The 3D hadron structure: from transverse momentum dependent factorization to parton distribution and fragmentation functions

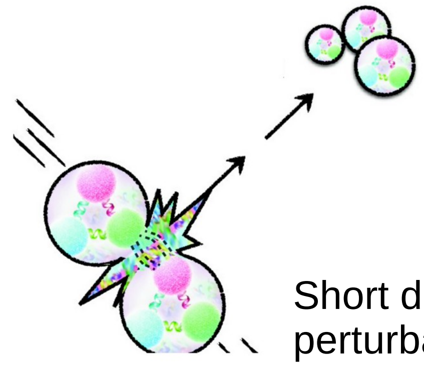
M. Boglione

In collaboration with O. Gonzalez and A. Simonelli

QNP 2022

QCD framework

QCD
- Asymptotic Freedom
- Confinement

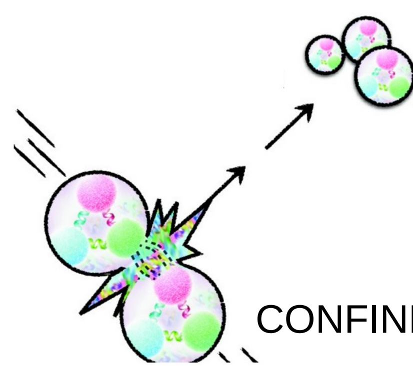


Long distance physics, non-perturbative structure functions.

Short distance effects, perturbative QCD

QCD
- Perturbative regime (computable but process dependent terms)
- Non perturbative regime (non computable but universal terms)

Strong interactions: **hadron structure** is a playground for 3D studies!



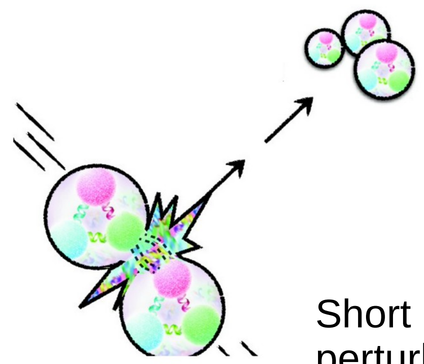
HADRONIZATION

CONFINEMENT

QCD framework

QCD

- Perturbative regime (computable but process dependent terms)
- Non perturbative regime (non computable but universal terms)



- The interplay between **perturbative** and **non-perturbative** regimes is currently one of the most challenging aspects in phenomenology.
- **Factorization** allows to separate the perturbative content of an observable from its non-perturbative content. At large *Q* and small *m*, the non-perturbative contributions are separated out from anything that can be computed by using perturbative techniques, and identified with universal quantities (structure functions).
- **Factorization** restores the predictive power of QCD

Factorization: region classification

J. Collins, Foundations of perturbative QCD (Cambridge University Press, 2011)

Particles are classified according to how they propagate in space,
i.e. according to their virtuality.

Momentum regions:

HARD
$k \sim (Q,\, Q,\, Q)$

Short-distance contributions → Large virtuality

Long-distance contributions → Low virtuality

SOFT
$k \sim (\lambda_S,\, \lambda_S,\, \lambda_S)$

$$\lambda_S = \lambda^2/Q$$

COLLINEAR
$k \sim (Q,\, \lambda^2/Q,\, \lambda)$

Factorization theorem

General structure of a generic factorization theorem:

$$\mathcal{O} = H \times \boxed{S \times \prod_j C_j} + p.s.$$

IR-safe hard contribution

Soft and collinear contributions, accounting for non-perturbative effects

Power suppressed terms

- Each term is equipped with proper subtractions.
- The soft factor S encodes the *correlation* among the various collinear parts.
- While H can be computed in pQCD, S and C have to be determined using non perturbative methods. For instance they can be modeled and extracted from experimental data, or computed in lattice QCD

TMD observables

The 3D hadron structure and transverse momentum dependence (TMD)

- Observables that carry information about the **transverse motion** of partons inside the hadrons are of primary interest in modern studies of QCD, as they encode very rich information about the 3D hadron structure and transverse spin effects.
- The **TMD factorization** of such observables is one of the most important and challenging approach to investigate the non-perturbative core of QCD, as well as spin-spin and spin-momentum correlations between the hadrons and their constituents.

Quark-quark correlation matrix

$$\Phi_{i\,j}(k,P,S) = \mathrm{F.T.}\langle P\,S|\overline{\psi}_j(0)\,W[0,\xi]\,\psi_i(\xi)|P\,S\rangle$$

See also talks by Harut Avakian, Matteo Cerutti, Marco Contalbrigo and Andrea Simonelli

- Dirac algebra expansion
- $\xi = \left(0, \xi^-, \vec{\xi}_T\right)$
- Leading Twist (Twist-2)

Contributions @ LP (power counting)

nucleon pol. \ quark pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

8 TMD PDFs at leading twist

hadron pol. \ quark pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_1,\ H_{1T}^\perp$

8 TMD FFs at leading twist

Collinear factorization

J. Collins, Foundations of perturbative QCD (Cambridge University Press, 2011)

$q_T \gtrsim Q$ → There is *enough* transverse momentum to produce jets at wide angles in the final state. → The low transverse momenta of the struck parton, the fragmenting parton and soft radiation are totally negligible

The soft factor becomes a trivial unit matrix in color space

All the hard jets are included into the hard part

Typical collinear factorization

Terms that encode non-perturbative effects:

- Collinear factor associated with the **target**
- Collinear factor associated with the **detected hadron**

None of them carry TMD information.

$$d\sigma = \sum_{j,j'} H_{j,j'} \otimes f_{j,\,h_1}(x) \otimes d_{j',\,h_2}(z)$$

TMD factorization

J. Collins, Foundations of perturbative QCD (Cambridge University Press, 2011)

$q_T \ll Q$ → The production of hard jets with high transverse momentum is strongly suppressed → The low transverse momenta of the struck parton, the fragmenting parton and soft radiation are *relevant*

The soft factor is a vacuum expectation value of Wilson lines

The hard part accounts for the sole virtual radiation.

Soft radiation correlates collinear terms

Terms that encode the non-perturbative effects:

- Collinear factor associated with the **target**
- Collinear factor associated with the **detected hadron**
- Soft factor correlating them.

All of them carry TMD information.

See also talks by Harut Avakian, Matteo Cerutti, Marco Contalbrigo and Andrea Simonelli

$$\widetilde{d\sigma} = \sum_{j,j'} H_{j,j'} F_{j,h_1}(x, b_T)\, S(b_T)\, D_{j',h_2}(z, b_T)$$

Rapidity divergences

- Rapidity divergences are introduced by the approximations induced by factorization
- Problematic for TMD factorization.
- Rapidity divergences are ultimately associated to Wilson lines along the light-cone:

Eikonal propagators $\sim \dfrac{1}{k^+ + i\,0}$ and $y = \dfrac{1}{2} \log \dfrac{k^+}{k^-}$

Collins' regulator: soft Wilson lines are tilted off the light-cone:

$(1,\, 0,\, \vec{0}_T) \to (1,\, -e^{-2\,y_1},\, \vec{0}_T),$ $\quad y_1$ Large and positive

$(0,\, 1,\, \vec{0}_T) \to (-e^{2\,y_2},\, 1,\, \vec{0}_T),$ $\quad y_2$ Large and negative

