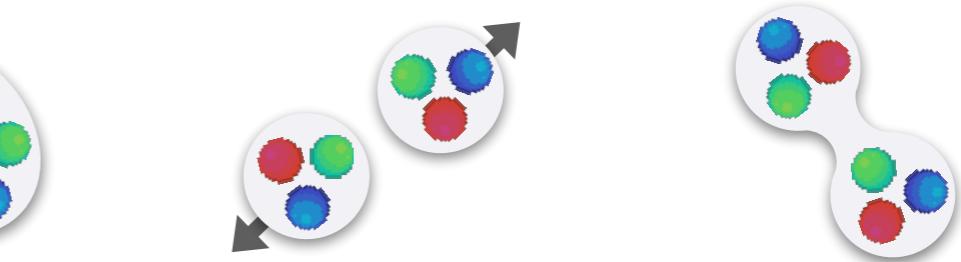
Dineutron channel GEVP spectrum



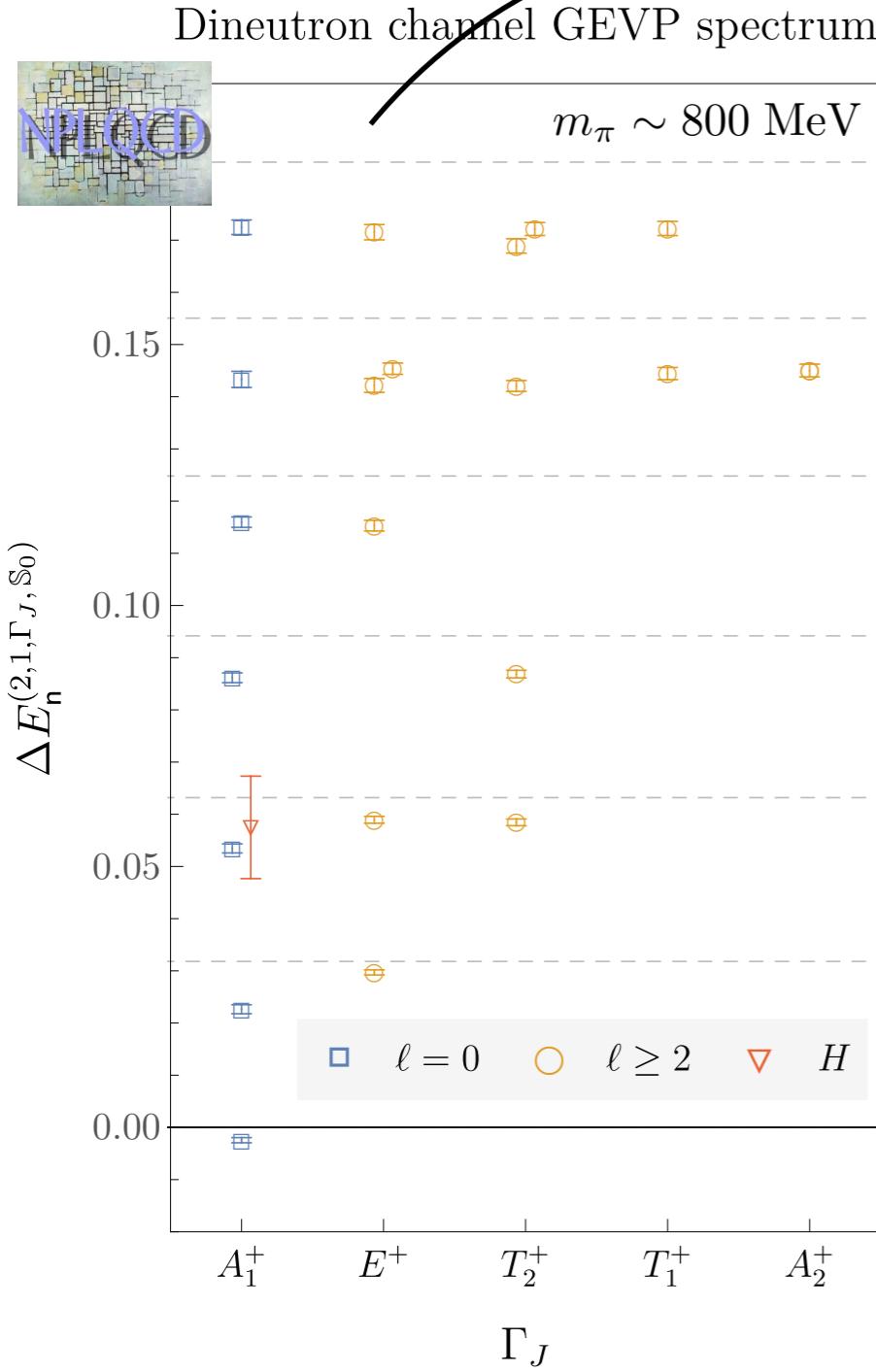
Amarasinghe et al. [NPLQCD],  
arXiv:2108.10835 [hep-lat]

