Simultaneous Extraction of Dihadron Beam Spin Asymmetry Modulations

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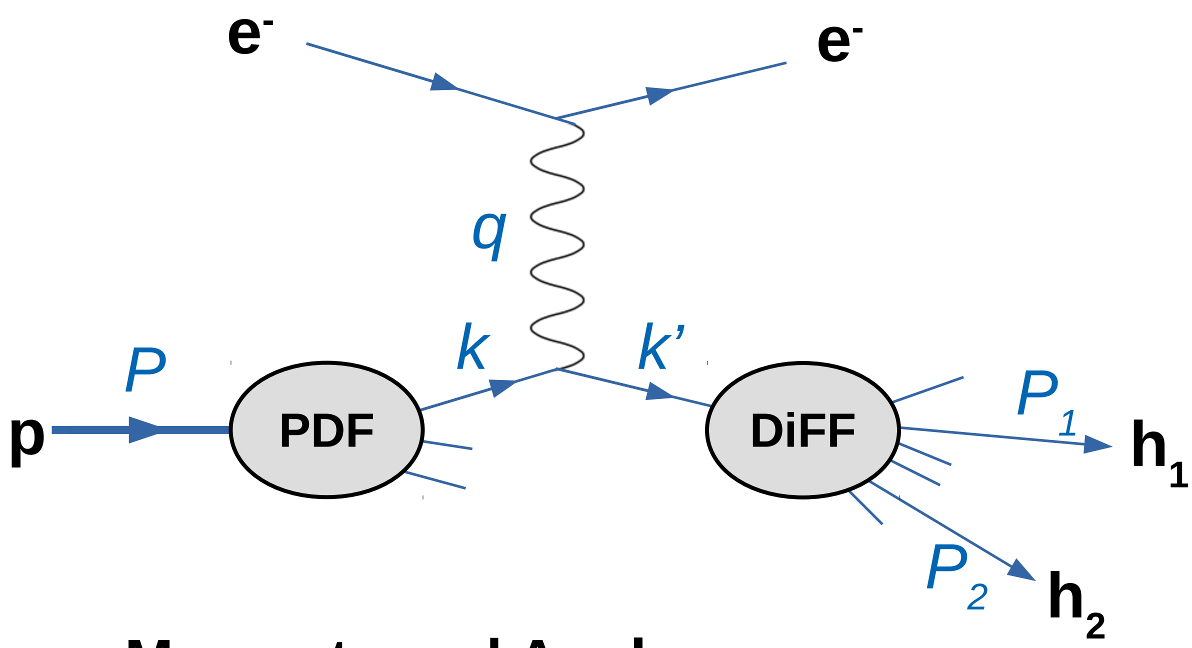
CLAS Collaboration Meeting

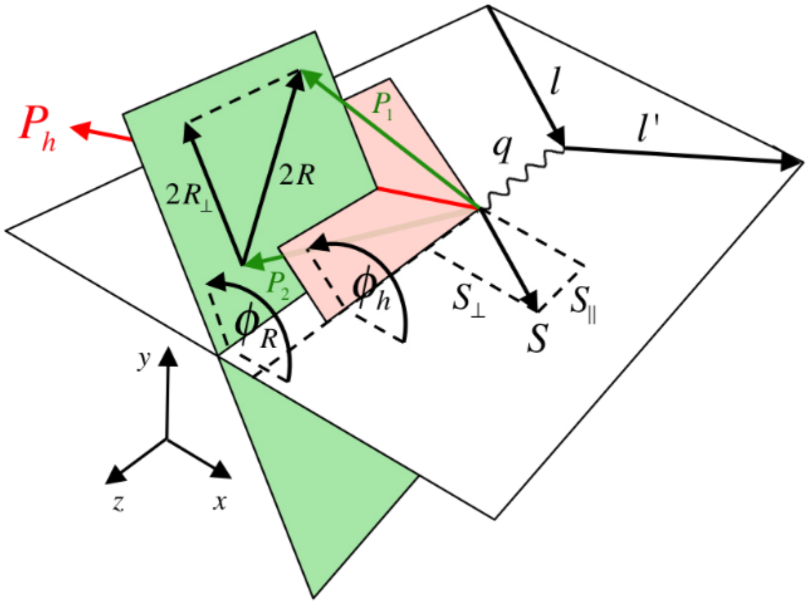
November 2019

Outline

- Intro and physics motivation
- DNP 2019 preliminary results recap
- Introducing additional modulations and partial waves
- Orthogonality of modulations
- Monte Carlo studies of asymmetries and orthogonality

Dihadron Process





Momenta and Angles

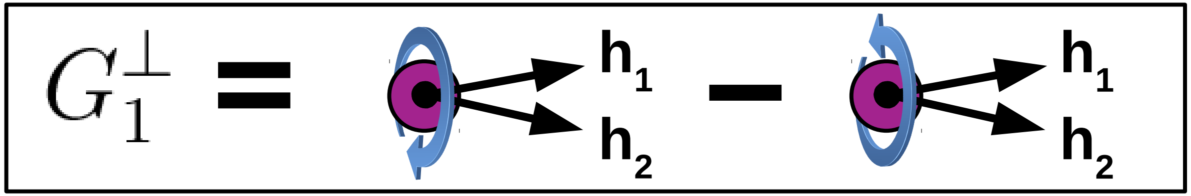
$$P_h = P_1 + P_2 \rightarrow \phi_h$$

$$R = \frac{1}{2}(P_1 - P_2) \rightarrow \phi_R$$

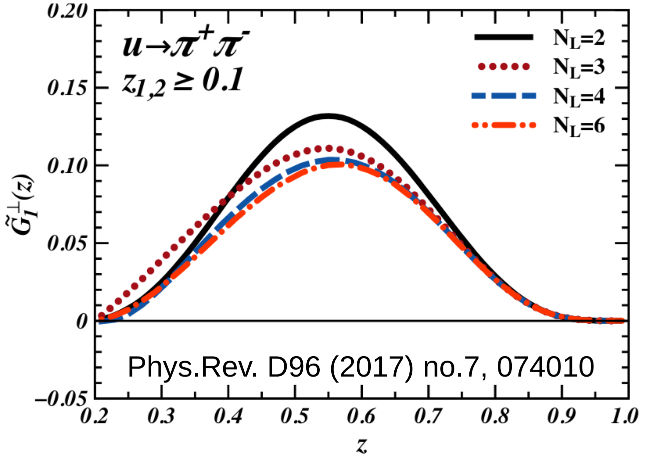
Kinematics:

- Scale: Q^2
- Bjorken-x: x
- Invariant Mass: M_h
- Fragmentation fraction: z

Dihadron Fragmentation Function $G_1^\perp$



$$\tilde{G}_1^\perp(z) \equiv \frac{1}{M_1 M_2} G_1^\perp(z)$$ (integrated over M_h)



- **Sometimes called handedness or helicity-dependent DiFF**
- **Accessible in the $\sin(\Phi_h - \Phi_R)$ modulation of dihadron longitudinal beam spin asymmetries**
- **Weighted by $P_h^\perp / M_h$**
- **Not yet constrained by data; quark-jet hadronization model predicts sizable $G_1^\perp$**

$$A_{LU}(x, y, z, M_h) = \frac{\langle P_h^\perp \sin(\phi_h - \phi_R) / M_h \rangle_{LU}}{\langle 1 \rangle_{UU}} = \lambda_l \frac{C'(y)}{A'(y)} \frac{\sum_a e_a^2 \boxed{f_1^a(x)}\, z\, \boxed{G_1^{\perp a}(z, M_h^2)}}{\sum_a e_a^2\, f_1^a(x)\, D_1^a(z, M_h^2)}$$

Matevosyan, et al.
– Phys.Rev. D96 (2017) no.7, 074010
– PoS DIS2018 (2018) 150

Event selection

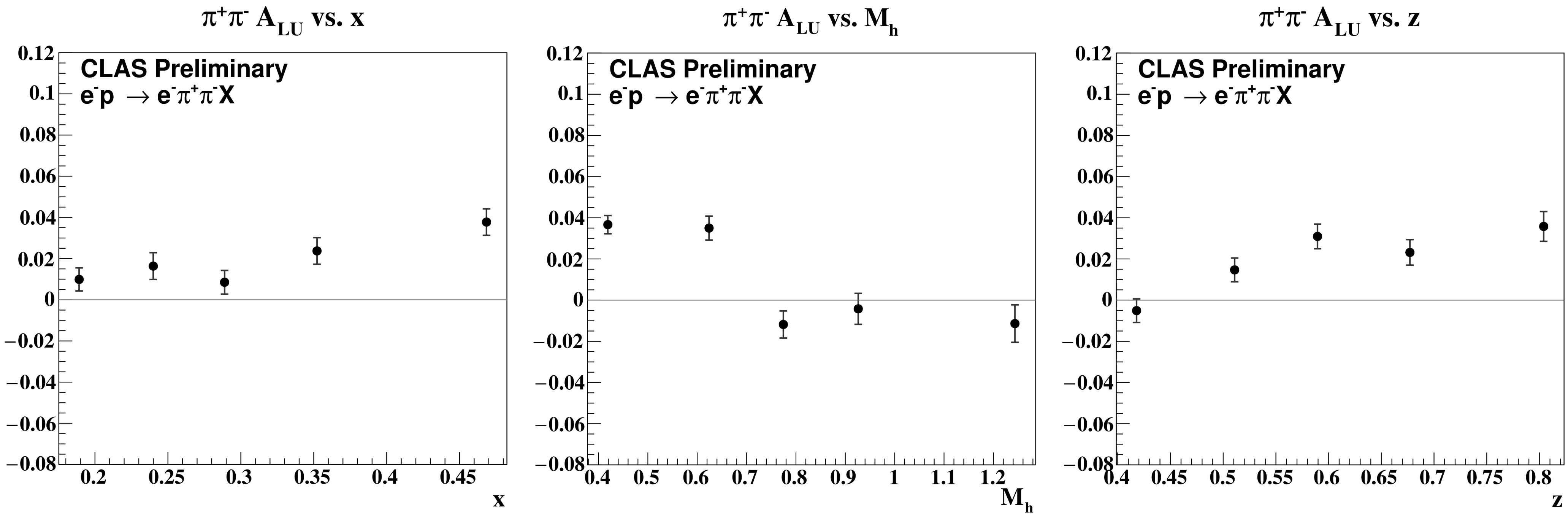
$$e^- p \rightarrow e^- \pi\pi X$$

- Both pions in CLAS Forward Detector
- Scattered e^- in DC and PCAL fiducial volume

Kinematic cuts:

DIS cuts	$Q^2 > 1$ GeV^2 $W > 2$ GeV
Omit radiative region	$p_\pi > 1.25$ GeV
Omit exclusive region	$M_X > 1.05$ GeV
Current fragmentation region	$x_F > 0$ $y < 0.8$ $z_\pi > 0.1$ $z_{\text{pair}} < 0.95$
Vertex cuts	$-8 < v_{z,e} < 3$ cm $-8 < v_{z,\pi} < 3$ cm

DNP2019 π+π- Asymmetries



$A_{hR}\sin(\phi_h-\phi_R)$

Beam spin asymmetry modulations

- **Dihadron structure functions:**
  - **twist 2** $$F_{LU,T}^{P_{\ell,m}\sin(m(\phi_h-\phi_{R_\perp}))} = -\mathcal{I}\left[2\cos\left(m(\phi_h-\phi_p)\right) f_1 G_1^{|\ell,m\rangle}\right]$$

m = 1 → $\sin(\phi_h-\phi_R)$		$f_1 G_1^{\perp}$

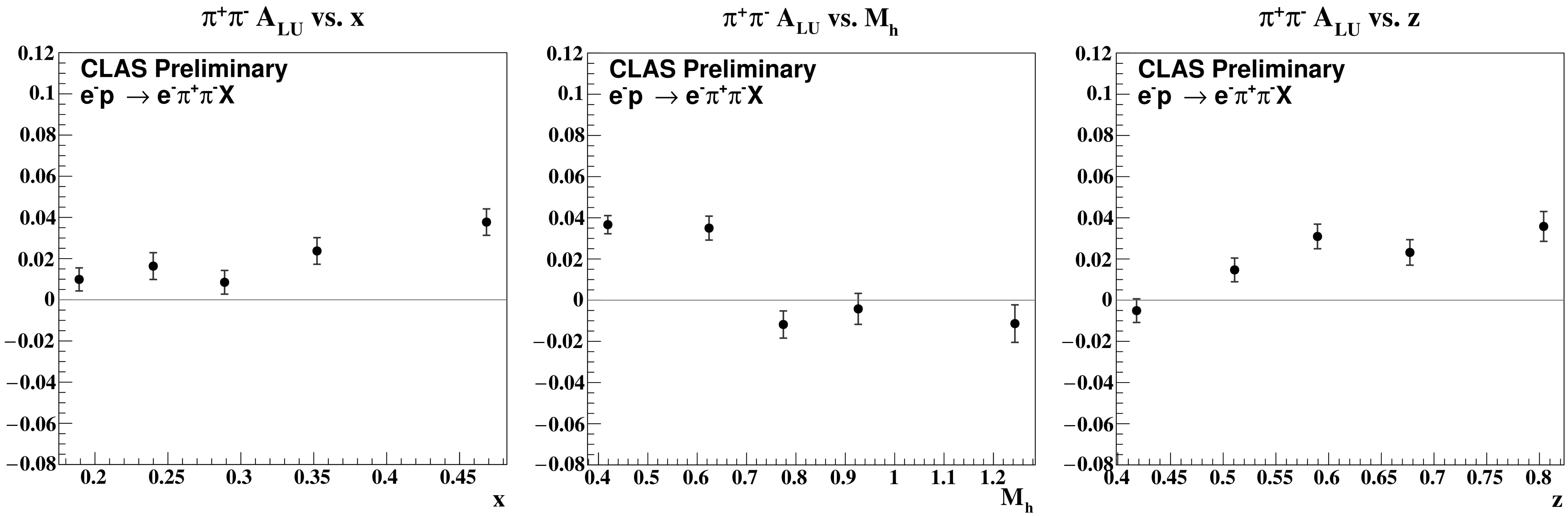
- Partial wave expansion of σ_{LU}
- DiFFs expand in { \|L,M> } basis
- ϕ-dependence only depends on M
- L=0 terms identifiable with 1h
- L=1 terms likely dominant for 2h