Analogous to introducing a **rapidity cut-offs**

$$\widetilde{d\sigma} = \sum_{j,j'} H_{j,\,j'} \underbrace{F_{j,\,h_1}(x, b_T)}_{-\infty < y < y_2} \underbrace{S(b_T)}_{y_2 < y < y_1} \underbrace{D_{j',\,h_2}(z, b_T)}_{y_1 < y < +\infty}$$

The dependence on rapidity cut-offs is cancelled in the final result

Soft factor and soft/collinear subtraction

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, S(b_T)\, D(b_T)$$

TMDs are defined through the **factorization definition**:

$$D(z,\, b_T,\, y_1) = \lim_{\hat{y}\to-\infty} \frac{D^{\text{uns.}}(z,\, b_T,\, y_P - \hat{y})}{S(b_T,\, y_1 - \hat{y})}$$

From quark-quark correlation matrix

Subtraction of soft-collinear overlapping

The soft factor (included the subtraction term) is defined as:

$$S(b_T,\, y_1 - y_2) = \frac{\text{Tr}}{N_C} \langle 0| W^\dagger_{n_2}[\vec{b_T}/2, \infty]\, W_{n_1}[\vec{b_T}/2, \infty] \times W_{n_2}[-\vec{b_T}/2, \infty]\, W^\dagger_{n_1}[-\vec{b_T}/2, \infty] |0\rangle$$

The soft factor of the process and the soft factor of subtractions are the same function!

Square root definition of TMDs

sqrt.

S. M. Aybat and T. C. Rogers, Phys.Rev. D83, 114042 (2011)

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, \cancel{\mathbb{S}_{2\text{-}h}}(b_T)\, D(b_T) =$$

sqrt.

Recasting terms

Parton model-like

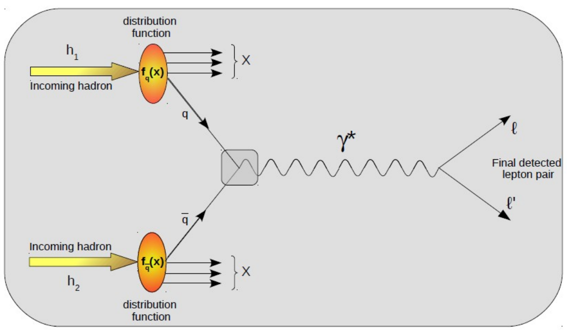
$$= \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F^{\text{sqrt}}(b_T)\, D^{\text{sqrt}}(b_T)$$

Square-root definition of the TMD:

$$D^{\text{sqrt}}(z,\, b_T,\, y_n) = \lim_{\substack{\widehat{y}_1\to+\infty \\ \widehat{y}_2\to-\infty}} D^{\text{uns.}}(z,\, b_T,\, y_P - \widehat{y}_2)\sqrt{\frac{S(b_T,\, \widehat{y}_1 - y_n)}{S(b_T,\, \widehat{y}_1 - \widehat{y}_2)\, S(b_T,\, y_n - \widehat{y}_2)}}$$

Where do we learn about TMDs?

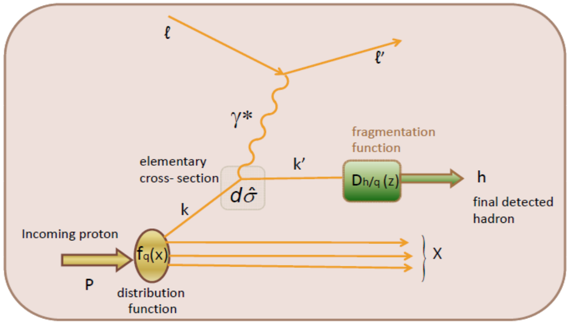
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

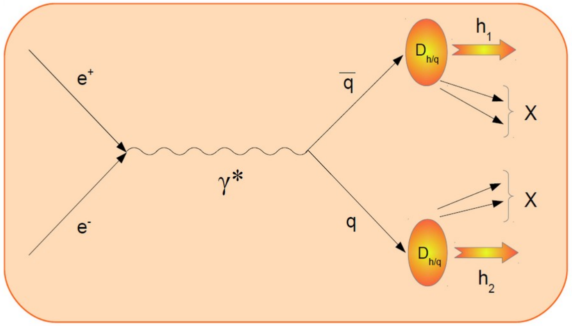
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

$e^+ e^- \to h_1 h_2 X$

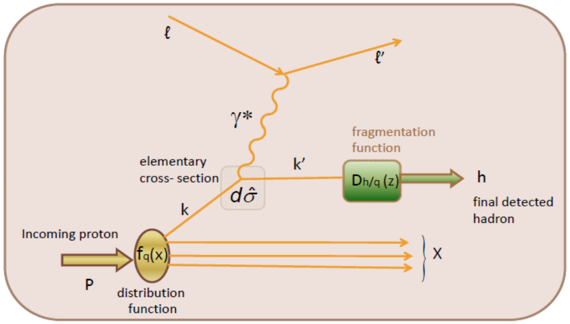


$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

Where do we learn about TMDs?

Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

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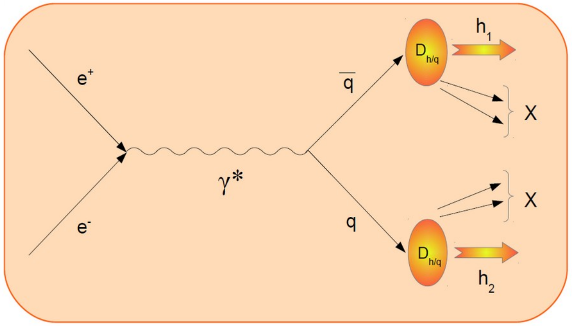
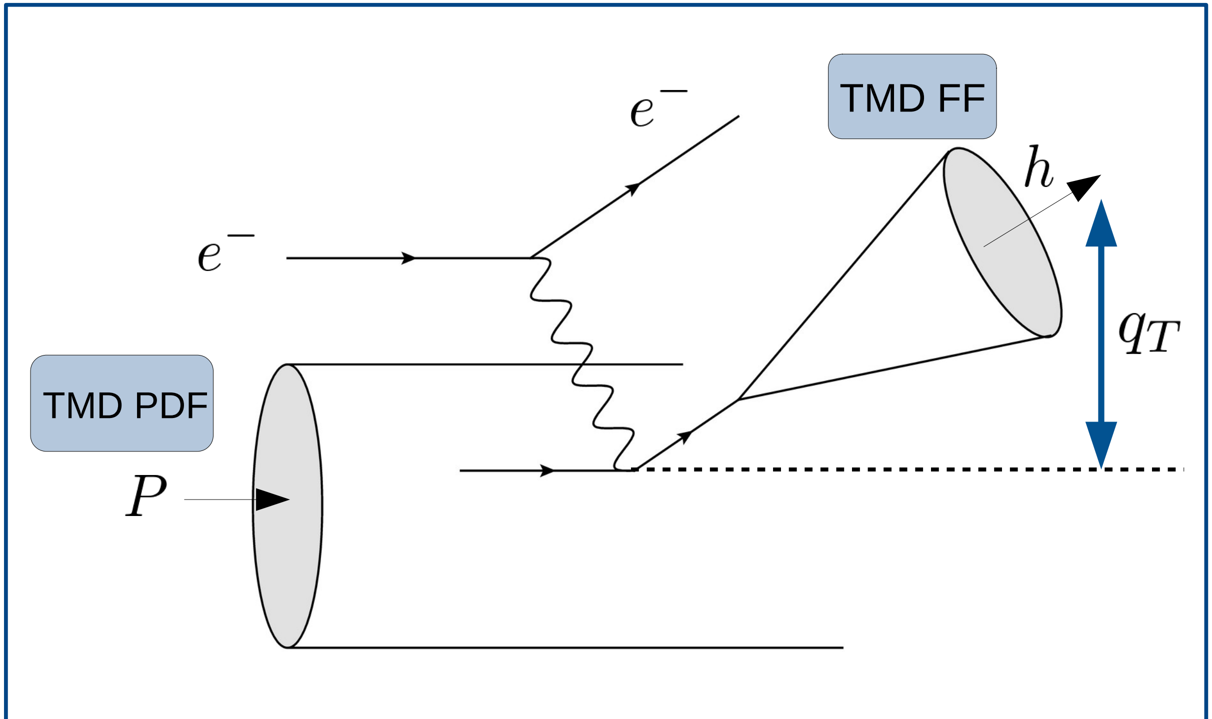
$e^+ e^- \rightarrow h_1 h_2 X$