Hermitian matrices with three operators:

- Hexaquark
- Dibaryon
- Quasilocal  $\rightarrow$  EFT inspired, with wavefunction that decays exponentially



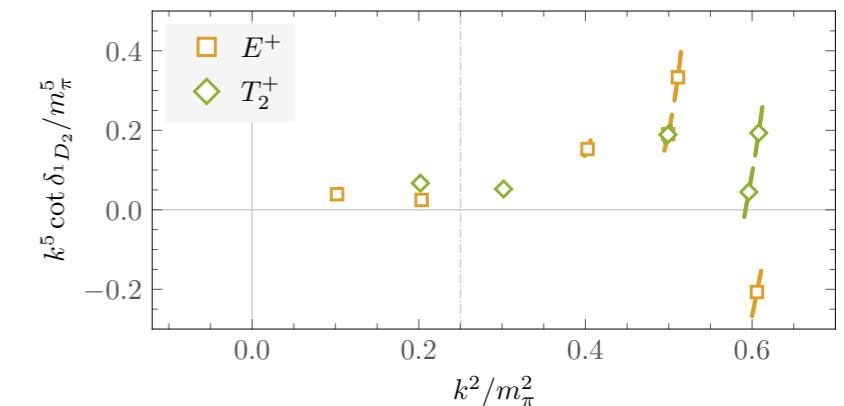
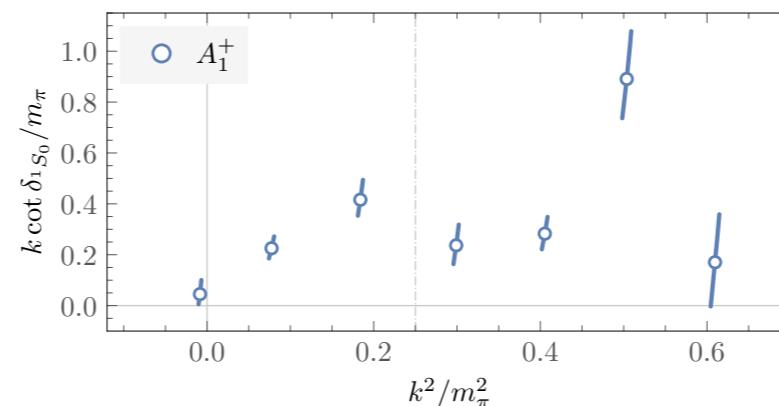
# First variational calculations



Amarasinghe et al. [NPLQCD],  
arXiv:2108.10835 [hep-lat]

## Using generalizations of Lüscher's QC (without mixing for now)

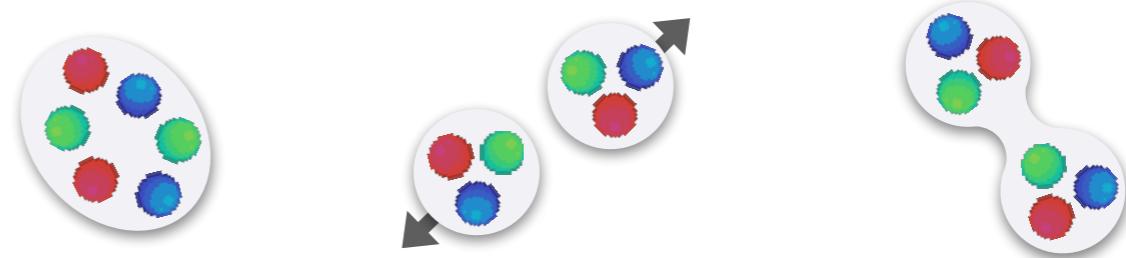
Luu and Savage, PRD83 (2011)  
Briceño, Davoudi and Luu, PRD88 (2013); +Savage, PRD88 (2013)



## Unbound $NN$ ( ${}^1S_0$ )

Hermitian matrices with three operators:

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# Robustness of variational approach

Large interpolating-operator dependence is observed

Energy levels disappear when the operator with the corresponding larger overlap is removed