  - **twist 3** $$F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})} = \frac{2M}{Q}\mathcal{I}\left[-\frac{|\boldsymbol{p}_T|}{M_h}\cos\left((1-m)(\phi_p-\phi_h)\right)\left(xe\,H_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}f_1\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right) + \frac{|\boldsymbol{k}_T|}{M}\cos\left((m-1)\phi_h+\phi_k-m\phi_p\right)\left(xg^{\perp}D_1^{|\ell,m\rangle}+\frac{M_h}{M}h_1^{\perp}\frac{\tilde{E}^{|\ell,m\rangle}}{z}\right)\right]$$

m = 1 → $\sin\phi_R$ $eH_1^{\sphericalangle}$	m = 0 → $\sin\phi_h$ $eH_1^{\perp}$	m = –1 → $\sin(2\phi_h-\phi_R)$ $eH_1^{\perp}$

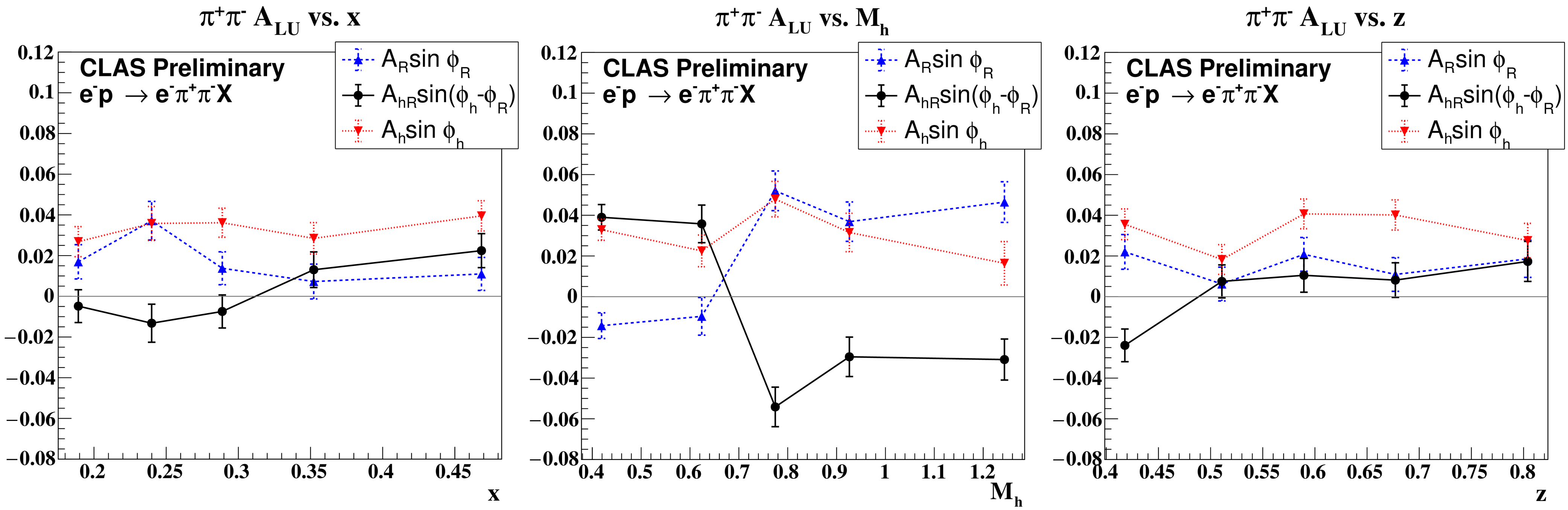
Gliske, Bacchetta, Radici
Phys.Rev. D90 (2014) no.11, 114027

DNP2019 π+π- Asymmetries



$A_{hR}\sin(\phi_h - \phi_R)$

DNP2019 π+π- Asymmetries



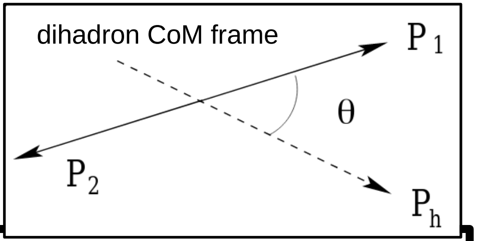
- Including additional modulations reveals more to the story
- There appears to be a sign change near M_ρ
- A_R has opposite M_h dependence to A_{hR}
- A_h is a constant 3-4%
- ±3.8% polarization scale uncertainty

$$\boxed{A_{hR}\sin(\phi_h-\phi_R) + A_R\sin\phi_R + A_h\sin\phi_h}$$

Partial wave expansion

Gliske, Bacchetta, Radici
Phys.Rev. D90 (2014) no.11, 114027

- **twist 2** $F_{LU,T}^{P_{\ell,m}\sin(m(\phi_h - \phi_{R_\perp}))} \longrightarrow f_1 \otimes G_1^{|\ell,m\rangle}$
- **twist 3** $F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp})} \longrightarrow e \otimes H_1^{\perp|\ell,m\rangle} + \ldots$



Modulations: product of two functions

Azimuthal modulation
– ϕ_h and ϕ_R dependence
– depends on M and Twist

- **twist 2** $\sin\left[m\left(\phi_h - \phi_R\right)\right]$
- **twist 3** $\sin\left[(1-m)\,\phi_h + m\phi_R\right]$

Associated Legendre polynomials
– θ dependence
– depends on L and M

$$P_{0,0} = 1\ , \qquad P_{2,0} = \frac{1}{2}\left(3\cos^2\vartheta - 1\right)$$

$$P_{1,0} = \cos\vartheta\ , \qquad P_{2,1} = \sin 2\vartheta\ ,$$

$$P_{1,1} = \sin\vartheta\ , \qquad P_{2,2} = \sin^2\vartheta$$

$$P_{\ell,-m} = P_{\ell,m}$$

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- **twist 3** $F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})} \rightarrow e \otimes H_1^{\perp|\ell,m\rangle} + \ldots$

$$G_1^{|0,0\rangle} = G_1^{|1,0\rangle} = G_1^{|2,0\rangle} = 0\ ,$$

$$G_1^{|1,1\rangle} = G_1^{|1,-1\rangle} = -\frac{|\boldsymbol{p}_T|\,|\boldsymbol{R}|}{2M_h^2} G_{1,OT}^{\perp}\ ,$$

$$G_1^{|2,1\rangle} = G_1^{|2,-1\rangle} = -\frac{|\boldsymbol{p}_T|\,|\boldsymbol{R}|}{4M_h^2} G_{1,LT}^{\perp}\ ,$$

$$G_1^{|2,2\rangle} = G_1^{|2,-2\rangle} = -\frac{|\boldsymbol{p}_T|\,|\boldsymbol{R}|}{4M_h^2} G_{1,TT}^{\perp}\ ,$$

$$H_1^{\perp|0,0\rangle} = \frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p}\ ,$$

$$H_1^{\perp|1,0\rangle} = H_{1,OL}^{\perp}\ , \qquad H_1^{\perp|2,0\rangle} = \frac{1}{2} H_{1,LL}^{\perp}\ ,$$

$$H_1^{\perp|1,1\rangle} = \frac{|\boldsymbol{R}|}{|\boldsymbol{p}_T|} H_{1,OT}^{\sphericalangle}\ , \qquad H_1^{\perp|1,-1\rangle} = H_{1,OT}^{\perp}\ ,$$

$$H_1^{\perp|2,1\rangle} = \frac{|\boldsymbol{R}|}{2\,|\boldsymbol{p}_T|} H_{1,LT}^{\sphericalangle}\ , \qquad H_1^{\perp|2,-1\rangle} = \frac{1}{2} H_{1,LT}^{\perp}\ ,$$

$$H_1^{\perp|2,2\rangle} = \frac{|\boldsymbol{R}|}{|\boldsymbol{p}_T|} H_{1,TT}^{\sphericalangle}\ , \qquad H_1^{\perp|2,-2\rangle} = H_{1,TT}^{\perp}\ .$$

Partial wave expansion

Gliske, Bacchetta, Radici
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- **twist 3** $F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp})} \rightarrow e \otimes H_1^{\perp|\ell,m\rangle} + \ldots$

$\lvert\ell, m\rangle$	twist-2	twist-3
$\lvert 0,0\rangle$	0	$\sin\phi_h$

$\lvert\ell, m\rangle$	twist-2	twist-3
$\lvert 1,1\rangle$	$\sin\theta\sin(\phi_h - \phi_R)$	$\sin\theta\sin\phi_R$
$\lvert 1,0\rangle$	0	$\cos\theta\sin\phi_h$
$\lvert 1,-1\rangle$		$\sin\theta\sin(2\phi_h - \phi_R)$

$\lvert\ell, m\rangle$	twist-2	twist-3
$\lvert 2,2\rangle$	$\sin^2\theta\sin(2\phi_h - 2\phi_R)$	$\sin^2\theta\sin(-\phi_h + 2\phi_R)$
$\lvert 2,1\rangle$	$\sin\theta\cos\theta\sin(\phi_h - \phi_R)$	$\sin\theta\cos\theta\sin\phi_R$
$\lvert 2,0\rangle$	0	$(3\cos^2\theta - 1)\sin\phi_h$
$\lvert 2,-1\rangle$		$\sin\theta\cos\theta\sin(2\phi_h - \phi_R)$
$\lvert 2,-2\rangle$		$\sin^2\theta\sin(3\phi_h - 2\phi_R)$

Orthogonality of Modulations

- **Let *m* and *n* be integers**
- **The azimuthal modulations form an orthogonal set**

- **twist-2 x twist-2**

$$\frac{1}{2\pi^2}\int_{-\pi}^{+\pi} d\phi_h \int_{-\pi}^{+\pi} d\phi_R \ \sin\left[m_1\left(\phi_h-\phi_R\right)\right]\ \sin\left[m_2\left(\phi_h-\phi_R\right)\right] = \delta_{m_2}^{m_1}$$

(for m_1,m_2>0)

- **twist-3 x twist-3**

$$\frac{1}{2\pi^2}\int_{-\pi}^{+\pi} d\phi_h \int_{-\pi}^{+\pi} d\phi_R \ \sin\left[\left(1-m_1\right)\phi_h+m_1\phi_R\right]\ \sin\left[\left(1-m_2\right)\phi_h+m_2\phi_R\right] = \delta_{m_2}^{m_1}$$

- **twist-2 x twist-3**

$$\frac{1}{2\pi^2}\int_{-\pi}^{+\pi} d\phi_h \int_{-\pi}^{+\pi} d\phi_R \ \sin\left[m_1\left(\phi_h-\phi_R\right)\right]\ \sin\left[\left(1-m_2\right)\phi_h+m_2\phi_R\right] = 0$$

General modulations are 2-dim Fourier series terms: $\Phi\left(\phi_h,\phi_R\right)=\sin\left(m\phi_h+n\phi_R\right)$

$$A\left(\phi_h,\phi_R\right)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}A_{mn}e^{i\left(m\phi_h+n\phi_R\right)}$$

Orthogonality of Modulations

- θ dependence is from associated Legendre polynomials $P_{l,m}(\cos\theta)$, which are orthogonal:

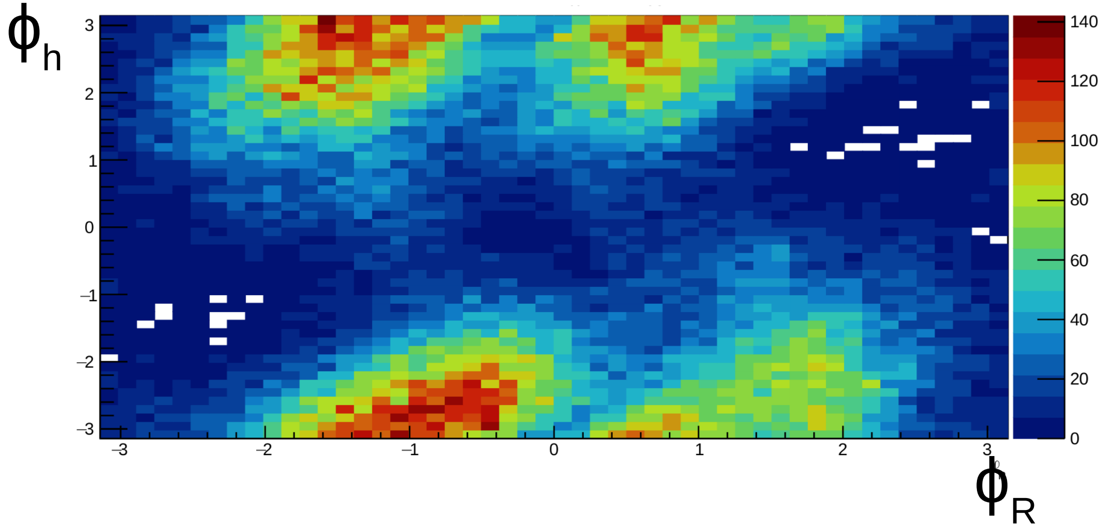
$$\int_0^\pi P_k^m(\cos\theta) P_\ell^m(\cos\theta)\ \sin\theta\, d\theta = \frac{2(\ell+m)!}{(2\ell+1)(\ell-m)!}\ \delta_{k,\ell}$$

