THE OHIO STATE UNIVERSITY

Center for Frontiers in Nuclear Science

Role of the chiral anomaly on the proton's spin

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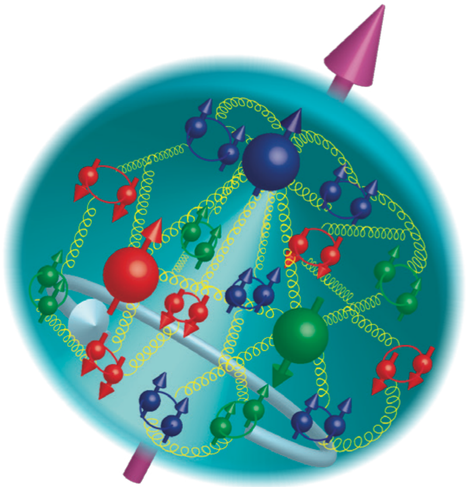
Based on Andrey Tarasov and Raju Venugopalan
Phys. Rev. D 102 (2020) 11, 114022 (arXiv:2008.08104), and
Phys. Rev. D 105 (2022) 1, 014020 (arXiv:2109.10370)

QNP2022, September 8, 2022

The proton's spin puzzle



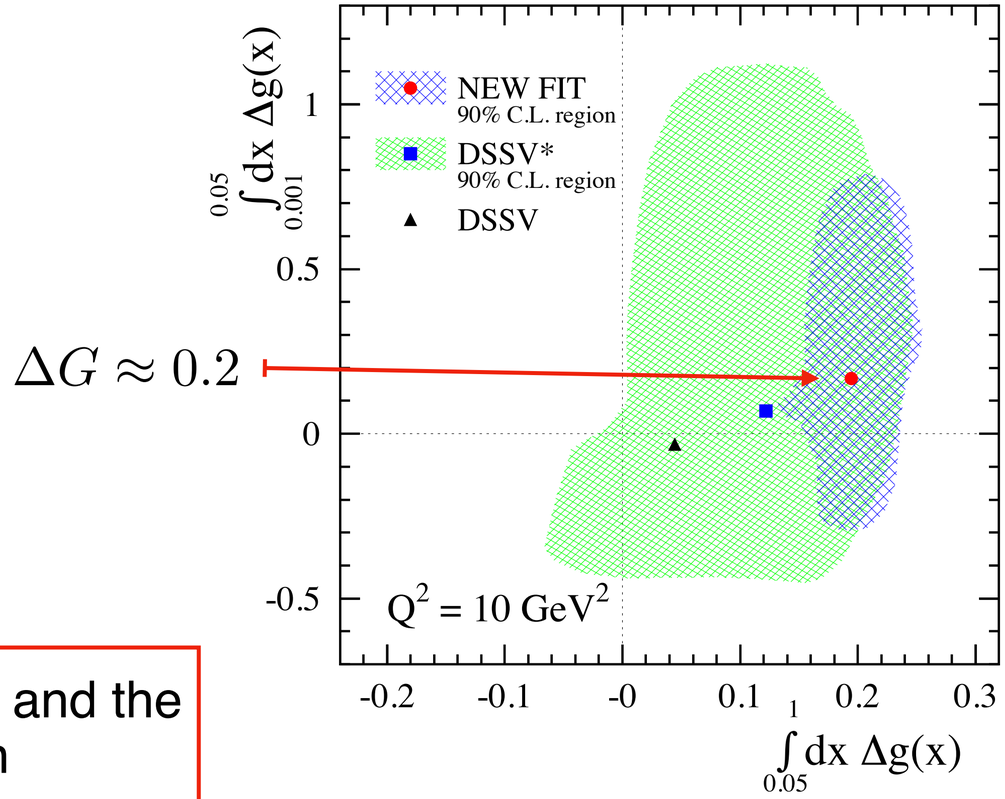
The fundamental properties of hadrons, and in particular its spin, are defined by the complex dynamics of quarks and gluons which form a strongly bonded many-body parton system

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Proton spin | Quark helicity (spin) | Gluon helicity | Orbital angular momentum

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton's spin: $\Delta\Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta\Sigma \approx 0.6$ - spin puzzle

The sum of quark and gluon helicities come short of 1/2 especially if one takes into account the error bars.



D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

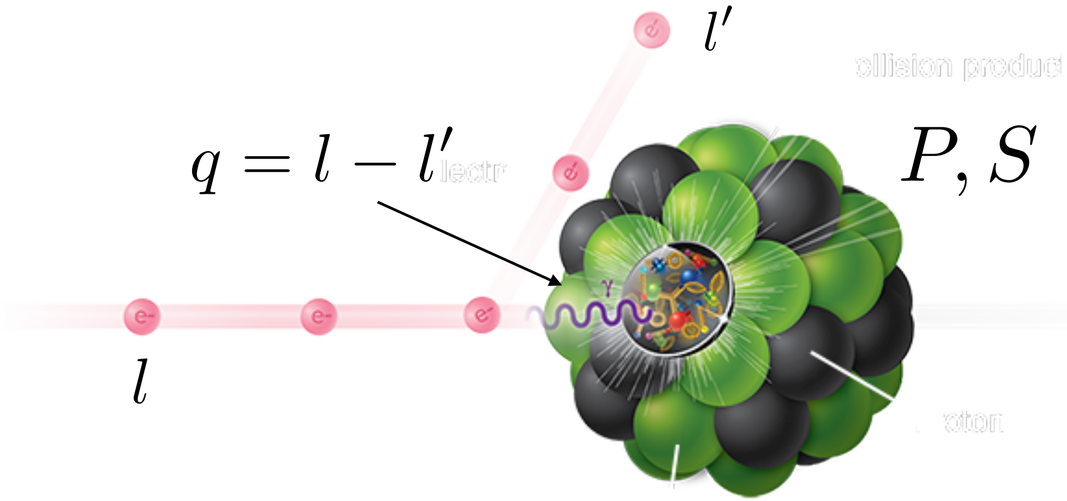
New tools for an old problem: interplay between parton dynamics and the topology of the QCD vacuum in the helicity structure of the proton

→ electroweak baryogenesis, sphaleron-like topological transitions

Deep inelastic scattering

We study the role of the chiral anomaly in the helicity structure of the proton which can be measured in the polarized DIS

$$e(l) + N(P,S) \to e(l') + X$$



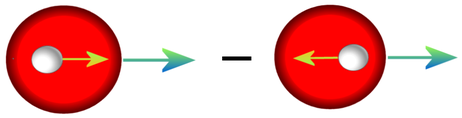
The process is characterized by its virtuality $Q^2 = -q^2$ and Bjorken variable $x_B = Q^2/(2P \cdot q)$.

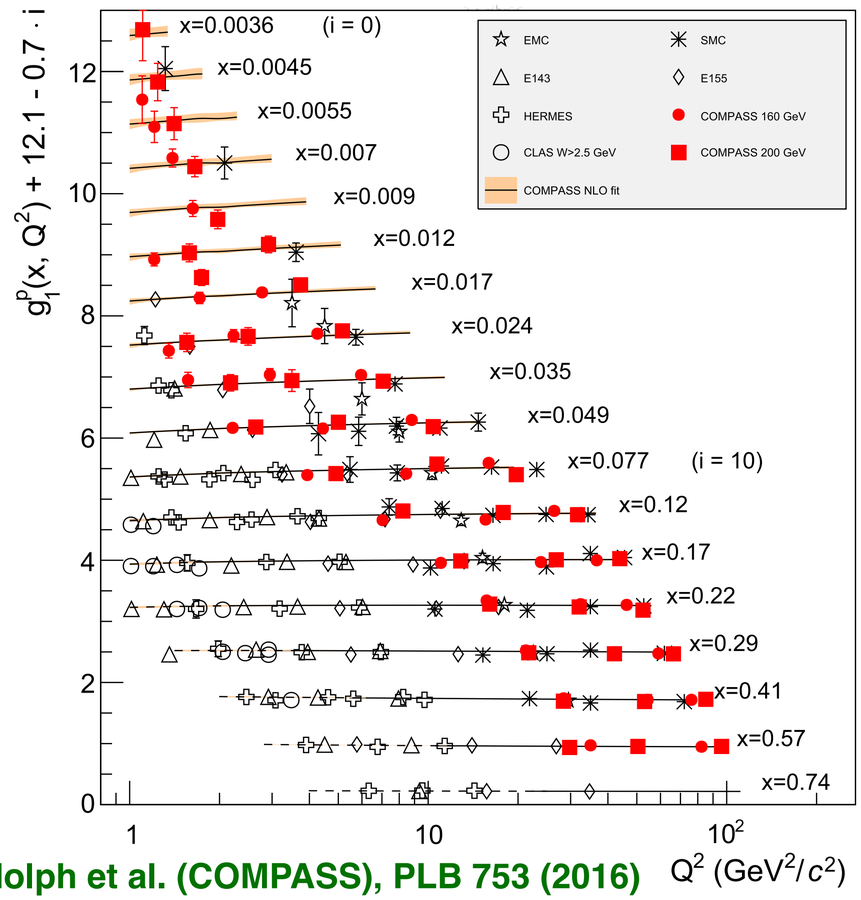
A key observable to study the proton helicity structure is the polarized structure function $g_1(x_B, Q^2)$:

$$\frac{1}{2}\left[\frac{\mathrm{d}^2\sigma^{\leftrightarrows}}{\mathrm{d}x_B\mathrm{d}Q^2} - \frac{\mathrm{d}^2\sigma^{\rightrightarrows}}{\mathrm{d}x_B\mathrm{d}Q^2}\right] \simeq \frac{4\pi\alpha^2}{Q^4} y(2-y) g_1(x_B, Q^2)$$