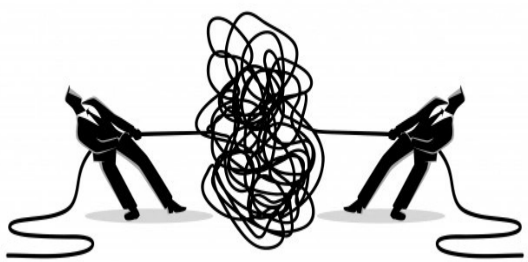
$$\sigma_{h1h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

SIDIS: $e\,p \to h\,X$



In e[+]e[-] cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?

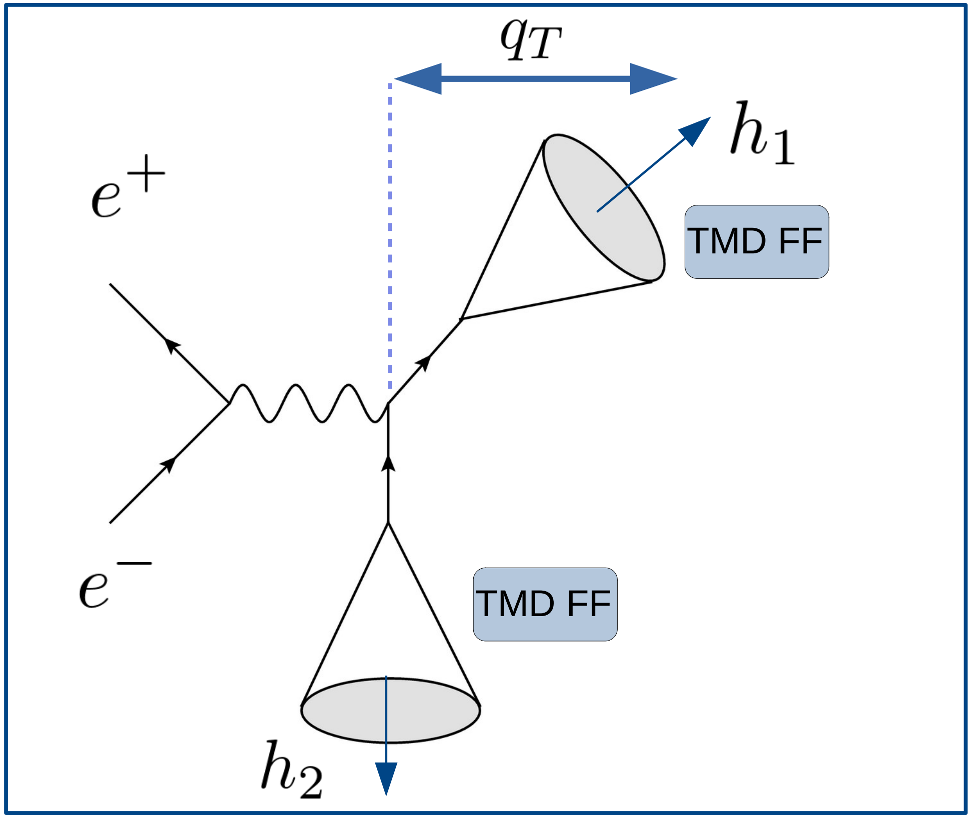


$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

3D-picture of partons inside the target hadron

3D-picture of partons hadronizing into the detected hadron

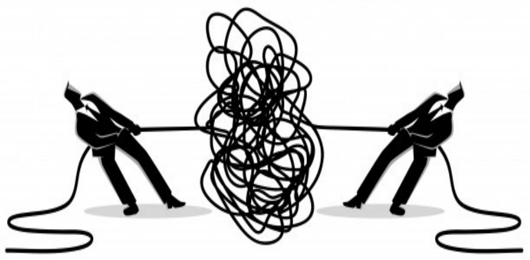
e^+e^- annihilations in two hadrons: $e^+ e^- \to h_1 h_2 X$



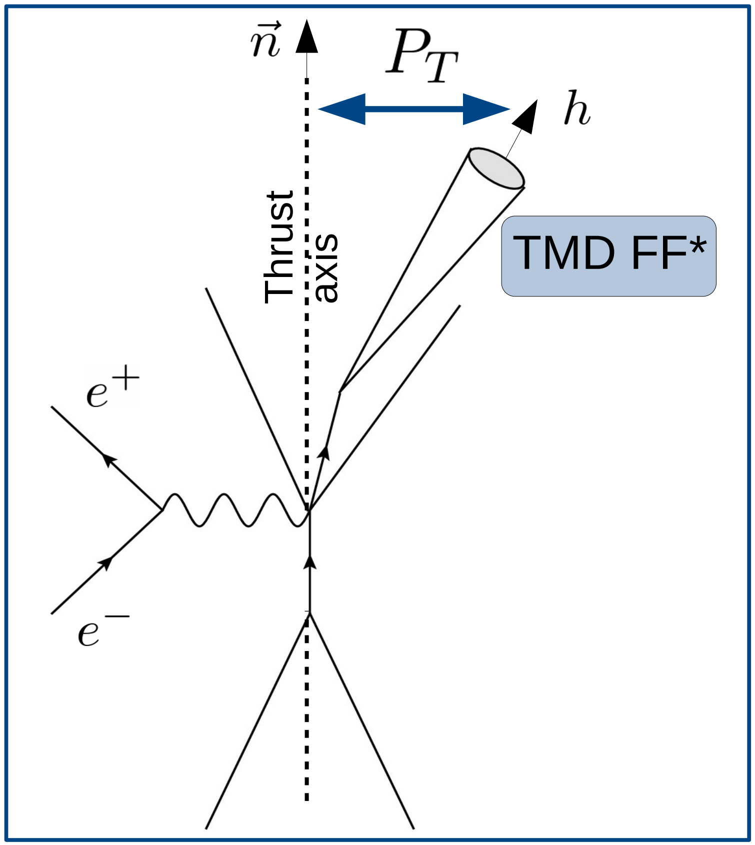
$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T} D_1(b_T)\, D_2(b_T)$$

3D-picture of the **hadronization** of partons into hadrons

In e^+e^- cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?



