QNP22- 9th conference on quarks and nuclear physics
5-9 September 2022

# GPDs of light nuclei

Sara Fucini

September 5, 2022

- **How do the nucleons interact to hold a nucleus together?**

  *Unfortunately, nuclear physics has not profited as much from analogy as has atomic physics. The reason seems to be that the nucleus is the domain of new and unfamiliar forces, for which men have not yet developed an intuitive feeling.*

  *V. L. Telegdi*

- **How does the nucleus emerge from QCD?**
  - Comparison of the behaviour of hadrons in nuclear matter with the one of hadrons in free space
  - Need to get a handle on medium modifications for a QCD based understanding of nuclei

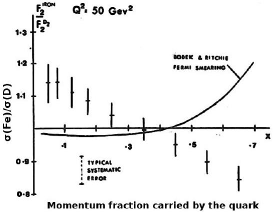
## Alliance of the two communities of QNP

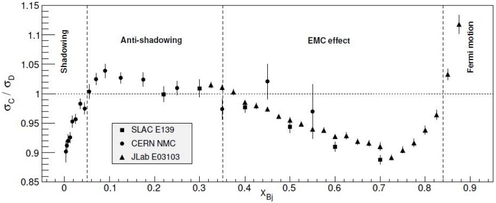
- High-energy physics
- Low-energy nuclear structure physics

**The nuclear medium modifies the structure of bound nucleons**

The European Muon Collaboration found

$$R(x) = \frac{F_2^A(x)}{F_2^d(x)} \neq 1 \;, x = \frac{Q^2}{2M\nu} \in \left[0; \frac{M_A}{M}\right]$$



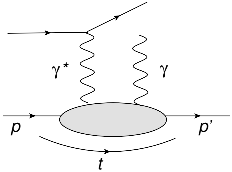



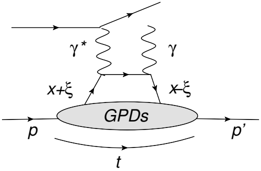

- $x \leq 0.05$: "Shadowing region"
- $0.3 \leq x \leq 0.85$ : "EMC region"
- $0.85 \leq x \leq 1$ : "Fermi motion region"

Collinear information led to many models but not yet to a complete explanation

(e.g., see **Cloët et al. JPG (2018)**, for a recent report)

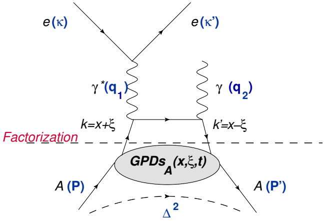
- **Exclusive electro-production of a real photon** → *clean access to* ***Generalized Parton Distributions***

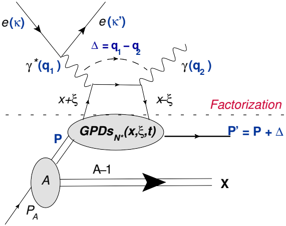



- **Two DVCS channels in nuclei**:

  - **Coherent channel** → GPDs of the **whole nucleus**
  - **Incoherent channel** ⟶ GPDs of the **bound nucleon**



# GPDs in a nutshell

- GPDs are function of:
  - $\Delta^2 = t = (p' - p)^2$
  - $x = \frac{\bar{k}^+}{\bar{p}^+}$
  - $\xi = -\frac{\Delta^+}{2\bar{P}^+} \approx \frac{x_B}{2-x_B}$, with $x_B = \frac{Q^2}{2M_A\nu}$
  - $Q^2 = -(\kappa - \kappa')^2$

- GPDS are defined in terms of **non-diagonal matrix elements of non-local operators** **(Ji, PRL (1997), Belitsky et al., Phys. Rept. (2005))**

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S'|\bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\psi\left(\frac{z^-}{2}\right)|PS\rangle$$

$$= \frac{1}{2\bar{P}^+}\left[\mathbf{H_q}(\mathbf{x},\xi,\mathbf{t})\bar{u}(P',S')\gamma^+u(P,S) + \mathbf{E_q}(\mathbf{x},\xi,\mathbf{t})\bar{u}(P',S')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P,S)\right]$$