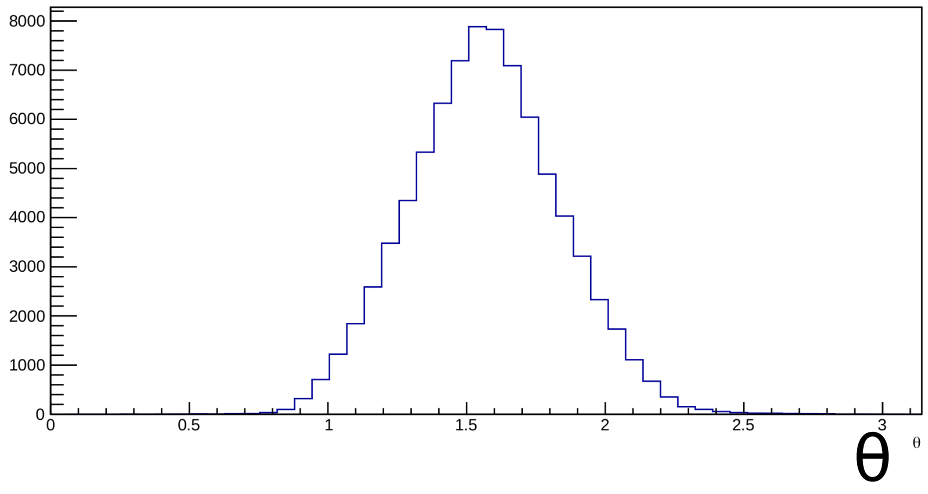
$$\int_0^\pi P_\ell^m(\cos\theta) P_\ell^n(\cos\theta) \csc\theta\, d\theta = \begin{cases} 0 & \text{if } m \neq n \\ \frac{(\ell+m)!}{m(\ell-m)!} & \text{if } m = n \neq 0 \\ \infty & \text{if } m = n = 0 \end{cases}$$

- Inner product includes a weight, otherwise there are some linear dependences
- If inner product is not weighted, there is only one nonzero overlap for L ≤ 2:

$$\langle 2,0|0,0\rangle \neq 0$$

- **CLAS acceptance limits the integration ranges, however, so these modulations may no longer be fully mutually orthogonal, given the *data* yield coverage:**





Orthogonality of Modulations

The integration limits can be applied by multiplying by an acceptance function

- Option 1: Model acceptance as a Fourier Expansion
- Option 2: Discretize the integrals and fold in the yield distribution over ϕ_h and ϕ_R

Discrete weighted inner product:

$$\langle fg \rangle = N_f N_g \sum_h \sum_r \sum_t w_h w_r w_t D_{hrt} f_{hrt} g_{hrt}$$

Normalization (ensures <ff>=1)

$$N_f = \left[\sum_h \sum_r \sum_t w_h w_r w_t D_{hrt} f_{hrt} f_{hrt} \right]^{-1/2}$$

- h, r, t = bin numbers, respectively ϕ_h, ϕ_R, θ
- D_{hrt} = yield in bin (h,r,t)
- f, g = modulations: $P_l^m(\cos\theta)\cdot\Phi_t^m(\phi_h,\phi_R)$
  - f_{hrt} = f evaluated for bin (h,r,t)
- w_h, w_r, w_t = bin width
- N = normalization

<fg> Matrices

- First verify orthonormality of modulations assuming fully uniform azimuthal acceptance
- θ-dependence not included
- Notation: $| L, M >_{\text{twist}}$
- Identity matrix → full mutual orthogonality

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

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<fg> matrix

	const	$\|L,0>_3$	$\|L,1>_2$	$\|L,1>_3$	$\|L,-1>_3$	$\|L,2>_2$	$\|L,2>_3$	$\|L,-2>_3$
$\|L,-2>_3$	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	1.000
$\|L,2>_3$	0.000	-0.000	0.000	0.000	0.000	0.000	1.000	-0.000
$\|L,2>_2$	0.000	0.000	-0.000	-0.000	-0.000	1.000	0.000	-0.000
$\|L,-1>_3$	0.000	0.000	-0.000	-0.000	1.000	-0.000	0.000	-0.000
$\|L,1>_3$	-0.000	0.000	0.000	1.000	-0.000	-0.000	0.000	-0.000
$\|L,1>_2$	0.000	-0.000	1.000	0.000	-0.000	-0.000	0.000	-0.000
$\|L,0>_3$	-0.000	1.000	-0.000	0.000	0.000	0.000	-0.000	0.000
const	1.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000

<fg> Matrices

- Weight with data yield distributions

- size of box proportional to |<fg>|
- X drawn on box if <fg> < 0

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

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<fg> matrix

	const	$\lvert L,0\rangle_3$	$\lvert L,1\rangle_2$	$\lvert L,1\rangle_3$	$\lvert L,-1\rangle_3$	$\lvert L,2\rangle_2$	$\lvert L,2\rangle_3$	$\lvert L,-2\rangle_3$
$\lvert L,-2\rangle_3$	-0.001	-0.099	0.003	-0.000	-0.255	-0.390	-0.058	1.000
$\lvert L,2\rangle_3$	0.043	0.062	-0.319	-0.316	0.034	0.397	1.000	-0.058
$\lvert L,2\rangle_2$	0.025	0.182	-0.246	-0.079	0.254	1.000	0.397	-0.390
$\lvert L,-1\rangle_3$	0.033	-0.266	-0.380	-0.142	1.000	0.254	0.034	-0.255
$\lvert L,1\rangle_3$	-0.082	-0.213	0.510	1.000	-0.142	-0.079	-0.316	-0.000
$\lvert L,1\rangle_2$	-0.067	0.247	1.000	0.510	-0.380	-0.246	-0.319	0.003
$\lvert L,0\rangle_3$	0.001	1.000	0.247	-0.213	-0.266	0.182	0.062	-0.099
const	1.000	0.001	-0.067	-0.082	0.033	0.025	0.043	-0.001

<fg> Matrices

Modulations of interest are at m=1

$f_1 G_1^{\perp}$ $\quad\quad$ $e H_1^{\sphericalangle}$

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\boxed{\sin(\phi_h - \phi_R)}$	$\boxed{\sin(\phi_R)}$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

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<fg> matrix

	const	$\lvert L,0\rangle_3$	$\lvert L,1\rangle_2$	$\lvert L,1\rangle_3$	$\lvert L,-1\rangle_3$	$\lvert L,2\rangle_2$	$\lvert L,2\rangle_3$	$\lvert L,-2\rangle_3$
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<fg> Matrices

- **The m=1 modulations are *not* orthogonal, given CLAS acceptance**
- **A fit to sinϕR modulation alone will be impacted by 50% of the actual sin(ϕh-ϕR) amplitude, and vice versa**

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

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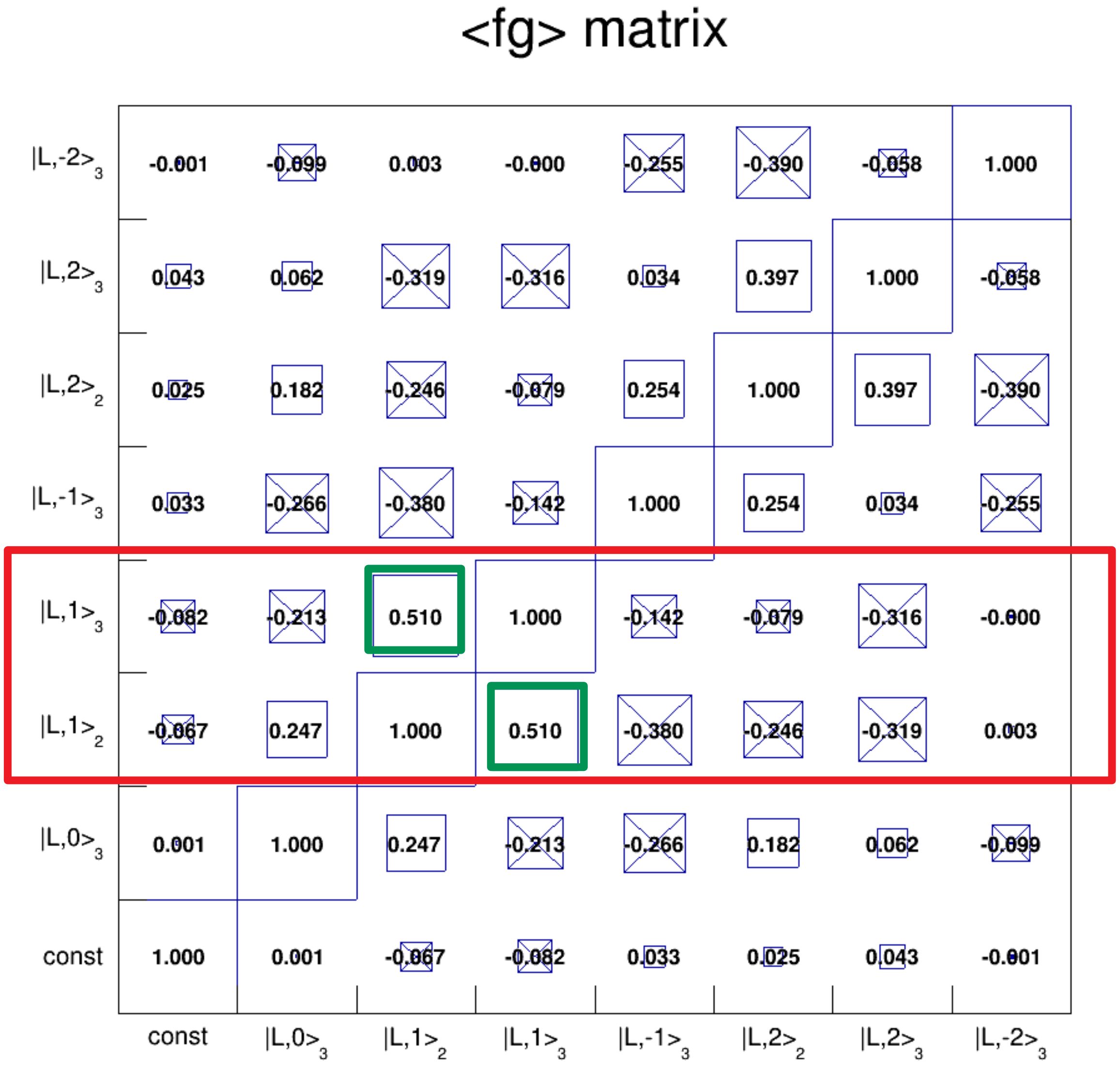

<fg> Matrices

- **Sinϕh modulation also contributes, to a lesser extent (20-25%)**
- **Impacts on m=1 modulations are opposite in sign**

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

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<fg> matrix

	const	$\vert L,0\rangle_3$	$\vert L,1\rangle_2$	$\vert L,1\rangle_3$	$\vert L,-1\rangle_3$	$\vert L,2\rangle_2$	$\vert L,2\rangle_3$	$\vert L,-2\rangle_3$
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$\vert L,0\rangle_3$	0.001	1.000	0.247	-0.213	-0.266	0.182	0.062	-0.099
const	1.000	0.001	-0.067	-0.082	0.033	0.025	0.043	-0.001

<fg> Matrices

- **m = –1 modulation also needs to be included, though some early looks show it may be ~0 in data**
- **-40% impact on sin(ϕh-ϕR)**
- **-14% impact on sin(ϕR)**

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

C. Dilks

<fg> matrix

	const	$\lvert L,0\rangle_3$	$\lvert L,1\rangle_2$	$\lvert L,1\rangle_3$	$\lvert L,-1\rangle_3$	$\lvert L,2\rangle_2$	$\lvert L,2\rangle_3$	$\lvert L,-2\rangle_3$
$\lvert L,-2\rangle_3$	-0.001	-0.099	0.003	-0.000	-0.255	-0.390	-0.058	1.000
$\lvert L,2\rangle_3$	0.043	0.062	-0.319	-0.316	0.034	0.397	1.000	-0.058
$\lvert L,2\rangle_2$	0.025	0.182	-0.246	-0.079	0.254	1.000	0.397	-0.390
$\lvert L,-1\rangle_3$	0.033	-0.266	-0.380	-0.142	1.000	0.254	0.034	-0.255
$\lvert L,1\rangle_3$	-0.082	-0.213	0.510	1.000	-0.142	-0.079	-0.316	-0.000
$\lvert L,1\rangle_2$	-0.067	0.247	1.000	0.510	-0.380	-0.246	-0.319	0.003
$\lvert L,0\rangle_3$	0.001	1.000	0.247	-0.213	-0.266	0.182	0.062	-0.099
const	1.000	0.001	-0.067	-0.082	0.033	0.025	0.043	-0.001