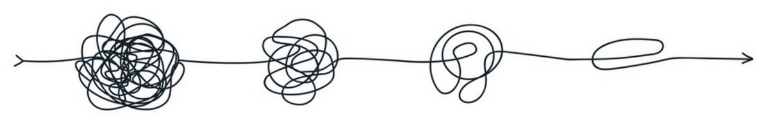
e^+e^- annihilations in one hadron: $e^+ e^- \to h\, X$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^{\star}(P_T)$$

3D-picture of the **hadronization** of partons into hadrons

In $e^+ e^- \to h\, X$ cross sections, only one fragmentation TMD appears



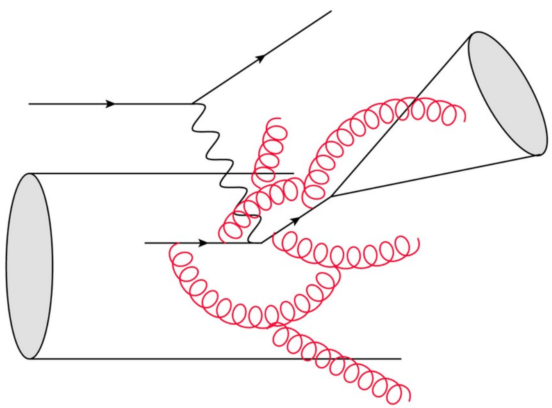
One of the cleanest ways to access TMD Fragmentation Functions***...**

BUT

D*(P_T) is not the same as D(P_T) !!!

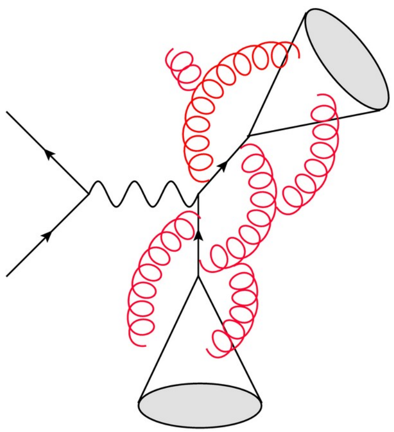
Soft Gluon contribution

SIDIS



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

Double hadron production



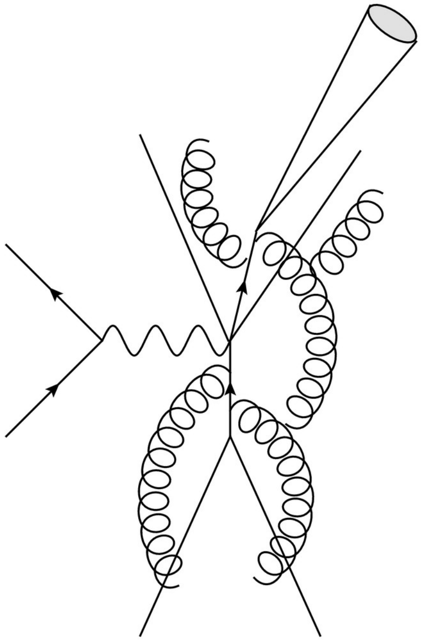
$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, D_1(b_T)\, D_2(b_T)$$

Soft Gluon Factor:
- Non-Perturbative contribution
- Evenly shared by the TMDs

Soft Gluons

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \rightarrow hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^\star(P_T)$$

Soft Gluon Factor:

- Perturbative contribution
- The TMD FF* is **free** from any soft gluon contributions

$D(P_T)$ and $D^*(P_T)$ are different,
BUT
the relation between D and D* is known!

We can perform combined analyses and disentangle non-perturbative terms.

Relation between FF and FF*

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$D = D^{\star}\sqrt{M_S}$$

SQUARE ROOT DEFINITION

Usual definition of TMDs.
Soft Gluon Factor contributing to the cross section are included in the two TMDS and equally shared between them.

FACTORIZATION DEFINITION

Purely collinear TMD, totally free from any soft gluon contribution.

SOFT MODEL

The Soft Gluon Factor appearing in the cross section (process dependent) is **not** included in the TMD

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).

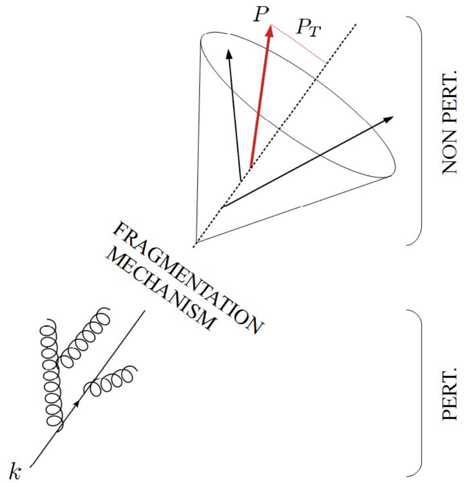
$e^+e^- \to hX$ cross section

M. Boglione, A. Simonelli, JHEP 02 (2021) 076

The hadronic cross section is written as a convolution of a partonic cross section with a TMD FF

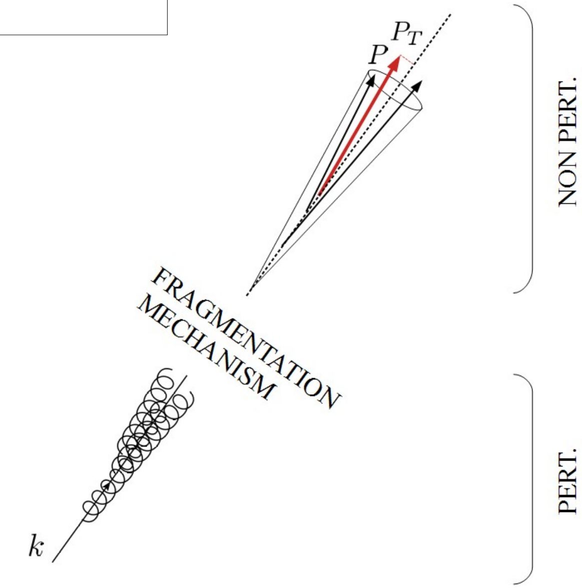
$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT} D_{1,\,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)$$

The TMD FF acquires a dependence on thrust through its rapidity cut-off.



2-jet limit

$T \to 1$



Partonic cross section (NLO)

M. Boglione, A. Simonelli, JHEP 02 (2021) 076

$$\frac{d\sigma}{dz_h\,dT\,dP_T^2} = \pi \sum_f \int_{z_h}^{1} \underbrace{\frac{dz}{z}\,\frac{d\widehat{\sigma}_f}{dz_h/z\,dT}}_{} D_{1,\,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\,Q^2)$$

$$\boxed{\frac{d\widehat{\sigma}_f}{dz\,dT} = \left[-\sigma_B e_f^2 N_C \frac{\alpha_S(Q)}{4\pi} C_F \delta(1-z)\left[\frac{3+8\log\tau}{\tau}\right] + \mathcal{O}\left(\alpha_S(Q)^2\right)\right] e^{-\frac{\alpha_S(Q)}{4\pi}\,3C_F(\log\tau)^2+\mathcal{O}\left(\alpha_S(Q)^2\right)}}$$

TMD Fragmentation Function

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)$$

Fourier Transform of:

Collinear FFs

$$\widetilde{D}_{1,\pi^\pm/f}(z,\, b_T;\, Q,\, \tau Q^2) = \frac{1}{z^2} \sum_k \left[\boxed{d_{\pi^\pm/k}} \otimes \mathcal{C}_{k/f}\right](\mu_b) \times$$
$$\times \exp\left\{\frac{1}{4}\widetilde{K} \log \frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D - \frac{1}{4}\gamma_K \log \frac{\tau Q^2}{\mu'^2}\right]\right\} \times$$
$$\times\, \underbrace{(M_D)_{f,\pi^\pm}(z,\, b_T)}\, \exp\left\{-\frac{1}{4}\underbrace{g_K(b_T)} \log\left(\tau \frac{Q^2}{M_H^2}\right)\right\}$$

Perturbative part (NLL)

Non-Perturbative part Phenomenological Model

Embeds the non-perturbative, long-range behavior of the TMD FF

Universal, independent of the TMD definition used

Phenomenological parametrization: M_D

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

$$M_D = \underbrace{\frac{2^{2-p}(b_T M)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_T M)}_{\text{Power-law model}} \times \underbrace{F(b_T, z_h)}_{\text{Multiplicative function modulating the z dependece}}$$

Power-law model

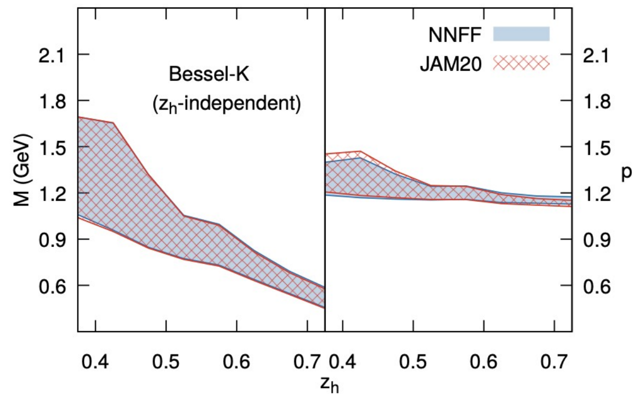
Multiplicative function modulating the z dependece

$\mathcal{FT}\{M_D\}$ reminiscent of a propagator in k_T space $\left.\right\}$ $\dfrac{1}{(k_T^2 + M^2)^p}$

Exponential behaviour at $b_T \to \infty$

Preliminary fits at fixed z show that

- the M and p parameters are VERY strongly correlated
- M requires some z-dependence while p does not vary much with z



Phenomenological parametrization: M_D

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

$$\boxed{M_{\mathrm{D}} = \underbrace{\frac{2^{2-p}(b_{\mathrm{T}} M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_{\mathrm{T}} M_0)}_{\text{BK parameters do not depend on z}} \times \underbrace{F(b_{\mathrm{T}}, z_h)}}$$

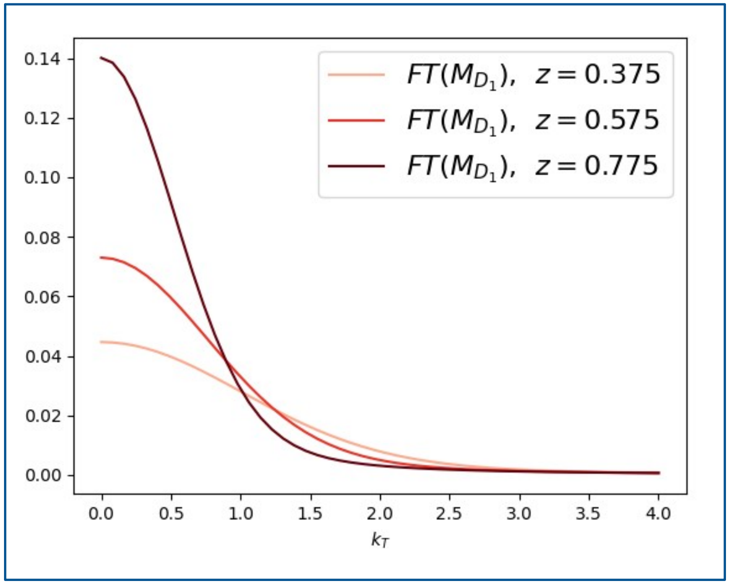
BK parameters do not depend on z

M_D MODEL 1

ID	M_{D} model
I	$F = \left(\dfrac{1 + \log\left(1 + (b_{\mathrm{T}} M_z)^2\right)}{1 + (b_{\mathrm{T}} M_z)^2} \right)^q$ $M_z = -M_1 \log(z_h)$

z-dependence controlled by the function F, through M_z

z-dependence controlled by F



Phenomenological parametrization: M_D

$$M_D = \frac{2^{2-p_z}(b_T M_z)^{p_z-1}}{\Gamma(p_z-1)} K_{p_z-1}(b_T M_z) \times F(b_T, z_h)$$

BK parameters depend on z

F = 1

M_D MODEL 2

II

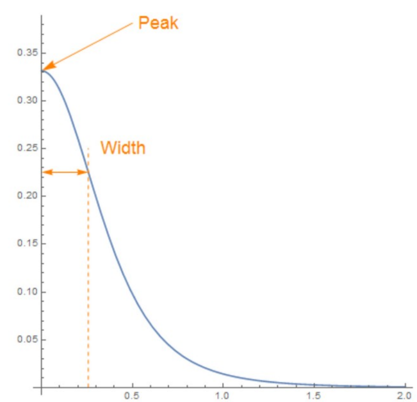
$$F = 1$$

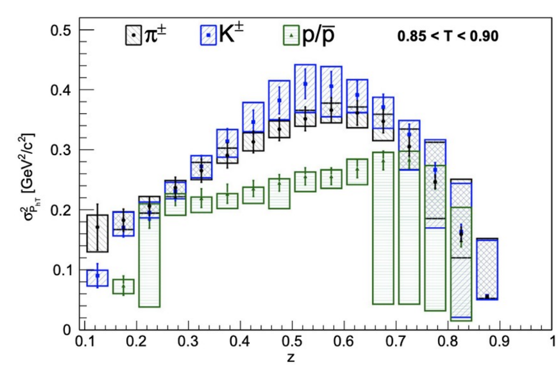
$$M_z = M_h \frac{1}{z\, f(z)^2} \sqrt{\frac{3}{1-f(z)}}$$

$$p_z = 1 + \frac{3}{2} \frac{f(z)}{1-f(z)}$$

$$f(z) = 1 - (1-z)^{\beta}, \quad \beta = \frac{1-z_0}{z_0}$$

The z behaviour of M_D is constrained by requiring that the theory lines appropriately reproduce the peak and the width of the measured cross sections, at each value of z.