- At LO, a system of spin $S$ has $2(2S+1)^2$ quark GPDs and $2(2S+1)^2$ gluon GPDs

$\longrightarrow$ **4(2S+1) x 4(2S+1) GPDs**

Special role played by **spin 0 nuclei**

## GPDs in a nutshell

- **Polinomiality property**, e.g. the first moment yields the **form factor**

$$\int dx H_q(x,\xi,t) = F_1^q(t)$$

- In the **forward limit**, one recovers the **parton distribution function**

$$H_q(x,\xi=0,t=0) = q(x), x>0$$

- GPDs have a **probabilistic interpretation** in the impact parameter space

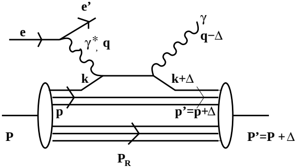
$$\rho_q(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H_q(x,0,\Delta_\perp^2)$$

- Need of a **variety of processes** to disentagle different GPDs (i.e. DVCS, Double DVCS, Timelike Compton Scattering ..)

- GPDs are **universal objects**, linked to the **Compton Form Factors**

$$\mathcal{H}_q(\xi,t) = \int_0^1 dx \left(\frac{1}{x+\xi} + \frac{1}{x-\xi}\right)\left(H_q(x,\xi,t) - H_q(x,-\xi,t)\right)$$

## Impulse approximation to the handbag approximation

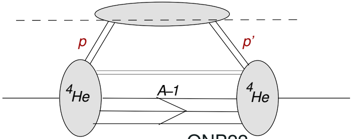


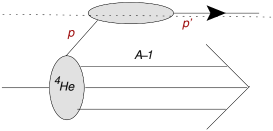

- Only nucleonic degrees of freedom
- The bound proton is **kinematically** off-shell

$$p_0 = M_A - \sqrt{M^{*2}_{A-1} + \vec{p}^2} \simeq M - E - T_{rec} \longrightarrow \mathbf{p^2 \neq M^2}$$

where the ***removal energy*** is $E = |E_A| - |E_{A-1}| - E^*$

- Possible final state interaction (**FSI**) effects are neglected
- Convolution formulas (for the cross section, for the GPD...) between nuclear $\Big($**spectral functions** obtained with **realistic potential and 3-body forces**, i.e. Argonne 18 (Av18) + Urbana IX$\Big)$ and nucleonic ingredients



# Coherent DVCS off $^4$He

# Our formalism for the nuclear GPD (S. F., S.Scopetta, M. Viviani, PRC 98 (2018) 015203)

In **IA**, a convolution formula for the chiral even GPD $H_q$ of the helium-4 can be obtained in terms of:

- **GPDs of the inner nucleons**

$$H_q^{4He}(x,\xi,\Delta^2) = \sum_N \boxed{\int_{|x|}^1 \frac{dz}{z} h_N^{4He}(z,\xi,\Delta^2)} \; \boxed{\mathbf{H_q^N}\left(\frac{x}{\zeta},\frac{\xi}{\zeta},\Delta^2\right)}$$

- **light-cone momentum distribution**

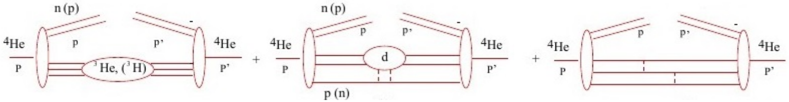
$$h_N^{4He}(z,\Delta^2,\xi) = \frac{M_A}{M}\int dE \int_{p_{min}}^{\infty} dp \int_0^{2\pi} d\phi \, p \, \tilde{M} P_N^{4He}(\vec{p},\vec{p}+\vec{\Delta},E)$$

$$\tilde{M} = \frac{M}{M_A}\left(M_A + \frac{\Delta^+}{\sqrt{2}}\right), \mathbf{H_q^N} = \sqrt{1-\xi^2}[H_q^N - \frac{\xi^2}{1-\xi^2}E_q^N]$$