In the parton model it can be related to the polarized parton distribution function (PDF) which represent parton dynamics of the proton:

$$g_1(x_B, Q^2) = \frac{1}{2}\sum_f e_f^2 \left(\Delta q_f(x_B, Q^2) + \Delta\bar{q}_f(x_B, Q^2)\right)$$

$\Delta q_f(x_B) =$ 



C.Adolph et al. (COMPASS), PLB 753 (2016)

First moment of the structure function

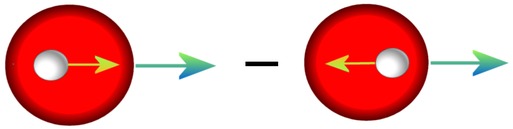
The helicity can be extracted from the first moment of the g_1 structure function

$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\boxed{\Sigma(Q^2)}\right)\left(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

quark helicity

In terms of quark PDFs the helicity can be defined as

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\, \left(\Delta q_f(x_B, Q^2) + \Delta\bar{q}_f(x_B, Q^2)\right)$$

$\Delta q_f(x_B) =$ 

Quark fields inside the proton

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f|P, S\rangle \equiv \frac{1}{M_N}\langle P, S|J_5^\mu(0)|P, S\rangle$$

Quark contribution to the proton spin is defined by the isosinglet axial vector current J_5^μ

Anomaly equation

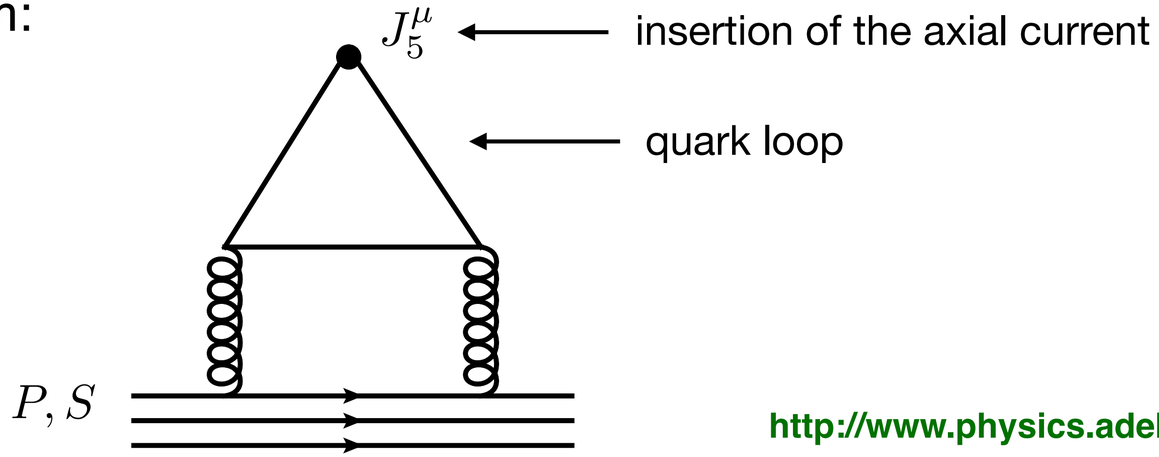
The fundamental property of the J_5^μ current is the anomaly equation:

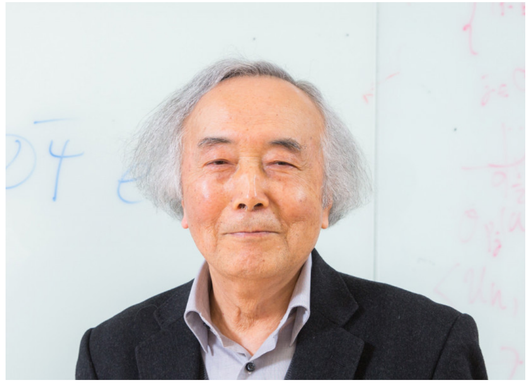
$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi} \, \mathrm{Tr}\left(F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)\right) = 2\, n_f\, \partial_\mu K^\mu$$

The isosinglet current couples to the topological charge density in the polarized proton!

The anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations. Topological properties of the QCD vacuum! **K. Fujikawa, PRL. 42, 1195 (1979)**

In the leading order the coupling is generated by the triangle diagram:

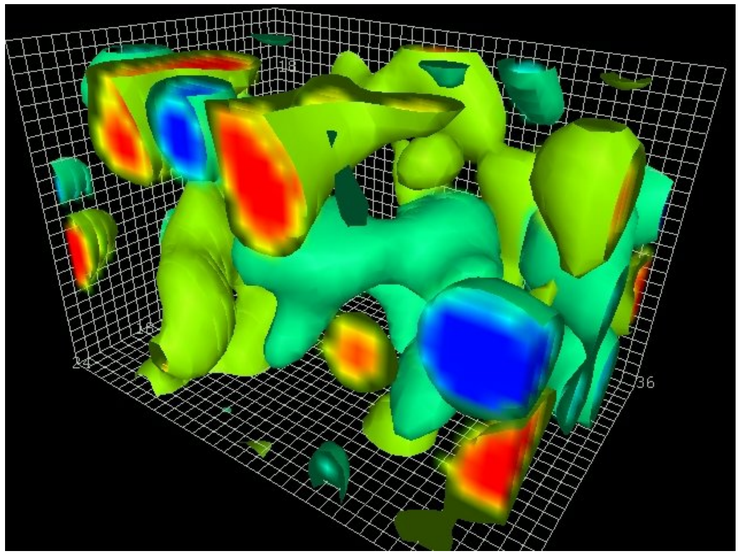




Kazuo Fujikawa

Chern-Simons current:

$$K_\mu = \frac{\alpha_S}{4\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma\right)\right]$$



Topological charge density

http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

The QCD effective action

More generally, the anomaly comes from the imaginary part of the effective action $\mathcal{W}$

We start with a general form of the QCD path integral (over fermion fields)

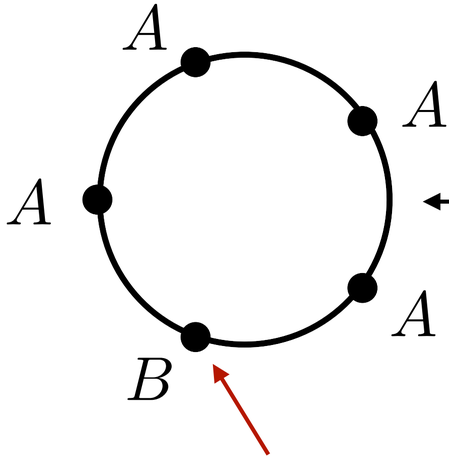
$$e^{i\mathcal{W}[A,B]} = \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}e^{iS[\bar{\Psi},A,B,\Psi]}$$

Coupling to vector and axial-vector currents

$$S[\bar{\Psi},A,B,\Psi] = \int d^4x\bar{\Psi}[i\not{\partial} + \not{A} + \gamma_5\not{B}]\Psi$$

which can be represented in terms of the functional determinant, i.e. for the effective action

$$-\mathcal{W}[A,B] = \ln \mathrm{Det}\ [\not{p} - \not{A} - \gamma_5\not{B}]$$



quark loop, see the triangle diagram

Imaginary and real parts of the effective action:

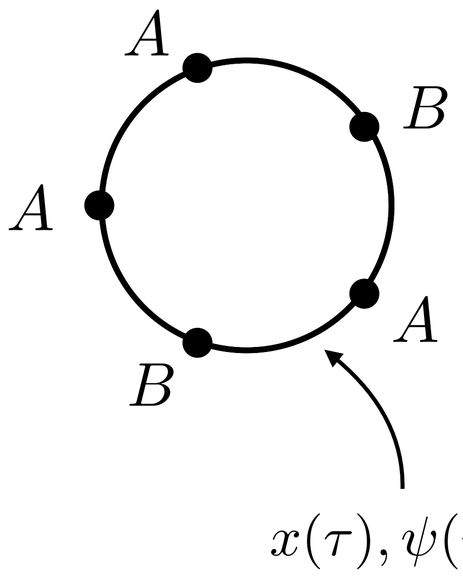
$$\mathcal{W}_R = -\frac{1}{2}\ln(\mathcal{D}^\dagger\mathcal{D}) \qquad \mathcal{W}_I = \frac{1}{2}\mathrm{ArgDet}(\mathcal{D}^2)$$

Insertion of the axial-vector current (γ_5). The anomaly is generated!

The anomaly couples to zero modes of the Dirac operator that must be treated in exact kinematics!

Worldline representation of the effective action

The calculation of the anomaly is very subtle. We use the worldline representation of the effective action which we find to be particularly suited to discussions of the anomaly

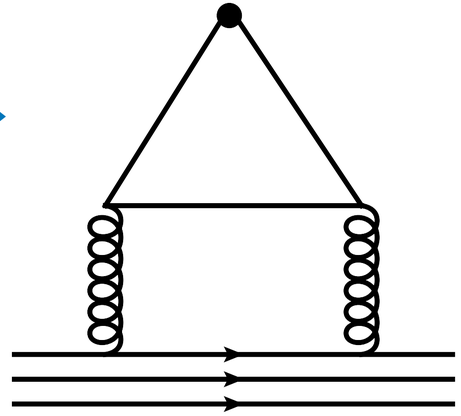


$$\mathcal{W}[A,B] = -\frac{1}{2}\mathrm{Tr_c}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \int_{AP} \mathcal{D}\psi \; e^{-S_{w.l.}(x,\psi,A,B)}$$

classical trajectories
τ is a proper-time variable

In this approach the effective action is rewritten as a quantum mechanical point particle path integrals over trajectories trajectories $x(\tau)$ and $\psi(\tau)$, where spin degrees of freedom of a particle (quark) are expressed in terms of Grassmann variables ψ.

$$\langle P,S|J_5^\mu(0)|P,S\rangle = \langle P,S|\int d^4y \, \frac{\partial \mathcal{W}_I[A,B]}{\partial B_\mu(y)}\Big|_{B_\kappa=0}|P,S\rangle$$



There is an effective techniques to calculate worldline functional integrals employing quantum mechanical worldline propagators

How does the anomaly manifest itself?