BELLE Phys. Rev. D99 (2019) 11 112006

Phenomenological parametrization: g_K

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

In this analysis we consider two different hypothesis for g_K for which, asymptotically, we have $g_K = o(b_T)$

J. Collins, T. Rogers, Phys. Rev. D 91, 074020 (2015)
C. Aidala et al., Phys.Rev. D89, 094002 (2014)
A.. Vladimirov Phys. Rev. Lett. 125, 192002 (2020).

g_{K} model		
A	$g_{\mathrm{K}} = \log\left(1 + (b_{\mathrm{T}} M_{\mathrm{K}})^{p_{\mathrm{K}}}\right)$	$M_{\mathrm{K}},\ p^*_{\mathrm{K}}$
B	$g_{\mathrm{K}} = M_{\mathrm{K}} b_{\mathrm{T}}^{(1-2p_{\mathrm{K}})}$	$M_{\mathrm{K}},\ p^*_{\mathrm{K}}$

Testing different b_T behaviors of g_K
allows us to give a reliable estimate
of the uncertainties affecting our analysis

Phenomenological results – correlations

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

Model I

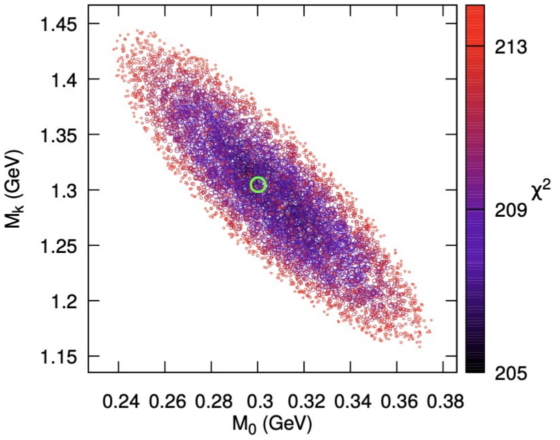
3 parameter fit

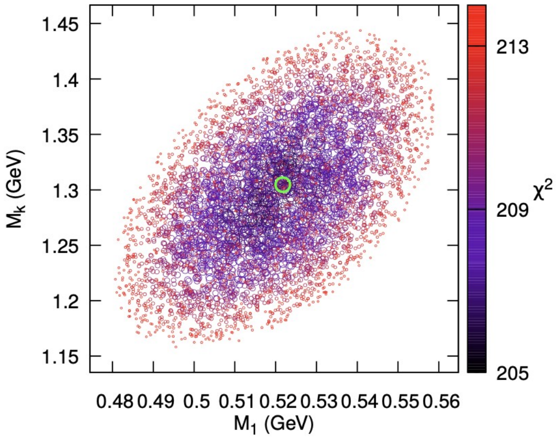
$q_T/Q < 0.15$ (pts = 168)		
	IA	IB
$\chi^2_{\rm d.o.f.}$	1.25	1.19
M_0(GeV)	$0.300^{+0.075}_{-0.062}$	$0.003^{+0.089}_{-0.003}$
M_1(GeV)	$0.522^{+0.037}_{-0.041}$	$0.520^{+0.027}_{-0.040}$
p^*	1.51	1.51
q^*	8	8
M_K(GeV)	$1.305^{+0.139}_{-0.146}$	$0.904^{+0.037}_{-0.086}$
p^*_K	0.609	0.229

Data selection

$$0.375 \le z_h \le 0.725\,, \qquad 0.750 \le T \le 0.875\,,$$

$$q_T/Q \le 0.15$$





Phenomenological results – correlations

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Model II

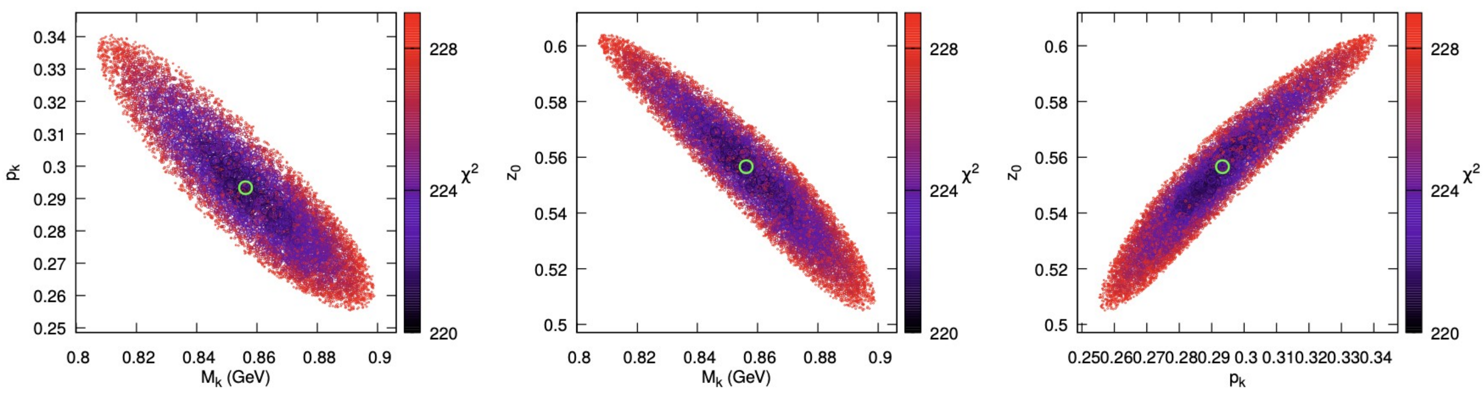
3 parameter fit

$q_{\mathrm{T}}/Q < 0.15$ (pts = 168)		
	IIA	IIB
$\chi^2_{\mathrm{d.o.f.}}$	1.35	1.33
z_0	$0.574^{+0.039}_{-0.041}$	$0.556^{+0.047}_{-0.051}$
$M_{\mathrm{K}}(\mathrm{GeV})$	$1.633^{+0.103}_{-0.105}$	$0.687^{+0.114}_{-0.171}$
p_k	$0.588^{+0.127}_{-0.141}$	$0.293^{+0.047}_{-0.038}$

Data selection

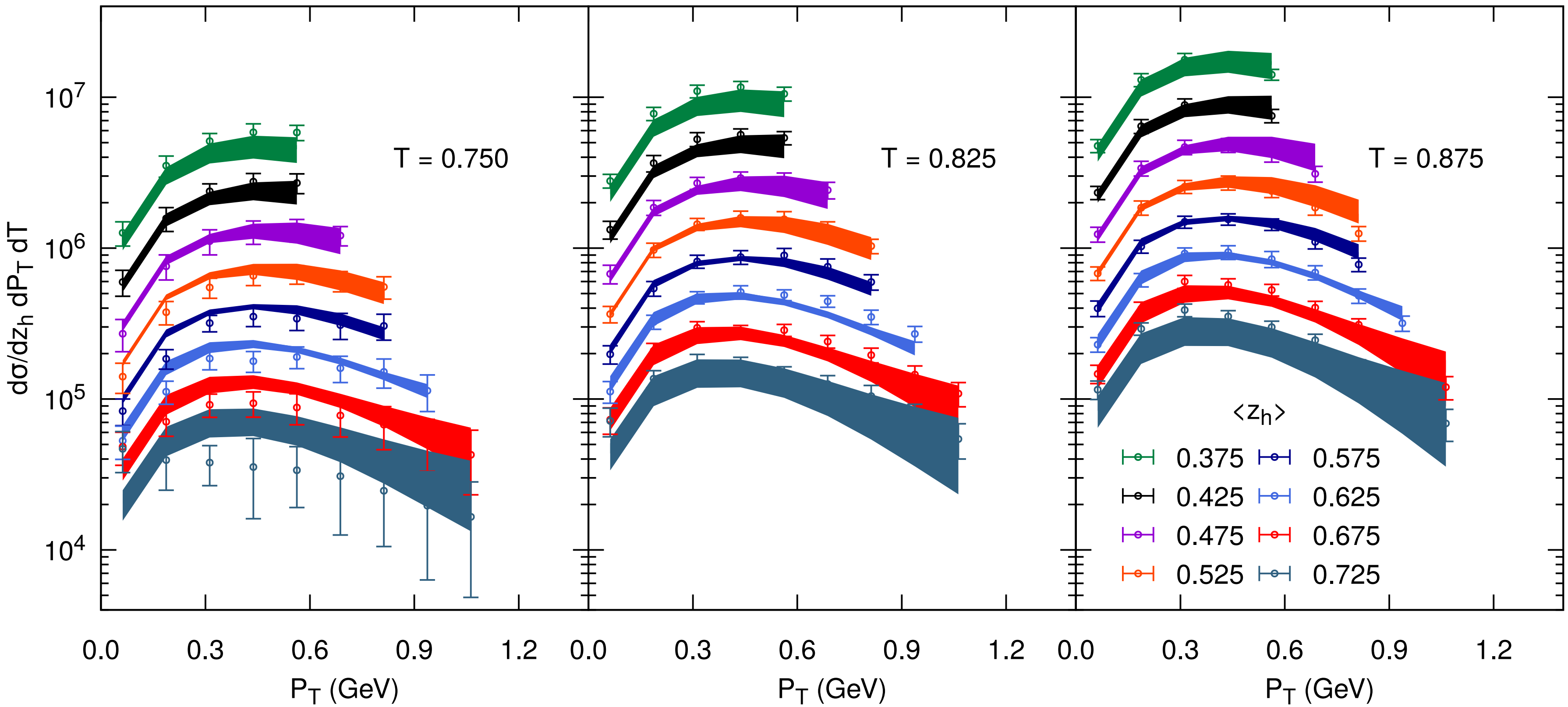
$$0.375 \leq z_h \leq 0.725\,, \qquad 0.750 \leq T \leq 0.875\,,$$