One needs the **non-diagonal spectral function** and the **nucleonic GPDs** (we used the **Goloskokov-Kroll** models **(EPJ C (2008)-EPJ C (2009))**

# Modelling the spectral function

$$P_N^{4He}(\vec{p}, \vec{p}+\vec{\Delta}, E) = \rho(E) \sum_{\alpha\,\sigma} \langle P+\Delta| -p\, E\, \alpha, p+\Delta\, \sigma\rangle \langle p\, \sigma_N, -p\, E\, \alpha|P\rangle$$




$$P^{4He}(\vec{p}, \vec{p}+\Delta, E) = \simeq a_0(|\vec{p}|)a_0(|\vec{p}+\vec{\Delta}|)\delta(E) + \sqrt{n_1(|\vec{p}|)n_1(|\vec{p}+\vec{\Delta}|)}\delta(E-\bar{E})$$

- the total momentum distribution is $n(p) \propto \int d\vec{r}_1 d\vec{r}_1' e^{i\vec{p}\cdot(\vec{r}_1-\vec{r}_1')}\rho_1(\vec{r}_1, \vec{r}_1')$
- the ground momentum distribution is $n_0(|\vec{p}|) = |a_0(|\vec{p}|)|^2$ with

$$a_0(|\vec{p}|) \approx \langle \Phi_{^3He/^3H}|\Phi_{^4He}\rangle \,.$$

- the **excited momentum distribution** is

$$\mathbf{n_1}(|\vec{p}|) = n(|\vec{p}|) - n_0(|\vec{p}|)$$

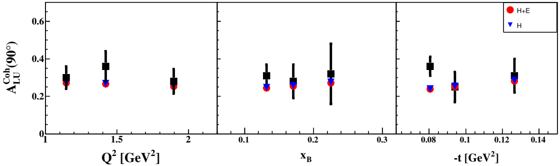
- $\boldsymbol{n(p), n_0(p)}$ have been evaluated within the Av18 NN interaction **(Wiringa et al., PRC (1995))** + UIX 3-body forces **(Pudliner et al., PRL (1995))**
- $\bar{E}$ is the average excitation energy comin from a realistic update of the model by **Ciofi et al., PRC (1996)**, i.e. $P_1^{\text{our model}} = N(p)P_{exc}^{\text{Ciofi's model}}$)

Beam spin asymmetry as a function of azimuthal angle

$$A_{LU}^{coh}(\phi) = \frac{\alpha_0(\phi)\,\Im m(\mathcal{H}_A)}{\alpha_1(\phi) + \alpha_2(\phi)\,\Re e(\mathcal{H}_A) + \alpha_3(\phi)\left(\Re e(\mathcal{H}_A)^2 + \Im m(\mathcal{H}_A)^2\right)}$$

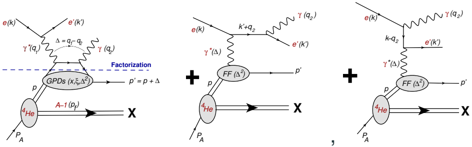
- $\alpha_i(\phi)$ are kinematical coefficients from **A. V. Belitsky et al., PRD (2009)**
- $H^{^4He}(x,\xi,t) = \sum_{q=u,d,s} \epsilon_q^2 H_q^{^4He}(x,\xi,t)$ comes **from our model**
  - $\Im m\mathcal{H}_A(\xi,t) = H^{^4He}(x=\xi,\xi,t) - H^{^4He}(x=-\xi,\xi,t)$
  - $\Re e\mathcal{H}_A(\xi,t) = \text{Pr}\int_{-1}^{1} dx \frac{H^{^4He}(x,\xi,t)}{x-\xi+i\epsilon}$