Quark helicity and the triangle anomaly

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S | \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f | P, S \rangle \equiv \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$

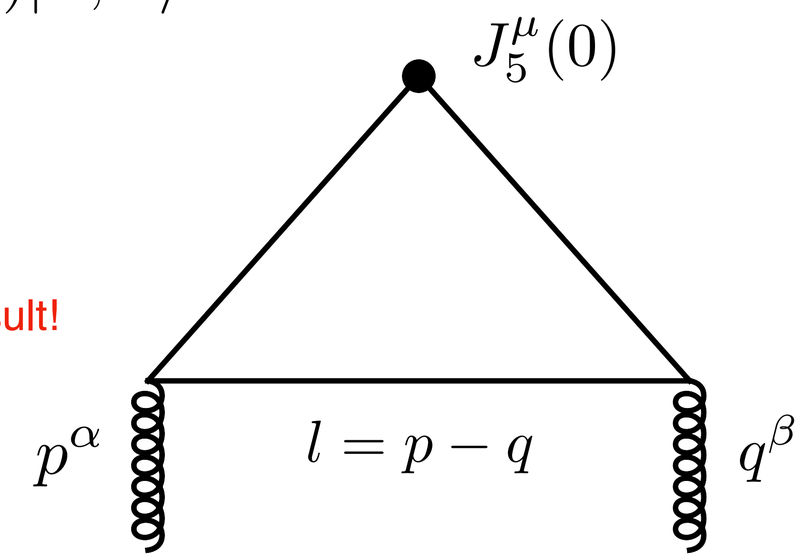
The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit. The exact worldline calculation gives

Exact result!

$$\langle P', S | J_5^\mu(0) | P, S \rangle = -i \frac{l^\mu}{l^2} \frac{\alpha_s n_f}{2\pi} \langle P', S | \mathrm{Tr}\left(F \tilde{F}\right) | P, S \rangle$$

infrared (anomaly) pole

topological charge density



Adler-Bell-Jackiw anomaly

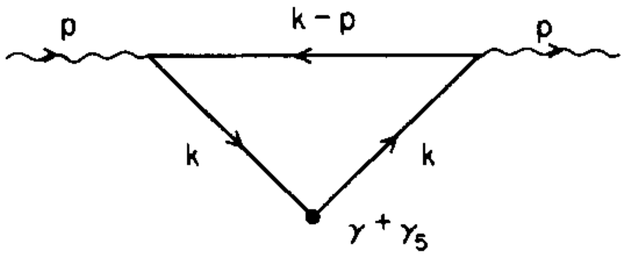
The triangle diagram is not local! The anomaly manifests itself as an infrared pole. Taking a divergence we obtain the anomaly equation

$$\partial^\mu J_\mu^5 = i l^\mu J_\mu^5 = \frac{n_f \alpha_s}{2\pi} \mathrm{Tr}\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)$$

R. L. Jaffe, A. Manohar
Nucl. Phys., B337, 509 (1990)
Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
K.-F. Liu (1992)

Calculation in the factorization approach

Calculation in the forward limit in the factorization ($k_\perp \gg p_\perp$) approach:

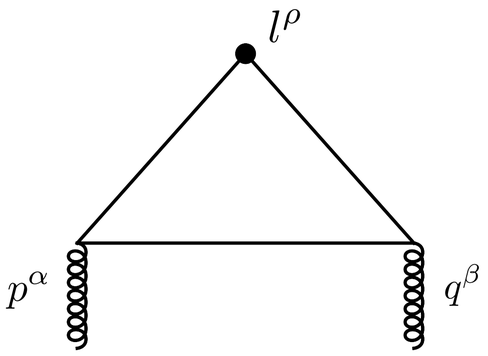


$$\frac{1}{2p^+}\langle p, \pm | j_5^+ | p, \pm \rangle = \mp \frac{\alpha_s N_f}{2\pi}$$

no infrared pole! There is an assumption that the anomaly is local

Carlitz, Collins, Mueller (1988)

However, calculation in exact kinematics contains the pole



$$\propto \frac{l^\mu}{l^2} F\tilde{F}$$

The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the η' a mass★.

R. L. Jaffe, A. Manohar
Nucl. Phys., B337, 509 (1990)

How is the pole cancelled? Interplay between perturbative and non-perturbative physics. The mechanism of the cancelation is deeply related to the $U_A(1)$ problem in QCD - topological mass generation of the $m^2_{\eta'}$.

Pseudoscalar contribution

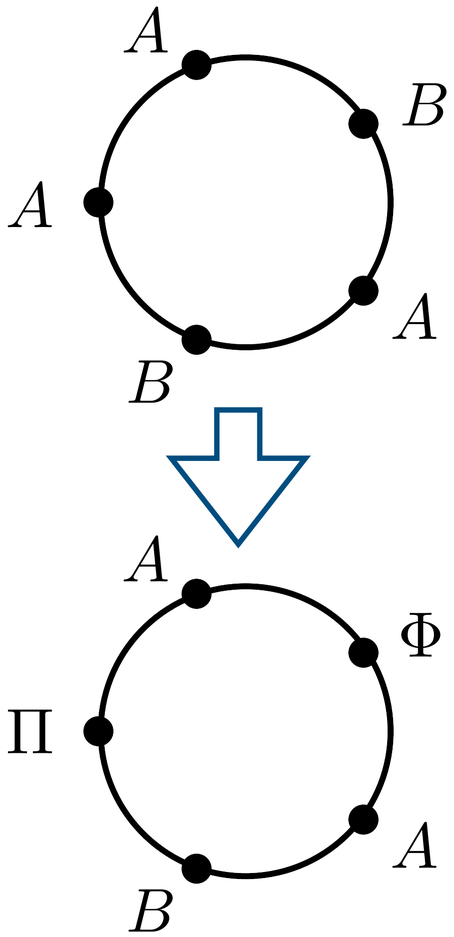
To see the mechanism of cancelation of the infrared pole
we have to extend the theory to the pseudoscalar sector

$$-\mathcal{W}[A,B] = \ln \mathrm{Det}\,\left[\not{p} - \not{A} - \gamma_5 \not{B}\right]$$ ← incomplete

$$-\mathcal{W}[A,B,\Phi,\Pi] = \ln \mathrm{Det}\,\left[\not{p} - i\Phi - \gamma_5\,\Pi - \not{A} - \gamma_5 \not{B}\right]$$ ← complete

scalar field

pseudoscalar field (mesons)



Functional integral representation of the imaginary part
of the effective action:

$$\mathcal{W}_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_0^{\infty} dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\ \mathrm{tr}\ \chi\,\bar{\omega}(0)\exp\Big[-\int_0^T d\tau\,\mathcal{L}_{(\alpha)}(\tau)\Big]$$

This is the most general parametrization of the phase of the
Dirac determinant in terms of scalar, pseudoscalar, vector
and axial vector fields

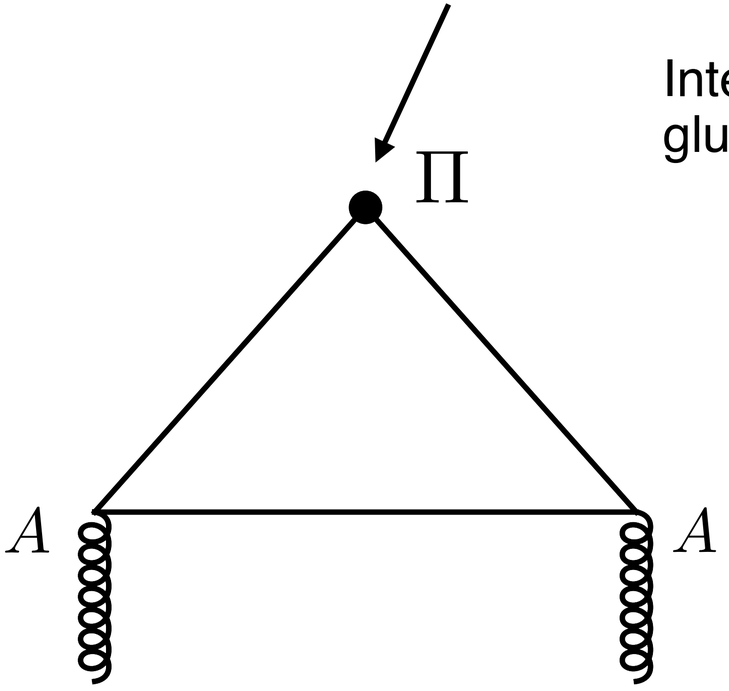
D'Hoker, Gagne, hep-th/9508131

Wess-Zumino-Witten coupling

Tarasov, Venugopalan
arXiv:2109.10370

$$W_{\mathcal{I}}[\Pi A^2] = -\frac{\mathcal{E}\Phi}{4}\int_{-1}^{1} d\alpha \int_0^\infty dT\mathcal{N}\int_P \mathcal{D}x\mathcal{D}\psi\ \exp\Big[-\int_0^T d\tau\Big\{\frac{\dot{x}^2}{2\mathcal{E}}+\frac{1}{2}\psi\dot{\psi}\Big\}\Big]\exp\Big[-T\frac{\mathcal{E}\alpha^2\Phi^2}{2}\Big]$$

$$\times\,\mathrm{tr}_c\Big(\psi_5(\tau_0)\Pi(x_0)-\psi^\mu(\tau_0)\dot{x}_\mu(\tau_0)\int_0^T d\tau_1\psi^\nu(\tau_1)\psi_5(\tau_1)\partial_\nu\Pi(x_1)\Big)V_2V_3$$