<fg> Matrices

- **Some impact from m=2 modulations, but not much from m=-2**

m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$

C. Dilks

<fg> matrix

	const	$\lvert L,0\rangle_3$	$\lvert L,1\rangle_2$	$\lvert L,1\rangle_3$	$\lvert L,-1\rangle_3$	$\lvert L,2\rangle_2$	$\lvert L,2\rangle_3$	$\lvert L,-2\rangle_3$
$\lvert L,-2\rangle_3$	-0.001	-0.099	0.003	-0.000	-0.255	-0.390	-0.058	1.000
$\lvert L,2\rangle_3$	0.043	0.062	-0.319	-0.316	0.034	0.397	1.000	-0.058
$\lvert L,2\rangle_2$	0.025	0.182	-0.246	-0.079	0.254	1.000	0.397	-0.390
$\lvert L,-1\rangle_3$	0.033	-0.266	-0.380	-0.142	1.000	0.254	0.034	-0.255
$\lvert L,1\rangle_3$	-0.082	-0.213	0.510	1.000	-0.142	-0.079	-0.316	-0.000
$\lvert L,1\rangle_2$	-0.067	0.247	1.000	0.510	-0.380	-0.246	-0.319	0.003
$\lvert L,0\rangle_3$	0.001	1.000	0.247	-0.213	-0.266	0.182	0.062	-0.099
const	1.000	0.001	-0.067	-0.082	0.033	0.025	0.043	-0.001

<fg> Matrices

- **Now include θ dependence**
- **Uniform acceptance → almost the identity matrix**

$$P_{0,0} = 1\ ,$$

$$P_{2,0} = \frac{1}{2}\left(3\cos^2\vartheta - 1\right)$$

$\|2,-2\rangle_3$	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	1.000
$\|2,2\rangle_3$	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	0.000	1.000	-0.000
$\|2,2\rangle_2$	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	1.000	0.000	-0.000
$\|2,-1\rangle_3$	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	1.000	0.000	-0.000	0.000
$\|2,1\rangle_3$	0.000	0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	1.000	-0.000	0.000	-0.000	0.000
$\|2,1\rangle_2$	0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	1.000	0.000	-0.000	0.000	-0.000	0.000
$\|2,0\rangle_3$	-0.000	$\boxed{0.426}$	-0.000	-0.000	-0.000	0.000	1.000	0.000	-0.000	-0.000	-0.000	0.000	-0.000
$\|1,-1\rangle_3$	0.000	0.000	-0.000	-0.000	-0.000	1.000	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000
$\|1,1\rangle_3$	-0.000	0.000	0.000	0.000	1.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	-0.000
$\|1,1\rangle_2$	-0.000	-0.000	0.000	1.000	0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	-0.000
$\|1,0\rangle_3$	0.000	-0.000	1.000	0.000	0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.000	0.000	-0.000
$\|0,0\rangle_3$	-0.000	1.000	-0.000	-0.000	0.000	0.000	$\boxed{0.426}$	0.000	0.000	-0.000	0.000	-0.000	0.000
const	1.000	-0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	-0.000
	const	$\|0,0\rangle_3$	$\|1,0\rangle_3$	$\|1,1\rangle_2$	$\|1,1\rangle_3$	$\|1,-1\rangle_3$	$\|2,0\rangle_3$	$\|2,1\rangle_2$	$\|2,1\rangle_3$	$\|2,-1\rangle_3$	$\|2,2\rangle_2$	$\|2,2\rangle_3$	$\|2,-2\rangle_3$

<fg> Matrices

- **With CLAS acceptance:**

	const	$\vert 0,0\rangle_3$	$\vert 1,0\rangle_3$	$\vert 1,1\rangle_2$	$\vert 1,1\rangle_3$	$\vert 1,-1\rangle_3$	$\vert 2,0\rangle_3$	$\vert 2,1\rangle_2$	$\vert 2,1\rangle_3$	$\vert 2,-1\rangle_3$	$\vert 2,2\rangle_2$	$\vert 2,2\rangle_3$	$\vert 2,-2\rangle_3$
$\vert 2,-2\rangle_3$	-0.002	-0.101	0.029	0.002	0.001	-0.257	0.104	0.046	0.014	-0.077	-0.385	-0.055	1.000
$\vert 2,2\rangle_3$	0.043	0.061	0.015	-0.322	-0.318	0.032	-0.058	-0.031	-0.071	0.023	0.391	1.000	-0.055
$\vert 2,2\rangle_2$	0.026	0.183	-0.016	-0.248	-0.080	0.255	-0.182	-0.075	-0.054	0.023	1.000	0.391	-0.385
$\vert 2,-1\rangle_3$	-0.007	-0.071	-0.264	0.017	0.012	0.035	0.046	-0.409	-0.138	1.000	0.023	0.023	-0.077
$\vert 2,1\rangle_3$	0.004	-0.060	-0.174	-0.031	0.018	0.012	0.036	0.528	1.000	-0.138	-0.054	-0.071	0.014
$\vert 2,1\rangle_2$	0.010	-0.028	0.230	0.005	-0.031	0.018	0.016	1.000	0.528	-0.409	-0.075	-0.031	0.046
$\vert 2,0\rangle_3$	-0.007	-0.950	-0.024	-0.243	0.216	0.257	1.000	0.016	0.036	0.046	-0.182	-0.058	0.104
$\vert 1,-1\rangle_3$	0.033	-0.266	-0.068	-0.377	-0.142	1.000	0.257	0.018	0.012	0.035	0.255	0.032	-0.257
$\vert 1,1\rangle_3$	-0.082	-0.215	-0.056	0.509	1.000	-0.142	0.216	-0.031	0.018	0.012	-0.080	-0.318	0.001
$\vert 1,1\rangle_2$	-0.066	0.247	-0.026	1.000	0.509	-0.377	-0.243	0.005	-0.031	0.017	-0.248	-0.322	0.002
$\vert 1,0\rangle_3$	0.001	0.016	1.000	-0.026	-0.056	-0.068	-0.024	0.230	-0.174	-0.264	-0.016	0.015	0.029
$\vert 0,0\rangle_3$	0.001	1.000	0.016	0.247	-0.215	-0.266	-0.950	-0.028	-0.060	-0.071	0.183	0.061	-0.101
const	1.000	0.001	0.001	-0.066	-0.082	0.033	-0.007	0.010	0.004	-0.007	0.026	0.043	-0.002

<fg> Matrices

m=1 modulations highlighted for 2 partial waves (L=1 and L=2)

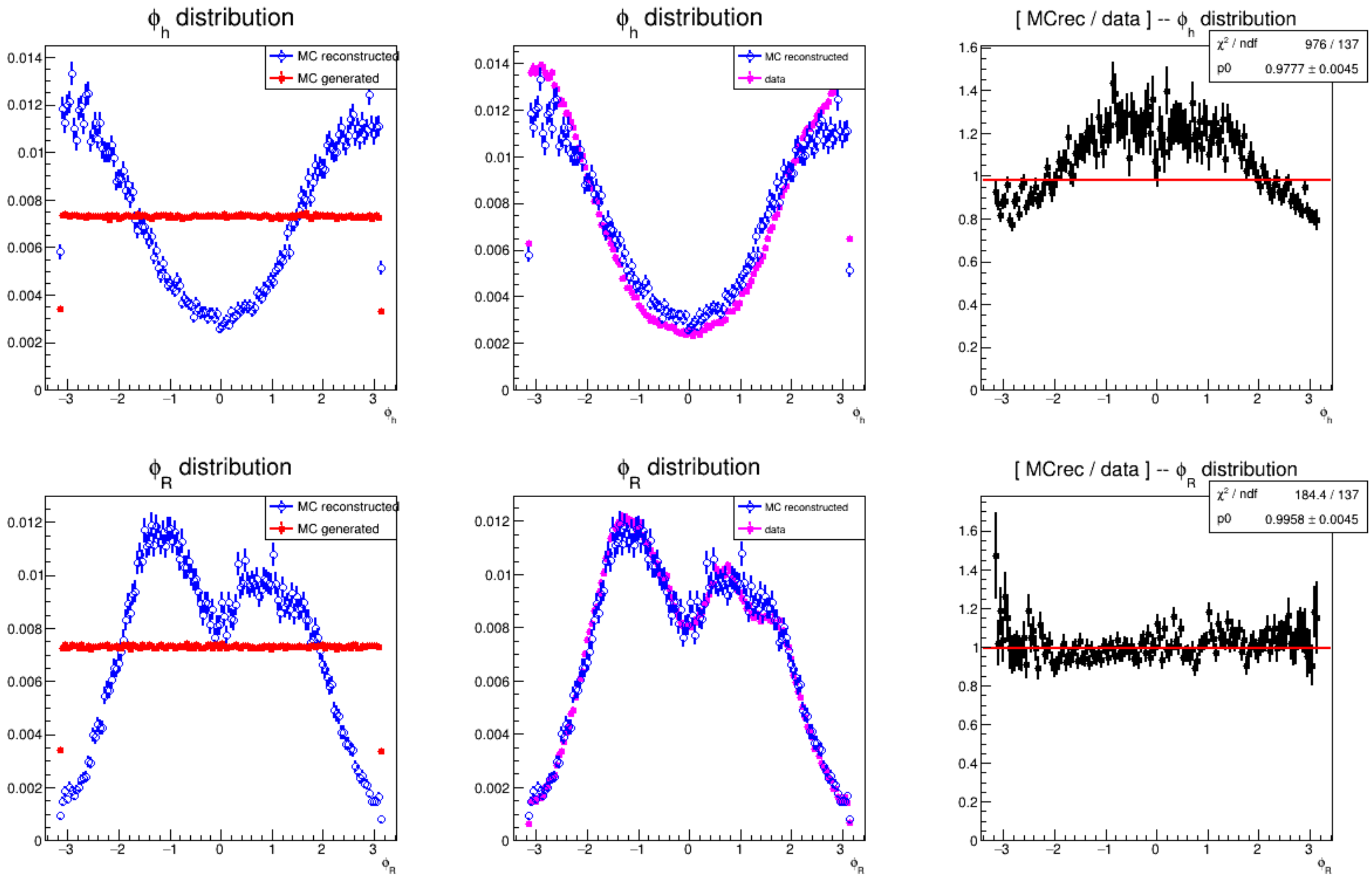
	const	$\|0,0\rangle_3$	$\|1,0\rangle_3$	$\|1,1\rangle_2$	$\|1,1\rangle_3$	$\|1,-1\rangle_3$	$\|2,0\rangle_3$	$\|2,1\rangle_2$	$\|2,1\rangle_3$	$\|2,-1\rangle_3$	$\|2,2\rangle_2$	$\|2,2\rangle_3$	$\|2,-2\rangle_3$
$\|2,-2\rangle_3$	-0.002	-0.101	0.029	0.002	0.001	-0.257	0.104	0.046	0.014	-0.077	-0.385	-0.055	1.000
$\|2,2\rangle_3$	0.043	0.061	0.015	-0.322	-0.318	0.032	-0.058	-0.031	-0.071	0.023	0.391	1.000	-0.055
$\|2,2\rangle_2$	0.026	0.183	-0.016	-0.248	-0.080	0.255	-0.182	-0.075	-0.054	0.023	1.000	0.391	-0.385
$\|2,-1\rangle_3$	-0.007	-0.071	-0.264	0.017	0.012	0.035	0.046	-0.409	-0.138	1.000	0.023	0.023	-0.077
$\|2,1\rangle_3$	0.004	-0.060	-0.174	-0.031	0.018	0.012	0.036	0.528	1.000	-0.138	-0.054	-0.071	0.014
$\|2,1\rangle_2$	0.010	-0.028	0.230	0.005	-0.031	0.018	0.016	1.000	0.528	-0.409	-0.075	-0.031	0.046
$\|2,0\rangle_3$	-0.007	-0.950	-0.024	-0.243	0.216	0.257	1.000	0.016	0.036	0.046	-0.182	-0.058	0.104
$\|1,-1\rangle_3$	0.033	-0.266	-0.068	-0.377	-0.142	1.000	0.257	0.018	0.012	0.035	0.255	0.032	-0.257
$\|1,1\rangle_3$	-0.082	-0.215	-0.056	0.509	1.000	-0.142	0.216	-0.031	0.018	0.012	-0.080	-0.318	0.001
$\|1,1\rangle_2$	-0.066	0.247	-0.026	1.000	0.509	-0.377	-0.243	0.005	-0.031	0.017	-0.248	-0.322	0.002
$\|1,0\rangle_3$	0.001	0.016	1.000	-0.026	-0.056	-0.068	-0.024	0.230	-0.174	-0.264	-0.016	0.015	0.029
$\|0,0\rangle_3$	0.001	1.000	0.016	0.247	-0.215	-0.266	-0.950	-0.028	-0.060	-0.071	0.183	0.061	-0.101
const	1.000	0.001	0.001	-0.066	-0.082	0.033	-0.007	0.010	0.004	-0.007	0.026	0.043	-0.002

<fg> Matrices

L=1 modulations → story is the same as it was without θ-dependence
L=2 modulations → different set of dominant non-orthogonal modulations
Could fit L=1 set *separately* from L=2 set for FF partial wave study!