**Results of our model (PRC(2018))** ● *VS* **JLab data** (**Hattawy et al., PRL (2017)**)



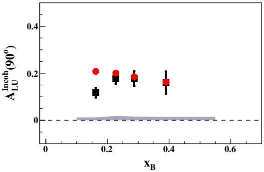
# Incoherent DVCS off $^4$He

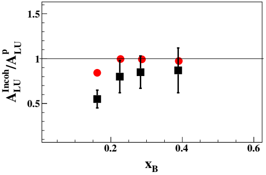
$$d\sigma^{\pm} \approx \int d\vec{p} dE P^{^4He}(\vec{p}, E) |\mathcal{A}^{\pm}(\vec{p}, E, K)|^2$$




$$A_{LU}^{Incoh}(K) = \frac{\mathcal{I}^{^4He}(K)}{T_{BH}^{2\,^4He}(K)} = \frac{\int_{exp} dE\, d\vec{p}\, P^{^4He}(\vec{p}, E)\, g(\vec{p}, E, K)\, \mathcal{I}(\vec{p}, E, K)}{\int_{exp} dE\, d\vec{p}\, P^{^4He}(\vec{p}, E)\, g(\vec{p}, E, K) T_{BH}^2(\vec{p}, E, K)}$$

- nuclear effects affect the motion of the proton in the nuclear medium (no modifications to the functional form of the GPDs and FFs)
- in $\mathcal{I}(\vec{p}, E, K) \propto \Im m \mathcal{H}(\xi', \Delta^2, Q^2)$, we used the nucleon **GPD model** evaluated for $\xi' = \frac{\mathbf{Q^2}}{(\mathbf{p}+\mathbf{p'})(\mathbf{q_1}+\mathbf{q_2})}$

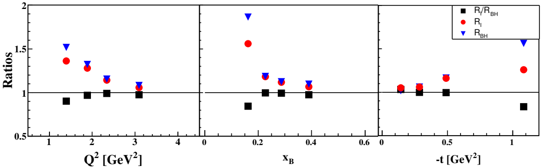


# Nuclear effects in $A_{LU}^{Incoh}$: S.F., S. Scopetta, M. Viviani PRC(2021)

**What kind of nuclear effects we are describing?** Let us consider the *super ratio*

$$A_{LU}^{Incoh}/A_{LU}^{p} = \frac{\mathcal{I}^{^4He}}{\mathcal{I}^{p}} \frac{T_{BH}^{2\ p}}{T_{BH}^{2\ ^4He}} = \frac{R_{\mathcal{I}}}{R_{BH}} \propto \frac{(nucl.eff.)_{\mathcal{I}}}{(nucl.eff.)_{BH}},$$

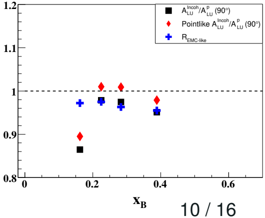



*These effects are due to the* ***dependence on the 4-momenta components*** *of the bound proton entering the amplitudes.*
*This behaviour hasn't to do with a modification of the* ***parton structure!***

It is confirmed by:

- the ratio $A_{LU}^{Incoh}/A_{LU}^{p}$ for **"pointlike" protons**
- the **"EMC-like" trend**

$$R_{EMC-like} = \frac{1}{\mathcal{N}} \frac{\int_{exp} dE\, d\vec{p}\, P^{^4He}(\vec{p}, E)\, \Im m\, \mathcal{H}(\xi', \Delta^2)}{\Im m\, \mathcal{H}(\xi, \Delta^2)}$$



# Incoherent DVCS off deuteron

- The nuclear ingredient is easier than for $^4$He: just **momentum distribution** (totally realistic!)
- $\Delta^2 = (p_{final} - p_{inner})^2$ or $\Delta^2 = (p_{final} - p_{rest})^2$
- Analitycal expression for $p'$

$$\begin{cases} \sqrt{|\vec{p}|^2 + |\vec{p}'|^2 + |q_1^z|^2 - 2|\vec{p}||\vec{p}'|\cos\theta_{pp'} - 2|\vec{p}'|q_1^z\cos\theta_N + 2|\vec{p}|q_1^z\cos\vartheta} - p_0 + E_2 - \nu \\ -\Delta^2 + M^2 + p_0^2 - |\vec{p}|^2 - 2p_0\sqrt{M^2 + |\vec{p}'|^2} + 2|\vec{p}'||\vec{p}|\cos\theta_{\widehat{pp'}} = 0 \end{cases}$$