Interaction with the background gluon field:

$$V_i \equiv \int_0^T d\tau_i\Big(\dot{x}^\rho(\tau_i)+\mathcal{E}\psi^\rho(\tau_i)\psi^\alpha(\tau_i)\partial_\alpha\Big)A_\rho(x_i)$$

Separation of the zero-modes:

$$x(\tau_i)=\bar{y}+y(\tau_i) \qquad\qquad \psi(\tau)=\psi+\xi(\tau)$$

Calculation of the the Grassmann functional integrals:

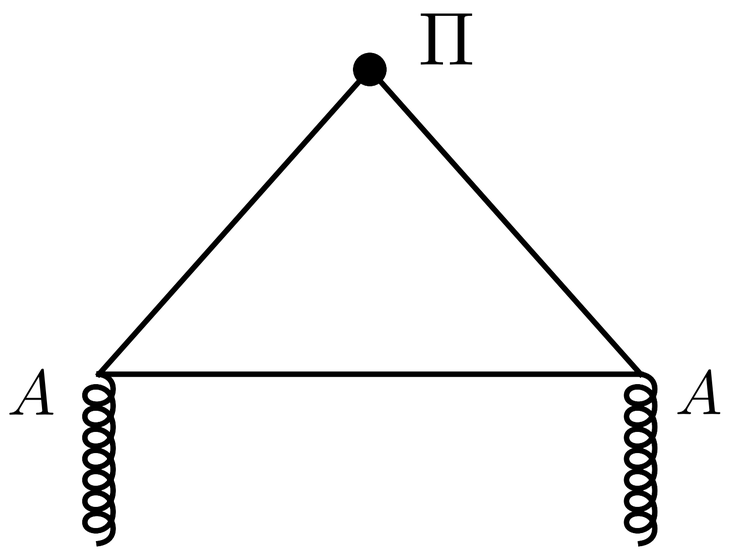
$$\int d^5\psi\,\psi^\mu\psi^\nu\psi^\rho\psi^\sigma\psi^5 = \epsilon^{\mu\nu\rho\sigma}$$

$$\int_P \mathcal{D}\xi\exp\Big[-\int_0^T d\tau\frac{1}{2}\xi\dot{\xi}\Big]\xi^\mu(\tau_1)\xi^\nu(\tau_2) = -\frac{1}{2}g^{\mu\nu}\dot{G}_B(\tau_1,\tau_2)$$

Wess-Zumino-Witten coupling

We calculate the imaginary part of the effective action in the leading order in Π. It generates the isosinglet Wess-Zumino-Witten coupling $\propto \bar{\eta} F \tilde{F}$

$$\mathcal{W}_{\mathcal{I}}[\Pi A^2] = \frac{i g^2\, 2 n_f}{16\pi^2} \frac{1}{\Phi}\, \mathrm{tr}_c \int d^4x\, \Pi(x)\, F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x)$$

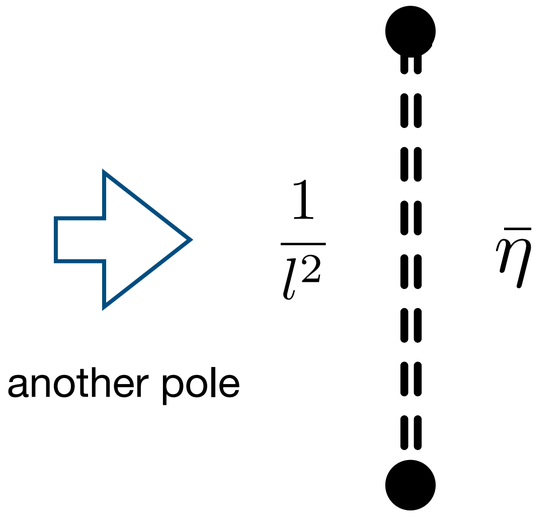


in agreement with the corresponding term in $\mathscr{L}_{\mathrm{WZW}}$ which was derived from chiral perturbation theory **Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)**

Tarasov, Venugopalan arXiv:2109.10370

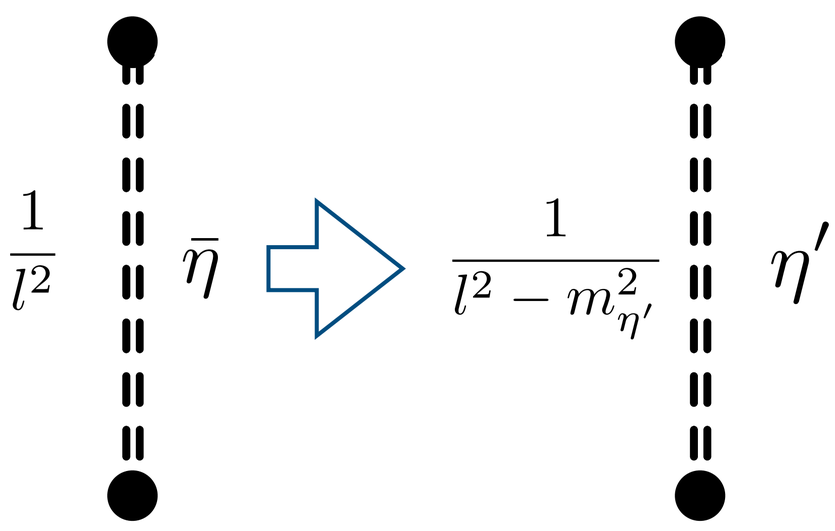
$$S^{\bar{\eta}}_{\mathrm{WZW}} = -i \frac{\sqrt{2\, n_f}}{F_{\bar{\eta}}} \int d^4x\, \bar{\eta}\, \Omega \qquad \Omega = \frac{\alpha_s}{4\pi} \mathrm{Tr}\left(F \tilde{F}\right)$$

where $\bar{\eta}$ is a massless "primordial" ninth Goldstone boson arising from the spontaneous symmetry breaking of the flavor group $U_L(3) \times U_R(3)$



Anomaly pole and the $U_A(1)$ problem

However $\bar{\eta}$ is not observed. Instead there is a heavy η' ($m_{\eta'} \approx 957 MeV$) - the famous $U_A(1)$ problem.

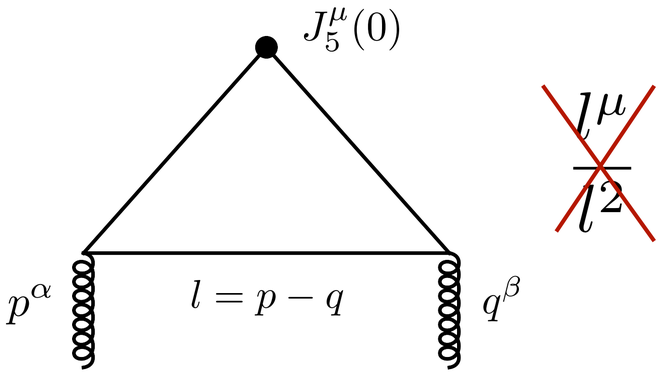


There is no Goldstone pole just as there is no anomaly pole in the QCD spectrum

the same mechanism

We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole

Tarasov, Venugopalan
arXiv:2109.10370

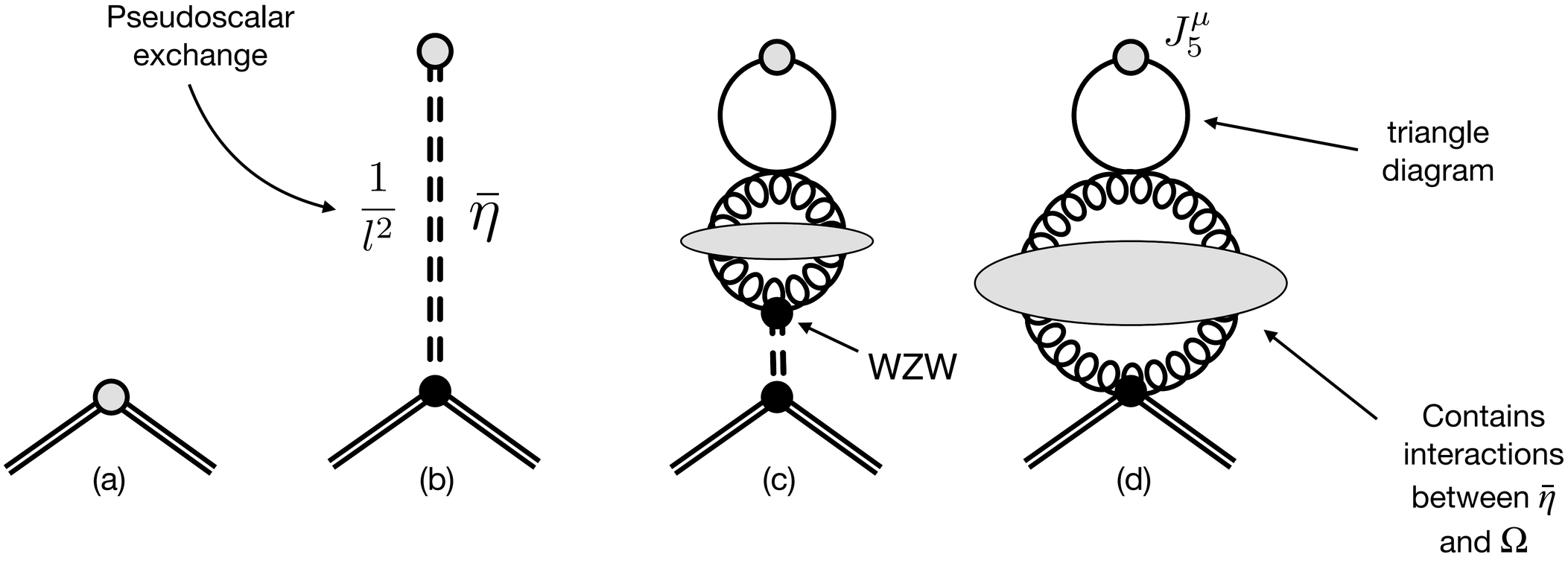


This mechanism relates the helicity structure of the proton to the topology of the QCD vacuum

How is the pole cancelled?