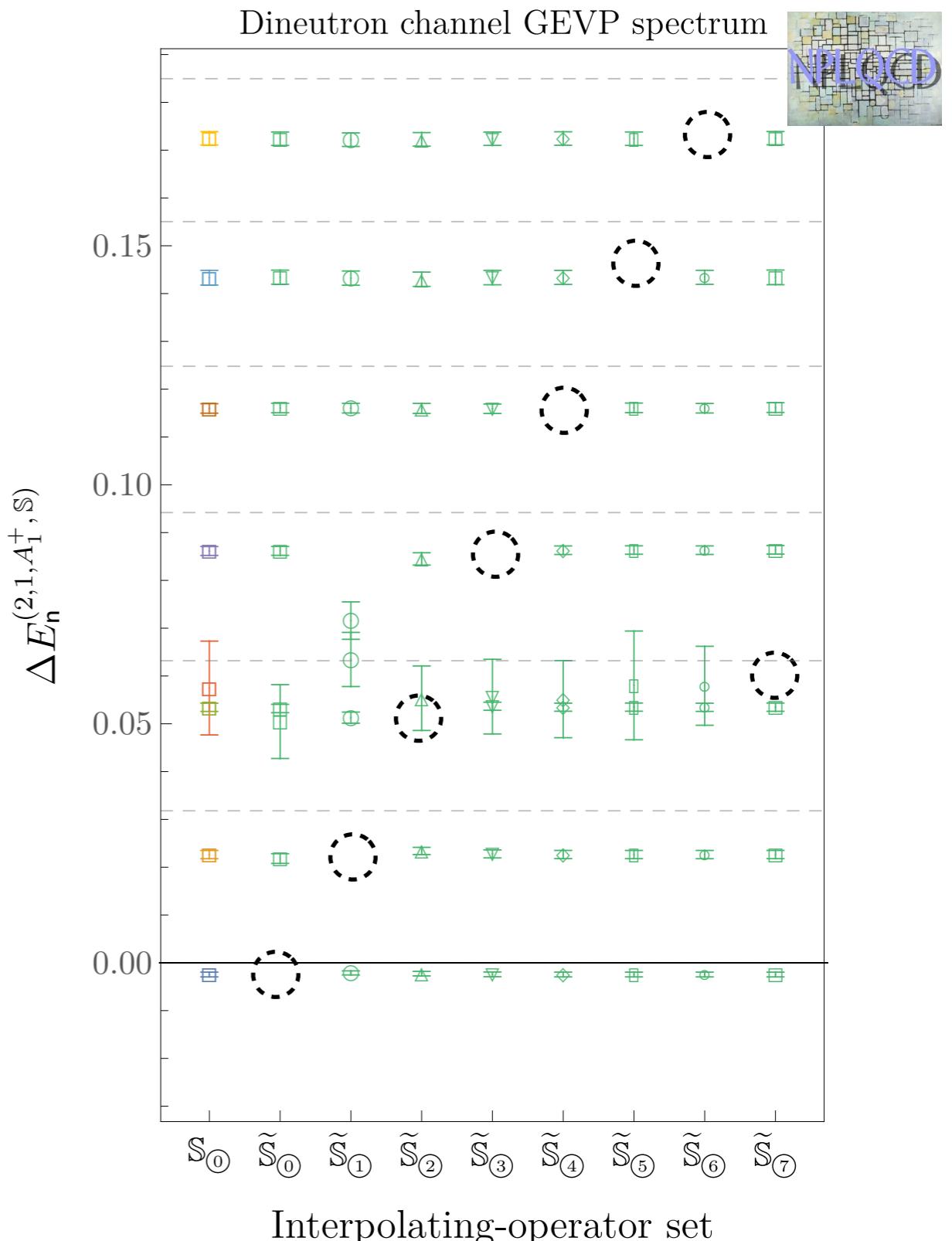
$\pi\pi$

Dudek et al. [HadSpec], PRD87 (2013)  
Wilson et al. [HadSpec], PRD92 (2015)

$N\pi$

Lang, Verduci, PRD87 (2013)  
Kiratidis et al., PRD91 (2015)

Are we still missing operators?