- Divergences in the BH cross section
- **Experimental data** for pDVCS and nDVCS are coming out at JLab using a 12 GeV electron beam (see the talk by Silvia Niccolai)

  In the meantime, our model can deliver

  - Predictions for **pDVCS**
  - Preliminary results for **nDVCS**: the GPD $E_q^N$ (**sizable!!!!**) is not enough constrained... **any clues about that?**

**Stay tuned for the comparison with CLAS data!**

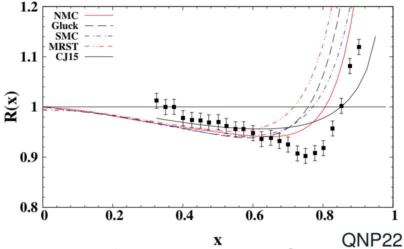
# EMC effect on light nuclei

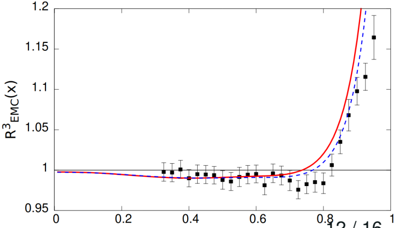
$$R(x) = \frac{R_2^A(x)}{R_2^d(x)} \quad \text{with} \quad R_2^A(x) = \frac{F_2^A(x)}{ZF_2^p(x) + (A-Z)F_2^n(x)} \quad x \in [0 : M_A/M]$$

where the **function structures** $F_2$ for $A = {}^4\text{He}, {}^3\text{He}$, d are defined as

$$F_2^A(x) = \sum_N \int_x^{M_A/M} dz f_N^A(z) F_2^N\left(\frac{x}{z}, Q^2\right)$$

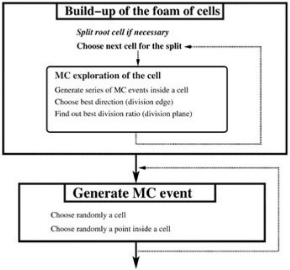
- For ${}^3$He (see **Pace E. et. al, e-Print: 2206.05485**), study the dependence upon the nuclear interaction
- For ${}^4$He (see **PRELIMINARY!**), study the dependence upon the nucleon model $F_2$



# From models to event generation

**TOPEG** is a Root based generator (**S. Jadach (2005)**) + **our model** for the coherent/incoherent DVCS off light nuclei




\+

**Our Impulse approximation models for DVCS off light nuclei (Fucini et al., PRC(2018)-PRC(2020) etc...)**

⇨

**TOPEG**

( The Orsay-Perugia Event Generator)

Use of the TFoam class to create and memorize a grid and then to generate events

## Putting these models in TOPEG

So far, we have results only for the coherent DVCS off $^4$He (version **1.0** released)

- **JLab**
  - Check for the events generated at the kinematics with 6 GeV electron beam
  - Good also for CLAS 12 GeV
- **EIC**
  - We generated events for the three electron - helium-4 beam energy configurations
    - **(5x41) GeV**
    - **(10x110) GeV**
    - **(18x110) GeV**
- These latter results are included in the **EIC Yellow Report** (e-Print: 2103.05419)

**Promising results**:

- the NUCLEAR DVCS can be observed at the EIC
- TOPEG is a flexible tool to do the GPDs phenomenology

# Targets

- $^4$He
- Deuteron
- Free proton/ free neutron (on going)
- Bound neutron/ free neutron (on going)

# Observables

- Unpolarized 4-differential cross section
- Beam spin asimmetries
- Beam charge asimmetries

# Reactions

- Coherent DVCS
- Incoherent DVCS
- Bethe Heitler
- Tagged DVCS (on going)

# Summing up

- **Coherent DVCS off $^4$He**
  - Improvement of the **$^4$He spectral function** (fully realistic calculation) **(in slow progress)**
  - Toward the semi-realistic description of the **EMC effect** in the helium-4 **(in progress)**
  - Impact of the **target mass corrections** on the observables and of **shadowing effects** **(planned)**