The fundamental role is played by the Wess-Zumino-Witten (WZW) coupling between the topological charge density Ω to a primordial massless isosinglet $\bar{\eta}$. We determine it in the worldline formalism, agrees with ChPT.

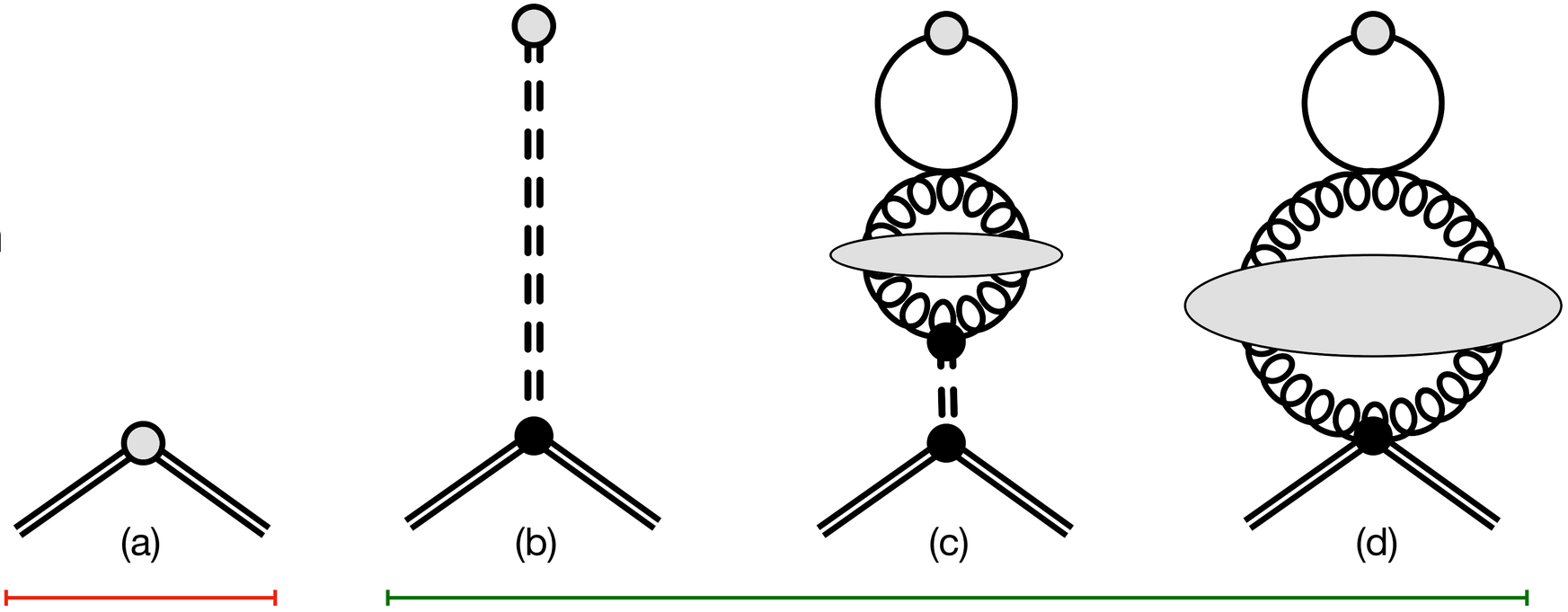
Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)



Pseudovector vs. pseudoscalar coupling

$$\langle P', S | J_5^\mu | P, S \rangle = \bar{u}(P', S) \Big[\gamma^\mu \gamma_5 G_A(l^2) + l^\mu \gamma_5 G_P(l^2) \Big] u(P, S)$$

We establish relations between diagrams using anomaly equation and absence of the infrared pole in $G_P(l^2)$



Direct axial-vector coupling

Pseudoscalar coupling to the polarized proton

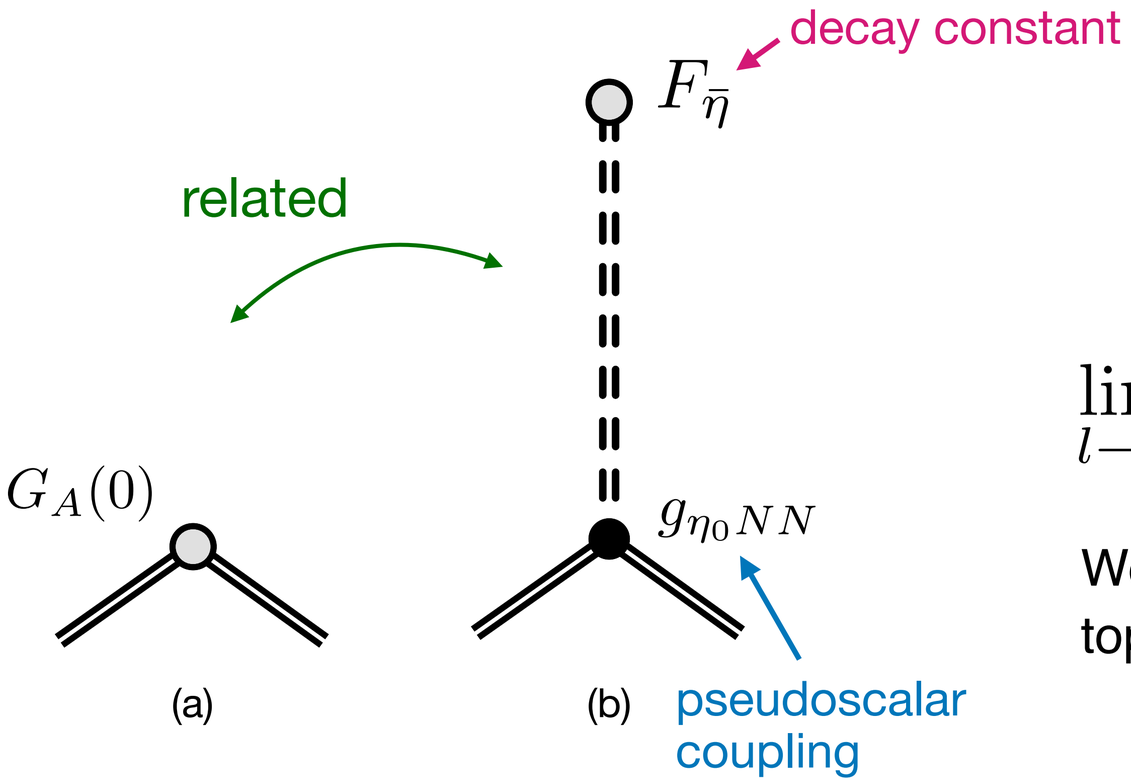
This relations will allow us to connect quark helicity Σ to the topological properties of QCD

Goldberger-Treiman relation

Since $G_P(l^2)$ form factor doesn't have a pole:

$$\lim_{l\to 0}\langle P', S|J_5^\mu|P, S\rangle = 2M_N\, G_A(0)\, S^\mu \quad \Rightarrow \quad \Sigma(Q^2) = 2G_A(0)$$

Using the anomaly equation we derive a generalization of the well-known Goldberger-Treiman which relates axial-vector and pseudoscalar sectors:



$$\boxed{G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}\, F_{\bar\eta}\, g_{\eta_0 NN}}$$

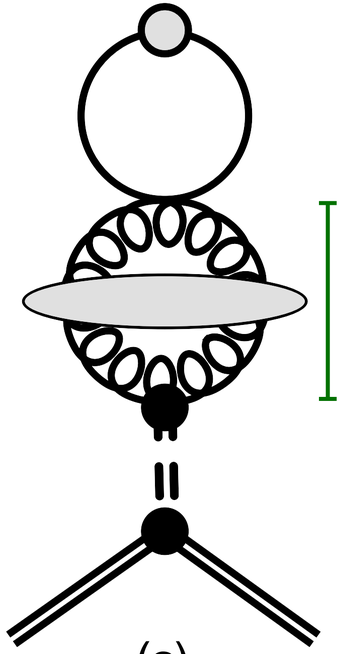
Veneziano (89)

$$\Downarrow$$

$$\lim_{l\to 0}\langle P', S|J_5^\mu|P, S\rangle = \sqrt{2\tilde{n}_f}\, F_{\bar\eta}\, g_{\eta_0 NN} S^\mu$$

We can further relate the decay constant $F_{\bar\eta}$ to the QCD topological susceptibility

Topological susceptibility



(c)

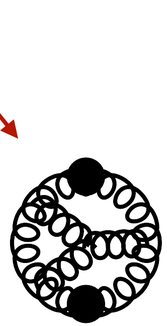
Topological susceptibility

$$\chi(l^2) = i \int d^4x\, e^{ilx} \langle 0|T\,\Omega(x)\Omega(0)|0\rangle$$

Yang-Mills topological susceptibility $\chi_{YM}(l^2)$

QCD topological susceptibility contains an infinite number of $\bar{\eta}$ insertions

$\chi(l^2) =$



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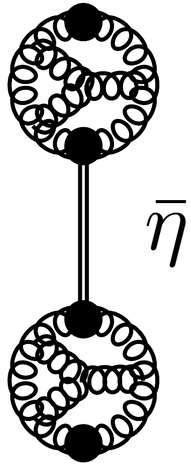


$\bar{\eta}$ + ...