$$q_{\mathrm{T}}/Q \leq 0.15$$



Phenomenological results – T dependence

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]



BELLE Collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

Phenomenological results

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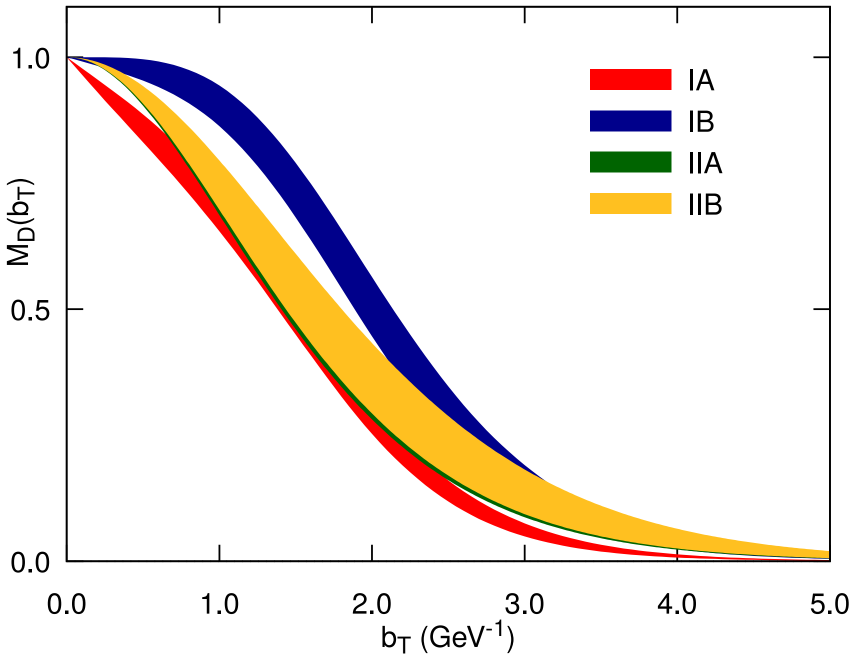
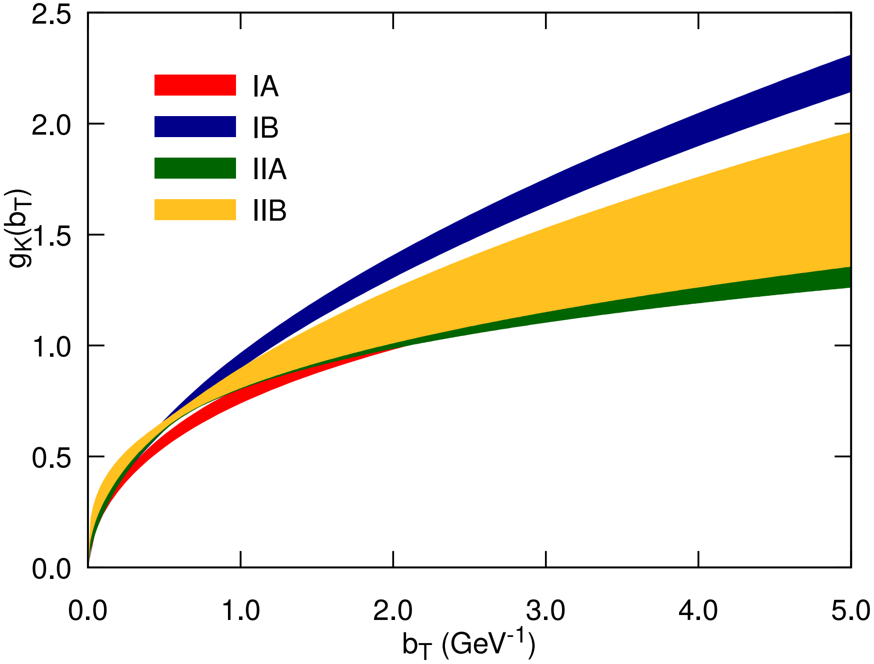
$M_D\,(b_T)$