# Are we missing operators?

Option a) There is a deep-bound state, but the current operators have a small overlap

Amarasinghe et al. [NPLQCD], arXiv:2108.10835 [hep-lat]

$$E_0^{(AB)} = \eta - \Delta \quad E_1^{(AB)} = \eta \quad E_2^{(AB)} = \eta + \delta$$

$$Z_n^{(A)} = (\epsilon, \sqrt{1 - \epsilon^2}, 0) \quad Z_n^{(B)} = (\epsilon, 0, \sqrt{1 - \epsilon^2})$$

$$\lambda_0^{(AB)} = e^{-(t-t_0)\eta} \quad \lambda_1^{(AB)} = e^{-(t-t_0)(\eta+\delta)}$$

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Option b) There is no deep-bound state, however...

Volume independence of the ground state

Analysis of the phase-shifts and checks on scattering parameters

Consistency in scalar ME extraction between different methods

Agreement with large- $N_c$  prediction of an SU(6) symmetry

Agreeing values for the magnetic moments and  $np \rightarrow d\gamma$  cross section

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Coincidence?

Volume independence of the ground state

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# Are we missing operators?

Hexaquark operators are expected to have large overlap to deep bound state

→ Large number of operators

$$H \sim T_{abcdef} (q_a^T C \Gamma_1 F_1 q_b) (q_c^T C \Gamma_2 F_2 q_d) (q_e^T C \Gamma_3 F_3 q_f)$$

↑ color                      ↑ spin                      ↑ flavor

5            x            32            x            5 (9)        = 800 (1440)

ways to create a  
color singlet

spin operators  
with correct parity

flavor operators  
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16

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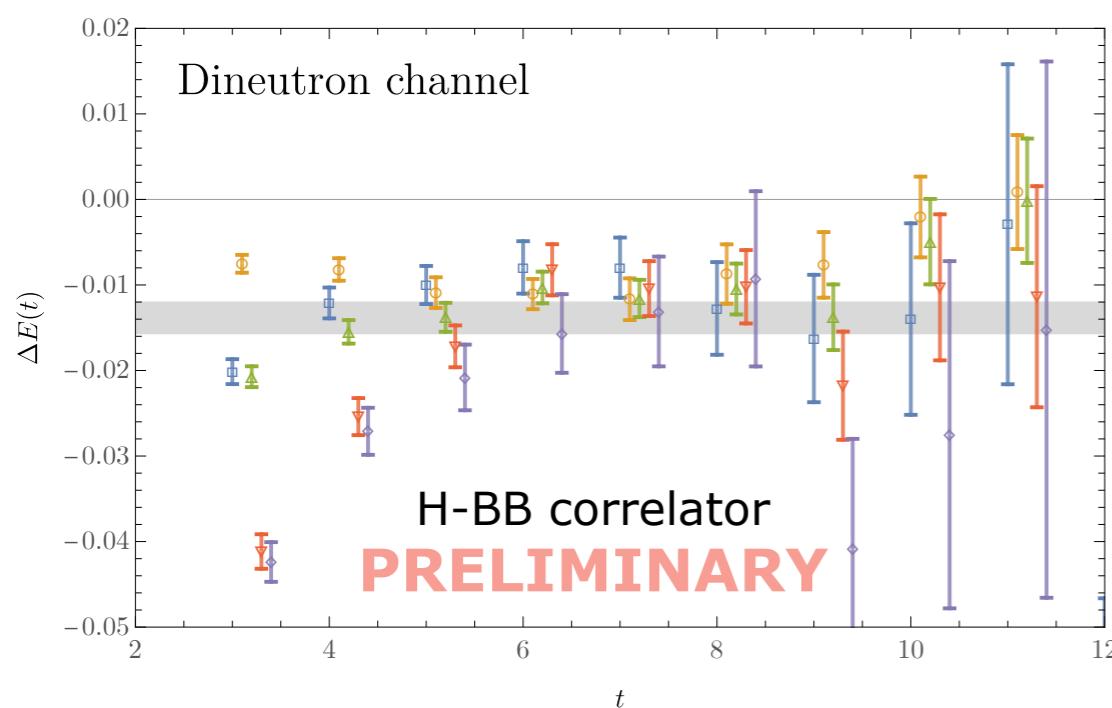
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Preliminary results show that after diagonalization, no bound state is found, but different off-diagonal correlators see the deep-bound state

# Summary

It is still unclear what the best operators are to include in a variational analysis for BB systems (significant interpolating-operator dependence)

While variational methods can provide reliable upper bounds on the energy levels, they don't rule out the existence of deep-bound states:

Ongoing study with additional operators and additional volumes at  $m_\pi \sim 800$  MeV for NN system

Ongoing production for different baryon-baryon systems, specifically the H-dibaryon system

Thank you for your attention