	const	$\|0,0\rangle_3$	$\|1,0\rangle_3$	$\|1,1\rangle_2$	$\|1,1\rangle_3$	$\|1,-1\rangle_3$	$\|2,0\rangle_3$	$\|2,1\rangle_2$	$\|2,1\rangle_3$	$\|2,-1\rangle_3$	$\|2,2\rangle_2$	$\|2,2\rangle_3$	$\|2,-2\rangle_3$
$\|2,-2\rangle_3$	-0.002	-0.101	0.029	0.002	0.001	-0.257	0.104	0.046	0.014	-0.077	-0.385	-0.055	1.000
$\|2,2\rangle_3$	0.043	0.061	0.015	-0.322	-0.318	0.032	-0.058	-0.031	-0.071	0.023	0.391	1.000	-0.055
$\|2,2\rangle_2$	0.026	0.183	-0.016	-0.248	-0.080	0.255	-0.182	-0.075	-0.054	0.023	1.000	0.391	-0.385
$\|2,-1\rangle_3$	-0.007	-0.071	-0.264	0.017	0.012	0.035	0.046	-0.409	-0.138	1.000	0.023	0.023	-0.077
$\|2,1\rangle_3$	0.004	-0.060	-0.174	-0.031	0.018	0.012	0.036	0.528	1.000	-0.138	-0.054	-0.071	0.014
$\|2,1\rangle_2$	0.010	-0.028	0.230	0.005	-0.031	0.018	0.016	1.000	0.528	-0.409	-0.075	-0.031	0.046
$\|2,0\rangle_3$	-0.007	-0.950	-0.024	-0.243	0.216	0.257	1.000	0.016	0.036	0.046	-0.182	-0.058	0.104
$\|1,-1\rangle_3$	0.033	-0.266	-0.068	-0.377	-0.142	1.000	0.257	0.018	0.012	0.035	0.255	0.032	-0.257
$\|1,1\rangle_3$	-0.082	-0.215	-0.056	0.509	1.000	-0.142	0.216	-0.031	0.018	0.012	-0.080	-0.318	0.001
$\|1,1\rangle_2$	-0.066	0.247	-0.026	1.000	0.509	-0.377	-0.243	0.005	-0.031	0.017	-0.248	-0.322	0.002
$\|1,0\rangle_3$	0.001	0.016	1.000	-0.026	-0.056	-0.068	-0.024	0.230	-0.174	-0.264	-0.016	0.015	0.029
$\|0,0\rangle_3$	0.001	1.000	0.016	0.247	-0.215	-0.266	-0.950	-0.028	-0.060	-0.071	0.183	0.061	-0.101
const	1.000	0.001	0.001	-0.066	-0.082	0.033	-0.007	0.010	0.004	-0.007	0.026	0.043	-0.002

Monte Carlo Comparison: Generated Reconstructed Data



MC Asymmetries

- Asymmetry injection: assign helicities using a random number generator, biased toward the desired asymmetry or linear combination of asymmetries
- Let $A(\phi_h, \phi_R, \ldots)$ be the asymmetry to be injected (dependent on azimuth and other kinematics)
- Generate random number r, with a uniform probability within [-1,+1]
- If $r < A(\ldots)$ then assign +1 helicity; assign –1 otherwise

MC-generated

- Asymmetry injection based on MC::Particle dihadrons
- Fit MC::Particle dihadrons asymmetries
- Uniform generation in ϕh and ϕR
- Asymmetry amplitudes should be completely linearly independent

MC-reconstructed

- Asymmetry injection based on REC::Particle dihadrons; no matching to generated particles
- Fit REC::Particle dihadrons asymmetries
- ϕh and ϕR distributions match those from data
- Asymmetry amplitudes' linear dependence can be studied

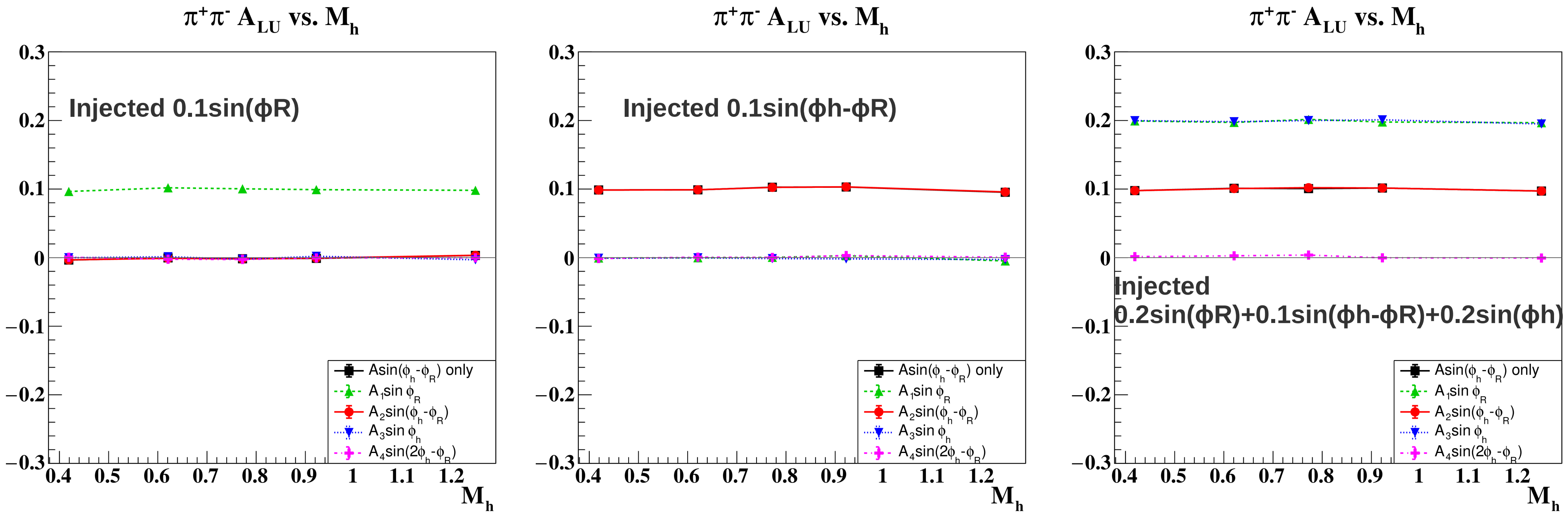
MC-reconstructed and matched

- Asymmetry injection based on MC::Particle dihadrons, which have been matched to REC::Particle dihadrons
- Fit REC::Particle dihadrons asymmetries
- Not much different from the full MC-reconstructed set
- Asymmetry amplitudes' linear dependence can be studied

Caveat: MC-reconstructed statistics are unexpectedly low… still investigating...

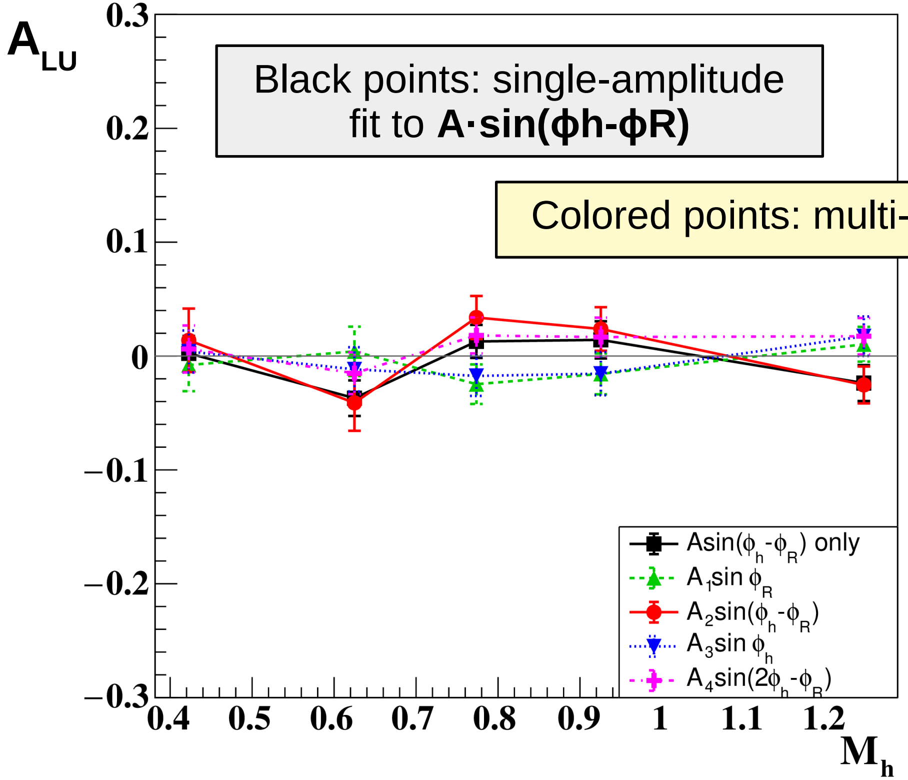
MC-generated Asymmetries

- Fit result always matches injected asymmetry (uncertainty smaller than point size)
- Single-amplitude fits (black points, here for sin(ϕh-ϕR)) agree with multi-amplitude fits (colored points), since all modulations are mutually orthogonal (uniform azimuthal acceptance)

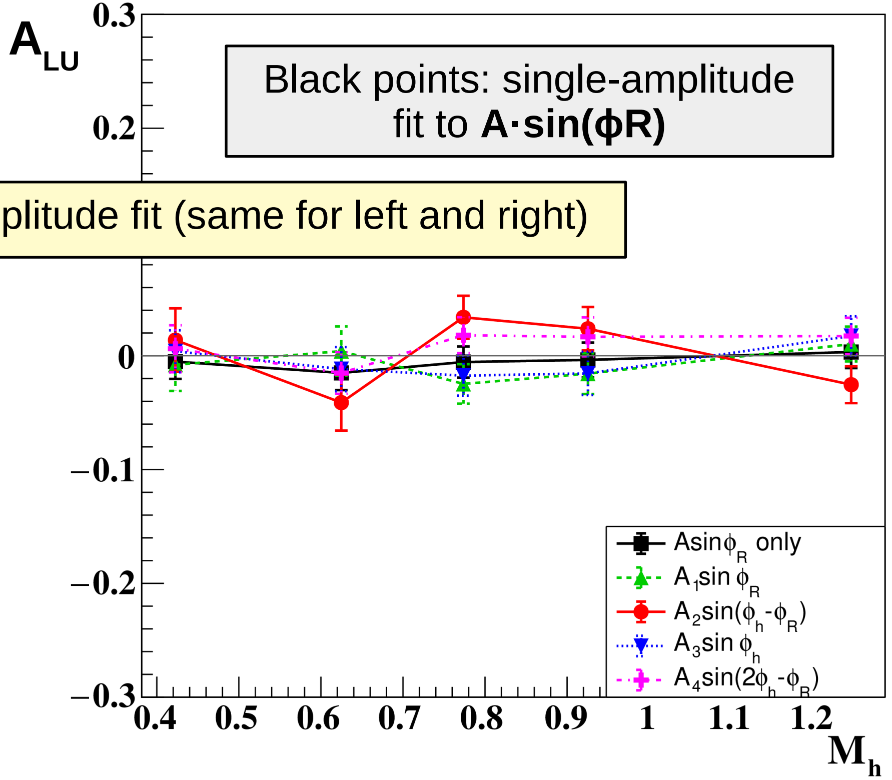


MC-reconstructed Asymmetries

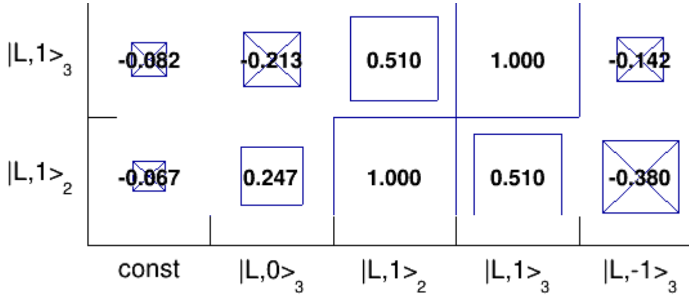
Injected Asymmetry: **A(ϕh,ϕR) = 0**



Colored points: multi-amplitude fit (same for left and right)