We compute the QCD topological susceptibility diagrammatically using the WZW term

$$\boxed{\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\rm YM}(l^2)}$$

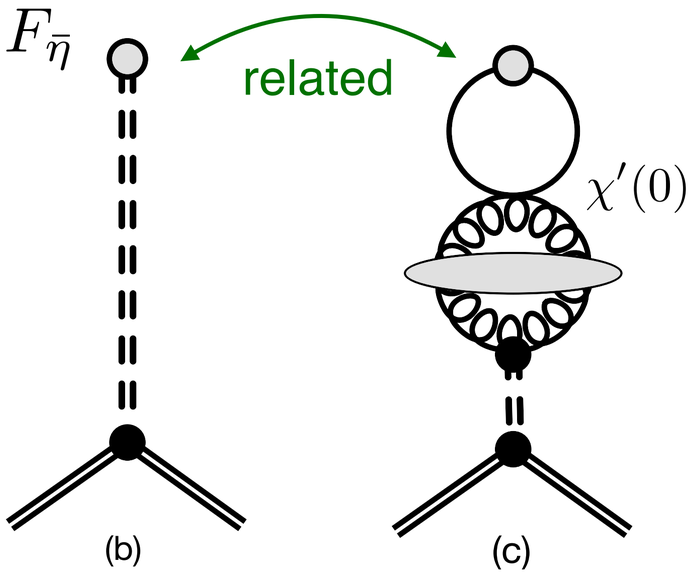
$$m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\bar{\eta}}^2} \chi_{\rm YM}(0)$$

Topological mass generation of η'.
Solution of the $U_A(1)$ problem

In agreement with Witten-Veneziano formula for $m_{\eta'}^2$

In the forward limit: $\chi(0) = 0$ but $\chi'(0) \neq 0$ ⇨ Diagram (c) is defined by $\chi'(0)$ and $F_{\eta'}$

Infrared pole cancelation



From the cancellation of the infrared pole we related diagrams (b) and (c) which yields a relation between the decay constant and the QCD topological susceptibility

topological screening

$$F_{\bar{\eta}}^2 = 2n_f \chi'(0) \Rightarrow \boxed{\Sigma(Q^2) = \sqrt{\frac{2}{3}}\frac{2n_f}{M_N} g_{\eta_0 NN}\sqrt{\chi'(0)}}$$

Goldberger-Treiman

Shore, Veneziano (1992)

Recover a Shore-Veneziano result (derived using anomalous functional chiral Ward identities)

$$\langle P,S|J_5^\mu|P,S\rangle = M_N S^\mu \Sigma(Q^2) = 2M_N S^\mu a_0$$

$$\boxed{a^0|_{Q^2=10GeV^2} = 0.33 \pm 0.05}$$

Gives a natural resolution of the spin crisis

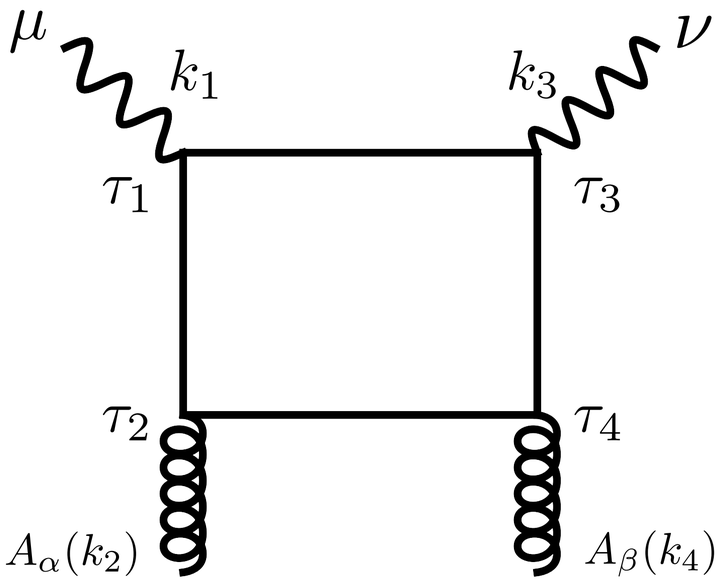
In agreement with COMPASS ($a^0|_{Q^2=3GeV^2} = 0.35 \pm 0.08$) and HERMES data ($a^0|_{Q^2=5GeV^2} = 0.330 \pm 0.064$)

Shore (2007), Narison (2021)

Shore

Veneziano

Infrared pole and the structure function g_1



We use powerful worldline QFT formalism to compute box diagram contribution to g_1 in exact kinematics of internal variables

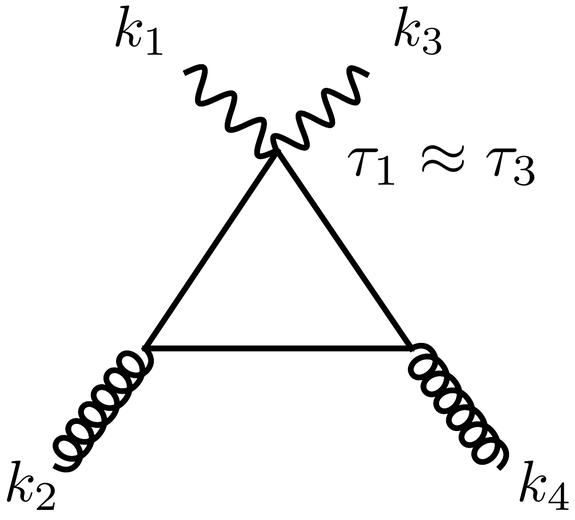
$$g_1(x_B, Q^2) \propto -\frac{g^2 e^2 e_f^2}{2} \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int \mathcal{D}\psi \ \exp\Big\{ - S_{w.l.}(x,\psi) \Big\}$$

$$\times \Big[V_1^\mu(k_1) V_3^\nu(k_3) V_2^\alpha(k_2) V_4^\beta(k_4) - (\mu \leftrightarrow \nu) \Big]$$

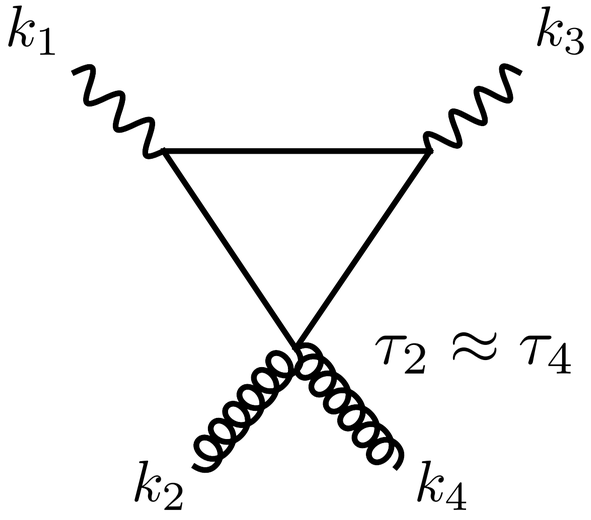
where the vertex corresponds to the interaction of a worldline with the external current

$$V_i^\mu(k_i) \equiv \int_0^T d\tau_i (\dot{x}_i^\mu + 2i\psi_i^\mu k_j \cdot \psi_j) e^{ik_i \cdot x_i}$$

This is the most general expression for the box diagram. We calculate it in Bjorken (large Q^2) and Regge (small x_B) asymptotic limits.



point-like interaction with a virtual photon at large Q^2



Lorentz contraction of background fields at small x_B

Infrared pole and the structure function g_1

We find that g_1 (box diagram) is dominated by the triangle anomaly - g_1 is topological in both asymptotic limits of QCD. This suggests that g_1 is governed by topology as in the case for the first moment

infrared pole

$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \boxed{\frac{l^\mu}{l^2}} \langle P', S| \boxed{\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)} |P, S\rangle + \text{non-pole terms}$$

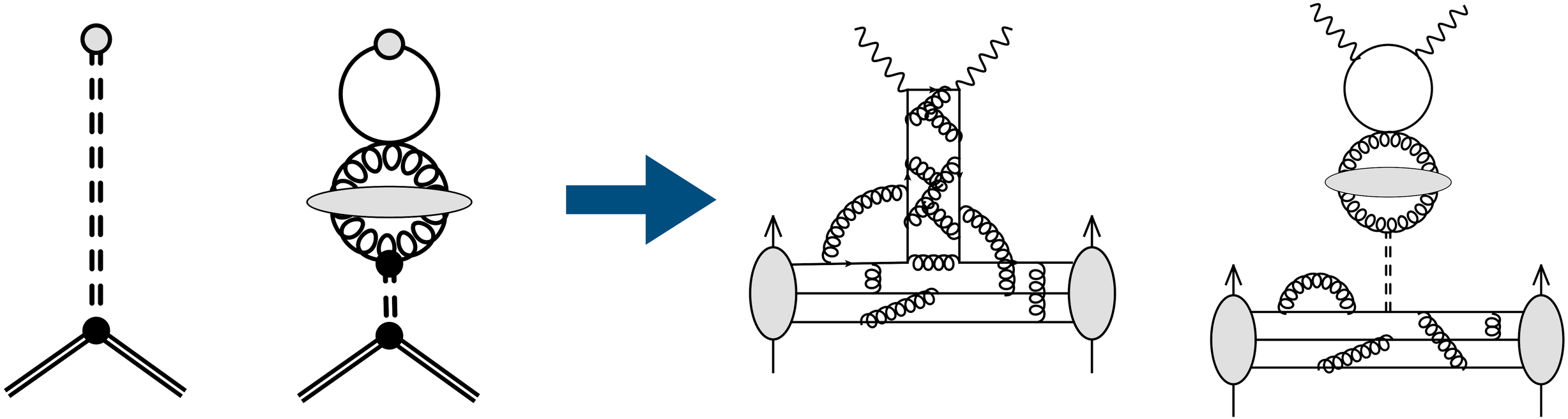
Tarasov, Venugopalan (arXiv:2008.08104)

$$S^\mu g_1(x_B, Q^2)\Big|_{x_{\mathrm{Bj}}\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \boxed{\frac{l^\mu}{l^2}} \langle P', S| \boxed{\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)} |P, S\rangle + \text{non-pole terms}$$

Note: interpretation of the r.h.s. as a contribution to g_1 is ambiguous since it's proportional l^μ and not S^μ. However, Goldberger-Treiman relation shows that axial and pseudoscalar sectors are tied.