- **Incoherent DVCS off $^4$He and $^2$H**
  - New formalism for **$^4$He** and the **deuteron** **(in progress)**
  - Introduction of some **final state interaction effects** **(TBD)**

- **TOPEG**
  - Nuclear DVCS can be performed at the EIC
  - Quark density profile extracted for the first time using TOPEG
  - TOPEG is a suitable phenomenological tool for the future studies on light nuclei **(in progress)**

# Backup slides

# DVCS off deuterium

## Incoherent channel

- Nuclear part: momentum distribution (it is exact: instant form or light front)
- Key study also for heavier nuclei

## Coherent channel

- 9 quark GPDs
- Formalism already developed and established (see **Cano, Pire EPJA (2004)**)
- there is a connection between the light-cone wave function of the deuteron (helicity amplitudes$\longrightarrow$ **GPDs**) in terms of light-cone coordinates and the ordinary (instant-form) relativistic wave function that fulfills a Schrödinger type equation (we can update the potential)
- we can compute

$$\chi(\vec{k};\mu_1,\mu_2) = \sum_{L;m_L;m_S} \langle \frac{1}{2}\frac{1}{2}1|\mu_1,\mu_2,m_S\rangle\langle L11|m_L m_S \lambda\rangle Y_{L,M_L}(\hat{k})u_L(k)$$

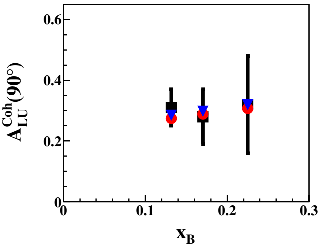
with AV18 and perform a Melosh rotation to relate the spin in the light-front with the spin in the instant-form frame of the dynamics

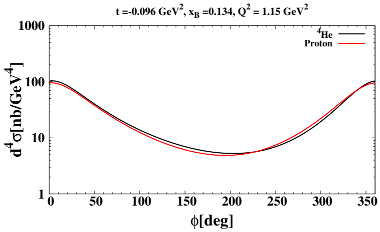
# Coherent DVCS off $^4$He

Model for the only one chiral-even GPD of $^4$He in **S. Fucini, S.Scopetta, M. Viviani, PRC 98 (2018)**

$$\frac{d^4\sigma^{\lambda=\pm}}{dx_A dt dQ^2 d\phi} = \frac{\alpha^3 x_A y^2}{8\pi Q^4\sqrt{1+\epsilon^2}}\frac{|\mathcal{A}|^2}{e^6};\, A_{LU} = \frac{d^4\sigma^+ - d^4\sigma^-}{d^4\sigma^+ + d^4\sigma^-}$$

$$T_{BH}^2 \propto F_A^2(t);\, T_{DVCS}^2 \propto \Im m\mathcal{H}^2 + \Re e\mathcal{H}^2;\, I_{BH-DVCS}^{\lambda} \propto F_A(t)\Im m\mathcal{H}$$




Data from **Hattawy et al., PRL (2017)**; our model including (red dots) or not (blue triangles) the real part of $\mathcal{H}$.
As an illustration, we plot $d^4\sigma_{^4He} \times (F_p^1/F_C^A)^2$ and $d^4\sigma_{proton} * 4$

# (18 x 110) GeV: analysis

**Is it possible to study the region around the first diffraction minimum in the $^4$He FF ($t_{\mathrm{dif.\,min}} = -0.48$ GeV$^2$)?** **YES, we can!**

- 99%+ electrons and photons are in the acceptance of the detector matrix
- This is true for all energy configurations

> Electrons and photons appear in easily accessible kinematics according to the detector matrix requirements (exceptions for small angles photons)

- Acceptance at low -t will be cut passing through the detectors
  - $t_{min}$ is set by the detector features
  - $t_{max}$ is fixed by the luminosity (billion of events to generate)

**From left to right, the kinematical distributions of the final particles: electron, photon and $^4$He**