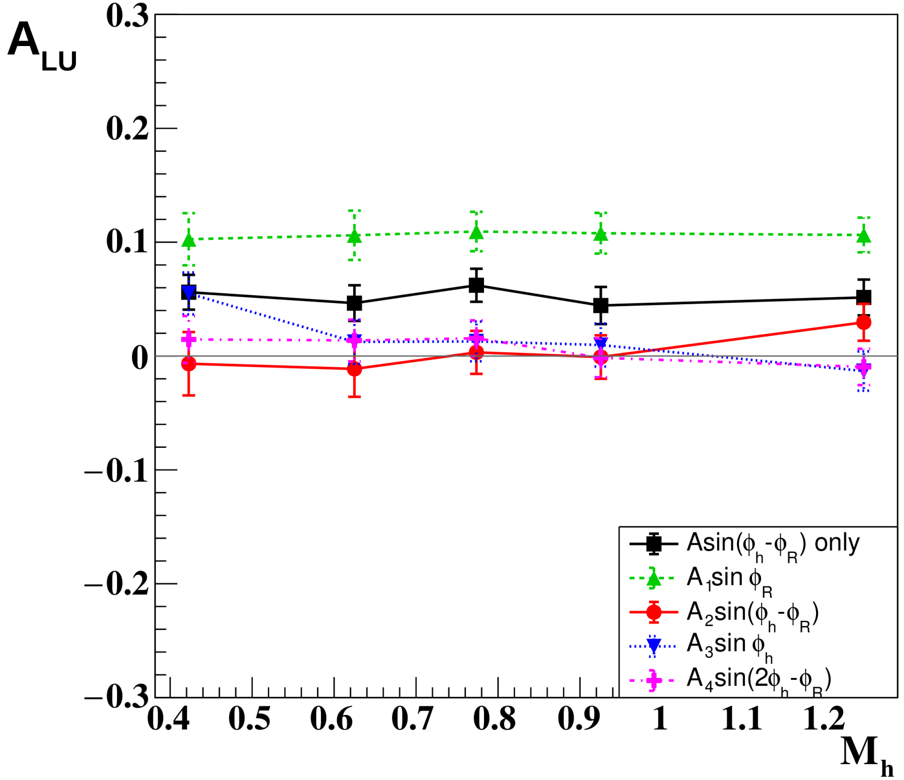


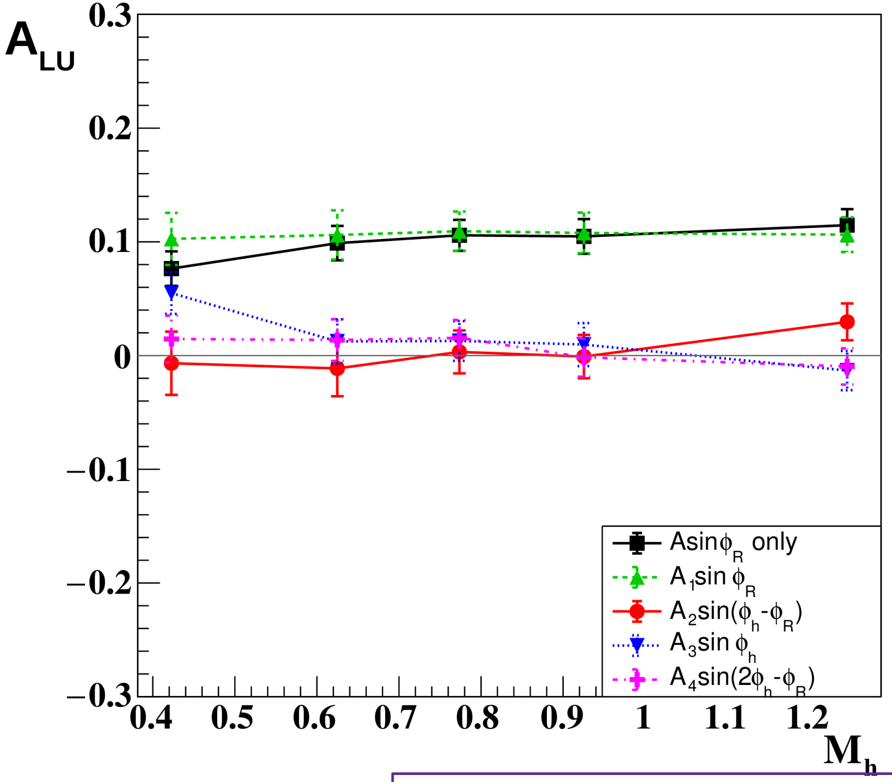
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$



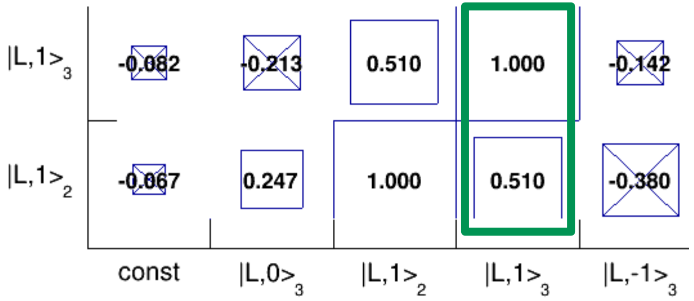
- Helicity is 50/50 random
- All asymmetries consistent with zero
- Using MC-reconstructed set, not matched to MC-generated

Injected Asymmetry: **A(ϕh,ϕR) = 0.10sin(ϕR)**



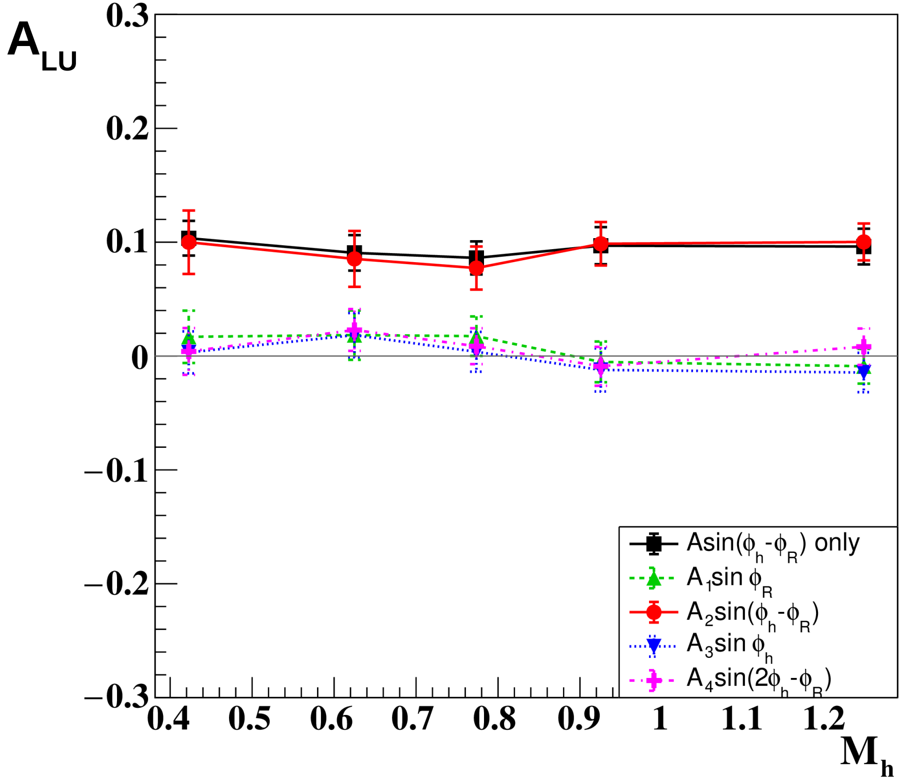


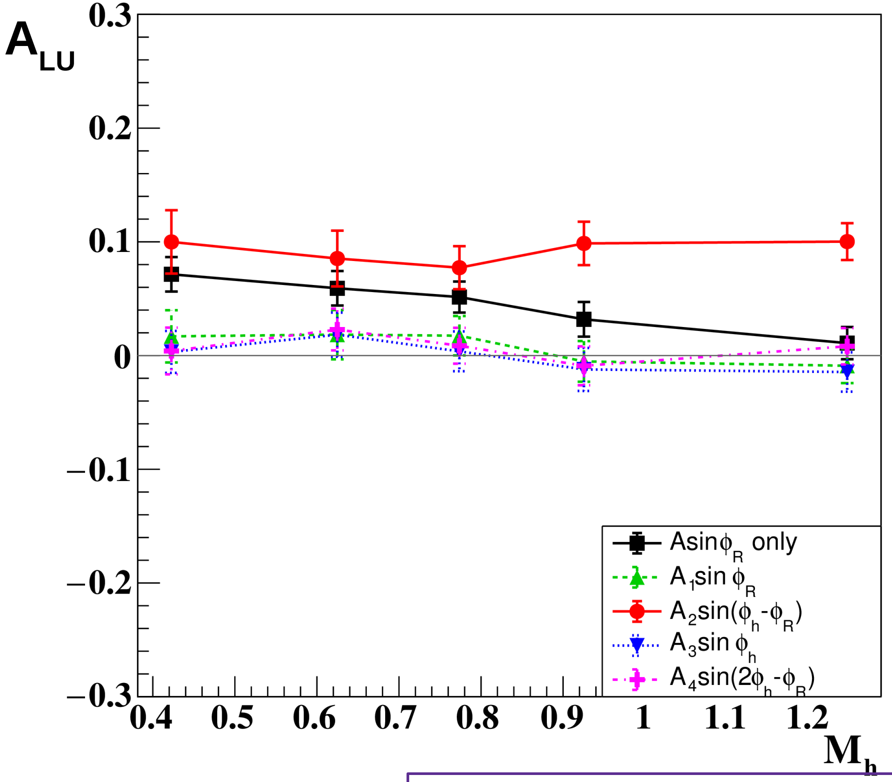
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\boxed{\sin(\phi_R)}$
0	0	$\sin(\phi_h)$
−1		$\sin(2\phi_h - \phi_R)$
−2		$\sin(3\phi_h - 2\phi_R)$



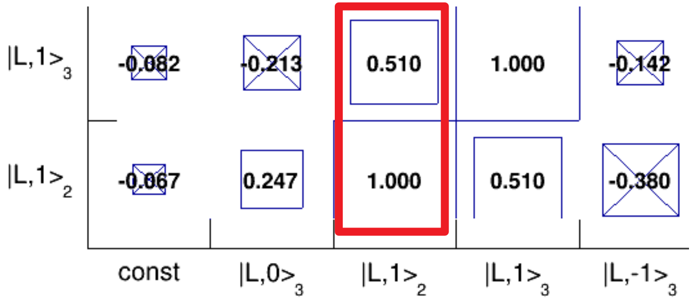
- Single fit to sin(ϕh-ϕR) amplitude pulled up by ~0.5 of sin(ϕR) amplitude, as expected from <fg> matrix
- Single fit to sinϕR agrees with multi-amp fit

Injected Asymmetry: **A(ϕh,ϕR) = 0.10sin(ϕh-ϕR)**



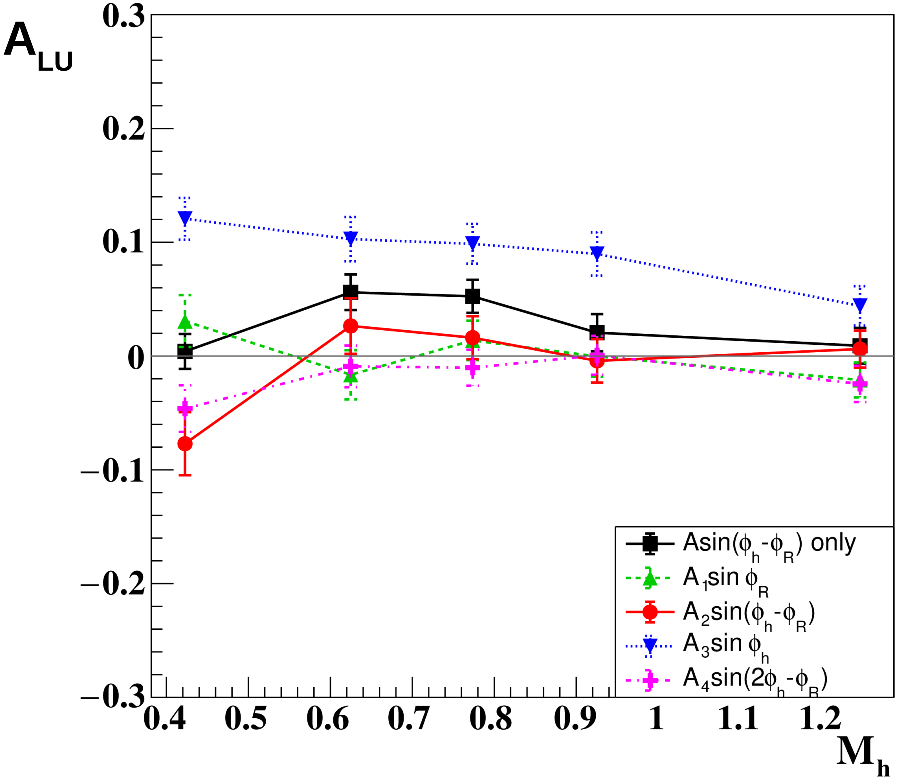


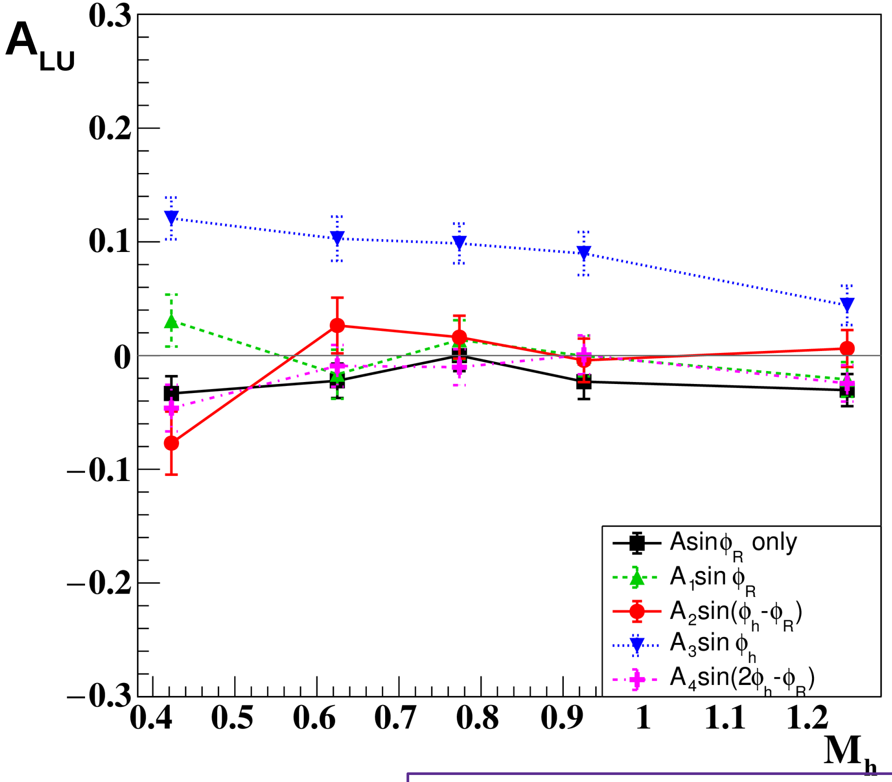
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$