Pole cancellation beyond the first moment

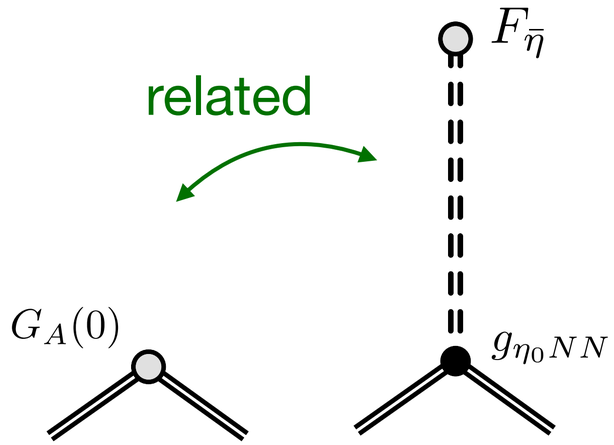
Isosinglet exchange is the only known mechanism to cancel anomaly pole. The absence of the physical pole must go through as for the first moment



Generalized Goldberger-Treiman relation

One requires the anomaly equation to be valid for the “smeared” topological charge density. If this generalized Goldberger-Treiman relation holds, one can relate g_1 directly to the coefficient of the $\frac{l^\mu}{l^2}$ divergent term in the box diagram calculation

$$\langle P,S|J_5^\mu|P,S\rangle = 2n_f \lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle\, g_{\eta_0 NN}\, S^\mu$$



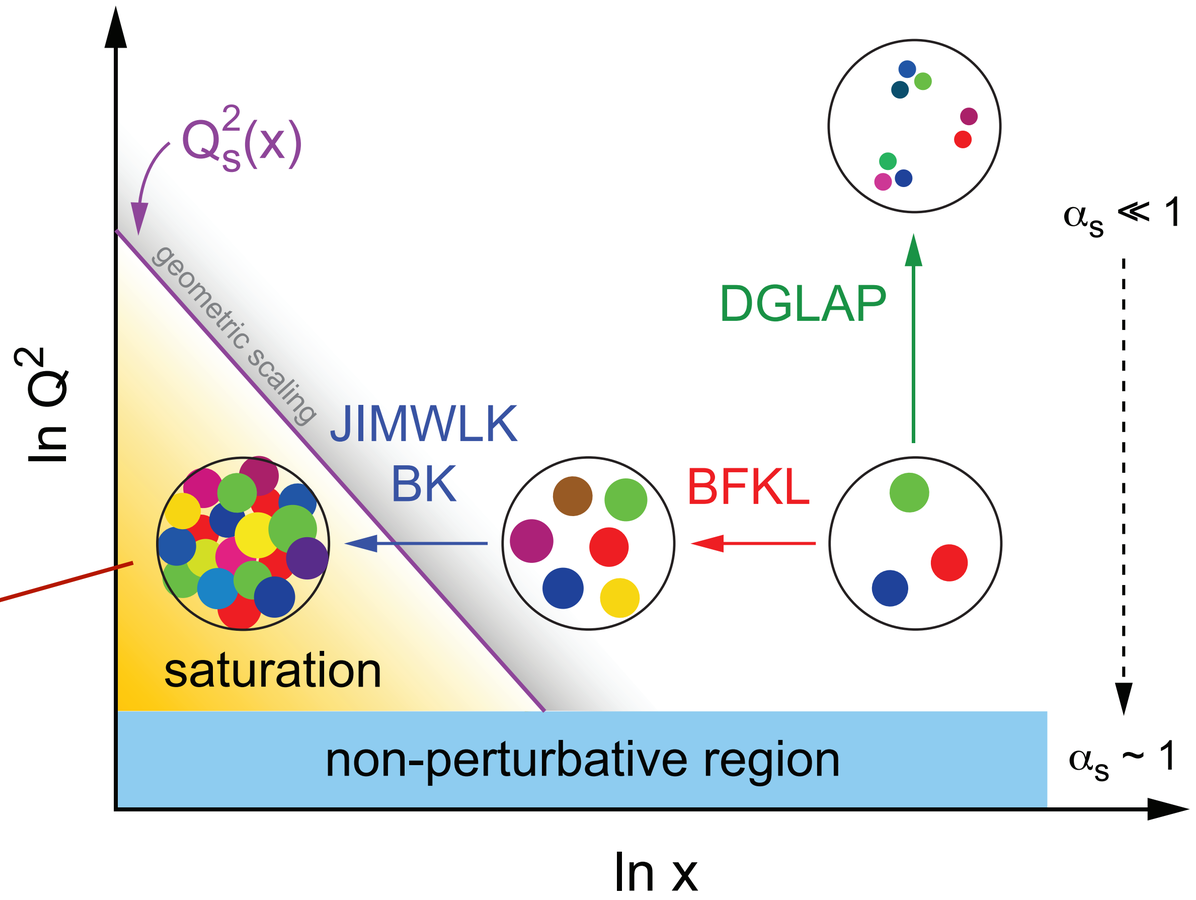
$$g_1(x_B,Q^2) = \left(\sum_f e_f^2\right) \frac{n_f\alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \int D\bar\eta\, \tilde W_{P,S}[\bar\eta] \int [DA]$$

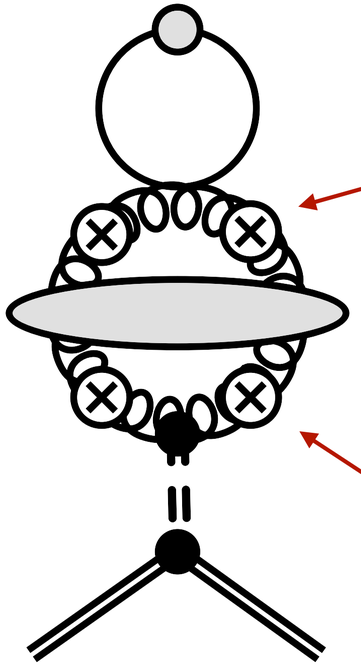
$$\times\, \mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde F^{\alpha\beta}(0)\,\eta_0(y)\,\exp\left(iS_{\mathrm{YM}} + i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\bar\eta\right)\left(\partial^\mu\bar\eta\right) - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega\right]\right)$$

Gluon saturation at small-x

The dynamics of the gauge fields in the small-x limit are qualitatively different. In the Regge asymptotics the gluon field saturate (strong classical field!). The saturation scale Q_s^2 emerge.

McLerran, Venugopalan (1994)





In the Regge limit, the YM topological susceptibility χ_{YM} , is dominated by a solution in a saturated small-x background field. It's not any more a vacuum solution. How do we describe the small-x gauge field?

Axion-like effective action at small-x

We construct an axion-like effective action at small x_B that describes the interplay between gluon saturation and the topology of the QCD vacuum. It contains the WZW coupling and a kinetic term for the $\bar{\eta}$ field. This dynamics is governed by the $m^2_{\eta'}$

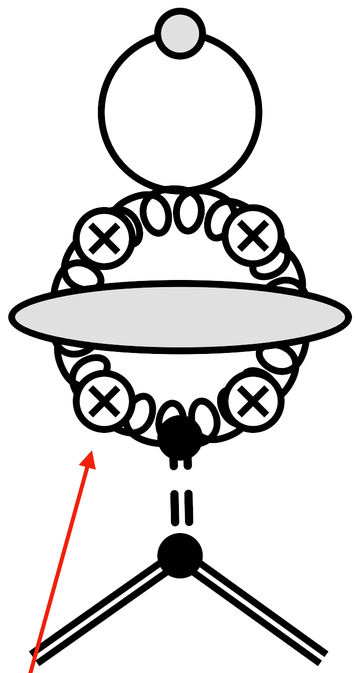
WZW coupling

$m^2_{\eta'}$

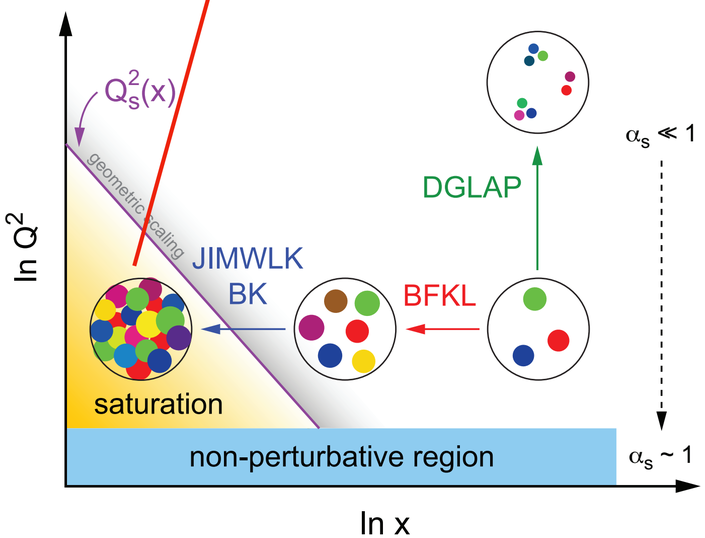
$$S_{\rm pCGC}[A,\rho,\bar{\eta}] = S_{\rm CGC}[A,\rho] + \int d^4x \left[\frac{1}{2}\left(\partial_\mu \bar{\eta}\right)\left(\partial^\mu \bar{\eta}\right) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}}\bar{\eta}\,\Omega \right]$$

Q_s^2

$\bar{\eta}$ "axion" propagation



The background gauge configurations are static classical configurations and their dynamics is described by the Color Glass Condensate (CGC) Effective Field Theory and controlled by the saturation scale $Q_s^2 > \Lambda^2_{QCD}$



Tarasov, Venugopalan arXiv:2109.10370

McLerran, Venugopalan (1994)

Two scales: Q_s^2 and $m^2_{\eta'}$!