$g_K\,(b_T)$



Phenomenological results

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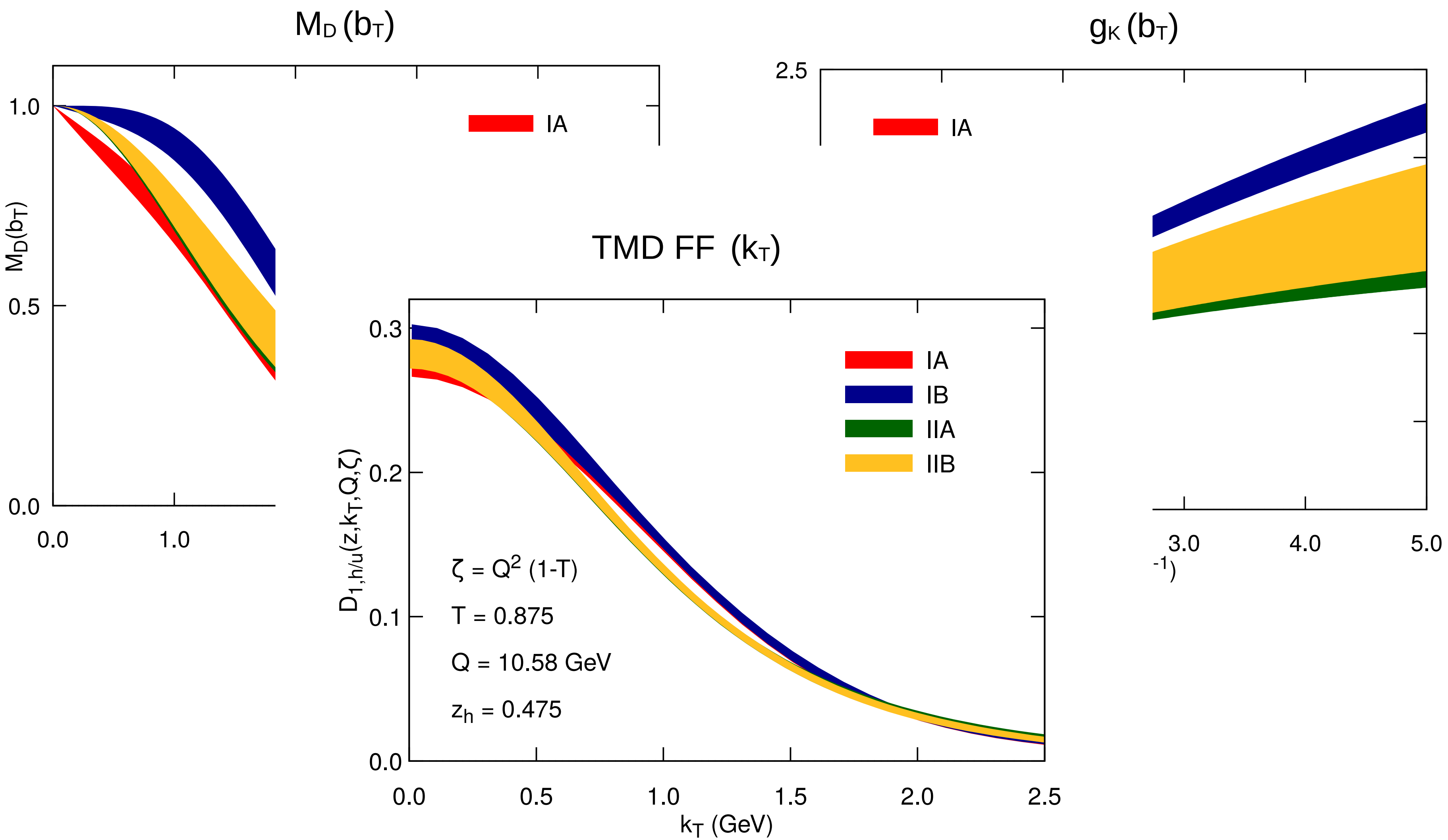
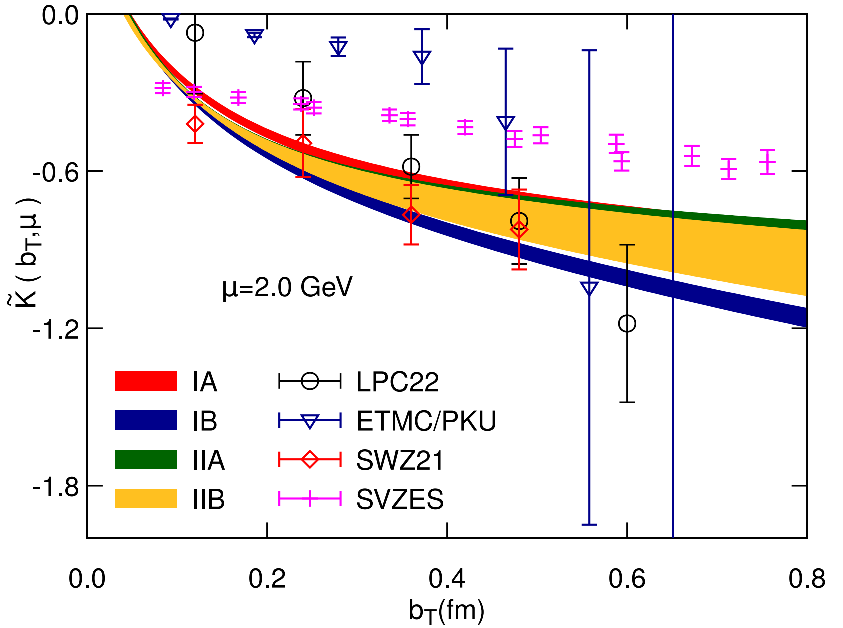


Collins-Soper kernel: comparison to other analyses

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

Our extraction of the Collins-Soper Kernel compared to corresponding lattice computations



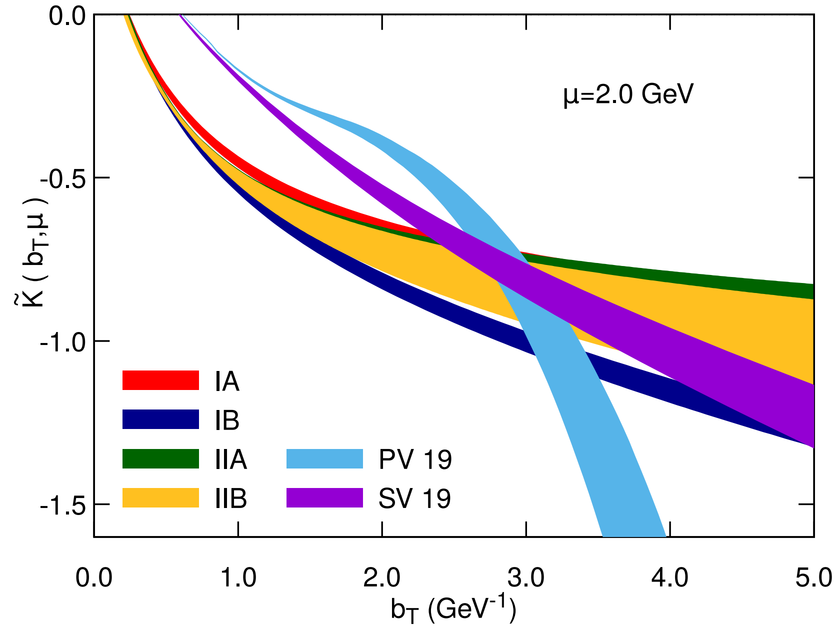
M.-H. Chu et al. (LPC22), arXiv:2204.00200 [hep- lat]

Y. Li et al.,(ETMC/PKU) Phys. Rev. Lett. 128, 062002 (2022),

P. Shanahan et al. (SVZ21) Phys. Rev. D 104, 114502 (2021),

M. Schlemmer et al. (SVZES) JHEP 08, 004 (2021),

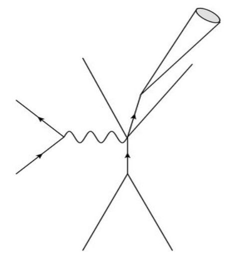
Our extraction of the Collins-Soper Kernel compared to other phenomenological analyses



I. Scimemi and A. Vladimirov, (SV19) JHEP 06, 137 (2020)

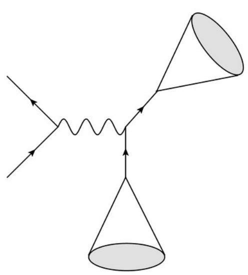
A. Bacchetta, et al. (PV19) JHEP 07, 117 (2020)

Outlook



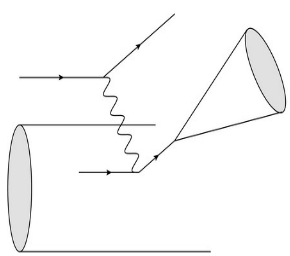
1. $e^+ e^- \to h\,X$

   Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. $e^+ e^- \to h_1 h_2\,X$

   Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_s, obtained as ratio: $M_S = D/D^\star$



3. $SIDIS$

   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_s, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, F*≠F).

Conclusions and Outlook

The Soft Factor acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the Soft Factor contribution (which encloses the full process dependent part of the TMD).

The Collins-Soper kernel acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the g_K function (which embeds the non-perturbative essence of the TMD evolution).