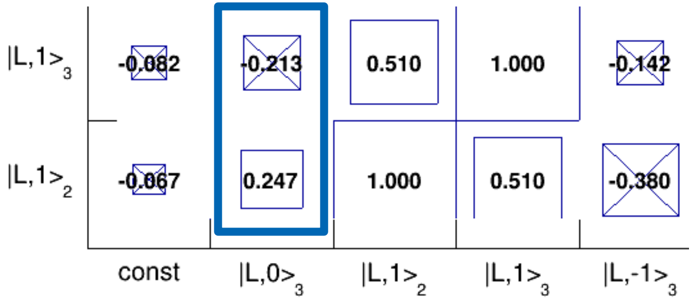
- Vice versa when we inject 10% sin(ϕh-ϕR) instead

Injected Asymmetry: **A(ϕh,ϕR) =** **0.10sin(ϕh)**



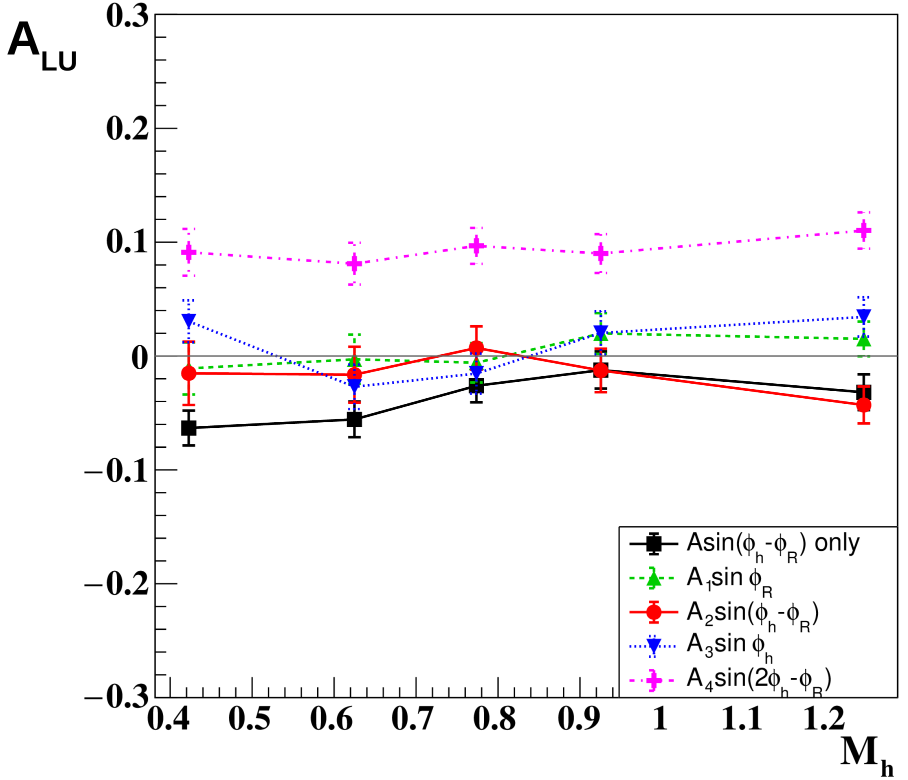


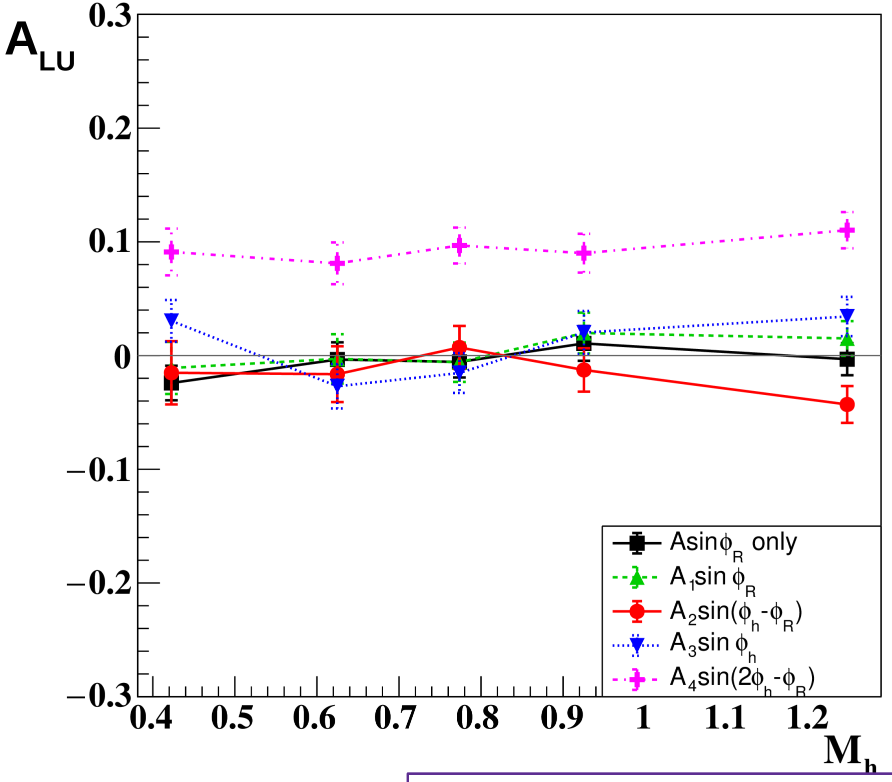
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\boxed{\sin(\phi_h)}$
-1		$\sin(2\phi_h - \phi_R)$
-2		$\sin(3\phi_h - 2\phi_R)$



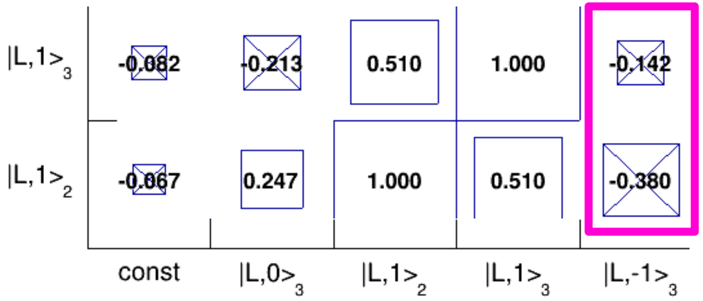
- sin(ϕh-ϕR) single-amp fit is pulled up a bit; <fg> matrix says ~25% of injected sin(ϕh) amplitude
- Similarly sin(ϕR) single-amp should be pulled *down* by *negative* ~21%

Injected Asymmetry: **A(ϕh,ϕR) = 0.10sin(2ϕh-ϕR)**



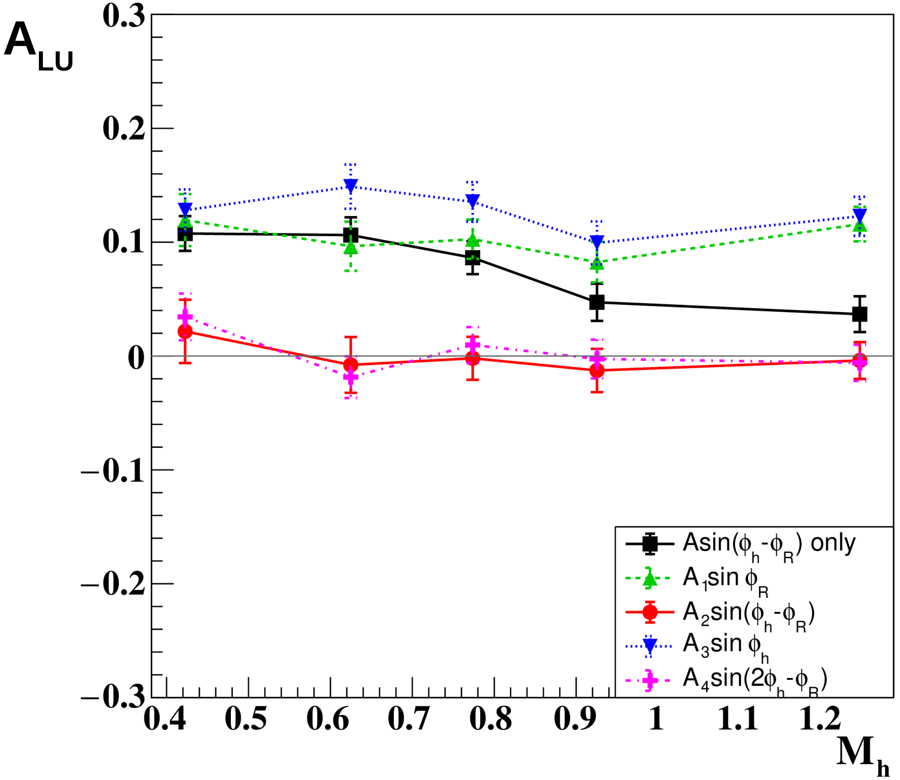


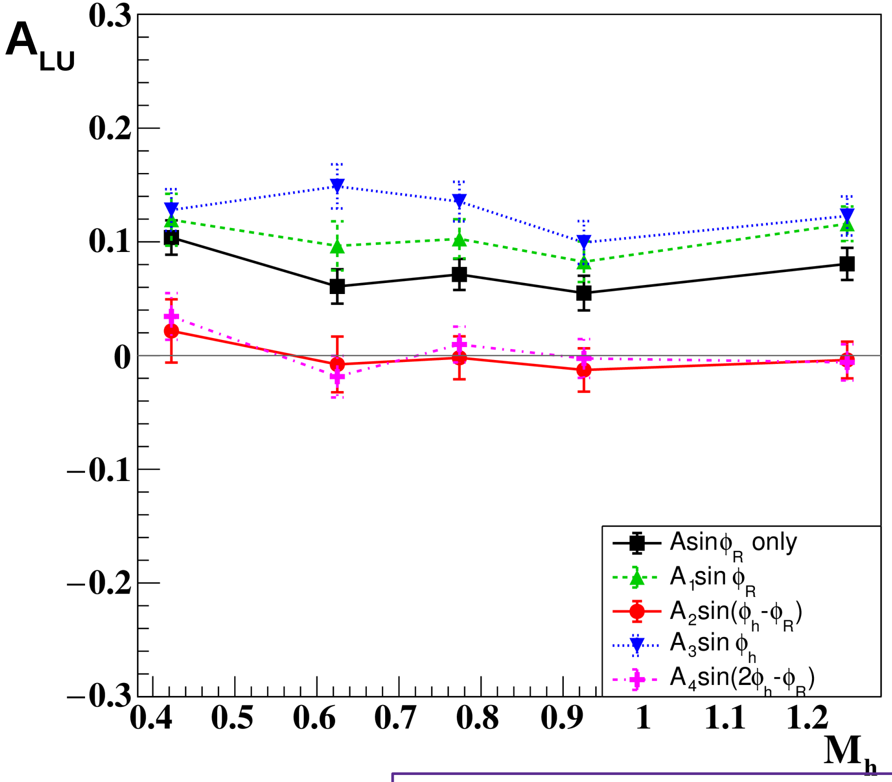
m	twist-2	twist-3
2	$\sin(2\phi_h-2\phi_R)$	$\sin(-\phi_h+2\phi_R)$
1	$\sin(\phi_h-\phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
−1		$\boxed{\sin(2\phi_h-\phi_R)}$
−2		$\sin(3\phi_h-2\phi_R)$



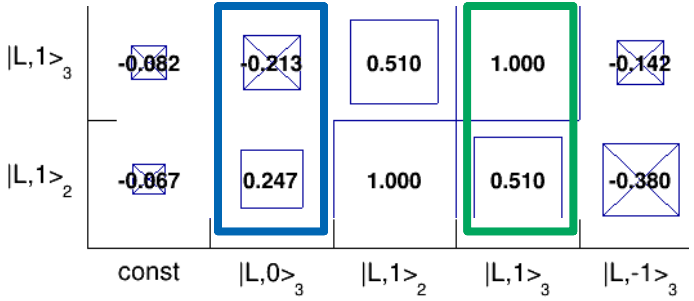
- sin(ϕh-ϕR) single-amp fit is pulled down a bit; <fg> matrix says negative 40% of injected sin(2ϕh-ϕR) amplitude, to -4%
- sin(ϕR) single-amp fit not pulled by much (expected –1.4%, but error bars too big to see it)