The $g_1(x_B, Q^2)$ structure function

We can detect the topological transitions in DIS by measuring g_1 structure function. We write an expression for g_1 using our axion-like effective action at small-x_B which describes the interplay between gluon saturation and the topology of the QCD vacuum

initial distribution of color sources

initial distribution of $\bar{\eta}$

$$g_1^{\text{Regge}}(x_B, Q^2) = \left(\sum_f e_f^2\right) \frac{n_f \alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \int \mathcal{D}\rho \boxed{W_Y[\rho]} \int D\bar{\eta} \boxed{\tilde{W}_{P,S}[\bar{\eta}]} \int [DA]$$

$$\times \, \text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) \, \eta_0(y) \, \exp\boxed{\left(iS_{\text{CGC}} + i \int d^4x \left[\frac{1}{2} (\partial_\mu \bar{\eta}) (\partial^\mu \bar{\eta}) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta} \, \Omega \right] \right)}$$

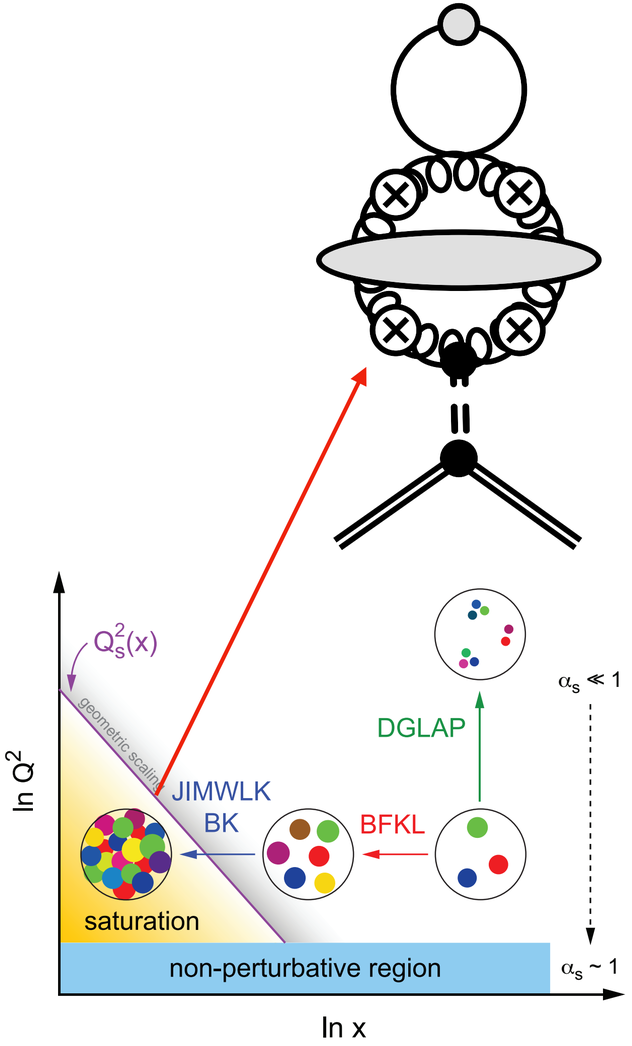
Tarasov, Venugopalan
arXiv:2109.10370

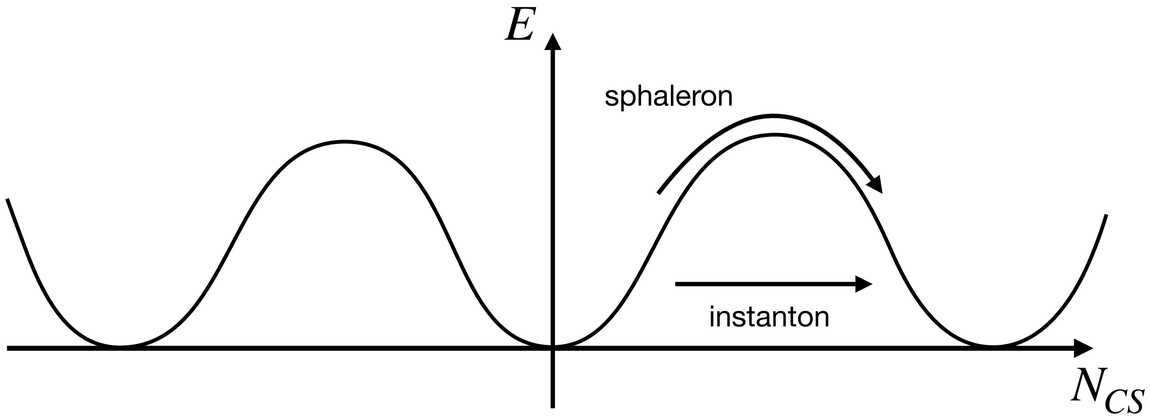
axion-like effective action

What effect do we expect to see in g_1 due to the topological transitions?

Sphaleron transitions

There are two scales in the problem! Gluon saturation (Q_s^2) can be treated as a perturbation around the instanton solution and induce over the barrier sphaleron-like transitions between different topological sectors of the QCD vacuum, each corresponding to distinct integer valued Chern-Simons number N_{CS}





plays a major role in electroweak baryogenesis!

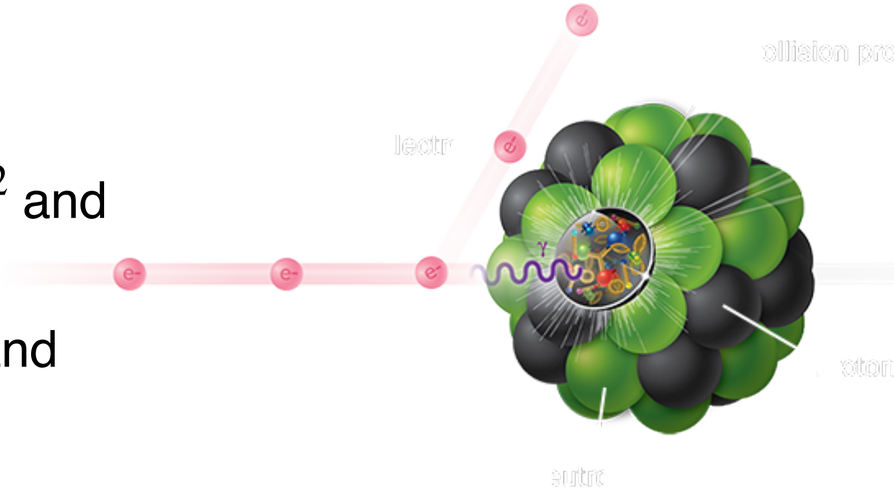
sphaleron transition rate: $$\Gamma_{\rm sphaleron} = \lim_{\delta t \to \infty} \frac{1}{V \delta t} \langle (N_{\rm CS}(t+\delta t) - N_{\rm CS}(t))^2 \rangle$$

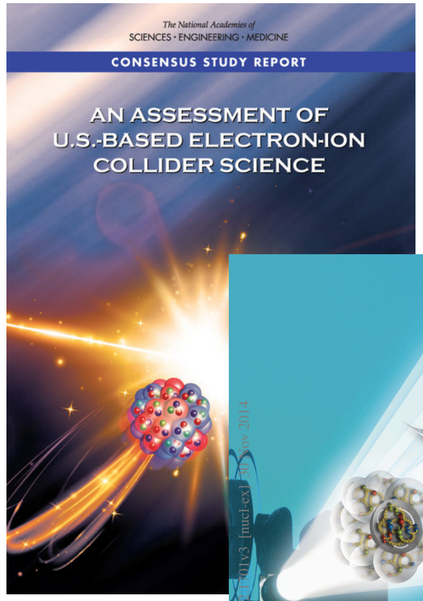
$$N_{\rm CS}(t) = \int d^3x \, K^0$$

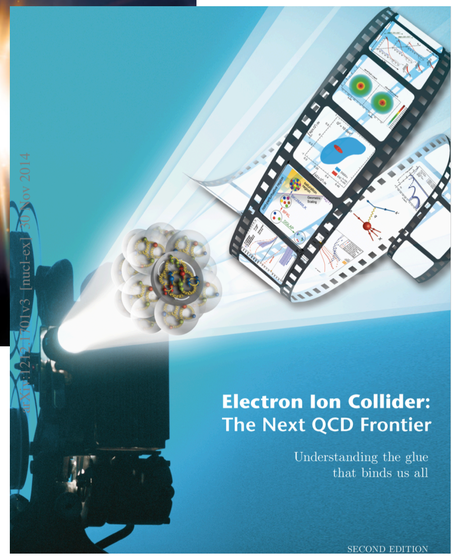
While at large x_B the gluon field is dominated by the instanton configurations, at small x_B the CGC background ($Q_s^2 > m_{\eta'}^2$) can induce over-the-barrier transitions. We predict over-the-barrier sphaleron transitions between different topological sectors of the QCD vacuum

Summary

- We show that the anomaly appears in both the Bjorken limit of large Q^2 and in the Regge limit of small x_B. g_1 is a topological quantity
- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD. This relates g_1 to the properties of the QCD vacuum
- We demonstrate the fundamental role of the WZW term both in topological mass generation of the η' and in the cancellation of the infrared pole
- We introduce an axion-like effective action at small-x which describes the interplay between gluon saturation and the topology of the QCD vacuum
- We outline the role of “over-the-barrier” sphaleron-like transitions in spin diffusion at small x_B. Such topological transitions can be measured in polarized DIS at a future Electron-Ion Collider.







Thank you for your attention!