Injected Asymmetry: **A(ϕh,ϕR) = 0.10sin(ϕR) + 0.12sin(ϕh)**



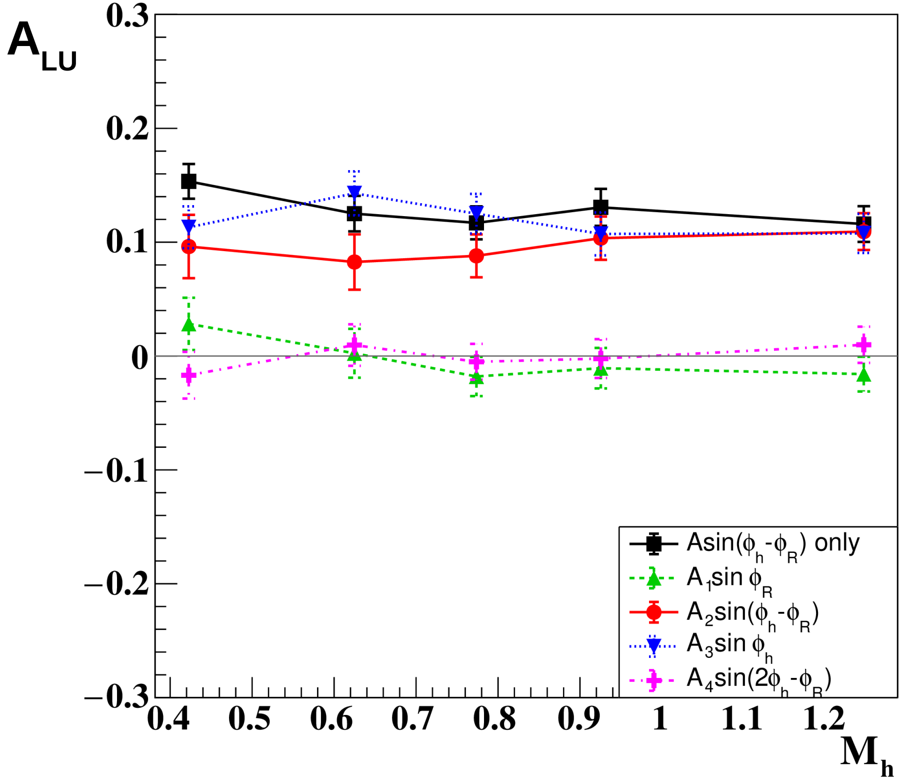


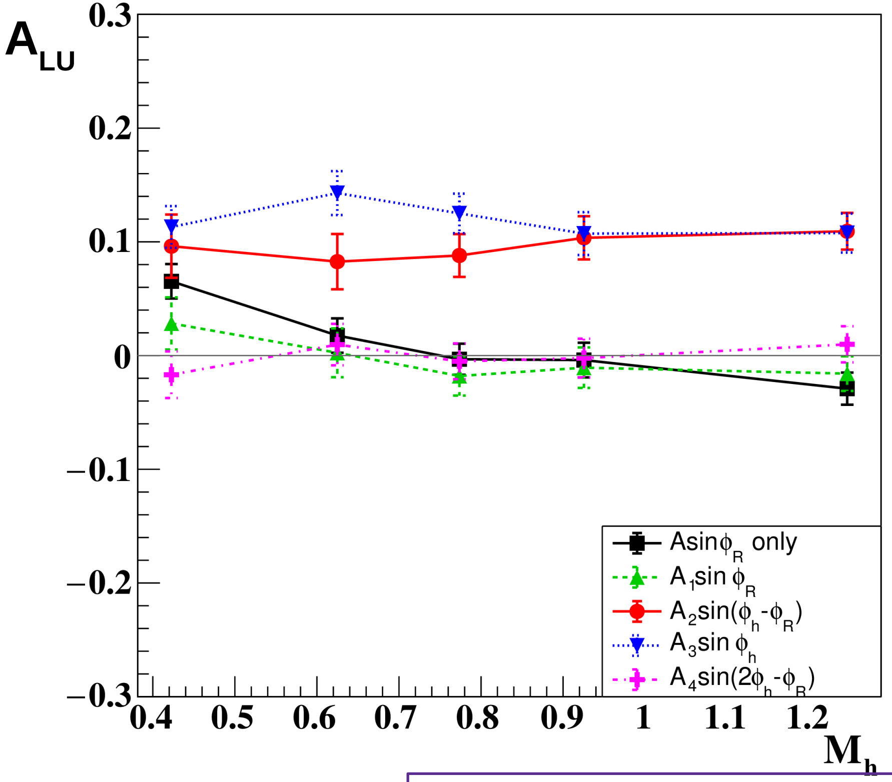
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
−1		$\sin(2\phi_h - \phi_R)$
−2		$\sin(3\phi_h - 2\phi_R)$



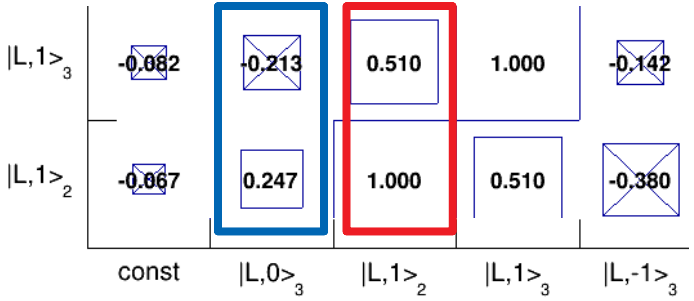
- sin(ϕh-ϕR) single-amp fit is pulled up by 50% of injected sin(ϕR) amp, plus 25% of injected sin(ϕh) amp
  - 0.5***0.1** + 0.25***0.12** = **8%**
- Similarly for sin(ϕR):
  - 1***0.1** – 0.21***0.12** = **7.5%**

Injected Asymmetry: **A(ϕh,ϕR) =** **0.10sin(ϕh-ϕR)** **+ 0.12sin(ϕh)**



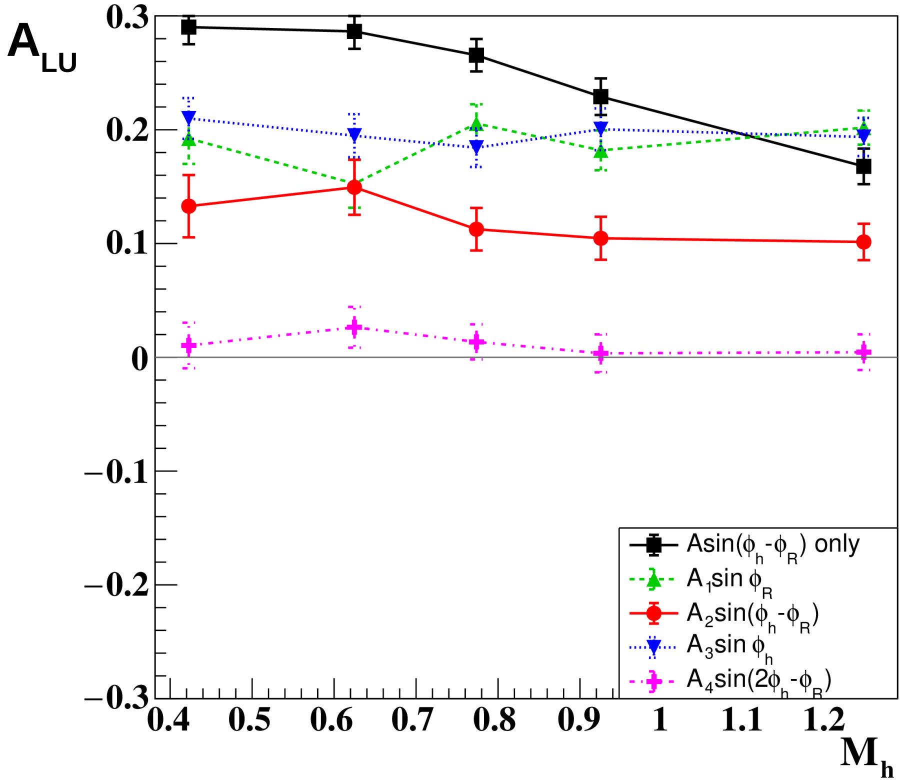


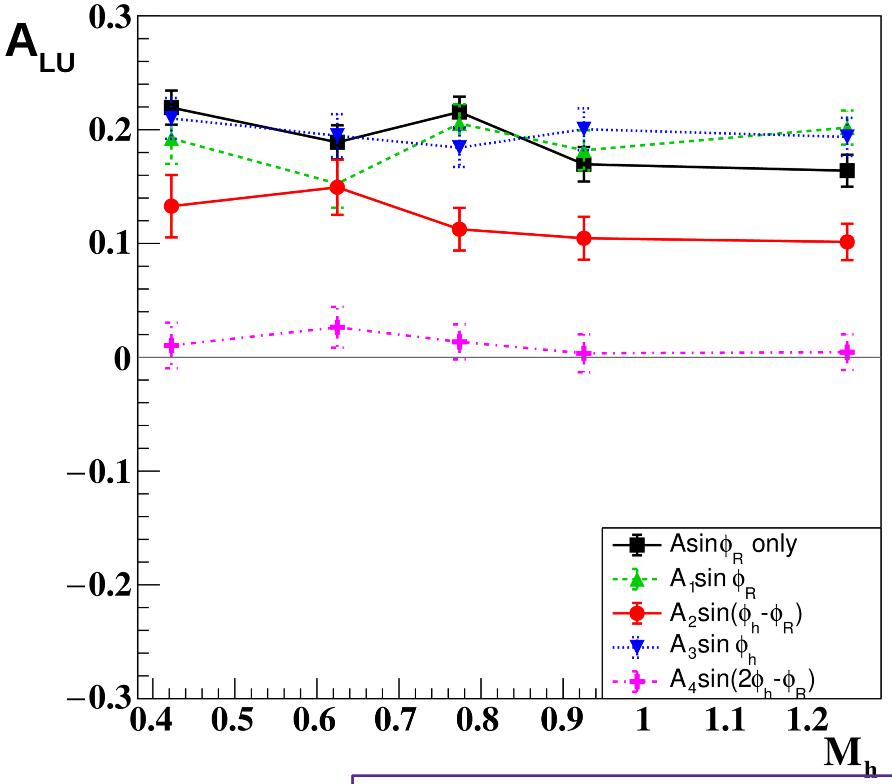
m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
−1		$\sin(2\phi_h - \phi_R)$
−2		$\sin(3\phi_h - 2\phi_R)$



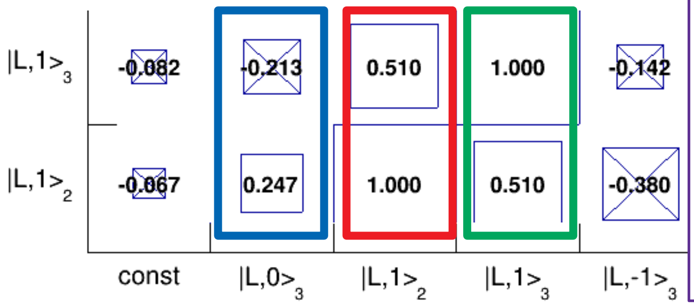
- sin(ϕh-ϕR) single-amp fit:
  - 1***0.1** + 0.25***0.12** = **13%**
- sin(ϕR) single-amp fit:
  - 0.5***0.1** – 0.21***0.12** = **2.5%**

Injected Asymmetry: **A(ϕh,ϕR) = 0.20sin(ϕR) + 0.10sin(ϕh-ϕR) + 0.20sin(ϕh)**





m	twist-2	twist-3
2	$\sin(2\phi_h - 2\phi_R)$	$\sin(-\phi_h + 2\phi_R)$
1	$\sin(\phi_h - \phi_R)$	$\sin(\phi_R)$
0	0	$\sin(\phi_h)$
−1		$\sin(2\phi_h - \phi_R)$
−2		$\sin(3\phi_h - 2\phi_R)$



- sin(ϕh-ϕR) single-amp fit:
  0.5***0.2** + 1***0.1** + 0.25***0.2** = **25%**
- sin(ϕR) single-amp fit:
  1***0.2** + 0.5***0.1** – 0.21***0.2** = **20%**

Conclusions and Outlook

- Single-amplitude fits can be biased by non-orthogonal modulations with nonzero amplitudes
  - Multi-amplitude fit, implemented via unbinned maximum-likelihood method, agrees well with injected amplitudes
  - **[vector of single-amp fit results] = [<fg> matrix] * [vector of true amplitudes]**
  - <fg> matrix is invertible, so it may be possible recover true amplitudes from single-amplitude fits
    - These ‘recovered’ amplitudes can be cross-checked with results from multi-amplitude fit
- The two most important amplitudes we would like to publish are the *least* orthogonal (50% overlap)
  - $\sin(\phi_h - \phi_R)$, for constraining $G_1^{\perp}$ (not yet constrained by *any* data!)
  - $\sin(\phi_R)$, for constraining $e(x)$ (see Timothy’s talk)
  - How to proceed toward publication of SIDIS dihadron BSA: 2 papers, or 1 paper?
- Up next:
  - Multi-amplitude fit to the data, including the most relevant modulations
  - Fits to the two partial waves (L=1 and L=2), which are fortunately orthogonal
  - Could model acceptance (Fourier series) to obtain ‘analytic’ <fg> matrix